Investigating the Mercury Lamp Yellow Spectral Line using Computational Fourier Interferometry

Blackett Laboratory, London, SW22AZ, UK
Logan C. Filipovich, Imperial College London, Department of Physics

***Abstract*—A Michelson Interferometer is used to produce interferograms of incoming light sources by changing the optical path length of one of the beams by moving a mirror. The mirror stage was moved at an optimal speed as to minimise the noise and increase resolution I investigated the mercury lamp as my light source and performed cosine Fourier transforms on the interferograms of filtered green and yellow light to produce a wavelength spectrum of each spectral line. To accurately measure the wavelength of the yellow spectral line, error from noise and detector sensitivity were removed by a filter function and the green line was used to correct the phase error in the data. The measured wavelength of the yellow spectral line was 579.1±0.1 compared to the true value of 579.067 nm corresponding to a 0.006% percentage difference. The yellow peak was also found to be a doublet peak with a secondary wavelength of 570.0±0.1 nm. The great accuracy is owing to the power of Fourier computation.**

I. Introduction

THE Interferometer is an extremely powerful tool in physics to measure distances on a nanoscopic scale with great precision. Interferometers produce interferograms – graphs of the interference pattern. One of the more common types of interferometers used in physics is the Michelson interferometer that changes the optical path length by moving a mirror on a stage. This interferogram can be used as a form of spectroscopy utilising Fourier analysis of the interferogram that returns the spectral distribution of the light source. In my investigation, I examined the interferogram of green and yellow light produced from a mercury lamp and used the green spectral line to correct for the true position of the yellow line to accurately measure the wavelength of the yellow line.

II. Theoretical Background

Interference roots from the principle of superposition. Mathematically, interference is simply the sum of the amplitudes of the interfering waves. Using this fact and the idea that intensity is the square of the amplitude it can be derived that the interference of monochromatic light can be expressed as [1]:

$$I(x) = A_0 \cos^2(\frac{2\pi x}{\lambda}) \quad (1)$$

However, in cases where there is two-beam interference of non-coherent light, that is of different wavelengths then the result is a product of sinusoidal functions with different frequency oscillations. For the interference of N electromagnetic waves with wavelengths λ_n, the interference pattern can be modelled as:

$$I_N(x) = \prod^{N} A_n \cos(\frac{2\pi x}{\lambda_n}) \quad (2)$$

In interferometry it is common to investigate interference patterns caused by two frequencies (or wavelengths). The result of this is a function that has a higher frequency sinusoid inside a lower frequency envelope. This is shown in Fig. 1.

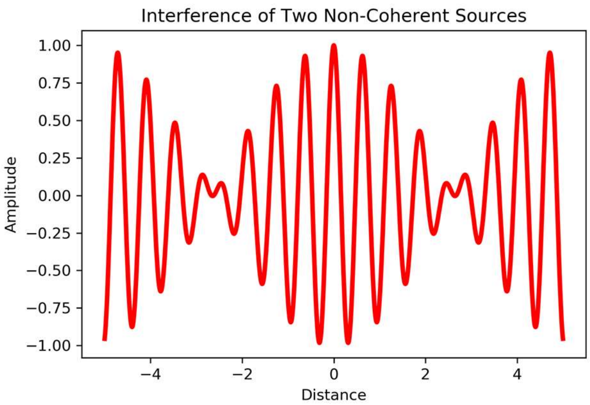

Fig. 1. The interference pattern produced with two light beams of different wavelengths producing a high-frequency sinusoid inside a lower frequency envelope.

These interference patterns can consist of many wavelengths of light and are called interferograms. We use Fourier analysis to determine the frequency of the incoming light by performing a Fourier transform of the interferograms which also provides mathematical means of reducing noise and removing unwanted data. For ideal Fourier interferometry [2], the interferogram $G(\tau)$ and frequency spectrum $g(\nu)$ are related as shown below.

$$g(\nu) = 2\int_0^{\infty} G(\tau)\cos(2\pi\nu\tau)d\tau \quad (3)$$

We sample at a finite rate $\delta\tau$ for digital computation hence the integral over the entire interferogram is replaced by a series.

$$g(\nu) = G(0) + 2\sum_{n=1}^{\infty} G(n\delta\tau)\cos(2\pi\nu\tau)d\tau \quad (4)$$

Finally, the use of interferometry is more interested in the wavelengths of the light that produces the interferogram to show spectral lines. This is done by finding the reciprocal of the frequency spectrum.

$$f(\lambda) = g(\nu)^{-1} \quad (5)$$

When processing the interferogram, it is common to remove noise or move the frequency spectrum through a convolution with a filter function that is some function of the frequency that we will call $\Phi(\lambda)$. This convolution takes the form:

$$f'(\lambda) = [f * \Phi](\lambda) \tag{6}$$

$$f'(\lambda) = \int_{-\infty}^{\infty} f(z)\Phi(\lambda - z)dz \tag{7}$$

Before, doing any Fourier interferometry experimentally, it is possible to (computationally) model the interference pattern produced by light of a specific wavelength (for this example we will use green 546 nm light) and hence determine the spectral line using the cosine transform method shown above. This can be seen in Fig. 2. By fitting a gaussian curve to the spectral line, the gaussian mean determines the wavelength of that spectral line.

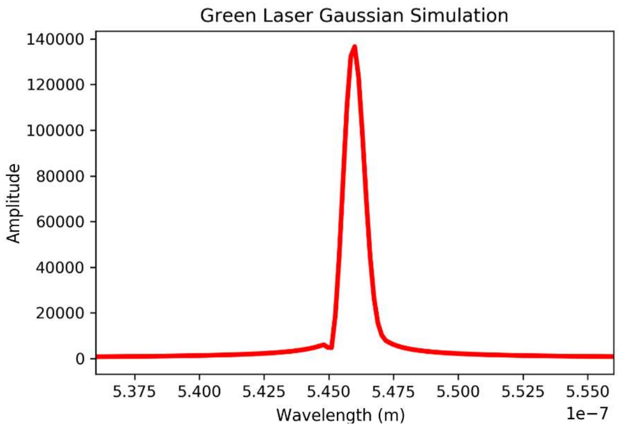

Fig. 2. A simulation of the spectral line of green light produced by performing a cosine Fourier transform of the green laser's interferogram.

III. Methodology

In my investigation of the Mercury (Hg) Lamp, I was given access to a Michelson interferometer [3]. The Michelson Interferometer worked by transmitting a beam from the Hg lamp source onto a beam splitter. The beamsplitter splits the beam towards two mirrors M1 and M2. The two beams are reflected by the mirrors respectively back to the beam splitter. This formed a single superposed beam with interference due to the difference in OPL in light going to M1 and M2. In my investigation of the mercury spectrum, I was only interested in the green and yellow wavelength peak hence the light was passed through a green filter and yellow filter separately, blocking any wavelength outside the green and yellow wavelength range respectively. The filtered light was then absorbed by a photodetector which measures the intensity of the light and changes in the intensity. The interferometer works by fixing one of the mirrors M1 and having a movable mirror M2 on a stage that can change the OPL of that beam of light.

The photodetector was simply a photodiode with an amplifier attached with a variable gain. This allowed me to vary the rate that the photodetector could sample and was set to the standard 50 Hz. At larger sampling rates, the photodetector proved to produce more noise. Sampling rates lower than 50 Hz took long periods to sample large amounts of data which

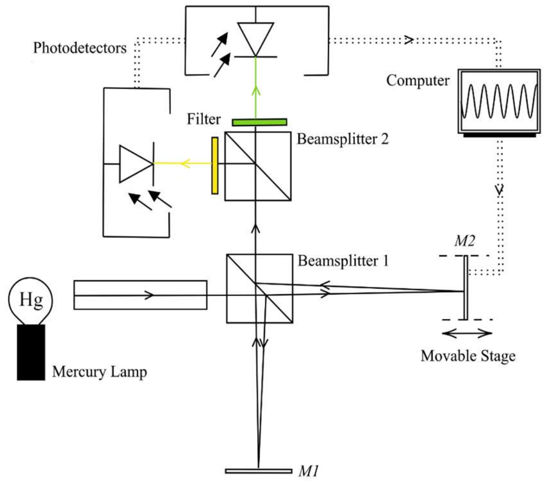

Fig. 3. The schematic of the experimental setup of the Michelson interferometer in my investigation.

is required to produce high-resolution spectral lines. Additionally, the interferometer was used remotely meaning it was controlled isolated from the apparatus. This was achieved by connecting to the laboratory computer via *NoMachine* (a programme that allows the remote control from a personal computer). As shown in Fig. 3, the computer could move the stage and the photodetectors would output the intensity of light. Hence, for each positional data x, that is the change in the OPL of one of the beams, there would be two output signals, one for the green mercury line and one for the yellow. These would need to be analysed separately.

To produce data from the interferometer that shows a high enough resolution of the interferogram, several parameters need to be set when moving the stage. Firstly, the stage needs to be set to a starting point – this is usually done by simply moving the stage very fast to our desired initial point. The second is the number of data points that we want to take. Evidently, the highest resolution interferogram is one with a large number of data points; typically, a single run of 100,000 data points is large enough to produce clear continuous spectral lines. Note that the reason this value is not higher is that at a 50 Hz sampling rate, the data can take 33 minutes to collect which is why anything larger than 100,000 would be a less efficient means of taking data.

Finally, the speed of the stage had to be set. This speed is set in millimetres per second and must be slow enough to record an interferogram by recording a minimum of 2 data points per coherence length. In other words, the finite interval of sampling $\delta\tau$ (as shown in (4)) must be less than the reciprocal of two times the spectral width $\Delta\nu$. However, in general, the slower does not equal better, as slower stage movements make the photodetectors more sensitive to noise. To optimise the speed of the stage, an interferogram of the white LED about a null point was taken. This was done using a similar setup shown in Fig. 3 but instead of a second beamsplitter, the light passes through a white filter. By plotting the spectral lines of the white LED, Gaussian curves could be fitted to the distribution. The errors in the fitted Gaussian curves could then be plotted against the stage speed to find the optimal stage speed for the coherence length of 10 μm and be adjusted based on the approximate coherence length of the spectral line under investigation. This was done for the white LED on a logarithmic scale 10^{-n} for n=2, 3, …,6. The data of error in μ and σ for the Gaussian distribution can be seen in Fig. 4.

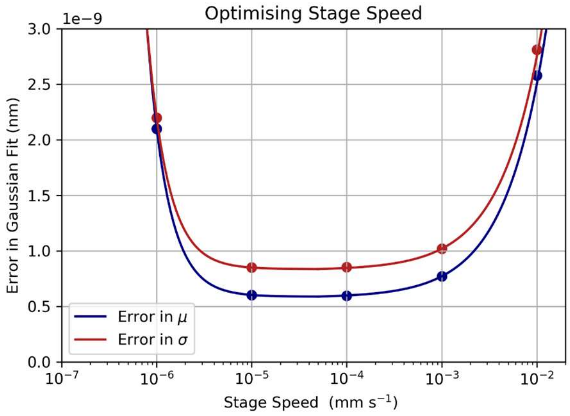

Fig. 4. A plot showing how the error in the Gaussian fit of a white LED spectral line varies with different magnitudes of stage speed.

By plotting various functions to the data, the stage speed decreases the error in the Gaussian fit linearly. However, the noise increases the fit reciprocally as shown above. This fit has proven to be accurate to ~0.2%. It was deduced, by finding the minimum of the fitted function, that the optimal stage speed was $(8.2 \pm 0.2) \times 10^{-5}$ mm s^{-1}. It was assumed that the optimal time interval $\delta\tau$ is inversely proportional to the coherence length, which is logical as a larger coherence length produces are more spread interferogram and hence a smaller $\delta\tau$. This allows us to deduce that for a typical spectral line of coherence length of 5 μm [4] that the optimal stage speed is approximately 1.6×10^{-5} mm s^{-1}. This was the speed used when producing the interferogram of the Hg lamp.

IV. Analysis and Results

Two interferograms are produced from the interferometer (Fig. 7a and 7b). The green line interferogram produces single frequency interference and the yellow line a two frequency 'beating' pattern. This shows that the yellow line is a doublet in the wavelength spectrum. However, the data contains many different sources of error. Firstly, noise. Noise in the data can be high frequency caused by very small fluctuations in the air or in the optical instruments through as well as higher frequency due to other light contamination. Secondly, the sensitivity of the detector i.e. the idea that photodetectors absorb different wavelengths more than others. Although they absorb in the visible range, a typical silicon photodiode can be up to 30% more sensitive to wavelengths at 600 nm than at 500 nm. Finally, systematic errors in the stage could mean that the position of the stage is not completely accurate and contains a phase error. For my interferograms of the mercury spectrum, these errors were taken into consideration by the means of a convolution of the spectral data with a filter function shown in (8).

$$\Phi(\lambda) = \mathrm{rect}(\phi(\lambda))e^{-i\psi(\lambda)}d(\lambda) \tag{8}$$

Each term in the filter function minimised the error in the data. Firstly, the rectangular function $\mathrm{rect}(\phi(\lambda))$ removed very high and low-frequency noise by removing all frequencies outside of the range $\Delta\lambda$ – for the analysis of the green line, a wider range of 500-600 nm is used prior to the phase error, however, this was narrowed to 550-590 nm for the yellow doublet as we expected a value of 578 nm once the phase error is removed. Subsequently, the term $d(\lambda)$ accounts for the sensitivity of the photodetector. Fig. (5) shows the relative spectral intensity of a silicon photodetector for different wavelengths. A convolution with this function adjusts the amplitude of the spectral line according to detector sensitivity.

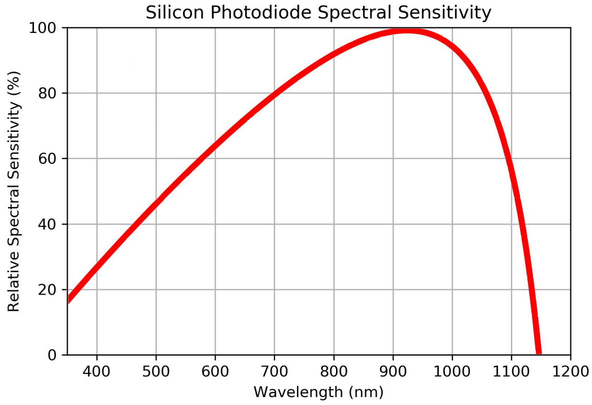

Fig. 5. A plot showing how the photodiode sensitivity varies with wavelength $d(\lambda)$ [5]

To fix the phase error, the data must undergo a common operation in Fourier interferometry known as phase correction. Phase correction is done by finding the spectral line position of a peak of known wavelength in the data and the data's value was compared to the known value. For my investigation of the yellow line, the green spectral line known to be 546 nm was used. The computationally convenient method that was performed on the wavelength spectrum was a convolution with the function $e^{i\psi'(\lambda)}$ that is seen in (8). The green peak, after noise reduction, was fitted by a Gaussian and the measured wavelength was 566.3 ± 0.1 nm compared to the actual value of 546.07 ± 0.01 nm [4]. Hence the phase error in the data is 20.2 ± 0.1 nm.

The filtered and unfiltered wavelength spectrum is shown in Fig. 7c and 7d. Here it is clear that the yellow spectral line is, in fact, a doublet. A function of two Gaussian curves was fitted to the yellow peak and determined the wavelengths of the two peaks to be 579.1 ± 0.1 nm for the higher peak and 577.0 ± 0.1 nm resulting in a peak difference $\Delta\lambda$ of 2.104 ± 0.003 nm (in the range of the true value of 2.106 nm [4]). This is a very good result compared to the true value of 579.067 nm - a percentage difference of 0.006%. The small error of 0.002% in the wavelength that propagates is likely due to very small phase errors that the filter could not correct.

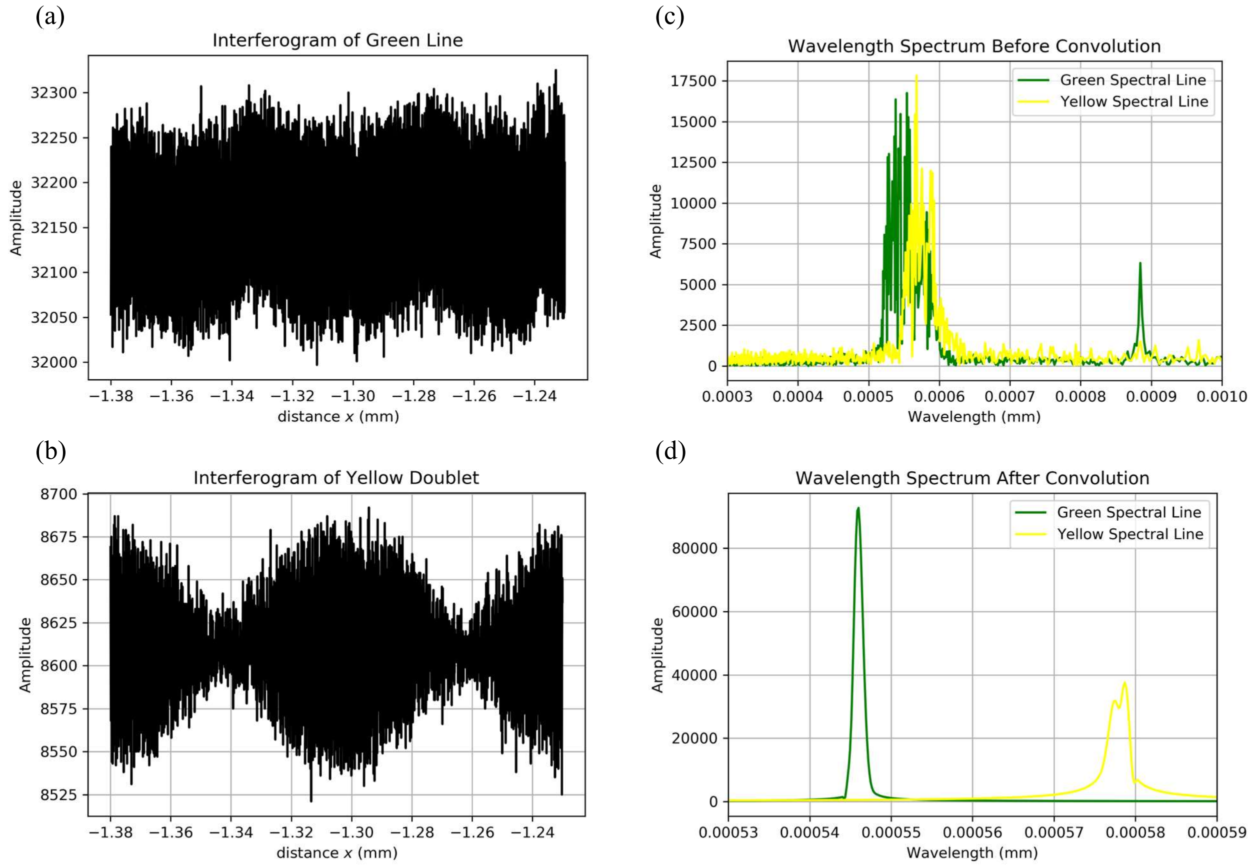

Fig 7a. and 7b show the interferogram produced from light from the green and yellow filter respectively and how the beating pattern of the yellow light indicates a doublet peak. Fig 7c and 7d show the wavelength spectrums before and after undergoing convolution with the filter function, respectively.

V. Conclusion

The method of using computational Fourier methods to determine the wavelength of the yellow doublet has proven extremely accurate with a very small error up of to 5σ. Not only does it show the power of Fourier analysis, how the convolution of filter functions can account for a variety of errors, but also shows the incredible power of interferometry. Over 100 years after its conception, when it was used as a foundation for special relativity, the Michelson interferometer still has large applications in measuring the world at nanoscales. However, it is more recently its combination with computational analysis, that the interferometer has found its true potential.

VI. References

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