Midterm 1 for Math 222, Lecture [redacted], Fall 2013

[redacted]

Your TA and Discussion (circle one):

[redacted]

Name: [redacted]

Total Score: 85

Problem 1 (15 pts): 8

Problem 2 (20 pts): 12

Problem 3 (20 pts): 20.

Problem 4 (15 pts): 15

Problem 5 (15 pts): 15

Problem 6 (15 pts): 15

CAUTION: No calculators are allowed. If you have question about the problems, please contact the proctor as soon as possible.

Formula Sheet

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$
$$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$$
$$2 \sin A \cos B = \sin(A + B) + \sin(A - B).$$

$$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C,$$
$$\int \sqrt{1-u^2}du = \frac{1}{2}u\sqrt{1-u^2} + \frac{1}{2}\arcsin u + C,$$
$$\int \frac{du}{\sqrt{1+u^2}} = \log(u + \sqrt{1+u^2}) + C,$$
$$\int \sqrt{1+u^2}du = \frac{1}{2}u\sqrt{1+u^2} + \frac{1}{2}\log(u + \sqrt{1+u^2}) + C,$$
$$\int \frac{du}{\sqrt{u^2-1}} = \log(u + \sqrt{u^2-1}) + C,$$
$$\int \sqrt{u^2-1}du = \frac{1}{2}u\sqrt{u^2-1} - \frac{1}{2}\log(u + \sqrt{u^2-1}) + C.$$

long division -7

1. (15pts) Compute

$$\int \frac{x^2+2}{x^2-3x+2}dx.$$

$$\int \frac{x^2+2}{x^2-3x+2}dx = \int \frac{x^2+2}{(x-2)(x-1)}dx = \int \frac{A}{x-2} + \frac{B}{x-1}$$

$$A(x-1) + B(x-2) = x^2+2$$

$x=1 \to$ $-B = 1+2$, $B=-3$ $\to 6\int \frac{1}{x-2}dx - 3\int \frac{1}{x-1}dx$

$x=2 \to$ $A = 2^2+2$, $A=6$

$$\boxed{= 6\ln|x-2| - 3\ln|x-1| + C}$$

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?

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2. (20pts) Is the following integral finite? If so, give an estimate of its value.

$$\int_0^\infty \frac{\sin x + x}{x^2+1}\,dx.$$

$$= \int_0^\infty \frac{\sin(x)}{x^2+1}\,dx + \int_0^\infty \frac{x}{x^2+1}\,dx$$

$$\int_0^\infty \frac{x}{x^2+1}\,dx = \int_0^1 \frac{x}{x^2+1}\,dx + \int_1^\infty \frac{x}{x^2+1}\,dx$$

$$\int_1^\infty \frac{x}{x^2+1}\,dx \geq \int_1^\infty \frac{x}{2x^2}\,dx \geq \frac{1}{2}\int_1^\infty \frac{1}{x}\,dx = \frac{1}{2}\left(\ln|x|\right)\Big|_1^\infty$$

$$\frac{1}{2} \cdot \lim_{M\to\infty} \left(\ln|x|\right)\Big|_1^M = \frac{1}{2}\left(\ln|M| - 0\right) = \frac{1}{2}(\infty)$$

$= \infty$, The integral is infinite because of the Tail Theorem

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$$4x - x^2 = -((2-x)^2 - 4) = 4 - (2-x)^2 = 4\left(1 - \left(\tfrac{2-x}{2}\right)^2\right)$$

$$= 4\left(1 - \frac{(2-x)^2}{4}\right) = 4 - (2-x)^2 = 4 - (4 - 4x + x^2) = 4x - x^2$$

3. (20pts) Compute

$$\int_0^4 \frac{1}{\sqrt{4x - x^2}}dx. \Rightarrow \lim_{x\to 0^+} \frac{1}{\sqrt{4x-x^2}} = \infty$$

$$\int_0^4 \frac{1}{\sqrt{4x-x^2}}dx = \int_0^1 \frac{1}{\sqrt{4x-x^2}}dx + \int_1^4 \frac{1}{\sqrt{4x-x^2}}dx \qquad \lim_{x\to 4^-} \frac{1}{\sqrt{4x-x^2}} = \infty$$

Indefinite: $\int \frac{1}{\sqrt{4x-x^2}} = \int \frac{1}{\sqrt{-((x-2)^2-4)}} = \int \frac{1}{\sqrt{4-(x-2)^2}} = \int \frac{1}{\sqrt{4\left(1-\left(\frac{x-2}{2}\right)^2\right)}}dx$

$$= \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{x-2}{2}\right)^2}}dx \qquad u = \frac{x-2}{2} \qquad 2\cdot du = \tfrac{1}{2}\cdot 2 \qquad = \int \frac{1}{\sqrt{1-u^2}}du = \arcsin(u) = \arcsin\left(\frac{x-2}{2}\right)$$

$$\int_0^1 \frac{1}{\sqrt{4x-x^2}}dx = \arcsin\left(\frac{x-2}{2}\right)\Big|_0^1 = \arcsin\left(\frac{-1}{2}\right) - \arcsin(-1)$$

$$\int_1^4 \frac{1}{\sqrt{4x-x^2}}dx = \arcsin\left(\frac{x-2}{2}\right)\Big|_1^4 = \arcsin(1) - \arcsin\left(\frac{-1}{2}\right)$$

$$= (\arcsin(1) - \arcsin(-1)) = \boxed{\pi}$$

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4. (15pts) Compute

$$\int x^2 \sin(3x)dx.$$

15

$f = \frac{-\cos(3x)}{3}$

$\int x^2 \sin(3x)dx$, $g = x^2$, $dg = 2x\,dx$, $df = \sin(3x)dx$

$f = \frac{\sin(3x)}{3}$

$= \frac{-x^2\cos(3x)}{3} + \frac{2}{3}\int x\cos(3x)dx$ $\quad g = x$, $dg = dx$, $df = \cos(3x)dx$

$= \frac{-x^2\cos(3x)}{3} + \frac{2}{3}\left(\frac{x\sin(3x)}{3} - \frac{1}{3}\int \sin(3x)dx\right)$

$$\boxed{= \frac{-x^2\cos(3x)}{3} + \frac{2x\sin(3x)}{9} + \frac{2\cos(3x)}{27} + C}$$

5. (15pts) Evaluate

$$\int \frac{sec^2\theta}{\sqrt{\tan^2\theta + 2\tan\theta + 2}} d\theta.$$

(Hint: $(\tan\theta)' = \sec^2\theta$.)

$u = \tan\theta + 1$

$du = \sec^2\theta$

$$\int \frac{\sec^2\theta}{\sqrt{\tan^2\theta + 2\tan\theta + 2}} = \int \frac{\sec^2\theta}{\sqrt{(\tan\theta+1)^2+1}}$$

$$= \int \frac{du}{\sqrt{u^2+1}} = \log\left(u + \sqrt{1+u^2}\right) + C$$

$$\boxed{= \log\left(\tan\theta + 1 + \sqrt{1 + (\tan\theta+1)^2}\right) + C}$$

6. (15pts) Solve the differential equation

$$(1+x^2)y' = e^y,$$

with initial value $y(0) = 0$.

$(1+x^2)y' = e^y$

$dy/dx = \frac{e^y}{1+x^2} \rightarrow e^y \neq 0$ (no Solutions lost)

$\int \frac{dy}{e^y} = \int \frac{dx}{1+x^2}$

$\frac{-1}{e^y} = \arctan(x) + C$

$\frac{-1}{e^0} = \arctan(0) + C$

$-1 = C$

$\frac{-1}{e^y} = \arctan(x) - 1$

$$\boxed{e^y = \frac{-1}{\arctan(x) - 1}}$$