



Image

Combinatoire

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This is the preface of the book...

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6.1 Catalan Numbers

Definition 6.1 (Catalan Numbers)

The n -th Catalan number C_n is given by:

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}.$$

The first ten Catalan numbers are:

$$\begin{aligned} C_0 &= 1, & C_1 &= 1, & C_2 &= 2, & C_3 &= 5, & C_4 &= 14, \\ C_5 &= 42, & C_6 &= 132, & C_7 &= 429, & C_8 &= 1430, & C_9 &= 4862. \end{aligned}$$



Catalan numbers is the answer to many combinatorial problems,

Path counting problem There is an $n \times n$ grid graph, with the bottom-left corner at $(0, 0)$ and the top-right corner at (n, n) . Starting from the bottom-left corner, and moving only right or up one unit at each step, the total number of paths to reach the top-right corner without going above the diagonal $y = x$ (but allowing touching it) is denoted as C_n .

Counting non-intersecting chords in a circle There are $2n$ points on a circle. The number of ways to pair these points with n chords such that no two chords intersect is the Catalan number C_n .

Triangulation counting problem The number of ways to divide a convex $(n+2)$ -sided region into triangular regions without intersecting diagonals is C_n .

Binary Tree Counting Problem The number of structurally different binary trees with n nodes is C_n . Equivalently, the number of structurally different full binary trees with n non-leaf nodes is C_n .

Counting problem of parenthesis sequences The number of valid parenthesis sequences consisting of n pairs of parentheses is C_n .

Stack popping sequence counting problem The push sequence of a stack (of infinite size) is $1, 2, \dots, n$, and the number of valid popping sequences is C_n .

Sequence Counting Problem The number of sequences a_1, a_2, \dots, a_{2n} consisting of $n+1$'s and $n-1$'s such that the partial sums satisfy $a_1 + a_2 + \dots + a_k \geq 0$ ($k = 1, 2, 3, \dots, 2n$) is C_n .

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Bibliography

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