2D S_N with Diffusion Acceleration

MATH 676: 1st Milestone Presentation

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One-group Linear Boltzmann Equation

Define the spatial domain $\mathcal{D} \in \mathbb{R}^2$ in which $\delta\mathcal{D}$ is on the boundary of $\mathcal{D}$. The set of propagation directions $\mathcal{S}$ is the unit disk. Introduce a quadrature rule $\{(\boldsymbol{\Omega}_d, \omega_d), d = 1, \ldots, N_\Omega\}$ where $\sum_d \omega_d = 2\pi$. The one-group S_N transport equation is then

$$\boldsymbol{\Omega}_d \cdot \nabla \psi_d(\mathbf{x}) + (\sigma_a(\mathbf{x}) + \sigma_s(\mathbf{x}))\, \psi_d(\mathbf{x}) - \sigma_s(\mathbf{x})\phi(\mathbf{x}) = q(\mathbf{x})\,, \qquad \forall \mathbf{x} \in \mathcal{D} \quad \text{(1a)}$$

$$\psi_d(\mathbf{x}) = \psi_d^{\mathrm{inc}}(\mathbf{x})\,, \qquad \forall \mathbf{x} \in \delta\mathcal{D}\,, \quad \boldsymbol{\Omega}_d \cdot \mathbf{n}(\mathbf{x}) < 0\,, \quad \text{(1b)}$$

where ϕ is the discrete scalar flux, defined by

$$\phi(\mathbf{x}) = \frac{1}{2\pi} \sum_{d=1}^{N_\Omega} w_j \psi_j(\mathbf{x})\,.$$

σ_a represents a probability of particle absorption and σ_s represents a probability of radiation scattering. For convenience, define $\sigma_t = \sigma_a + \sigma_s$.

Problem: Lots of scattering

We commonly solve Eq. (1) by *source iteration*, a form of Richardson iteration. Cast Eq. (1) with iterative index ℓ as

$$\mathbf{\Omega}_d \cdot \nabla \psi_d^{(\ell+1)} + \sigma_t \psi_d^{(\ell+1)} = \sigma_s \phi^{(\ell)} + q\,, \tag{2}$$

where ℓ is the iterative index, $\psi_d^{(0)} = \phi^{(0)} = \vec{0}$. After solving each direction, d, for an iteration ℓ in Eq. (2), update the scalar flux with

$$\phi^{(\ell+1)} = \frac{1}{2\pi} \sum_{d=1}^{N_\Omega} w_d \psi_d^{(\ell+1)}\,.$$

$\psi^{(\ell+1)}$ is the particles that have scattered at most ℓ times. Recall that $\sigma_t = \sigma_s + \sigma_a$. As $\sigma_s/\sigma_t \to 1$, particles scatter more before they are absorbed $\to$ **the number of source iterations becomes significant!**

Example: Lots of scattering

Introduce $\mathcal{D} = [0, 10]^2$, $N_\Omega = 20$, $q = 1$, $\sigma_a + \sigma_s = \sigma_t = 100$, and 64^2 elements. Increase the scattering ratio, σ_s/σ_t and observe results.