Time Series Home work 3

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Question 1

(a) No, the expected value must be independent of t

(b) $E[Y_t] = E[7 - 3t + X_t] = 7 - 3t + 3t = 7$

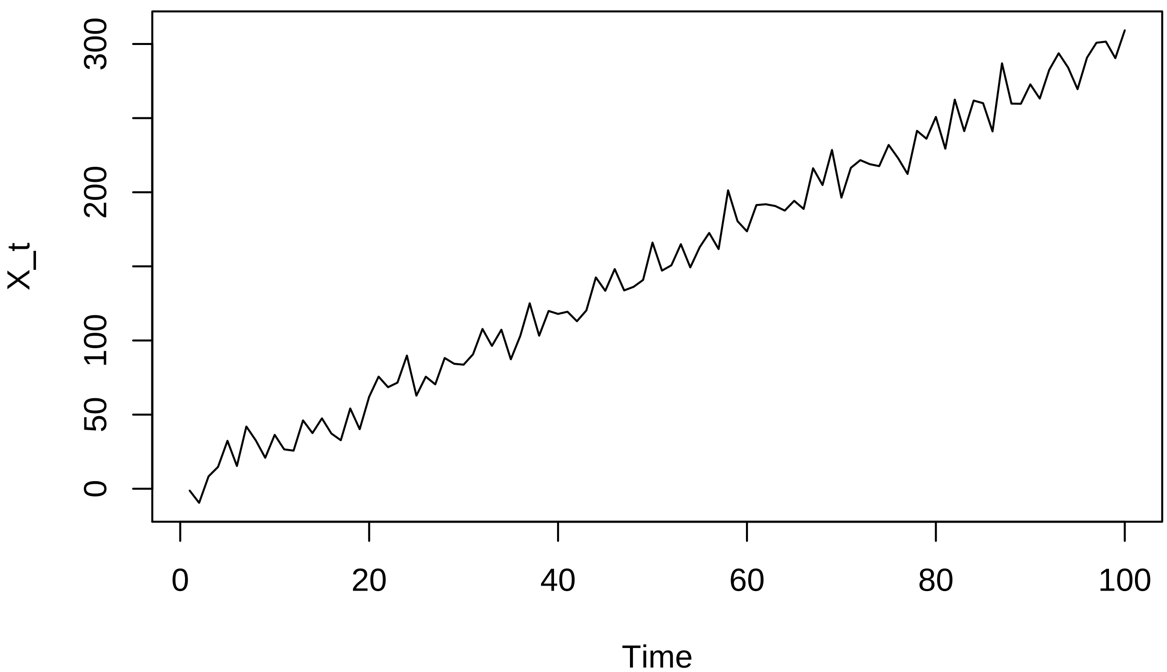
$Var(Y_t) = Var(7 - 3t + X_t) = Var(X_t) = Cov(X_t, X_t) = \gamma_0$

$Cov(Y_t, Y_{t-k}) = Cov(7 - 3t + X_t, 7 - 3t + X_{t-k}) = \gamma_k$

Yes, γ_t is stationary because mean, variance, and autocovariance are all independent of t

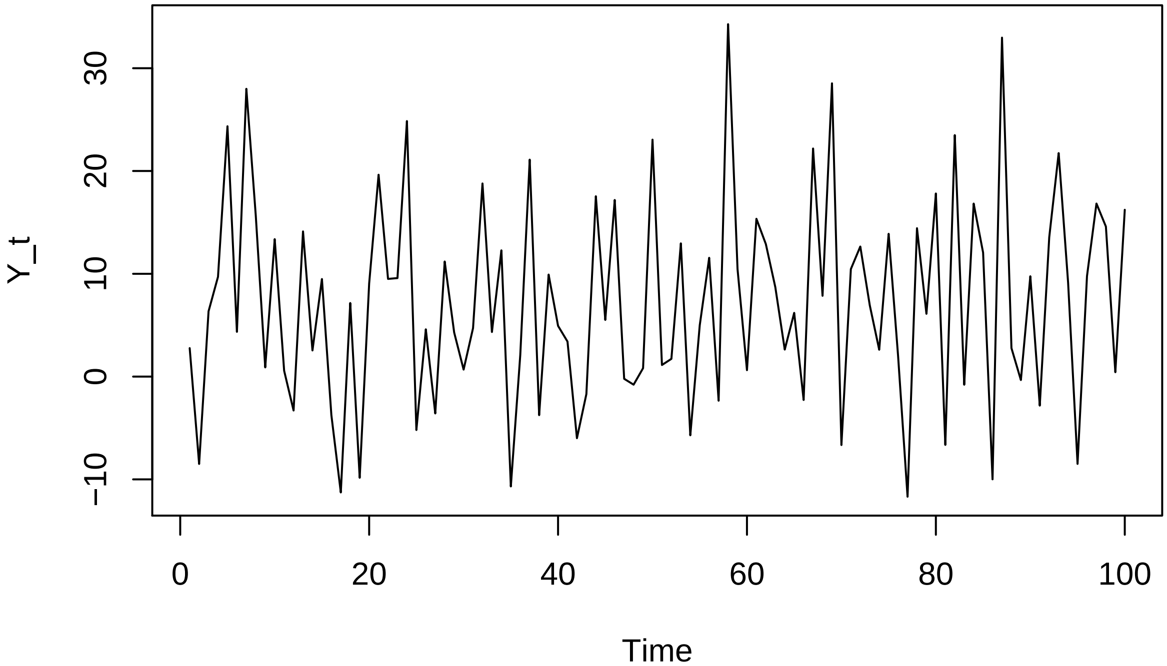
(c)

```
set.seed(2018)
time <- 1:100
noise <- rnorm(100, mean=0, sd=10)
X_t <- c()

for (t in time) {
  X_t[t] <- 3*t + noise[t]
}

plot.ts(X_t)
```




```
Y_t <- c()
for (t in time){
```

```r
  Y_t[t] <- 7 - 3*t + X_t[t]
}

plot.ts(Y_t)
```



Simulation is consistent with analytical solution.

Question 2

(a) odd t and k $cov(X_t, X_{t-k} + 3) = cov(X_t, X_{t-k}) = \gamma_k$

odd k even t or odd t even k $Cov(X_t, X_{t-k}) = \gamma_k$ even t and k $Cov(X_t + 3, X_{t-k} + 3) = cov(X_t, X_{t-k}) = \gamma_k$

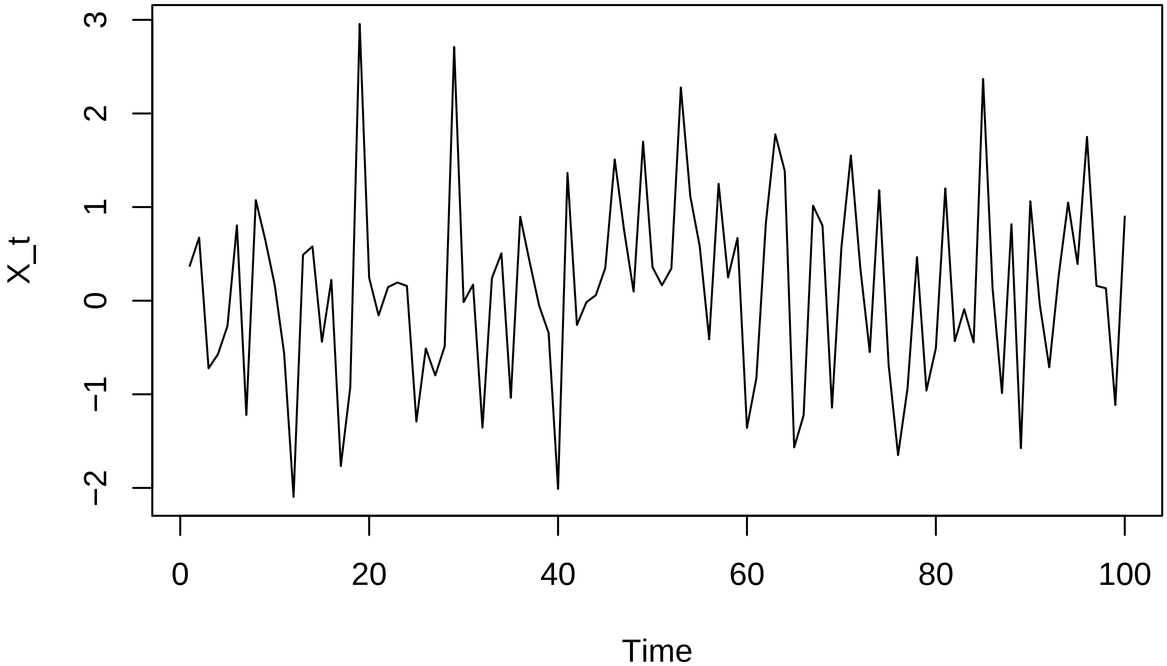
(b) t is odd $E[Y_t] = E[X_t] = constant$ t is odd $E[Yt] = E[Xt + 3] = 3 + constant$

$Var(Y_t) = Var(X_t + 3) = Var(X_t) = constant$

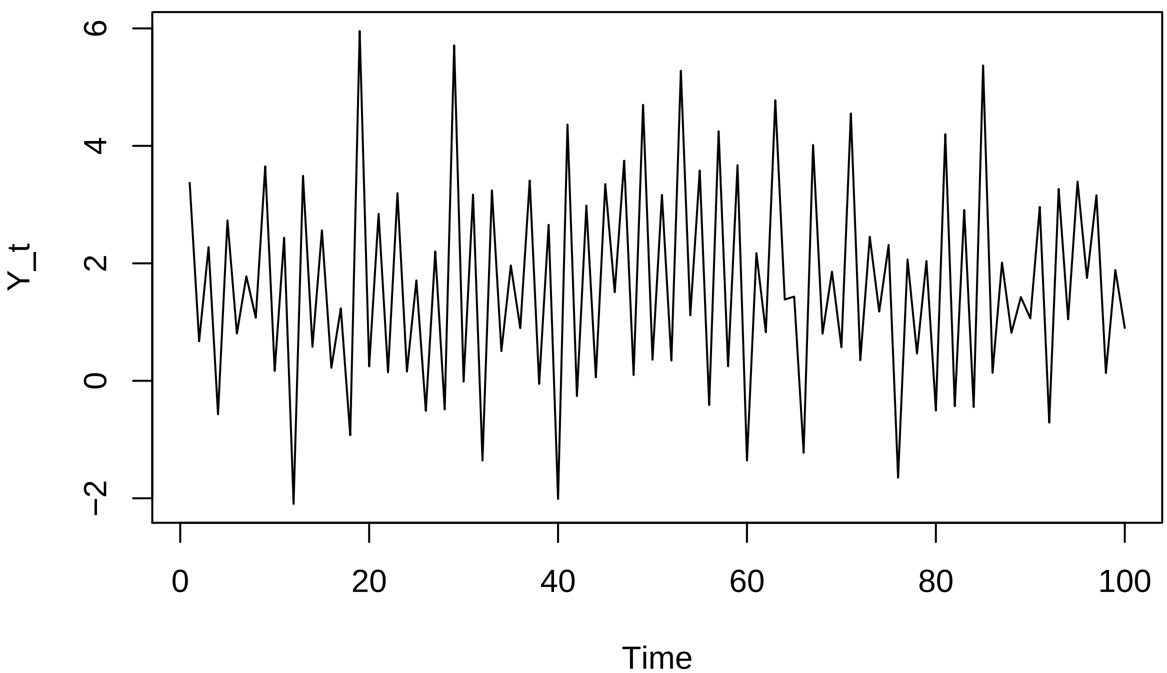
Yes, Y_t is stationary because mean, variance, and autocovariance are all independent of t

```r
X_t <- c()
for (t in time) {
  X_t[t] <- rnorm(1)
}

Y_t <- c()
for (t in time) {
  if (t %% 2 == 0) {
    Y_t[t] <- X_t[t]
  }
  else {
    Y_t[t] <- X_t[t] + 3
  }
}
```

```r
plot.ts(X_t)
```




```r
plot.ts(Y_t)
```



Simulation shows Y_t is indeed stationary using $X_t \sim N(0,1)$

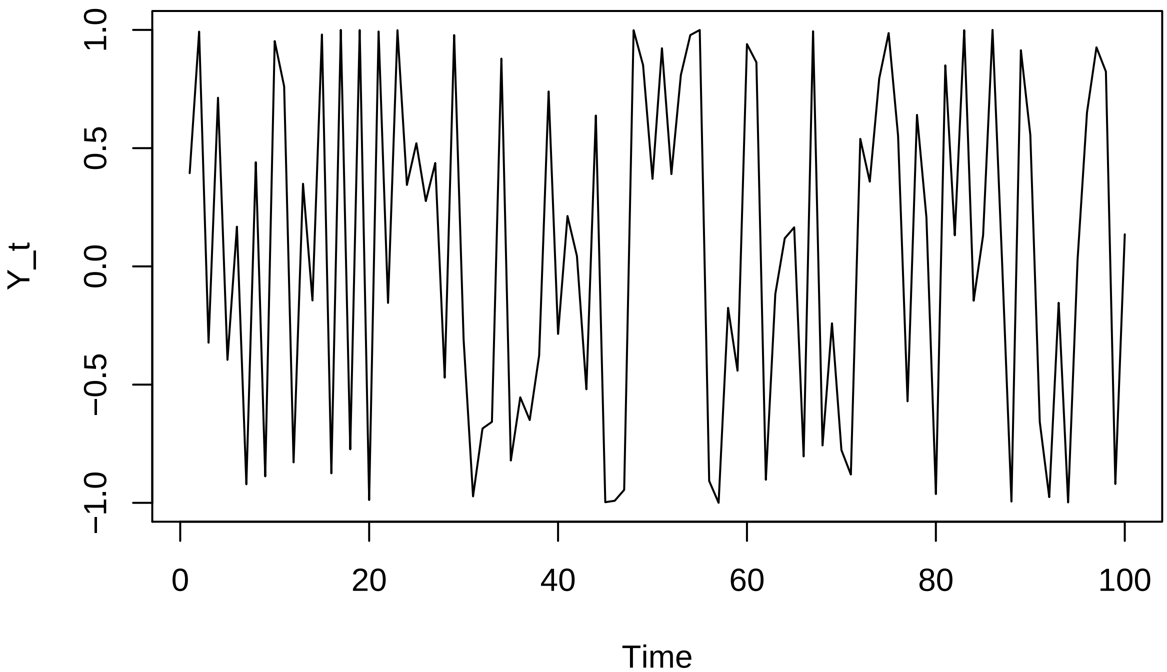
Question 3

(a) $E[Y_t] = \int Y_t d\varphi = \int cos(2\pi(\frac{t}{12} + \varphi)d\varphi = \frac{1}{2}\pi sin[2\pi(\frac{t}{12}) + \varphi]$ evaluated from 0 to 1 $= \frac{1}{2}\pi[sin(2\pi(\frac{t}{12} + 1)) - sin(2\pi(\frac{t}{12}))] = \frac{1}{2}\pi[0] = 0$

$Var[Y_t] = E[{Y_t}^2] - (E[Y_t])^2 = E[Y_t^2] = \int cos^2(2\pi(\frac{t}{12} + \varphi))d\varphi = \frac{1}{2}\int cos(4\pi(\frac{t}{12} + \varphi)) + 1d\varphi = \frac{1}{2}[\frac{1}{4\pi}sin(4\pi(\frac{t}{12} + \varphi)) + 1]$ evaluated from 0 to 1$= \frac{1}{8}\pi[sin(4\pi(\frac{t}{12} + 1)) - sin(4\pi(\frac{t}{12}))] + \frac{1}{2} = \frac{1}{2}$

(b)

```
Y_t <- c()
noise <- runif(100, min = 0, max = 1)
for (t in time) {
  Y_t[t] <- cos(2*pi*(t/12 + noise[t]))
}

plot.ts(Y_t)
```



Y_t does not show any trend, as it is stationary.