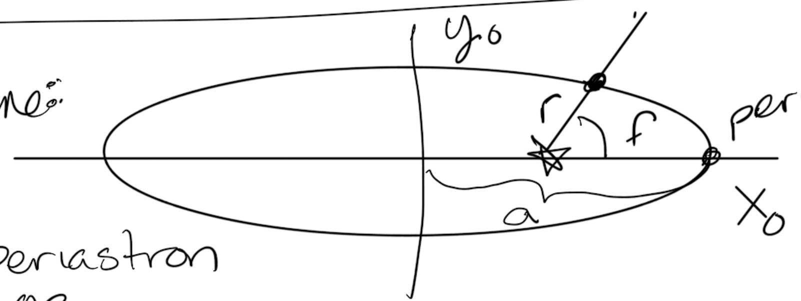
My equations for orbit fitting

$\boxed{\text{Determing observables from orbital elements:}}$

Orbital plane:



t_0 = time of periastron passage

t = time of observation

$n = \sqrt{\frac{\mu}{ma^3}} \leftarrow$ mean motion -or- orbital angular frequency

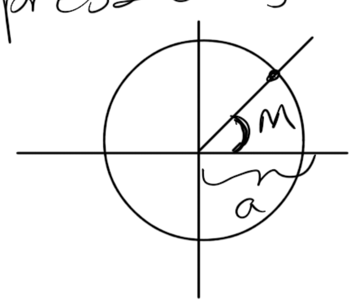
$n\left[\frac{1}{s}\right]$ $\quad n = \frac{2\pi}{P}$ $\quad P$ = period

$P = \sqrt{\frac{a^3}{m}} \leftarrow$ Kepler's 3rd law $a[AU]$ $m[M_\odot]$

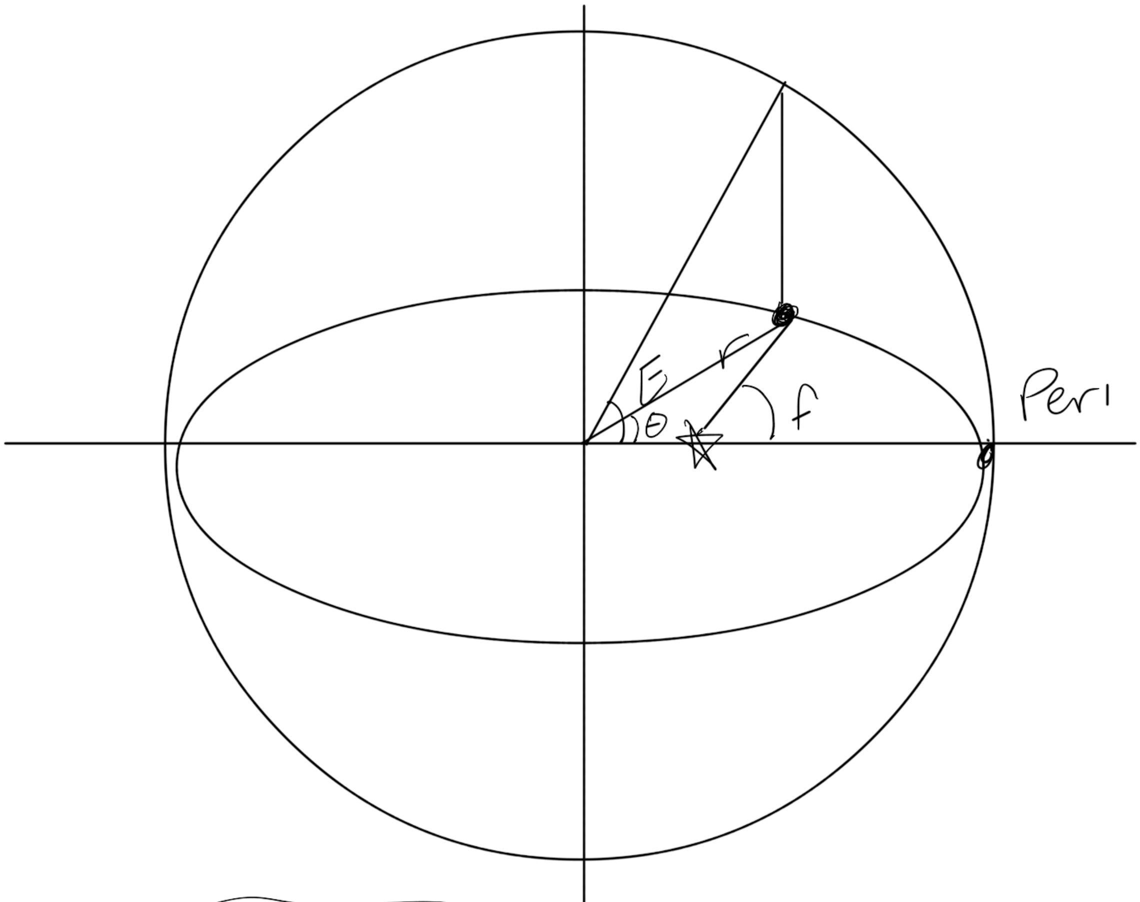
$M = n(t - t_0) \leftarrow$ mean anomaly

$= E - e\sin E$

= fraction of orbit completed at time of observation expressed as an angle in radians if on a circular orbit of radius a



E = eccentric anomaly deviation from circular due to eccentricity



$$\begin{cases} r\cos f = a(\cos E - e) \\ r\sin f = a\sqrt{1-e^2}\sin E \end{cases}$$

Kepler's constant:

$K = \frac{\mu}{m}$ where $\mu = Gm_1m_2$

$m = \frac{1}{m_1^{-1} + m_2^{-1}}$ reduced mass

$\left[\frac{m^3}{s^2}\right]$

$\boxed{\text{In the orbital plane:}}$

$$X_0 = (r\cos f, r\sin f, 0)$$
$$= (a(\cos E - e), a\sqrt{1-e^2}\sin E, 0)$$

$$V_0 = \left(-\frac{na\sin E}{1-e\cos E}, \frac{na\sqrt{1-e^2}\cos E}{1-e\cos E}, 0\right)$$

$\boxed{\text{In the plane of the sky, perform 3 rotations:}}$

$$X = R_z(\Omega) R_x(i) R_z(\omega) X_0$$
$$V = R_z(\Omega) R_x(i) R_z(\omega) V_0$$

where
$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

accelerations

I wasn't able to get accel via rot matricies, but it derive them explicitly and got correct answers, so these eqns are correct. These derivations were not trivial.

$$\ddot{x} = (\ddot{r} - r\dot{f}^2)(\cos\Omega\cos(\omega+f) - \sin\Omega\sin(\omega+f)\cos i$$
$$-(2\dot{r}\dot{f} + r\ddot{f})(\cos\Omega\sin(\omega+f) + \sin\Omega\cos(\omega+f)\cos i$$
$$\ddot{y} = (\ddot{r} - r\dot{f}^2)(\sin\Omega\cos(\omega+f) + \cos\Omega\sin(\omega+f)\cos i$$
$$+(2\dot{r}\dot{f} + r\ddot{f})(\sin\Omega\sin(\omega+f) + \cos\Omega\cos(\omega+f)\cos i$$
$$\ddot{z} = \left[(\ddot{r} - r\dot{f}^2)\sin(\omega+f) + (2\dot{r}\dot{f} + r\ddot{f})\cos(\omega+f)\right]\sin i$$

where

$$r = a(1-e\cos E) \qquad f = 2\arctan\left(\frac{\sqrt{1+e}}{\sqrt{1-e}}\tan\left(\frac{E}{2}\right)\right)$$
$$\dot{r} = \frac{aen\sin f}{\sqrt{1-e^2}} \qquad \dot{f} = \frac{n(1+e\cos f)}{1-e^2}\ \frac{\sin f}{\sin E}$$
$$\ddot{r} = ae\cos E\,\dot{E}^2 + ae\sin E\,\ddot{E}$$
$$\ddot{f} = \ddot{E}\frac{\sin f}{\sin E} - \dot{E}^2\ \frac{e\sin f}{1-e\cos E}$$

$$\dot{E} = \frac{n}{(1-e\cos E)} \qquad \ddot{E} = -\frac{n e \sin f}{(1-e^2)}\dot{f}$$

Compute Keplerian elements from observables

$h = \|\vec{r} \times \vec{v}\|$ where $\vec{r} = (x, y, z)$

$\vec{v} = (v_x, v_y, v_z)$

specific angular momentum

$\hat{n} = \dfrac{\vec{r} \times \vec{v}}{h}$ normal vector to orbital plane

$i = \cos^{-1}(\hat{n}_z)$

$\Omega = \arctan(\hat{n}_x, \hat{n}_y)$

$a = \left(\dfrac{2}{r} - \dfrac{v^2}{k}\right)$ where $r = \|\vec{r}\|$

$v = \|\vec{v}\|$

k = kepler's constant

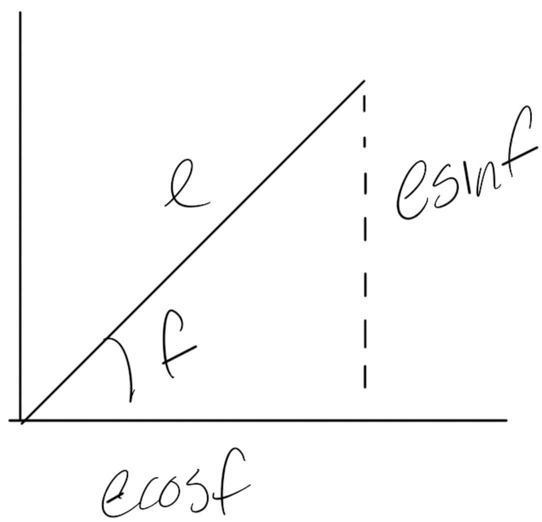
$k = \dfrac{\mu}{M} \left[\dfrac{m^3}{s^2}\right]$

parameter $p = \frac{P_\theta^2}{\mu m}$

$$h = \frac{P_\theta}{m} \Rightarrow h^2 = \frac{P_\theta^2}{m^2}\left[\frac{m}{\mu}\right] = \frac{P_\theta^2}{\mu m}$$

$\frac{1}{K}$

$$\Rightarrow p = \frac{h^2}{K}$$



$$e\sin f = \frac{p}{r} - 1$$

$$e\cos f = \frac{h\dot{r}}{K}$$

$$\dot{r} = \frac{(\vec{r} \cdot \vec{V})}{r}$$

$$e = \mathrm{sqrt}\left[e\sin f^2 + e\cos f^2\right]$$

$$f = \tan^{-1}\left(\frac{e\sin f}{e\cos f}\right)$$

define $u = \omega + f$

$$r\cos u = x\cos\Omega + y\sin\Omega$$

$$r\sin u = (-x\sin\Omega + y\cos\Omega)/\cos i$$

$$u = \tan^{-1}\left(\frac{r\sin u}{r\cos u}\right)$$

$$\omega = u - f$$

$$E = 2\arctan\left(\frac{\sqrt{1-e}}{\sqrt{1+e}}\tan\left(\frac{f}{2}\right)\right)$$

$$M = E - e\sin E$$

Done.

All of the above are a combination of Jack Wisdoms text and my own derivations.

Observationally:

Decl $= +x$

RA $= +y$

Towards observer $= +z$