$\frac{21.5}{30}$

Na   Discussion Sectio

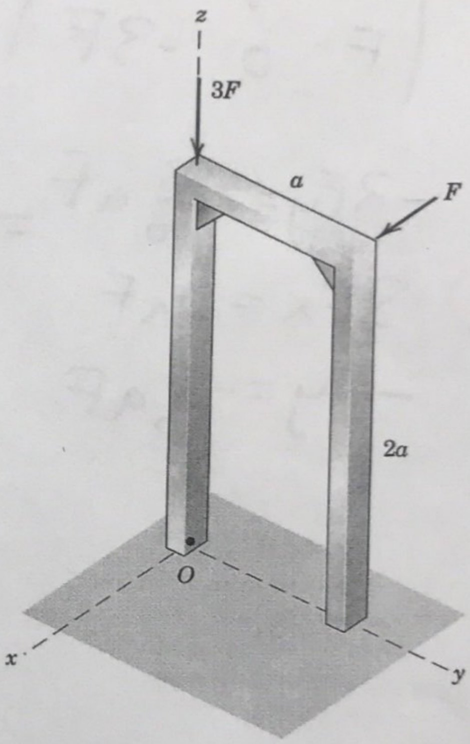
EMA 201: Exam #2

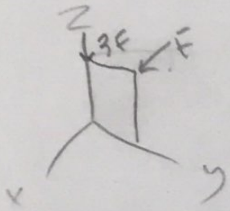
November 15th, 2017

This exam is closed book and closed notes except for the "cheat sheet" provided. Each problem is equally weighted. All work must be shown, ***including free body diagrams where appropriate***.

$\frac{9.5}{10}$

1. A door frame is subjected to two forces as shown in the figure.
   (a) Find an equivalent force system at the origin, *O*.
   (b) Replace the system in part (a) with a wrench and find where its line of action passes through the *yz*-plane. (Coordinate locations should be written in terms of length scale, *a*).





$\vec{F}_R = F\hat{i} - 3F\hat{k}$ (+1)

$\hat{U}_R = \frac{F}{\sqrt{F^2+(3F)^2}}\hat{i} - \frac{3F}{\sqrt{F^2+3F^2}}\hat{k}$ (+1)

$\vec{M}_{R_0} = (a\hat{j} + 2a\hat{k}) \times (F\hat{i}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & a & 2a \\ F & 0 & 0 \end{vmatrix} = 2aF\hat{j} - aF\hat{k}$ (+1)

$M_{\parallel} = \vec{M}_{R_0} \cdot \hat{U}_R = \frac{3aF^2}{\sqrt{F^2+3F^2}}$ (+.5)

(+1.5)

$\vec{M}_{\parallel} = \frac{3aF^2}{\sqrt{F^2+3F^2}}\left(\frac{F}{\sqrt{F^2+3F^2}}\hat{i} - \frac{3F}{\sqrt{F^2+3F^2}}\hat{k}\right) = \frac{3aF^3}{F^2+9F^2}\hat{i} - \frac{9aF^3}{F^2+9F^2}\hat{k}$

$= \frac{3}{10}aF\,\hat{i} - \frac{9}{10}aF\,\hat{k}$

$\vec{M}_{\perp} = \vec{M}_{R_0} - \vec{M}_{\parallel} = -\frac{3}{10}aF\,\hat{i} + 2aF\,\hat{j} - \frac{1}{10}aF\,\hat{k}$ (+1)

a) $\boxed{\begin{aligned}\vec{F}_R &= F\hat{i} - 3F\hat{k} \\ \vec{M}_{R_0} &= 2aF\hat{j} - aF\hat{k}\end{aligned}}$

b) $(x\hat{i} + y\hat{j}) \times (F\hat{i} - 3F\hat{k}) = -\frac{3}{10}aF\hat{i} + 2aF\hat{j} - \frac{1}{10}aF\hat{k}$

Interest in y-z plane not xy. $\rightarrow$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cancel{x}\ 0 & y & \cancel{0}\ z \\ F & 0 & -3F \end{vmatrix} = -3Fy\hat{i} + 3Fx\hat{j} - Fy\hat{k}$$ (+1)

$-3Fy = -\frac{3}{10}aF$

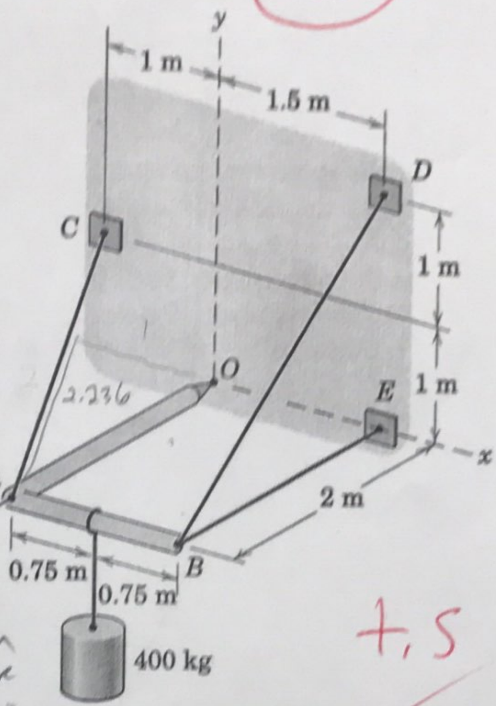
(+1.5) $3Fx = 2aF$ $\Rightarrow$ $\boxed{\begin{aligned}x &= \tfrac{2}{3}a \\ y &= \tfrac{1}{10}a\end{aligned}}$ (+1)

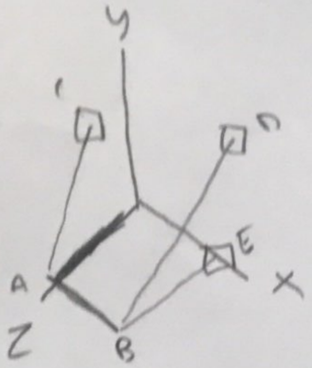
$-Fy = -\frac{1}{10}aF$

Nice!

$\boxed{6}$

2. The *L*-shaped boom *OAB* is supported by a ball-and-socket joint at *O* and by three cables. Given that the boom is supporting a mass of 400 kg, determine the cable tensions and the reactions at *O*.



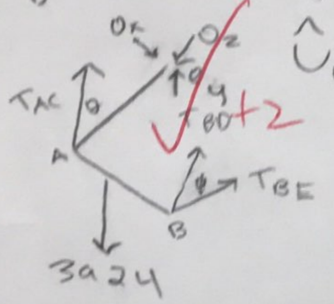


$T_{AC} = -1\hat{i} + 1\hat{j} - 2\hat{k}$

$\hat{U}_{AC} = -.4083\hat{i} + .4083\hat{j} - .8167\hat{k}$

$T_{BD} = 2\hat{j} - 2\hat{k}$

$\hat{U}_{BD} = .7072\hat{i} - .7072\hat{k}$ +.5



+2

$\phi = \tan^{-1}\left(\frac{2}{2}\right) = 45°$

+.5

$W = 400\,kg \cdot 9.81\,m/s^2$
$= 3924\,N$

$\Sigma F_y = 0: T_{AC}\sin(\cancel{26.56}) + T_{BD}\sin(45) - 3924 + O_y = 0$

$\Sigma F_z = 0: O_z - T_{AC}\cos(\cancel{26.56}) - T_{BD}\cos(45) - F_{BE}$

$\Sigma F_x = 0: \cancel{O_x - \frac{1}{\sqrt{2}} T_{AC} = 0}$

+1.5

$\Sigma M_o = 0: (2\hat{k} \times T_{AC}(-.4083\hat{i} + .4083\hat{j} - .8167\hat{k})) + (.75\hat{i} + 2\hat{k} \times -3924\hat{k}) +$
$(1.5\hat{i} + 2k \times T_{BD}(.7072\hat{i} - .7072\hat{k})) + (1.5\hat{i} + 2k \times T_{BE}(-2\hat{k})) = 0$

$= -T_{AC}\,.8166\hat{j} + 2943 + 2.475\,T_{BD}\hat{j} + 1.5T_{BE}\hat{j}$

$\Sigma M_{o_y} = 0: 2943 = -.8166T_{AC} + 2.475\,T_{BD} + 1.5T_{BE} \Rightarrow T_{BE} = 1962 - 1.65T_{BD}$

$\Sigma M_{o_x} = 0: -.8166T_{AC} = 0$

+2

$O_z = -.9429T_{BD} - 1962$

ΣM_{o_z}?

$\cancel{T_{AC} = 0}$

$\cancel{O_x = 0}$

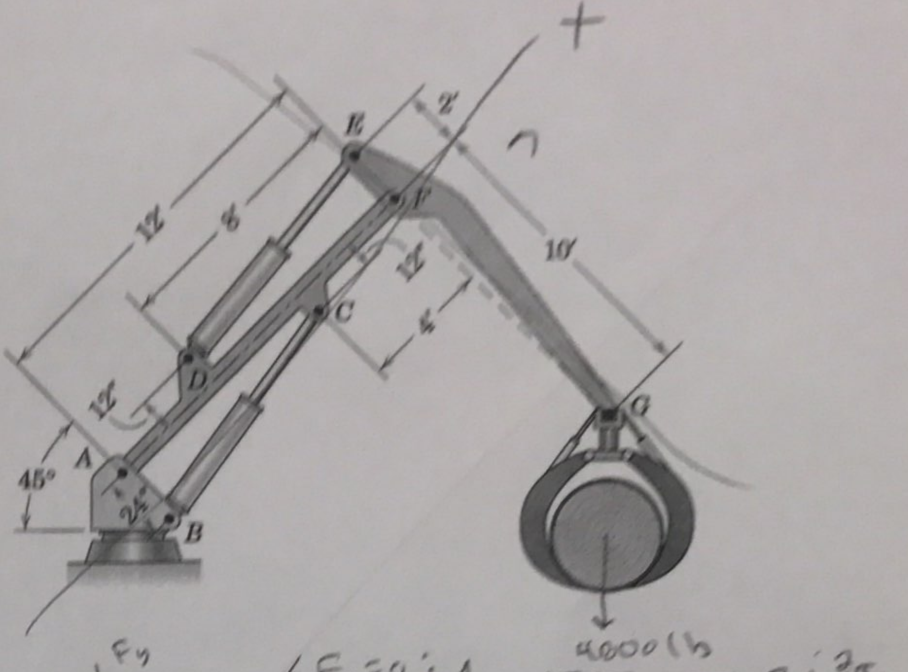
units? -.5

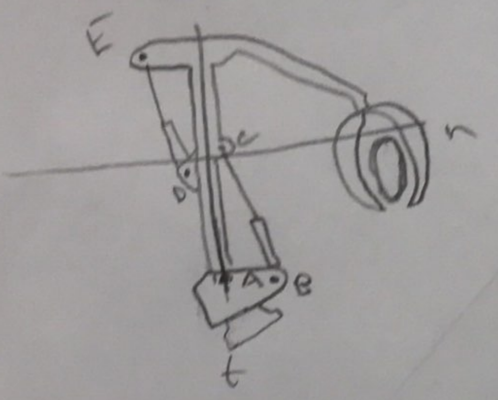
$O_y = 3924 - .7071T_{BD}$

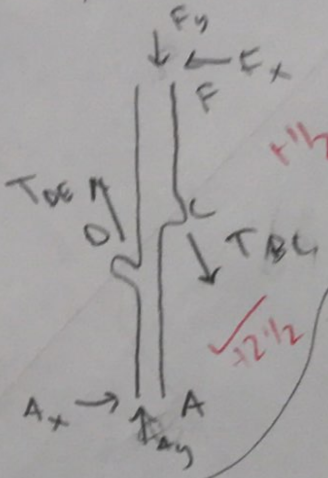
solutions? $T_{BD} = 1189 - .6061T_{BE}$

6/10

(3) This device is used to move logs in a lumber mill. At this instant, the booms *ADCF* and *EFG* are perpendicular to each other, and *AB* is also perpendicular to *ADCF*. What forces are supported by the hydraulic cylinders in this configuration for a log that weighs 4800 lb?
Hint: Given the geometry, it may be helpful to use a coordinate system normal and tangent to ADCF.
Note: The notation 12" means 12 *inches*, and the notation 4' means 4 *feet*.



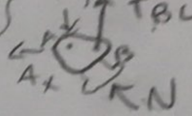




$\Sigma F_x = 0: A_x - 1753 + 1753 + \frac{2}{8} T_{BC}$

+1 1/2 $\Sigma F_y = 0: A_y + 3394 - 6788 - \frac{7.746}{8} T_{BC}$

$\Sigma M_A = 0:$

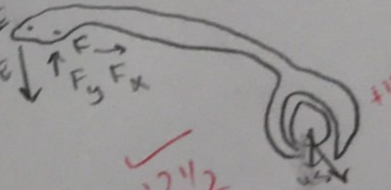


$\Sigma F_x = 0: -\frac{2}{8} T_{BC} - A_x$

$\Sigma F_y = 0: \frac{7.746}{8} T_{BC} - A_y$

$\Sigma M_A = 0: 15.492 T_{BC}$

not needed



+1 1/2 $\Sigma F_y = 0: F_y - 3394 - \frac{7.746 T_{ED}}{8}$

$\Sigma F_x = 0: F_x + \frac{2}{8} T_{ED}$

$\Sigma M_F = 0: 7.746 T_{DE} + 13576 + 40728$

From where do you get all these numbers?!

$T_{DE} = -7011$ ~~N~~ lb

$F_y = -3394$ ~~N~~

$F_x = 1753$ ~~N~~