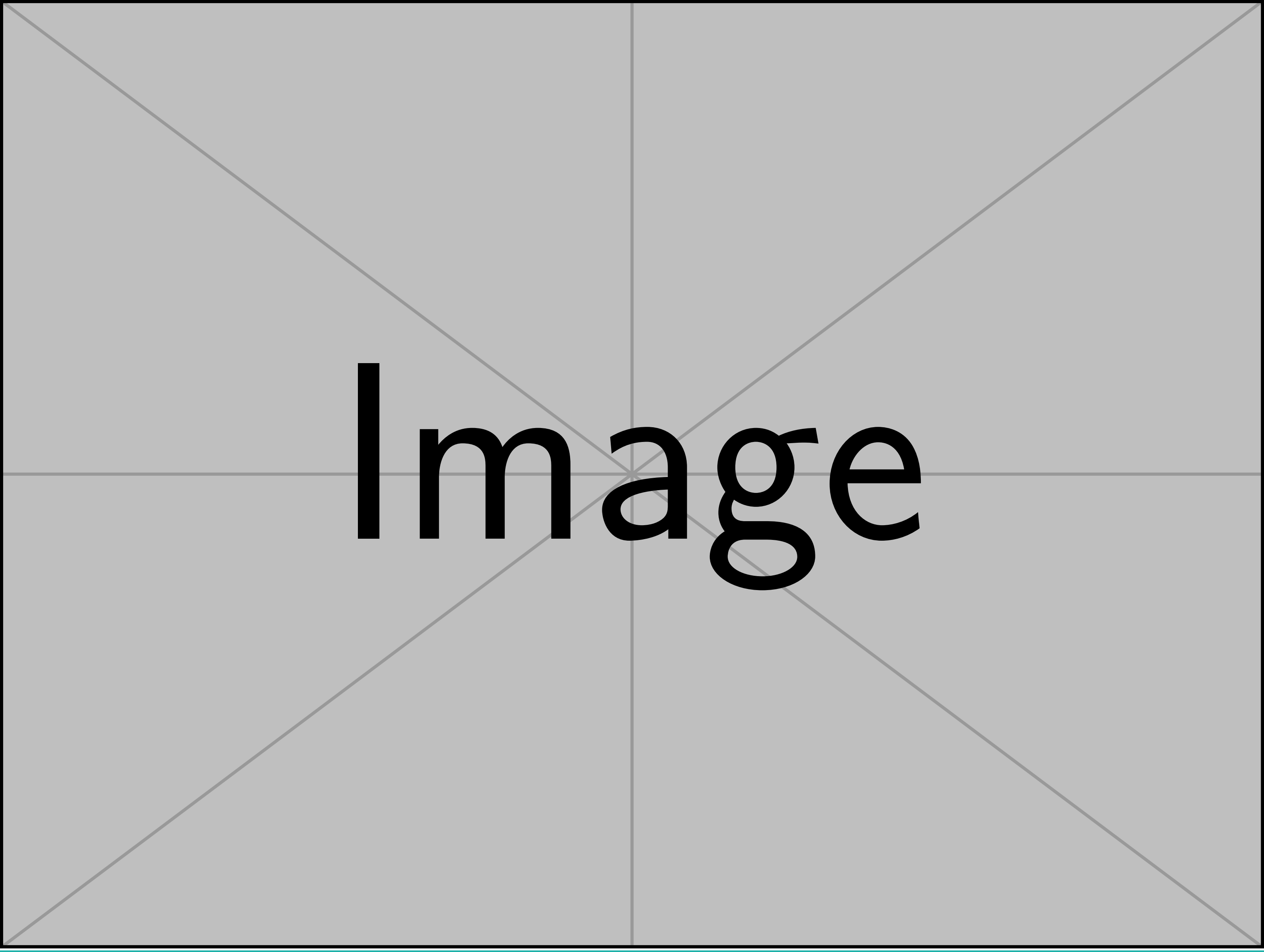

Combinatoire

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Contents

Preface

This is the preface of the book…

Chapter 1 Permutations and Combinations

Chapter 2 Basic Counting Principles

2.1 Pigeonhole Principle

Chapter 3 Binomial Coefficients

Chapter 4 Inclusion-Exclusion Principle

Chapter 5 Recurrence Relations and Generating Functions

Chapter 6 Special Counting Sequences

6.1 Catalan Numbers

Definition 6.1 (Catalan Numbers)

The n-th Catalan number C_n is given by:

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}.$$

The first ten Catalan numbers are:

$$C_0 = 1, \quad C_1 = 1, \quad C_2 = 2, \quad C_3 = 5, \quad C_4 = 14,$$
$$C_5 = 42, \quad C_6 = 132, \quad C_7 = 429, \quad C_8 = 1430, \quad C_9 = 4862.$$

♣

Catalan numbers is the answer to many combinatorial problems,

Path counting problem There is an $n \times n$ grid graph, with the bottom-left corner at $(0, 0)$ and the top-right corner at (n, n). Starting from the bottom-left corner, and moving only right or up one unit at each step, the total number of paths to reach the top-right corner without going above the diagonal $y = x$ (but allowing touching it) is denoted as C_n.

Counting non-intersecting chords in a circle There are $2n$ points on a circle. The number of ways to pair these points with n chords such that no two chords intersect is the Catalan number C_n.

Triangulation counting problem The number of ways to divide a convex $(n + 2)$-sided region into triangular regions without intersecting diagonals is C_n.

Binary Tree Counting Problem The number of structurally different binary trees with n nodes is C_n. Equivalently, the number of structurally different full binary trees with n non-leaf nodes is C_n.

Counting problem of parenthesis sequences The number of valid parenthesis sequences consisting of n pairs of parentheses is C_n.

Stack popping sequence counting problem The push sequence of a stack (of infinite size) is $1, 2, \ldots, n$, and the number of valid popping sequences is C_n.

Sequence Counting Problem The number of sequences $a_1, a_2, \ldots, a_{2n}$ consisting of n $+1$'s and n -1's such that the partial sums satisfy $a_1 + a_2 + \ldots + a_k \geq 0$ ($k = 1, 2, 3, \ldots, 2n$) is C_n.

6.2 Stirling Numbers

Chapter 7 Extremal Principle

Chapter 8 Ramsey Theory

Chapter 9 Design Theory

Chapter 10 Pólya Counting

Bibliography

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[2] Author2, Title2, Journal2, Year2. *This is another example of a reference.*