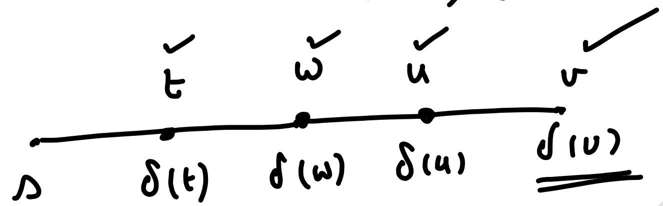
Note Title

Dijkstra's Algorithm.

$G = (V, E, w, s)$.



This path in the SPT is a sh. path from s to v.

$w(e) > 0$

$\delta(t) < \delta(w) < \delta(u) < \delta(v)$

Find $\delta(\;)$ in their increasing order.

$$\delta(v) = \min_{(u,v)} \delta(u) + \underbrace{w(u,v)}_{>0}$$

for v, we need only the $\delta(\;)$, $\delta(u)$ where

$\delta(u) < \delta(v)$

$(n-1)$ stages: iterations.

$|V| = n$, $\delta(s) = 0$

$\delta(v)$? $v \neq s$.

S $\qquad$ $V-S$

set of all vertices for which $\delta(\,)$ are known. $\qquad$ NOT known

Find a vertex v in $V-S$ for which $\delta(v)$ is computed / known.

Move v to S.

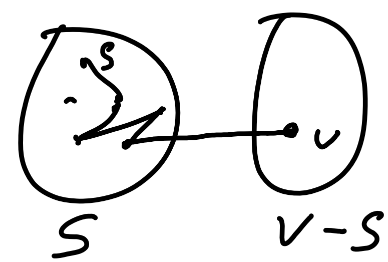
$\boxed{\delta(s) = 0}$

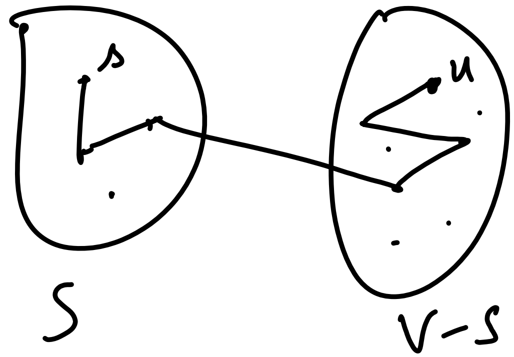
S $\qquad$ $V-S$

s $\qquad$ $V - \{s\}$

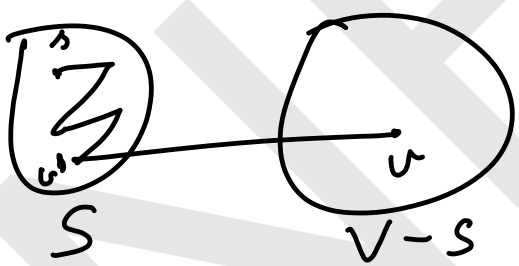
for each $v \in V-S$, we maintain "some info"

Call a path from s to v in $V-S$ a special path if all vertices other than v ~~is~~ are in S.



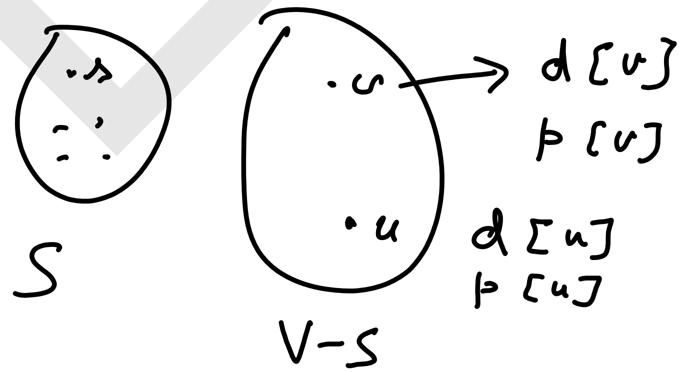


Let $d[v]$ be the weight of shortest special path from s to v. $\{$ for each $v \in V-S \}$



$p(v)$ is the vertex in S that is previous to v.

$\underline{\underline{p(v) = v'}}$ (in this picture)

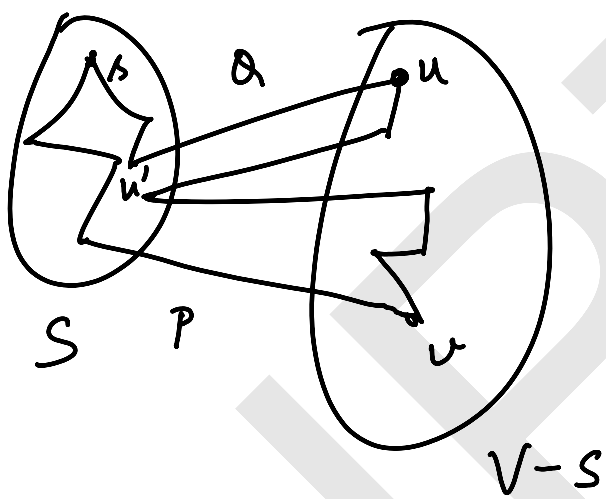


Find a vertex v in $V-S$ with minimum $d[\ ]$ value.

$d[v] \leq d[u] \;\; \forall \; u \in V-S$.

$d[v] = \delta(v)$!!!

Move v to S.



Let P be the special path from s to v with $w(P) = d[v]$.

We claim P is a shortest path.

If not, let Q be a shortest path from s to v.

$\longrightarrow$ $w(Q) < w(P) = d[v]$.

Let (u', u) be the first crossing edge in Q.

$$Q[s,v] = \underbrace{Q[s,u]}_{\text{special path from } s \text{ to } u} + Q[u,v]$$

$$\underline{w(Q[s,v])} = w(Q[s,u]) + \underline{w(Q[s,v])}.$$

$$> w(Q[s,u])$$

$$\geq d[u]$$

$$\geq d[v] \quad \text{(by the property of } v\text{.}$$

$$= w(P)$$

$w(Q) > w(P)$

This is a contradiction to the property of Q.

Hence, such a Q can not exist.

Hence P is the shortest path

Thus $d[v] = \delta(v)$.

Hence we can move v from $V-S$ to S.