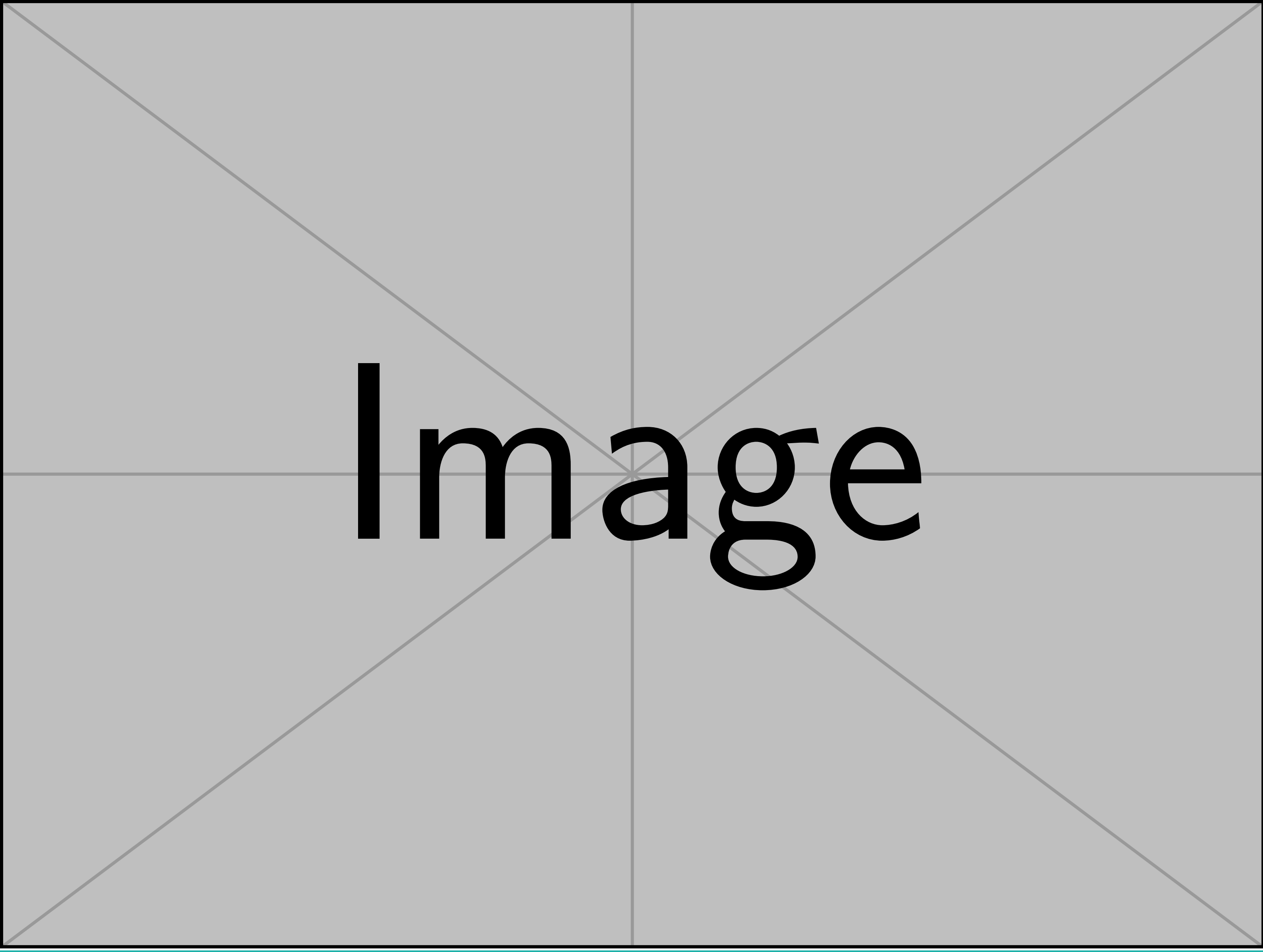

Géométrie Analytique

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Date: October, 2025

Version: 0.1

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Preface

This is the preface of the book…

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2.1 Coordinate Systems

Definition 2.1 (Coordinate Frame)

A fixed point O in $\mathbb{R}^3$ space, together with three non-coplanar ordered vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, is called a **coordinate frame** (or **reference frame**) in space, denoted by $\{O; \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$.

If $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are unit vectors, then the frame is called a **Cartesian frame**. Furthermore, if $\mathbf{e}_1 \perp \mathbf{e}_2, \mathbf{e}_2 \perp \mathbf{e}_3, \mathbf{e}_3 \perp \mathbf{e}_1$, then the frame is called a **rectangular Cartesian frame**, or simply a **rectangular frame**.

Generally, $\{O; \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is called **affine frame**. ♣

2.2 Theorems about Vectors

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¶ `Inner Product (Dot Product)`

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4.1 Equations of Planes

¶ Point-Vector Form

In space, fix a point $M_0 = (X_0, Y_0, Z_0)$ and two non-collinear vectors $\mathbf{a} = (X_1, Y_1, Z_1)$ and $\mathbf{b} = (X_2, Y_2, Z_2)$. The equation of the plane passing through the point M_0 and parallel to the vectors $\mathbf{a}$ and $\mathbf{b}$ is given by:

$$\mathbf{r} = O\vec{M} + \lambda\mathbf{a} + \mu\mathbf{b},$$

or in coordinate form:

$$\begin{cases} x = X_0 + \lambda X_1 + \mu X_2 \\ y = Y_0 + \lambda Y_1 + \mu Y_2 \\ z = Z_0 + \lambda Z_1 + \mu Z_2 \end{cases}$$

where $\lambda, \mu \in \mathbb{R}$.

Taking the dot product of both sides of the parametric vector equation with $\mathbf{a} \times \mathbf{b}$, we eliminate λ and μ to obtain $(\mathbf{r} - O\vec{M}_0, \mathbf{a}, \mathbf{b}) = 0$, that is,

$$\begin{vmatrix} x - X_0 & y - Y_0 & z - Z_0 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = 0. \tag{4.1}$$

All above forms are called the **point-vector form** of the plane equation.

Given three non-collinear points $M_1(X_1, Y_1, Z_1)$, $M_2(X_2, Y_2, Z_2)$ and $M_3(X_3, Y_3, Z_3)$, the equation of the plane passing through these three points is given by:

$$\mathbf{r} = O\vec{M}_1 + \lambda M_1\vec{M}_2 + \mu M_1\vec{M}_3,$$

or in coordinate form:

$$\begin{cases} x = X_1 + \lambda(X_2 - X_1) + \mu(X_3 - X_1) \\ y = Y_1 + \lambda(Y_2 - Y_1) + \mu(Y_3 - Y_1) \\ z = Z_1 + \lambda(Z_2 - Z_1) + \mu(Z_3 - Z_1) \end{cases}$$

where $\lambda, \mu \in \mathbb{R}$. And the determinant form is:

$$\begin{vmatrix} x - X_1 & y - Y_1 & z - Z_1 \\ X_2 - X_1 & Y_2 - Y_1 & Z_2 - Z_1 \\ X_3 - X_1 & Y_3 - Y_1 & Z_3 - Z_1 \end{vmatrix} = 0,$$

or equivalently,

$$\begin{vmatrix} x & y & z & 1 \\ X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{vmatrix} = 0.$$

All above forms are also called the **three-point form** of the plane equation.

If plane intersects the three coordinate axes at $M_1(X_1, 0, 0)$, $M_2(0, Y_2, 0)$, $M_3(0, 0, Z_3)$ (where $X_1, Y_2, Z_3 \neq$

0), then the equation of the plane can be expressed in the form:

$$\frac{x}{X_1} + \frac{y}{Y_2} + \frac{z}{Z_3} = 1,$$

which is called the **intercept form** of the plane equation.

¶ General Form

The general equation is obtained by expanding the determinant form of the parametric equation 4.1 of a plane:

$$Ax + By + Cz + D = 0,$$

where

$$A = \begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}, \quad B = \begin{vmatrix} Z_1 & X_1 \\ Z_2 & X_2 \end{vmatrix}, \quad C = \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}, \quad D = -\begin{vmatrix} X_0 & Y_0 & Z_0 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix}.$$

Special cases include:

Theorem 4.1

Any plane in space can be represented by a linear equation in three variables x, y, and z, and conversely, every such equation represents a plane in space. ♡

¶ Point-Normal Form

4.2 Linear Equations

4.3 Relative Positions of Points, Lines and Planes

4.4 Pencil of Planes and Lines

Chapter 5 Common Surfaces

Chapter 6 Conic Sections

6.1 General Equation of Conic Sections

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

$$F_1(x, y) \equiv a_{11}x + a_{12}y + a_{13}$$

$$F_2(x, y) \equiv a_{12}x + a_{22}y + a_{23}$$

$$F_3(x, y) \equiv a_{13}x + a_{23}y + a_{33}$$

$$\Phi(x, y) \equiv a_{11}x^2 + 2a_{12}xy + a_{22}y^2$$

6.2 Conic Sections and Lines

6.3 Simplification of Conic Equations

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