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# ENGSCI 355 Project 1

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# 1 Formulation

## 1.a Parameters

$$\begin{aligned} S &= \{A, P, N, Z, X, O\}; && \text{Set of shift types} \\ W &= \{1, 2, 3, 4, 5, 6\}; && \text{Set of weeks in the roster cycle} \\ D &= \{\text{Mon}, \dots, \text{Sun}\}; && \text{Set of days in a week} \end{aligned}$$

Note that for continuity, a dummy week 0 and dummy day Sun<sub>dummy</sub> also exist but are not part of the sets  $W$  and  $D$ .

## 1.b Decision Variables

$X$  is the array of binary variables determining if a type of shift belongs to a given week and day in the roster:

$$x_{s,w,d} \in \{0, 1\} \quad \forall s \in S, w \in W \cup \{0\}, d \in D \cup \{\text{Sun}_{\text{dummy}}\}$$

$Y$  determines if a given week in the roster is the night shift week. Note the night shift also includes the final three days of the preceding week:

$$y_w \in \{0, 1\} \quad \forall w \in W \cup \{\text{Sun}_{\text{dummy}}\}$$

$V$  denotes whether registrars are forced to take a weekend off:

$$v_w \in \{0, 1\} \quad \forall w \in W \cup \{\text{Sun}_{\text{dummy}}\}$$

## 1.c Constraints

Create a dummy week 0 that is equal to week 6.

$$x_{s,0,d} = x_{s,|W|,d} \quad \forall s \in S, d \in D \quad (1)$$

Create a dummy day 0 that is equal to Sunday of the previous week, allowing wrap-around from Sunday to Monday.

$$x_{s,w-1,|D|} = x_{s,w,0} \quad \forall s \in S, w \in W \quad (2)$$

Ensure every slot has a shift assigned by summing over all shift types, except for the night-shift week which must have two.

$$\sum_{s \in S} x_{s,w,d} - y_w = 1 \quad \forall w \in W, d \in D \quad (3)$$

Every day must have a single registrar assigned to each A, P and N shift.

$$\sum_{w \in W} x_{s,w,d} = 1 \quad \forall s \in \{A, P, N\}, d \in D \quad (4)$$

Every P shift must follow an A shift, except for Sunday where an A must follow.

$$x_{P,w,d} = x_{A,w,d-1} \quad \forall w \in W, d \in D \quad (5)$$

$$x_{A,w,d} = x_{A,w,d-1} \quad \forall w \in W, d \in \{\text{Sun}\} \quad (6)$$

Setting up the night shift: only one week can be the full 'night shift' week.

$$\sum_{w \in W} y_w = 1 \quad (7)$$

The Friday to Sunday before the full night shift week are also night shifts.

$$x_{N,w-1,d} - y_w = 0 \quad \forall w \in W, d \in \{\text{Fri, Sat, Sun}\} \quad (8)$$

The Monday to Thursday of the full night shift week are night shifts.

$$x_{N,w,d} - y_w = 0 \quad \forall w \in W, d \in \{\text{Mon, Tue, Wed, Thu}\} \quad (9)$$

The Friday to Sunday of the full night shift week are sleep shifts.

$$x_{Z,w,d} - y_w = 0 \quad \forall w \in W, d \in \{\text{Fri, Sat, Sun}\} \quad (10)$$

Ensure no one else has a sleep shift and is slacking off!

$$\sum_{w \in W, d \in D} x_{Z,w,d} = \text{n.o. allowed rests (i.e., 3)} \quad (11)$$

To give people weekends off: no weekdays are allowed to be taken off:

$$x_{X,w,d} = 0 \quad \forall w \in W, d \in \{\text{Mon}, \dots, \text{Fri}\} \quad (12)$$

Weekends must be taken off if scheduled as a 'weekend off' (but may be taken off on other weekends):

$$x_{X,w,d} \geq 0 \quad \forall w \in W, d \in \{\text{Sat, Sun}\} \quad (13)$$

No two consecutive weekends can pass without a weekend off being forced.

$$v_w + v_{w-1} \geq 1 \quad \forall w \in W \quad (14)$$