Estimation of Vehicle Frame Extrinsics

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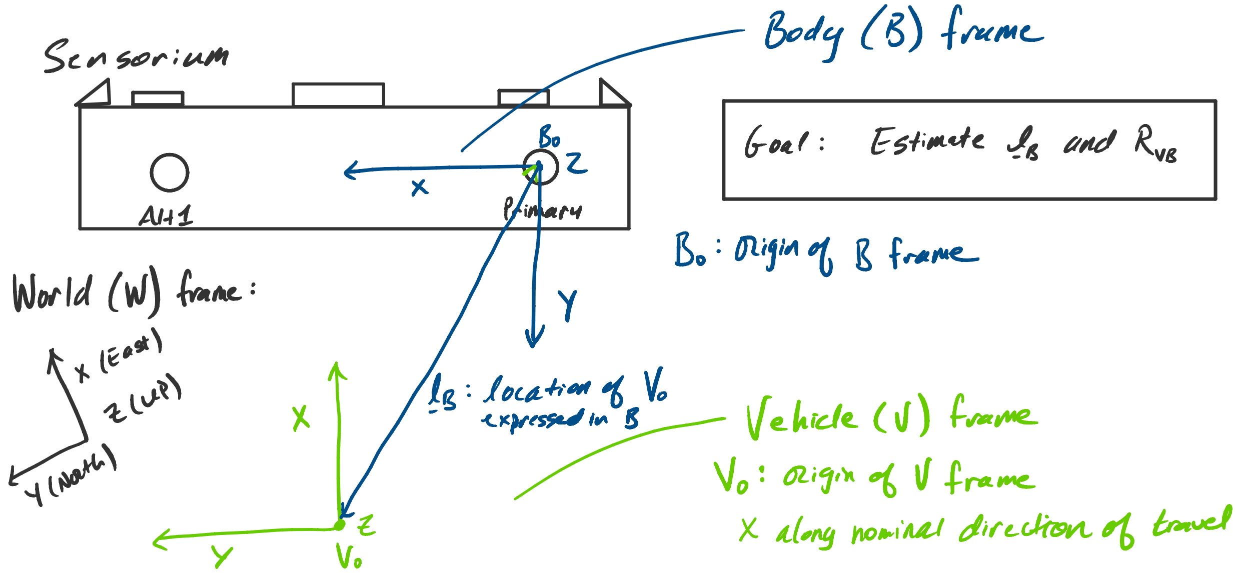

Goal: Estimate $\underline{l}_B$ and R_{VB}

<u>Kinematics</u>:
$$\underline{V}_{V_oB} = \underline{V}_{B_oB} + \underline{\omega}_B \times \underline{l}_B$$

- $\underline{\omega}_B$: angular rate of B wrt W
- $\underline{V}_{B_oB}$: velocity of B_o wrt W expressed in B
- $\underline{V}_{V_oB}$: velocity of V_o wrt W expressed in B

Let R_{VB} be the B-to-V attitude matrix. Then

$$R^T_{VB}\,\underline{V}_{V_oV} = \underline{V}_{B_oB} + \underline{\omega}_B \times \underline{l}_B$$

Velocity of V_o wrt W expressed in V: $\underline{V}_{V_oV} = R_{VB}\left[\underline{V}_{B_oB} + \underline{\omega}_B \times \underline{l}_B\right]$

Let our parameter vector be $\underline{x} = \begin{bmatrix} \underline{e} \\ \underline{l}_B \end{bmatrix}$ where $\underline{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$

is a vector of small Euler angles that relate the nominal R_{VB}, denoted $\bar{R}_{VB}$, and the true R_{VB}:
$$R_{VB} = R(\underline{e})\,\bar{R}_{VB}$$

- R_{VB}: true attitude
- $\bar{R}_{VB}$: nominal attitude
- $R(\underline{e})$: attitude matrix formed by 3-1-2 sequence of Euler angles in $\underline{e}$.

<u>Vehicle</u> <u>velocity</u> <u>Constraints</u>:

(1) Zero sideslip: $V_{V_oV_y} = 0$

(2) Zero vertical: $V_{V_oV_z} = 0$

Meas. model: $$\underline{V}_{V_oV} = \begin{bmatrix} V_{V_oV_x} \\ 0 \\ 0 \end{bmatrix} = R(\underline{e})\,\bar{R}_{VB}\left[\underline{V}_{B_oB} + \underline{\omega}_B \times \underline{l}_B\right] + \underline{w}' = \underline{h}'(\underline{x}) + \underline{w}'$$

$\underline{w}'$: noise

$\underbrace{\qquad}_{\underline{z}'}$

We don't know $V_{V_0V_x}$ exactly, nor do we want to estimate it. So we will take only the lower two rows of the above measurement equation. Write this as

$$\underline{z} = \underline{h}(\underline{x}) + \underline{w} \quad , \quad \underline{z} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

We will apply a Gauss-Newton approach to solving for $\underline{x}$. This will require us to obtain Jacobians $\left[\frac{\partial \underline{h}}{\partial \underline{e}}\right], \left[\frac{\partial \underline{h}}{\partial \underline{l}_B}\right]$

First obtain Jacobians for full 3-element measurement equation:

$$H_1' \triangleq \left[\frac{\partial \underline{h}'}{\partial \underline{e}}\right]\Bigg|_{\underline{l}_B = \bar{\underline{l}}_B} = \frac{\partial}{\partial \underline{e}} R(\underline{e}) \underbrace{\bar{R}_{VB}\left[\underline{V}_{B_0B} + \underline{\omega}_B \times \underline{l}_B\right]\Big|_{\underline{l}_B = \bar{\underline{l}}_B}}_{\underline{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}}$$

$$\cong \frac{\partial}{\partial \underline{e}} \overbrace{\begin{bmatrix} 1 & e_3 & -e_2 \\ -e_3 & 1 & e_1 \\ e_2 & -e_1 & 1 \end{bmatrix}}^{\text{approx. of } R(\underline{e}) \text{ for small } \underline{e}} \underline{a}$$

$$= \frac{\partial}{\partial \underline{e}} \begin{bmatrix} a_1 + e_3 a_2 - e_2 a_3 \\ -e_3 a_1 + a_2 + e_1 a_3 \\ e_2 a_1 - e_1 a_2 + a_3 \end{bmatrix}$$

$$\Rightarrow H_1' = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} , \quad H_1 = \begin{bmatrix} a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

$$H_2' \triangleq \frac{\partial \underline{h}'}{\partial \underline{l}_B}\Bigg|_{\underline{e} = \bar{\underline{e}}} = \frac{\partial}{\partial \underline{l}_B} R(\bar{\underline{e}}) \bar{R}_{VB}\left[\underline{\omega}_B \times\right] \underline{l}_B = R(\bar{\underline{e}}) \bar{R}_{VB}\left[\underline{\omega}_B \times\right]$$

$$H_2 = \left[R(\bar{\underline{e}}) \bar{R}_{VB}\left[\underline{\omega}_B \times\right]\right]_{[2:3,:]}$$ rows 2-3, all columns

$$H_1 \in \mathbb{R}^{2\times 3} , \quad H_2 \in \mathbb{R}^{2\times 3}$$

$$\underline{z} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cong \underline{h}(\bar{\underline{x}}) + \underbrace{\left[H_1, H_2\right]}_{H} (\underline{x} - \bar{\underline{x}}) + \underline{w}$$

$$\underbrace{\underline{z} - \underline{h}(\bar{\underline{x}}) + H\bar{\underline{x}}}_{\tilde{\underline{z}}} = H\underline{x} + \underline{w}$$

$$\boxed{\tilde{\underline{z}} = H\underline{x} + \underline{w}, \quad E[\underline{w}] = 0, \; E[\underline{w}\underline{w}^T] = R_w}$$

Stack N of these measurement equations and solve using standard least squares.