Epileptic Seizure Prediction (ESP) Using Non-Linear Dynamics And Machine Learning

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Abstract

The aim of this project is to be able to predict epileptic seizures using non-linear dynamics and machine learning on EEG recordings (CHB-MIT database).

1 Introduction

The term epilepsy is used to describe neurological diseases which are all based on repetitive *epileptic* seizures. Epileptic seizures are transient abnormal electrical activity of neurons. It is quite easily visible in the Figure 1 showing an EEG recording with a seizure starting at 2996 seconds and ending at 3036 seconds. Those seizures are of different impacts : screaming, falling, unconsciousness, stiffness, twitching, etc... Sadly, in France, 500000 people are subject to epilepsy. Actually, those seizures are said to be *spontaneous* and thus hard to predict. Our research aims to try to find underlying patterns on EEG recordings that could lead to a prediction model. We based our research on Bartosz Swidersk studies [4] that emphasize how the synchronization of the maximum Lyapunov exponent through many recording channels is an efficient predictor. We will also used extensively the famous '*Nonlinear Time Series Analysis*' book (abbreviated NTSA after) written by Holger Kantz.

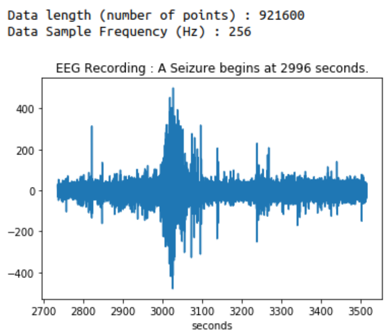


Figure 1: One EEG Recording Channel Data With An Epileptic Seizure

2 About The Database

The '*CHB-MIT Scalp EEG'* Database (`https://physionet.org/content/chbmit/1.0.0`) has been used to make some experimentation. The data consist of many recordings from 22 young people suffering from epilepsy. Each case (called chb01, chb02, etc.) contains between 9 and 42 continuous .edf files (time series signals) from a single subject. For each experimentation, signals are recorded from different parts of the brain during one hour in average. Different parts of the brain recorded are presented in Figure 2. All signals were sampled at 256 samples per second with 16-bit resolution. For each experiment, the epileptic seizure starting and ending time is annotated in a separate file. The data can be read and plotted with Python tools: for instance, the Figure 1 is the representation of the signal measured between the FP1 position and the F7 position on the skull. It appears that the difference between recordings from different parts of the skull are not very important evaluating the shape of the amplitudes as you can see in Figure 3:

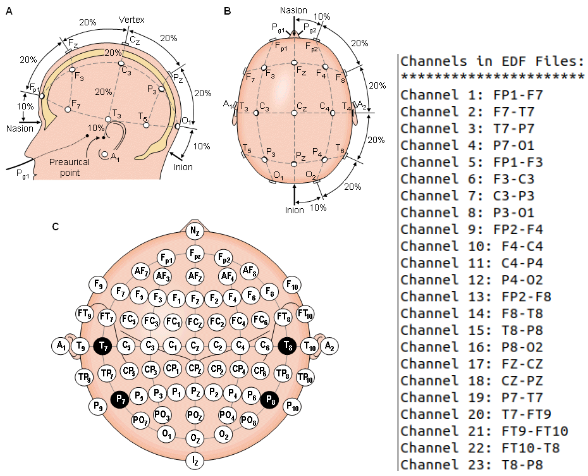


Figure 2: Possible EEG recording channels and information provided for each experimentation

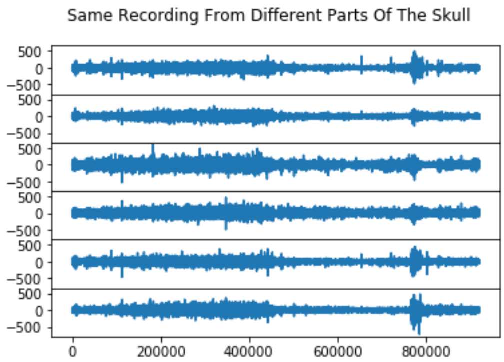


Figure 3: Data from different parts of the skull: (From Top To Bottom) FP1-F7, T7-P7, FZ-TZ, CZ-PZ, FP2-F8, T8-P8

3 Data Analysis

The underlying assumption of almost every non linear dynamics tools for time series analysis is that the data are stationary. It means that the statistical property of the process that generates the times series do not change over time. In a more formal way of speaking, it means that the joint probabilities of finding the system in a certain state is shift-invariance (in time) within the observation period (see Chapter 2 and 13 from the NTSA book). Unfortunately, stationary recordings from living beings are impossible because they are sensitive to the environment. Visual, sounds, skin stimuli add time dependency to the neuronal activity. Most stable statistical quantities for stationary time series are the mean and the variance. It is easy to see in Figure 1 and 3 that the variance of EEG recordings is not constant through time which is characteristic of non-stationary time series. Thus, it will be difficult to assume that the results found in this report will characterize the underlying system.

A first requirement for our time series to be considered as valid is that it must have enough data to cover the underlying mechanism over time. Indeed, it would be extremely dangerous to characterize a periodic system with only data recorded over a time under its period. The second problem is the aliasing effect of the sampling period. According to the Nyquist Sampling Rate theorem, the sampling rate must be two times the highest frequency of the input signal. If we consider obviously that neurons are the source of the signals we measure, it is hard to know exactly at which frequency they work but an estimate would be around 2 Hz (`https://aiimpacts.org/rate-of-neuron-firing/`) and 200 Hz as an upper bound. This estimation may be confirmed in Figure 4. Thus, 256 Hz as the sampling frequency respects the Nyquist Sampling Rate theorem. Because the longest relevant time scale can be estimated as the inverse of the lowest frequency which still contains a significant fraction of the total power of the signal, the time series must be longer to that period. The power spectrum is the most accurate tool to evaluate that kind of measurement: Figure 4 shows that the lowest frequency which still contains a significant fraction of the total power is 10^{-1} Hertz and so data are sufficiently sampled (one hour in average) to evaluate correctly the hidden system.

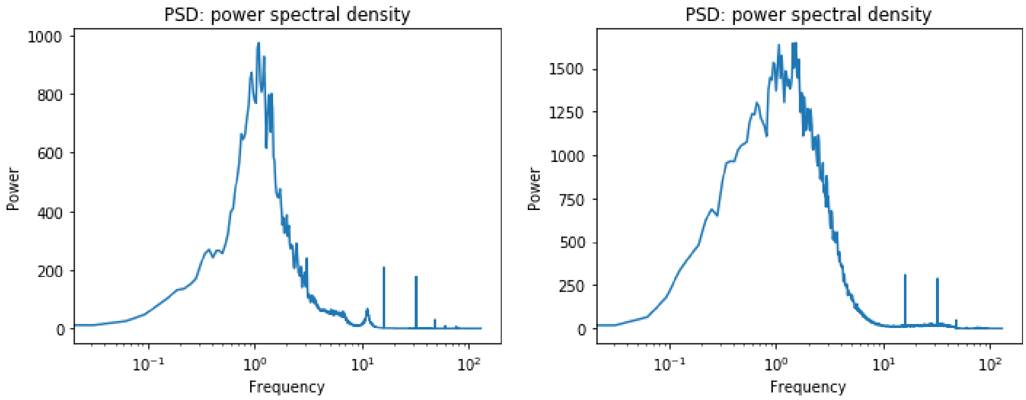


Figure 4: Example of the power spectrum of the channel 1 with an epileptic seizure (Left Figure) and without an epileptic seizure with the same patient: Welch method with the 'Hann' window of length 8192 points

One considerably problem is that our data are strongly non-stationary as we can see with the running mean and the running variance of one recording on one channel without epileptic seizure (weak stationary principle: same mean and variance through time) in Figure 5. In fact, non stationary data cannot be modeled and thus predicted. It is most of the time a real expertise to transform non stationary data into stationary ones. Our challenge lies in extracting properties of non stationary data while "even slight non-stationarity can sometimes lead to severe mis-interpretations" (Holger Kantz, NTSA book p.277).

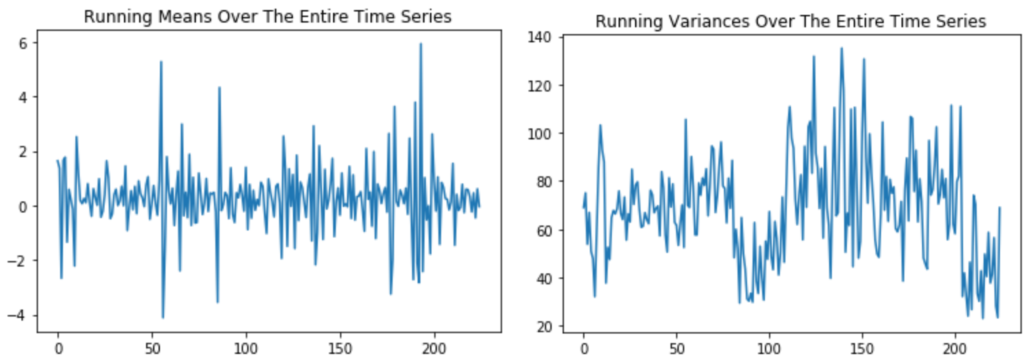


Figure 5: Example of the Running Mean and Variance of non-overlapping segments of length 4096 points (total number of points: 921600)

The power spectrum from Figure 4 is an important tool to extract dominant frequencies and harmonics. Because we are maybe studying a chaotic system, it is hard to distinguish chaoticity and noise. According to the NTSA book (p.21-22), "deterministic chaotic signals may also have sharp spectral lines but even in the absence of noise there will be a continuous part of the spectrum" (see Taylor-Couette flow experiment). When we studied the power spectrum, we lose all time information: in order to preserve this information, the spectogram performs a spectral analysis on consecutive segments of the time series (see Figure 6).

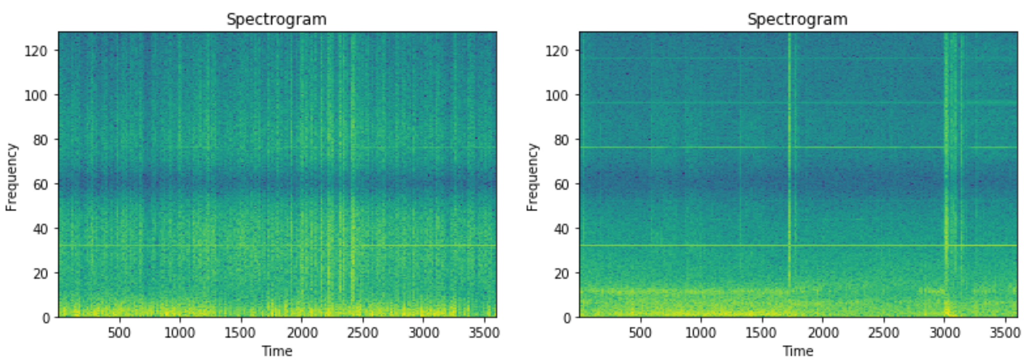


Figure 6: Example of the spectrogram of the channel 1 with an epileptic seizure (Right) and without seizure (Left)

Even if the second spectogram seems very different than the first one. There is no immediate correlation between the shape of the spectogram and the epilepsy before the seizure. Indeed, the seizure is, by observation, an abnormal activity of the brain which implies more sharp variations of the neuronal signal and so more diffuse energy across frequencies (or more energy in high frequencies) as you can see in the right figure at 3000 during the seizure. But before the crisis, it is not clear if there is a pattern in the energy dispersion in frequencies. Even if some intuitions have been made, it has been hard to develop a strong and robust analysis beyond done by eyes. Thus, because of time constraint, we will not focus our research on the FFT while it could be interesting but we will stick to the approximate but rigorous method of nonlinear dynamics.

4 Phase Space Analysis

This part aims to transform times series into a spatial and time dependent *phase space*. This mixing between dynamics and geometry is at the heart of the nonlinear times series analysis. The notion of phase space comes from the Taken's theorem. Precisely, "[it] tells us that information about the hidden states of a dynamical system can be preserved in a time series output" (definition from URL-1T:`https://cnx.org/contents/k57_M8Tw@2/Takens-Embedding-Theorem`). This theorem has large implications because it allows us to say that the time series output we have studied from the beginning can embed information to reconstruct a phase space, called the *reconstruction space* or the *embedded space*, that is a *one-to-one image* of the hidden space. *One-to-one* means that distinct system states are not mapped to the same point in the reconstruction space. Moreover, the reconstruction space is *topology preserving* : "by preserving the topology of the manifold in the reconstruction space, many properties of the manifold and the dynamical system are retained, including its dimensionality and its Lyapunov exponents, just to name a few"(URL-1T).

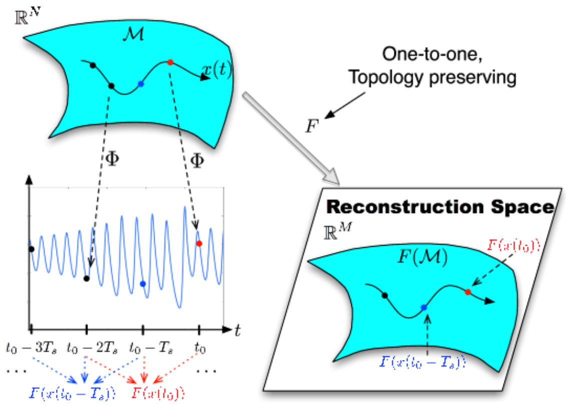


Figure 7: Embedding Theorem

Knowing and analyzing the phase space is not immediate. Indeed, we do not know in which space our system is evolving (the hidden space of dimension N in figure 7). What we observe when recording the ECG is not a phase space object but a time series, in other words, a sequence of scalar measurements. So, here, we are facing a problem of phase space reconstruction. Therefore, we need to convert the observations (times series) into state vectors (reconstruction space), well-designed to fully describe our system. One property of the Taken's theorem that must satisfy the reconstructed space is that $M > 2N$ (see figure 7).The problem of phase space reconstruction is technically solved by the method of delays and that is what we used. A delay reconstruction in m dimensions is then formed by the vectors S_n. We create a vector for every scalar observation, S_n, with $n > (m-1)\tau$.

Observations $\qquad\qquad\qquad\qquad$ Delay state vectors

$$s_n = s(\mathbf{x}(n\Delta t)) + \eta_n. \quad \longleftrightarrow \quad \mathbf{s}_n = (s_{n-(m-1)\tau}, s_{n-(m-2)\tau}, \dots, s_{n-\tau}, s_n) \quad \textbf{(1)}$$

The embedding theorems guarantee there exists a dimension m such that the vectors S_n are equivalent to phase space vectors. Under quite general circumstances, that we assume verified, the attractor formed by *S_n is equivalent to the attractor in the unknown space in which the original system is living if the embedding dimension m is sufficiently large.* This transformation involves two important parameters: τ which is the time difference in number of samples between two adjacent components in the delay vector and m which represents the embedding dimension of the system. So now, let's discuss qualitatively how to choose τ and m. The delay time τ needs to be sufficiently large to ensure that the correlation between adjacent points is not due to noise. Obviously, choosing a very large τ do not make sense since the points will then be completely uncorrelated. On the other hand, the dimension needs to be sufficiently large for the equivalence (1) to be true. It is important to find the first sufficiently large m since we want to exploit determinism with minimal computational effort.

Now, what criteria for the choice of tau can be recommended ? First of all, one can apply a geometrical argument. The attractor should be unfolded, i.e. the extension of the attractor in all space dimensions should be roughly the same. The NTSA book gives us a visual method to choose the right time lag. In figure 8, the method described in the book is represented in the line 'Theory'. In this line, analysts must choose the time lag used in the middle because the attractor is expended in all direction quite evenly. From experiments, we tried to use the same method. And by visual inspection, we found that the time lag $15\delta t$ (with δt the sampling period of the time series) is a good one. But, we faced different time lag for each channel which put hard work into the task of the Lyapunov exponent computation. Somewhat, we made an important assumption: a good time lag for a 2-dimensional embedding space has high probability to be a good time lag for higher dimensional embedding space. However, the visual inspection could give us hint about the time lag but it would be better to use other precised techniques for better precision. For instance, the first minimum of the *mutual information* and the crossing zero of the auto-correlation could be used to determine the time lag.

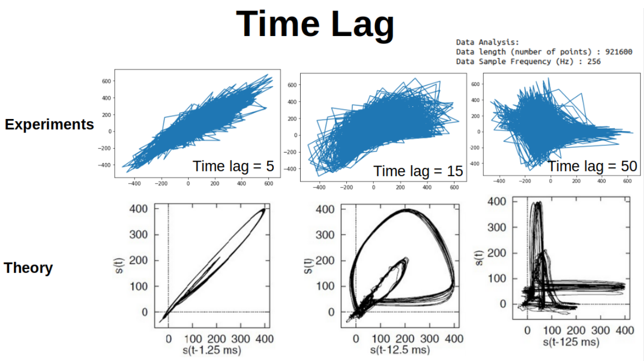
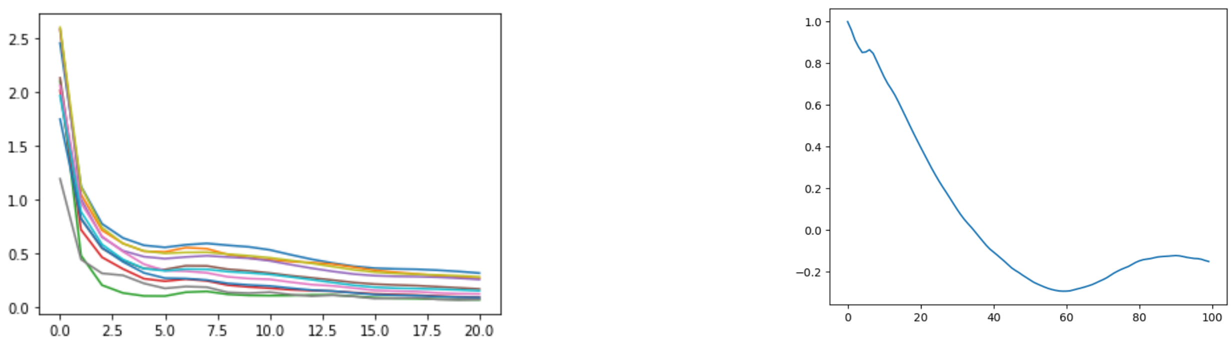


Figure 8: Time Lag

The mutual information is based on the Shannon entropy and tells us how much information we know about the measured data at time $t + \tau$ if we know the measurement at time t. The first minimum of the mutual information marks the time lag where the signal at $t + \tau$ "adds maximal information to the knowledge"(NTSA book) we have from the signal at t, "or, in other words, the redundancy is least" (NTSA book). Even if this time lag still applies strictly to two dimensional embedding phase space only, it is still the most accurate indicator of the *time lag* comparing to the visual analysis. However, even if the mutual information seems more elaborated than the auto-correlation, it has been experimentally found that the auto-correlation gives better results according to visualization in two dimension (as in figure 8).



(a) Mutual Information

(b) Auto-correlation

Figure 9: Mutual Information from many channels and the auto-correlation from one channel

After having compute the time lag, it is possible to figure out the minimum embedding dimension of the reconstructed space by using the 'false nearest' algorithm. This algorithm is based on the theory that the attractor has no intersection. If k is the embedding dimension by the Taken's theorem, then any two points which stay close in the k-dimensional reconstructed space will be still close in the k+1-dimensional reconstructed space. However, the false neighbor algorithm has two shortcomings. It is sensitive to noise and is affected by subjective parameters. Pragmatically, we compute the ratio of the distance in the dimension k+1 and the dimension k and we compare to a threshold (if larger, it was a false neighbors). The percentage of false neighbors should decrease as the dimension increase. But choosing a too large value of the embedding dimension will add redundancy and thus degrade the performance of algorithms such as the Lyap exponent. Thus, we choose a dimension that satisfies a chosen fraction of false neighbors.

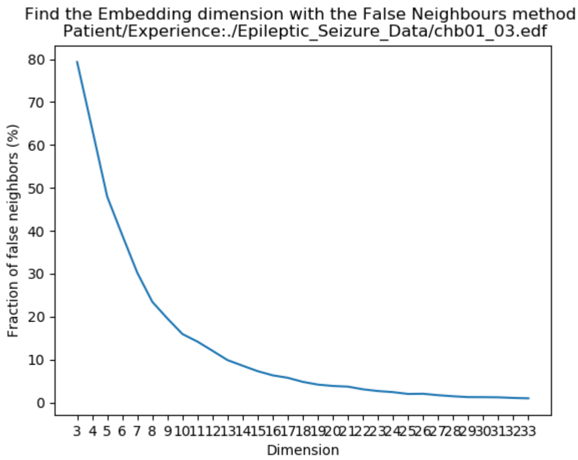


Figure 10: False Neighbor Algorithm Result

Even if the theory seems to be suitable for our problem if we didn't manage the non stationarity of our data, great challenges occurred during our experimentation. Indeed, obviously the auto-correlation and the mutual information should give the same dimension but it appears that it is not often the case if we consider an analysis on the whole data. Moreover, the false nearest algorithm was very long to compute the right dimension for the whole time series. It has been found that there are no precised parameters that could defined a patient, or a recording. It is quite reassuring that we can't define someone brain dynamics by two fixed parameters (time lag, dimension). It has been not easy to find a framework that allows us to continue using non linear dynamics tools such as the maximum Lyapunov exponent as the phase space was hard to build precisely. But we had to find a model that could fix our lack of precision. First of all, we admitted that it is somewhat impossible to embed all states of the brain *interictally* (an ictal is a physical event like a seizure, thus interictally time is time between events) [2]. Thus, we defined three states: *pre-ictal* (before the seizure), *seizure*, *post-ictal* (after the seizure). It appeared that the separation helped us to determine phase space parameters with more precision. In fact, during the seizure, the brain dynamics tends to a low dimensional chaos [1]: lower time lag and dimension than during the pre-ictal or post-ictal stages which is more manageable with Tisean tools. Even if the parameters are not chosen precisely for all states of the brain, "it should be adequate for detection of the transition of the brain toward the ictal stage if the epileptic attractor is active in its phase prior to the occurrence of the epileptic seizure."[2] Finally, it has been decided to use non linear dynamics tools with parameters computed during the seizure in order to be able to predict an epileptic phase times before the seizure.

5 Lyapunov Exponent

The core of the prediction lies in the synchronization of the maximum Lyapunov exponents across channels [3][4].
What is the Lyapunov exponent ? About the window stationary 4 5?
How to compute it ?
How to use the result : stretching factor?

(a) Seizure

(b) Post-ictal

Figure 11: Maximum Lyapunov exponent Tisean results

6 Synchronization Index

7 Machine Learning

References

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