Strong Connected Components.

Directed graph $(G = V, E)$.

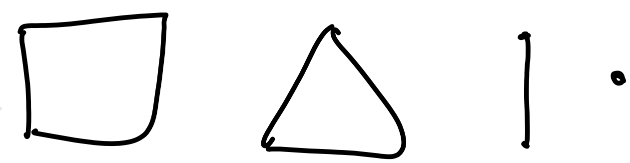
G is said to be strongly connected <u>iff</u> for every pair of vertices (u, v) there is a path from u to v.

$\Rightarrow$ $\{v, u\}$, there is a path from v to u & from u to v.

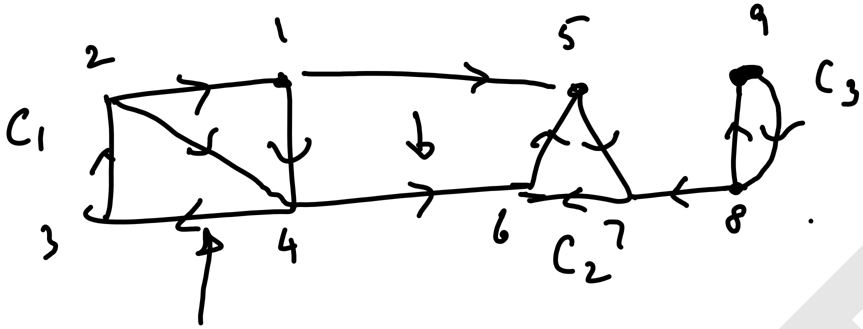
$$v \, R \, u \text{ iff } \left.\begin{array}{l} v \longrightarrow u \ \& \\ \text{from } u \longrightarrow v \end{array}\right\} \text{paths.}$$

This relation induces a partition on the vertex set.

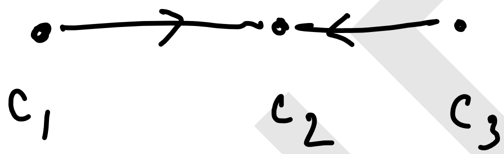
Each such partition is called a strongly connected component.

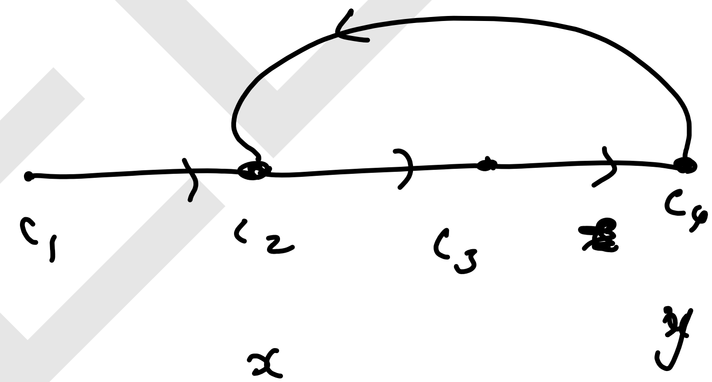


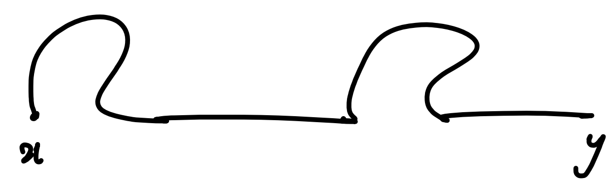
4 components.



No path from 5 to 1







Condensed graph is acyclic graph.

DAG

$u, v \in V$

Start dfs from u.

if v is reachable then DFS will reach v.

Start a dfs from v

check if u is reached?

$2\binom{n}{2} = n(n-1)$ times DFS is invoked

to check if G is strongly connected or NOT? $n(n-1)(m)$

Total cost $O(n^2 m)$

Let $G = (V, E)$ be a directed graph.

G is strongly connected iff.

for any fixed $s \in V$, $\forall v \in V, v \neq s$, there is a path from s to v & v to s.

Perform DFS from s.

if DFS can reach all vertices, then one part is proved.

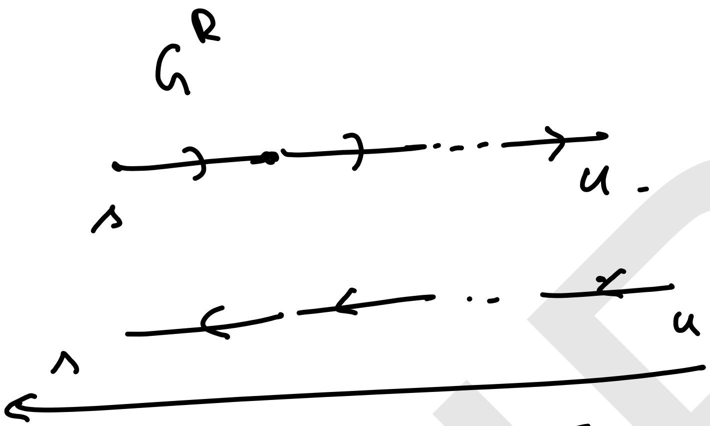
Consider G^R (G-reverse).

(each edge is reversed its direction.

$a \to b \leftarrow c$ $\qquad$ $a \leftarrow b \to c$.

In G^R, suppose there is a path from s to u.

$\Rightarrow$ there is a path from u to s in the original graph.

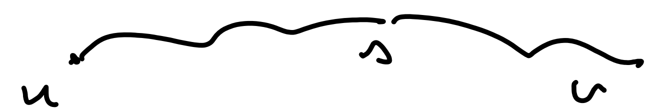


if there is a path from s to u in G^R, then there is a path from u to s in G.

The improvement from $n(n-1)$ dfs calls to 2 dfs is really interesting.

If there is a path from s to u and u to s $\forall u \neq s$, then, there is a path from u to v $\forall (u,v)$. $u \neq v$, $(u,v) \in V \times V$.

$\Rightarrow$ G is strongly connected.



There will be a path from u to v.

Given $G = (V, E)$

we can determine if G is a strongly connected graph or not by performing 2 DFS (one over G and another over G^R).

If G is NOT strongly connected, How do we find the SCC ?