Homework 1

Logan Hall

I. Problem 1

This section performs and discusses the simulation of the motion of three connected spheres falling inside viscous fluid.

A. Problem 1.1

Prompt: Show the shape of the structure at t = 0, 0.01, 0.05, 0.10, 1.0, 10.0

All of the following graphs in this section (Problem 1.1) are shown with the same x and y scale as one another for ease of comparison. The following graphs are from the implicit method. As seen in the graphs, the middle node starts moving downwards faster than the side nodes at first until the turning angle has significantly increased. Then during the simulation, the terminal velocity for all three nodes has been reached and they no longer move relative to one another, but appear to be moving downwards through the medium

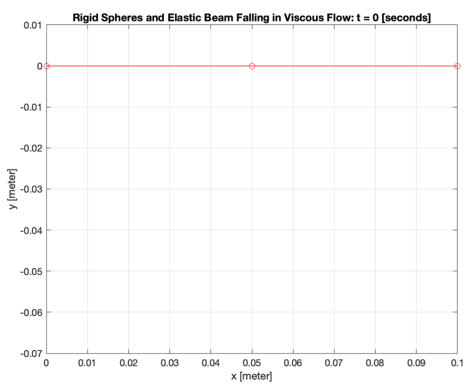

Fig. 1. Shape of the Structure at t = 0 seconds

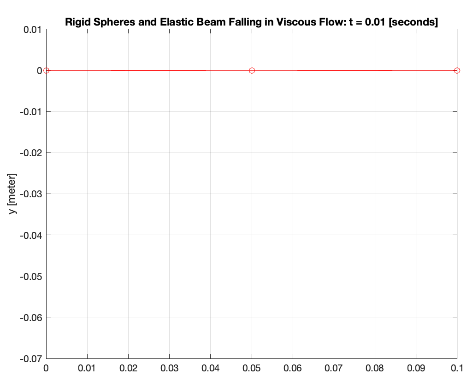

Fig. 2. Shape of the Structure at t = 0.01 seconds

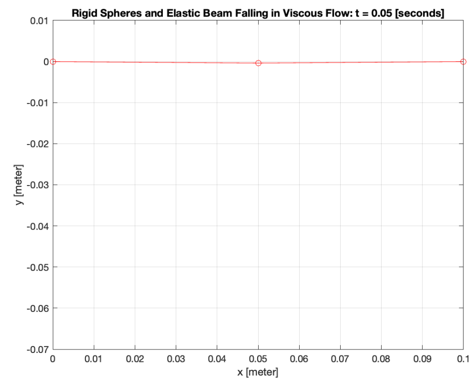

Fig. 3. Shape of the Structure at t = 0.05 seconds

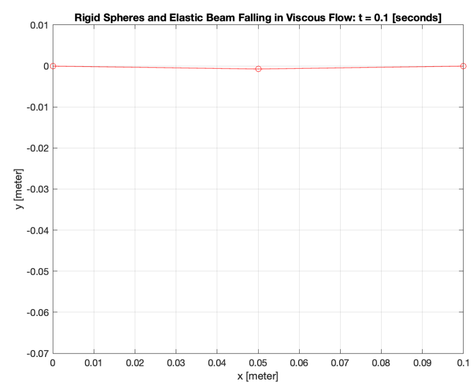

Fig. 4. Shape of the Structure at t = 0.1 seconds

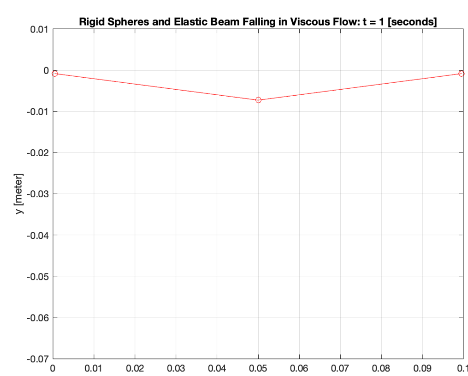

Fig. 5. Shape of the Structure at t = 1 second

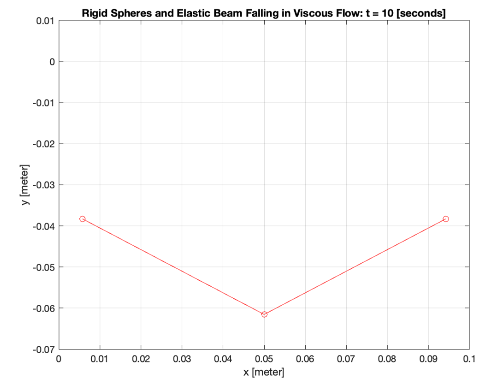

Fig. 6. Shape of the Structure at t = 10 seconds

Prompt: Plot the position and velocity (along y-axis) of R_2 as a function of time. As seen here in figure 7, the position travels negative, at a nonlinear rate, until the terminal velocity is approached and it becomes linear. This is supported by figure 8, where it shows the velocity becoming constant, at the terminal velocity.

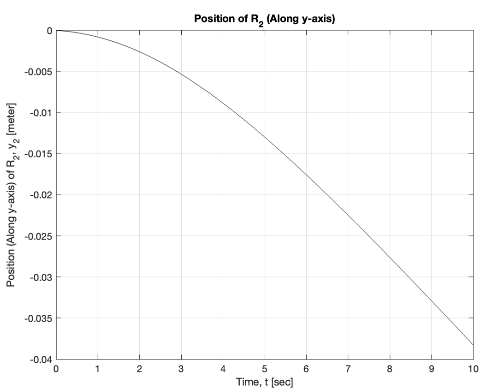

Fig. 7. Position (Along y-axis) of R_2 as a Function of Time

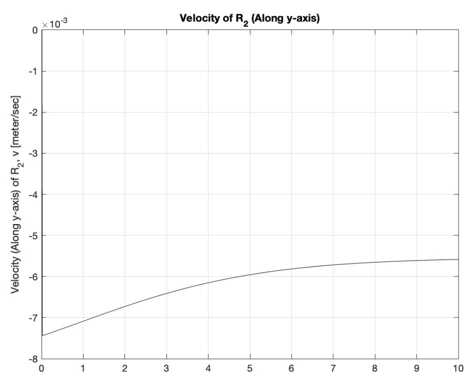

Fig. 8. Velocity (Along y-axis) of R_2 as a Function of Time

B. Problem 1.2

The terminal velocity along the y-axis is visualized in figure 8, as the velocity begins to approach a constant value, which is the terminal velocity of R_2. The terminal velocity of R_2 is equal to the terminal velocity of the body. The terminal velocity as calculated from the implicit method is $v_{terminal} = -.00558$ meters/second. The terminal velocity as calculated from the explicit method is $v_{terminal} = -.00597$ meters/second.

C. Problem 1.3

If all the radii (R_1, R_2, R_3) are the same, the turning angle remains constant at $\theta = 0$. This does agree with my intuition, as on a surface level view, the spheres are identical and thus the forces acting on each sphere are identical. Each sphere is now governed by the same equations, as they all start at y=0, and have the same mass, viscous damping and weight. This is shown below in figure 9.

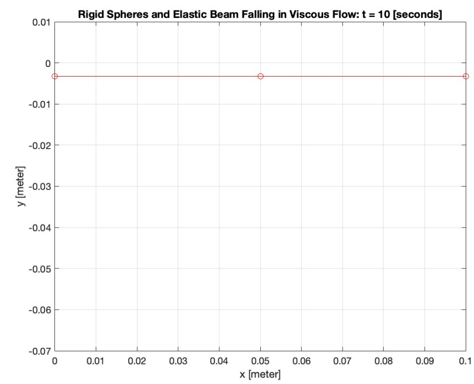

Fig. 9. Locations of Spheres with Constant Radii

D. Problem 1.4

Although explicit methods are usually easier and simpler, they require a smaller time step size than implicit methods. When taking the explicit method approach used in this problem and lowering the time step from 10^{-5} to 10^{-4}, the simulation fails. It begins to give results that deviate too far, and eventually results in equations that can not be solved. When reducing the time step for the implicit method from 10^{-2} to 10^{-1}, it still arrives at a representative result. This is further evidence that the implicit method allows for a larger time step size and results in a shorter computation time.

II. Problem 2

The section performs and discusses the simulation of the motion of N-connected spheres falling inside viscous fluid

A. Problem 2.1

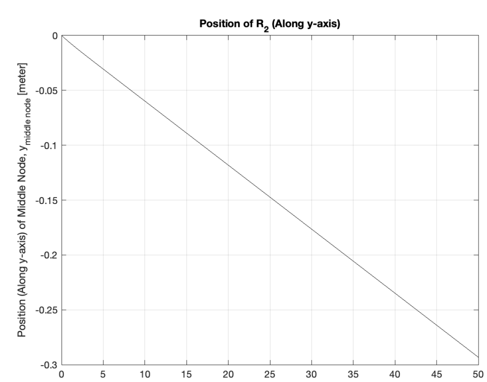

Fig. 10. Position (Along y-axis) of Middle Node as a Function of Time

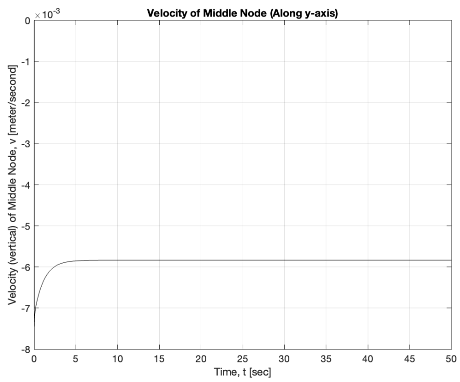

Fig. 11. Velocity (Along y-axis) of Middle Node as a Function of Time

Both figure 10 and figure 11 show that for the majority of this simulation, the middle node is traveling at a constant velocity, and thus it reached its terminal velocity very quickly. The terminal velocity in this simulation is

$$v_{terminal} = -0.00583 \text{ meters/sec}$$

B. Problem 2.2

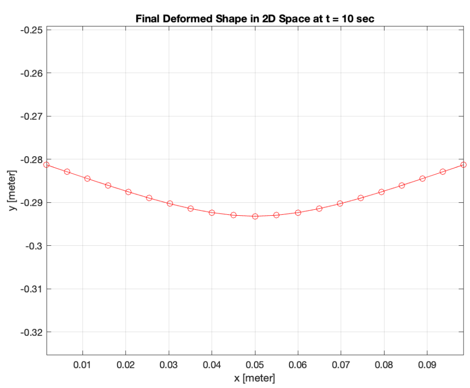

Fig. 12. Final Deformed Shape of the Beam at t = 50 Seconds

Figure 12 above shows the final deformed shape of the beam, showing that it has remained symmetric about x = 0.05 m and the end conditions have both decreased and moved away from 0.

C. Problem 2.3

Per the problem statement for this assignment, any simulation should be discretized such that the quantifiable metrics (such as terminal velocity) do not vary by much if N is increased and Δ t is decreased.

From figure 13, the terminal velocity does appear to be asymptotic, and looking at our prompted value for this simulation of N = 21, we are very close to that asymptote and the difference between N = 21 and increasing the value of N appears to be negligable. However, once we start to decrease the value of N from N=21, the terminal velocity decreases at a faster rate. Thus in this problem, we are sufficiently discretized at N=21 nodes

From figure 14, we can see that there is no appreciable difference when the time step is changed. This is likely due to the efficiency of the implicit method. When Δ t is decreased, there is no significant difference in the calculated terminal velocity. Therefore, the simulation performed with the given parameters, we are sufficiently temporally discretized.

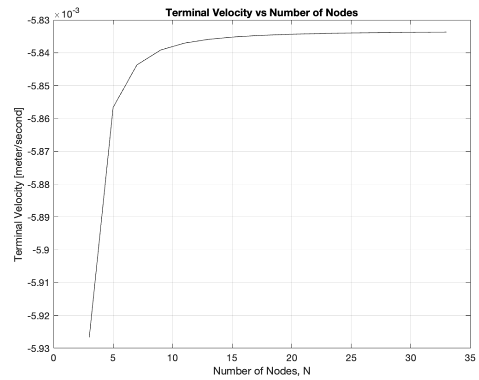

Fig. 13. Spacial Discretization: Terminal Velocity vs Number of Nodes

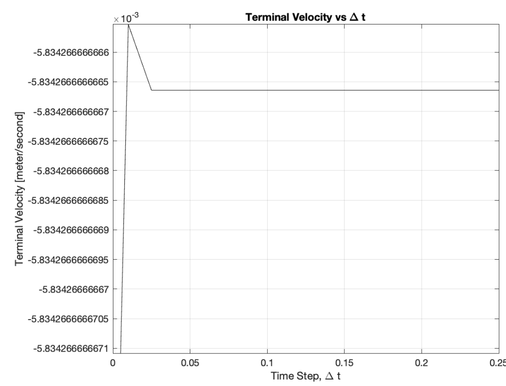

Fig. 14. Temporal Discretization: Terminal Velocity vs Time Step Size

III. Problem 3

This section performs and discusses the simulation of the deformation of elastic beams and compares it with Euler-Bernoulli beam theory.

A. Problem 3.1

In this case, the maximum vertical displacement of the simulated beam does reach a steady state value. For the simulation, the steady state value is $y_{max} = -0.0371m$ For the theoretical prediction from Euler beam theory, $y_{max} = -0.0380m$. The percent error between these two values is .24%., Thus showing the accuracy between both the theoretical and simulated methods. Figure 15 shows how quickly the simulation converged to the maximum beam deflection

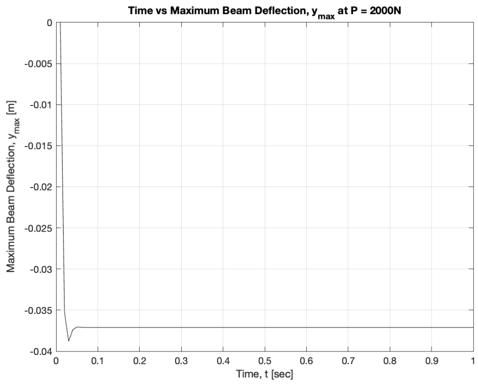

Fig. 15. Maximum Vertical Displacement vs Time, P = 2000N

B. Problem 3.2

In this case, the maximum vertical displacement of the simulated beam still does reach a steady state value. For the simulation, the steady state value is $y_{max} = -0.2353m$ For the theoretical prediction from Euler beam theory, $y_{max} = -0.3804m$. The percent error between these two values is 61%., This error is extremely large and if the theoretical approach is used in this case, it will clearly not yield correct results. Euler beam theory is only valid for small deformations, however, this simulation is able to handle much larger deformations. Figures 17 and 18 show a plot comparing the theoretical and simulated vertical displacements. It is seen that at an approximate load of P = 3600 n, the two methods are equal. For all values below approximately P = 5200 N, the percent error is less than 3 %. This is shown in figure 19. As the value of P continues to increase from this approximately P = 5200 N, the two methods continue to diverge, showing how the theoretical method cannot handle larger deformations.

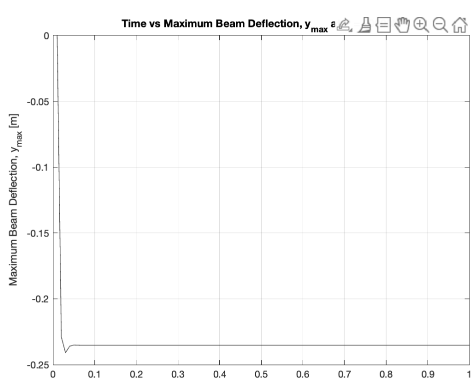

Fig. 16. Maximum Vertical Displacement vs Time, P = 20000N

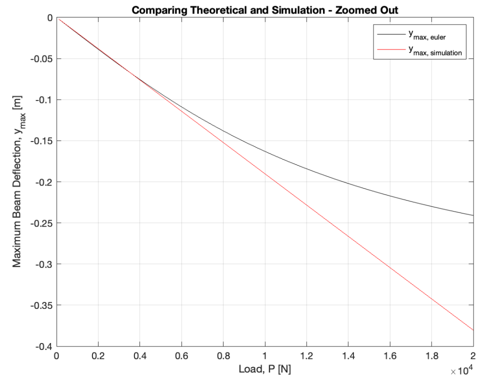

Fig. 17. Comparing Theoretical vs Simulated Maximum Vertical Displacements - Zoomed Out

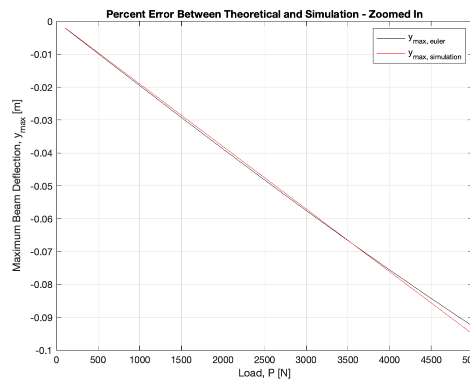

Fig. 18. Comparing Theoretical vs Simulated Maximum Vertical Displacements - Zoomed In

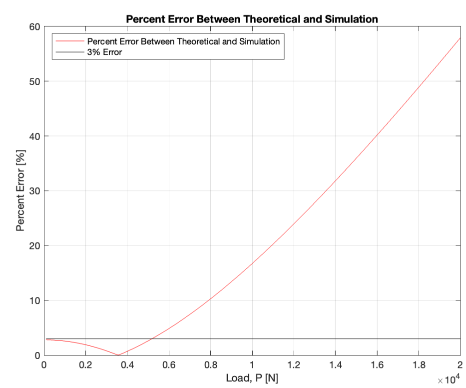

Fig. 19. Percent Error Between Theoretical and Simulation

REFERENCES

[1] Jawed, Khalid. "Lecture 3", 9 Oct 2023,
[2] Jawed, Khalid. "Lecture 4", 16 Oct 2023,
[3] Jawed, K., Lim, S. "Discrete Simulation of Slender Structures