

# Recursive Optimization

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## 1 Kalman Filter as State Observer

Okay so first of all We restrict ourselves to steady-state stuff, so no more  $A(k)$ , now We just have the same  $A$  for all time. Same for  $H$ , same for  $Q, R$ .

Let's skip to the interesting bits:

### 1.1 Detectability

Is defined as  $\lim_{k \rightarrow \infty} z(k) = 0$  implies  $\lim_{k \rightarrow \infty} x(k) = 0$  for any initial condition  $x_0 \in \mathbb{R}^n$ .

So things are detectable is measurement going to zero also means that the state is going to zero. I suppose this implies that  $z$  is capturing information about  $x$ .

To formalize this, recall  $x(k+1) = Ax(k), z(k) = Hx(k)$ .

The idea then is that the system is detectable if and only if  $Hw \neq 0$  for any eigenvector  $w$  corresponding to an eigenvalue of the matrix  $A$  with magnitude greater or equal to one.

So what this mean?

Basically this just means that if there is some unstable direction for the system, which is characterized by the eigenvalue of  $A$  being more than one, We have to be able to detect it and keep track of it. If We can't, game over.

More linear algebra-y way of saying this is that no unstable bits of  $A$  can be in the null-space of  $H$ .

The formalization of this is as follows: subtract an eigenvalue of  $A$  from  $A$ , and then stack the result with  $H$ . If We still have full... Well I think  $A$  is a square matrix.

So how does that work? Well We are looking for full column rank. If We don't have full column rank, one of the columns must be a linear combination of the rest. According to this theorem, this must be true if one of the vectors from the top half is in the nullspace

This is known as the PBH test and actually it's pretty neat.

So, first of all, think of  $H$  in terms of eigenvectors of  $A$ . Just change the basis.

Now, by subtracting eigenvalues, We're knocking one eigenvector out at a time. So, if  $H$  does not have anything that goes in the direction of that eigenvector, We've just lost rank! And so the test will fail. Super neat.

## 1.2 Observability

Okay so detection was about being able to detect changes in the state, I think is something like a summary.

So what's observability? Well here We are dealing with a completely deterministic system. No noise what so ever.

The idea then is that given  $n$  measurements, We are able to determine the initial state, so everything is reversible. You can see everything. I suppose in detectability We could detect stuff, but there may be a range of initial states that lead us to our converged variance.

PBH test still applies, wee. Only difference is that We have to check all eigenvalues, not just poorly behaved ones.

## 1.3 asd