Problem Set #4

MMAE 350 – Computational Mechanics

Unless stated otherwise, complete all problems using
Python, Matlab, or Mathematica.

Submit Problem Set via Blackboard as a single pdf file.

Problem #1

Consider the following data:

x	-7	-5	-1	0	2	5	6
y	15	12	5	2	0	-5	-9

a) Perform linear regression of this data set using hand calculations with the equations derived in the lecture notes to obtain the coefficients in the linear trial function

$$f(x) = c_0 + c_1 x.$$

b) Check your results for the coefficients in the trial function using a built-in function in Matlab, Python, or Mathematica.

c) Plot the data points as dots and the best-fit line as a solid line on the same figure.

Problem #2

Recall from the lecture notes that the coefficients in an exponential function

$$y = a_0 e^{a_1 x}$$

can be determined by converting the exponential into a linear regression problem.

a) Write a user-defined function that takes two vectors as its input arguments that contain the values of x_i and y_i $i = 1, \ldots, N$ in the data set, performs the linear regression, and returns the values of the coefficients a_0 and a_1.

b) Consider the following data giving the population of the earth in billions for selected years between 1850 and 2000:

Year	1850	1900	1950	1980	2000
Population	1.3	1.6	3.0	4.4	6.0

Assuming that this population growth data follows roughly an exponential growth curve, use your user-defined function from part (a) to determine the best-fit values of the coefficients a_0 and a_1 in the exponential function.

c) Use a built-in function in Matlab (lsqcurvefit), Python (curve_fit), or Mathematica(Fit) to determine the coefficients in the exponential function directly, i.e. without using linear regression, and compare with your result in part (b).

d) Plot the data and your best-fit curve. Using the best-fit curve [from part (b) or (c)], estimate the population in the year 1970.

Problem #3

A specimen is placed in an axial-load test machine. The applied tensile force F on the specimen leads to the length of the specimen becoming L. The axial stress σ_x and strain ϵ_x are defined by

$$\sigma_x = \frac{F}{A_0}\frac{L}{L_0}, \quad \text{and} \quad \epsilon_x = \ln\frac{L}{L_0},$$

where A_0 is the original unloaded cross-sectional area of the specimen, and L_0 is the original unloaded length of the specimen. Beyond the yield point of the stress-strain curve, the relationship between stress and strain can often be modeled by the relationship

$$\sigma_x = K\epsilon_x^m.$$

The values of force F and length L measured in an experiment are given by

F(kN)	24.6	29.3	31.5	33.3	34.8	35.7	36.6
L (mm)	12.58	12.82	12.91	12.95	13.05	13.21	13.35

F(kN)	37.5	38.8	39.6	40.4
L (mm)	13.49	14.08	14.21	14.48

The initial cross-sectional area is $A_0 = 1.25\times10^{-4}\ m^2$, and the original specimen length is $L_0 = 0.0125\ m$. Use a built-in function in Matlab (lsqcurvefit), Python (curve_fit), or Mathematica(Fit) to determine the best-fit coefficients K and m in the trial function. Plot the data and your best-fit curve.

Problem #4

Values of specific enthalpy h of an Argon plasma in equilibrium as a function of temperature are given by:

T(K)	5000	7500	10000	12500	15000	17500	20000
h (MJ/kg)	3.3	7.5	41.8	51.8	61	101.1	132.9

T(K)	22500	25000	27500	30000
h (MJ/kg)	145.5	171.4	225.8	260.9

Determine the cubic spline interpolating function for this data using a built-in function in Matlab, Python, or Mathematica. Using this spline function, interpolate the value of h for $T = 13000\ K$.