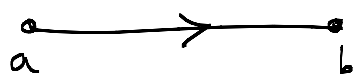
Shortest Path Algorithms.

Note Title

Directed graphs. $G = (V, E)$

$E \subseteq V \times V$.

$(a, b) \in E$. directed edge.



① Walk
② Path

Weighted, Directed graph.

Edge-weighted, Directed Graphs.

$w : E \to \mathbb{I}$.

$\underline{\underline{w(u,v)}}$, $(u,v) \in E$.

$w(u,v)$ — positive, zero, negative.

$(a, b) \in V \times V$

but $(a, b) \notin E$

$w(a, b) = \infty$.

Weight function is defined on E.

Extended weight function is defined on $V \times V$.

$G = (V, E, w)$

w ↑ weight function.

Sequence of edges

$\langle e_1, e_2, \dots, e_k \rangle$ is

said to be a walk from vertex $\underline{\underline{u}}$ to vertex v.

if

$e_i = (u_{i-1}, u_i) \in E$.

$u_0 = u$

$u_k = v$.

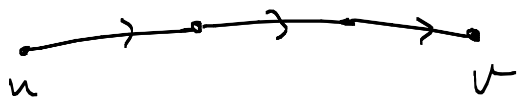
$$\overset{u}{u_0} \xrightarrow{e_1} u_1 \xrightarrow{e_2} u_2 \xrightarrow{e_3} \overset{v}{u_3}$$

k, the number of edges in the walk, is called its <u>length</u>.

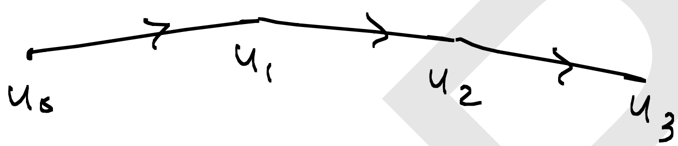
u, v are called end vertices.

u_0, u_k

if $k \geq 2$



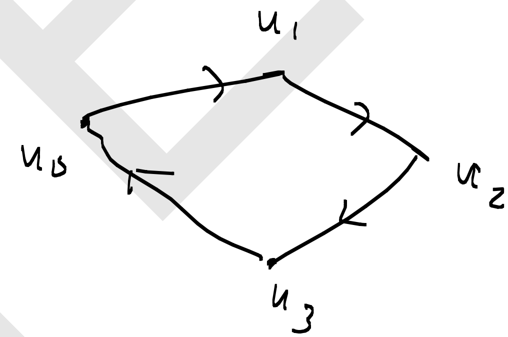
All vertices in the walk, other than end vertices are called intermediate vertices.



u_1, u_2 are intermediate vertices.

$\langle u_0, u_1, u_2, u_3 \rangle$, $(u_{i-1}, u_i) \in E$.

Closed Walk. "$u = v$" 'end vertices' are same



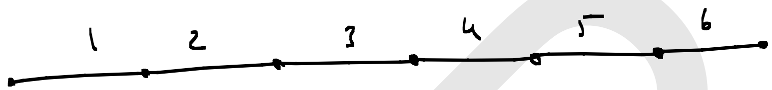
$\boxed{u_4 = u_0}$

$\langle u_0, u_1, u_2, u_3, u_4 \rangle$
where $u_4 = u_0$

Path is a walk in which all intermediate vertices are distinct.

A closed path, Cycle.

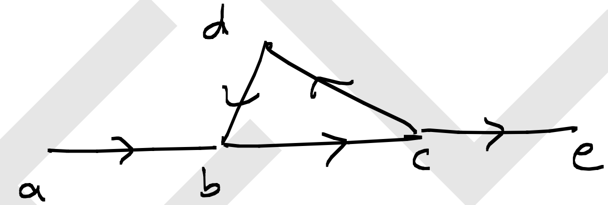
$|V| = n$



Length of any path $\leq |V| - 1$

$= (n-1)$

There may not be any bound on the length of a walk.



$\langle a, b, c, e \rangle$ — 3

$\langle a, b, c, d, b, c, e \rangle$ — 6

$\langle a, b, c, d, b, c, d, b, c, e \rangle$ — 9.

Walks from a to e.

Weight of a Walk W

$$w(W) = \sum_{e \in W} w(e)$$

Weight of a Path P

$$w(P) = \sum_{e \in P} w(e)$$

Weight of a cycle C

$$w(C) = \sum_{e \in C} w(e)$$

A cycle is said to be

positive $\quad w(C) > 0$

zero $\quad w(C) = 0$

negative $\quad w(C) < 0$.

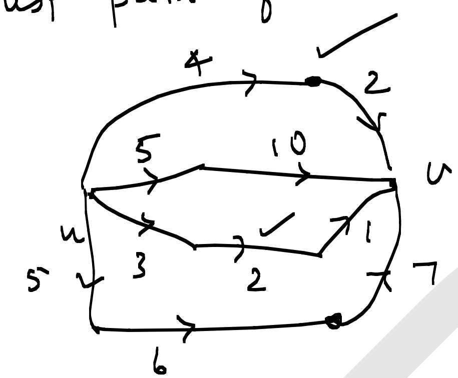
Shortest path weight $\delta(u,v)$

$$\delta(u,v) = \mathrm{Min}\{ w(P) : P \text{ is a path from } u \text{ to } v \}.$$

Since, number of paths from u to v is finite.

$\delta(u,v)$ is well-defined & finite.

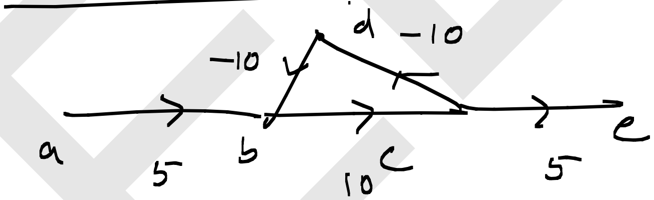
~~Any~~ Any path p with $w(p) = \delta(u, v)$ is called a shortest path from u to v.



13, 6, 15, 6

$\delta(u, v) = 6$.

Shortest Walk



$\langle a, b, c, e \rangle$ — $5 + 10 + 5 = \underline{20}$.

$\langle a, b, c, d, b, c, e \rangle$ $+5 + 10 - 10 - 10 + 10 + 5$

$= \underline{10}$

Wt of $\langle b, c, d, b \rangle = 10 - 10 - 10$
$= -10$

Negative cycle.

$\langle a, b, c, d, b, c, d, b, c, e \rangle$ — 0 //.

Min wt walk has $-\infty$ as its wt. Using ∞ number of edges.

" shortest walk is not well-defined "
" if G has a negative cycle :

if $\alpha(u,v)$ is the weight of shortest walk, then
$\alpha(u,v)$ could become $-\infty$.
while $\delta(u,v)$ is always well-defined
$\alpha(u,v)$ may become $-\infty$.