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$E(X_i)=\mu$, $V(X_i)=\sigma^2=E(X_i^2)-\mu^2$

$E(\bar{X})=\mu$, $V(\bar{X})=\frac{\sigma^2}{n}=E(\bar{X}^2)-\mu^2$

$E(\hat{\theta}_1)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$

$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\frac{n-1}{n}\sigma^2$

$E(\hat{\theta}_2)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$

$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\sigma^2$

∴ $\hat{\theta}_2=\sum_{i=1}^{n}(X_i-\bar{X})^2/(n-1)$ 為 σ^2 的不偏估計量

$\hat{\theta}_1=\sum_{i=1}^{n}(X_i-\bar{X})^2/n$ 為 σ^2 的偏誤估計量

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(1) $t_{0.025}(10)=2.228$

(2) $t_{0.95}(8)=-1.86$

(3) $\chi^2_{0.05}(12)=21.03$

(4) $\chi^2_{\alpha}(15)=7.26$ 求 α $\alpha=0.95$

(5) $\chi^2_{0.95}(10)=3.94$

(6) $F_{0.05}(5,8)=3.69$

(7) $F_{0.95}(6,7)=0.238$

(8) $F_{\alpha}(6,6)=4.28$ 求 α $\alpha=0.05$