Introduction to Graphs.

Note Title

Communication Networks
Social Network.
Transportation Network.
Distributed System.
!

Graph:

Directed and undirected graphs.

Directed graph.

$G = (V, E)$

V is a finite, non empty set.

$E \subseteq V \times V$.

$V \times V =$ { set of all ordered pairs of elements of V }.

$|V| = n$

$|V \times V| = n^2$

$V = \{1,2,3\}$, $V \times V = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$.

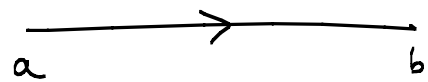
$(1,2) \neq (2,1)$

$\{1,2\} = \{2,1\}$.

V — Vertex set

E — edge set.

$(a,b) \in E$ is called a directed edge.



(Pictorial representation of Directed edge)

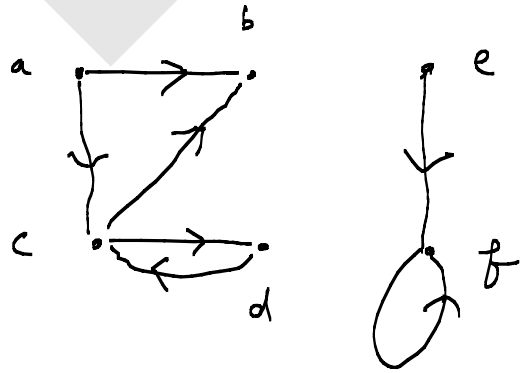
The edge (a,b) — leave a

— arrive b.

an arrow head is used to indicate the order (direction).

$V = \{a, b, c, d, e, f\}$

$E = \{(a,b), (a,c), (c,b), (c,d), (d,c), (b,f), (e,f)\}$

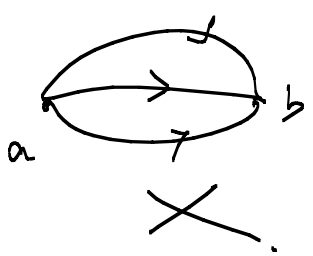


Out_degree (v) — No of edges leaving v.

In_degree (v) — No of edges arriving at v.

out_degree (a) = 2, in_degree (a) = 0.

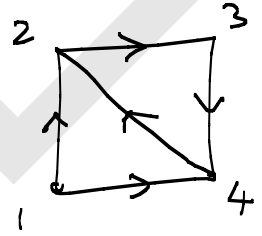
A Directed graph is said to be simple, if it has no self-loops or multiple edges between a pair of nodes.



$$\sum_{v \in V} \text{out\_degree}(v) = |E| = m.$$

$$\sum_{v \in V} \text{in\_degree}(v) = |E| = m.$$

$V = \{1, 2, 3, 4, 5, 6\}$



$A_{n \times n}$ $\quad |V| = n$

$A[i,j] = 1$, if $(i,j) \in E$

$= 0$ if $(i,j) \notin E$.

0 1 0 1 0 0
0 0 1 0 0 0

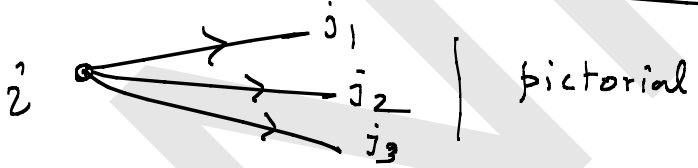
Adjacency List Representation.

Array of Linked List,
one for each vertex.

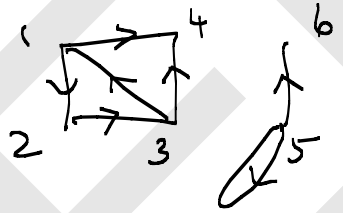
(n linked lists), $|V| = n$.

The list corresponding to i contains all $j \ni (i,j) \in E$.

$(i, j_1), (i, j_2), (i, j_3) \in E$.



pictorial



1 — 2, 4

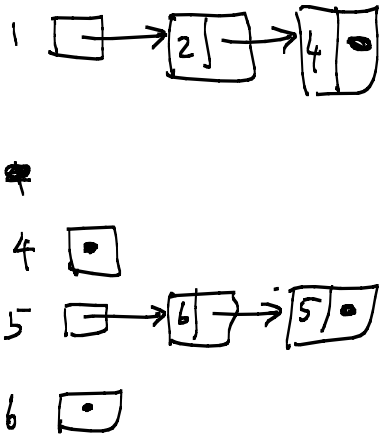
2 — 3

3 — 4, 1

4 —

5 — 6, 5

6 —



Undirected graph :