MATH 676 PROJECT S$_N$ SUMMARY

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Problem definition

Begin with the spatial domain $\mathcal{D} \in \mathbb{R}^2$ in which $\delta\mathcal{D}$ is on the boundary of $\mathcal{D}$. The set of propagation directions $\mathcal{S}$ is the unit disk.

The linear Boltzmann equation for one-group transport is

$$\mathbf{\Omega} \cdot \nabla\Psi(\mathbf{\Omega}, \mathbf{x}) + \sigma_t(\mathbf{x})\Psi(\mathbf{\Omega}, \mathbf{x}) - \sigma_s(\mathbf{x})\Phi(\mathbf{x}) = q(\mathbf{x})\,, \qquad \forall(\mathbf{\Omega}, \mathbf{x}) \in \mathcal{S} \times \mathcal{D}\,, \tag{1a}$$

$$\Phi(\mathbf{\Omega}, \mathbf{x}) = \Phi^{\text{inc}}(\mathbf{\Omega}, \mathbf{x})\,, \qquad \forall(\mathbf{\Omega}, \mathbf{x}) \in \mathcal{S} \times \delta\mathcal{D}\,,\ \mathbf{\Omega} \cdot \mathbf{n}(\mathbf{x}) < 0\,, \tag{1b}$$

where Φ is the scalar flux, defined by

$$\Phi = \frac{1}{2\pi}\int_{\mathcal{S}} \Phi(\mathbf{\Omega}, \mathbf{x})d\Omega\,.$$

S$_N$ discretization

Introduce the S$_N$ discretization, which replaces the angular flux with a discrete angular flux, as

$$\psi(\mathbf{x}) = [\psi_1(\mathbf{x}), \psi_2(\mathbf{x}), \dots \psi_{N_\Omega}(\mathbf{x})]^T\,. \tag{2}$$

We then introduce the quadrature rule $\{(\mathbf{\Omega}_d, \omega_d), d = 1, \dots, N_\Omega\}$ where $\sum_d \omega_d = 2\pi$. With said quadrature rule, we have

$$\int_{\mathcal{S}} f(\mathbf{\Omega}, \mathbf{x})d\Omega \approx \sum_{d=1}^{N_\Omega} w_d f(\mathbf{\Omega}_d, \mathbf{x})\,.$$

This discretization allows us to write the system in Equation (1) as

$$\mathbf{\Omega}_d \cdot \nabla + \sigma_t(\mathbf{x})\phi_d(\mathbf{\Omega}, \mathbf{x}) - \sigma_s(\mathbf{x})\phi(\mathbf{x}) = q(\mathbf{x})\,, \qquad \forall\mathbf{x} \in \mathcal{D} \tag{3a}$$

$$\psi_d(\mathbf{x}) = \Psi_j^{\text{inc}}(\mathbf{x})\,, \qquad \forall\mathbf{x} \in \delta\mathcal{D}\,,\ \mathbf{\Omega}_d \cdot \mathbf{n}(\mathbf{x}) < 0\,, \tag{3b}$$

where the discrete scalar flux, ϕ, is

$$\phi(\mathbf{x}) = \frac{1}{2\pi}\sum_{d=1}^{N_\Omega} w_j\psi_j(\mathbf{x})\,.$$

Discontinuous Galerkin discretization