

# Richly Parameterized Linear Models

Hodges (2014). Richly Parameterized Linear Models: Additive, Time Series, and Spatial Models Using Random Effects.

## 1 An Opinionated Survey of Methods for Mixed Linear Models

The general notation of mixed linear models is  $y = X\beta + Zu + \epsilon$ , where the observation  $y$  is explained through a design matrix  $X$  connected to fixed effects  $\beta$ , a design matrix  $Z$  connected to normal random effects  $u$ , and a normally distributed error  $\epsilon$ . The covariance matrix of the random effects  $u$  (the errors  $\epsilon$ ) is a function of unknowns  $\phi_G$  ( $\phi_R$ ). “All of the oddities and inconveniences examined in this book arise because  $\phi_G$  is unknown.” The author distinguishes between old-style (the underlying distribution is of interest) versus new-style random effects (also the levels themselves are of interest). Three examples are given, which can be reproduced in R with `{lme4}`. Estimation results have some oddities (non-convergence, zero variance estimates), probably due to non-identified models, too less data points, or numerical problems.

Estimation with conventional analysis 1. estimates  $\phi$  (via maximizing the restricted likelihood to avoid bias) and 2. treats  $\phi$  as known and estimates  $\beta$  and  $u$  (via maximizing the likelihood). This approach has some problems ... while Bayesian analysis alleviates some (?) problems.

## 2 Two More Tools: Alternative Formulation, Measures of Complexity

## 3 Penalized Splines as Mixed Linear Models

## 4 Additive Models and Models with Interactions