

# A Matrix Primer

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A matrix of size  $m \times n$  is a 2D grid of values, typically decimal numbers, with  $m$  rows and  $n$  columns. Element  $E[i, j]$  of a matrix, refers to the value in row  $i$  and column  $j$ . Element  $E[1, 1]$  is in the top left corner of the matrix and element  $E[m, n]$  is the bottom right corner of the matrix. A matrix can be added with another matrix, multiplied by a scalar, or multiplied by another matrix.

Two matrices can be added only if they have the same size. Addition is performed element-wise, meaning that if  $C = A + B$ , where  $C$ ,  $B$ , and  $A$ , are matrices, then element  $C[i, j] = A[i, j] + B[i, j]$  for all elements of  $C$ . E.g.,

$$\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Any matrix can be multiplied by a scalar, which is typically a decimal value. Scalar multiplication is performed element-wise, meaning that if  $C = sA$ , where  $s$  is a scalar value and  $A$  is a matrix, then  $C[i, j] = s \times A[i, j]$ . E.g.,

$$\begin{bmatrix} -3 & -6 \\ -9 & -12 \end{bmatrix} = -3 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Two matrices can be multiplied only if the width of the first matrix is equal to the height of the second matrix. If  $C = AB$ , where  $A$  is a matrix of size  $m \times n$ ,  $B$  is a matrix of size  $n \times p$ , and  $C$  is a matrix of size  $m \times p$ , then

$$C[i, j] = A[i, 1] \times B[1, j] + A[i, 2] \times B[2, j] + A[i, 3] \times B[3, j] + \dots + A[i, n] \times B[n, j]$$

E.g.,

$$\begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 29 & 40 & 51 \\ 39 & 54 & 69 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$