

# Binary tree properties

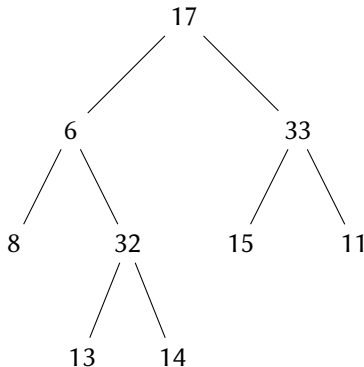
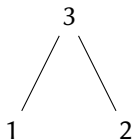
Goldsmiths Computing

## Height-balanced property

In a height-balanced tree:

- the heights of left- and right-subtrees of every node differ by at most 1

### Example height-balanced trees

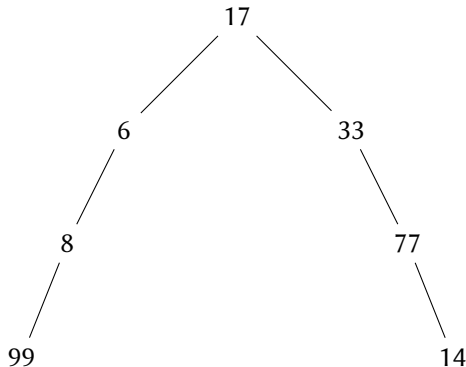
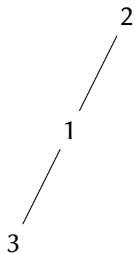


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### Example non-height-balanced trees



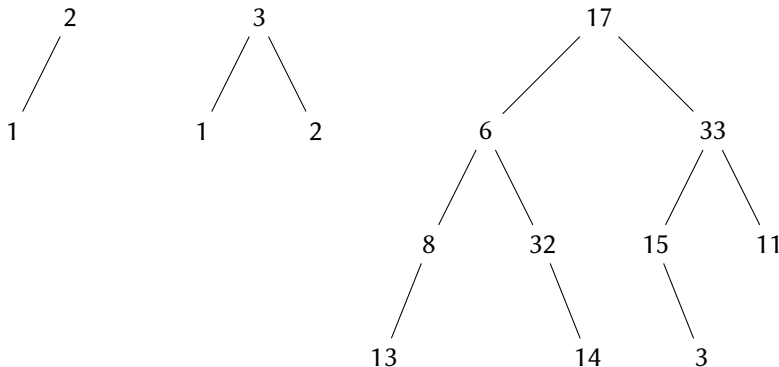
## Weight-balanced property

In a weight-balanced tree:

- the number of nodes of left- and right-subtrees of every node differ by at most 1

Weight-balanced trees are automatically height-balanced.

### Example weight-balanced trees



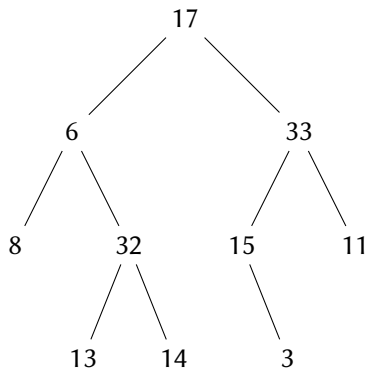
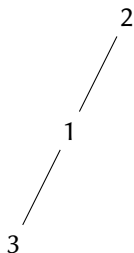
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### Example non-weight-balanced trees



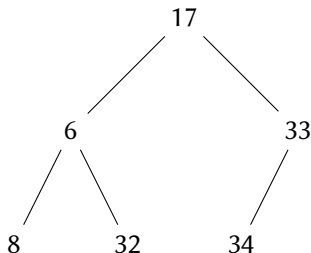
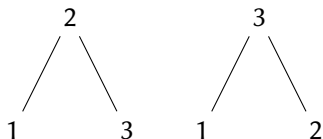
## Nearly-complete property

In a nearly-complete tree:

- all levels except *possibly* the lowest level are completely filled;
- the lowest level is filled from the left;
- a complete tree (lowest level filled) is by convention also a nearly-complete tree.

Nearly-complete trees are automatically height-balanced (but not necessarily weight-balanced)

### Example nearly complete trees



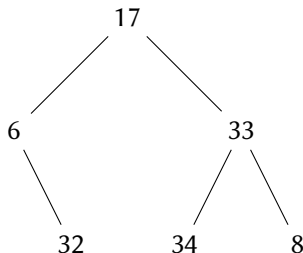
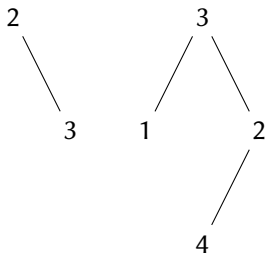
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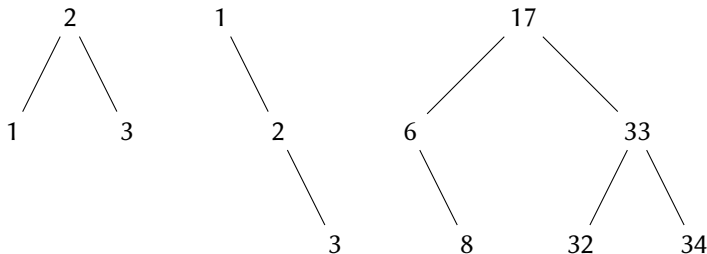
### Example non-nearly complete trees



## Binary search tree property

Let  $x$  be a node in a binary search tree. If  $y$  is a node in the left subtree of  $x$ , then  $y.\text{key} < x.\text{key}$ . If  $z$  is a node in the right subtree of  $x$ , then  $z.\text{key} \geq x.\text{key}$ .

### Example binary search trees





## Heap property

Let  $x$  be a node in a max-heap. If  $y$  is a (generalised) parent of  $x$ , then  $y.\text{key} \geq x.\text{key}$ .