# Richly Parameterized Linear Models

Hodges (2014). Richly Parameterized Linear Models: Additive, Time Series, and Spatial Models Using Random Effects.

## 1 An Opinionated Survey of Methods for Mixed Linear Models

The general notation of mixed linear models is $y = X\beta + Zu + \epsilon$, where the observation $y$ is explained through a design matrix $X$ connected to fixed effects $\beta$, a design matrix $Z$ connected to normal random effects $u$, and a normally distributed error $\epsilon$. The covariance matrix of the random effects $u$ (the errors $\epsilon$) is a function of unknowns $\phi_G$ ($\phi_R$). "All of the oddities and inconveniences examined in this book arise because $\phi_G$ is unknown." The author distinguishes between old-style (the underlying distribution is of interest) versus new-style random effects (also the levels themselves are of interest). Three examples are given, which can be reproduced in `R` with `{lme4}`. Estimation results have some oddities (non-convergence, zero variance estimates), probably due to non-identified models, too less data points, or numerical problems.

Estimation with conventional analysis 1. estimates $\phi$ (via maximizing the restricted likelihood to avoid bias) and 2. treats $\phi$ as known and estimates $\beta$ and $u$ (via maximizing the likelihood). This approach has some problems ... while Bayesian analysis alleviates some (?) problems.