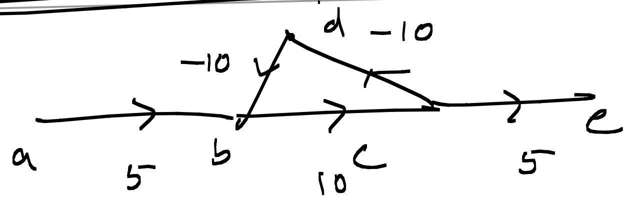
Note Title

Shortest Walk



$\langle a, b, c, e \rangle$ — $5 + 10 + 5 = 20$.

$\langle a, b, c, d, b, c, e \rangle$ $+5 + 10 - 10 - 10 + 10$
$+5$

$= 10$

wt of $\langle b, c, d, b \rangle = 10 - 10 - 10$
$= -10$

Negative cycle.

$\langle a, b, c, d, b, c, d, b, c, e \rangle$ — 0 //.

Min wt walk has $-\infty$ as its wt. Using ∞ number of edges.

"Shortest walk is not well-defined" if G has a negative cycle:

if $\alpha(u, v)$ is the weight of shortest walk, then $\alpha(u, v)$ could become $-\infty$.

While $\delta(u, v)$ is always well-defined $\alpha(u, v)$ may become $-\infty$.

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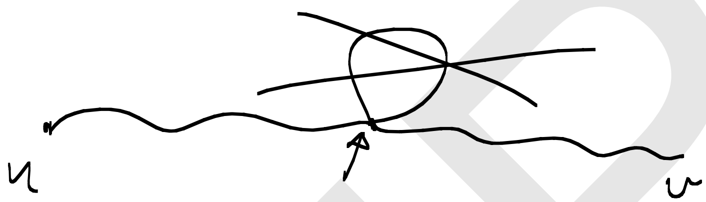
The weight of a shortest walk can be reduced to $-\infty$ if there is a negative cycle in the walk.

$\boxed{P1}$ If W is a walk from u to v, then W contains a path from u to v.



path

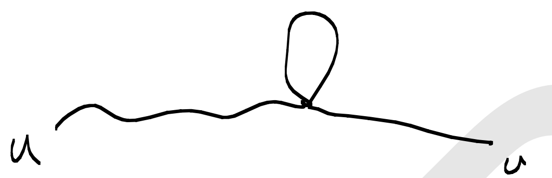
We obtain the path by removing the cycles from the walk.

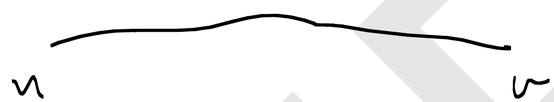
$\boxed{P2}$ Assume that G has no negative cycles. Let W be a walk from u to v. Then W contains a path P from u to v $\ni$

$$w(P) \leq w(W)$$

$\boxed{P3}$ If G has no negative cycle, no zero cycle, then the shortest walk from u to v will be a shortest path from u to v.

Proof: if the shortest walk has any cycle, it can be removed and a shorter walk is obtained.
(the weight of the cycle is > 0)

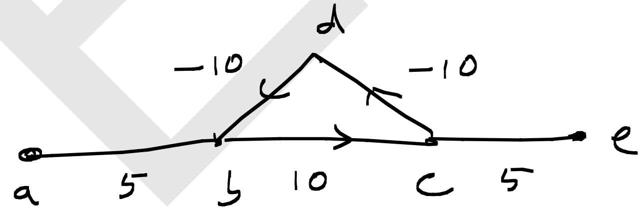




This contradiction shows that shortest walk can not contain any cycle : shortest walk is indeed a shortest path.

Hence if G has no negative / zero cycle

$$\underline{\alpha(u,v)} = \underline{\delta(u,v)} \quad //.$$

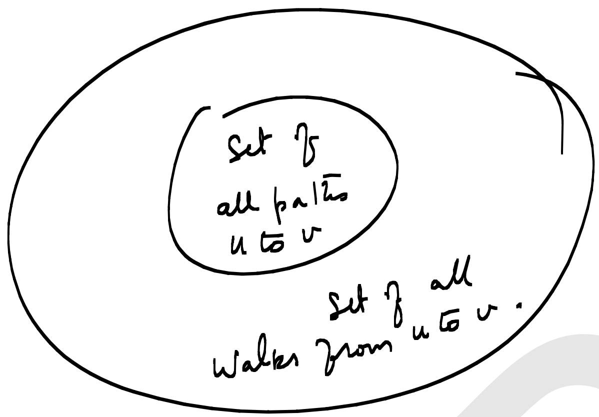


$\langle a, b, c, e \rangle$ - Path - only ~~the~~ one path
- shortest path.

$$\delta(a,e) = 5 + 10 + 5 = 20.$$

$$\alpha(a,e) = -\infty.$$

Since every path is a walk,



$$\alpha(u,v) \leq \delta(u,u)$$