
ENGSCI 355 Project 1

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AUGUST 14, 2017 The University of Auckland

1 Formulation

Our model can be conceptualised as two, linked linear programs. The first is a roster of shifts, and the second one is a cyclical network representing ward occupancies during the 42 days of the roster.

1.a Parameters

$$\begin{aligned} S &= \{A, P, N, Z, X, O\}; && \text{Set of shift types} \\ W &= \{1, \dots, 6\}; && \text{Set of weeks in the roster cycle} \\ D &= \{\text{Mon}, \dots, \text{Sun}\}; && \text{Set of days in a week} \end{aligned}$$

Note that for continuity, a dummy week 0 and dummy day Sun_{dummy} also exist but are not part of the sets W and D .

1.b Decision Variables

X is the array of binary variables determining if a type of shift belongs to a given week and day in the roster:

$$x_{s,w,d} \in \{0, 1\} \quad \forall s \in S, w \in W \cup \{0\}, d \in D \cup \{\text{Sun}_{\text{dummy}}\}$$

Y determines if a given week in the roster is the night shift week. Note the night shift also includes the final three days of the preceding week:

$$y_w \in \{0, 1\} \quad \forall w \in W \cup \{\text{Sun}_{\text{dummy}}\}$$

V denotes whether registrars are forced to take a weekend off:

$$v_w \in \{0, 1\} \quad \forall w \in W \cup \{\text{Sun}_{\text{dummy}}\}$$

1.c Constraints

Create a dummy week 0 that is equal to the final week.

$$x_{s,0,d} = x_{s,|W|,d} \quad \forall s \in S, d \in D \quad (1)$$

Create a dummy day 0 that is equal to Sunday of the previous week, allowing wrap-around from Sunday to Monday.

$$x_{s,w-1,|D|} = x_{s,w,0} \quad \forall s \in S, w \in W \quad (2)$$

Ensure every slot has a shift assigned by summing over all shift types, except for the night-shift week which must have two.

$$\sum_{s \in S} x_{s,w,d} - y_w = 1 \quad \forall w \in W, d \in D \quad (3)$$

Every day must have a single registrar assigned to each A, P and N shift.

$$\sum_{w \in W} x_{s,w,d} = 1 \quad \forall s \in \{A, P, N\}, d \in D \quad (4)$$

Every P shift must follow an A shift, except for Sunday where an A must follow.

$$x_{P,w,d} = x_{A,w,d-1} \quad \forall w \in W, d \in D \quad (5)$$

$$x_{A,w,d} = x_{A,w,d-1} \quad \forall w \in W, d \in \{\text{Sun}\} \quad (6)$$

Setting up the night shift: only one week can be the full ‘night shift’ week.

$$\sum_{w \in W} y_w = 1 \quad (7)$$

The Friday to Sunday before the full night shift week are also night shifts.

$$x_{N,w-1,d} - y_w = 0 \quad \forall w \in W, d \in \{\text{Fri, Sat, Sun}\} \quad (8)$$

The Monday to Thursday of the full night shift week are night shifts.

$$x_{N,w,d} - y_w = 0 \quad \forall w \in W, d \in \{\text{Mon, Tue, Wed, Thu}\} \quad (9)$$

The Friday to Sunday of the full night shift week are sleep shifts.

$$x_{Z,w,d} - y_w = 0 \quad \forall w \in W, d \in \{\text{Fri, Sat, Sun}\} \quad (10)$$

Ensure no one else has a sleep shift and is slacking off!

$$\sum_{w \in W, d \in D} x_{Z,w,d} = \text{n.o. allowed rests (i.e., 3)} \quad (11)$$

To give people weekends off: no weekdays are allowed to be taken off:

$$x_{X,w,d} = 0 \quad \forall w \in W, d \in \{\text{Mon}, \dots, \text{Fri}\} \quad (12)$$

Weekends must be taken off if scheduled as a ‘weekend off’ (but may be taken off on other weekends):

$$x_{X,w,d} \geq 0 \quad \forall w \in W, d \in \{\text{Sat, Sun}\} \quad (13)$$

No two consecutive weekends can pass without a weekend off being forced.

$$v_w + v_{w-1} \geq 1 \quad \forall w \in W \quad (14)$$

1.d Objective Parameters

There are several new definitions that are particular to this section of the formulation.

$WA = \{\text{lime, navy, yellow}\}$; Set of wards

$R = W \times D$; Set of all days in the roster, to simplify $r+1$ and $r-1$ subscripts

ρ = Poisson constant for discharges

1.e Objective Variables

A is a unimodular matrix determining whether a ward is admitting on a certain day.

$$a_{wa,r} \in \{0, 1\} \quad \forall wa \in WA, r \in R$$

O is a matrix of each ward's mean expected occupancy for each day in the roster.

$$o_{wa,r} \geq 0 \quad \forall wa \in WA, r \in R$$

P is the mean daily admission rate for each day of the week.

$$p_d \in \mathbb{Z}^+ \quad \forall d \in D$$

Δ is the maximum pairwise difference between all wards for a given day of the roster.

$$\delta_r \geq 0 \quad \forall r \in R$$

1.f Objective Constraints and Objective Function

These constraints calculate the expected mean occupancy of each ward throughout the 42-day roster. % is the modulo function and $\lceil x \rceil$ rounds up to the nearest integer.

Control whether a ward is admitting on a given day of the 6-week cycle. The absolutely insane subscripts adjust for starting week offsets; i.e. on week one, day one, a registrar who started on week 6 will actually be looking at week six, day one of the roster.

$$a_{wa,(r+1)\%|W \times D|+1} = \sum_{re \in \{1 \dots n_{\text{registrars}}\}} x_{A, \lceil \frac{r+|D|s_{wa,re}-1}{|D|} \rceil \% |W|+1, (r-1)\%|D|+1} \quad \forall wa \in WA, r \in R \quad (15)$$

Calculate the expected occupancy based on the discounted occupancy from the previous day, plus the previous day's admissions if the ward was admitting. We used a mean stay of 4.5 days to model discharges as a Poisson process with a rate of 0.22.

$$o_{wa,r\%|R|+1} = o_{wa,r} \times (1 - \rho) + a_{wa,r} \times \dot{p}_{(r-1)\%|D|+1} \quad \forall wa \in WA, r \in R \quad (16)$$

For each day calculate the maximum pairwise difference, between the most and least occupied wards.

$$\delta_r \geq o_{wa,r} - o_{wb,r} \quad \forall r \in W \times D, wa \in WA, wb \in WA \quad (17)$$

Minimize the maximum pairwise occupant difference, summing over all days in the roster.

$$\text{minimize } \sum_{r \in R} \delta_r \quad (18)$$