Jagged stock investment strategy

Loc Nguyen
Independent scholar, Vietnam
Email: ng_phloc@yahoo.com
Homepage: www.locnguyen.net

Abstract
There are many investment ways such as bank depositing, enterprise business, and stock investment. Bank depositing is a safe and easy way to method and hence, it is known as reference tool to compare or make decision on other investment methods. Alternately, stock investment is preferred method with good feeling about its preeminence. However, according to mathematical model, stock investment and bank depositing have the same benefit if their growth rate and interest rate are the same. Therefore, I propose a so-called jagged stock investment (JSI) strategy in which the chain of buying stock in the given time interval is modeled as a saw with expectation that JSI strategy gets frequently profitable.
Keywords: stock investment, bank depositing, share, return-on-investment.

1. Stock investment
Let X_0 be the beginning price of a given share *A* and let *X* be the late price of *A* after a time interval *t*. Let *D* be the dividend of *A* after the time interval *t*.

$$X_0 > 0$$
$$X \geq 0$$
$$D \geq 0$$

Of course, the value of *A* after the time interval *t* is:

$$Y = X + D \tag{2.1}$$

Let *p* be price-bias ratio or interest rate:

$$p = \frac{X + D - X_0}{X_0} = \frac{Y - X_0}{X_0} \tag{2.2}$$

Let *l* be financial leverage ratio which supports investors.

$$0 < l < 1$$

If $l = 0$, there is no leverage support. Actually, an investor buys share *A* with payment lX_0. As a result, the smaller the leverage ratio *l* is, the better the investor is supported. The return-on-investment (ROI) received by the investor with support of the leverage ratio *l* after the time interval *t* is:

$$r = \frac{X + D - X_0 - lX_0}{lX_0} = \frac{1}{l}\left(\frac{X + D - X_0}{X_0}\right) - 1$$

This implies:

$$r = \frac{1}{l}\left(\frac{Y - X_0}{X_0}\right) - 1 = \frac{p}{l} - 1 \tag{2.3}$$

If $r > 0$, the investor gets profit. If $r = 0$, the investor breaks even. If $r < 0$, the investor gets loss. The *r* is also called profit ratio. According to equation 2.3, if *l* is much smaller than *p* ($l \ll$ p), the leverage ratio *l* affects seriously on the ROI *r*. In other words, if *l* is small enough, the investor will get much profit with a small amount of invested money (lX_0) when share *A* goes up ($Y - X_0 > 0$), but he will also lose much profit when share *A* goes down ($Y - X_0 < 0$) even though the price deviation ($Y – X_0$) is small. However, the leverage ratio l always helps the investor to save invested money because he only pays the amount lX_0 for share A with price X_0 where $lX_0 < X_0$.

Let x be growth rate of A and let d be ratio of D to X, we have:

$$\begin{aligned} X &= xX_0 \\ D &= dX_0 \end{aligned} \quad (2.4)$$

Let y be entire growth rate of A, we have:

$$\begin{aligned} y &= x + d \\ Y &= yX_0 \end{aligned} \quad (2.5)$$

The ROI r is rewritten as follows:

$$r = \frac{y - 1}{l} - 1 \quad (2.6)$$

The investor receives profit if $r > 0$, which implies:

$$r = \frac{1}{l}\left(\frac{Y - X_0}{X_0}\right) - 1 > 0$$

Hence, profitable condition is specified as follows:

$$p = \frac{Y - X_0}{X_0} > l \quad (2.7)$$

It is easy to deduce that the investor receives profit if the profitable condition is satisfied. In other words, the investor receives profit if the interest rate p is larger than the leverage ratio l. Let r_0 be least ROI. That the investor receives profit with least ROI $r_0 > 0$ is implied as follows:

$$r = \frac{1}{l}\left(\frac{Y - X_0}{X_0}\right) - 1 \geq r_0$$

Hence, least profitable condition is specified as follows:

$$p = \frac{Y - X_0}{X_0} \geq l(1 + r_0) \quad (2.8)$$

According to the least profitable condition, the investor receives profit with r_0 at least if the interest rate p is larger than or equal to the product $l(1+r_0)$.

Although the leverage ratio l helps the investor to save invested money, there is a question that whether it causes high pressure on investment. By reviewing equation 2.7 without the leverage ratio l, the profitable condition is $p > 0$ whose pressure is lower due to $r_0 = 0 < l$, obviously. Hence, the leverage ratio increases pressure on investment when $r_0 = 0$. By reviewing equation 2.8 without the leverage ratio l, the least profitable condition is $p \geq r_0$ with note that $r_0 > 0$. In this case, the pressure on investment will be lower if $r_0 < l/(1-l)$ such that $r_0 < l(1+r_0)$. However, the inequality $r_0 < l/(1-l)$ does not occur if r_0 is too large or l is too small. Hence, the leverage ratio decreases pressure on investment when it is small enough. Anyway, a small enough leverage ratio is always good for both saving invested money and decreasing pressure on investment if the investor manages well risk of falling share price. Of course, a so large r_0 (> 0) is always not good.

2. Bank depositing and jagged stock investment

Suppose the investor deposits money into bank instead, the future value Y for the starting capital X_0 given interest rate p over n periods of a given time interval is (Wikipedia, Compound interest, 2021):

$$Y = X_0(1 + p)^n = X_0 y^n \quad (2.1)$$

Where,

$$\begin{aligned} 0 &< p \\ y &= 1 + p \end{aligned}$$

Note, X_0 and Y also denote the beginning price and the value of a share in stock investment, respectively. As a result, the ROI of bank depositing is:

$$r = \frac{Y - X_0}{X_0} = (1 + p)^n - 1 = y^n - 1 \quad (2.2)$$

With the same growth, stock investment does not benefit significantly more than bank depositing. Therefore, I propose a so-called jagged stock investment (JSI) strategy in which the chain of buying stock in the given time interval t is modeled as a saw. The reason is that the price function of a share is not always increased or decreased. Exactly, it varies like a jagged saw. However, if t is large enough so that it can be divided into n periods so that the price at the beginning of each period is smaller than the price at the end of such period. For instance, suppose t is split into n periods with n+1 time points t_0, t_1, t_2,…, t_n along with n+1 prices X_0, X_1, X_2,…, X_n. According to JSI strategy, the share A is bought repeatedly and consequently at t_1, t_2,…, t_{n-1} so that $X_i > X_{i-1}$ for all i. It is possible that many prices after X_{i-1} and before X_i are larger than X_i. Moreover, many prices after X_{i-1} and before X_i may be smaller than X_{i-1}. This JSI strategy is first order JSI strategy in which A is only bought at the first level in the chain t_1, t_2, t_{n-1} with note that the starting purchase (X_0) is excluded because it is default. In other words, given the chain t_1, t_2, t_{n-1}, there are n–1 lines of bought replications of A. Note, JSI strategy assumes that in the overview, stock price is increased when the own company gets profitable in the time interval t. Figure 1 illustrates JSI strategy.

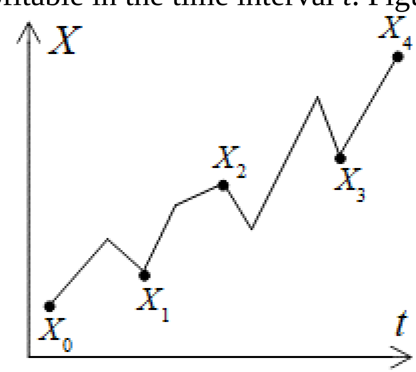

Figure 1. JSI strategy

Let y_1, y_2,…, y_n be entire growth rates of A over n time points t_1, t_2,…, t_n, the final value given of A is:

$$X_n = X_0 \prod_{j=1}^{n} y_j$$

Where,

$$y_j > 1, \forall j \geq 1$$

With leverage ratio $0 < l < 1$, the interest from buying A is:

$$\Delta(X_n) = X_n - lX_0 - X_0 = X_0 \prod_{j=1}^{n} y_j - X_0 - lX_0 = X_0 \left(\prod_{j=1}^{n} y_j - (1 + l) \right)$$

At time point t_i where $1 \leq i \leq n$–1, the new share A_i which is a replication of A is bought with beginning price $X_i - X_{i-1}$ which is equal to the benefit $X_i - X_{i-1}$. Let $m = X_i - X_{i-1}$ then m is the new amount of invested money to buy A_i. The final value of A_i is:

$$X_n(i) = X_0 \prod_{j=1}^{n} \begin{cases} y_j \text{ if } j \neq i \\ y_j - 1 \text{ otherwise} \end{cases}$$

With leverage ratio l, the interest from buying A_i is:

$$\Delta\big(X_n(i)\big) = X_n(i) - (X_i - X_{i-1}) - l(X_i - X_{i-1})$$
$$= X_0\left(\prod_{j=1}^{n}\begin{cases} y_j \text{ if } j \neq i \\ y_j - 1 \text{ otherwise}\end{cases} - (1+l)\prod_{j=1}^{i}\begin{cases} y_j \text{ if } j \neq i \\ y_j - 1 \text{ otherwise}\end{cases}\right)$$

Hence, the final value of all shares A and A_i is:

$$Y = X_n + \sum_{i=1}^{n-1} X_n(i) = X_0 \sum_{i=0}^{n-1}\prod_{j=1}^{n}\begin{cases} y_j \text{ if } i = 0 \text{ or } j \neq i \\ y_j - 1 \text{ otherwise}\end{cases} \tag{2.3}$$

With leverage ratio l, the interest from buying all shares A and A_i is:

$$\Delta Y = X_0 \sum_{i=0}^{n-1}\left(\prod_{j=1}^{n}\begin{cases} y_j \text{ if } i = 0 \text{ or } j \neq i \\ y_j - 1 \text{ otherwise}\end{cases} - (1+l)\prod_{j=0}^{i}\begin{cases} y_j \text{ if } i = 0 \text{ or } j \neq i \\ y_j - 1 \text{ otherwise}\end{cases}\right) \tag{2.4}$$

As a convention $y_0 = 1$. For easy explanation, let be the average growth ratio of share A as follows:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

As a result, the final value from buying all shares A and A_i is re-formulated as follows:

$$Y = X_0 \sum_{i=0}^{n-1}\prod_{j=1}^{n}\begin{cases} y_j \text{ if } i = 0 \text{ or } j \neq i \\ y_j - 1 \text{ otherwise}\end{cases}$$
$$= X_0\left(\bar{y}^n + \sum_{i=1}^{n-1}\prod_{j=1}^{n}\begin{cases} \bar{y} \text{ if } j \neq i \\ \bar{y} - 1 \text{ otherwise}\end{cases}\right) = X_0\left(\bar{y}^n + (n-1)(\bar{y}-1)\bar{y}^{n-1}\right)$$

Equation 2.5 summarize how to calculate Y.

$$Y = X_0\left(\bar{y}^n + (n-1)(\bar{y}-1)\bar{y}^{n-1}\right) \tag{2.5}$$

Consequently, the interest from buying all shares A and A_i is re-formulated as follows:

$$\Delta Y = X_0 \sum_{i=0}^{n-1}\left(\prod_{j=1}^{n}\begin{cases} y_j \text{ if } i = 0 \text{ or } j \neq i \\ y_j - 1 \text{ otherwise}\end{cases} - (1+l)\prod_{j=0}^{i}\begin{cases} y_j \text{ if } i = 0 \text{ or } j \neq i \\ y_j - 1 \text{ otherwise}\end{cases}\right)$$
$$= X_0\left(\bar{y}^n + (n-1)(\bar{y}-1)\bar{y}^{n-1} - (1+l) - (1+l)\sum_{i=1}^{n-1}\prod_{j=1}^{i}\begin{cases} \bar{y} \text{ if } j \neq i \\ \bar{y} - 1 \text{ otherwise}\end{cases}\right)$$
$$= X_0\left(\bar{y}^n + (n-1)(\bar{y}-1)\bar{y}^{n-1} - (1+l) - (1+l)\frac{\bar{y}-1}{\bar{y}}\sum_{i=1}^{n-1}\bar{y}^i\right)$$
$$= X_0\left(\bar{y}^n + (n-1)(\bar{y}-1)\bar{y}^{n-1} - (1+l) - (1+l)\frac{\bar{y}-1}{\bar{y}}\sum_{i=0}^{n-1}\bar{y}^i + (1+l)\frac{\bar{y}-1}{\bar{y}}\right)$$
$$= X_0\left(\bar{y}^n + (n-1)(\bar{y}-1)\bar{y}^{n-1} - (1+l) - (1+l)\frac{\bar{y}-1}{\bar{y}}\frac{1-\bar{y}^n}{1-\bar{y}} + (1+l)\frac{\bar{y}-1}{\bar{y}}\right)$$
$$= X_0\left(\bar{y}^n + (n-1)(\bar{y}-1)\bar{y}^{n-1} - (1+l) + (1+l)(1-\bar{y}^{n-1})\right)$$
$$= X_0\left(\bar{y}^n + (n-1)(\bar{y}-1)\bar{y}^{n-1} - (1+l)\bar{y}^{n-1}\right) = X_0\left(n\bar{y}^n - (n+l)\bar{y}^{n-1}\right)$$

Equation 2.6 summarizes how to calculate Y.

$$\Delta Y = X_0\left(n\bar{y}^n - (n+l)\bar{y}^{n-1}\right) \tag{2.6}$$

The ROI of JSI strategy is:

$$r = \frac{\Delta Y}{lX_0} = \begin{cases} \dfrac{n\bar{y}^n - (n+l)\bar{y}^{n-1}}{l} & \text{if } l \neq 0 \\ n\bar{y}^n - n\bar{y}^{n-1} & \text{if } l = 0 \end{cases} \tag{2.7}$$

Suppose the growth rate is same to the interest rate such that

$$\bar{y} = y = 1 + p$$

The ROI of JSI strategy becomes as follows:

$$r = \begin{cases} \dfrac{ny^n - (n+l)y^{n-1}}{l} & \text{if } l \neq 0 \\ ny^n - ny^{n-1} & \text{if } l = 0 \end{cases} \tag{2.8}$$

By comparing equation 2.2 and equation 2.8, the following inequation must be satisfied so that ROI of JSI is larger than the ROI of bank depositing.

$$\frac{ny^n - (n+l)y^{n-1}}{l} > y^n - 1 \Rightarrow (n-l)y^n - (n+l)y^{n-1} + l > 0 \tag{2.9}$$

We prove that inequation 2.9 is satisfied with some condition. In fact, the left side of inequation 2.9 is extended as follows:

$$(n-l)y^n - (n+l)y^{n-1} + l = y^{n-1}\big((n-l)y - n - l\big) + l$$
$$= y^{n-1}\big((n-l)(1+p) - n - l\big) + l = y^{n-1}\big(np - l(p+2)\big) + l$$
$$= y^{n-1}\big(n(y-1) - l(y+1)\big) + l$$

Inequation 2.9 will be satisfied if

$$y^{n-1}\big(n(y-1) - l(y+1)\big) > -l$$

Due to $y > 1$, this inequation is equivalent to:

$$n(y-1) - l(y+1) > -l$$

Therefore, profitable condition for JSI strategy is:

$$n > \frac{ly}{y-1} \tag{2.10}$$

According to inequation 2.10, if l is small enough or y is large enough, the profitable condition for JSI is always satisfied, which implies that the ROI of JSI from equation 2.8 is larger than the ROI of bank depositing from equation 2.2. If $l \leq y-1$, the JSI profitable condition specified by inequation 2.10 is simplified as follows:

$$n > y \text{ if } l \leq y - 1$$

Due to:

$$\text{if } l \leq y - 1 \text{ then } \frac{ly}{y-1} \leq \frac{(y-1)y}{y-1} = y$$

Especially, in the worst case that there is no leverage support ($l = 0$), inequation 2.9 becomes:

$$ny^n - ny^{n-1} > y^n - 1 \Rightarrow ny^{n-1}(y-1) > \frac{y-1}{\sum_{k=0}^{n-1} y^k} \Rightarrow ny^{n-1}\sum_{k=0}^{n-1} y^k > 1$$

Which is obvious because y is always larger than 1. In general, JSI strategy is better than bank depositing. From equation 2.8 and inequation 2.10, the magnitude of r depends on the magnitude of n and thus it is easy to recognize that whether JSI strategy is preeminent is dependent on how to split the time interval. The finer the splitting process is (n is large enough), the better JSI strategy is. Note, we cannot buy all shares A and A_i at the starting time point t_0 as an alternative way for JSI because we cannot predict interests $X_i - X_{i-1}$.

3. Conclusions

In theory, stock investment and bank depositing have the same benefit because they share the same interest rate (growth rate) in mathematical model. However, stock investment often has the growth rate which is higher than interest rate unless the own company is unprofitable. Remind that the fine splitting time interval is most important to JSI strategy in order to get always profitable. The solution for this problem is not so difficult because it is totally possible to determine the vicinity of the bottom of a given share price with acceptable error rate. Note that we only predict such vicinity and so it is not necessary to determine exactly the bottom price in some interval. The JSI strategy here is the first order one in which the share A is only bought at the first level in the chain t_1, t_2, t_{n-1}. It is not difficult to develop k^{th} JSI strategy where replications A_i are in turn replicated till $(k-1)^{th}$ time. If we deposit repeatedly and continuously many money items into bank by the same way of jagged strategy, we can get benefits like JSI. Unfortunately, banking interest rate is constant whereas share growth rate varies, which is the reason that bank depositing is stable but has not many profitable chances.

References

Wikipedia. (2021, March 30). *Compound interest*. (Wikimedia Foundation) Retrieved April 1, 2021, from Wikipedia website: https://en.wikipedia.org/wiki/Compound_interest