ENGSCI 355 Project 1

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Abstract

In this report, we describe the mixed-integer formulation to balance patient numbers at a hospital by rostering the registrars in each ward. We predict the infinite-horizon mean occupancies of each ward assuming a Poisson distribution of discharges.

We estimate that our program can reduce the average daily imbalance (the difference in patients between the most and least occupied wards each day) by approximately 42%. We believe most of this effect is due to the equidistant placement of each ward's registrars around the weeks of the roster; however, if this is not possible then our program is necessary to control spikes in ward admissions.

1 Formulation

Our model can be conceptualised as two coupled linear programs. The first is a roster of shifts to ensure feasibility, and the second one is a cyclical network representing ward occupancies during the 42 days of the roster.

1.a Parameters

$S = \{\text{A, P, N, Z, X, O}\};$ Set of shift types

$W = \{1, \ldots, 6\};$ Set of weeks in the roster cycle

$D = \{\text{Mon}, \ldots, \text{Sun}\};$ Set of days in a week

Note that for continuity, a dummy week 0 and dummy day $\text{Sun}_{\text{dummy}}$ also exist but are not part of the sets W and D.

1.b Decision Variables

X is the array of binary variables determining if a type of shift belongs to a given week and day in the roster:

$$x_{s,w,d} \in \{0,1\} \quad \forall s \in S,\ w \in W \cup \{0\},\ d \in D \cup \{\text{Sun}_{\text{dummy}}\}$$

Y determines if a given week in the roster is the night shift week. Note the night shift also includes the final three days of the preceding week:

$$y_w \in \{0,1\} \quad \forall w \in W \cup \{\text{Sun}_{\text{dummy}}\}$$

V denotes whether registrars are forced to take a weekend off:

$$v_w \in \{0,1\} \quad \forall w \in W \cup \{\text{Sun}_{\text{dummy}}\}$$

1.c Constraints

Create a dummy week 0 that is equal to the final week.

$$x_{s,0,d} = x_{s,|W|,d} \quad \forall s \in S,\ d \in D \tag{1}$$

Create a dummy day 0 that is equal to Sunday of the previous week, allowing wrap-around from Sunday to Monday.

$$x_{s,w-1,|D|} = x_{s,w,0} \quad \forall s \in S,\ w \in W \tag{2}$$

Ensure every slot has a shift assigned by summing over all shift types, except for the night-shift week which must have two.

$$\sum_{s \in S} x_{s,w,d} - y_w = 1 \quad \forall w \in W,\ d \in D \tag{3}$$

Every day must have a single registrar assigned to each A, P and N shift.

$$\sum_{w \in W} x_{s,w,d} = 1 \quad \forall s \in \{\text{A, P, N}\},\ d \in D \tag{4}$$

Every P shift must follow an A shift, except for Sunday where an A must follow.

$$x_{\mathrm{P},w,d} = x_{\mathrm{A},w,d-1} \quad \forall w \in W,\ d \in D \tag{5}$$

$$x_{\mathrm{A},w,d} = x_{\mathrm{A},w,d-1} \quad \forall w \in W,\ d \in \{\text{Sun}\} \tag{6}$$

Setting up the night shift: only one week can be the full ‘night shift’ week.

$$\sum_{w \in W} y_w = 1 \tag{7}$$

The Friday to Sunday before the full night shift week are also night shifts.

$$x_{\mathrm{N},w-1,d} - y_w = 0 \quad \forall w \in W,\ d \in \{\text{Fri, Sat, Sun}\} \tag{8}$$

The Monday to Thursday of the full night shift week are night shifts.

$$x_{\mathrm{N},w,d} - y_w = 0 \quad \forall w \in W,\ d \in \{\text{Mon, Tue, Wed, Thu}\} \tag{9}$$

The Friday to Sunday of the full night shift week are sleep shifts.

$$x_{\mathrm{Z},w,d} - y_w = 0 \quad \forall w \in W,\ d \in \{\text{Fri, Sat, Sun}\} \tag{10}$$

Ensure no one else has a sleep shift and is slacking off!

$$\sum_{w \in W,\ d \in D} x_{\mathrm{Z},w,d} = \text{n.o. allowed rests (i.e., 3)} \tag{11}$$

To give people weekends off: no weekdays are allowed to be taken off:

$$x_{\mathrm{X},w,d} = 0 \quad \forall w \in W,\ d \in \{\text{Mon}, \ldots, \text{Fri}\} \tag{12}$$

Weekends must be taken off if scheduled as a ’weekend off’ (but may be taken off on other weekends):

$$x_{\mathrm{X},w,d} \geq 0 \quad \forall w \in W,\ d \in \{\text{Sat, Sun}\} \tag{13}$$

No two consecutive weekends can pass without a weekend off being forced.

$$v_w + v_{w-1} \geq 1 \quad \forall w \in W \tag{14}$$

1.d Objective Parameters

There are several new definitions that are particular to this section of the formulation.

$$\begin{aligned} WA &= \{\text{lime},\ \text{navy},\ \text{yellow}\}; \quad \text{Set of wards} \\ R &= W \times D; \quad \text{Set of all days in the roster, to simplify } r+1 \text{ and } r-1 \text{ subscripts} \\ \rho &= \text{Poisson constant for discharges} \\ s_{wa,re} &\in \{1, \ldots, |W|\}, \quad re \in \{1, \ldots, n_{\text{registrars}}\} \end{aligned}$$

S is the vector of starting weeks for each registrar, where $n_{\text{registrars}}$ is the number of registrars per ward. S is not a decision variable of the IP; rather, it is enumerated by a tree with 15 unique combinations.

1.e Objective-Related Variables

A is a unimodular matrix determining whether a ward is admitting on a certain day.

$$a_{wa,r} \in \{0,\ 1\} \quad \forall wa \in WA,\ r \in R$$

O is a matrix of each ward's mean expected occupancy for each day in the roster.

$$o_{wa,r} \geq 0 \quad \forall wa \in WA,\ r \in R$$

Ṗ is the mean daily admission rate for each day of the week.

$$\dot{p}_d \in Z^+ \quad \forall d \in D$$

Δ is the maximum pairwise difference between all wards for a given day of the roster.

$$\delta_r \geq 0 \quad \forall r \in R$$

1.f Objective Constraints and Objective Function

These constraints calculate the expected mean occupancy of each ward throughout the 42-day roster. % is the modulo function and $\lceil x \rceil$ rounds up to the nearest integer.

Control whether a ward is admitting on a given day of the 6-week cycle. The absolutely insane subscripts adjust for starting week offsets; i.e. on week one, day one, a registrar who started on week 6 will actually be looking at week six, day one of the roster.

$$a_{wa,(r+1)\%|W\times D|+1} = \sum_{re\in\{1\ldots n_{\text{registrars}}\}} x_{\text{A},\lceil \frac{r+|D|\text{s}_{wa,re}-1}{|D|} \rceil \%|W|+1,(r-1)\%|D|+1} \quad \forall wa \in WA,\ r \in R \tag{15}$$

Calculate the expected occupancy based on the discounted occupancy from the previous day, plus the previous day's admissions if the ward was admitting. We used a mean stay of 4.5 days to model discharges as a Poisson process with a rate of 0.22.

$$o_{wa,r\%|R|+1} = o_{wa,r} \times (1-\rho) + a_{wa,r} \times \dot{p}_{(r-1)\%|D|+1} \quad \forall wa \in WA,\ r \in R \tag{16}$$

For each day calculate the maximum pairwise difference, between the most and least occupied wards.

$$\delta_r \geq o_{wa,r} - o_{wb,r} \quad \forall r \in W \times D,\ wa \in WA,\ wb \in WA \tag{17}$$

Objective function: Minimize the maximum pairwise occupant difference, summing over all days in the roster.

$$\text{minimize} \sum_{r\in R} \delta_r \tag{18}$$

2 Introduction

Finally, after a lengthy formulation:

This linear program generates feasible rostering solutions with the aim of minimising the mean expected disparity at any given day between the occupancies of the hospital's wards. The program is flexible, easily accommodating different staff arrangements and historical data with small modifications to a data file.

3 Assumptions

As the conditions for a feasible roster are given, all assumptions come from how we calculate the mean expected number of patients in each ward.

3.a People are Identical

To calculate the outcome of a roster, we take advantage of symmetry in the problem and take the liberty of adding some when convenient.

We assume that all ward teams act equally and independently, i.e. there is no difference if we swap the Navy and Lime teams. The state of one ward also does not affect the future state of another. Then we assume that all registrars are equal. Both of these are fair assumptions because registrars usually all have the same background and ward teams are separated.

A trickier assumption is that all patients are equal and independent. This allows us to model their stays with a Poisson distribution where the mean stay is 4.5 days. In reality this should be a heavy-tail distribution (if a patient is going to stay for a while, they will be there for a *long* time), but the memoryless property is a convenient approximation.

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3.b Occupation Numbers are Periodic

We assume that the mean admission for each day of the week does not change between weeks, so our input data is an average of each day from the past admissions data.

We also assume that patient numbers are periodic over the six-week roster. This avoids any discontinuities and checks that the hospital is not critical (even though by using a memoryless patient model, this is not possible).

4 Results

Table 1: Final optimised roster. Ward 1 registrars begin on weeks 1 and 4; Ward 2 on 2 and 5, and Ward 3 on 3 and 6. Notably, each ward has its registrars three weeks apart.

Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	O	A	P	O	N	N	N
2	NO	NO	NO	NO	ZO	ZX	ZX
3	O	O	O	O	A	P	P
4	O	O	O	A	P	X	X
5	A	P	A	P	O	A	A
6	P	O	O	O	O	X	X

By comparing our optimised roster with a series of random, unoptimised rosters (objective function fixed to zero), we see a significant improvement in the daily ward imbalances. On a day-to-day basis using a random roster, the most occupied ward will have approximately 46 people more people than the least occupied. After optimisation, this decreases to 20 - a 42% improvement.

Interestingly (noticed in the raw output), the random unoptimised sample with an identical starting week distribution to the optimal solution where registrars are equally spaced was only 6% worse. Although this is only one sample, it could indicate that equal registrar spacing is the most important factor. When registrars are not equally spaced, the unoptimised rosters are around 40% worse.

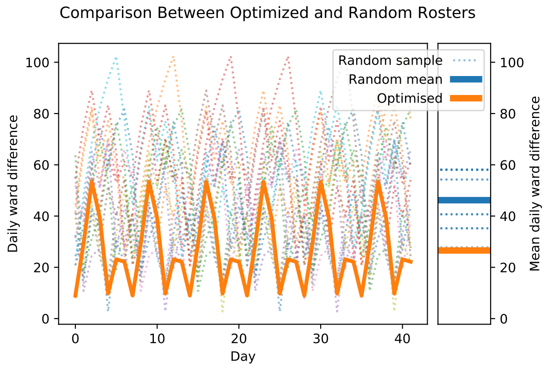

Figure 1: Effect of optimisation on day-to-day patient imbalances

5 Conclusion

We believe that our optimisation technique can drastically reduce hospital patient imbalances compared to a random roster. If we had more pages, we would like to display the results from our analysis of the contributions from different techniques (the roster itself vs registrar starting weeks). We also cannot include the small amount of sensitivity analysis we performed. However, we note that because we are using predictions for the long-term mean, our figures will be resilient to random fluctuations in admissions.