

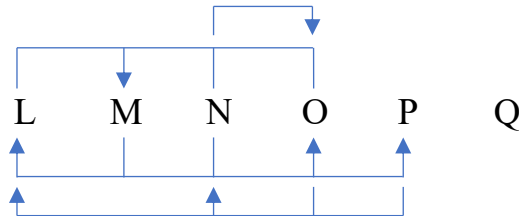
## Part C: Functional Dependencies

### C-1 Transitive Dependency and Keys

You have a relation  $R(L,M,N,O,P,Q)$  and a set of functional dependencies  $F = \{LNO \rightarrow M, MN \rightarrow LO, N \rightarrow O, OP \rightarrow LN\}$ .

- [2pt] Can we infer  $NP \rightarrow LM$  from  $F$  ?
- [3pt] Can we infer  $NQ \rightarrow LO$  from  $F$  ?

**Solution:**



Iterations:

$X = \{N, P, O\}$       Use:  $N \rightarrow O$

$X = \{N, P, O, L\}$       Use:  $OP \rightarrow LN$

$X = \{N, P, O, L, M\}$       Use:  $LNO \rightarrow M$

$X = \{N, P, O, L, M\}$       Use:  $MN \rightarrow LO$

$X = \{N, P, O, L, M\}$       No more changes to  $X$  are possible so  $X = \{N, P\}^+$

So, we can infer  $NP \rightarrow LM$  from  $F$ .

Iterations:

$X = \{N, Q, O\}$       Use:  $N \rightarrow O$

$X = \{N, Q, O\}$       No more changes to  $X$  are possible so  $X = \{N, Q\}^+$

So, we cannot infer  $NQ \rightarrow LO$  from  $F$ .

### C-2 Keys

(i) [5pt] Find **all** the candidate keys of the Relation  $R(ABCDE)$  with FD's:

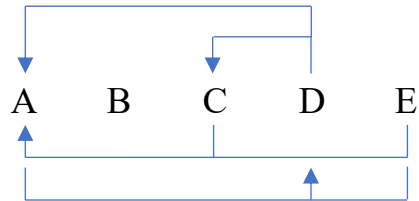
$D \rightarrow C$ ,  $CE \rightarrow A$ ,  $D \rightarrow A$ , and  $AE \rightarrow D$

(ii) [5pt] Determine **all** the candidate and superkeys of the relation  $R(ABCDEF)$  with FD's:

$AEF \rightarrow C$ ,  $BF \rightarrow C$ ,  $EF \rightarrow D$ , and  $ACDE \rightarrow F$

### Solution:

i.



There is no incoming arrow to B and E, so BE cannot be determined by others. It has to be on the left side.

$BE \rightarrow BE$

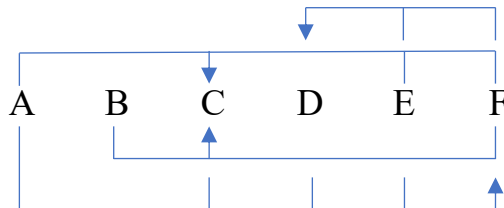
$BEA \rightarrow BEADC$

$BEC \rightarrow BECAD$

$BED \rightarrow BEDCA$

So, the candidate keys are BEA, BEC, BED.

ii.



There is no incoming arrow to A, B and E, so ABE cannot be determined by others. It has to be on the left side.

$ABE \rightarrow ABE$

$ABEC \rightarrow ABEC$

$ABED \rightarrow ABED$

$ABEF \rightarrow ABEFDC$

$ABECD \rightarrow ABECDF$

$ABECF \rightarrow ABECFD$

$ABEDF \rightarrow ABEDFC$

$ABECDF \rightarrow ABECDF$

So, the candidate keys are ABEF and ABECD. Superkeys are ABEF, ABECD, ABEFC, ABEFD, ABECDF.

### C-3 Minimal Cover

[5pt] Find a minimal cover for the following set F of functional dependencies.

$A \rightarrow BC$

$AB \rightarrow D$

$C \rightarrow AD$

$D \rightarrow B$

Show your working clearly. Points will be deducted if you do not show the extraneous attributes, and their elimination.

#### Solution:

| F   | F' <sub>0</sub>   | F' <sub>1</sub>  | F' <sub>2</sub>   | F' <sub>3</sub>   | F' <sub>4</sub>   | F' <sub>5</sub>   |
|---|---|--|---|---|---|---|
|   |   | Is $A \rightarrow B$ ?   | Is $A \rightarrow C$ ?  | Is $C \rightarrow A$ ?  | Is $C \rightarrow D$ ?  | Is $D \rightarrow B$ ?  |
| $A \rightarrow BC$<br>$AB \rightarrow D$<br>$C \rightarrow AD$<br>$D \rightarrow B$ | $A \rightarrow B$<br>$A \rightarrow C$<br>$AB \rightarrow D$<br>$C \rightarrow A$<br>$C \rightarrow D$<br>$D \rightarrow B$ | $A \rightarrow C$<br>$AB \rightarrow D$<br>$C \rightarrow A$<br>$C \rightarrow D$<br>$D \rightarrow B$ | $AB \rightarrow D$<br>$C \rightarrow A$<br>$C \rightarrow D$<br>$D \rightarrow B$ | $A \rightarrow C$<br>$AB \rightarrow D$<br>$C \rightarrow D$<br>$D \rightarrow B$ | $A \rightarrow C$<br>$AB \rightarrow D$<br>$C \rightarrow A$<br>$D \rightarrow B$ | $A \rightarrow C$<br>$AB \rightarrow D$<br>$C \rightarrow A$<br>$C \rightarrow D$ |
|   |   | $A \rightarrow C \rightarrow$<br>$D \rightarrow B$   | NO  | NO  | NO  | NO  |

$F'_5 = \{A \rightarrow C, AB \rightarrow D, C \rightarrow A, C \rightarrow D, D \rightarrow B\}$ . For  $AB \rightarrow D$  we can easily tell  $A \rightarrow B$ , so it's  $A \rightarrow D$ . But  $A \rightarrow D$  can be derived from  $A \rightarrow C$  and  $C \rightarrow D$ .

Finally,  $F_c = \{A \rightarrow C, C \rightarrow AD, D \rightarrow B\}$

### C-4 Equivalence (15 points)

[10pt] Consider the following set of F.Ds. Determine if FD1 is equivalent to FD2 or to FD3:

FD1:

$\{BC \rightarrow D, ACD \rightarrow B, CG \rightarrow B, CG \rightarrow D, AB \rightarrow C, C \rightarrow A, D \rightarrow E, BE \rightarrow C, D \rightarrow G, CE \rightarrow A, CE \rightarrow G\}$

FD2:

$\{AB \rightarrow C, C \rightarrow A, BC \rightarrow D, CD \rightarrow B, D \rightarrow E, D \rightarrow G, BE \rightarrow C, CG \rightarrow D\}$

FD3:

$\{AB \rightarrow C, C \rightarrow A, D \rightarrow G, BE \rightarrow C, CG \rightarrow D, CE \rightarrow G, BC \rightarrow D, CD \rightarrow B, D \rightarrow E\}_s$

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FD1<sub>FD1</sub>:  $(BC)^+ = \text{BCADEC}$ ,  $(ACD)^+ = \text{ACDBEG}$ ,  $(CG)^+ = \text{CGBDAE}$ ,  $(AB)^+ = \text{ABCDEG}$ ,  $C^+ = \text{CA}$ ,  $D^+ = \text{DEG}$ ,  $(BE)^+ = \text{BECADG}$ ,  $(CE)^+ = \text{CEAGBD}$

FD1<sub>FD2</sub>:  $(BC)^+ = \text{BCADEC}$ ,  $(ACD)^+ = \text{ACDEGB}$ ,  $(CG)^+ = \text{CGADEB}$ ,  $(AB)^+ = \text{ABCDEG}$ ,  $C^+ = \text{CA}$ ,  $D^+ = \text{DEG}$ ,  $(BE)^+ = \text{BECADG}$ ,  $(CE)^+ = \text{CEA}$

FD1<sub>FD3</sub>:  $(BC)^+ = \text{BCADEC}$ ,  $(ACD)^+ = \text{ACDEGB}$ ,  $(CG)^+ = \text{CGADEB}$ ,  $(AB)^+ = \text{ABCDEG}$ ,  $C^+ = \text{CA}$ ,  $D^+ = \text{DGE}$ ,  $(BE)^+ = \text{BECADG}$ ,  $(CE)^+ = \text{CEAGDB}$

FD2<sub>FD2</sub>:  $(AB)^+ = \text{ABCDEG}$ ,  $C^+ = \text{CA}$ ,  $(BC)^+ = \text{BCADEC}$ ,  $(CD)^+ = \text{CDBAEG}$ ,  $D^+ = \text{DEG}$ ,  $(BE)^+ = \text{BECADG}$ ,  $(CG)^+ = \text{CGADEB}$

FD2<sub>FD1</sub>:  $(AB)^+ = \text{ABCDEG}$ ,  $C^+ = \text{CA}$ ,  $(BC)^+ = \text{BCDEGA}$ ,  $(CD)^+ = \text{CDAEGB}$ ,  $D^+ = \text{DEG}$ ,  $(BE)^+ = \text{BECADG}$ ,  $(CG)^+ = \text{CGBDEA}$

FD3<sub>FD3</sub>:  $(AB)^+ = \text{ABCDEG}$ ,  $C^+ = \text{CA}$ ,  $D^+ = \text{DEG}$ ,  $(BE)^+ = \text{BECADG}$ ,  $(CG)^+ = \text{CGADEB}$ ,  $(CE)^+ = \text{CEAGDB}$ ,  $(BC)^+ = \text{BCDEGA}$ ,  $(CD)^+ = \text{CDAEGB}$ ,  $D^+ = \text{DEG}$

FD3<sub>FD1</sub>:  $(AB)^+ = \text{ABCDEG}$ ,  $C^+ = \text{CA}$ ,  $D^+ = \text{DEG}$ ,  $(BE)^+ = \text{BECADG}$ ,  $(CG)^+ = \text{CGDEAB}$ ,  $(CE)^+ = \text{CEAGBD}$ ,  $(BC)^+ = \text{BCDEGA}$ ,  $(CD)^+ = \text{CDAEGB}$ ,  $D^+ = \text{DEG}$

$(CE)^+$  in FD1 doesn't have the same closure as that in FD2. So FD1 and FD2 are not equivalent. FD1 and FD3 have the same search power. So FD1 is equivalent to FD3.