Epileptic Seizure Prediction (ESP) Using Non-Linear Dynamics And Machine Learning

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February 2020

Abstract

The aim of this project is to be able to predict epileptic seizures using non-linear dynamics and machine learning on EEG recordings (CHB-MIT database).

1 Introduction

The term epilepsy is used to describe neurological diseases which are all based on repetitive *epileptic* seizures. Epileptic seizures are transient abnormal electrical activity of neurons. It is quite easily visible in the Figure 1 showing an EEG recording with a seizure starting at 2996 seconds and ending at 3036 seconds. Those seizures are of different impacts : screaming, falling, unconsciousness, stiffness, twitching, etc... Sadly, in France, 500000 people are subject to epilepsy. Actually, those seizures are said to be *spontaneous* and thus hard to predict. Our research aims to try to find underlying patterns on EEG recordings that could lead to a prediction model. We based our research on Bartosz Swidersk studies [4] that emphasize how the synchronization of the maximum Lyapunov exponent through many recording channels is an efficient predictor. We will also used extensively the famous '*Nonlinear Time Series Analysis*' book (abbreviated NTSA after) written by Holger Kantz.

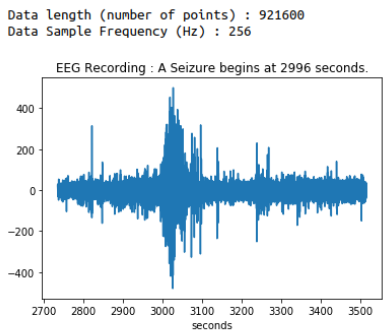


Figure 1: One EEG Recording Channel Data With An Epileptic Seizure

2 About The Database

The '*CHB-MIT Scalp EEG*' Database (`https://physionet.org/content/chbmit/1.0.0`) has been used to make some experimentation. The data consist of many recordings from 22 young people suffering from epilepsy. Each case (called chb01, chb02, etc.) contains between 9 and 42 continuous .edf files (time series signals) from a single subject. For each experimentation, signals are recorded from different parts of the brain during one hour in average. Different parts of the brain recorded are presented in Figure 2. All signals were sampled at 256 samples per second with 16-bit resolution. For each experiment, the epileptic seizure starting and ending time is annotated in a separate file. The data can be read and plotted with Python tools: for instance, the Figure 1 is the representation of the signal measured between the FP1 position and the F7 position on the skull. It appears that the difference between recordings from different parts of the skull are not very important evaluating the shape of the amplitudes as you can see in Figure 3:

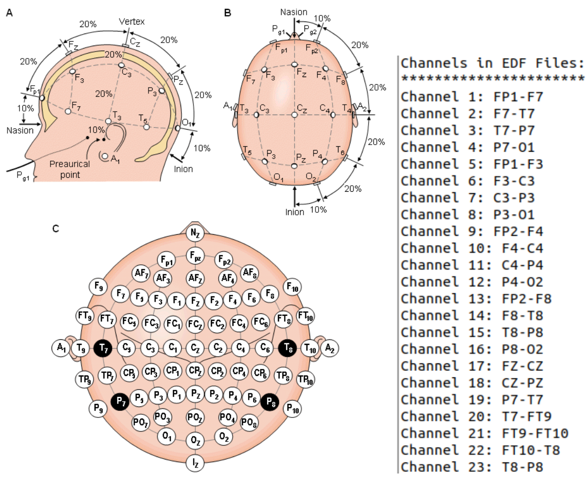


Figure 2: Possible EEG recording channels and information provided for each experimentation

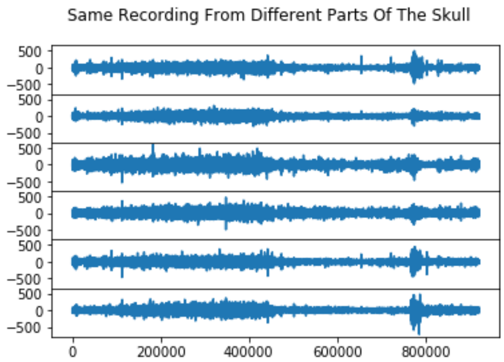


Figure 3: Data from different parts of the skull: (From Top To Bottom) FP1-F7, T7-P7, FZ-TZ, CZ-PZ, FP2-F8, T8-P8

3 Data Analysis

The underlying assumption of almost every non linear dynamics tools for time series analysis is that the data are stationary. It means that the statistical property of the process that generates the times series do not change over time. In a more formal way of speaking, it means that the joint probabilities of finding the system in a certain state is shift-invariance (in time) within the observation period (see Chapter 2 and 13 from the NTSA book). Unfortunately, stationary recordings from living beings are impossible because they are sensitive to the environment. Visual, sounds, skin stimuli add time dependency to the neuronal activity. Most stable statistical quantities for stationary time series are the mean and the variance. It is easy to see in Figure 1 and 3 that the variance of EEG recordings is not constant through time which is characteristic of non-stationary time series. Thus, it will be difficult to assume that the results found in this report will characterize the underlying system.

A first requirement for our time series to be considered as valid is that it must have enough data to cover the underlying mechanism over time. Indeed, it would be extremely dangerous to characterize a periodic system with only data recorded over a time under its period. The second problem is the aliasing effect of the sampling period. According to the Nyquist Sampling Rate theorem, the sampling rate must be two times the highest frequency of the input signal. If we consider obviously that neurons are the source of the signals we measure, it is hard to know exactly at which frequency they work but an estimate would be around 2 Hz (`https://aiimpacts.org/rate-of-neuron-firing/`) and 200 Hz as an upper bound. This estimation may be confirmed in Figure 4. Thus, 256 Hz as the sampling frequency respects the Nyquist Sampling Rate theorem. Because the longest relevant time scale can be estimated as the inverse of the lowest frequency which still contains a significant fraction of the total power of the signal, the time series must be longer to that period. The power spectrum is the most accurate tool to evaluate that kind of measurement: Figure 4 shows that the lowest frequency which still contains a significant fraction of the total power is 10^{-1} Hertz and so data are sufficiently sampled (one hour in average) to evaluate correctly the hidden system.

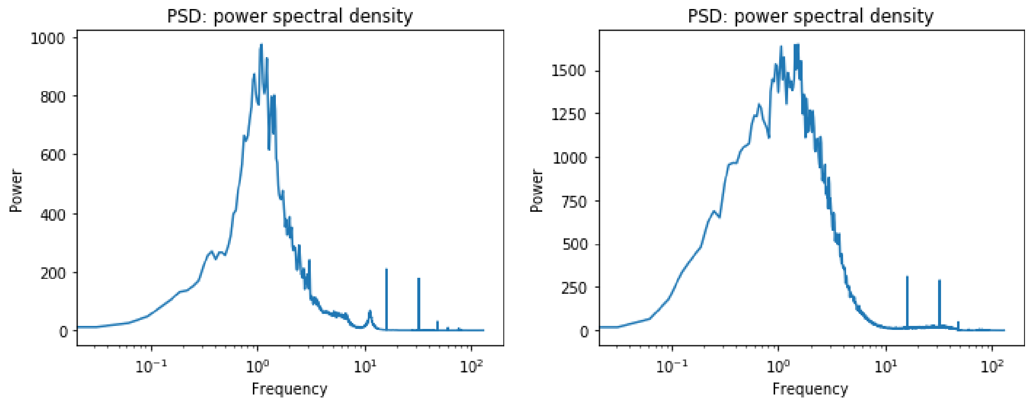


Figure 4: Example of the power spectrum of the channel 1 with an epileptic seizure (Left Figure) and without an epileptic seizure with the same patient: Welch method with the 'Hann' window of length 8192 points

One considerably problem is that our data are strongly non-stationary as we can see with the running mean and the running variance of one recording on one channel without epileptic seizure (weak stationary principle: same mean and variance through time) in Figure 5. In fact, non stationary data cannot be modeled and thus predicted. It is most of the time a real expertise to transform non stationary data into stationary ones. Our challenge lies in extracting properties of non stationary data while "even slight non-stationarity can sometimes lead to severe mis-interpretations" (Holger Kantz, NTSA book p.277).

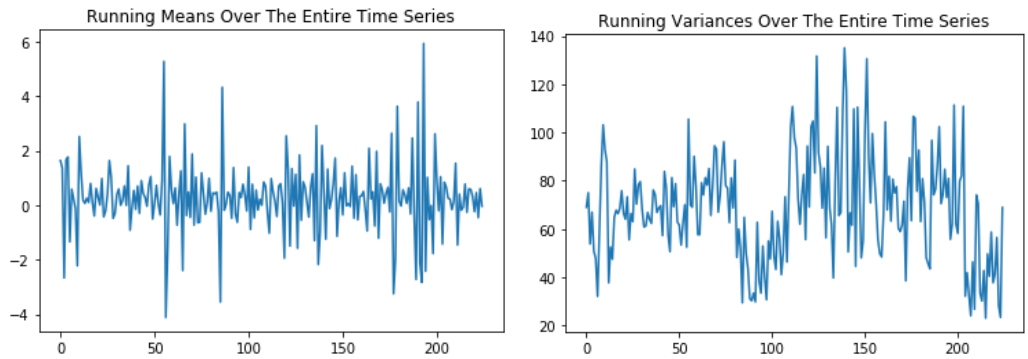


Figure 5: Example of the Running Mean and Variance of non-overlapping segments of length 4096 points (total number of points: 921600)

The power spectrum from Figure 4 is an important tool to extract dominant frequencies and harmonics. Because we are maybe studying a chaotic system, it is hard to distinguish chaoticity and noise. According to the NTSA book (p.21-22), "deterministic chaotic signals may also have sharp spectral lines but even in the absence of noise there will be a continuous part of the spectrum" (see Taylor-Couette flow experiment). When we studied the power spectrum, we lose all time information: in order to preserve this information, the spectogram performs a spectral analysis on consecutive segments of the time series (see Figure 6).

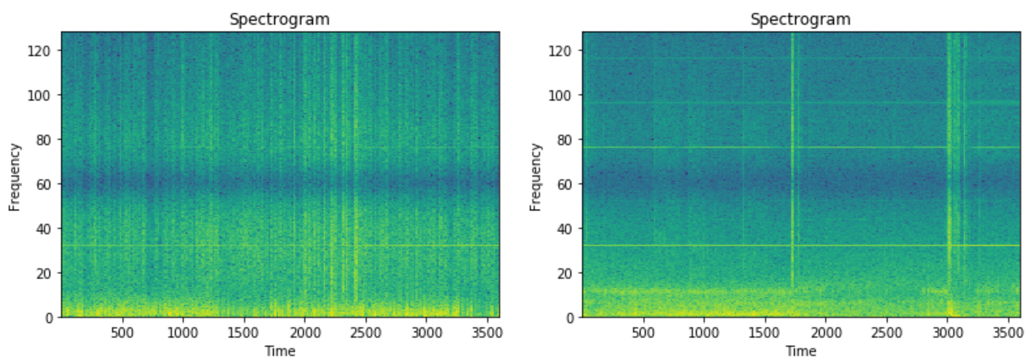


Figure 6: Example of the spectrogram of the channel 1 with an epileptic seizure (Right) and without seizure (Left)

Even if the second spectogram seems very different than the first one. There is no immediate correlation between the shape of the spectogram and the epilepsy before the seizure. Indeed, the seizure is, by observation, an abnormal activity of the brain which implies more sharp variations of the neuronal signal and so more diffuse energy across frequencies (or more energy in high frequencies) as you can see in the right figure at 3000 during the seizure. But before the crisis, it is not clear if there is a pattern in the energy dispersion in frequencies. Even if some intuitions have been made, it has been hard to develop a strong and robust analysis beyond done by eyes. Thus, because of time constraint, we will not focus our research on the FFT while it could be interesting but we will stick to the approximate but rigorous method of nonlinear dynamics.

4 Phase Space Analysis

This part aims to transform times series into a spatial and time dependent *phase space*. This mixing between dynamics and geometry is at the heart of the nonlinear times series analysis. The notion of phase space comes from the Taken's theorem. Precisely, "[it] tells us that information about the hidden states of a dynamical system can be preserved in a time series output" (definition from URL-1T:`https://cnx.org/contents/k57_M8Tw@2/Takens-Embedding-Theorem`). This theorem has large implications because it allows us to say that the time series output we have studied from the beginning can embed information to reconstruct a phase space, called the *reconstruction space* or the *embedded space*, that is a *one-to-one image* of the hidden space (like with conjugacy maps). *One-to-one* means that distinct system states are not mapped to the same point in the reconstruction space. Moreover, the reconstruction space is *topology preserving* : "by preserving the topology of the manifold in the reconstruction space, many properties of the manifold and the dynamical system are retained, including its dimensionality and its Lyapunov exponents, just to name a few"(URL-1T).

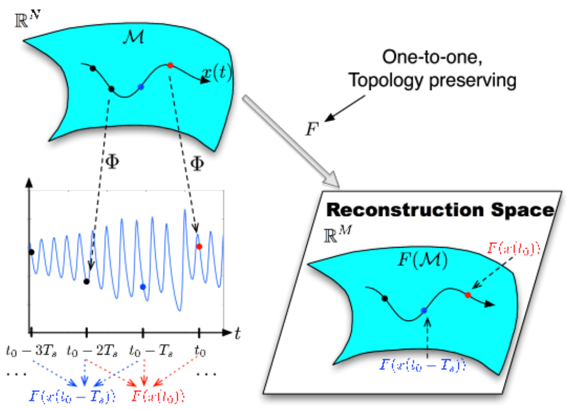


Figure 7: Embedding Theorem

Knowing and analyzing the phase space is not immediate. Indeed, we do not know in which space our system is evolving (the hidden space of dimension N in figure 7). What we observe when recording the ECG is not a phase space object but a time series, in other words, a sequence of scalar measurements. So, here, we are facing a problem of phase space reconstruction. Therefore, we need to convert the observations (times series) into state vectors (reconstruction space), well-designed to fully describe our system. One property of the Taken's theorem that must satisfy the reconstructed space is that $M > 2N$ (see figure 7).The problem of phase space reconstruction is technically solved by the method of delays and that is what we used. A delay reconstruction in m dimensions is then formed by the vectors S_n. We create a vector for every scalar observation, S_n, with $n > (m-1)\tau$.

Observations — Delay state vectors

$$s_n = s(\mathbf{x}(n\Delta t)) + \eta_n. \quad \Longleftrightarrow \quad \mathbf{s}_n = (s_{n-(m-1)\tau}, s_{n-(m-2)\tau}, \ldots, s_{n-\tau}, s_n) \quad \textbf{(1)}$$

The embedding theorems guarantee there exists a dimension m such that the vectors S_n are equivalent to phase space vectors. Under quite general circumstances, that we assume verified, the attractor formed by *S_n is equivalent to the attractor in the unknown space in which the original system is living if the embedding dimension m is sufficiently large.* This transformation involves two important parameters: τ which is the time difference in number of samples between two adjacent components in the delay vector and m which represents the embedding dimension of the system. So now, let's discuss qualitatively how to choose τ and m. The delay time τ needs to be sufficiently large to ensure that the correlation between adjacent points is not due to noise. Obviously, choosing a very large τ do not make sense since the points will then be completely uncorrelated. On the other hand, the dimension needs to be sufficiently large for the equivalence (1) to be true. It is important to find the first sufficiently large m since we want to exploit determinism with minimal computational effort.

Now, what criteria for the choice of tau can be recommended ? First of all, one can apply a geometrical argument. The attractor should be unfolded, i.e. the extension of the attractor in all space dimensions should be roughly the same. The NTSA book gives us a visual method to choose the right time lag. In figure 8, the method described in the book is represented in the line 'Theory'. In this line, analysts must choose the time lag used in the middle because the attractor is expended in all direction quite evenly. From experiments, we tried to use the same method. And by visual inspection, we found that the time lag $15\delta t$ (with δt the sampling period of the time series) is a good one. But, we faced different time lag for each channel which put hard work into the task of the Lyapunov exponent computation. Somewhat, we made an important assumption: a good time lag for a 2-dimensional embedding space has high probability to be a good time lag for higher dimensional embedding space. However, the visual inspection could give us hint about the time lag but it would be better to use other precised techniques for better precision. For instance, the first minimum of the *mutual information* and the crossing zero of the auto-correlation could be used to determine the time lag.

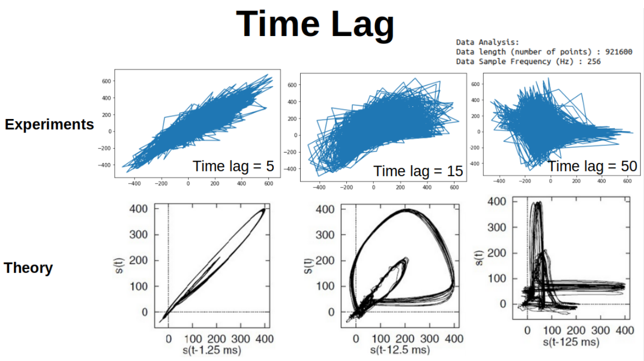
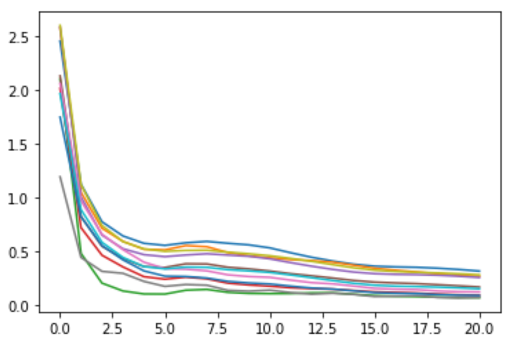
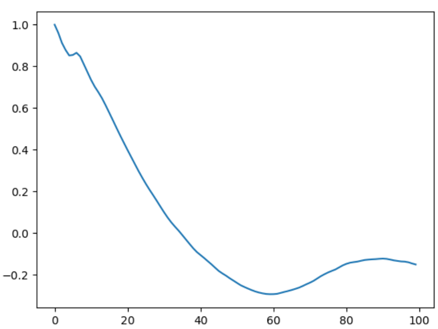


Figure 8: Time Lag

The mutual information is based on the Shannon entropy and tells us how much information we know about the measured data at time $t + \tau$ if we know the measurement at time t. The first minimum of the mutual information marks the time lag where the signal at $t + \tau$ "adds maximal information to the knowledge"(NTSA book) we have from the signal at t, "or, in other words, the redundancy is least" (NTSA book). Even if this time lag still applies strictly to two dimensional embedding phase space only, it is still the most accurate indicator of the *time lag* comparing to the visual analysis. However, even if the mutual information seems more elaborated than the auto-correlation, it has been experimentally found that the auto-correlation gives better results according to visualization in two dimension (as in figure 8).



(a) Mutual Information



(b) Auto-correlation

Figure 9: Mutual Information from many channels and the auto-correlation from one channel

After having compute the time lag, it is possible to figure out the minimum embedding dimension of the reconstructed space by using the 'false nearest' algorithm. This algorithm is based on the theory that the attractor has no intersection. If k is the embedding dimension by the Taken's theorem, then any two points which stay close in the k-dimensional reconstructed space will be still close in the k+1-dimensional reconstructed space. However, the false neighbor algorithm has two shortcomings. It is sensitive to noise and is affected by subjective parameters. Pragmatically, we compute the ratio of the distance between points in the dimension (k+1) and the dimension k and we compare it to a threshold (over the threshold implies a false neighbors). The percentage of false neighbors should decrease as the dimension increase. But choosing a too large value of the embedding dimension will add redundancy and thus degrade the performance of algorithms such as the Lyapunov exponent. Thus, we chose a dimension that satisfies a chosen fraction of false neighbors.

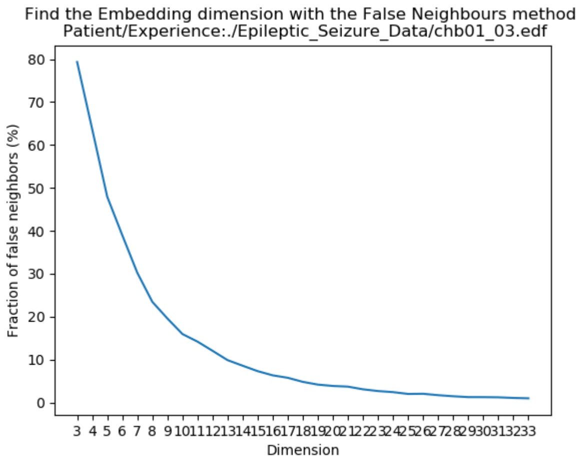


Figure 10: False Neighbor Algorithm Result

The theory seems to be suitable for our problem but if we didn't manage the non stationarity of our data, great challenges occurred during our experimentation. Indeed, obviously the auto-correlation and the mutual information should give the same dimension but it appeared that it is not often the case if we consider an analysis on the whole data. Moreover, the false nearest algorithm was very long to compute the right dimension for the whole time series. It has been found that there are no precised parameters that could defined a patient, or a recording. It is quite reassuring that we can't define someone brain dynamics by two fixed parameters (time lag, dimension). It has been not easy to find a framework that allows us to continue using non linear dynamics tools such as the maximum Lyapunov exponent because the phase space was hard to build precisely. But we had to find a model that could fix our lack of precision. First of all, we admitted that it is somewhat impossible to embed all states of the brain *interictally* (an ictal is a physical event like a seizure, thus interictally time is time between events) [2]. Thus, we defined three states: *pre-ictal* (before the seizure), *seizure*, *post-ictal* (after the seizure). It appeared that the separation helped us to determine phase space parameters with more precision. In fact, during the seizure, the brain dynamics tends to a low dimensional chaos [1]: lower time lag and dimension than during the pre-ictal or post-ictal stages which is more manageable with Tisean tools. For instance, in the pre-ictal and post-ictal state, the time lag found was near 50 iterations while during the ictal phase, it has been found to be around 30 iterations.

Even if the parameters are not chosen precisely for all states of the brain, "it should be adequate for detection of the transition of the brain toward the ictal stage if the epileptic attractor is active in its phase prior to the occurrence of the epileptic seizure."[2] Finally, it has been decided to use non linear dynamics tools with parameters computed during the seizure in order to be able to predict an epileptic transition before the seizure. In fact, we are interested about the transition toward the epileptic state which has the property to have a low dimensional chaotic dynamics. It has been the only way for us to reduce the problem into a more manageable one. However, we may conclude with false assumption or conclusion if the overall dimension chosen (time lag and dimension) are under the threshold given by Taken's theorem (two times the dimension of the hidden phase space).

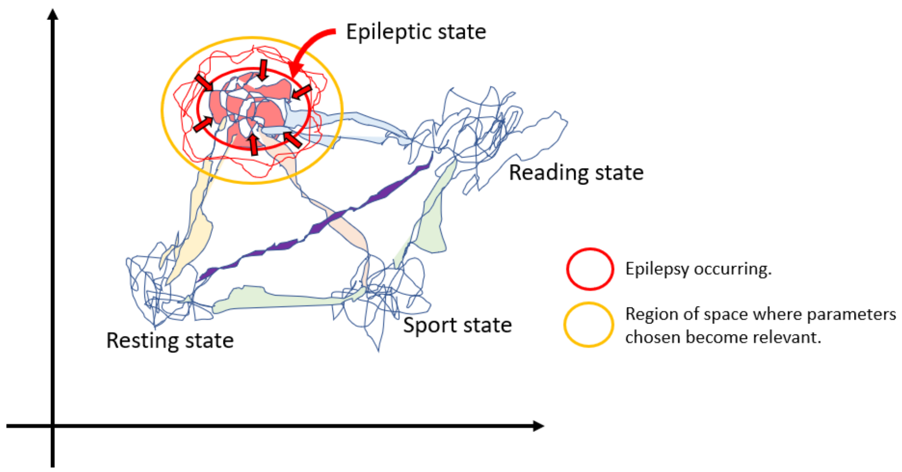


Figure 11: Transition to Epilepsy - Intermittency Interpretation

5 Lyapunov Exponent

The geometrical object (the epilepsy state) has been reconstructed from a finite sample of data points which are most likely to contain some errors but we have still to enhance our knowledge about the underlying system. The core of the prediction lies in the synchronization of the maximum Lyapunov exponents across channels [3][4]. The maximum Lyapunov exponent is used to quantify the chaoticity of the trajectory of the time series. A positive maximum Lyapunov exponent is the most striking evidence for chaos [5]. We used the algorithm provided by Tisean to compute the maximum Lyapunov exponent. In order to compute the maximum Lyapunov exponent, we provided the dimension, the time lag, the Theiler window length and the number of iterations. The dimension and the time lag have been defined before and have been refined for each channel according to the values found during the crisis. The Theiler window length has been chosen according to the space-time separation plot thanks to the *stp* command of Tisean (see figure 12).

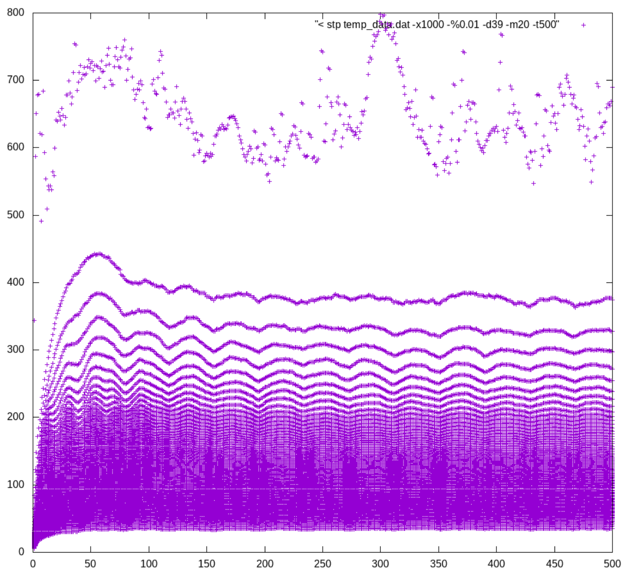


Figure 12: Space-time Separation Plot

The Theiler window aims to identify neighbors that are not temporal ones (neighbors that are closed in time): "The most important temporal correlations are caused by the fact that data close in time are also close in space, a fact which is not only true for purely deterministic systems systems but also for many stochastically driven processes." (NTSA book). The separation plot gives the number of pairs as the function of two variables, the time separation and the spatial distance. The oscillation shown should not affect computation as long as the observation period is much longer than a cycle length. Thus, we can take 400 to be safe. The loss in statistics is marginal.

Now that we have all parameters to compute the maximum Lyapunov exponent, it lacks the window length of the data set to compute and the shifting mechanism. Indeed, we can't take the whole data to get the maximum Lyapunov exponent because we want a dynamic maximum Lyapunov exponent through time. Thus, we have to separate the time series into small chunks of data that can be overlapping. We will use them to have the dynamics of the maximum Lyapunov exponent. The best window length is the one that gives stationary data. Thus, we tried different window lengths and we calculate the variance of the means in function of the window length. This gave us an idea of the speed of the decrease (see figure 13) and a possible value of the length that gives *weak-stationary* chunks of data.

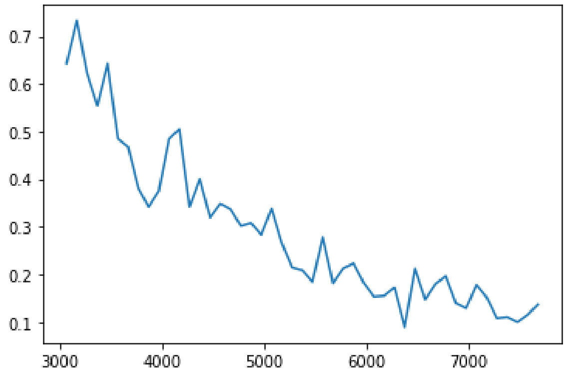


Figure 13: Variance of the means according to the window length

We choose to take a window length near 7000 which is closed to 27 seconds ($\frac{7000}{256} = 27.4$). Now, we computed thanks to Tisean tools the stretching factor given by the Rosenstein algorithm. The slope of the stretching factor is the Lyapunov exponent if it is a straight line. One issue that we faced is that the Lyapunov exponent is oscillatory that maybe does not give us a right value. But, we continue although this result in order to see if we can still keep the dynamic of the Lyapunov exponent synchrony through channels.

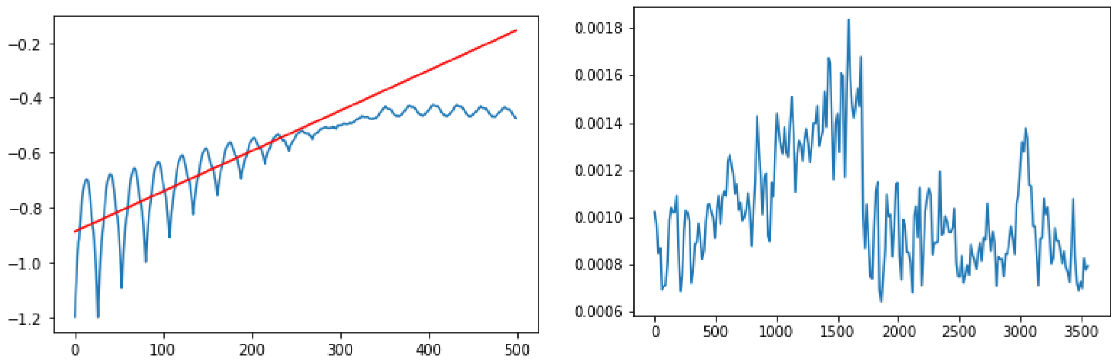


Figure 14: Maximum Lyapunov Exponent for a single window

In the right image of the figure 14, we can see the Lyapunov dynamics through one experience for one channel. This dynamics appears different to what most papers found. Indeed, the maximum Lyapunov exponent seems to bump during the seizure while papers experiments found a slight decrease of the Lyapunov exponent due to the low-dimensional chaos appearing. But, we don't use the same Lyapunov exponent computation technique. While they often use the Short-Term Lyapunov exponent described by A.Wolf, we used the Rosenstein algorithm. However, the Lyapunov dynamics is quite correlated to the original signal. In the right figure, we can see four parts: an increase, a sharp decrease, a plateau and a bump. Those four parts have a corresponding signal shape. In the first part, the original signal are moving with high amplitude. Then, the sharp decrease appears when the signal begins to have low amplitude dynamic. Finally, the bump corresponds to the seizure occurring.

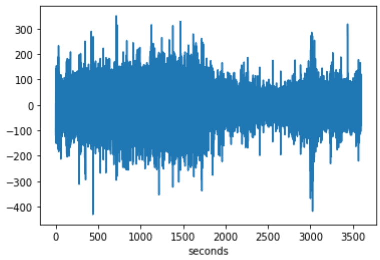


Figure 15: Original Signal

This observation could be the beginning of a new way of seeing things and maybe we can developed a theory about the route to epilepsy but it is too premature. The analysis must be done for many patients and for many experiences for each which has not been done because it is very long to compute all parameters even for one experience. Thus, we stuck to what papers have found and especially the peculiar behavior of the brain neurons synchronization.

6 Synchronization Index

Since the results were not as satisfactory as we had hoped, we turned to the synchronization index. This additional parameter is the one that allows us to check the correlation between the different Lyapunov. The purpose of adding this parameter is still to monitor the chaoticity of the time series. Let's calculate them: the final index will be calculated as the average values of the Tijs. Tij which will compare observations as the distance between 2 Lyapunovs over time between the sites of the electrodes i and j defined as :

$$T_{ij}(t) = \frac{E\left\{ \left| STL_{\max,i}(t) - STL_{\max,j}(t) \right| \right\}}{\sigma_{i,j}(t)/\sqrt{N}}$$

Figure 16: Index formula

With E which represents the mean value of the absolute values of the difference between the Lyapunov profiles of the sites of electrodes i and j, then, N represents the width of the sliding window, finally, the denominator is the standard sample deviation of the Lyapunov differences between electrode sites i and j.

So, the final value researched is the averaged of the Tij. With that, we were hoping for an almost constant index during the time series without any epileptic seizures. In time series with seizures, we hoped to first be able to see the characteristic low peak of the seizure. Before predicting a seizure, we need to be able to see when one is occurring at the moment. Here's an example:

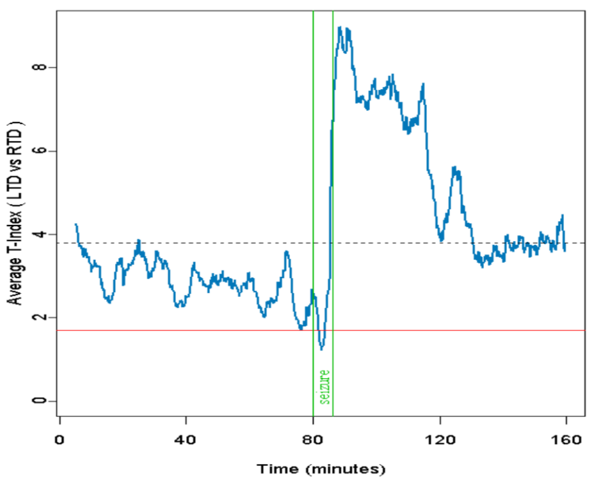


Figure 17: Seizure detected by the Synchronization index

So, we calculated this index of synchronization with this set of channels : [0, 5, 10, 15, 16, 20] and we had this result:

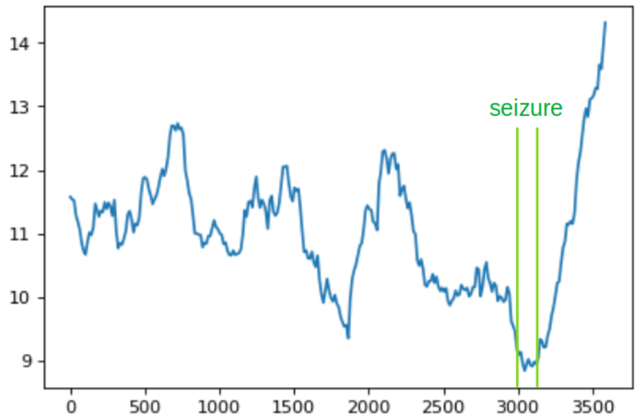


Figure 18: Index of synchronization from our experience

So, we can see a low peak at the seizure time which is pretty interesting, We can notice that we should had tested the same thing for a time series without any seizure thing didn't done because of time. We found that the system is very sensitive to the channels chosen.

7 Conclusion

To conclude, the work done during this project has initiated the use of non linear dynamics tools for the brain state analysis without giving important conclusions. However, the fact that we succeeded to build a framework that allows us to interpret data is promising for our understanding of a part of the underlying brain mechanism. We have found lots of dissimilarities between our results and what it has been found in the literature. However, there is a path where the framework could be deepened and give better recurrent results. The dissimilarities can be explained by the fact that we don't use the same experiments and strictly the same method. Moreover, we have studied one patient and few experiments (3) for that patient. It is not enough to get the overall picture of the mechanism. The first part of the report warns us about the non-stationarity of the data which theoretically would stop us to use non linear tools. Then, lots of parameters must be found and filled in some TISEAN functions that threatens at each step the final result. There are some parameters that we maybe did not tuned precisely enough. Finally, we must recall the complexity of the brain dynamics. Do we know exactly what triggered an epileptic seizure ? The epileptic state seems to be a transient brain chaotic state which the route is hard to find. Our data are scrambled with many perturbations from the outside. We do not know the conditions of the experiments: are patients watching movies ? listening musics ? Any external perturbations will push the patient into a new brain state. What if a particular perturbation attracts the brain state toward the epileptic state ? Lots of questions remain but are of the essence of the century's studies about the brain. In order to have better answers, we maybe could have used filters, machine learning. We have also been limited by our computation capacity.

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