2D S_N with Diffusion Acceleration

MATH 676: 1^{st} Milestone Presentation

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One-group Linear Boltzmann Equation

Define the spatial domain $\mathcal{D} \in \mathbb{R}^2$ in which $\delta\mathcal{D}$ is on the boundary of $\mathcal{D}$. The set of propagation directions $\mathcal{S}$ is the unit disk.

$$\boldsymbol{\Omega} \cdot \nabla\Psi(\boldsymbol{\Omega}, \mathbf{x}) + \sigma_t(\mathbf{x})\Psi(\boldsymbol{\Omega}, \mathbf{x}) - \sigma_s(\mathbf{x})\Phi(\mathbf{x}) = q(\mathbf{x})\,, \qquad \forall(\boldsymbol{\Omega}, \mathbf{x}) \in \mathcal{S} \times \mathcal{D}\,, \tag{1a}$$

$$\Phi(\boldsymbol{\Omega}, \mathbf{x}) = \Phi^{\text{inc}}(\boldsymbol{\Omega}, \mathbf{x})\,, \qquad \forall(\boldsymbol{\Omega}, \mathbf{x}) \in \mathcal{S} \times \delta\mathcal{D}\,, \quad \boldsymbol{\Omega} \cdot \mathbf{n}(\mathbf{x}) < 0\,, \tag{1b}$$

where Φ is the scalar flux, defined by

$$\Phi = \frac{1}{2\pi} \int_{\mathcal{S}} \Phi(\boldsymbol{\Omega}, \mathbf{x}) d\Omega\,.$$

S_N Discretization

Introduce the S_N discretization, which replaces the angular flux with a discrete angular flux, as

$$\psi(\mathbf{x}) = \left[\psi_1(\mathbf{x}), \psi_2(\mathbf{x}), \dots \psi_{N_\Omega}(\mathbf{x})\right]^T . \tag{2}$$

Introduce a quadrature rule $\{(\mathbf{\Omega}_d, \omega_d), d = 1, \dots, N_\Omega\}$ where $\sum_d \omega_d = 2\pi$ to cast the linear Boltzmann equation as

$$\mathbf{\Omega}_d \cdot \nabla \psi_d(\mathbf{x}) + \sigma_t(\mathbf{x})\psi_d(\mathbf{x}) - \sigma_s(\mathbf{x})\phi(\mathbf{x}) = q(\mathbf{x}) , \qquad \forall \mathbf{x} \in \mathcal{D} \tag{3a}$$

$$\psi_d(\mathbf{x}) = \Psi_j^{\text{inc}}(\mathbf{x}) , \qquad \forall \mathbf{x} \in \delta\mathcal{D} , \mathbf{\Omega}_d \cdot \mathbf{n}(\mathbf{x}) < 0 , \tag{3b}$$

where the discrete scalar flux, ϕ, is

$$\phi(\mathbf{x}) = \frac{1}{2\pi} \sum_{d=1}^{N_\Omega} w_j \psi_j(\mathbf{x}) .$$