

Evaluation of discrete-element method granular flow simulation

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Abstract

This work evaluates a discrete-element method simulation of granular hopper flow, implemented in Python using Taichi, by comparing its behavior to established properties of granular materials, including the Beverloo equation, depth-independent mass flow rate, and funnel flow. Depth-independent mass flow rate and funnel flow were both observed. Nine trials with different particle diameters were taken ($2 \cdot 10^5$ particles), and the mass flow rate was measured in each trial. While the 2.5 power of the Beverloo equation was recovered, the empirical constants C and k were not, possibly due to computational limitations and simplified contact modeling.

I. Introduction

This work evaluates the physical accuracy of a discrete-element granular hopper flow simulation implemented in Python using the Taichi programming language (see [1] for the simulation's code) [2]. Granular hopper flow has many well-known characteristics. These include the Beverloo equation, $W = C\rho_b\sqrt{g}(D - kd)^{2.5}$, where W represents mass flow rate of the granular material through the aperture of the hopper, C is a dimensionless constant typically in the range ~ 0.5 - 0.7 , ρ_b is the bulk density of the material, D is the diameter of the aperture, k is a dimensionless constant dependent on the shape of each grain (typically ~ 1.4 for spheres), and d is the diameter of each grain [3]. A consequence of the Beverloo equation is that mass flow rate in this situation is, unlike most liquids, independent of the depth of the granular material.

For a conical hopper with half angle $\Theta > \sim 20^\circ - 30^\circ$ (the exact value depends on the parameters of the materials involved), rather than every grain of the material flowing as one in a similar fashion to fluid flow, a flow column appears in a phenomenon called funnel flow [4]. This behavior is observed in a cross section of the simulated hopper flow (see [5]) [1].

II. Results

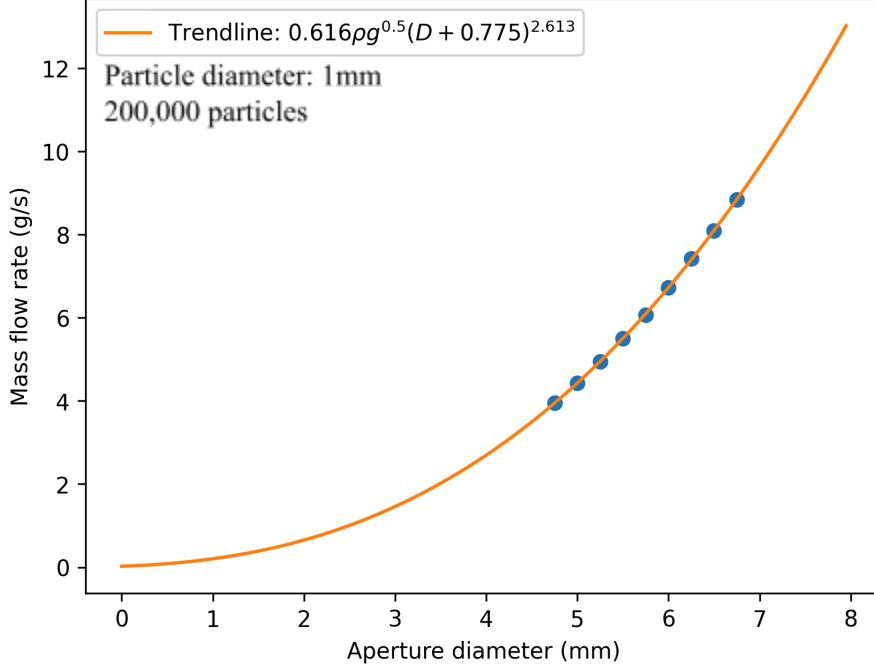


Figure 1: Variation of mass flow rate of conical hoppers with aperture diameter. Data were analyzed using `scipy.optimize.curve_fit()` in Python. Parameter uncertainties were estimated from the covariance matrix returned by the fit. Particles are uniform spheres all of diameter 1mm. See [1] for raw data and other parameters used.

$$C = 0.616 \pm 0.520 \quad k = -0.775 \pm 0.526 \quad \exp = 2.61 \pm 0.21$$

As shown in Figure 1, the parameters \exp and C agree with the accepted values within the stated uncertainty. The exceptionally large relative uncertainty in C , approximately 84%, indicates that the result should be interpreted with caution. This is likely due primarily to computational limitations that make it impractical to run many trials, as the simulations were executed on consumer-grade hardware.

The offset, k , is known to decrease with particle deformability [6]. Again due to computational limitations, the spring constant used to generate this data was 50 N/m. A higher spring constant, while more physically accurate, would have required too low of a dt value for stability to simulate practically. While the 50 N/m value used is high enough not to significantly interfere with the recovery of the 5/2 power, it is likely the primary cause of the deviation of the offset from the expected value. Common spring constant values in the literature are often close to 10^6 N/m [7]. The low spring constant causes the effective diameter of the aperture to be larger than the actual diameter, as particles are able to partially pass through the edges of the orifice. The relative uncertainty of about 147% in k indicates a highly unreliable result, but this could also be due to the aforementioned computational limitations. A definite conclusion cannot be drawn without additional data.

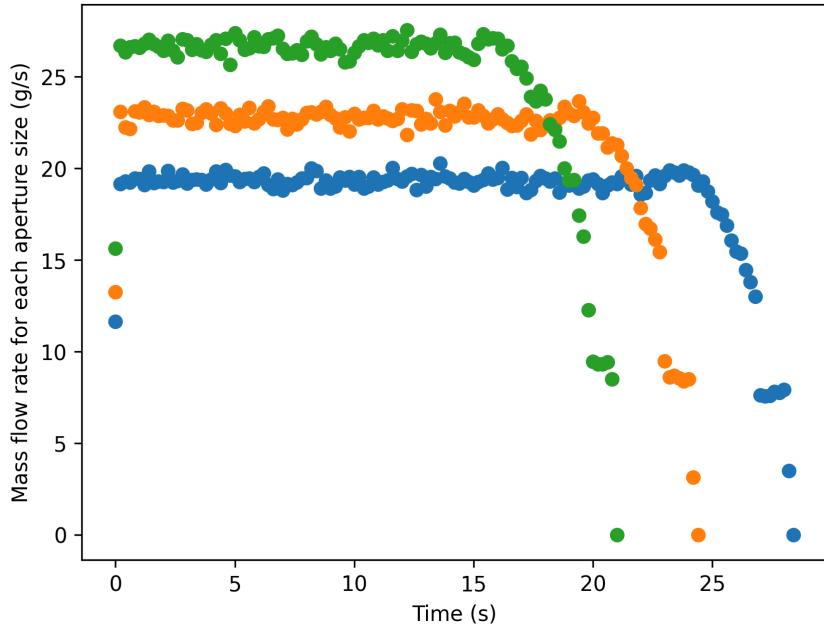


Figure 2: Mass flow rate of conical hoppers over time for varying aperture sizes. Particle diameter of 1mm, 200,000 particles. See [1] for raw data and other parameters used.

The mass flow rate is indeed shown by Figure 2 to be independent of depth (which varies continuously with time). The gradual drop to zero at the end is only due to the transition to mass flow visible in [5].

III. Discussion

Other than the inaccuracy of the offset discussed above, this simulation seems to match reality quite well, as shown by the adherence to the power of the Beverloo equation, the depth-independent mass flow rate, and the appearance of funnel flow. However, there are many ways in which it could be improved.

Particle-surface static friction is calculated in this simulation using the Cundall-Strack model,

$$F = -k_t \int v_t dt - c_t v_t [8], \text{ where } k_t \text{ is the tangential spring constant, } v_t \text{ is tangential velocity, and } c_t \text{ is}$$

the tangential friction damping coefficient, and appears sufficient to reproduce macroscopic flow characteristics; for comparison, see [9] for a simulation run with no particle-surface static friction, only kinetic friction. On the other hand, particle-particle static friction has not yet been added. It is likely not as vital to the recovery of the key characteristics explained above as particle-surface static friction because most particles — those in the static bulk of the material, outside of the flowing region — are cradled by normal forces from other particles in all directions (and so static friction is not as necessary to hold them up). However, it does play a role in accurate particle behavior near the aperture. Particle-particle static friction contributes to a phenomenon called the free-fall arch, in which transient arches of particles continuously form and fail above the aperture [10]. Below these arches, particles go into free fall. This behavior is not significantly observed in this simulation, likely due to a lack of particle-particle static friction. It is also entirely possible that the omission of particle-particle static friction contributes to the deviation of the offset, k , from the expected value.

Particles also seem to adhere to the walls of the cylinder as the hopper drains [5]. While particles are expected to stack near the walls to some extent, stacks as high as 20 particles have been observed in some trials of this simulation. This is clearly unphysical behavior, and while the cause is unclear, a possible candidate may be an issue with the normal force dashpot damping. This issue warrants further investigation.

Currently, the simulation uses a linear spring-dashpot model to describe normal forces. A Hertzian contact ($F \propto x^{3/2}$) would be more physically accurate [11]. The simulation would also benefit from the addition of particle rotation.

References

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