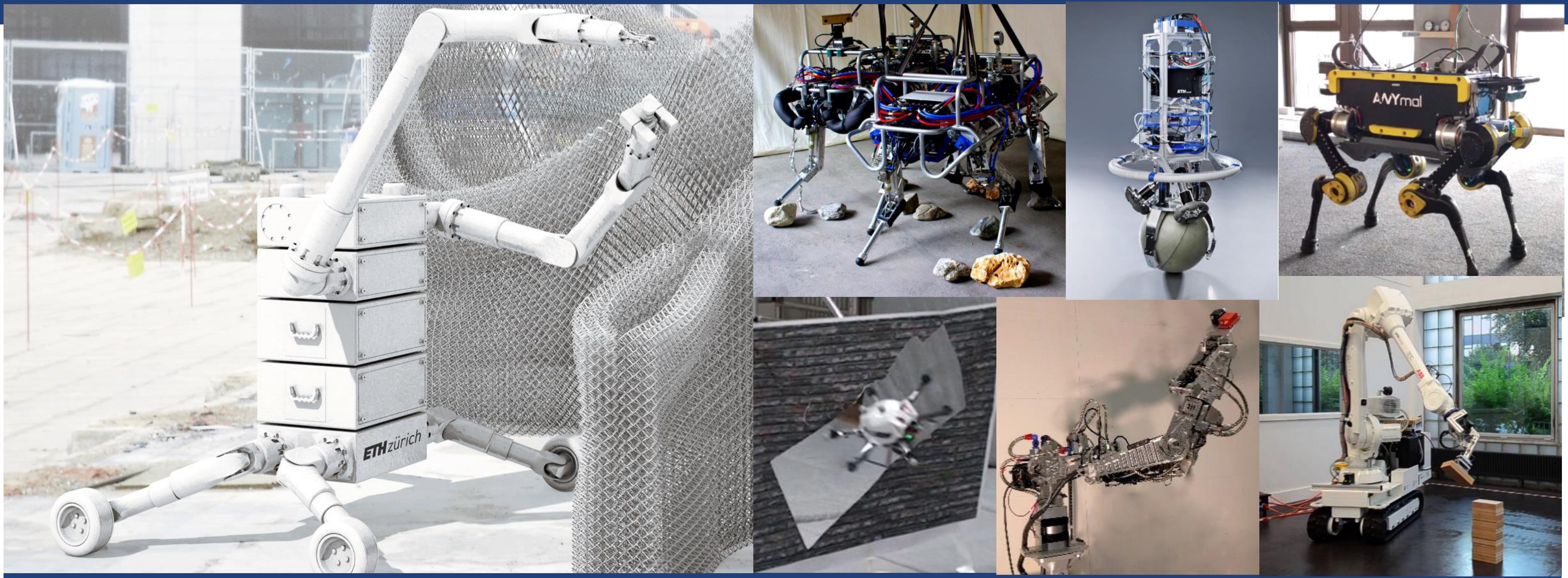


Efficient Numerical Optimal Robot Control Across Different Domains and Platforms

From UAVs to Quadrupeds – Markus Giftthaler and Michael Neunert

Agile & Dexterous Robotics Lab
Institute of Robotics and Intelligent Systems
ETH Zurich

Who we are

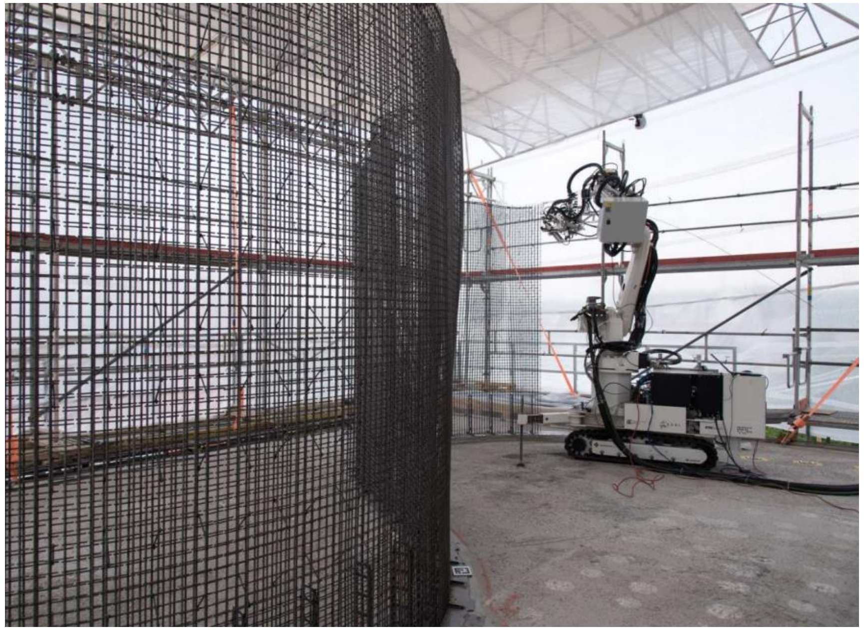
M. Giftthaler

M. Neunert

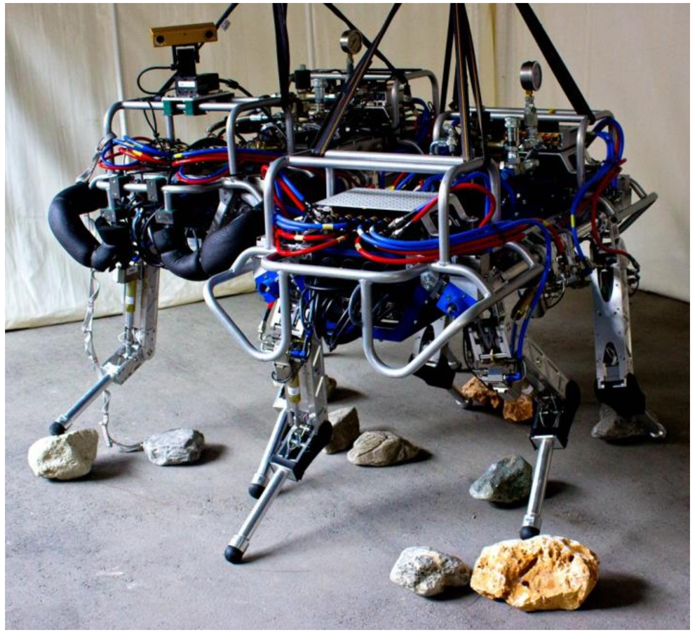
J. Buchli

ADRL

Our focus of research:



Construction Robotics and Digital Fabrication



Legged Locomotion

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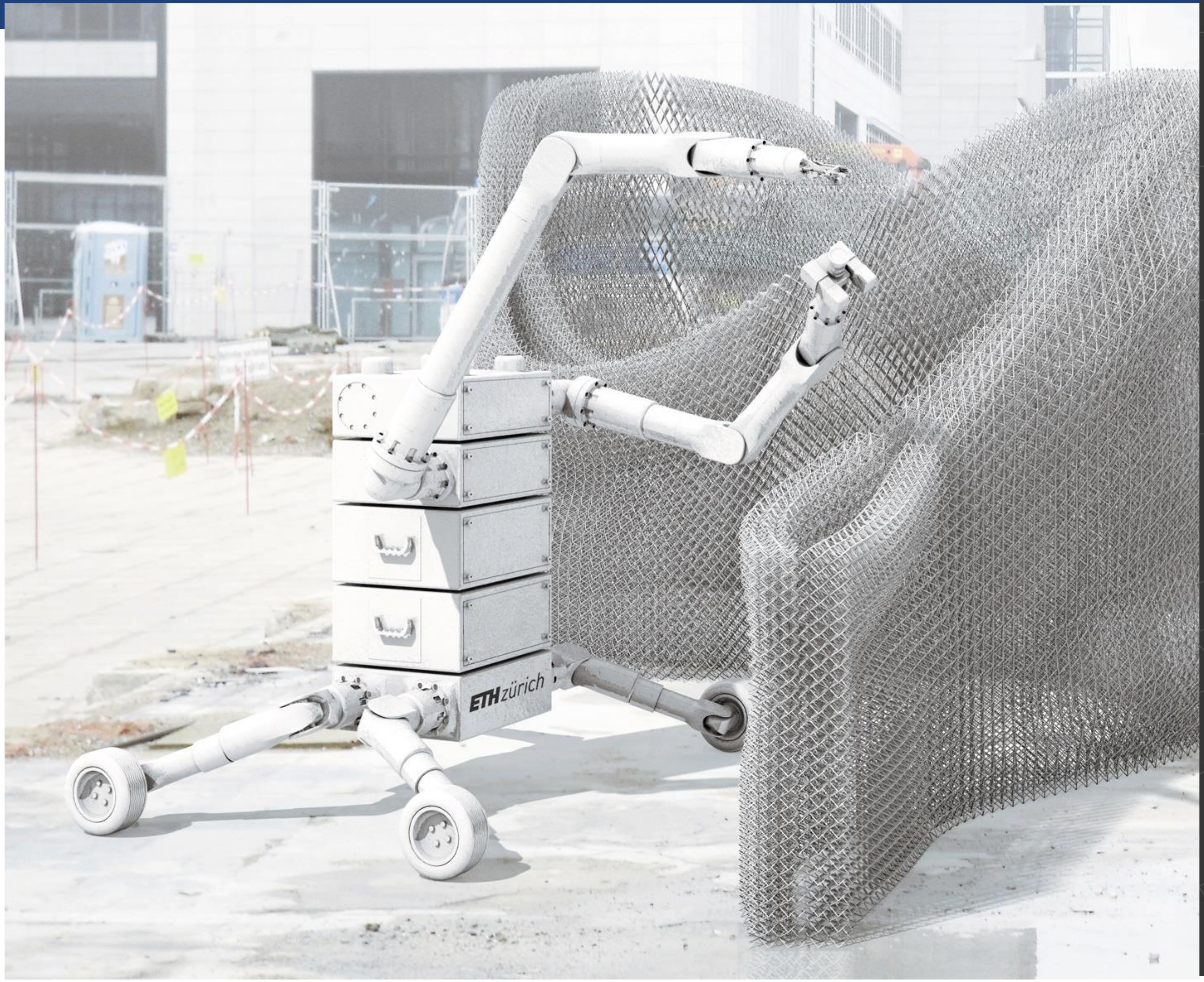
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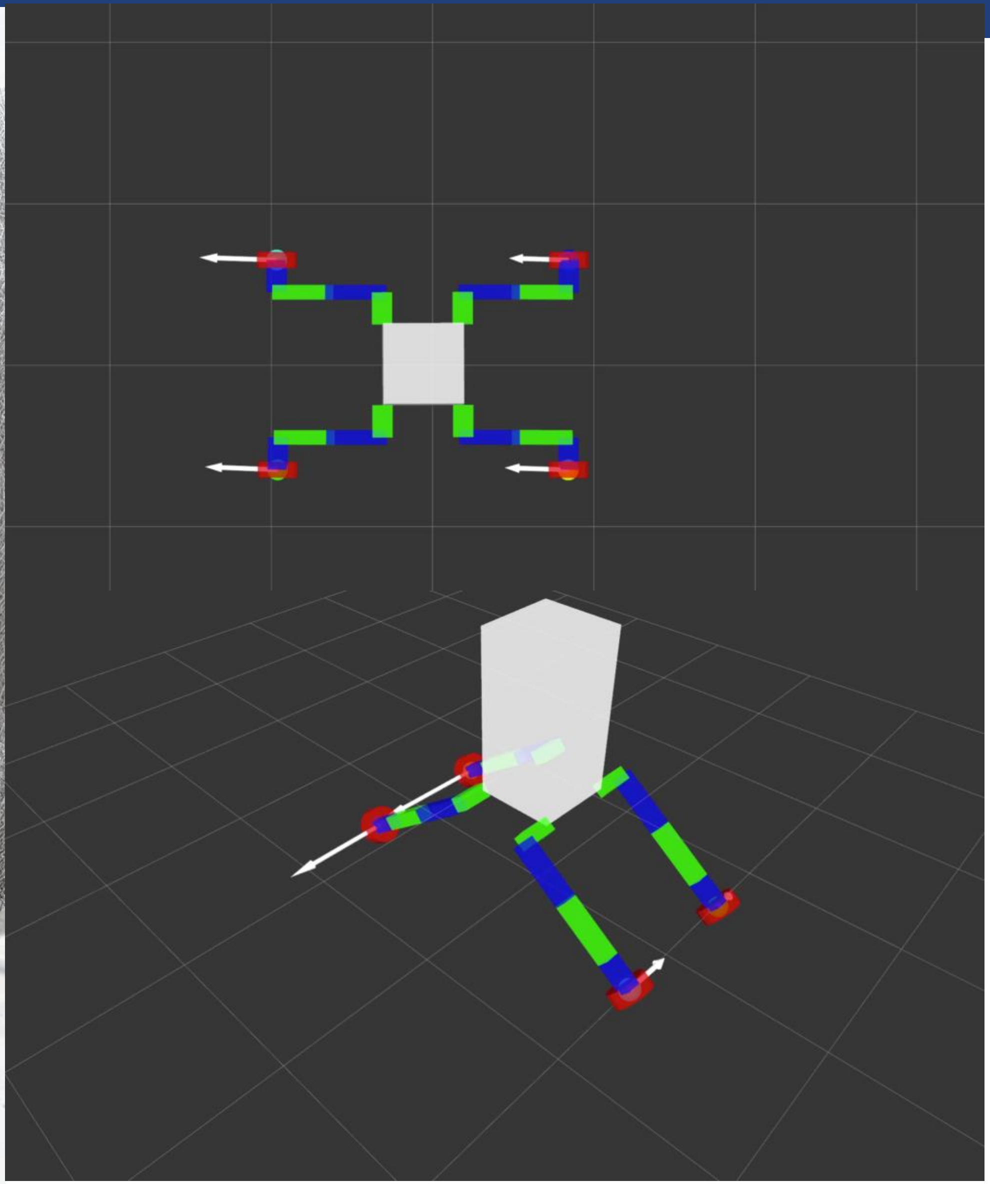


Rigid Body Dynamics

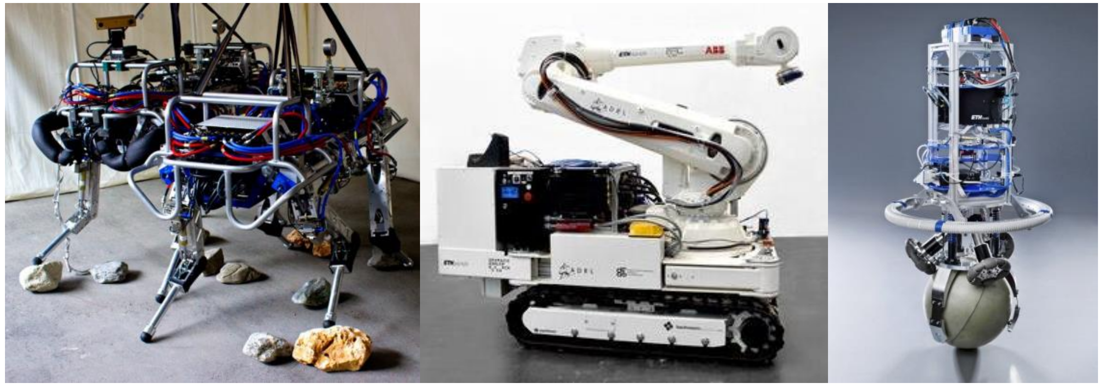
+ Kinematics

+ Actuator models

+ Friction

+ Contact dynamics

+ Aerodynamics



$$\underset{x(\cdot),\,u(\cdot),\,T}{\text{minimize}} \quad \int_0^T L(x(t),u(t))\,dt \;+\; E\,(x(T))$$

subject to

$$\begin{aligned}
x(0) - x_0 &= 0, & & \text{(fixed initial value)}\\
\dot{x}(t) - f(x(t),u(t)) &= 0, & t \in [0,T], \quad & \text{(ODE model)}\\
h(x(t),u(t)) &\geq 0, & t \in [0,T], \quad & \text{(path constraints)}\\
r\,(x(T)) &= 0 & & \text{(terminal constraints).}
\end{aligned}$$

(Continuous-time) Optimal Control Problem

One Optimal Control Framework - Different Regimes

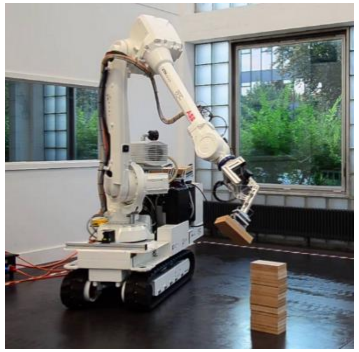
How to achieve NMPC at >100 Hz ?

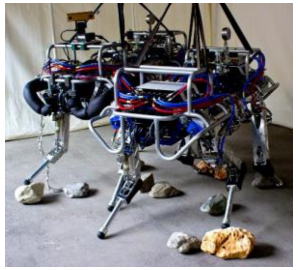
Two key points:

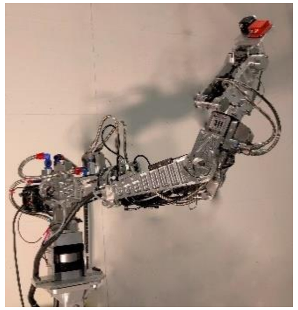
- Algorithmic and Numerical Engineering
- Efficient software implementation

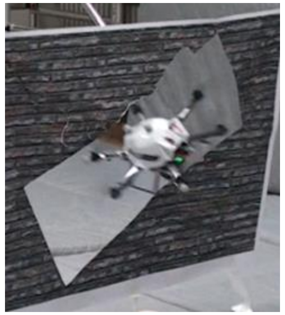
Framework benefits:

- From a morphological description to MPC
- Extensively tested on Hardware









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A (Simplified) Overview of Numerical Optimal Control in Robotics

$$\underset{x(\cdot),u(\cdot),T}{\text{minimize}} \quad \int_0^T L(x(t),u(t))\,dt \; + \; E(x(T))$$

subject to

$$\begin{aligned} x(0) - x_0 &= 0, && && \text{(fixed initial value)} \\ \dot{x}(t) - f(x(t),u(t)) &= 0, && t \in [0,T], && \text{(ODE model)} \\ h(x(t),u(t)) &\geq 0, && t \in [0,T], && \text{(path constraints)} \\ r(x(T)) &= 0 && && \text{(terminal constraints).} \end{aligned}$$

Shooting & Collocation

Nonlinear Program (NLP)

↓

Off-the shelf, general purpose NLP solvers

- IPOPT
- SNOPT
- …

↓

- ➢ Good constraint handling
- ➢ Open-loop Trajectories
- ➢ Insufficient sparsity exploitation
- ➢ Comp. complexity ~O(N^2)-O(N^3)

Differential Dynamic Programming

- DDP
- iLQR, SLQ

↓

- ➢ Trajectories + Feedback law
- ➢ Riccati-based LQ problem solving
- ➢ Computational complexity ~O(N)

“Lifting” [2, A]

↓

Multiple-shooting DDP & iLQR

~O(N)

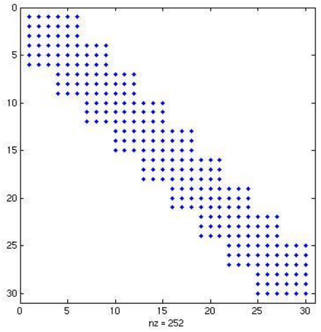
highly optimized Riccati solvers:

- “Forces Pro” [3]
- “HPIPM”

↑

Sparse (constrained) LQ Optimal Control problems

Riccati Factorization [1] →



← **add constraints**

Main “ingredients” for fast NMPC

Control Toolbox

- Auto Differentiation for Dynamics and Cost Function Derivatives
- Optimized Rigid Body Dynamics
- Custom Linear Quadratic Optimal Control Solver
- Software Engineering (Vectorization, Multi Threading)

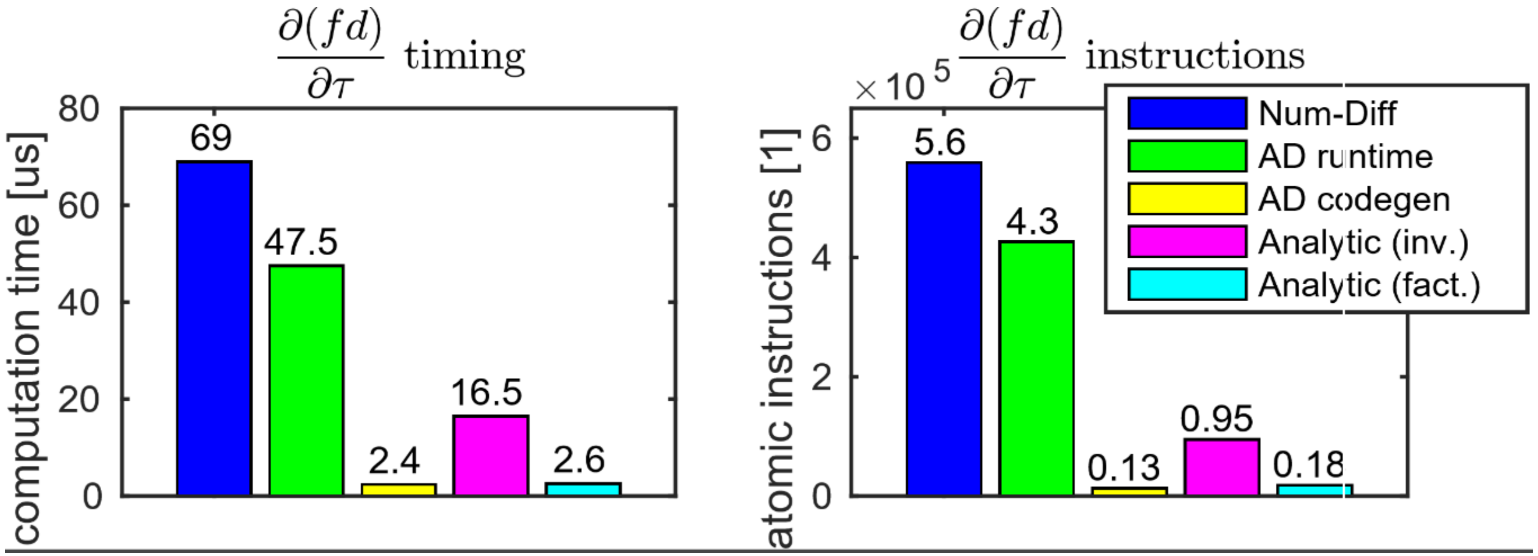
Differentiation Methods Compared

Method	Accuracy	Computation Speed	Setup Time	Error Safety
Numeric Differentiation	-	-	+++	+++
Analytical Derivatives	+++	++	-	-
Symbolic Math Engine	+++	+	+	++
Automatic Differentiation	+++	+	++	++
Auto-Diff Code Generation	+++	+++	++	++

Automatic Differentiation: Efficiently Computing Derivatives

Automatic Differentiation

- As **accurate** as analytic derivatives
- As **fast** or faster than analytic derivatives
- **Convenient** to use and error-safe
- **Code generation** and JIT compilation add extra speed



Comparison – 1st order forward dynamics derivative for a quadruped

An Auto-differentiable Rigid Body Dynamics Engine

- First **fully automatically differentiable** Rigid Body Dynamics Engine
- Generates **highly optimized C++ code** for Rigid Body Dynamics and Kinematics
- Input: **Simple parametric description** of the robot
- **Error safe**

ROB CO GEN

https://bitbucket.org/robcogenteam/
(original author: M. Frigerio)

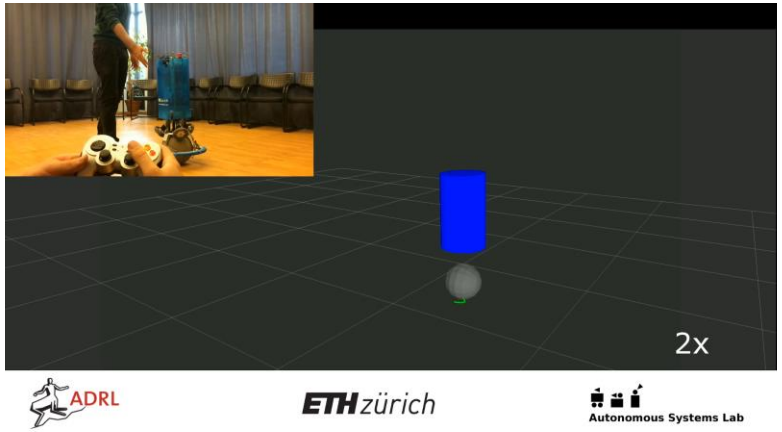
See also:

M. Giftthaler, M. Neunert, et al. “Automatic Differentiation of Rigid Body Dynamics for Optimal Control and Estimation”, *Advanced Robotics*, November 2017, Taylor and Francis.

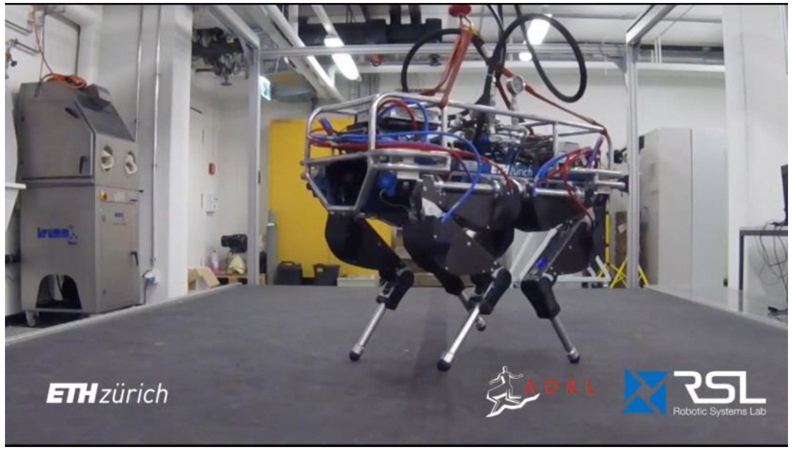
M. Frigerio, J. Buchli, and D. Caldwell, “Code generation of algebraic quantities for robot controllers,” in IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Oct 2012.

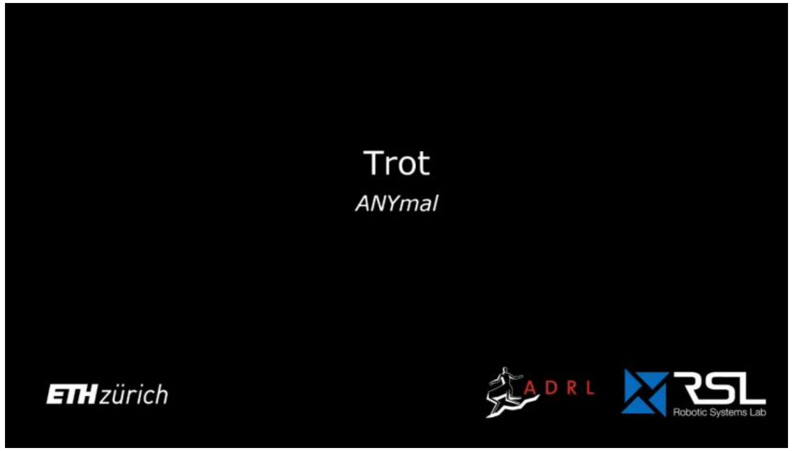
NMPC Application Examples Across Domains

Ground Robots



Legged Robots

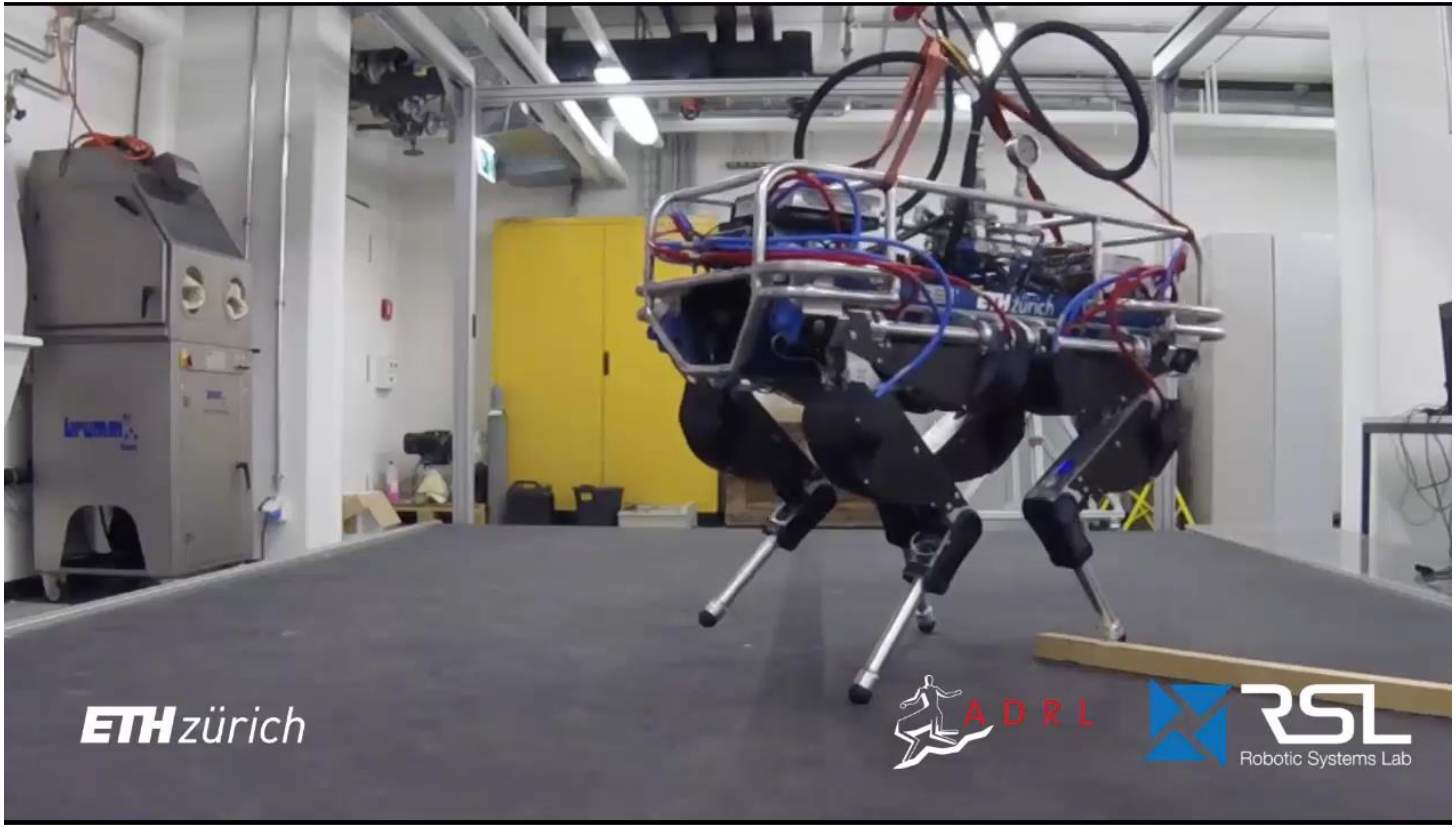




UAVs

- **Full dynamic** models
- MPC rates: **>100 Hz**
- Time horizons: **0.5 – 8s**
- No contact timings, locations or gait patterns specified
- Desktop **quad-core CPUs**

Non-Linear Full Dynamics MPC



- **Full dynamic** model
- **36 states**
- **12 control inputs**
- MPC rates: **>150 Hz**
- Time horizons: **0.5 s**
- No **contact timings, locations or gait patterns specified**
- **quad-core CPUs**
- tested on two quadrupedal robots

Optimal Control Tools

	Control Toolbox	ACADO	MUSCOD-II	Drake	MuJoCo
Free software	✓	✓	✗	✓	✗
Auto-Diff (w. codegen)	2^{nd} order (✓)	2^{nd} order (✓)	✗	1^{st} order (✗)	✗
Optimal Control Solvers	LQR, TVLQR, DMS, SLQ	DMS	DMS	LQR, TVLQR, DirCol	iLQR
Rigid Body Dynamics engine	✓	✗	✗	✓	✓
Embedded/ realtime applications	✓	✓	✗	✗	✗

Questions and Discussion

Links

- www.adrl.ethz.ch
- www.bitbucket.org/adrlab/ct

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Additional References

[1] G. Frison, Algorithms and Methods for Fast Model Predictive Control. PhD thesis, Technical University of Denmark, 2015

[2] J. Albersmeyer et al. “The Lifted Newton Method and Its Application in Optimization,” SIAM Journal on Optimization, vol. 20, no. 3, pp. 1655–1684, 2010

[3] Embotech Forces Pro. https://www.embotech.com/FORCES-Pro

Related Publications

[A] M. Giftthaler, M. Neunert, et al. “A Family of iterative Gauss-Newton Shooting Methods for Nonlinear Optimal Control”. Submitted to IEEE ICRA 2018 (pre-print online soon)

[B] M. Giftthaler, M. Neunert, et al. “Automatic Differentiation of Rigid Body Dynamics for Optimal Control and Estimation”, *Advanced Robotics*, November 2017, Taylor and Francis.

[C] M. Giftthaler, et al. “Mobile Robotic Fabrication at 1:1 scale: the In situ Fabricator”. Construction Robotics (2017), Springer.

[D] M. Giftthaler et al. “Efficient Kinematic Planning for Mobile Manipulators with Non-holonomic Constraints Using Optimal Control”, IEEE Int. Conf. on Robotics and Automation (ICRA), May 2017, Singapore

[E] M. Neunert, M. Giftthaler, et al. “Fast Derivatives of Rigid Body Dynamics for Control, Optimization and Estimation”. In 2016 IEEE International Conference on Simulation, Modeling, and Programming for Autonomous Robots (SIMPAR) 2016.

[F] M. Neunert, M. St¨auble, M. Giftthaler, C. D. Bellicoso, J. Carius, C. Gehring, M. Hutter, and J. Buchli, “Whole-Body Nonlinear Model Predictive Control Through Contacts for Quadrupeds,” 2017. Submitted to IEEE Robotics and Automation Letters. (pre-print online soon)

[G] M. Neunert, C. de Crousaz, F. Furrer, M. Kamel, F. Farshidian, R. Siegwart, and J. Buchli, “Fast nonlinear model predictive control for unified trajectory optimization and tracking,” in IEEE International Conference on Robotics and Automation (ICRA), 2016