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Preface

This is the preface of the book...

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Chapter 2 Systems of Linear Equations

Chapter 3 Matrices

Chapter 4 Linear Spaces

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- 4.1.2 Dimension, Basis, and Coordinates
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- **4.2 Subspaces**
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Chapter 5 Linear Mappings

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- **5.1.1** Definition of Linear Mappings
- 5.1.2 Existence and Uniqueness of Linear Mappings
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- 5.2 Kernel and Image of Linear Mappings
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- **5.4 Linear Functions and Dual Spaces**

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- 6.2 Eigenvectors and Diagonalization
- 6.2.1 Eigenvalues and Eigenvectors
- 6.2.2 Necessary and Sufficient Conditions for Diagonalization
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 - 6.3 Space Decomposition and Diagonalization
 - 6.3.1 Invariant Subspace
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Chapter 7 Jordan Forms

- 7.1 Polynomial Matrices
- **7.2 Invariant Factors**
- 7.3 Rational Canonical Form
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Chapter 8 Quadratic Forms

- 8.1 Quadratic Forms and Their Standard Forms
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Chapter 9 Inner Product Spaces

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- 9.7 Unitary Spaces and Unitary Transformations
- 9.8 Symplectic Spaces

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