ML

25/1/25

Given a dataset for performing linear regression with one variable.

Example:- Used car price prediction

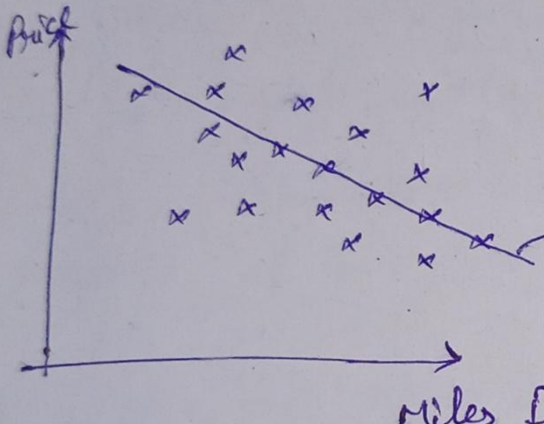
	Features		O/p
I/p →	Miles Driven	Engine Capacity	Price

But for simplicity purpose we are taking only one i/p.

Miles Driven	Price
1000	2.5
2520	3.6
⋮	⋮

let, 2D matrix
(1000, 2.5)
(2520, 3.6)

$y = mx + c$
$= \theta_1 x + \theta_0$



→ hypothesis
$h_\theta(x) = \underbrace{\theta_0 + \theta_1 x}_{}$
⇓ ⇓
Find Find

CSV (Comma Separate Vector) can store huge no. of datas in tabular form.

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1) What is machine learning?

2) What is learning?

3) What are the different types of m/l? → Supervised Learning → Unsupervised Learning → Semisupervised learning

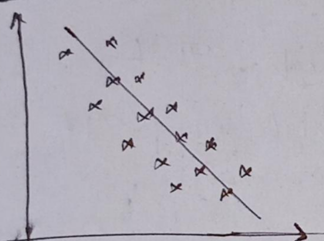
4) What is Supervised Learning?

5) What type of problem we can solve using Supervised learning?

↓ Regression ↓ Classification

6) What is Regression problem explain with a real-life example?

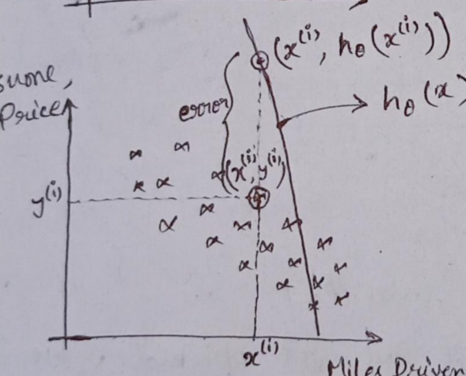
- Linear Regression with one variable



$$h_\theta(x) = \theta_0 + \theta_1 x$$

Initially choose θ_0 and θ_1 randomly.

Assume,



$\rightarrow h_\theta(x) = \theta_0 + \theta_1 x$

$$\text{error} = \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

~~We only need +ve values~~

The distance can be -ve but we only need magnitude, but we can't do mod(| |). Because, for later we have to do derivative but mod can make problem at then. So, we can do square.

Now, assume there are m data samples.

So, total no. of errors $= \underbrace{\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2}_{\text{mean squared error (MSE)}}$ → loss function

Minimize Error

$$\underset{\theta_0,\,\theta_1}{\text{Minimize}}\; J(\theta_0, \theta_1)$$

$$J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

When we have more than one variable then we need to do partial derivative.

$y = mx + c$
$\Downarrow$
slope

$$m = \frac{dy}{dx}$$

$$\frac{dJ(\theta_0, \theta_1)}{d\theta_0} \qquad \frac{dJ(\theta_0, \theta_2)}{d\theta_1}$$

Updating the value of θ_0, θ_1:

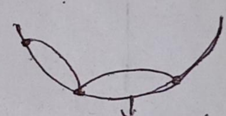
$$\theta_0 = \theta_0 - \alpha \underbrace{\frac{dJ(\theta_0, \theta_1)}{d\theta_0}}_{\text{gradiant}}$$

$$\theta_1 = \theta_1 - \alpha . \frac{dJ(\theta_0, \theta_1)}{d\theta_1}$$

This is called Gradiant Decent Algorithm for learning.

α = Learning rate

If $\alpha = 1$ then it means α is too high. That means it is seeing the slope and making a high jump. It can make prolelem.



reqd value

Steps:

(i) Take Random points
(ii) Calculate errors
(iii) Calculate gradiants
(iv) Calculate α - value
(v) Calculate new value for θ_0 and θ_1
(vi) Repeat from step (i) until we get optimal value

Performing Partial Derivative:

For θ_0

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2 \qquad h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)}\right)^2$$

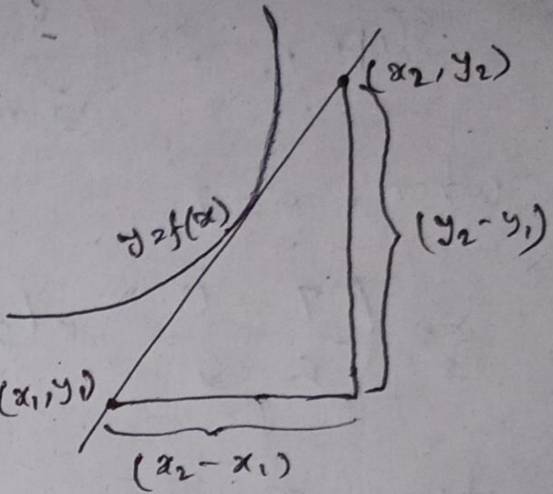
$$\frac{d}{d\theta_0} J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \frac{d}{d\theta_0} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)}\right)^2$$

$$= \frac{1}{2m} \sum_{i=1}^{m} \frac{d(\theta_0 + \theta_1 x^{(i)} - y^{(i)})^2}{d(\theta_0 + \theta_1 x^{(i)} - y^{(i)})} \times \frac{d(\theta_0 + \theta_1 x^{(i)} - y^{(i)})}{d\theta_0}$$

$$= \frac{1}{2m} \sum_{i=1}^{m} 2\left(\theta_0 + \theta_1 x^{(i)} - y^{(i)}\right)(1 + 0 - 0)$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)}\right)$$

$$\boxed{\frac{d}{d\theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)}$$

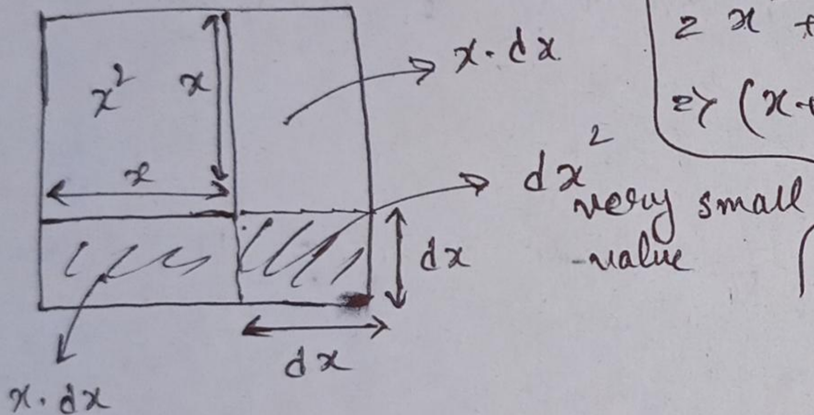


Now, for θ_1

$$\frac{d\, J(\theta_0, \theta_1)}{d\theta_1} = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}$$

$$(x + dx)^2$$
$$= x^2 + x\,dx + x\,dx + dx^2$$
$$\Rightarrow (x + dx)^2 - x^2 = 2x\,dx$$
$$\Rightarrow \frac{(x + dx)^2 - x^2}{dx} = 2x$$

$$\tan\theta = \frac{\text{height}}{\text{base}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$$



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Total Error $\rightarrow \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$

Avg. err

Mean Square Error $\rightarrow \boxed{\frac{1}{m}\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2} \rightarrow$ Loss function

(*) Gradiant Descent Algorithm

Linear Regression with multiple features:

Input — output

Miles Driven (x_1)	Engine Capacity (x_2)	Fuel Type (x_3)	Price (y)

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$

Fix $x_1 = 1$

$h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + \dots + \theta_n x_n$

$= \theta^T x$

$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1}$ $\quad x = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$

$\underset{(1 \times n)}{\begin{bmatrix} \theta_0 & \theta_1 & \dots & \theta_n \end{bmatrix}} \underset{(n \times 1)}{\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}}$

Repeat Control Convergent :

$$\theta_j = \theta_j - \alpha \frac{d}{d\theta_j} J(\theta) \qquad J = 0 \text{ to } n$$

ONE HOT ENCODING :

- Non-numerical Feature $\xrightarrow{\text{to}}$ vector

$$\begin{array}{cccc} C & P & D & E \\ \end{array}$$
$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

Fuel Type

→ Convert to

C	P	D	E

Normalization of Features : / Feature Scaling

[0 – 1]

min (age)
max (age)

$$\frac{x - \min}{\max} \rightarrow [0 - 1]$$

Age	Salary
31	80000
32	80001
40	240000

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Miles Driven	Engine Capacity	Year	No. of Pre owners	Fuel Types
2000	[-1 to 1]	[-1 to 1]	[-1 to 1]	CNG Des. Pet. Bat.

C	D	P	B
0	1	0	0
1	0	0	0

□ ≤ 0.5 → Class 0

□ > 0.5 → Class 1

0.5 is threshold value. If the probability value is 0.5 then the machine will think that there has 50% chances for both.

Qn:) What is the role of the sigmoid / logistic fn to determine the class?

- Cost fn and loss fn are same.

Regularization : Overfitting Problem

Under fit, Perfect fit, Overfit

ML

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Classification using Decision-Tree

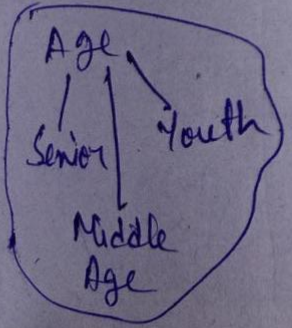
Feautes

Age, Income, Student? Creadit Rating

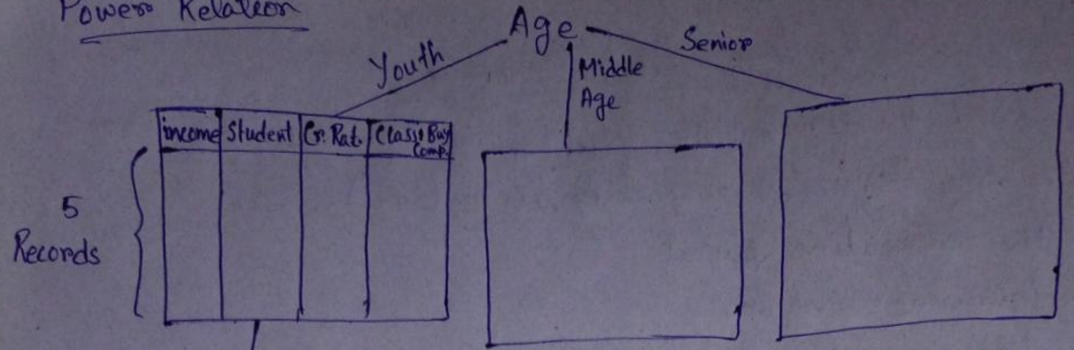
The prediction is basically buy computer or not.

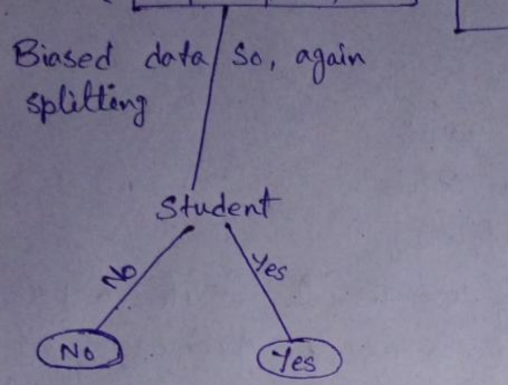
Example :

Age	Income	Student	Credit Rating	~~By~~ buy Comp.
Youth	High	No	Fare	No



Power Relation





① if age = 'middle age' then
Buy PC = Yes

② if age = 'youth' and
student = 'no'
Buy PC = No

If after splitting all the attributes, there left no attribute then go for majority voting. If the majority is 'yes' then all will be 'yes' and vice-versa.

Attribute Selection for Splitting

Information gain / Entropy

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

∴ m = no. of classes

• The unit of entropy is bits.

For our case,

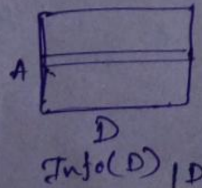
m = 2 → Buy computer = Yes 4 (p_1)

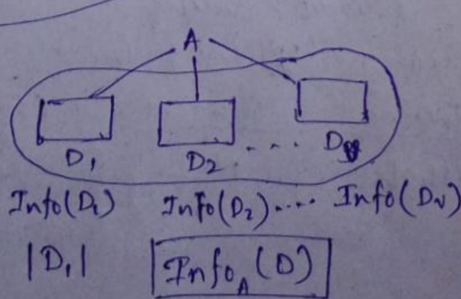
" " = No (p_2)

$IC \propto \frac{1}{P(A)}$

↓

Information Content





$$\text{Info}_A(D) = \frac{|D_1|}{|D|}\text{Info}(D_1) + \frac{|D_2|}{|D|}\text{Info}(D_2) + \dots\dots + \frac{|D_v|}{|D|}\text{Info}(D_v)$$

$$= \sum_{j=1}^{v} \frac{|D_j|}{|D|}\text{Info}(D_j)$$

$$\boxed{\text{Gain}(A) = \text{Info}(D) - \text{Info}_A(D)}$$

$$\text{Gain}(Age) = \text{Info}(D) - \text{Info}_{Age}(D)$$

$$\text{Info}(D) = -\sum_{i=1}^{2}(p_i \times \log_2(p_i))$$

$$= -p_1\log_2(D_1) - p_2\log_2(D_2)$$

$$= -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.94$$

$$= -0.64\log_2(0.64) - 0.36\log_2(0.36)$$

$$= (-0.64)\times(-0.45) - (0.36)\times(-1.02)$$

$$= 0.288 + 0.3672$$

$$= 0.28$$

$$\text{Info}_{Age}(D) = \frac{5}{14}\left[-\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right)\right] + \frac{5}{14}\left[-\frac{3}{5}\log_2\left(\frac{3}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right)\right] + \frac{4}{14}\left[0 - \frac{4}{4}\log_2\left(\frac{4}{4}\right)\right]$$

$$= \frac{10}{14}(0.97 + 0.97)$$

$$= 0.69285$$

$$\text{gain}(age) = 0.97 - 0.69 = 0.2465$$

$$\text{Info}_{Age}(D) = \sum_{j=1}^{v}\frac{|D_j|}{|D|}\text{Info}(D_j)$$

$$= \sum_{j=1}^{3}\frac{|D_j|}{|D|}\text{Info}(D_j)$$

$$= \frac{|D_1|}{|D|}\text{Info}(D_1) + \frac{|D_2|}{|D|}\text{Info}(D_2) + \frac{|D_3|}{|D|}\text{Info}(D_3)$$

$$= \frac{5}{14}\left[-\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log\frac{3}{5}\right] + \frac{4}{14}\left[-\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4}\right]$$

$$+ \frac{5}{14}\left[-\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5}\right]$$

$$= 0.6928$$

$$\text{gain}(age) = 0.97 - 0.69 = 0.28$$

Gain of Income = 0.029 bits
Gain of Student = 0.151 bits
Credit Rating = 0.048 bits

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Entropy

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

m : no. of classes

$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \cdot Info(D_j)$$

$$Gain(A) = Info(D) - Info_A(D)$$

$$Split\ Info_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2 \frac{|D_j|}{|D|}$$

According to Income attribute

$$= -\sum_{j=1}^{3} \frac{|D_j|}{|D|} \times \log_2 \frac{|D_j|}{|D|}$$

$$= -\frac{|D_1|}{|D|} \times \log_2 \frac{|D_1|}{|D|} - \frac{|D_2|}{|D|} \times \log_2 \frac{|D_2|}{|D|} - \frac{|D_3|}{|D|} \times \log_2 \frac{|D_3|}{|D|}$$

$$= -\frac{4}{14} \log_2 \frac{4}{14} - \frac{6}{14} \log_2 \frac{6}{14} - \frac{4}{14} \log_2 \frac{4}{14}$$

~~= 1.5564~~ = 1.5564

~~= 0.155455 + 0.15770 + 0.155455~~

$$Gain\ Ratio(A) = \frac{Gain(A)}{Split\ Info(A)}$$

$$= \frac{0.28}{1.5564} = 0.1799$$ Ans

$$Gini(D) = 1 - \sum_{i=1}^{m} p_i^2$$

$$Gain_A(D) = \frac{|D_1|}{|D|} Gini(D_1) + \frac{|D_2|}{|D|} Gini(D_2)$$

$$\Delta Gini(A) = Gini(D) - Gini_A(D)$$

$$\sum_{i=1}^{m} \underbrace{y^{(i)}}_{\text{Actual class}} \log_2 \underbrace{h_\theta(x^{(i)})}_{\text{Predicted class}}$$

Binary cross entropy loss

$$\left(\underbrace{\sum_{i=1}^{m} y^{(i)} \log_2 h_\theta(x^{(i)})}_{\text{Binary cross entropy for +ve class}} + \underbrace{\sum_{i=1}^{m} (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))}_{\text{Binary class entropy for -ve class}} \right)$$

ML

Receiver Operating Characteristics (ROC) Curve

if $g(\theta^T x) \geq 0.5 \Rightarrow$ Class 1 $\qquad h_\theta(x) = g(\theta^T x)$

$g(\theta^T x) < 0.5 \Rightarrow$ Class 0 $\qquad 0 \leq h_\theta(x) \leq 1$

Consider T_1 as thresold value

$$T_1 \longrightarrow S_1 \begin{cases} \rightarrow \text{TPR (True Positive Rate)} \\ \rightarrow \text{FPR (False Positive Rate)} \end{cases}$$