Lecture 7

Algorithms & Data Structures

Goldsmiths Computing

November 19, 2018

Outline

Outline

Lecture

- Binary search trees
  - contents property
  - improved find
- Mergesort
- Master theorem

Lab

- Be a data structure implementor
  1. even more methods on linked lists
  2. understand how mergesort behaves
- Use data structures
  1. populate stack and queue appropriately

VLE activities

Dynamic arrays quiz

Statistics so far:

- 242 attempts: average mark 4.36
- 98 students: average mark 4.18
  - 57 under 4.00, 21 over 6.99, 7 at 10.00

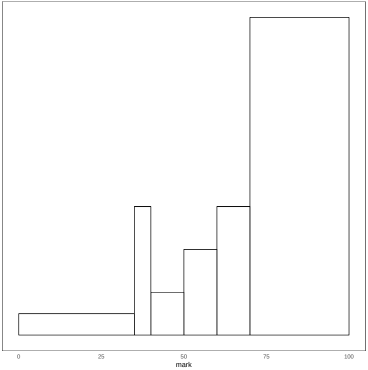
Quiz closes at 16:00 on Friday 23rd November

- no extensions
- grade is
  - 0 (for no attempt)
  - $30 + 70 \times (\text{score}/10)^2$

VLE activities (cont'd)

Mergesort submission

- 126 final uploads: average mark 83.06



Outline

Motivation

We have seen a number of data structures for storing data by now. Is there a unifying concept behind storing data items?

Definition

collection a grouping of some variable number of data items. aka: “container” (C++)

linear collection a collection with an underlying linear order

collection	linear?
linked list	✓
dynamic array	✓
binary tree	?
set	✗
multiset	✗
stack	✓
queue	✓
priority queue	✓
deque	✓

Operations

Generic collection

size	how many elements does the collection contain?
insert[o]	add o to the collection
find[o]	is the object o in the collection?
remove[o]	return a collection with all instances of o removed
count[o]	how many times is o stored in the collection?
sum	what is the sum of the objects in the collection?

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Linear collection

position[o]	what index is o at, if any?
get[i]	get the object at index i

Work

1. Reading
   - Drozdek [C++], section 1.7.1 (Containers), 3.7 (Lists in the STL), 4.4-4.7 (Stacks, Queues, Priority Queues, Deques in the STL)
   - Drozdek [Java], section 1.5 (Vectors in `java.util`), 3.7 (Lists in `java.util`), 4.1.1 (Stacks in `java.util`)

Outline

Introduction

Collections

Hash tables

Cycle detection

Motivation

A different way to implement a collection, with different performance implications

Definition

A hash table is a data structure that can represent a set, or more generally a map of keys to values (an associative array), by computing a numeric value for each key using a hash function and then using that numeric value to compute an index into an array to look up the value.

Set operations

insert[o]	insert the object o into the set
find[o]	is the object o in the set?

and also

delete[o]	delete the object o from the set

Sets of small integers

Represent sets of non-negative integers smaller than N using an array of size N. e.g. for domain [0,5]:

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✓	✗	✗	✓	✗	✓

represents the set $\{0, 3, 5\}$

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Represent sets of non-negative integers smaller than N using an array of size N. e.g. for domain [0,5]:

✓	✗	✗	✓	✗	✓

represents the set $\{0, 3, 5\}$

insert[o]	S[o] $\leftarrow$ true
find[o]	**return** S[o]
delete[o]	S[o] $\leftarrow$ false

Sets of unbounded integers

Apply the same representation?

Sets of unbounded integers

Apply the same representation?

✓	✗	✗	✓	✗	✓	⋯											

2^{32} integers? 2^{29} bytes of RAM (512MB)

Sets of unbounded integers

If the expected size of the sets is small (even if the range of possible values is large):

1. choose a reasonable size for the array, say twice expected size
2. reduce the integer to within the range of array indices using a function f(n)
3. store the (unreduced) integer in the array slot

Then

insert[o] $S[f(o)] \leftarrow o$

find[o] **return** $S[f(o)] = o$

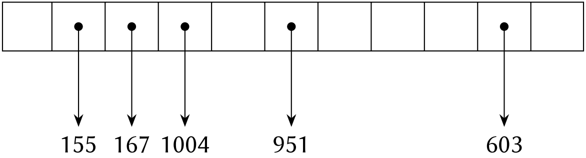
delete[o] $S[f(o)] \leftarrow \mathrm{NIL}$

Example

range $[0,2^{10})$

set size 5

Choose array size of (say) 11 and compute index as f(n) = n mod 11



represents the set {155, 167, 603, 951, 1004}

Complexity analysis

Provided the reducing function f(n) is $\Theta(1)$

insert

$\Theta(1)$ reduction and $\Theta(1)$ memory operations

$$\implies \Theta(1)$$

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delete

$\Theta(1)$ reduction and $\Theta(1)$ memory operations

$$\Longrightarrow \Theta(1)$$

So what am I not telling you?

Sets of arbitrary things

- compute an integer (a *hash code*) for the things using a *hash function*
  - equal things must have equal hash codes
  - unequal things should be unlikely to share hash codes

computing an integer for the things:

Java `public int hashcode()`

C++ `operator()` functor second template argument to container

equal things must have equal integer codes:

Java `public boolean equals(Object o)`

C++ `operator()` functor third template argument to container

Work

1. Reading
   - CLRS, sections 11.1 and 11.2
   - DPV, section 1.5
   - Drozdek, sections 10.1

Outline

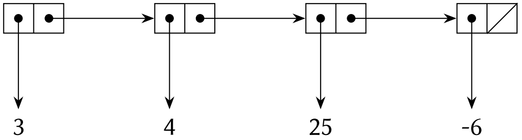
Motivation

Did your SLList code suffer from baffling infinite loops at any point?

Definition

Cycle detection algorithms detect whether there are loops in a graph, graph, or repeated values in a function with same domain and range, and where and how long those loops are.

Linked list implementation



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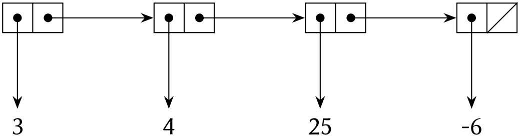
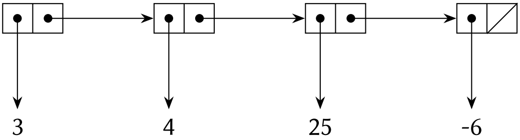


SET-REST!(REST(REST(REST(list))), REST(list))

Linked list implementation



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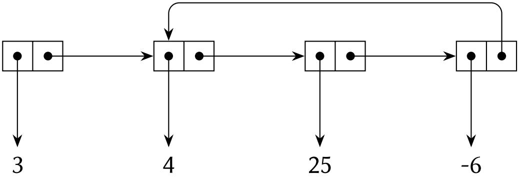

Naïve circularity detection

Algorithm: at each step, check all previous steps for repeat

Helper function

```
function IS-IN?(sequence,node,end)
    j ← 0
    x ← sequence
    while j < end do
        if x = node then
            return true
        end if
        x ← REST(x)
        j ← j + 1
    end while
    return false
end function
```

Naïve circularity detection

Algorithm: at each step, check all previous steps for repeat

Main function

Require: sequence :: list
  node ← REST(sequence)
  end ← 0
  while node ≠ NIL **do**
    end ← end + 1
    if IS-IN?(sequence,node,end) **then**
      return true
    end if
    node ← REST(node)
  end while
  return false

Naïve circularity detection

Algorithm: at each step, check all previous steps for repeat

Complexity analysis

Space

No extra space required

$$\implies \Theta(1)$$

Time

$$T_k = T_{k-1} + \Theta(k)$$

If there is no cycle, the algorithm traverses the entire list, checking an increasing amount of the entire list each time

$$\implies \Theta(N^2)$$

If there is a cycle, the algorithm stops at the first repeated node after once round the cycle

$$\implies \Theta((j + l)^2)$$

Less naïve circularity detection

Algorithm: at each step, check all previous steps for repeat

Hash-table memory

Require: sequence :: list
  table $\leftarrow$ new Hashtable
  node $\leftarrow$ sequence
  while node $\neq$ NIL **do**
    if node $\in$ table **then**
      return true
    end if
    table[node] $\leftarrow$ true
  end while
  return false

Less naïve circularity detection

Algorithm: at each step, check all previous steps for repeat

Complexity analysis

Space

Hash table with N entries required

$$\implies \Theta(N) \text{ extra space}$$

Time

$$T_k = T_{k-1} + \Theta(1)$$

If there is no cycle, the algorithm traverses the entire list, doing a constant-time lookup each time

$$\implies \Theta(N)$$

If there is a cycle, the algorithm stops at the first repeated node

$$\implies \Theta(j + l)$$

(assumes hash-table lookup is $\Theta(1)$)

Hare and tortoise

Also known as Floyd's cycle-finding algorithm

Key insight

for circularity of length l beginning at position j

$$L[k + nl] = L[k]$$

for all $k > j, n \geq 0$.

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... or in words

If two nodes at different positions in the list are identical, the difference in positions is an integer multiple of the circularity length.

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... or in words

If two nodes at different positions in the list are identical, the difference in positions is an integer multiple of the circularity length.

Converse

If there is a circularity and two iterators are each within it, incrementing the *difference* between two list iterators by 1 will always lead to the two iterators arriving at the same list node.

Hare and tortoise

Also known as Floyd's cycle-finding algorithm

Algorithm

Require: sequence :: list
  tortoise $\leftarrow$ REST(sequence)
  hare $\leftarrow$ REST(tortoise)
  while hare $\neq$ NIL **do**
    if hare = tortoise **then**
      return true
    end if
    tortoise $\leftarrow$ REST(tortoise)
    hare $\leftarrow$ REST(REST(hare))
  end while
  return false

Hare and tortoise

Also known as Floyd’s cycle-finding algorithm

Complexity analysis

Space

No extra space needed

$$\implies \Theta(1)$$

Time

If there is no cycle, the hare traverses the list; in that time, the tortoise traverses half the list:

$$\implies \Theta(N)$$

If there *is* a cycle, the hare and tortoise meet no more than l steps after the tortoise is in the cycle

$$\implies \Theta(j + l)$$

Additional information from algorithm

Position of first repeat

1. reset tortoise to the head of the list
2. move hare and tortoise one step at a time (same speed)
3. count steps until hare and tortoise are equal

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Length of circularity

1. hold tortoise still
2. move hare one step at a time
3. count steps until hare and tortoise are equal again