MATH 676 Project – Bilinear Form

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Begin with the spatial domain $\mathcal{D} \in \mathbb{R}^2$ in which $\delta\mathcal{D}$ is on the boundary of $\mathcal{D}$. The set of propagation directions $\mathcal{S}$ is the unit disk.

The linear Boltzmann equation for one-group transport is

$$\mathbf{\Omega} \cdot \nabla\Psi(\mathbf{\Omega}, \mathbf{x}) + \sigma_t(\mathbf{x})\Psi(\mathbf{\Omega}, \mathbf{x}) - \sigma_s(\mathbf{x})\Phi(\mathbf{x}) = q(\mathbf{x})\,, \qquad \forall(\mathbf{\Omega}, \mathbf{x}) \in \mathcal{S} \times \mathcal{D}\,, \tag{1a}$$

$$\Phi(\mathbf{\Omega}, \mathbf{x}) = \Phi^{\text{inc}}(\mathbf{\Omega}, \mathbf{x})\,, \qquad \forall(\mathbf{\Omega}, \mathbf{x}) \in \mathcal{S} \times \delta\mathcal{D}\,,\ \mathbf{\Omega} \cdot \mathbf{n}(x) < 0\,, \tag{1b}$$

where Φ is the scalar flux, defined by

$$\Phi = \frac{1}{2\pi}\int_{\mathcal{S}} \Phi(\mathbf{\Omega}, \mathbf{x})\; d\Omega\,.$$

Introduce the S_N discretization by