Growth of functions

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Motivation

Turning empirical measurements into scaling hypotheses, or *vice versa*

Common functional classes

1. power-law
2. logarithmic and linear-logarithmic
3. exponential

Power-law

$$f(n) \propto n^k$$

$$f(n) = An^k$$

Given $f(n_1)$ and $f(n_2)$, estimate k (and A):

$$\frac{f(n_1)}{f(n_2)} = \left(\frac{n_1}{n_2}\right)^k$$

$$k = \frac{\log\left(\frac{f(n_1)}{f(n_2)}\right)}{\log\left(\frac{n_1}{n_2}\right)}$$

Logarithmic

$$f(n) \propto \log(Bn)$$

$$f(n) = A\log(Bn)$$

Given $f(n_1)$ and $f(n_2)$, estimate A (and B):

$$f(n_1) - f(n_2) = A(\log(n_1) - \log(n_2))$$

$$A = \frac{f(n_1) - f(n_2)}{\log(n_1) - \log(n_2)}$$

Exponential

$$f(n) \propto 2^{cn}$$

$$f(n) = A2^{cn}$$

Given $f(n_1)$ and $f(n_2)$, estimate c (and A):

$$\log(f(n_1)) - \log(f(n_2)) = c(n_1 - n_2)$$

$$c = \frac{\log(f(n_1)) - \log(f(n_2))}{n_1 - n_2}$$

Work

1. Do growth of functions quiz
    - open until 16:00 9th November 2018
    - no extensions