DS284-Numerical Linear Algebra - Assignment 4

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Answer 1

error= $2 - norm(p - f(t))$ where $p, f(t) \in \mathbb{R}^{100}$
p is the vector that consists of the predicted values of the 100 input values evaluated via polynomial fitting.
$f(t)$ is the vector that consists of the actual values of the 100 input values evaluated via the original function $f(t) = sin(10t)$.

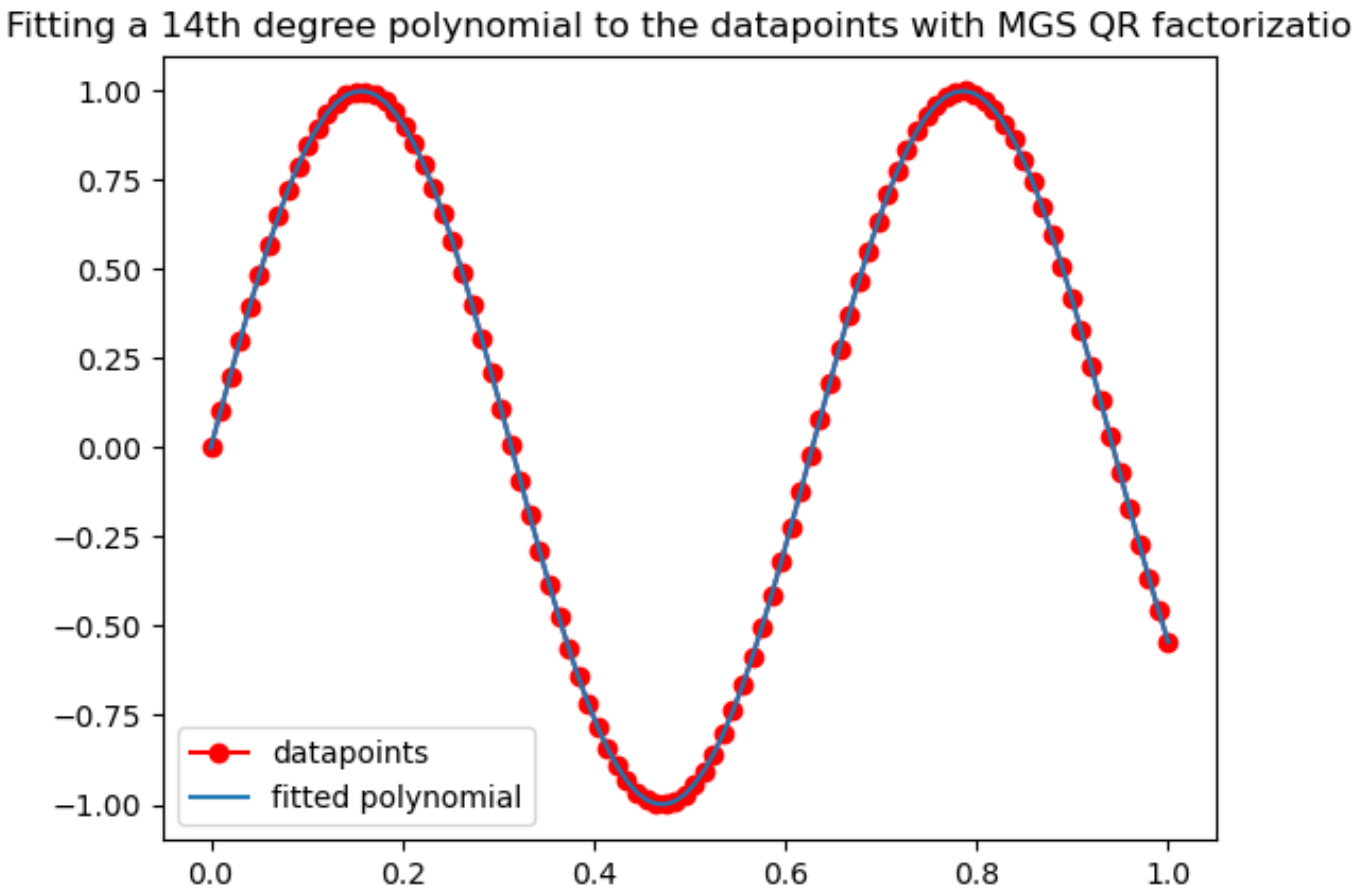


Figure 1: **(a)**error= 66.47188248164618

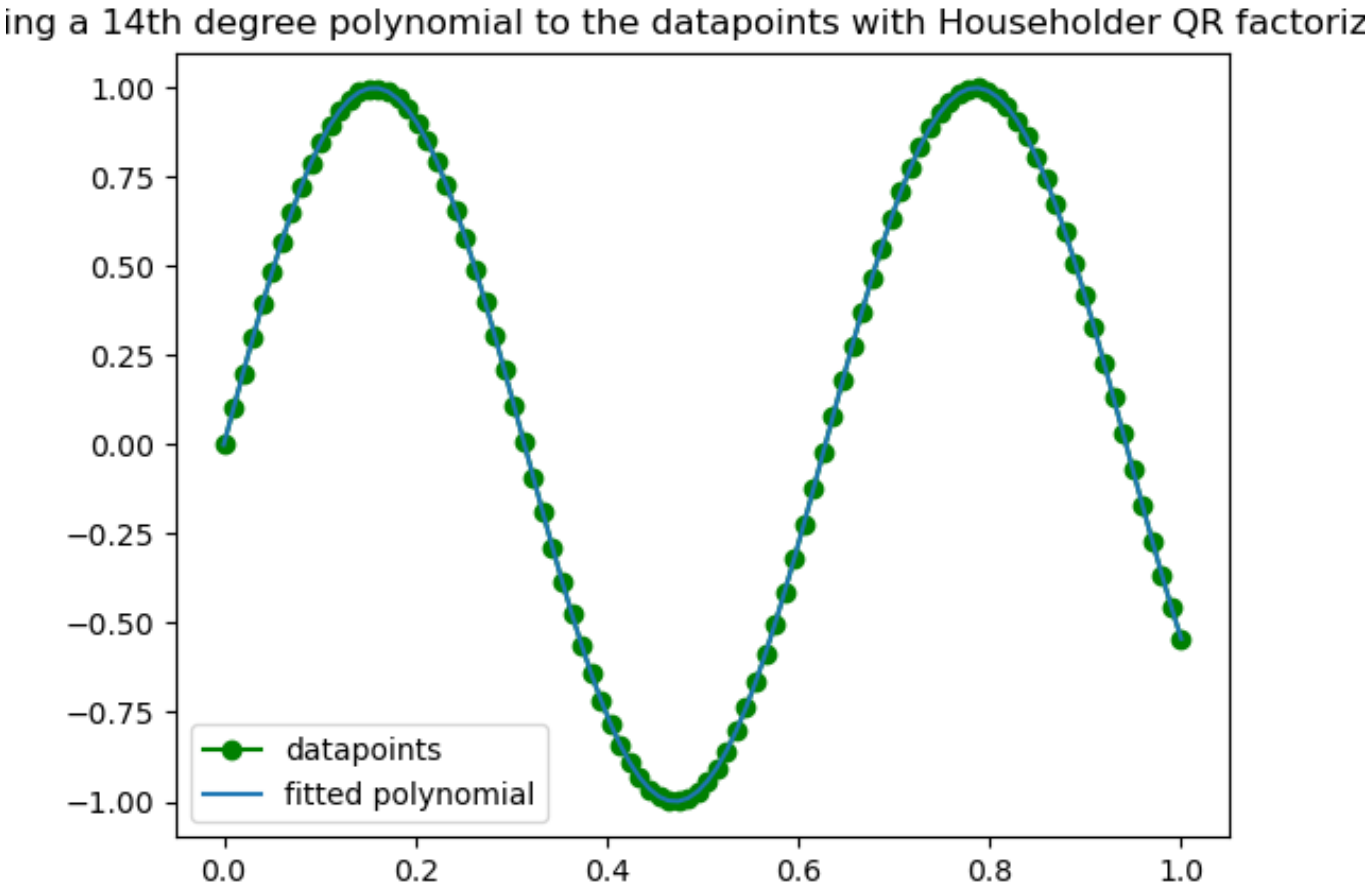


Figure 2: **(b)**error= 66.47188248158425

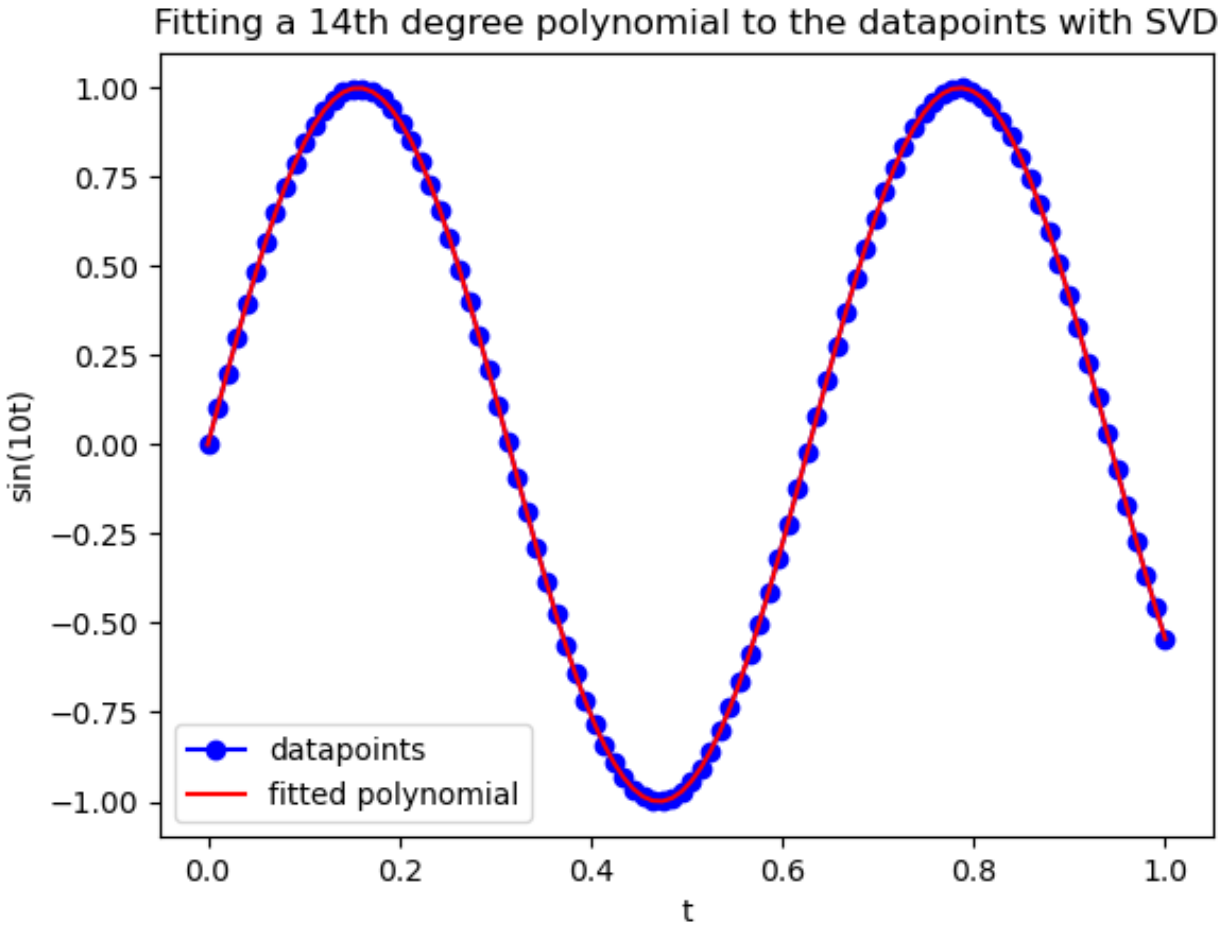


Figure 3: **(c)**error= 66.47188248158271

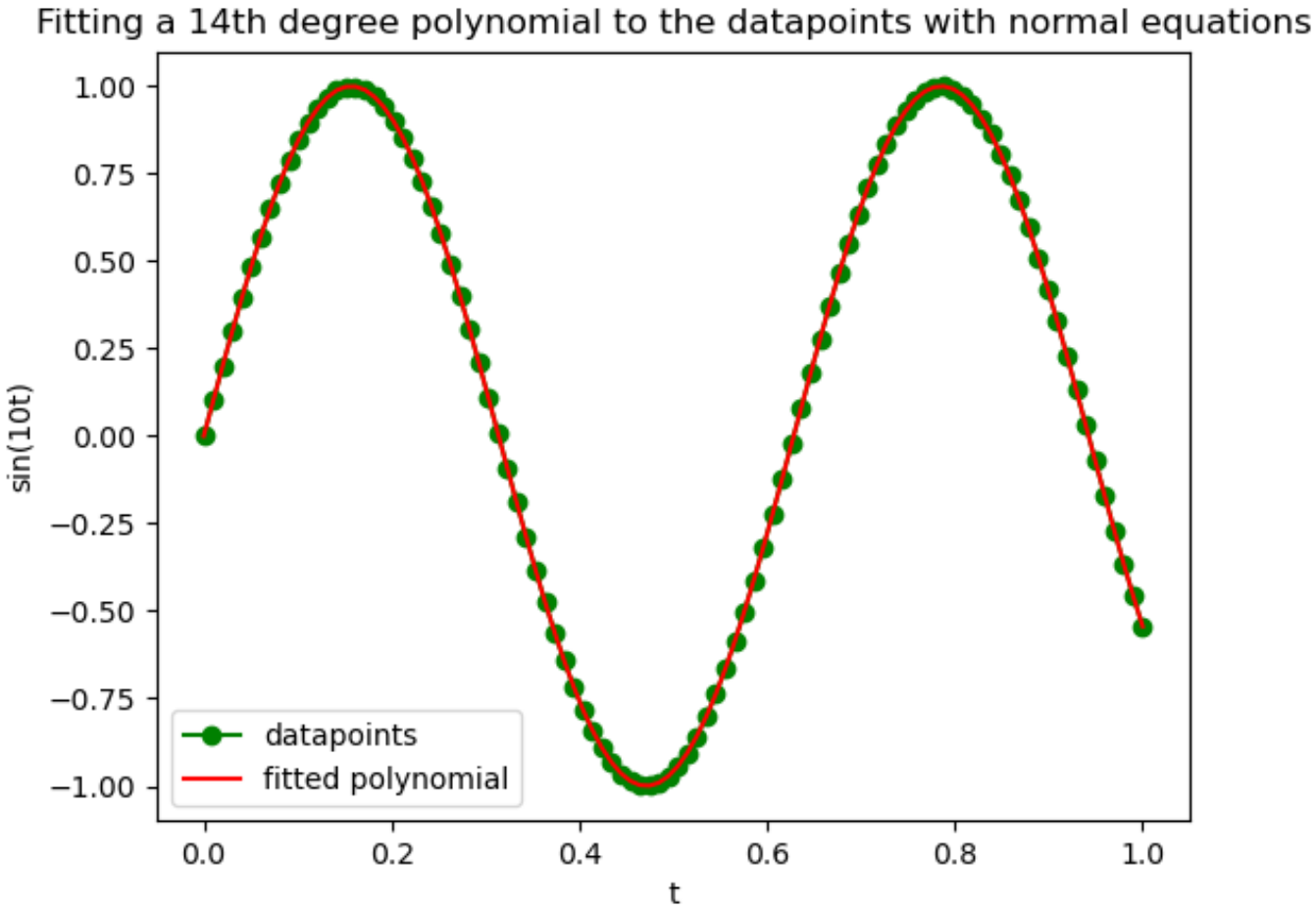


Figure 4: **(d)**error= 66.47188275756871

Coefficients of the 14 degree polynomial for the function $f(t) = sin(10t)$ approximation:

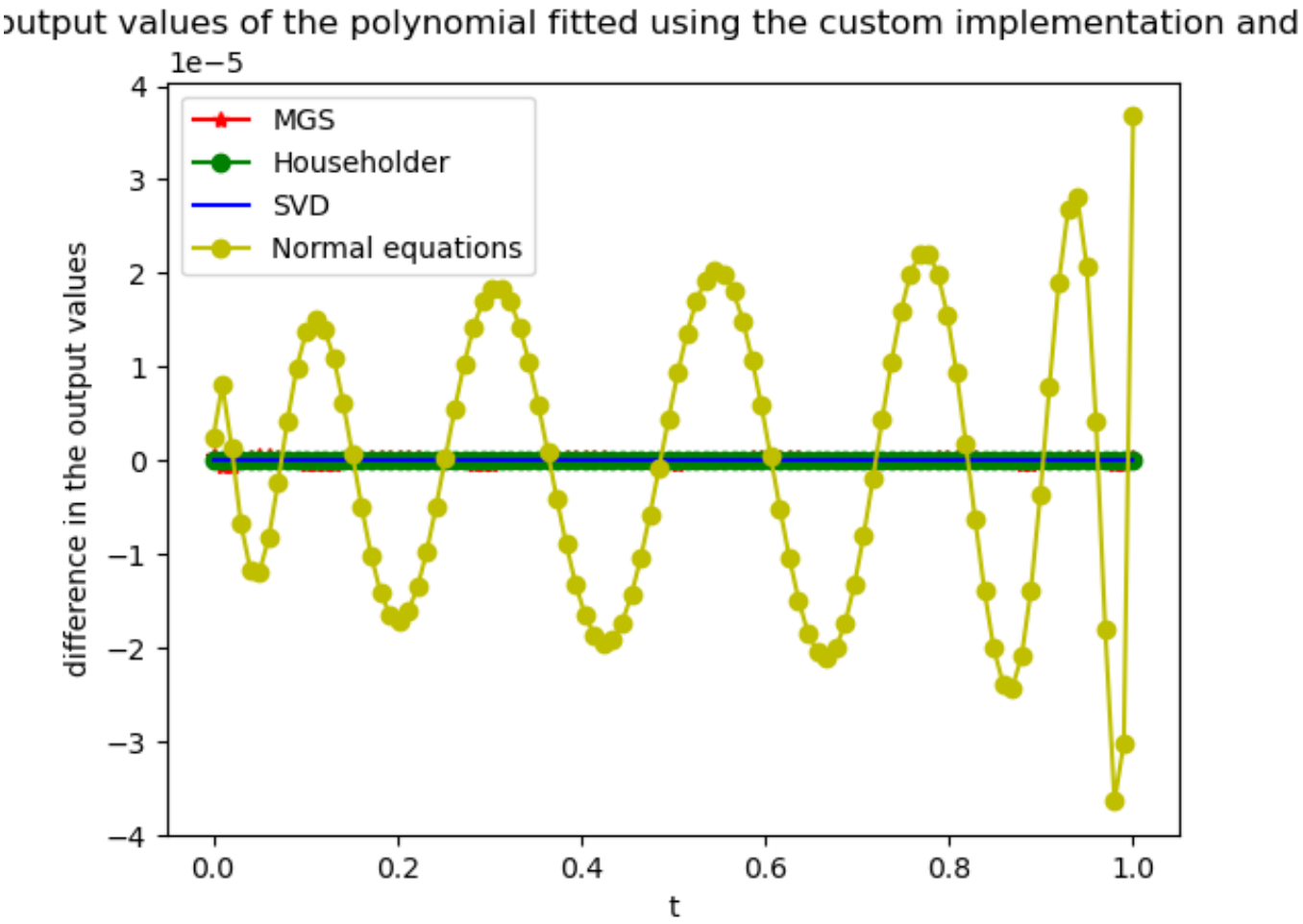
$p_{14} = a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5+a_6x^6+a_7x^7+a_8x^8+a_9x^9+a_{10}x^{10}+a_{11}x^{11}+a_{12}x^{12}+a_{13}x^{13}+a_{14}x^{14}$

	MGS	**Householder**	**normal equations**	**SVD**
a_0	4.13e-7	-1.604e-7	0.0000022188	-1.604e-7
a_1	9.9998424247	10.0000600749	10.0017959484	10.0000600747
a_2	0.0095821262	-0.0031492289	-0.1558430004	-0.0031492228
a_3	-166.9113404969	-166.6106191969	-162.496532603	-166.6106192929
a_4	3.4840726795	-0.2959493046	-53.2919958934	-0.2959482308
a_5	801.5976640372	830.4565467514	1219.9893792245	830.4565385493
a_6	199.3841776048	55.2238419012	-1734.0963064463	55.2238843827
a_7	-2884.4179382225	-2392.0438688348	3003.4347635999	-2392.0440199592
a_8	2979.4731872136	1801.1100507212	-9120.6421024678	1801.1104252418
a_9	-4503.7856651089	-2508.830344659	12323.3515274771	-2508.8309956998
a_{10}	12907.8718186067	10530.107804164	-2596.3576586784	10530.1085943036
a_{11}	-18825.4162729622	-16874.0560276336	-9907.4873359055	-16874.0566831362
a_{12}	13836.7980869608	12787.2865803221	-9907.4873359055	12787.2869345805
a_{13}	-5133.4271555276	-4800.4499588965	-4908.1733685353	-4800.4500712968
a_{14}	774.7959199589	727.5610133192	832.8919068456	727.5610292058

Table 1: Coefficients of the fitted polynomial.

Comparing the models based on the error:

The difference is calculated by taking the difference between the values obtained from the custom implemented algorithm and the values obtained from the in-built function.



You can see from the graph that Normal Equations performed the worst among the implemented algorithms.
maximum error is 66.47188275756871, occured in Normal equations.
minimum error is 66.47188248158271, occured in SVD.
On comparing the obtained solutions of the different models with solution obtained using the in-built function, **"np.linalg.solve()"**, I got the maximum difference, calculated in terms of 2-norm, in the case of normal equations.
Upto 6 digits after decimal in the values of error, its the same. Thus, we need to set the tolerance more than 10^{-6} to achieve high degree of accuracy to compare among the several methods for solving the least squares problem.