



# Image

## Combinatoire

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This is the preface of the book...

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## 6.1 Catalan Numbers

### Definition 6.1 (Catalan Numbers)

The  $n$ -th Catalan number  $C_n$  is given by:

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)n!}.$$

The first ten Catalan numbers are:

$$\begin{aligned} C_0 &= 1, & C_1 &= 1, & C_2 &= 2, & C_3 &= 5, & C_4 &= 14, \\ C_5 &= 42, & C_6 &= 132, & C_7 &= 429, & C_8 &= 1430, & C_9 &= 4862. \end{aligned}$$



Catalan numbers is the answer to many combinatorial problems,

**Path counting problem** There is an  $n \times n$  grid graph, with the bottom-left corner at  $(0, 0)$  and the top-right corner at  $(n, n)$ . Starting from the bottom-left corner, and moving only right or up one unit at each step, the total number of paths to reach the top-right corner without going above the diagonal  $y = x$  (but allowing touching it) is denoted as  $C_n$ .

**Counting non-intersecting chords in a circle** There are  $2n$  points on a circle. The number of ways to pair these points with  $n$  chords such that no two chords intersect is the Catalan number  $C_n$ .

**Triangulation counting problem** The number of ways to divide a convex  $(n + 2)$ -sided region into triangular regions without intersecting diagonals is  $C_n$ .

**Binary Tree Counting Problem** The number of structurally different binary trees with  $n$  nodes is  $C_n$ . Equivalently, the number of structurally different full binary trees with  $n$  non-leaf nodes is  $C_n$ .

**Counting problem of parenthesis sequences** The number of valid parenthesis sequences consisting of  $n$  pairs of parentheses is  $C_n$ .

**Stack popping sequence counting problem** The push sequence of a stack (of infinite size) is  $1, 2, \dots, n$ , and the number of valid popping sequences is  $C_n$ .

**Sequence Counting Problem** The number of sequences  $a_1, a_2, \dots, a_{2n}$  consisting of  $n + 1$ 's and  $n - 1$ 's such that the partial sums satisfy  $a_1 + a_2 + \dots + a_k \geq 0$  ( $k = 1, 2, 3, \dots, 2n$ ) is  $C_n$ .

## 6.2 Stirling Numbers

## Chapter 7 Extremal Principle

## Chapter 8 Ramsey Theory

## Chapter 9 Design Theory

## Chapter 10 Pólya Counting

## Bibliography

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- [2] Author2, Title2, Journal2, Year2. *This is another example of a reference.*