Geothermal Strategy Optimisation: A Summary

ENGSCI 700 Literature Review and Statement of Research Intent

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***Abstract*—The Wairakei geothermal field is one of the oldest geothermal electricity producers in the world and has been instrumental in advancing the utilisation of lower enthalpy fluids. Contact Energy Ltd. is the current operator, and they wish to find ways to increase the productivity of their assets and staff.**

The optimisation of geothermal well operation is not well-studied; it is often done on a case-by-case basis for each steam field. However, there are many techniques such as non-linear analysis, mixed-integer programming and Bayesian inference that could improve the current workflow applied by Contact Energy. This document contains a review of previous research in these techniques applied to different fields, and proposes future research toward implementation at the Wairakei network.

I. Introduction

Geothermal power is a hallmark of New Zealand's renewable generation alongside hydro and wind. It makes up 13% of the nation's electricity supply, with the first generator being commissioned in Wairakei in 1958. Although New Zealand is not the world's largest geothermal producer, it is unique in that Wairakei is liquid-dominated, and most of the geothermal production is two-phase flow or 'wet' steam, which must be filtered to avoid damaging the turbines. This contrasts with the 'dry' steam in places such as Italy, which contains no water and is easier to handle [1].

This review will begin with the current and historical physical state of the Wairakei geothermal surface network, along with some of the processes used by the current operators, Contact Energy Ltd. A summary of literature surrounding operational management and modelling of geothermal networks will follow, along with areas of potential research.

The scope of this review is the modelling of the surface network from the wellhead to the generator. We are less interested in subsurface effects such as subsidence, scaling (deposition of impurities in the wellbore) and the possible reoccurrence of injected fluid at production wells, which are typically analysed with subsurface modelling and wellbore simulators.

II. Objectives of the Wairakei Geothermal Field

Currently, the Wairakei geothermal field has over 200 wells [2]. While many are retired, the wells in operation must primarily be maintained to a long-term operational standard, and also support a baseload New Zealand power generation.

There is currently no automatic process in place to optimise the surface network flows. While Contact Energy has tools to store, analyse and present data, these do not make recommendations, and all operational decisions such as workovers for de-scaling are made by experienced staff or by heuristics (e.g. the highest enthalpy wells in decreasing order are directed to Te Mihi, while lower enthalpy wells are directed to Wairakei).

A. Long-Term Sustainability

The significance of sustainability was demonstrated by a period in 1960-1970 when reservoir temperatures and pressures declined rapidly, affecting production at some wells. The environmental impact of discharge was also not taken into account during commission, with 4500 T/h discharged into the Waikato River until 1997. Re-injection of fluids was introduced but must be done carefully with regard to its effect on the pressure of the reservoir and potential suppression of hot fluids by cooler, re-injected fluid. Many of the limitations regarding environmental sustainability are quantified by resource consents held by Contact Energy. These will impose environmental constraints in any potential research such as:

1. River discharge (e.g. temperature, mass, arsenic, hydrogen sulphide) [3].
2. Long-term pressure drawdown in the reservoir with:
    a. Daily mass uptake (280 kT/d).
    b. Average mass take over three months (245 kT/d [4]).
    c. A proposal to replace three-month limit with an annual limit.
3. Subsidence in the Taupo region as a result of pressure loss [4].

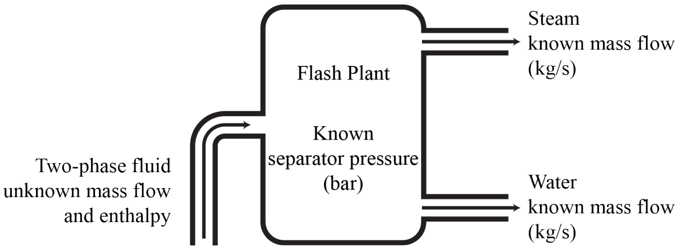

Figure 1: Schematic diagram of a flash plant, showing the known and unknown inputs and outputs.

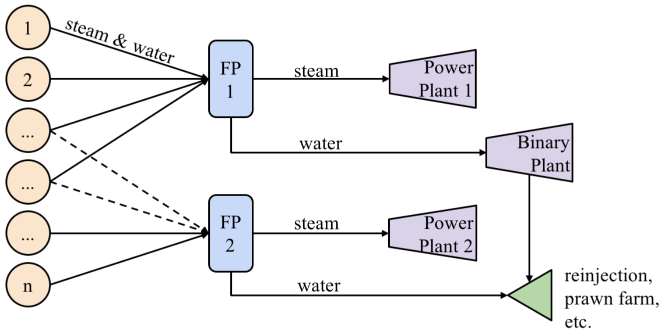

Figure 2: High-level representation of the Wairakei surface network.

B. Short-term Power Generation

Although the field is known to recharge over time, balancing sustainable production in the future with satisfying current energy needs is a difficult challenge because the recharge rate is slow. The resource consents held by Contact Energy also include measures to maximise long-term power generation. Factors that have previously been taken into account include, but are not limited to:

1. Base-load power generation requirements.
2. Scaling and work-overs of wells.
3. Operating conditions of wells and separators, e.g. enthalpy and pressure limits.
4. Start-up and shut-down times of wells.
5. Safety valve pressures in pipelines.

The most important facilities of the surface network are wells, flash plants and generators.

a) Well bores

Well bores are cylindrical shafts with a valve at the surface. The valve has a pressure gauge attached, which collects data in real time. However, they are not paired with a flow meter. Well bores in the field have varying enthalpy and steam qualities, which are measured occasionally but must otherwise be estimated. Superheated steam or a two-phase steam and brine mixture flows from the wells to flash plants (sometimes called separators when located at individual well-heads).

b) Flash Plants

The role of a flash plant or separator is to extract pure steam from the two-phase flow as water and chemical impurities cause pitting in turbine blades. Extra steam can be extracted by a maximum of three pressure drops, causing water to flash and increase the total steam flow. The inputs and outputs of a flash plant are shown in Figure 1, where we can observe the outputs but do not know the individual mass inflows.

Some wells can output to one of a selection of flash plants. This scale of this decision is on the order of months, and does not affect the day-to-day operation. Inactive connections are shown in Figure 2 with dashed lines.

Extra water is released into the Wairakei River or reinjected at another bore.

c) Generators

The resulting, highly purified steam drives generators at the Wairakei, Te Mihi and Poihipi power plants, which have different efficiencies. Some hot water is redirected to a binary plant that uses pentane as a working fluid.

The surface network currently feeds a PI database with a combination of real-time flow sensors and manual testing. By using the connectivity of the graph, manual corrections (offsets) can be applied to readings where the sensors are inaccurate by ensuring mass flow is preserved. This process is not automated and is done on an ad-hoc basis.

III. Current Research

The operation of geothermal and petroleum wells is discussed extensively in the scientific literature. We are mainly interested in three aspects of modelling and simulation: short-term predictions for production, methods to optimise a long-term strategy for a set of wells in a reservoir, and methods to cope with uncertainty in the network.

A. Network Modelling Techniques

Modelling of the Wairakei field has taken place at the University of Auckland since the 1970s. However, the majority of this work is focused on subsurface flows within the reservoir; models of the surface pipe network are developed by Contact Energy.

1) Graph Representation

Figure 2 shows a simplified representation of the connectivity of the surface network, which can be represented as an acyclic directed graph. In general, the network takes on a tree structure with wells, flash plants and generators as the nodes and pipelines as the arcs.

2) Heat Loss

Loss of heat to the environment is a common cause of inefficiency for power systems. This will occur especially when there are events such as contact with machinery, creating friction and inducing conduction and convection. Zarrouk states that in pipes, heat loss is negligible at around 0.6% [5]. Heat loss for more complex components is much more difficult to estimate, and is likely contained within the overall efficiency for the component.

3) Nonlinear Modelling

Linear equations are common when modelling a network, making the assumption that flows can be linearly super-imposed on each other and exist in an acyclic graph. This makes finding the state of a system and optimisation of decision variables possible by techniques such as linear and mixed-integer programming.

However, there are limitations outlined by Y. Huang and D. H. Freeston [6]. For instance, head loss in a pipe is often of the non-linear form:

$$h_L = \frac{kLQ^n}{D^m}$$

where h_L is the head loss, k is the friction factor, L is the pipe length, Q is the volumetric flow rate and D is the pipe diameter.

Inclusion of non-linear effects can more accurately capture the physical processes that affect power generation at Wairakei. These sets of non-linear equations can be solved using derivative methods but rely on convergence, so Huang and Freeston had to be careful to ensure their method would converge. Optimality is also difficult to prove, compared with a linear or mixed-integer program.

4) Outage Scheduling

Both the geothermal and petroleum extraction industries require well *workovers*, which are labourious interventions to restore the function of a corroded, damaged or otherwise impaired well. Workovers are limited by the number of rigs available. In a 1976 paper [7], workovers are scheduled to minimise the loss of having each well offline for a time t_i, plus lost production from before the workover, p_i. The paper presents two heuristics for finding good solutions within 2% of optimality within half an hour for using a 'desk calculator'. Optimality may be possible using modern branch-and-bound techniques.

B. Production Prediction

An IPENZ report by K. Wigram [8], first presented at the New Zealand Geothermal Workshop in 2012, outlines the need for prediction at Wairakei and gives an overview of current (pre-2012) methods. A brief summary follows:

1) Predicting Generation Using the Past

Historical macro-trends can be extrapolated into the future. Trends such as pressure loss in each well are due to effects such as scaling and reservoir drawdown, and are roughly linear. More complex trends such as the pressure restoration after a workover depend on the engineer's knowledge of the well and are difficult to predict accurately.

Simple linear models also fail to capture the interaction between wells. Wigram demonstrates that if a 3 MW well were added to an existing field, the actual marginal power increase may only be 2 MW if pressure at existing wells is negatively affected.

2) Building an Excel Model

Consistent with our own observations of the methods used at Contact, Wigram details the use of an Excel workbook to predict power output by tracking flows to and from each facility. This workbook contains basic physical and thermodynamic calculations, and also affords the operator some diagnostic capability when checked for behaviours such as conservation of mass. Wigram concludes that this model gives good predictions for events such as new wells and outages; however, it is not accurate for long-term predictions. The greatest benefit of this model is the ability to test hypothetical scenarios; e.g. restarting a high-pressure turbine, or estimating heating in the river.

A few of the conclusions drawn by Wigram include:

1. The ability to test scenarios to maximise economic gain or minimise loss.
2. Even if accuracy cannot be achieved, consistency is still useful as two outcomes can be compared.
3. There is no substitution for real experience. Model validation and testing is always necessary.
4. A good model can result in huge economic benefits.

C. Uncertainty

Current models used at Contact Energy do not take uncertainty into account. One source of uncertainty is due to measurement noise. For instance, measurements of mass flows from wells entering a flash plant do not always sum to the measured mass flow leaving the flash plant. These suggest calibration or systematic errors and require the operator to add a correction so that mass is conserved.

A second source of uncertainty comes from variation in the well test data. *Bore tests* take a well offline, apply a testing apparatus to the well-head, and run the disconnected well at different pressures to evaluate the function $\dot{m} = f(P_{\text{wh}})$, where $\dot{m}$ is the mass extraction rate and P_{wh} is the well-head pressure. Bore tests are repeated at multiple pressures on a given day to create data for a production curve. *Tracer flow tests* (TFTs) inject a tracer dye at the well-head under normal operating conditions. The downstream concentrations of separate liquid and vapour tracers reveal the mass flow rates of their respective phases of the water they are mixed with. The well can continue production during TFTs and mass flow is measured under realistic conditions, but this only provides one data point which cannot describe a curve. TFTs are therefore used to adjust the curve set by a previous bore test. The usage of bore test and TFT data is discussed further in Section III.C.3).

Some statistical methods are available to account for incomplete information and generate parameter estimates/distributions. While Contact Energy use regression methods, including a model for variance is a potential improvement on point estimates.

1) Bayesian Framework

Z. Poulakis et al. applied Bayesian methods to detect leaks in pipes based on pressure and flow measurements [9]. With K possible leak locations, analogous to K forms of decline in the

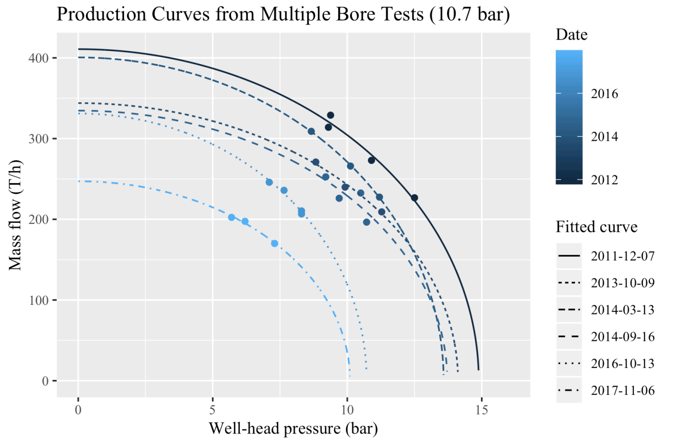

Figure 3: Elliptic production curves fitted to WK261 bore tests. Data supplied by Contact Energy. The curves tend to decline, but do not model time as a continuous variable.

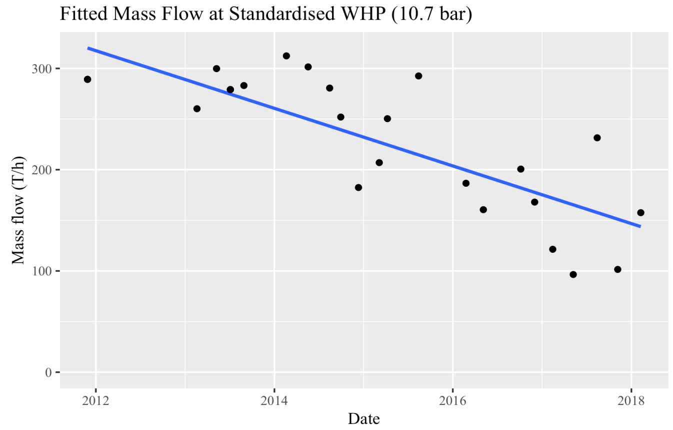

Figure 4: Pressure decline for well WK261 with a linear approximation. Mass flows are estimated from production curves at a fixed well-head pressure.

network such as scaling or pressure decline, solving a multi-dimensional optimisation problem to find the maximum of the probability density function for leaks in the K pipes allows the most likely locations for the leak given the data to be estimated. Similarly, we could use Bayesian analysis to estimate the true condition of the wells given erroneous data. This method requires simulation over a large parameter space to find the best fitting set(s) of parameters, and/or expert knowledge to create realistic prior distributions.

Bayesian probabilities also offer the advantage of being able to identify data points or wells that have low precision and are likely to be erroneous. The current operators rely on experience to detect bad data.

2) Monte Carlo Methods

Relating to the Bayesian framework, the graph structure of the geothermal network lends itself to simulation under uncertainty with methods such as sampling using the open-source JAGS (Just Another Gibbs Sampler). This is either used to compute posterior distributions, or to generate synthetic prior estimates under the empirical Bayes method for further Bayesian inference.

Specifically, Monte Carlo (MC) methods are applicable to scenario analysis under uncertainty. When making decisions involving significant capital, Contact Energy is obligated to carry out due diligence on its investments. Sampling distributions offer management a better idea of the risk involved than point estimates. A method for estimating the probability of failure using net present value (NPV) is described by M. Goumas et al. [10] in their report on how to incorporate a range of technical, economic, social and environmental parameters into the decision making process. They conclude that MC methods require less prior information because guesses can be made, they make interpretation accessible for a layperson, and sensitivity analysis is easy to perform.

Scenario evaluation with simulation also enables rudimentary multi-objective optimisation to be performed by comparing posteriors under different scenarios. Common conflicting objectives include the tradeoff between maximising the expected value and minimising the variance.

3) Model Fitting

Wells do not have flow meters attached during normal operation. Currently, Contact Energy estimates a production curve of mass flow as a function of well-head pressure at a single point in time:

$$\dot{m} = f(P_{\mathrm{wh}} | t)$$

Grant and Bixley [11] propose two different equations that can be fit to a series of three points at different pressures.

$$W = \beta_1(\beta_2^2 - P_{\mathrm{wh}}^2)^{\beta_3}$$

$$\frac{(\dot{m} - \beta_1)^2}{\beta_2^2} + \frac{P_{\mathrm{wh}}^2}{\beta_3^2} = 1$$

They suggest that the fitted performance of both equations is similar; therefore, the elliptical equation is used by Contact Energy because the parameters relate to real-world metrics:

$$\dot{m}_{\min} = \beta_1$$
$$\dot{m}_{\max} = \beta_1 + \beta_2$$
$$P_{\mathrm{wh,\,max}} = \beta_3$$

Fitting three parameters requires the bore test to be run at three different pressures each time. An example of production curves fitted at discrete dates is given in Figure 3, with each date having a discrete date.

Each production curve is used to estimate mass flow if it were measured at a fixed well-head pressure, $\dot{m}|P_{\mathrm{wh}}$. With multiple bore tests taken at different points in time, a plot of standardised mass flow over time is created, $\dot{m}|P_{\mathrm{wh}} = g(t)$. An example from data presented by Contact Energy is shown in Figure 4, where they fit a linear approximation to the decline rate. A similar linear regression is fitted to the trend in enthalpy.

This method allows Contact Energy to observe the changing conditions of the well, independent of operating pressure. It is used by the operator to indicate whether maintenance is required. With experience, the operator can also diagnose characteristics such as whether it is correlated with nearby wells or the cause of decline, such as scaling, instrumentation error or reservoir pressure loss.

Figure 4 is used to gauge the magnitude of decline for a single well-head pressure and can indicate whether the production curve needs to be re-tested and re-fit. It does not give a representation of how the production curve changes over time, and therefore cannot be used to forecast production in the future. Each curve also does not incorporate data from soon before or after the bore test, inflating variance in the estimates shown in Figure 3 and Figure 4.

In their current state, each independent model is only supported by three points when typically, thirty or forty are available. To improve the model, a production curve incorporating time as a covariate could be of the form:

$$\frac{(\dot{m}-\beta_1(t))^2}{\beta_2^2(t)}+\frac{P_{\mathrm{wh}}^2}{\beta_3^2(t)}=1$$

where $\beta_i(t)$ is typically a model such as ordinary linear regression $\beta_i(t) = mt + c$ or a time-series model such as the auto-regressive integrated moving average (ARIMA) family, which does not assume independence between consecutive observations. This would increase the effective sample size of each production curve and enable the entire curve to be forecasted.

IV. Potential Areas of Development

Research around optimisation and stochastic modelling has seen little application to the operational strategy of geothermal networks. It has focused more on the prediction of pipe flows than operational decisions. This gives us several opportunities to research the applications of existing mathematical techniques to geothermal networks.

A. *Decision-making in a Pipe Network*

Current research focuses on solving for pipe flows where flows and pressures are the only variables. There is not much research on the discrete optimisation of pipe networks; i.e. which wells to activate and which pipe sections to route flows through.

Further optimisation can also include a scheduling component. The most disruptive activity that takes place at the geothermal field is wellbore workovers, which must be planned carefully. We also know that if a well is shut down, the pipes cool and this incurs a warm-up delay before the well can be restarted. An unscheduled but potentially preventable blowing of the trip valves also incurs a time delay.

B. *Time-series Prediction of Well Parameters*

The current operational spreadsheet generates production curves at discrete points in time in the past. However, there is no parametric representation of time, so the relationship of the production curve with time cannot be extrapolated into the future.

C. *Bayesian Analysis on a Transient Network of Pipes*

Geothermal fluids contain more impurities than a city's network of potable water pipes. This, combined with extreme changes in pressure and temperature, contributes to high levels of scaling and corrosion. Pressure is therefore expected to decline over time, compounded by the possibility of temperature pressure loss in the reservoir itself.

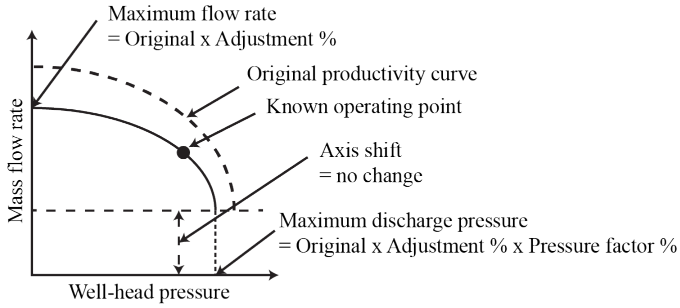

Figure 5: Demonstration of how production curves change over time and can be updated with a new data point.

Bayesian analysis (or, as an alternative, frequentist maximum likelihood estimation) can offer probabilistic distributions of the network parameters such as maximum flow or actual flow, and the effects of taking certain actions on forecasts. This has the potential to improve on the point-estimates currently made by the Contact Energy operators, and the single snapshots in time, such as those used in research such as Poulakis et al. [9].

V. Statement of Research Intent

The work we intend to carry out centers around the implementation and adaptation of engineering science methods to the management of the Wairakei geothermal field. The overall objectives for this project are:

1. Develop a mathematical/computational model of the Wairakei network that is flexible and can be reused in further research.
2. Create an optimisation program that generates policy recommendations in the near future for decisions such as bringing wells online/offline, redirecting flows and maintenance events.
3. Simulate network states and power plant performance under uncertainty to augment the existing forecasting workbook.
4. Develop a time-series forecast to predict changes in the production curves as illustrated by Marsh in Figure 5 [12], enthalpies and propagate changes through to future power generation estimates.

A. *Resources*

Historical flow meter data, schematics and details of selected past events such as bore tests have been provided by Contact Energy in the form of several Excel workbooks.

An AMPL licence is desired for the optimisation component of this project because of its compatibility with multiple different solvers. However, there are also open-source alternatives, detailed in the methodology.

Guidelines for uncertainty estimates and thermodynamic equations are available from both Grant and Bixley [11] and personal contact with Contact Energy.

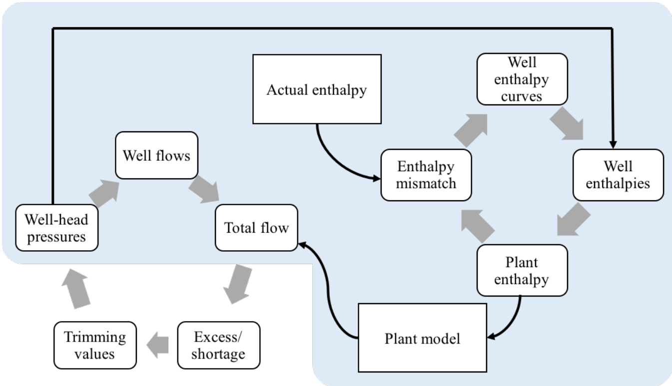

Figure 6: Information flow for calculations of the Wairakei surface network. The blue shaded region is the scope of our proposed research.

B. Methodology

The four objectives listed above provide a framework for tracking the progress of this project in a relatively linear fashion. Although our research team is a pair of undergraduates, the separate tasks have dependencies and will likely be done in a series of steps.

1) Develop a Model

Previous models of pipe networks have shown linear equations give good representations of the flows and physics in the network. The field can therefore be modelled by a directed acyclic graph where a series of stochastic or deterministic conversions take place at the nodes. Variables are not just limited to flows – they can also include decisions on which routes to activate to maximise power and probabilities of events such as constraint violation. Constraints are provided by equipment limitations and council resource consents.

The models seen in previous research contain a mix of linear and non-linear relationships. Nonlinear equations can be used in Monte Carlo methods, but may need to be linearised, discretised, or replaced by other approximations for use in linear programs.

2) Optimisation

A linear representation of the network makes solving to optimality possible and sometimes computationally cheap using commercially available solvers. We want to make recommendations for the daily or weekly operations of the network in order to maximise revenue while remaining within environmental and operational constraints.

Decision variables include whether a well is enabled or disabled, and for some wells, which flash plants their flows are directed to when there is a choice e.g. Te Mihi [13]. This will be done on a discretised time scale, starting large and decreasing to the daily level if the optimisation is performant.

Candidates for implementing a linear program include AMPL and Python via the PuLP package [14]. Pipes can be represented as arcs with flows between nodal components where thermodynamic conversions take place. Linear programming requires all functions to be linear in terms of the decision variables. For example, the mass of steam resulting from a drop to a pressure P is:

$$\dot{m}_{\text{steam},j} = \dot{m}_j \frac{h_j - h_{f@P}}{h_{fg@P}}$$

where $\dot{m}_j$ is the total mass flow (a decision variable) and h are enthalpic constants of the flow at a given pressure, resulting in a linear equation.

A secondary goal for the optimisation is to take into account the scheduling of maintenance activities. Two considerations raised by Contact Energy and a third that may be of importance are:

1. The necessity of activities on a well: will they make a difference?
2. Impact of maintenance on power output of the field.
3. Scheduling and availability of resources; this will require additional techniques such as branch and bound on top of linear programming if the problem is non-linear.

Stochastic optimisation will be considered, although it is not always tractable with linear/mixed-integer programming alone. Successful optimisation can be verified by the Contact Energy operators, as they have expert knowledge about running the field at optimal or near-optimal performance.

3) Simulation

Marsh describes a high-level overview of an iterative scheme, adapted in Figure 6, where the state of the full surface network could theoretically be calculated [12]. While the current methods include deterministic calculations and ad-hoc estimates by operators, automated stochastic modelling can streamline the process and offer more representative precision.

Simulation of the network according to the shaded region in Figure 6 will not involve novel methods, but instead an application of proven statistical methods on a model. It will also evaluate the performance of different solutions to confirm optimality and aid analysis.

As with previous studies on simulation to evaluate geothermal strategies [10], this will consist of a Bayesian approach to generate posterior densities of parameters such as flow, probabilities of constraint violations such as mass limits, and predicted power output.

Tools to perform analysis include the RJAGS package for R, various MCMC (Markov Chain Monte Carlo) packages in Python, or a custom Metropolis-Hastings algorithm in Python. Custom implementations require additional code verification, but the additional control makes it possible to integrate an object-oriented approach into both the sampling and the optimisation simultaneously.

If using a package to perform analysis, several alternatives need to be explored. While both JAGS [15]/RJAGS [16] and PyMC [17] are relatively new, WinBUGS is better established but is not cross-platform.

4) Validation and Verification

We will know our if our optimisation and simulation are effective by:

1. Comparisons between our optimal decisions with historical decisions as well as between optimal and historical flows can verify that our linear program is set up correctly if we find strong similarities. It can also confirm whether Contact Energy is already making optimal decisions.
2. DIC (Deviance Information Criteria) is one method to compare fits between different models and find the most likely one that fits the data. The amount of detail we can extract from a model depends on having enough data to support it, so checking DIC can help check that a model fits well without making claims that have too much uncertainty associated with them.
3. To quantify prediction accuracy at a point in time in the future, we can train on historical data up to a certain time in the past and make predictions from that time up to the most recent data. A comparison between predicted values and the ground truth can verify that our model can forecast to unknown scenarios and give an idea of how its accuracy decreases the further into the future we look.
4. Finally, we will validate and verify our model by generating sample outputs such as a visual representation of our graph/matrix showing which facilities are connected, which types of fluid flow along each arc and typical distributions of the parameters such as enthalpy and mass flow. An engineer familiar with the actual operation of the Wairakei geothermal field can confirm that our model is a useful representation of reality, and offer validation by confirming whether our outputs are a useful extension to the existing Excel spreadsheet.

Once performance is confirmed, a working simulation model will be turned into a tool to aid operators.

C. Limitations of Scope

There are aspects we would like to explore given more time but are not necessary for this research.

1) Integration with Excel

Excel is a standard within most businesses, including Contact Energy [8]. Their current methods are implemented in Excel. We have observed that Excel is also the cause of much dissatisfaction, with the workbook taking several minutes to launch and requiring its own desktop to not delay other tasks. To develop a working demonstration sooner, we will use software more suited to mathematical analysis such as Python and R.

2) Optimisation of the Entire Network

It has been made clear by Contact Energy that optimisation in the field involves all facilities. However, there are many facilities and many different variations of each facility that would have to be manually implemented. This research will first focus on a section of the field with a smaller amount of distinct facilities for simplicity, and if time allows, may be expanded to the entire network.

3) Global Optimisation

Our stochastic optimisation will first focus on optimisation of the expected value using linear programming. If the distributions of uncertain parameters can be estimated and computation time is not too long, global optimisation methods beginning with direct enumeration will also be experimented with but are not guaranteed.

D. Significance of Research

With this research, we will deliver a proof-of-concept and development in modelling of geothermal surface networks.

1) Proof-of-Concept

A proof-of-concept will illustrate three main benefits to Contact Energy:

1. Reduce the time spent updating data and performing tasks that can be automated such as regression, freeing staff to perform value-add tasks.
2. Access to a transparent model that gives estimates of the current and forecasted surface network states, and is not dependent on a single operator.
3. A better understanding of uncertainty around sensor data, the inferred state of the system and outcomes of potential actions.

2) Development of the Field on Geothermal Fields

This work will establish a precedent for the applications of optimisation and uncertainty techniques in any system where there are flows in a network with mechanical components, thermodynamic reactions, time-series drift and the need for corrective maintenance over time.

References

[1] NZ Geothermal Association, "Geothermal Energy & Electricity Generation," NZ Geothermal Association, 27 April 2018. [Online]. Available: http://nzgeothermal.org.nz/elec_geo/. [Accessed 27 April 2018].

[2] P. Bixley, A. Clotworthy and W. Mannington, "Evolution of the Wairakei geothermal reservoir during 50 years of production," *Geothermics,* no. 38, pp. 145-154, 2009.

[3] A. L. Wymer and S. A. Nicholson, "Wairakei Powerstation Discharges," in *Water NZ Annual Conference*, 2014.

[4] Waikato Regional Council, "Consents granted for contact geothermal operations," New Zealand Government, 15 October 2004. [Online]. Available: https://www.waikatoregion.govt.nz/community/whats-happening/news/media-releases-archived/consents-granted-for-contact-geothermal-operations/. [Accessed 9 May 2018].

[5] S. J. Zarrouk and H. Moon, "Efficiency of geothermal power plants: A worldwide review," *Geothermics,* no. 51, pp. 142-153, 15 January 2014.

[6] Y. Huang and D. H. Freeston, "Non-Linear Modelling of a Geothermal Steam Pipe Network," in *New Zealand Geothermal Workshop*, Auckland, 1992.

[7] J. W. Barnes, J. J. Brennan and R. M. Knapp, "Scheduling a Backlog of Oilwell Workovers," *Journal of Petroleum Technology,* no. 29, pp. 1651-1653, 1977.

[8] K. Wigram, "Modelling the Surface Plant of a Geothermal Power Station," *IPENZ Transactions,* no. 42, pp. 12-17, 2015.

[9] Z. Poulakis, D. Valougeorgis and C. Papadimitriou, "Leakage detection in water pipe networks using a Bayesian probabilistic framework," *Probabilistic Engineering Mechanics,* no. 18, pp. 315-327, 2003.

[10] M. G. Goumas, V. A. Lygerou and L. E. Papayannakis, "Computational methods for planning and evaluating geothermal energy projects," *Energy Policy,* no. 27, pp. 147-154, 1999.

[11] M. A. Grant and P. F. Bixley, Geothermal Reservoir Engineering, 2nd Edition ed., Taupo: Academic Press, 2011, p. 163.

[12] A. Marsh, "Modelling a Geothermal Steam Field to Evaluate Well Capacities and Assist Operational Decisions," in *World Geothermal Congress*, Melbourne, 2015.

[13] K. Harwood, K. Koorey and C. Mann, "Te Mihi Steamfield Production Development Design," in *World Geothermal Congress*, Melbourne, 2015.

[14] S. Mitchell, M. O'Sullivan and I. Dunning, *PuLP: A Linear Programming Toolkit for Python,* Auckland: The University of Auckland, 2011.

[15] M. Plummer, "JAGS: A Program for Analysis of Bayesian Graphical Models Using Gibbs Sampling," in *Proceedings of the 3rd International Workshop on Distributed Statistical Computing*, Vienna, 2003.

[16] M. Plummer, A. Stukalov and M. Denwood, "rjags: Bayesian Graphical Models using MCMC," CRAN, 19 February 2016. [Online]. Available: https://cran.r-project.org/web/packages/rjags/index.html. [Accessed 10 May 2018].

[17] C. Fonnesbeck, A. Patil, D. Huard and J. Salvatier, "PyMC Documentation," GitHub, 15 September 2015. [Online]. Available: https://pymc-devs.github.io/pymc/README.html#history. [Accessed 10 May 2018].