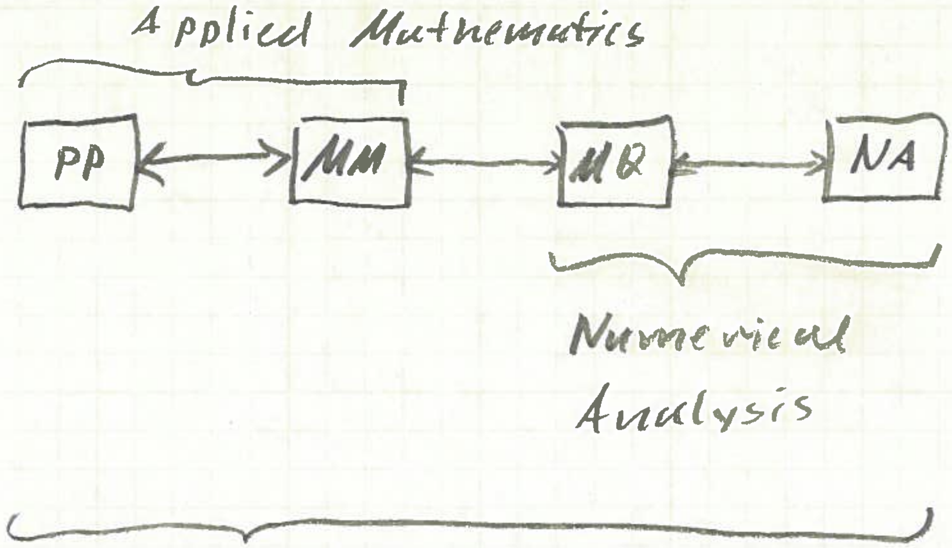
Math 5600 5/12/14

- Discuss Syllabus

- What is Numerical Analysis?



Scientific Computing
Computational Engineering & Science

- Focus: Design and Analysis of computer algorithms.

PP: Physical Problem

MM: Mathematical Model

MQ: Mathematical Question

NA: Numerical Answer

Specific subjects:

Linear Systems $Ax = b$
Eigenvalue Problems $Ax = \lambda x$

$$\left.\begin{array}{l}\text{Discrete}\\ \text{Continuous}\end{array}\right\} \left.\begin{array}{l}\text{Linear}\\ \text{Nonlinear}\end{array}\right\} \text{Approximation}$$

Interpolation $p(x_i) = f(x_i)$

Numerical Differentiation and Integration

Nonlinear Equations and systems

constrained, unconstrained optimization

IVP, BVP of ODEs, PDEs

Prerequisites:

- Solid Calculus, particularly Calculus of several variables
- Linear Algebra
- Programming

- Designing or studying algorithms without programming is like designing cars without ever driving one – nobody would buy the car you designed.

- For this class you need some ability to program in a language that allows Unix standard input and output, such as C, C++, java, python, Fortran ...

- 5600: surveys all of Num. Ana.

  5610-20: covers the same material in greater depth.

  6610-20 graduate level, focus on analysis

  6630 Numerical PDEs

  6875 Optimization.

- no official textbook

- If you want a basic text I recommend K.E. Atkinson, An Introduction to Numerical Analysis, Wiley, 1989, ISBN 0-471-62489-6

- It has the right emphasis and covers the standard topics, but is a bit dated
- Notes will be online

www.math.utah.edu/~pa/5600

and I'll mention, and show in class, many books.

- we'll spend the rest of today, and the next two days, reviewing some prerequisites
- On Friday, and next Monday, we'll discuss the centerpiece of this class, the term project on the global positioning system.

- vectors: $\mathbb{R}^n$
- vectors can be added and multiplied with scalers (numbers)
- linear combination

$$\alpha_i \in \mathbb{R} \quad i = 1, \dots, n$$

$$v_i \in \mathbb{R}^n \quad i = 1, \dots, n$$

$\sum_{i=1}^{n} \alpha_i v_i$ is a linear combination of $v_1, \dots, v_n$

The linear combination is "trivial" if $\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$

- The set $\{v_i\}$ is <u>linearly independent</u> if

$$\sum_{i=1}^{n} \alpha_i v_i = 0 \implies \alpha_1 = \alpha_2 = \dots = \alpha_n = 0$$

"implies that"

- Given a (finite) set B of vectors the <u>span</u> of B is the set of all linear combinations of vectors in B

- A <u>spanning set</u> B of a vector space S is a set of vectors in S such that every vector in S can be written as a linear combination of vectors in B
- A <u>basis</u> of S is a linearly independent spanning set
- All bases of a given space S have the same number of vectors.
- that number is the <u>dimension</u> of S
- functions form vector spaces.
- for example, polynomials of degree d form a vector space (linear space) of dimension $d+1$

- An $m \times n$ matrix defines a linear function

$$T(x) = Ax$$

domain is $\mathbb{R}^n$, range is $\mathbb{R}^m$

- $T(u+v) = T(u) + T(v)$ $\quad A(u+v) = Au + Av$

$T(\alpha u) = \alpha T(u)$ $\quad A(\alpha u) = \alpha A u$

- moreover, every linear function can be written as a matrix

$$A = [T(e_1), T(e_2), \dots, T(e_n)]$$

$$x = [x_i]_{i=1,\dots,n}$$

- Then $T(x) = T(\sum_{i=1}^{n} x_i e_i)$

by linearity,

$$= \sum_{i=1}^{n} x_i T(e_i)$$

by definition of Ax

$$= Ax$$

- $A = [a_{ij}]_{\substack{i=1,\dots,m \\ j=1,\dots,n}}$ is an $m \times n$ matrix
  "m by n"

columns
rows
entries
rank
square
singular
column space
row space
kernel

- Matrix Multiplication

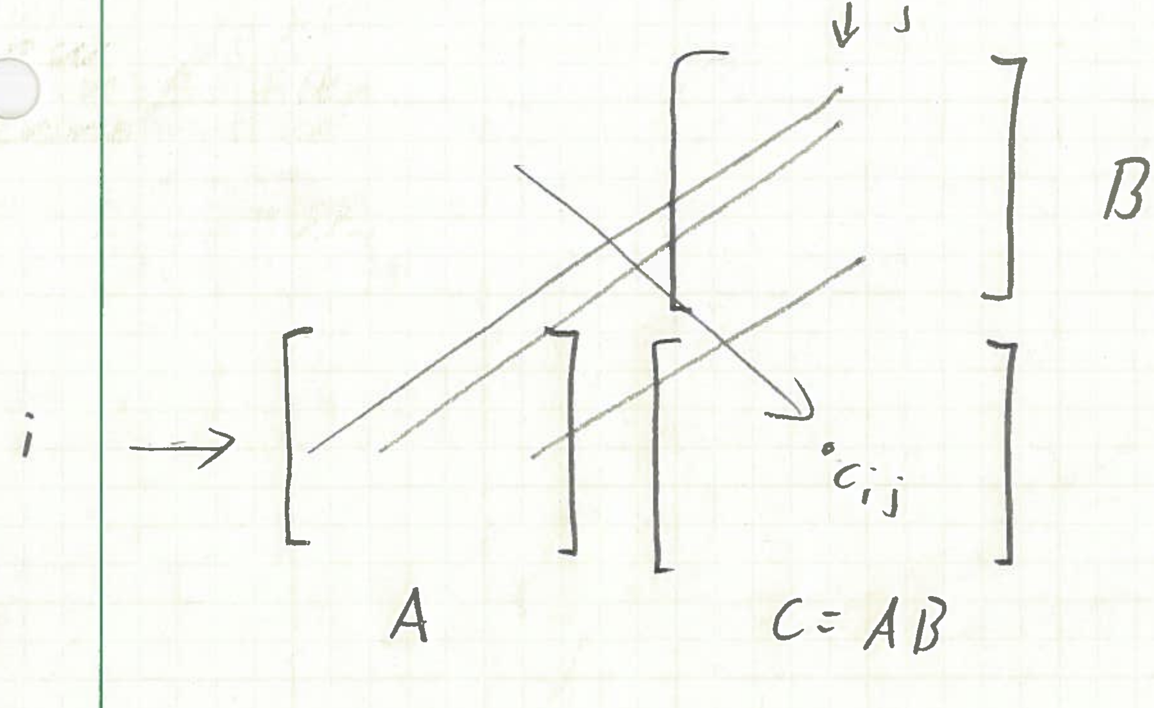
A $m \times p$
B $p \times n$
$C = AB$ $m \times n$

$$c_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj}$$

- why?

- linear functions can be composed
- the composition is also linear
- It can be represented by a matrix
- That matrix is the product of the two matrices representing the two composed functions

- to multiply two matrices write them like this



$$
\begin{array}{ccc}
 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\
\begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} & \begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix} & \begin{bmatrix} 25 & 36 \\ 11 & 16 \end{bmatrix}
\end{array}
$$

- several interpretations of $C = AB$

- The i,j entry of c is the dot product of the i-th row of A and the j-th column of B

- The j-th column of C is A times the j-th column of B

- The i-th row of C is the i-th row of A times B

- $$C = \sum_{i=1}^{p} \underbrace{\text{row}_i(A)\ \text{column}_j(B)}_{\text{rank 1 matrix}}$$

uv^T is a matrix!

$u \in \mathbb{R}^m \quad v \in \mathbb{R}^n \quad uv^T \in \mathbb{R}^{m \times n}$