2D S_N Radiation Transport with Diffusion Acceleration

MATH 676 – Final Presentation

Logan H. Harbour

Department of Nuclear Engineering
Texas A&M University

NUCLEAR ENGINEERING
TEXAS A&M UNIVERSITY

Introduction

- My Ph.D. work involves various acceleration techniques for method of characteristics (MOC) radiation transport
- Many acceleration methods for discrete ordinates (S_N) transport are similar to those in MOC transport
- Therefore, the overarching goal for this course is to investigate and implement a common acceleration method for S_N transport utilizing Deal.ii
- In specific, the focus is on diffusion synthetic acceleration (DSA) of source iteration
- Additional goals were added as I was able to complete the primary goal early

One-group Linear Boltzmann Equation

Start with the one-group S_N transport equation for a single direction d (neglecting boundary conditions for simplicity), as

$$\boldsymbol{\Omega}_d \cdot \nabla \psi_d(\mathbf{x}) + \left(\sigma_a(\mathbf{x}) + \sigma_s(\mathbf{x})\right) \psi_d(\mathbf{x}) - \frac{\sigma_s(\mathbf{x})}{2\pi} \sum_{d=1}^{N_\Omega} \omega_d \psi_d(\mathbf{x}) = q(\mathbf{x})\,, \tag{1}$$

Let $\mathbb{T}_h$ be the set of all cells of the triangulation in a discontinuous approximation space. The DG weak form with test function v_d is

$$\sum_{K \in \mathbb{T}_h} \Big[\left(-\boldsymbol{\Omega}_d \cdot \nabla v_d, \psi_d\right)_K + \left(\psi_d^+ \boldsymbol{\Omega}_d \cdot \mathbf{n}, v_d\right)_{\delta K} + \left(\sigma_t \psi_d, v_d\right)_K - \left(\sigma_s \phi, v_d\right)_K = \left(q, v_d\right)_K \Big]\,, \tag{2}$$

where ϕ is the *scalar flux*, $\phi = \sum_d^{N_\Omega} \omega_d \psi_d$, and ψ_d^+ is the upwind value of ψ_d (the value from the side of the face in which $\boldsymbol{\Omega} \cdot \mathbf{n} \geq 0$).

Source Iteration

To solve, cast Eq. (1) with iterative index ℓ as

$$\mathbf{\Omega}_d \cdot \nabla \psi_d^{(\ell+1)} + \sigma_t \psi_d^{(\ell+1)} = \sigma_s \phi^{(\ell)} + q\,, \tag{3}$$

where ℓ is the iterative index, $\psi_d^{(0)} = \phi^{(0)} = \vec{0}$. After solving each direction, d, for an iteration ℓ in Eq. (3), update the scalar flux with

$$\phi^{(\ell+1)} = \frac{1}{2\pi} \sum_{d=1}^{N_\Omega} w_d \psi_d^{(\ell+1)}\,.$$

As $\sigma_s/\sigma_t \to 1$, particles scatter more before they are absorbed $\to$ **the number of source iterations becomes significant!**

Diffusion Acceleration

Simple algebraic manipulations can show that the error in $\psi^{\ell+1}$ satisfies the transport equation with a source equal to:

$$R^{\ell+1} = \frac{\sigma_s}{2\pi}(\phi^{\ell+1} - \phi^{\ell}) \,.$$

Fourier analysis shows that the angular flux error has a linearly anisotropic angular dependence. The diffusion approximation is exact for such a dependence, therefore we can cast the diffusion problem with the source above to form an error equation with the diffusion approximation that will attenuate the errors most poorly attenuated by the transport solve. The approximation is cast as:

$$-\nabla \cdot D\nabla\delta e^{\ell+1} + \sigma_a \delta e^{\ell+1} = \sigma_s \left(\phi^{\ell+1} - \phi^{\ell}\right) \,, \tag{4}$$

where $e^{\ell+1}$ is the approximated error in $\phi^{\ell+1}$ and $D = 1/3\sigma_t$.

Diffusion Acceleration (cont.)

Casting Equation (4) in the same DG space with interior edges $\mathcal{E}_h^i$ and boundary edges $\mathcal{E}_h^b$ using a modified interior penalty method for the face terms we obtain

$$\int_{\mathbb{T}_h} (D\nabla e \cdot \nabla v + \sigma_a e v) + \int_{\mathcal{E}_h^i} (\{\!\{D\delta_n e\}\!\}[\![v]\!] + \{\!\{D\delta_n v\}\!\}[\![e]\!] + \kappa[\![e]\!][\![v]\!]) \\ + \int_{\mathcal{E}_h^b} (\kappa e v - D v \delta_n e - D e \delta_n v) = \int_{\mathbb{T}_h} (\phi^{\ell+1} - \phi^{\ell}) v \,, \quad (5)$$

where

$$\{\!\{u\}\!\} \equiv \frac{u^+ + u^-}{2} \quad \text{and} \quad [\![u]\!] \equiv u^+ - u^- \,,$$

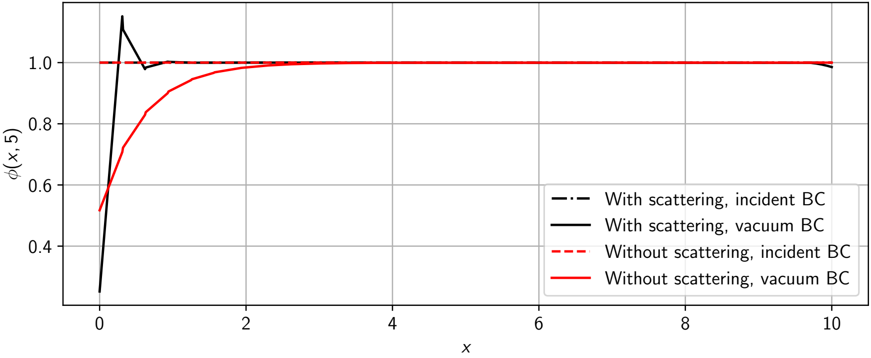
in which the penalty coefficient is

$$\kappa = \begin{cases} 2\left(\frac{D^+}{h_\perp^+} + \frac{D^-}{h_\perp^-}\right) & \text{for interior edges}\,, \\ 8\frac{D^-}{h_\perp^-} & \text{for boundary edges}\,, \end{cases}$$

and $h_\perp^\pm$ is a characteristic length of the cell in the direction orthogonal to the edge.

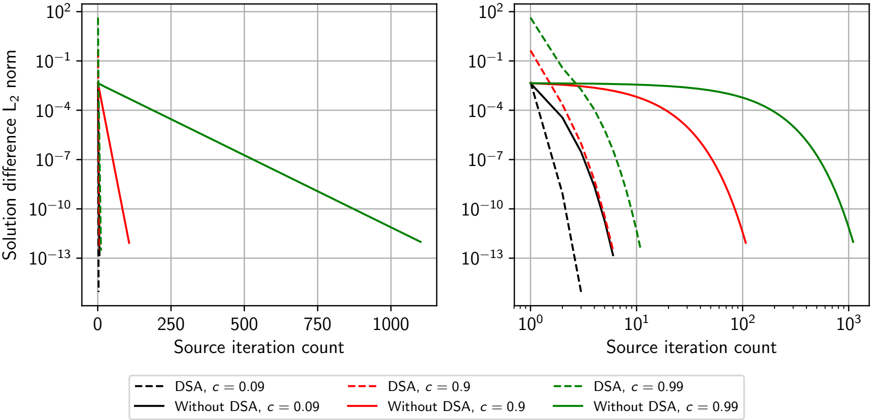
Constant Solution Verification

Consider $\mathcal{D} = [0, 10]^2$, $N_\Omega = 4$, $q = 1$, $\sigma_t = 100$, and 36^2 elements. Top and right boundary conditions are reflective. Bottom and left boundary conditions are either vacuum or incident isotropic flux of q/σ_a.



Diffusion Acceleration Results

Consider $\mathcal{D} = [0, 10]^2$, $N_\Omega = 20$, $q = 1$, $\sigma_a + \sigma_s = \sigma_t = 100$, and 16^2 elements. Increase the scattering ratio, $c = \sigma_s/\sigma_t$, with and without diffusion acceleration and observe the results:



Additional Goals Completed

The primarily implementation (transport with diffusion acceleration) was completed earlier than expected. Therefore, additional goals were added (and completed!) to round out the project:

Parallel support

- Supports parallel solves using MPI and Trillinos wrappers, completed primarily by following step-40
- Transport is solved with GMRES and the AMG preconditioner
- Diffusion is solved with CG and the AMG preconditioner

Reflecting boundary conditions

- Reflecting boundaries require storing the outgoing flux on the boundaries and then reflecting on the incoming boundaries (bit of a pain, but it works)
- Also supported in the diffusion acceleration scheme through adding an additional source term for boundary flux error

Whoop