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**BookName**

**Subtitle**

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**Date:** August, 2025

**Version:** 0.1

**Custom Key:** Custom Value

*extra information*

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# Preface

This is the preface of the book...

# Chapter 1 The Propositional Calculus

## 1.1 Propositional Connectives and Truth Tables

A proposition is a statement whose truth value can be determined (represented by 1 for true and 0 for false<sup>1</sup>), and a proposition is either true or false (within the framework of classical binary logic). We use lower-case letters such as  $p, q, r \dots$ , to denote propositions.

There are five commonly used propositional connectives:

### ¶ Negation

If  $p$  is a proposition, then the **negation** of  $p$  is denoted by  $\neg p$ , which is true if and only if  $p$  is false. Its truth table is as follows:

$p$	$\neg p$
0	1
1	0

### ¶ Conjunction

The **conjunction** of two propositions  $p$  and  $q$  is denoted by  $p \wedge q$ , which is true if and only if both  $p$  and  $q$  are true. Its truth table is as follows:

$p$	$q$	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

### ¶ Disjunction

The **disjunction** of two propositions  $p$  and  $q$  is denoted by  $p \vee q$ , which is true if and only if at least one of  $p$  or  $q$  is true. Its truth table is as follows:

$p$	$q$	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

### ¶ Implication

Proposition  $p$  **implies**  $q$ , namely "if  $p$ , then  $q$ ", is denoted by  $p \rightarrow q$ , which is false if and only if  $p$  is true and  $q$  is false. Its truth table is as follows:

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<sup>1</sup>The set of logical truth values can be represented in various ways, such as  $\{T, F\}$ ,  $\{\top, \perp\}$ , or  $\{\text{True}, \text{False}\}$ .  $\{0, 1\}$  is applied in this book for simplicity.

$p$	$q$	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

### Biconditional

The **biconditional** of two propositions  $p$  and  $q$  is denoted by  $p \leftrightarrow q$ , which is true if and only if  $p$  and  $q$  have the same truth value. Its truth table is as follows:

$p$	$q$	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

## 1.2 Propositional Calculus

Propositional calculus is a formal system  $\mathcal{L} = (A, \Omega, Z, I)$ , whose formulas are constructed as follows:

- A The infinite set consisting of propositional variables or constants.
- $\Omega$  The infinite set consisting of logical connectives<sup>2</sup>.
- Z The infinite set consisting of inference rules.
- I The infinite set consisting of axioms(start point).

<sup>2</sup>It is divided into the following mutually disjoint subsets:

$$\Omega = \Omega_0 \cup \Omega_1 \cup \Omega_2 \cup \dots \cup \Omega_m,$$

where  $\Omega_j$  is the set of  $j$ -ary logical connectives(operators). In general,  $\Omega_0 = \{0, 1\}$ ,  $\Omega_1 = \{\neg\}$ ,  $\Omega_2 = \{\wedge, \vee, \rightarrow, \leftrightarrow\}$ .

## Chapter 2 First-Order Logic

## Chapter 3 Second-Order Logic

# Bibliography

[1] 作者, Title1, Journal1, Year1. *This is an example of a reference.*

[2] Author2, Title2, Journal2, Year2. *This is another example of a reference.*