HW3

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Homework 3
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1. Suppose cov(Xt,Xtk) = gammak is free of t but that E(Xt) = 3t.

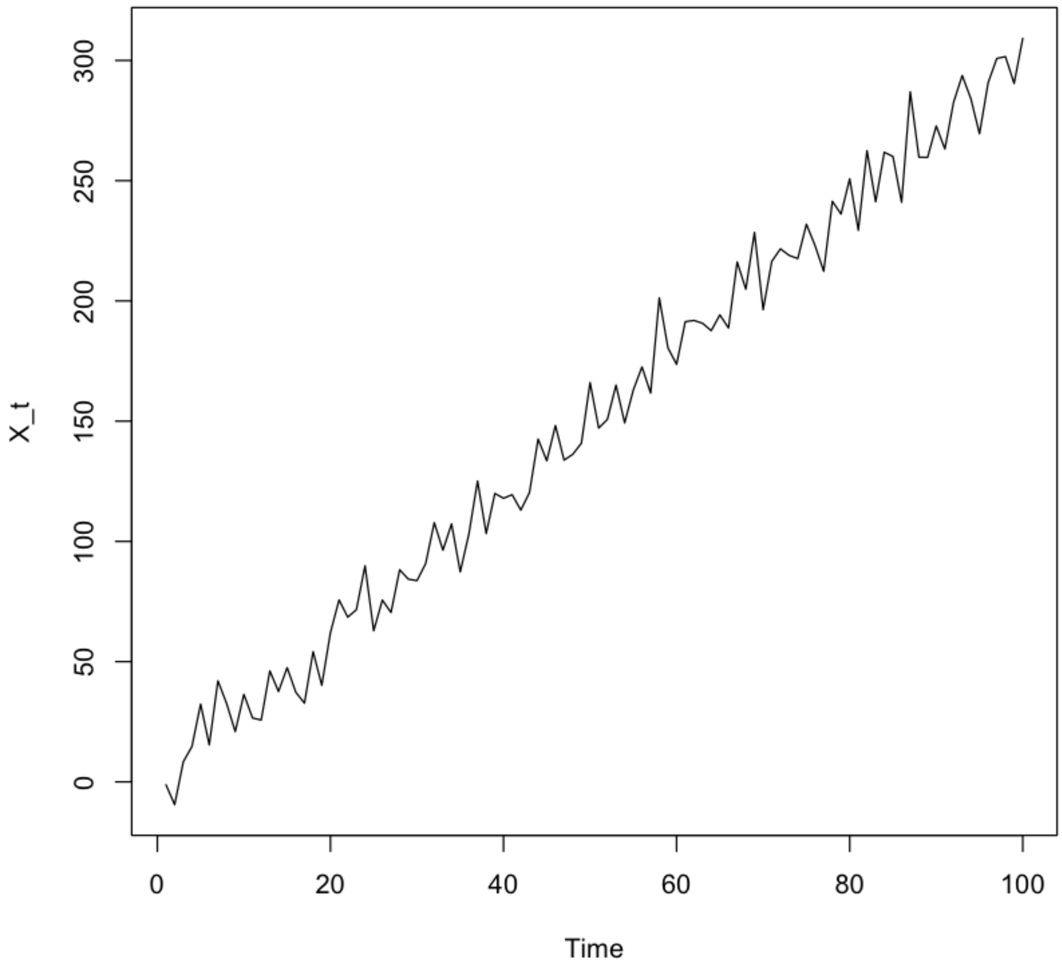
(a) Is Xt stationary? No, the expected value must be independent of t

(b) Let Yt=7-3t+Xt. Is Yt stationary? E[Yt] = E[7-3t+Xt] = 7 - 3t + 3t = 7

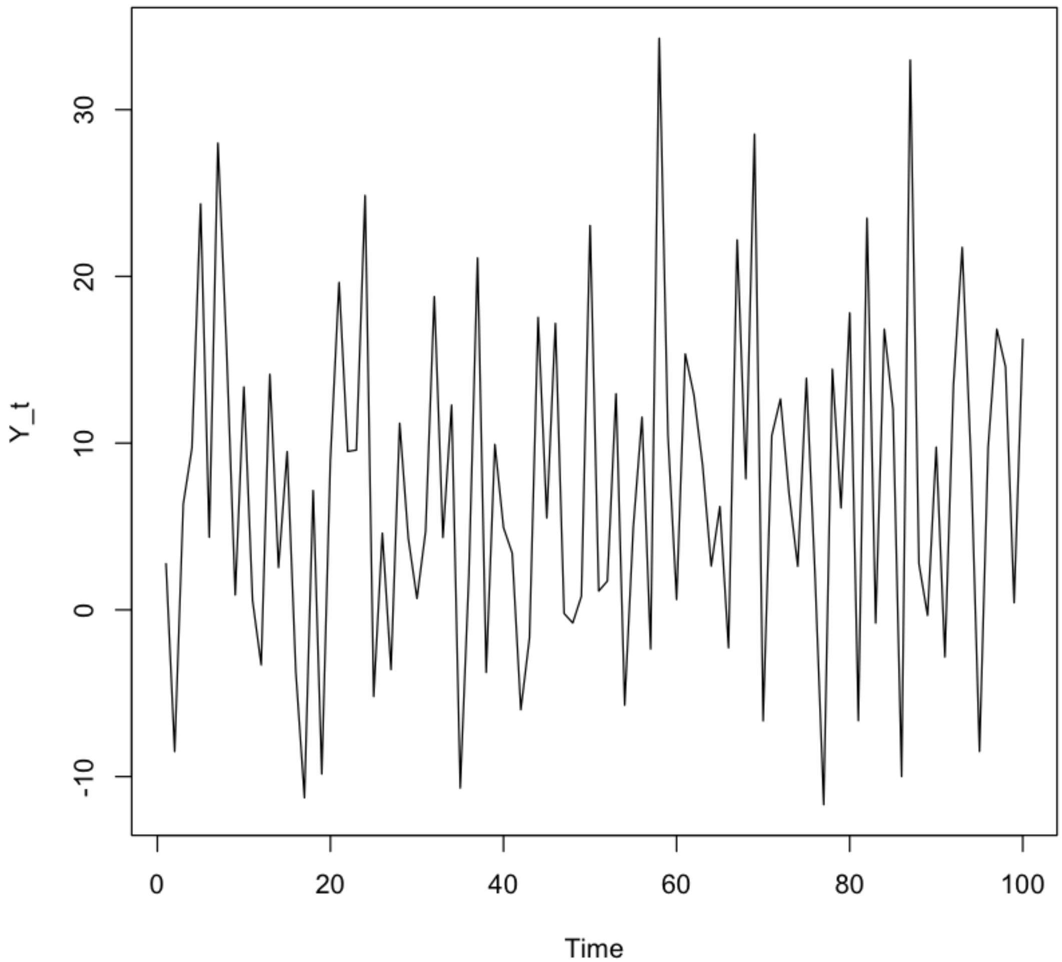
var(Yt) = var(7-3t+Xt) = var(Xt) = cov(Xt, Xt) = gamma0
cov(Yt, Yt-k) = cov(7-3t+Xt, 7-3t+Xt-k) = cov(Xt, Xt-k) = gammak
Yes, Yt is stationary because mean, variance, and autocovariance are all independent of t

(c) Simulate 100 observations for both Xt and Yt and plot as a time series. Check if your answer for parts a and b are consistent with your simulation.

```
In [6]: set.seed(2018)
        time <- 1:100
        noise <- rnorm(100, mean=0, sd=10)
        X_t <- c()

        for (t in time) {
          X_t[t] <- 3*t + noise[t]
        }

        plot.ts(X_t)


        Y_t <- c()
        for (t in time){
          Y_t[t] <- 7 - 3*t + X_t[t]
        }

        plot.ts(Y_t)
```



X_t
0
50
100
150
200
250
300
0
20
40
60
80
100
Time



Simulation is consistent with analytical solution.

2. Let Xt be a stationary time series, and define Yt = Xt when t is odd and Xt + 3 when t is even

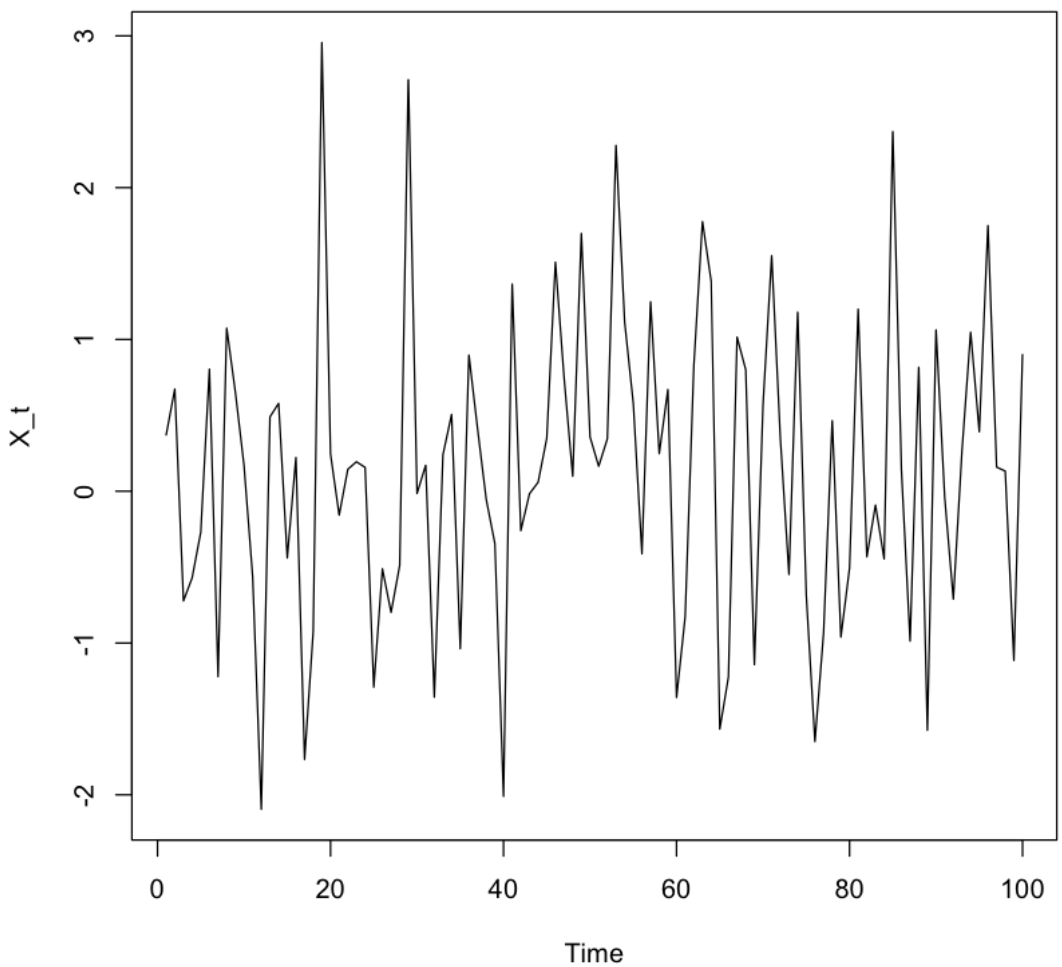
(a) Show that cov(Yt,Yt-k) is free of t for all lags k. odd t and k cov(Xt, Xt-k + 3) = cov(Xt, Xt-k) = gammak odd k even t or odd t even k cov(Xt, Xt-k) = gammak even t and k cov(Xt + 3, Xt-k + 3) = cov(Xt, Xt-k) = gammak

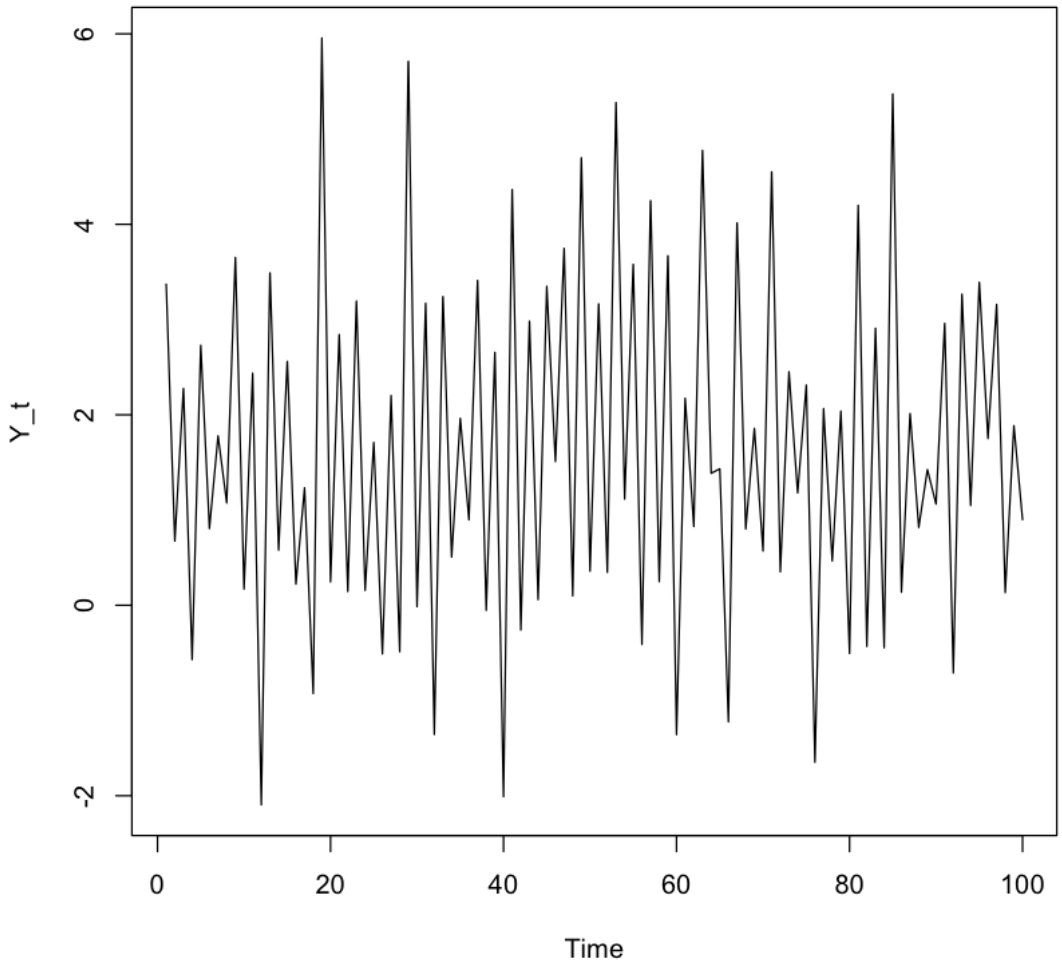
(b) Is Yt stationary? t is odd E[Yt] = E[Xt] = constant t is odd E[Yt] = E[Xt+3] = 3 + constant

var(Yt) = var(Xt + 3) = var(Xt) = constant
Yes, Yt is stationary because mean, variance, and autocovariance are all independent of t

(c) Plot 100 simulated observations for the random variable Yt and comment on your finding.

In [4]:
```
X_t <- c()
for (t in time) {
  X_t[t] <- rnorm(1)
}

Y_t <- c()
for (t in time) {
  if (t %% 2 == 0) {
    Y_t[t] <- X_t[t]
  }
  else {
    Y_t[t] <- X_t[t] + 3
  }
}

plot.ts(X_t)
plot.ts(Y_t)
```


X_t
Time
3
2
1
0
-1
-2
0
20
40
60
80
100



Simulation shows Yt is indeed stationary using Xt~N(0,1)

3. Random Cosine Wave Let Yt = cos(2pi(t/12 + phi)) where phi is selected from a uniform distribution on the interval 0 to 1.

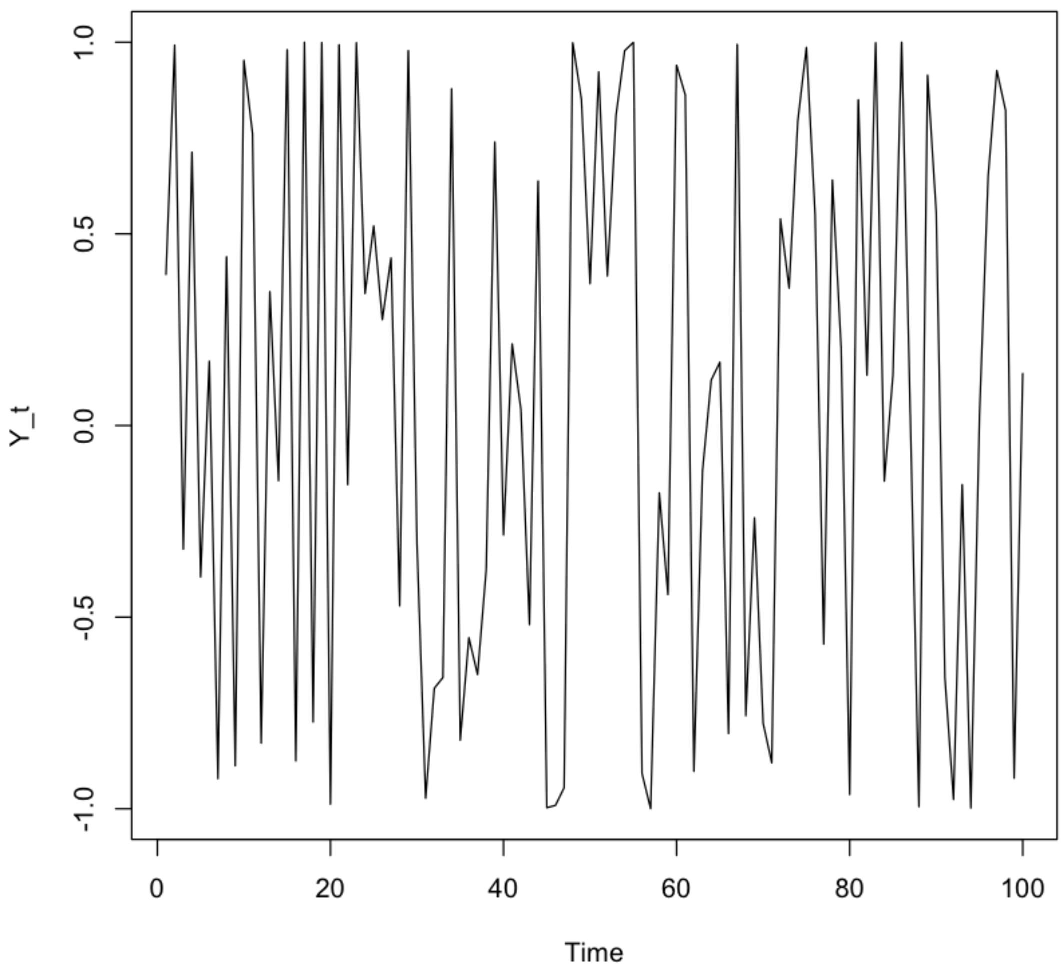
(a) Find the expected value and variance of Yt

Let I(x) means integral of x from 0 to 1 E[Yt] = I(Yt)dphi = I(cos(2pi(t/12 + phi))) dphi = 1/2pi sin[2pi(t/12)+phi] evaluated from 0 to 1 = 1/2pi [sin(2pi(t/12+1)) - sin(2pi(t/12))] = 1/2pi[0] = 0

Var[Yt] = E[Yt^2] - (E[Yt])^2 = E[Yt^2] = I(cos^2(2pi(t/12+phi)))dphi = 1/2 I(cos(4pi(t/12+))+1)dphi = 1/2[1/4pi sin(4pi(t/12+phi)) + 1] evaluated from 0 to 1 = 1/8pi[sin(4pi(t/12+1)) - sin(4pi(t/12))] + 1/2 = 1/2

(b) plot 100 simulated observations and check if there is any trend.

In [5]:
```
Y_t <- c()
noise <- runif(100, min = 0, max = 1)
for (t in time) {
  Y_t[t] <- cos(2*pi*(t/12 + noise[t]))
}

plot.ts(Y_t)
```



Yt does not show any trend, as it is stationary.