

DS284-Numerical Linear Algebra - Assignment 4

LOCHAN SURYA TEJA NEELI(06-18-01-10-22-23-1-23198)

October 22, 2023

Answer 1

error = $2 - \text{norm}(p - f(t))$ where $p, f(t) \in \mathbb{R}^{100}$

p is the vector that consists of the predicted values of the 100 input values evaluated via polynomial fitting.

$f(t)$ is the vector that consists of the actual values of the 100 input values evaluated via the original function $f(t) = \sin(10t)$.

Fitting a 14th degree polynomial to the datapoints with MGS QR factorization

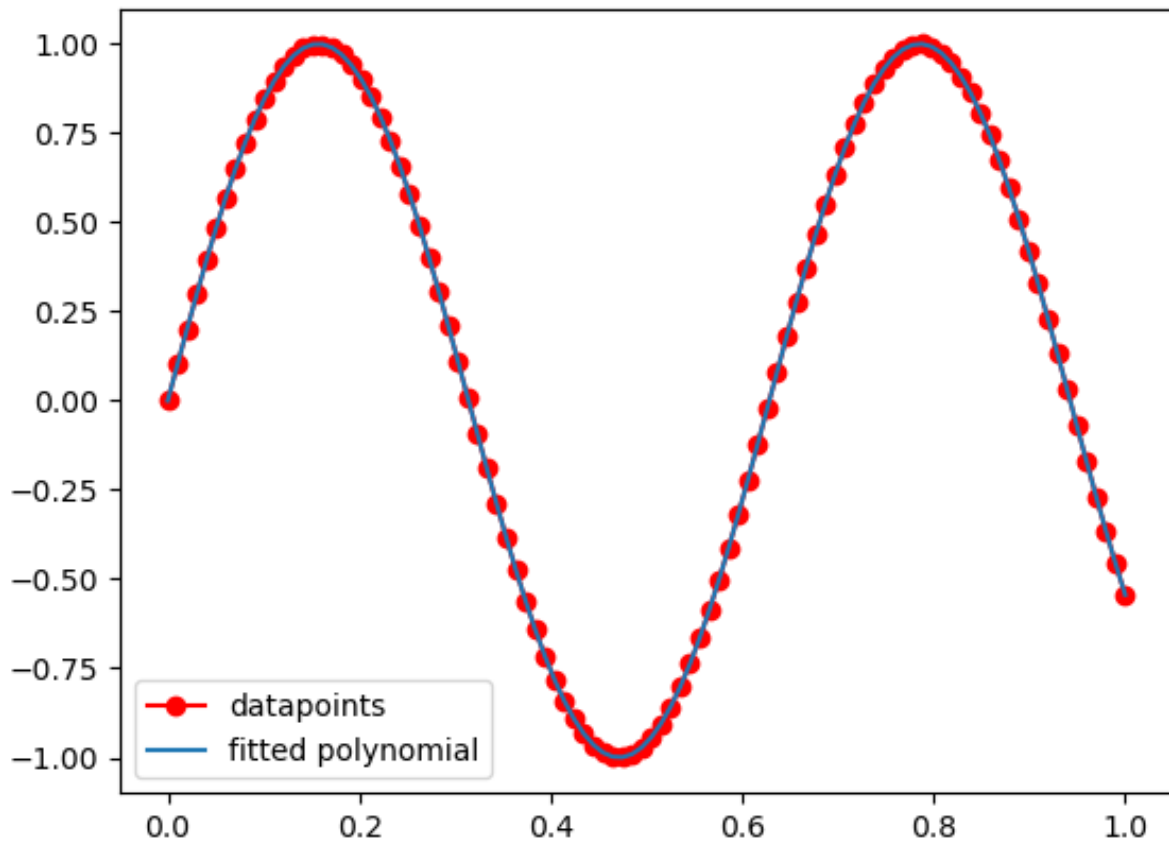


Figure 1: error= 66.47188248164618

ing a 14th degree polynomial to the datapoints with Householder QR factoriz

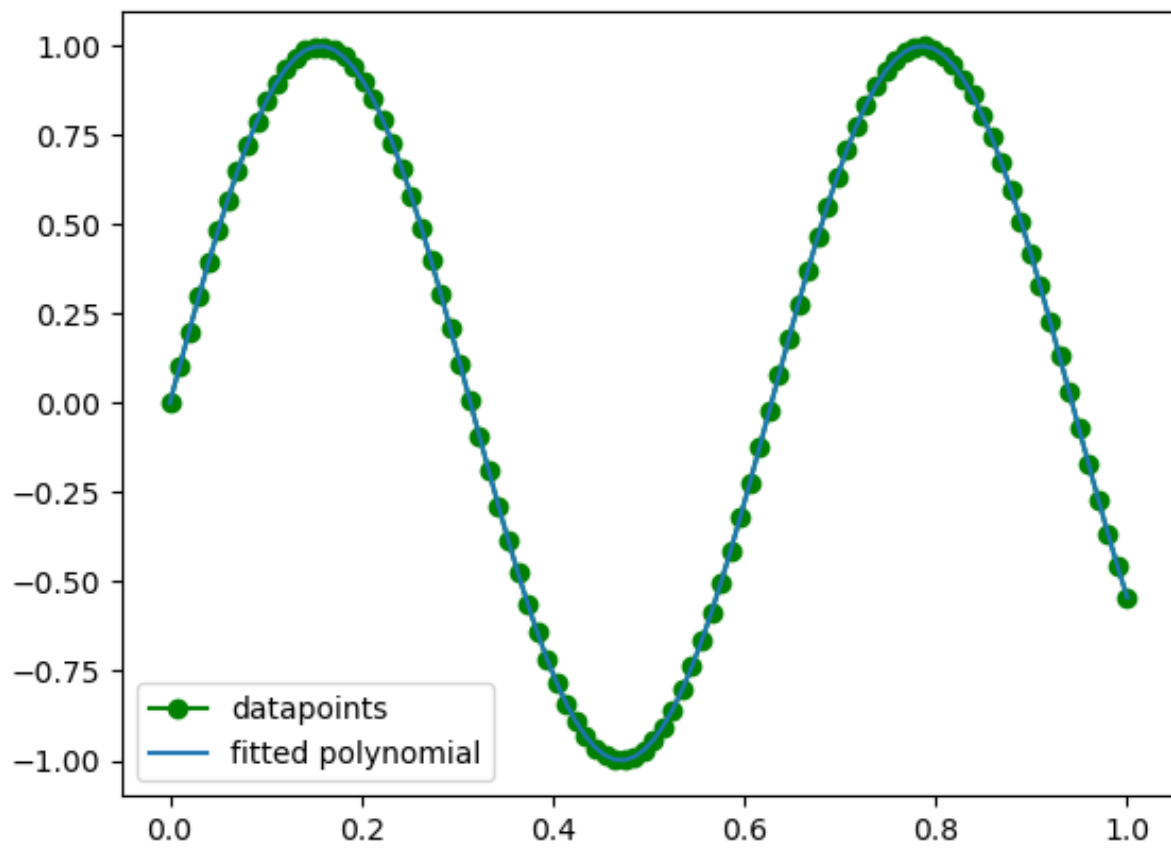


Figure 2: error= 66.47188248158425

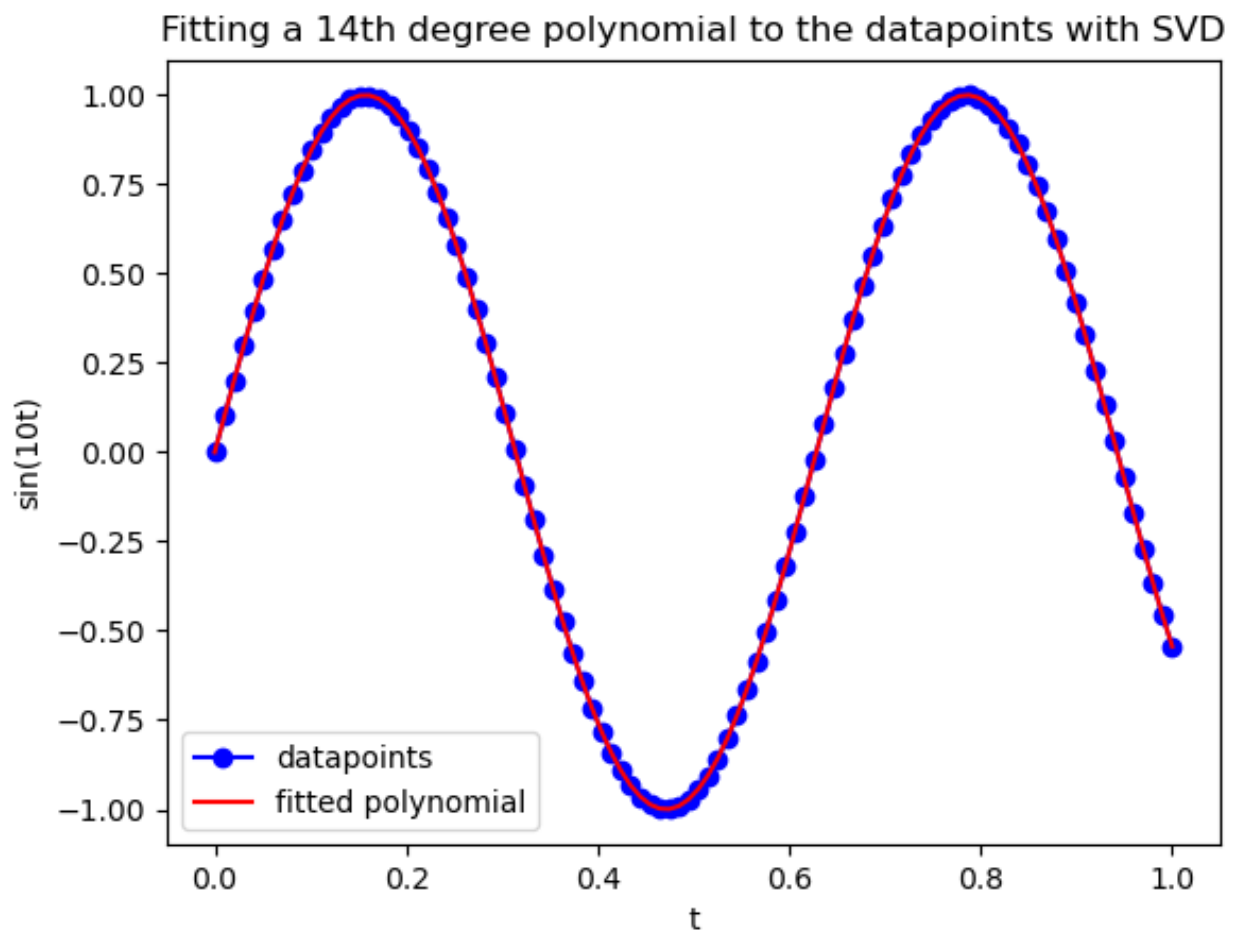


Figure 3: error= 66.47188248158271

Fitting a 14th degree polynomial to the datapoints with normal equations

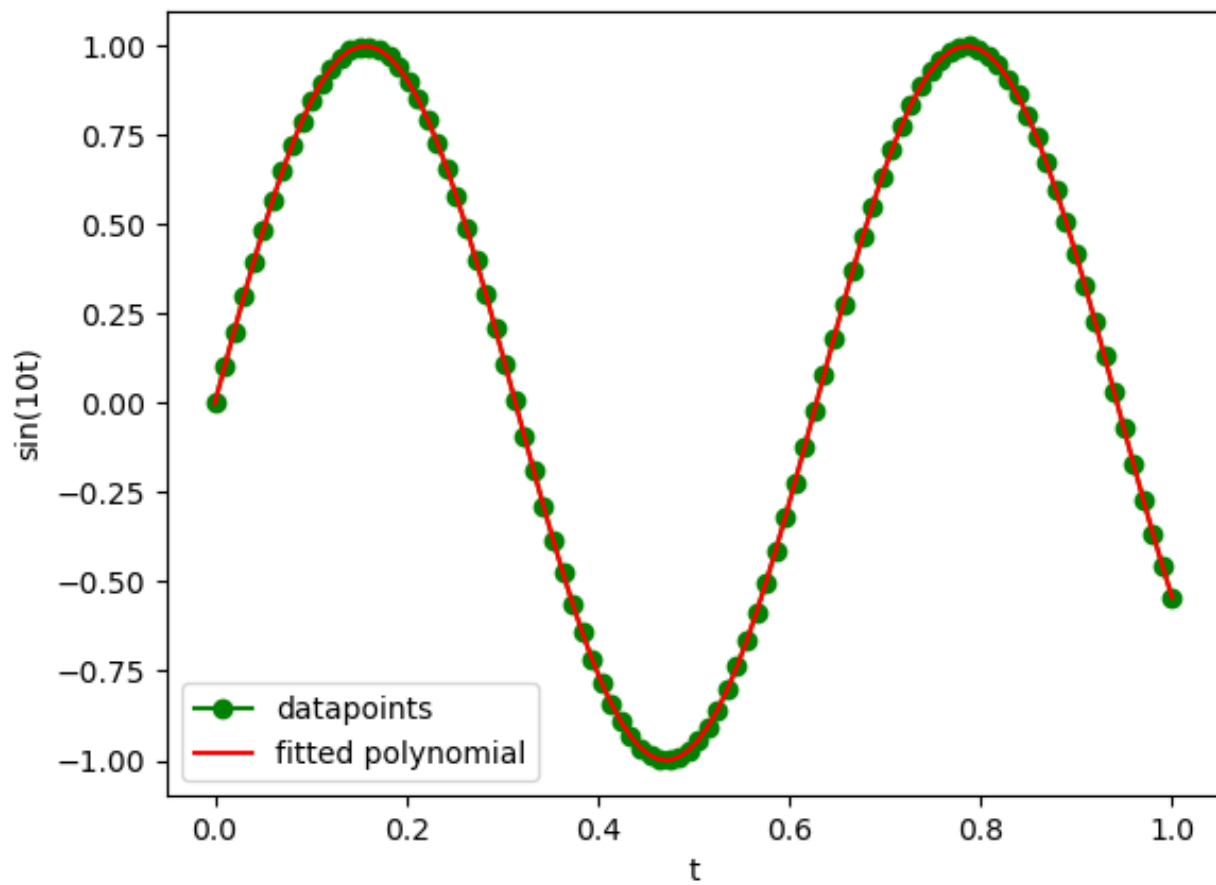


Figure 4: error= 66.47188275756871

Coefficients of the 14 degree polynomial for the function $f(t) = \sin(10t)$ approximation:

$$p_{14} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10} + a_{11}x^{11} + a_{12}x^{12} + a_{13}x^{13} + a_{14}x^{14}$$

	MGS	Householder	normal equations	SVD
a_0	4.13e-7	-1.604e-7	0.0000022188	-1.604e-7
a_1	9.9998424247	10.0000600749	10.0017959484	10.0000600747
a_2	0.0095821262	-0.0031492289	-0.1558430004	-0.0031492228
a_3	-166.9113404969	-166.6106191969	-162.496532603	-166.6106192929
a_4	3.4840726795	-0.2959493046	-53.2919958934	-0.2959482308
a_5	801.5976640372	830.4565467514	1219.9893792245	830.4565385493
a_6	199.3841776048	55.2238419012	-1734.0963064463	55.2238843827
a_7	-2884.4179382225	-2392.0438688348	3003.4347635999	-2392.0440199592
a_8	2979.4731872136	1801.1100507212	-9120.6421024678	1801.1104252418
a_9	-4503.7856651089	-2508.830344659	12323.3515274771	-2508.8309956998
a_{10}	12907.8718186067	10530.107804164	-2596.3576586784	10530.1085943036
a_{11}	-18825.4162729622	-16874.0560276336	-9907.4873359055	-16874.0566831362
a_{12}	13836.7980869608	12787.2865803221	-9907.4873359055	12787.2869345805
a_{13}	-5133.4271555276	-4800.4499588965	-4908.1733685353	-4800.4500712968
a_{14}	774.7959199589	727.5610133192	832.8919068456	727.5610292058

Table 1: Coefficients of the fitted polynomial.

maximum error is 66.47188275756871, occurred in Normal equations.

minimum error is 66.47188248158271, occurred in SVD.

Upto 6 digits after decimal in the values of error, its the same. Thus, we need to set the tolerance more than 10^{-6} to achieve high degree of accuracy to compare among the several methods for solving the least squares problem.