Modeling Continuous Opacities of Vega and a Sun-like Star

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1 Introduction

The opacity of a star can provide many different insights about the composition and various processes occurring within the stellar atmosphere. By determining how different elements affect light at different temperatures and pressures, one can see just how these elements absorb and attenuate photons coming from within the star. There are essentially four different processes by which photons are attenuated: an electron transitioning from a bound state to a free state (bound-free), an electron transitioning from a free state to a different free state (free-free), an electron transitioning from a bound state to a different bound state (bound-bound), and the scattering of electrons and photons due to Rayleigh and Thomson scattering.

The bound-free states occur when a bound electron transitions to a free state with velocity v. This creates a continuous absorption since free electrons can have a continuous range of velocities. The free-free states occur when a free electron transitions to a different free state. This essentially means that there is a change in the free electron's kinetic energy, due to a change in it's acceleration, resulting in radiation which then effects the absorption coefficient of the star. This is also a continuous process, since the electron has a continuous range of velocities, much like the bound-free transition. The bound-bound transitions occur when an electron transitions from one bound state to another bound state. This results in a single line absorption since the transition happens at a discrete energy value.

Scattering processes effect how light interacts with particles and therefore, how the light is attenuated. Rayleigh scattering can be thought of as the oscillation of an electron due to an electromagnetic field. Since light is made of electromagnetic waves, electrons experience an oscillation, and therefore a change in acceleration, in the presence of light. The change in acceleration produces a radiation which is observed as scattered light. Thomson scattering is associated with free electrons and occurs when a photon accelerates a free electron, resulting in radiation. Thomson scattering is independent of wavelength whereas Rayleigh scattering is dependent on wavelength ($\propto \lambda^{-4}$). Both of these scattering processes are important for determining how they effect the stellar atmosphere, although Thomson scattering is significantly smaller than Rayleigh scattering since hydrogen atoms are much more abundant than free electrons.

For this report, only hydrogen (H), the negative hydrogen ion (H-), and a metal will be considered to make up the composition of the star. While helium also makes up a part of the stellar opacity, it will be ignored here for simplification. Also, only the free-free and bound-free transition states (i.e., only the continuous opacities) will be considered when calculating the absorption coefficients of H and H-. On top of that, this model for opacity assumes an atmosphere which is made of a parallel plane geometry, the model is assumed to be in hydrostatic equilibrium, the total flux from the star remains constant, and the source function equals the Planck function. The final assumption allows me to use the Saha equation and state that the distribution of particle velocities if Maxwellian [1].

The ultimate purpose of this project is to calculate the various components of the opacity of a sun-like star and of Vega, an A star. Specifically, the absorption coefficient per neutral hydrogen, the absorption coefficient per ionized hydrogen, and the total scattering coefficient per hydrogen is calculated for a specific wavelength for each star and plotted against a range of temperatures. First, I will present the calculations which represent the absorption coefficients and scattering processes. I will then present the data and results for both the sun-like star and Vega. Finally, the code I wrote to calculate these results will be attached and referenced at the end of the paper.

2 Calculations

In order to calculate the relative absorption coefficients, a few parameters and constants need to be defined. They are listed below with the correct units. The equations used for these calculations are from Dimitri Mihalas's book, *Methods in Computational Physics* [1]. I have reproduced them here for ease.

Abbreviation, Descriptions, and Values of various Constants and Parameters		
A	Ratio of the number of hydrogen to metal atoms	10^4
B	Ratio of the number of helium to hydrogen atoms	0.1
m_h	Proton mass	$1.67 \times 10^{-24} g$
σ_t	Thomson cross-section for an electron	$6.65 \times 10^{-25} cm^2$
c	Speed of light	29979245800 $^{cm}/_{s}$
h	Planck's constant	$6.62 \times 10^{-27} erg \cdot s$
k	Boltzmann's constant	$1.38 \times 10^{-16} {}^{erg}/_{K}$
χ	Ionization energy of H	$2.195 \times 10^{-11} erg$

Table 1: Variables as defined throughout the upcoming calculations.

2.1 Calculating ionization ratios

In order to calculate the absorption coefficients, the fraction of ionized hydrogen and the fraction of ionized metal needs to be determined. They are labelled as X and Y, where X is the fraction of H ionized and Y is the fraction of metal ionized:

$$X = \frac{\frac{n_H^+}{n_H^o}}{1 + \frac{n_H^+}{n_H^o}} \tag{1}$$

$$Y = \frac{\frac{n_m^+}{n_m^o}}{1 + \frac{n_m^+}{n_m^o}} \tag{2}$$

The ratios in the above equations ($\frac{n_H^+}{n_H^o}, \frac{n_m^+}{n_m^o}$) can be calculated from the corresponding Saha equations. The Saha equations relate the ionization state of a certain element with given temperatures and pressures.
Saha equation for neutral hydrogen:

$$log_{10}(n_H^+/n_H^o) = -log_{10}\, P_e - 13.595\, \Theta + 2.5\, log_{10}\, T - 0.4772 \tag{3}$$

Saha equation for neutral state of metal:

$$log_{10}(n_m^+/n_m^o) = -log_{10}\, P_e - 7.9\, \Theta + 2.5\, log_{10}\, T - 0.0971 \tag{4}$$

Where:

$$\Theta = \frac{5040K}{T} \tag{5}$$

2.2 Finding the absorption coefficient for neutral hydrogen

To correctly calculate the absorption coefficients for neutral hydrogen, a few quantum-mechanical fudge factors need to be calculated. These are called "Gaunt Factors" and there is one for both the free-free transition and the bound-free transition. λ should be in microns for the following calculations.
The Gaunt factor for free-free transitions is defined as:

$$\begin{aligned} g_{ff}(\lambda,\Theta) = &1.084 + \frac{0.0188}{\Theta} + \left(0.00161 + \frac{0.02661}{\Theta}\right)\lambda \\ &- \left(0.0192 - \frac{0.03889}{\Theta} + \frac{0.02833}{\Theta^2} - \frac{0.007828}{\Theta^3} + \frac{0.007304}{\Theta^4}\right)\lambda^2 \end{aligned} \tag{6}$$

The Gaunt Factor for bound-free transitions is defined as:

$$g_{bf}(m,\lambda) = a_m + b_m\lambda + c_m\lambda^2 \tag{7}$$

Where a_m, b_m, c_m are the coefficients as defined by the following table:

Bound-Free Coefficients			
m	a_m	b_m	c_m
1	0.9916	0.09068	-0.2524
2	1.105	-0.7922	0.4536
3	1.101	-0.329	0.1152
4	0.9736	0.	0.
5	1.03	0.	0.
6	1.097	0.	0.
7	1.098	0.	0.
8	1.	0.	0.
9	1.	0.	0.
10	1.	0.	0.

Table 2: Bound-Free coefficients for determining the bound-free Gaunt Factor.

The equation for the absorption coefficient per neutral H atom:

$$\alpha_{\lambda,H} = \frac{2.0898\times10^{-14}e^{-u_1}\lambda^3}{U_o(\Theta,P_e)}\left(1-e^{\frac{-h\nu}{kT}}\right)\times\left\{\left(\sum_{m_o}^{m^\star} g_{bf}(m,\lambda)\frac{e^{u_m}}{m^3}\right) + \frac{1}{2u_1}\left[e^{u_{m^\star}} - 1 + g_{ff}(\lambda,\Theta)\right]\right\} \tag{8}$$

Where $U_o(\Theta, P_e)$ is the partition function of neutral hydrogen (equal to 2) and u_m is defined as:

$$u_m = \frac{\left(\chi/kT\right)}{m^2} \tag{9}$$

Where m is the quantum number of the mth state. The conditions for m_o and $m^\star$ are as follows:

- m_o is the largest integer such that $u_m \leq h\nu/kT$
- $m^\star$ is the value of the highest bound state considered (here, 10 is the highest bound state considered)

For both the sun-like star and Vega, $m^\star = 10$ and $m_o = 3$.
To get the mass absorption coefficient per gram, multiply the absorption coefficient per neutral H atom by the following multiplier:

$$k_{\lambda,H} = \alpha_{\lambda,H} \frac{1-X}{(1+4B)m_h} \tag{10}$$

Where X is the fraction of H ionized calculated above (eqn. 1). The log of this value ($\kappa_{\lambda,H}$) is plotted against a range of temperatures in Figure 1 and Figure 2 (colored dark blue) for a certain wavelength value, which I will define below.

2.3 Finding the absorption coefficient for the H^- ion

The equation for the bound-free absorption coefficient:

$$\alpha_{bf}(\lambda,\Theta) = 10^{-26} \times P_e \times 0.4158 \times \Theta^{5/2} \times e^{1.726\Theta}\left(1 - e^{-h\nu/kT}\right)k^\star \tag{11}$$

Where $k^\star$ is defined as:

$$k^\star = 0.00680133 + 0.178708\Lambda + 0.164790\Lambda^2 - 0.024842\Lambda^3 + 5.95244 \times 10^{-4}\Lambda^4 \tag{12}$$

Here, Λ is the wavelength in angstroms. The equation for the free-free absorption coefficient is:

$$\begin{aligned}\alpha_{ff}(\lambda,\Theta) = 10^{-26}P_e \Big[&0.0053666 - 0.011493\,\Theta + 0.027029\,\Theta^2 \\ &- (3.2062 - 11.924\,\Theta + 5.939\,\Theta^2)\left(\frac{\lambda}{10^6}\right) \\ &- (0.40192 - 7.0355\,\Theta + 0.34592\,\Theta^2)\left(\frac{\lambda^2}{10^9}\right)\Big]\end{aligned} \tag{13}$$

The total absorption coefficient for the H^- ion is then defined as:

$$\alpha_{\lambda,H^-} = \alpha_{bf} + \alpha_{ff} \tag{14}$$

To get the mass absorption coefficient per gram, multiply the absorption coefficient per H^- ion by the following multiplier:

$$k_{\lambda,H^-} = \alpha_{\lambda,H^-} \frac{1-X}{(1+4B)m_h} \tag{15}$$

Where X is the fraction of H ionized calculated above (eqn. 1). The log of this value (κ_{λ,H^-}) is plotted against a range of temperatures in Figure 1 and Figure 2 (colored green) for a certain wavelength value, which I will define below.

2.4 Finding the Scattering Coefficients

2.4.1 Rayleigh Scattering

The cross section of hydrogen per neutral hydrogen atoms in the ground state can be calculated with the following equation:

$$\sigma_r = {}^{5.799\times10^{-13}}/_{\Lambda^4} + {}^{1.422\times10^{-6}}/_{\Lambda^6} + {}^{2.784}/_{\Lambda^8} \tag{16}$$

Here, Λ is in angstroms. To get this in terms of mass absorption coefficient per gram, multiply σ_r by the multiplier in equations 10 and 15. This gives the mass absorption coefficient per gram for Rayleigh scattering:

$$\sigma_R = \sigma_r \frac{1-X}{(1+4B)m_h} \tag{17}$$

2.4.2 Thomson Scattering

The equation for Thomson scattering is as follows:

$$\sigma_T = \sigma_t \frac{(X + {}^{Y}/_{A})}{(1+4B)m_H} \tag{18}$$

2.4.3 Total Scattering

The total scattering is defined as:

$$\sigma_{total} = \sigma_R + \sigma_T \tag{19}$$

The log of this value ($log_{10}(\sigma_{total})$) is then plotted against a range of temperatures in Figure 1 and Figure 2 (colored red) for a certain wavelength value, which I will define below.

3 Data and Results

After calculating the values above in the order presented for a single wavelength, $\kappa_{\lambda,H}$, κ_{λ,H^-}, $log_{10}(\sigma_{total})$, as well as the total kappa value ($\kappa_{total} = \kappa_{\lambda,H} + \kappa_{\lambda,H-}$), they are plotted against a range of temperatures for each star.
I used a wavelength of $\lambda = 5000$ angstroms for both stars. The calculated values are plotted in Figures 1 and 2, respectively. The tables of data for both the sun-like star and Vega are located near the end of the paper, in Table 3 and Table 4 respectively. The code I wrote in order to create these tables is located in Section 5.

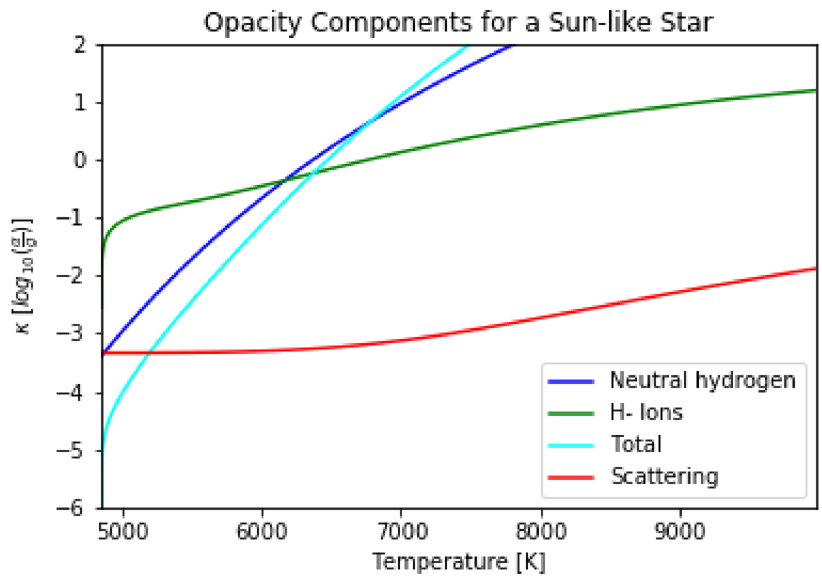

Figure 1: The log of the various kappa components of a sun-like star vs. temperature. This is calculated for a wavelength of $\lambda = 5000$ angstroms. The components seen here are neutral hydrogen in mass absorption coefficient per gram (dark blue), the H- ion in mass absorption coefficient per gram (green), the neutral hydrogen and the H- ion added together (light blue), and finally the log of the total scattering (red).

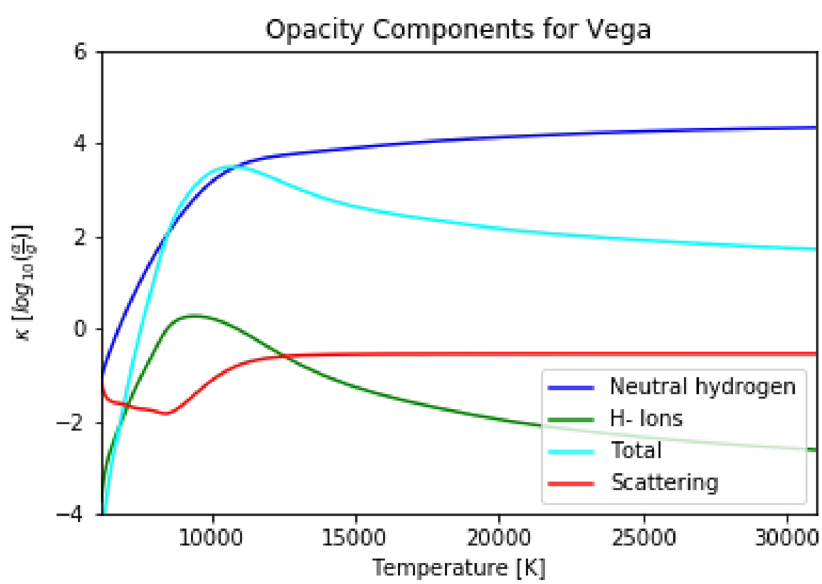

Figure 2: The log of the various kappa components of Vega vs. temperature. This is calculated for a wavelength of $\lambda = 5000$ angstroms. The components seen here are neutral hydrogen in mass absorption coefficient per gram (dark blue), the H- ion in mass absorption coefficient per gram (green), the neutral hydrogen and the H- ion added together (light blue), and finally the log of the total scattering (red).

4 Conclusion

The plots above represent the continuous opacities of a sun-like star and Vega, an A star. The lower the κ value, the more "translucent" the stellar atmosphere is. Looking at Figures 1 and 2, the κ values for neutral hydrogen and ionized hydrogen begin to fall below zero right around 6000 K and 10000 K, respectively. This makes sense considering the effective temperature of the sun is about 5,700 K and the effective temperature of Vega is about 10,000 K. At these temperatures, the spectrum of the sun and Vega peak, indicating that the most light can be seen in these regions of the electromagnetic spectrum. One would expect that if the spectrum of a star peaks at a certain wavelength, then the absorption coefficients should be relatively low.

While my data for the sun-like star appears to be correct, compared to the actual data that Professor Larson provided, the neutral hydrogen values appear to be off by an average of 1.58 $log_{10}(\frac{\alpha}{g})$, while the values for the hydrogen ion and sigma total are almost exact. The reason for this is unknown. It must be something to do with my code, although I have looked over it many times and cannot find the culprit.

This project has shown that I have successfully (for the most part) calculated the absorption coefficients for neutral hydrogen, ionized hydrogen, and the total scattering coefficient for both Vega and a sun-like star. The graphs I produced provide information about how opaque a star is at different temperature values in terms of the absorption coefficient per gram.

Results of the Opacity components of a Sun-Like Star				
Temp [K]	$log_{10}(P_e)$	$log_{10}(H)$	$log_{10}(H-)$	$log_{10}(\sigma_{total})$
4852.0	-0.9857	-3.39203616384	-2.55679820878	-3.32099137095
4852.18	-0.9482	-3.39150709977	-2.51936608103	-3.32186126848
4852.2	-0.9127	-3.3914472258	-2.48387253067	-3.32263494872
4852.23	-0.8787	-3.39135818574	-2.44988297504	-3.32332400827
4852.26	-0.846	-3.39126926727	-2.4171935398	-3.32394316758
4852.3	-0.8144	-3.39115114006	-2.38560805507	-3.32450355606
4852.34	-0.7837	-3.39103309899	-2.35492265435	-3.32501551667
4852.38	-0.7537	-3.39091513044	-2.32493732405	-3.32548724517
4852.43	-0.7244	-3.39076791834	-2.29565590678	-3.32592233495
4852.48	-0.6956	-3.39062076249	-2.26687454246	-3.32632762541
4852.55	-0.6673	-3.39041504251	-2.23860092508	-3.32670512927
4852.62	-0.6395	-3.3902093725	-2.21082735119	-3.32705824053
4852.7	-0.612	-3.38997443918	-2.18335766265	-3.32739126842
4852.78	-0.5848	-3.38973954743	-2.15618800724	-3.32770635428
4852.88	-0.558	-3.38944608822	-2.12942608157	-3.32800336741
4853.0	-0.5314	-3.38909406314	-2.10287187929	-3.32828611103
4853.12	-0.505	-3.38874208053	-2.07651770047	-3.3285564806
4853.26	-0.4788	-3.38833153975	-2.05037124008	-3.32881505254
4853.42	-0.4529	-3.38786245034	-2.02453249657	-3.32906199821
4853.59	-0.4271	-3.38736411516	-1.99879761688	-3.32930055918
4853.79	-0.4014	-3.38677795293	-1.97317429463	-3.32953109142
4854.01	-0.376	-3.3861332764	-1.9478586811	-3.32975292803
4854.26	-0.3506	-3.38540080986	-1.92255461769	-3.32996932368
4854.54	-0.3254	-3.38458058133	-1.89746210278	-3.33017926993
4854.85	-0.3002	-3.38367261841	-1.87238113073	-3.33038518368
4855.2	-0.2752	-3.38264767953	-1.8475155453	-3.33058580993
4855.6	-0.2503	-3.38147654231	-1.82276918567	-3.33078241075
4856.04	-0.2255	-3.38018854415	-1.79813819921	-3.33097578841
4856.54	-0.2008	-3.37872522796	-1.7736302684	-3.33116607332
4857.09	-0.1762	-3.37711595467	-1.74924153856	-3.3313539892
4857.72	-0.1516	-3.37527308487	-1.72488352893	-3.33154021914
4858.42	-0.1271	-3.37322603785	-1.70065238029	-3.33172461158

4859.2	-0.1027	-3.37094576201	-1.67655191728	-3.33190738497
4860.08	-0.0784	-3.36837404721	-1.65258979972	-3.33208853005
4861.07	-0.0541	-3.36548200438	-1.62866983676	-3.33226902576
4862.17	-0.0298	-3.36227002551	-1.60479199177	-3.33244920533
4863.41	-0.0056	-3.35865101403	-1.58106773529	-3.33262794498
4864.8	0.0185	-3.3545964334	-1.55750084459	-3.33280528732
4866.36	0.0427	-3.35004874763	-1.53389891658	-3.33298276665
4868.11	0.0668	-3.34495066705	-1.51046952736	-3.33315846105
4870.07	0.0908	-3.33924518789	-1.48722022572	-3.33333208525
4872.26	0.1149	-3.33287563922	-1.46395852899	-3.333505115
4874.72	0.139	-3.32572765201	-1.440799569	-3.33367593284
4877.47	0.1631	-3.31774557589	-1.41775076985	-3.33384414358
4880.56	0.1873	-3.30878738427	-1.39473095931	-3.3340096431
4884.01	0.2115	-3.29879894336	-1.37184742197	-3.33417100619
4887.88	0.2358	-3.28761133281	-1.34902261179	-3.33432783334
4892.2	0.2602	-3.27514378442	-1.32626741513	-3.33447934801
4897.04	0.2847	-3.26120168844	-1.30360780107	-3.33462387658
4902.46	0.3094	-3.24562157755	-1.28096571981	-3.33476091932
4908.51	0.3342	-3.22827120216	-1.25845909643	-3.33488810141
4915.28	0.3593	-3.20890671117	-1.23592073032	-3.33500525677
4922.84	0.3846	-3.18734560207	-1.21347547029	-3.33510948227
4931.28	0.4102	-3.16335296089	-1.19105528759	-3.33519951546
4940.7	0.4361	-3.13667140153	-1.16869536852	-3.3352727787
4951.2	0.4624	-3.10705065723	-1.14633023723	-3.33532777581
4962.91	0.4892	-3.07416454853	-1.12390484897	-3.3353623512
4975.95	0.5164	-3.03772574537	-1.10155942017	-3.33537231693
4990.46	0.5441	-2.99740340066	-1.07924045672	-3.33535525734
5006.59	0.5724	-2.95285395932	-1.05689676164	-3.33530852898
5024.5	0.6014	-2.90372402192	-1.03447916333	-3.33522922699
5044.37	0.6311	-2.8496264263	-1.01204385702	-3.33511272968
5066.4	0.6616	-2.79014496347	-0.98955195505	-3.33495490453
5090.77	0.6929	-2.72494625356	-0.967054572748	-3.33475055525
5117.7	0.7251	-2.65362212951	-0.94451421254	-3.33449408805
5147.43	0.7584	-2.57575055179	-0.921796825678	-3.33417958038
5180.18	0.7928	-2.49100534981	-0.898957082831	-3.3337975328
5216.2	0.8284	-2.39902984228	-0.875955707302	-3.3333362497
5255.75	0.8655	-2.29949569939	-0.852551704971	-3.33278222104
5299.09	0.9043	-2.19213318184	-0.82859837585	-3.33211478136
5346.48	0.945	-2.07673458166	-0.803942930697	-3.3313045084
5398.2	0.9881	-1.95310973627	-0.77813279154	-3.3303132233
5454.51	1.034	-1.82118581992	-0.750801748744	-3.32908524392
5515.68	1.0834	-1.68093607387	-0.721279854448	-3.32754733889
5581.98	1.1369	-1.53240511169	-0.688989855182	-3.32559663039
5653.66	1.1954	-1.37575182612	-0.653040858466	-3.32310379729
5730.97	1.2594	-1.21120037563	-0.612935048473	-3.31989021244
5814.15	1.3296	-1.03905793728	-0.567964802325	-3.31573665937
5903.42	1.4063	-0.859727920419	-0.517810303193	-3.31035928642
5998.98	1.4896	-0.673699198556	-0.462340494744	-3.30340444827
6101.03	1.5795	-0.481497296785	-0.401519790348	-3.29444413816
6209.76	1.6755	-0.283696329204	-0.335805922878	-3.28293245596
6325.32	1.7771	-0.080961161501	-0.265642763986	-3.26823243748
6447.86	1.8835	0.126038116812	-0.19177288323	-3.24956944074
6577.52	1.9942	0.336616555772	-0.114635571595	-3.22612867965
6714.42	2.1084	0.55006781071	-0.0349656183878	-3.19700341369

6858.68	2.2253	0.765704890034	0.0465004008596	-3.16127286405
7010.39	2.3444	0.982822631265	0.129330622532	-3.11820394017
7169.66	2.4649	1.20077681327	0.212784072424	-3.06713187539
7336.58	2.5863	1.41892969105	0.296419795482	-3.00776956078
7511.23	2.7083	1.63666990921	0.379993278538	-2.94028705187
7693.7	2.8302	1.85343766948	0.462849500519	-2.86495728111
7884.09	2.9517	2.06872274707	0.54472345485	-2.78253909669
8082.48	3.0724	2.28202789481	0.625243883336	-2.69394369439
8288.96	3.1921	2.49290316125	0.704226328331	-2.60031747765
8503.65	3.3105	2.70095762634	0.781366143188	-2.50271257072
8726.63	3.4273	2.90578583677	0.856349951071	-2.40214443323
8958.02	3.5422	3.10704005935	0.928836354646	-2.29947852977
9197.96	3.6551	3.30440258427	0.998659927544	-2.19561873231
9446.55	3.7657	3.49752454708	1.06543211821	-2.09120813474
9703.94	3.8738	3.68609522698	1.12882311569	-1.9868330894
9971.29	3.9791	3.87044224952	1.18816900878	-1.88216709352

Table 3: Results from my continuous opacity model for a sun-like star. Temperature in Kelvin, the log of the electron pressure in $dynes/cm^2$, the log of the neutral hydrogen absorption coefficient per gram, the log of the H- ion absorption coefficient per gram, and the log of the total scattering per gram for H.

Results of the Opacity components of Vega				
Temp [K]	$log_{10}(P_e)$	$log_{10}(H)$	$log_{10}(H-)$	$log_{10}(\sigma_{total})$
6176.7	-1.606775884	-1.0912273429	-3.73144650306	-1.15337289374
6203.1	-1.478992748	-1.02625210672	-3.59347717625	-1.21141801056
6223.7	-1.345438445	-0.971023890757	-3.44713730844	-1.28574217396
6253.7	-1.207048292	-0.904421044865	-3.30347477321	-1.35197116782
6289.0	-1.066057397	-0.83196276254	-3.16147319689	-1.41404660798
6331.3	-0.924818145	-0.751022340873	-3.02390400069	-1.46680434848
6379.5	-0.785156152	-0.663179382792	-2.89150111305	-1.51022798148
6433.3	-0.648590248	-0.568966051212	-2.76526169494	-1.54344515789
6491.4	-0.515985037	-0.470450812558	-2.64517368872	-1.56789255723
6552.8	-0.387534036	-0.369225658571	-2.53078891251	-1.58513540752
6616.0	-0.263603498	-0.267588375656	-2.42175455721	-1.59732436957
6679.9	-0.143815073	-0.167099613168	-2.31715795431	-1.60654151921
6744.1	-0.027889772	-0.0682733220266	-2.21652002945	-1.61362933683
6807.8	0.084576278	0.0278797549995	-2.11909346097	-1.62009581111
6870.5	0.19368103	0.120792307649	-2.02460009619	-1.62666437147
6931.7	0.300160537	0.20998768759	-1.9320858621	-1.63459188846
6991.2	0.404149249	0.295357772832	-1.84136967722	-1.64413741785
7049.0	0.50609896	0.377076967124	-1.75200383095	-1.65556381799
7105.8	0.606058749	0.456152296928	-1.66421494181	-1.66782420868
7162.0	0.704150517	0.533171093401	-1.57802879014	-1.68040109261
7218.1	0.800373355	0.608803470302	-1.49362364036	-1.69256482851
7274.4	0.894758994	0.683447173485	-1.41106424805	-1.70391544215
7331.5	0.98708503	0.75781794555	-1.33077422377	-1.71345490363
7389.8	1.077367905	0.832373475938	-1.25284686101	-1.72070161793
7448.9	1.165541077	0.906616129391	-1.17721311741	-1.72613624537
7508.1	1.25163822	0.979741500675	-1.10361638616	-1.73067448146

7566.9	1.335858911	1.05122643452	-1.03169660862	-1.73508871735
7625.8	1.418632687	1.12169381956	-0.961152172381	-1.7391214469
7683.5	1.499824496	1.18974587662	-0.891751741894	-1.7441764878
7741.6	1.57989787	1.25720044596	-0.82347812428	-1.7487176072
7798.8	1.658964843	1.32269797581	-0.755844573815	-1.7543499007
7855.6	1.737113094	1.38682339909	-0.688911241432	-1.76051985128
7911.9	1.814513952	1.44952028106	-0.622477361757	-1.76745980736
7967.6	1.891258617	1.51073080201	-0.556428886145	-1.7753247702
8022.6	1.967501175	1.57040370327	-0.490587085288	-1.78432371209
8077.7	2.043362278	1.62937702852	-0.425046336025	-1.7936534599
8132.7	2.118925753	1.68747060918	-0.359669483674	-1.80359454335
8188.6	2.193958978	1.74565284487	-0.294960796732	-1.81285387766
8244.4	2.268811904	1.8029722162	-0.230294902304	-1.82282081944
8301.9	2.343408594	1.86113308828	-0.16620119174	-1.8315445051
8362.6	2.417471693	1.92146426288	-0.103321953291	-1.83728983512
8427.8	2.490800952	1.9850345363	-0.042152975799	-1.83867890684
8501.4	2.562887381	2.05514167923	0.0158741189438	-1.8315600926
8583.0	2.633367445	2.13102253214	0.0705162207241	-1.81637690433
8674.6	2.70182693	2.21394278998	0.120881025399	-1.79125189095
8780.0	2.767971721	2.3064222513	0.165737082271	-1.75297062573
8900.5	2.831357785	2.40845211933	0.204204933074	-1.70091130819
9038.7	2.891760401	2.52068091114	0.235176211394	-1.63386317903
9198.7	2.948852906	2.64417934077	0.256778935613	-1.54993193302
9385.5	3.002597981	2.77947624837	0.266671044123	-1.44796459708
9605.1	3.053078443	2.92588929058	0.261356227927	-1.32807625224
9858.3	3.101059355	3.07733966532	0.238169667073	-1.19710007056
10153.7	3.147367108	3.22959195944	0.190210519645	-1.05936452082
10494.3	3.193402903	3.37262773016	0.111231287189	-0.926267116778
10887.7	3.241297387	3.49764746512	-0.00586589465361	-0.808333893611
11340.3	3.293362555	3.59776984236	-0.16399524357	-0.714585590576
11862.5	3.353531559	3.67363713606	-0.358447324943	-0.647845618011
12456.6	3.424718337	3.7313253413	-0.574056405439	-0.605372785521
13138.2	3.509605705	3.780048697	-0.799824232395	-0.579894668995
13903.0	3.609807769	3.82838041938	-1.01539486164	-0.565607327573
14763.7	3.723537762	3.87916347029	-1.21857482855	-0.557559910837
15701.6	3.847634344	3.93408897283	-1.3974302657	-0.553133257419
16731.4	3.976991545	3.98927618051	-1.56269768187	-0.550556484798
17847.4	4.107888025	4.04282700526	-1.71470168577	-0.549032161805
19064.7	4.236033147	4.09044982035	-1.86372396355	-0.548083633648
20384.7	4.361916619	4.13337330964	-2.00564394301	-0.54749212351
21829.7	4.490520309	4.17597059247	-2.13344105171	-0.547120587245
23387.8	4.619302076	4.21713625238	-2.24581489587	-0.546884508681
25080.2	4.744292983	4.25236607329	-2.3530814893	-0.546725194672
26903.2	4.866228247	4.28321740145	-2.45047976461	-0.546617931695
28880.3	4.985067033	4.30909213655	-2.54012938691	-0.546543372764
31003.8	5.100370545	4.33049870019	-2.61956211733	-0.546491378275

Table 4: Results from my continuous opacity model for Vega. Temperature in Kelvin, the log of the electron pressure in $dynes/cm^2$, the log of the neutral hydrogen absorption coefficient per gram, the log of the H- ion absorption coefficient per gram, and the log of the total scattering per gram for H.

5 Python Opacity Model

```python
import numpy as np
import matplotlib.pyplot as plt
import astropy.units as u
from tabulate import tabulate

#Definining a function to make units easier:
def SI(l, units):
    SI_I = (l * units).decompose()
    return SI_I.value

##DEFINING CONSTANTS
#Constants:
T_test = 5730.97 #Kelvin
P_test = 1.2594 #log(P)
T_test_vega = 6744.1
P_test_vega = -0.027889772
B = 0.1 #ratio of the #He/#H
A = 10**4 #ratio of #H/#metal
sigma_t = 6.655*10**(-25) #cm^2 -- thompson
m_h = 1.6726219*10**(-24) #grams -- mass of ionized hydrogen, eg. a proton
c = SI(29979245800, u.cm/u.s) #cm / s -- speed of light
h = 6.62606896*10**(-27) #ergs * s -- Planck's constant
k = 1.38065*10**(-16) #ergs / Kelvin -- Boltzmann constant
Chi = 2.195*10**(-11) #ergs -- ionization energy of hydrogen

#Constants for the sun-like star:
L = SI(5000, u.AA) #angstroms
l = ((L*u.m).to(u.micron)).value #microns
v = c/(L) # s^-1 -- frequency associated with L

#Constants for Vega:
L_vega = SI(7000, u.AA) #angstroms -- I will consider this wavelength for Vega
l_vega = ((L_vega*u.m).to(u.micron)).value
v_vega = c/(L_vega)

##READING INPUT DATA
#input data - sun-like star:
check_ans = np.genfromtxt('opacities.dat', dtype='f8', names = ['temp', 'log_P', 'logH', 'logHminus', 'logsigma'])
temp_pelog_dat = np.genfromtxt('temp_pelog.dat', dtype='f8', names = ['temp','log_P' ])
temp = check_ans['temp']
pe_log = check_ans['log_P']

#ANSWERS for sun-like star opacities
check_H = check_ans['logH']
check_Hminus = check_ans['logHminus']
check_sigma = check_ans['logsigma']

#input data - vega:
opacities_vega = np.genfromtxt('vega-atmos-grid.dat', dtype='f8', names = ['temp', 'P'])
temp_vega = opacities_vega['temp']
p_vega = opacities_vega['P']

##BEGINNING CALCULATIONS
#Defining theta:
def Theta(T):
    theta = 5040./T
    return theta

#Defining the Saha equation for hydrogen. Returns the ratio of n/n:
def Saha_H(T, P):
    Saha_H_ans =  (5.0/2.0)*np.log10(T) - 13.595*(Theta(T)) - P - 0.4772
    return 10**Saha_H_ans

#Defining the Saha equation for metals. Returns the ratio of n/n:
```

```
def Saha_metals(T, P):
    Saha_metals_ans = (5.0/2.0)*np.log10(T) - 7.9*(Theta(T)) - P - 0.0971
    return 10**Saha_metals_ans

#Defining X = fraction of H ionized:
def X(H_ratio):
    X_ans = H_ratio / (1 + H_ratio)
    return X_ans

#Defining Y = faction of metal ionized:
def Y(m_ratio):
    Y_ans = m_ratio / (1 + m_ratio)
    return Y_ans

#Sun - H/m_ratio for X and Y:
H_ratio = Saha_H(temp, pe_log)
m_ratio = Saha_metals(temp, pe_log)

#Vega - H/m_ratio for X and Y:
H_ratio_vega = Saha_H(temp_vega, p_vega)
m_ratio_vega = Saha_metals(temp_vega, p_vega)

#Defining Gaunt free-free factor:
def Gaunt_ff(T,l):
    Gaunt_ff_ans = 1.084 + 0.0188/Theta(T) + (0.00161 + (0.02661/Theta(T)))*l - (0.0192 -
    (0.03889/Theta(T)) + (0.02833/((Theta(T))**2)) - (0.007828/((Theta(T))**3)) +
    (0.0007304/((Theta(T))**4)))*(l**2)
    return Gaunt_ff_ans

#Defining coefficients for a, b, c for the Gaunt bound-free factor:
m = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
a_m = [0.9916, 1.105, 1.101, 0.9736, 1.03, 1.097, 1.098, 1., 1., 1.]
b_m = [0.09068, -0.7922, -0.329, 0., 0., 0., 0., 0., 0., 0.]
c_m = [-0.2524, 0.4536, 0.1152, 0., 0., 0., 0., 0, 0., 0.]

#Defining Gaunt bound-free factor:
def Gaunt_bf(l,m):
    Gaunt_bf_ans = a_m[m-1] + b_m[m-1]*l + c_m[m-1]*(l**2)
    return Gaunt_bf_ans

#Defining u, used in the absorption coefficient equation:
def u(T, m):
    u_ans = (Chi/(k*T))/np.power(m, 2)
    return u_ans

#Finding m_o and m_star - for the sun-like star and Vega.
test_m = (h * v) / (k*T_test)
for i in m:
    if test_m <= u(T_test,i):
        continue
    if test_m >= u(T_test, i):
        m_0 = i
        break
m_star =  m[9]

#Defining a new array of m's
new_m = np.arange(m_0, m_star + 1, 1)

#Doing the summation over new m's for the sun-like star
summ = []
for i in new_m:
    factor = Gaunt_bf(l, i) * (np.exp(u(temp, i))/(i**3))
    summ.append(factor)
summ = np.array(summ)
summ = np.sum(summ)

#Doing the summation over new m's for Vega
summ_vega = []
```

```python
for i in new_m:
    factor_vega = Gaunt_bf(l, i) * (np.exp(u(temp_vega, i))/(i**3))
    summ_vega.append(factor_vega)
summ_vega = np.array(summ_vega)
summ_vega = np.sum(summ_vega)

#Atomic hydrogen absorption coefficient per neutral hydrogren: alpha_lambda
def neutral_H(summ,T,l,v,m_star):
    one = ((2.0898*10**(-14.))* (l**3) * np.exp(-1. * u(T,1))) * (1./2.)
    two = 1. - np.exp(-(h*v)/(k*T))
    three = (1./(2*u(T,1))) * (np.exp(u(T,m_star))) - 1. + Gaunt_ff(T,l)
    answer = one * two * (summ + three)
    return answer

#mass absorption coefficient per gram - for the sun-like star
multiplier = (1.-X(H_ratio))/((1.+(4.*B))*m_h)
kappa_atomic = np.log10(neutral_H(summ, temp, l, v, m_star)*multiplier)

#mass absorption coefficient per gram - for Vega
multiplier_vega = (1.-X(H_ratio_vega))/((1.+(4.*B))*m_h)
kappa_atomic_vega = np.log10(neutral_H(summ_vega, temp_vega, l, v, m_star)*multiplier_vega)

#Defining k^* for a wavelength of 5000 Angstroms
k_star = 0.00680133 + 0.178708*(5.) + 0.164790*(5.**2) - 0.024842*(5.**3) + (5.**4)
    *5.95244*10**(-4)

#Bound-free absorption coefficient
def absorp_bf(P,T,v,k_star):
    absorp_bf_ans = (10.**(-26)) * (10.**P) * 0.4158*((Theta(T))**(5./2.)) * np.exp(1.726*(
    Theta(T))) * (1-np.exp((-h*v)/(k*T))) * k_star
    return absorp_bf_ans
#Free-free absorption coefficient
def absorp_ff(P,T,l):
    absorp_ff_ans = (10.**(-26)) * (10.**P) * (0.0053666 - 0.011493*(Theta(T)) + 0.027029*((
    Theta(T))**(2)) - (3.2062 - 11.924*(Theta(T)) + 5.939*((Theta(T))**(2)))*(l/(10**6)) -
    (0.40192 - 7.0355*(Theta(T)) + 0.34592*((Theta(T))**(2)))*((l**2)/(10**9)))
    return absorp_ff_ans

#ionized hydrogen absorption coefficient for the sun-like star
absorp_H_neg = absorp_bf(pe_log, temp, v, k_star) + absorp_ff(pe_log, temp, l)
kappa_ion = np.log10(absorp_H_neg*multiplier)
kappa_total = kappa_atomic + kappa_ion

#ionized hydrogen absorption coefficient for Vega
absorp_H_neg_vega = absorp_bf(p_vega, temp_vega, v, k_star) + absorp_ff(p_vega, temp_vega, l
    )
kappa_ion_vega = np.log10(absorp_H_neg_vega*multiplier_vega)
kappa_total_vega = kappa_atomic_vega + kappa_ion_vega

#Calculating rayleigh scattering cross section, l is in angstroms
def cross_section_r(l):
    sigma_r_ans = (5.799*10**(-13))/(l**4) + 1.422*10**(-6)/(l**6) + 2.784/(l**8)
    return sigma_r_ans

#Rayleigh scattering for the sun-like star
sigma_R_ang = cross_section_r(5000)*multiplier

#Rayleigh scattering for Vega
sigma_R_ang_vega = cross_section_r(5000)*multiplier_vega

#Calculating Thompson scattering for the sun-like star
sigma_T = sigma_t * ((X(H_ratio) + Y(m_ratio)/A)/((1.+4.*B)*m_h))

#Calculating total scattering for the sun-like star
sigma_total = sigma_R_ang + sigma_T

#Calculating Thompson scattering for Vega
sigma_T_vega = sigma_t * ((X(H_ratio_vega) + Y(m_ratio_vega)/A)/((1.+4.*B)*m_h))
```

```

#Calculating total scattering for Vega
sigma_total_vega = sigma_R_ang_vega + sigma_T_vega

#print 'Official ANSWERS'
#print tabulate(check_ans, headers = ['Temp [K]', 'Log10(P_e)', 'Log10(H)', 'Log10(H-)', 'Total Scatter'])
#print 'My values'
#print tabulate(my_ans, headers = ['Temp [K]', 'Log10(P_e)', 'Log10(H)', 'Log10(H-)', 'Total Scatter'])

#Writing my values to files to read in and for Sharelatex
f = open('my_answers.dat', 'w')
f_vega = open('my_answers_vega.dat','w')
write_kappa_atomic = kappa_atomic
write_kappa_ion = kappa_ion
write_sigma_total = np.log10(sigma_total)

write_kappa_atomic_vega = kappa_atomic_vega
write_kappa_ion_vega = kappa_ion_vega
write_sigma_total_vega = np.log10(sigma_total_vega)

for i in range(len(check_ans['temp'])):
    f.write(str(check_ans['temp'][i]) + ' ' + str(check_ans['log_P'][i]) + ' ' + str(write_kappa_atomic[i]) + ' ' + str(write_kappa_ion[i]) + ' ' + str(write_sigma_total[i]) + ' \n')
f.close()

for i in range(len(opacities_vega['temp'])):
    f_vega.write(str(opacities_vega['temp'][i]) + ' ' + str(opacities_vega['P'][i]) + ' ' + str(write_kappa_atomic_vega[i]) + ' ' + str(write_kappa_ion_vega[i]) + ' ' + str(write_sigma_total_vega[i]) + ' \n')
f_vega.close()


#For Sharelatex
latex = open('latex.dat', 'w')
for i in range(len(check_ans['temp'])):
    latex.write(str(check_ans['temp'][i]) + ' & ' + str(check_ans['log_P'][i]) + ' & ' + str(write_kappa_atomic[i]) + ' & ' + str(write_kappa_ion[i]) + ' & ' + str(write_sigma_total[i]) + '\\\ \n')
latex.close()

latex_vega = open('latex_vega.dat', 'w')
for i in range(len(opacities_vega['temp'])):
    latex_vega.write(str(opacities_vega['temp'][i]) + ' & ' + str(opacities_vega['P'][i]) + ' & ' + str(write_kappa_atomic_vega[i]) + ' & ' + str(write_kappa_ion_vega[i]) + ' & ' + str(write_sigma_total_vega[i]) + '\\\ \n')
latex_vega.close()

#Reading my answers in to print
my_ans = np.genfromtxt('my_answers.dat', dtype='f8', names = ['temp', 'log_P', 'logH', 'logHminus', 'logsigma'])

#Plotting opacity values for a sun-like star
plt.plot(temp, kappa_atomic, color = 'blue', label = 'Neutral hydrogen')
plt.plot(temp, kappa_ion, color = 'green', label = 'H- Ions')
plt.plot(temp, kappa_total, color = 'cyan', label = 'Total')
plt.plot(temp, np.log10(sigma_total), color = 'red', label = 'Scattering')
plt.xlim(np.min(temp), np.max(temp))
plt.ylim(-6, 2)
plt.legend(loc=4, prop={'size':10})
plt.xlabel('Temperature [K]')
plt.ylabel(r'$\kappa \,\, [log_{10}(\frac{\alpha}{g})]$')
plt.title('Opacity Components for a Sun-like Star')
plt.savefig('Sun_opacity.png')
plt.show()

```

```python
#Plotting opacity values for Vega
plt.plot(temp_vega, kappa_atomic_vega, color = 'blue', label = 'Neutral hydrogen')
plt.plot(temp_vega, kappa_ion_vega, color = 'green', label = 'H- Ions')
plt.plot(temp_vega, kappa_total_vega, color = 'cyan', label = 'Total')
plt.plot(temp_vega, np.log10(sigma_total_vega), color = 'red', label = 'Scattering')
plt.xlim(np.min(temp_vega), np.max(temp_vega))
plt.ylim(-4, 6)
plt.legend(loc=4, prop={'size':10})
plt.xlabel('Temperature [K]')
plt.ylabel(r'$\kappa \,\, [log_{10}(\frac{\alpha}{g})]$')
plt.title('Opacity Components for Vega')
plt.savefig('Vega_opacity.png')

#Plotting official results
plt.plot(temp, check_H, color = 'blue', label = 'Neutral hydrogen')
plt.plot(temp, check_Hminus, color = 'green', label = 'H- Ions')
plt.plot(temp, check_H + check_Hminus, color = 'cyan', label = 'Total')
plt.plot(temp, check_sigma, color = 'red', label = 'Scattering')
plt.xlim(np.min(temp), 10000)
plt.ylim(-6, 6)
plt.legend(loc=4, prop={'size':10})
plt.xlabel('Temp (K)')
plt.ylabel(r'$Kappa $')
plt.title('Opacity Components for a Sun-like Star -- ANSWER')
delta_H = kappa_atomic - check_H
delta_Hion = kappa_ion - check_Hminus
delta_sigma = np.log10(sigma_total) - check_sigma

#Check average change in H values, they are off by about this much
print 'Your neutral H values are off by ', np.sum(delta_H)/len(delta_H)
print 'Your H- values are off by ', np.sum(delta_Hion)/len(delta_Hion)
print 'Your sigma values are off by ', np.sum(delta_sigma)/len(delta_sigma)
```

References

[1] Dimitri Mihalas. *The Calculation of Model Stellar Atmospheres*. Methods in Computational Physics, Vol. 7, 1967.