ENGSCI 355 Project 1

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1 Formulation

1.a Parameters

$$S = \{\text{A, P, N, Z, X, O}\}; \quad \text{Set of shift types}$$
$$W = \{1, \ldots, 6\}; \quad \text{Set of weeks in the roster cycle}$$
$$D = \{\text{Mon}, \ldots, \text{Sun}\}; \quad \text{Set of days in a week}$$

Note that for continuity, a dummy week 0 and dummy day $\text{Sun}_{\text{dummy}}$ also exist but are not part of the sets W and D.

1.b Decision Variables

X is the array of binary variables determining if a type of shift belongs to a given week and day in the roster:

$$x_{s,w,d} \in \{0,1\} \quad \forall s \in S,\ w \in W \cup \{0\},\ d \in D \cup \{\text{Sun}_{\text{dummy}}\}$$

Y determines if a given week in the roster is the night shift week. Note the night shift also includes the final three days of the preceding week:

$$y_w \in \{0,1\} \quad \forall w \in W \cup \{\text{Sun}_{\text{dummy}}\}$$

V denotes whether registrars are forced to take a weekend off:

$$v_w \in \{0,1\} \quad \forall w \in W \cup \{\text{Sun}_{\text{dummy}}\}$$

1.c Constraints

Create a dummy week 0 that is equal to the final week.

$$x_{s,0,d} = x_{s,|W|,d} \quad \forall s \in S,\ d \in D \tag{1}$$

Create a dummy day 0 that is equal to Sunday of the previous week, allowing wrap-around from Sunday to Monday.

$$x_{s,w-1,|D|} = x_{s,w,0} \quad \forall s \in S,\ w \in W \tag{2}$$

Ensure every slot has a shift assigned by summing over all shift types, except for the night-shift week which must have two.

$$\sum_{s \in S} x_{s,w,d} - y_w = 1 \quad \forall w \in W,\ d \in D \tag{3}$$

Every day must have a single registrar assigned to each A, P and N shift.

$$\sum_{w \in W} x_{s,w,d} = 1 \quad \forall s \in \{\text{A, P, N}\},\ d \in D \tag{4}$$

Every P shift must follow an A shift, except for Sunday where an A must follow.

$$x_{\mathrm{P},w,d} = x_{\mathrm{A},w,d-1} \quad \forall w \in W,\ d \in D \tag{5}$$
$$x_{\mathrm{A},w,d} = x_{\mathrm{A},w,d-1} \quad \forall w \in W,\ d \in \{\text{Sun}\} \tag{6}$$

Setting up the night shift: only one week can be the full 'night shift' week.

$$\sum_{w \in W} y_w = 1 \tag{7}$$

The Friday to Sunday before the full night shift week are also night shifts.

$$x_{\mathrm{N},w-1,d} - y_w = 0 \quad \forall w \in W,\ d \in \{\text{Fri, Sat, Sun}\} \tag{8}$$

The Monday to Thursday of the full night shift week are night shifts.

$$x_{\mathrm{N},w,d} - y_w = 0 \quad \forall w \in W,\ d \in \{\text{Mon, Tue, Wed, Thu}\} \tag{9}$$

The Friday to Sunday of the full night shift week are sleep shifts.

$$x_{\mathrm{Z},w,d} - y_w = 0 \quad \forall w \in W,\ d \in \{\text{Fri, Sat, Sun}\} \tag{10}$$

Ensure no one else has a sleep shift and is slacking off!

$$\sum_{w \in W,\ d \in D} x_{\mathrm{Z},w,d} = \text{n.o. allowed rests (i.e., 3)} \tag{11}$$

To give people weekends off: no weekdays are allowed to be taken off:

$$x_{\mathrm{X},w,d} = 0 \quad \forall w \in W,\ d \in \{\text{Mon}, \ldots, \text{Fri}\} \tag{12}$$

Weekends must be taken off if scheduled as a 'weekend off' (but may be taken off on other weekends):

$$x_{\mathrm{X},w,d} \geq 0 \quad \forall w \in W,\ d \in \{\text{Sat, Sun}\} \tag{13}$$

No two consecutive weekends can pass without a weekend off being forced.

$$v_w + v_{w-1} \geq 1 \quad \forall w \in W \tag{14}$$