2D S_N with Diffusion Acceleration

MATH 676 – Milestone 1 Presentation

Logan H. Harbour

Department of Nuclear Engineering
Texas A&M University

One-group Linear Boltzmann Equation

Begin with the one-group S_N transport equation for a single direction d (neglecting boundary conditions for simplicity), as

$$\boldsymbol{\Omega}_d \cdot \nabla \psi_d(\mathbf{x}) + (\sigma_a(\mathbf{x}) + \sigma_s(\mathbf{x}))\, \psi_d(\mathbf{x}) - \frac{\sigma_s(\mathbf{x})}{2\pi} \sum_{d=1}^{N_\Omega} \omega_d \psi_d(\mathbf{x}) = q(\mathbf{x})\,, \quad (1)$$

where σ_a represents a probability of particle absorption and σ_s represents a probability of radiation scattering. Let $\mathbb{T}_h$ be the set of all cells of the triangulation in a discontinuous approximation space. The DG weak form with test function v_d is

$$\sum_{K \in \mathbb{T}_h} \Big[(-\boldsymbol{\Omega}_d \cdot \nabla v_d, \psi_d)_K + \left(\psi_d^+ \boldsymbol{\Omega}_d \cdot \mathbf{n}, v_d \right)_{\delta K} + (\sigma_t \psi_d, v_d)_K - (\sigma_s \phi, v_d)_K = (q, v_d)_K \Big]\,, \quad (2)$$

where ϕ is the *scalar flux*, $\phi = \frac{1}{2\pi} \sum_d^{N_\Omega} \omega_d \psi_d$, and ψ_d^+ is the upwind value of ψ_d (the value from the side of the face in which $\boldsymbol{\Omega} \cdot \mathbf{n} \geq 0$).

Issue: Source Iteration

We commonly solve the transport equation by *source iteration*, a form of Richardson iteration. Cast Eq. (1) with iterative index ℓ as

$$\mathbf{\Omega}_d \cdot \nabla \psi_d^{(\ell+1)} + \sigma_t \psi_d^{(\ell+1)} = \sigma_s \phi^{(\ell)} + q\,, \tag{3}$$

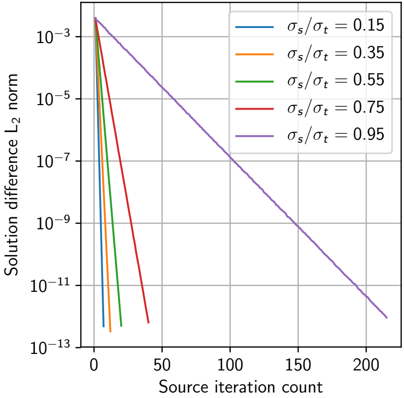
where ℓ is the iterative index, $\psi_d^{(0)} = \phi^{(0)} = \vec{0}$. After solving each direction, d, for an iteration ℓ in Eq. (3), update the scalar flux with

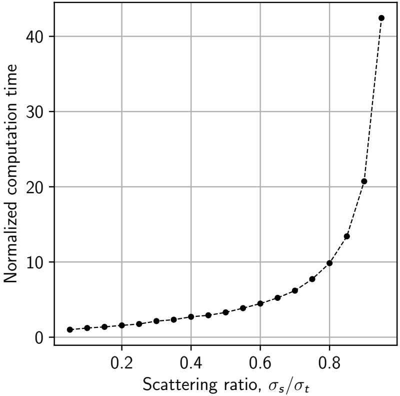
$$\phi^{(\ell+1)} = \frac{1}{2\pi} \sum_{d=1}^{N_\Omega} w_d \psi_d^{(\ell+1)}\,.$$

$\psi^{(\ell+1)}$ is the particles that have scattered at most ℓ times. As $\sigma_s/\sigma_t \to 1$, particles scatter more before they are absorbed $\to$ **the number of source iterations becomes significant!** This problem becomes the goal of this work: introduce a diffusion problem as a preconditioner for Eq. (3).

Example: Lots of Scattering

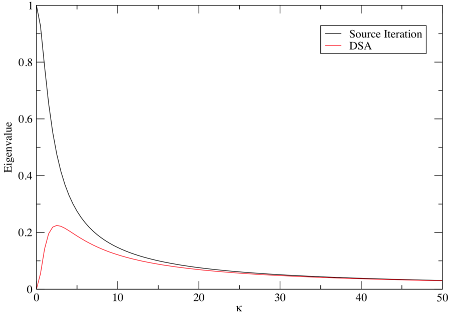
Introduce $\mathcal{D} = [0, 10]^2$, $N_\Omega = 20$, $q = 1$, $\sigma_a + \sigma_s = \sigma_t = 100$, and 64^2 elements. Increase the scattering ratio, σ_s/σ_t and observe results.





Diffusion Acceleration

- The source iteration process will converge quickly whenever particles scatter just a few times on average before being absorbed or escaping.
- It can converge very slowly in *diffusive* problem, as particles scatter an arbitrary number of times on average before being absorbed or escaping.



Project Progress and Future Goals

Completed works

- A one-group, 2D neutron transport code using the S_N approximation has been developed using linear discontinuous finite elements in Deal.ii.
- Verified using known constant source solutions and MMS.
- Primarily uses the `MeshWorker` interface as discussed in step-12.
- Utilizes `downstream_renumbering` to precondition the within-direction solve.