ML

25/1/25

Given a dataset for performing linear regression with one variable.

Example:- Used car price prediction

| I/p → Miles Driven | Engine Capacity | Price (o/p) |
|---|---|---|
| | | |

Features

But for simplicity purpose we are taking only one i/p.

| Miles Driven | Price |
| --- | --- |
| 1000 | 2.5 |
| 2520 | 3.6 |
| ⋮ | ⋮ |

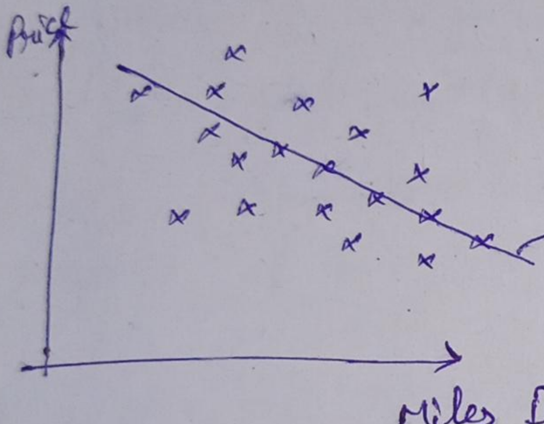
let, 2D matrix

(1000, 2.5)

(2520, 3.6)

$y = mx + c$

$= \theta_1 x + \theta_0$




→ hypothesis

$h_\theta(x) = \theta_0 + \theta_1 x$

$\underbrace{\underset{\text{Find}}{\Downarrow} \quad \underset{\text{Find}}{\Downarrow}}$

CSV (Comma Separate Vector) can store huge no. of datas in tabular form.

25/1/25

1) What is machine learning?

2) What is learning?

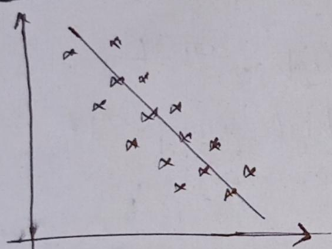
3) What are the different types of m/l? → Supervised Learning → Unsupervised Learning → Semisupervised learning

4) What is Supervised Learning!

5) What type of problem we can solve using Supervised learning!
↓ Regression ↓ Classification

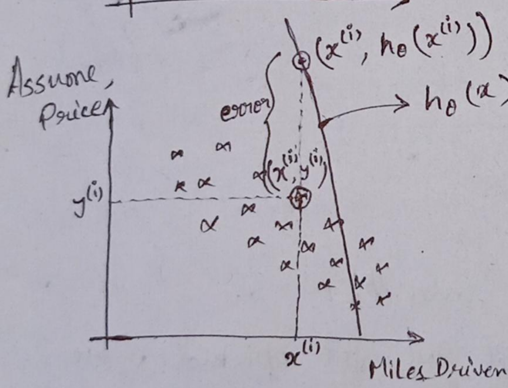
6) What is Regression problem explain with a real-life example?

- <u>Linear Regression with one variable</u>



$h_\theta(x) = \theta_0 + \theta_1 x$

Initially choose $\theta_0$ and $\theta_1$ randomly.




→ $h_\theta(x) = \theta_0 + \theta_1 x$

$\text{error} = \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

~~We only need +ve values~~

The distance can be -ve but we only need magnitude, but we can't do mod(| |). Because, for later we have to do derivative but mod ~~make~~ can make problem at then. So, we can do square.

Now, assume there are m data samples.

So, total no. of errors $= \underbrace{\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2}_{\text{mean squared error (MSE)}}$ → loss function

<u>Minimize Error</u>

$\underset{\theta_0, \theta_1}{\text{Minimize}}\ J(\theta_0, \theta_1)$

$J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left( h_\theta(x^{(i)}) - y^{(i)} \right)^2$

When we have more than one variable then we need to do partial derivative.

$y = mx + c$
$\Downarrow$
slope

$m = \frac{dy}{dx}$

$\frac{d\,J(\theta_0, \theta_1)}{d\theta_0}$ $\frac{d\,J(\theta_0, \theta_2)}{d\theta_1}$

**Updating the value of $\theta_0, \theta_1$:**

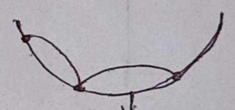
$\theta_0 = \theta_0 - \alpha \frac{d\,J(\theta_0, \theta_1)}{d\theta_0} \rightarrow$ gradiant

$\theta_1 = \theta_1 - \alpha . \frac{d\,J(\theta_0, \theta_1)}{d\theta_1}$

This is called Gradiant Decent Algorithm for learning.

$\alpha$ = Learning rate

If $\alpha = 1$ then it means $\alpha$ is too high. That means it is seeing the slope and making a high jump. It can make problem.



reqd value

**Steps:**

(i) Take Random points
(ii) Calculate errors
(iii) Calculate gradiants
(iv) Calculate $\alpha$ - value
(v) Calculate new value for $\theta_0$ and $\theta_1$
(vi) Repeat from step (i) until we get optimal value

**Performing Partial Derivative:**

For $\theta_0$/

$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left( h_\theta(x^{(i)}) - y^{(i)} \right)^2$ $h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$

$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left( \theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)^2$

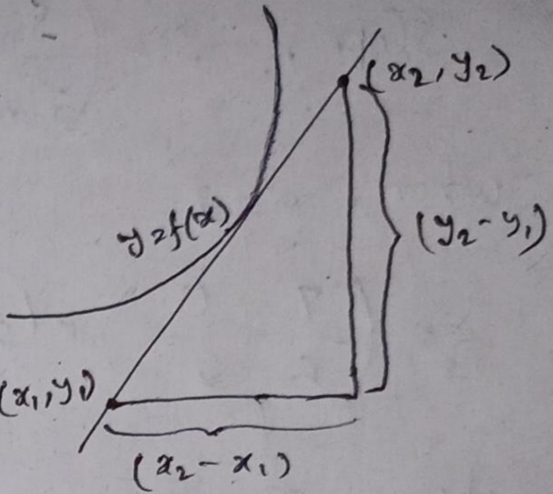
$\frac{d}{d\theta_0} J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \frac{d}{d\theta_0} \left( \theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)^2$

$= \frac{1}{2m} \sum_{i=1}^{m} \frac{d(\theta_0 + \theta_1 x^{(i)} - y^{(i)})^2}{d(\theta_0 + \theta_1 x^{(i)} - y^{(i)})} \times \frac{d(\theta_0 + \theta_1 x^{(i)} - y^{(i)})}{d\theta_0}$

$= \frac{1}{2m} \sum_{i=1}^{m} 2(\theta_0 + \theta_1 x^{(i)} - y^{(i)})(1 + 0 - 0)$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)}\right)$$

$$\boxed{\frac{d}{d\theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)}$$

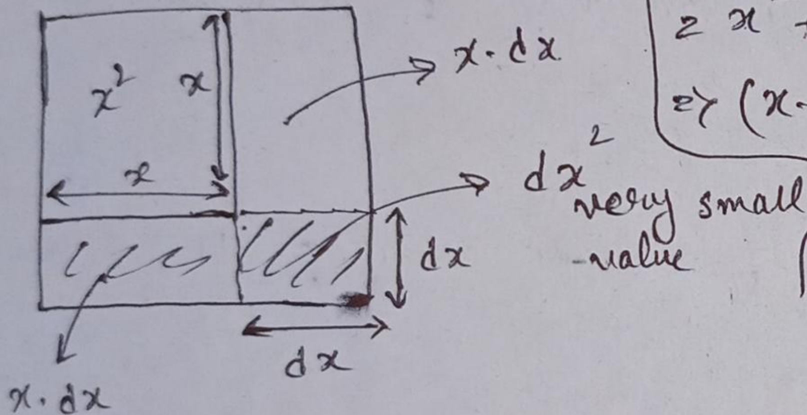



Now, for $\theta_1$

$$\frac{d J(\theta_0, \theta_1)}{d\theta_1} = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}$$

$$(x + dx)^2$$
$$= x^2 + x\,dx + x\,dx + dx^2$$
$$\Rightarrow (x + dx)^2 - x^2 = 2x\,dx$$
$$\Rightarrow \frac{(x + dx)^2 - x^2}{dx} = 2x$$

$$\tan\theta = \frac{\text{height}}{\text{base}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$$



30/1/2025

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Total Error $\rightarrow \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Avg. err

Mean Square Error $\rightarrow \boxed{\frac{1}{m}\sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2} \rightarrow$ Loss function

(*) Gradiant Descent Algorithm

Linear Regression with multiple features:

Input | output

| Miles Driven ($x_1$) | Engine Capacity ($x_2$) | Fuel Type ($x_3$) | Price ($y$) |
|---|---|---|---|
| | | | |

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ldots + \theta_n x_n$

Fix $x_1 = 1$

$h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + \ldots + \theta_n x_n$

$= \theta^T x$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1} \qquad x = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \theta_0 & \theta_1 & \cdots & \theta_n \end{bmatrix}}_{(1 \times n)} \underbrace{\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}}_{(n \times 1)}$$

## Repeat Control Convergent :

$$\theta_j = \theta_j - \alpha \frac{d}{d\theta_j} J(\theta) \qquad J = 0 \text{ to } n$$

## ONE HOT ENCODING :

- Non-numerical Feature $\xrightarrow{\text{to}}$ vector

$$\begin{array}{cccc} C & P & D & E \\ \end{array}$$
$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

| Fuel Type |
| --- |
| |

Convert to

| C | P | D | E |
| --- | --- | --- | --- |
| | | | |

## Normalization of Features :/ Feature Scaling

[0 - 1]

min (age)

max (age)

| Age | Salary |
| --- | --- |
| 31 | 80000 |
| 32 | 80001 |
| 40 | 240000 |

$$\frac{x - min}{max} \rightarrow [0 - 1]$$

ML

| Miles Driven | Engine Capacity | Year | No. of Pre owners | Fuel Types |
|---|---|---|---|---|
| 2000 | [-1 to 1] | [-1 to 1] | [-1 to 1] | CNG<br>Des.<br>Pet.<br>Bat. |

| C | D | P | B |
|---|---|---|---|
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 |

$\square \leq 0.5 \longrightarrow$ Class 0

$\square > 0.5 \longrightarrow$ Class 1

0.5 is threshold value. If the probability value is 0.5 then the machine will think that there has 50% chances for both.

Qn:) What is the role of the sigmoid / logistic fn to determine the class?

- Cost fn and loss fn are same.

Regularization : Overfitting Problem

Under fit, Perfect fit, Overfit

ML

8/2/25

## Classification using Decision-Tree

Feautes

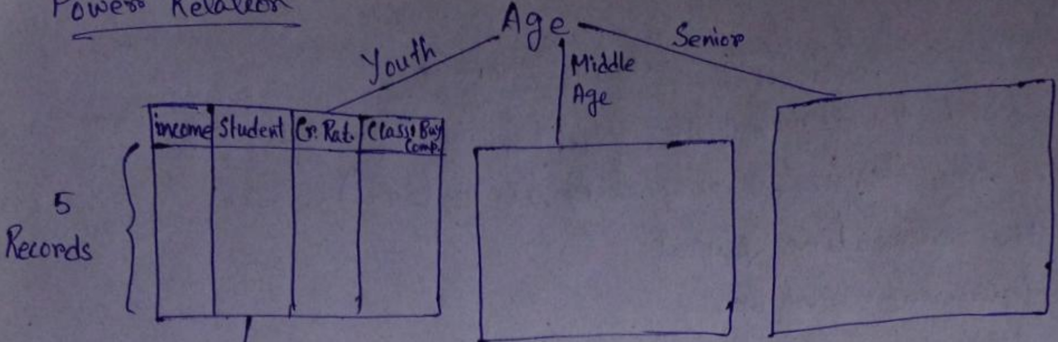
Age, Income, Student? Creadit Rating
The prediction is basically buy computer or not.

Example:

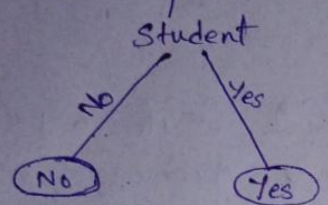
| Age | Income | Student | Credit Rating | ~~By~~ buy Comp. |
|---|---|---|---|---|
| Youth | High | No | Fare | No |

Age
- Senior
- Middle Age
- Youth

# Power Relation




Biased data/ So, again splitting




① if age = 'middle age' then
Buy PC = Yes

② if age = 'youth' and
student = 'no'
Buy PC = No

If ~~ax~~ after splitting all that attributes, there left no attribute then go for majority voting. If the majority is 'yes' then all will be 'yes' and vice-versa.

## Attribute Selection for Splitting

Information gain / Entropy

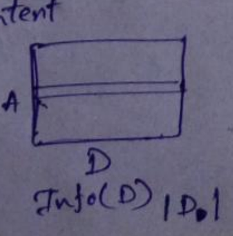
$\text{Info}(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$ ∴ m = no. of classes

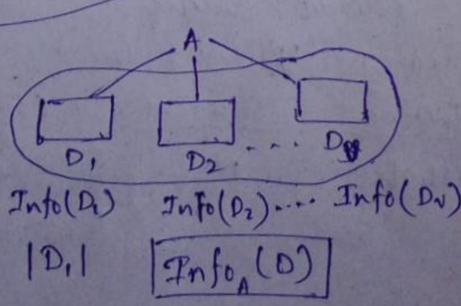
For our case,

$m = 2$ → Buy computer = Yes &
$p_1$ " " = No
$p_2$

- The unit of entropy is bits.

$IC \propto \frac{1}{P(A)}$
↓
Information Content



$$Info_A(D) = \frac{|D_1|}{|D|} Info(D_1) + \frac{|D_2|}{|D|} Info(D_2) + \dots + \frac{|D_v|}{|D|} Info(D_v)$$

$$= \sum_{j=1}^{v} \frac{|D_j|}{|D|} Info(D_j)$$

$$\boxed{Gain(A) = Info(D) - Info_A(D)}$$

$$Gain(Age) = Info(D) - Info_{Age}(D)$$

$$Info(D) = -\sum_{i=1}^{2} (p_i \times \log_2(p_i))$$

$$= -p_1 \log_2(D_1) - p_2 \log_2(D_2)$$

$$= -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.97$$

$$= -0.64 \log_2(0.64) - 0.36 \log_2(0.36)$$

$$= (-0.64)\times(-0.45) - (0.36)\times(-1.02)$$

$$= 0.288 + 0.3672$$

$$= 0.28$$

$$Info_{Age}(D) = \frac{5}{14}\left[-\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right)\right] + \frac{5}{14}\left[-\frac{3}{5}\log_2\left(\frac{3}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right)\right] + \frac{4}{14}\left[0 - \frac{4}{4}\log_2\left(\frac{4}{4}\right)\right]$$

$$= \frac{10}{14}(0.97 + 0.97)$$

$$= 0.69285$$

$$\cancel{gain(age) = 0.97 - 0.69 = 0.2465}$$

$$Info_{Age}(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} Info(D_j)$$

$$= \sum_{j=1}^{3} \frac{|D_j|}{|D|} Info(D_j)$$

$$= \frac{|D_1|}{|D|} Info(D_1) + \frac{|D_2|}{|D|} Info(D_2) + \frac{|D_3|}{|D|} Info(D_3)$$

$$= \frac{5}{14}\left[-\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log\frac{3}{5}\right] + \frac{4}{14}\left[-\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4}\right]$$

$$+ \frac{5}{14}\left[-\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5}\right]$$

$$= 0.6928$$

$$gain(age) = 0.97 - 0.69 = 0.28$$

Gain of Income = 0.029 bits
Gain of Student = 0.151 bits
Credit Rating = 0.048 bits

13/2/2025

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Entropy

$$\text{Info}(D) = -\sum_{i=1}^{m} P_i \log_2(P_i)$$

m: no. of classes

$$\text{Info}_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \cdot \text{Info}(D_j)$$

$$\text{Gain}(A) = \text{Info}(D) - \text{Info}_A(D)$$

$$\text{Split Info}_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2 \frac{|D_j|}{|D|}$$

According to Income attribute

$$= -\sum_{j=1}^{3} \frac{|D_j|}{|D|} \times \log_2 \frac{|D_j|}{|D|}$$

$$= -\frac{|D_1|}{|D|} \times \log_2 \frac{|D_1|}{|D|} - \frac{|D_2|}{|D|} \times \log_2 \frac{|D_2|}{|D|} - \frac{|D_3|}{|D|} \times \log_2 \frac{|D_3|}{|D|}$$

$$= -\frac{4}{14} \log_2 \frac{4}{14} - \frac{6}{14} \log_2 \frac{6}{14} - \frac{4}{14} \log_2 \frac{4}{14}$$

~~= 1.5564~~ = 1.5564

~~= 0.155465 + 0.15770 + 0.155465~~

$$\text{Gain Ratio}(A) = \frac{\text{Gain}(A)}{\text{Split Info}(A)}$$

$$= \frac{0.28}{1.5564} = 0.1799 \text{ Ans}$$

$$\text{Gini}(D) = 1 - \sum_{i=1}^{m} P_i^2$$

$$\text{Gain}_A(D) = \frac{|D_1|}{|D|} \text{Gini}(D_1) + \frac{|D_2|}{|D|} \text{Gini}(D_2)$$

$$\Delta \text{Gini}(A) = \text{Gini}(D) - \text{Gini}_A(D)$$

$$\sum_{i=1}^{m} \underbrace{y^{(i)}}_{\text{Actual class}} \log_2 \underbrace{h_\theta(x^{(i)})}_{\text{Predicted class}}$$

Binary cross entropy loss

$$\Big(\underbrace{\sum_{i=1}^{m} y^{(i)} \log_2 h_\theta(x^{(i)})}_{\text{Binary cross entropy for +ve class}} + \underbrace{\sum_{i=1}^{m} (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))}_{\text{Binary class entropy for -ve class}}\Big)$$