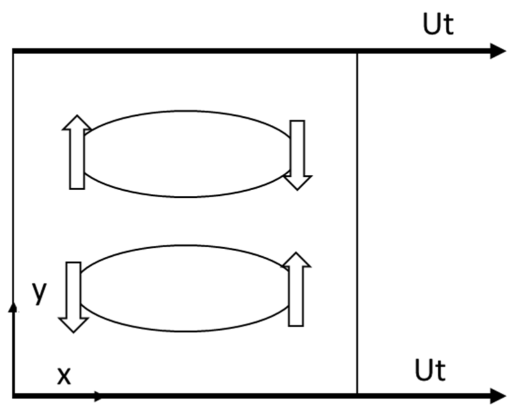
MEEN 644 – Numerical Heat Transfer and Fluid Flow
Spring 2019
Homework # 4

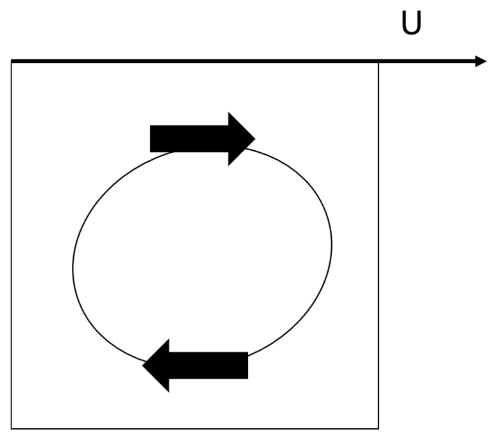
Name ____

Instructor: N. K. Anand

Due Date: March 21, 2019 Maximum points: 100

A viscous fluid (Water) is trapped in a square 2-D cavity of dimension 0.2 m by 0.2 m. Either top or bottom walls are pulled to the right at a uniform velocity on purpose.





Left, flow for problem (a) to verify symmetry and code. Right, flow for problem to compare with Roy et a. (2015)

Write a finite volume-based computer program to predict the 2-D steady laminar flow field for Re = 400. Solve the velocity and pressure fields by linking them through the SIMPLE algorithm in a staggered grid. Represent solution to the one-dimensional convection-diffusion problem using the power law scheme.

(a) In order to verify your code for symmetry, make calculations using 5 x 5 uniformly sized control volumes (CVs). Declare convergence when (R_U & $R_V < 10^{-6}$) and ($R_P < 10^{-5}$). Print your velocity and pressure fields up to 5 decimal places. (E.g. 9.12345e-6) **(60 points)**

(b) With the top plate pulled to right at constant velocity at Re = 400, calculate velocity and pressure fields using 8x8, 16x16, 32x32, 64x64, and 128x128 CVs.

   i) Plot the centerline U and V velocities for each case (For centerline U, plot @ x=0.1m while for centerline V, plot @y = 0.1m). **(20 Points)**

   ii) Compare your solutions of 128x128 CV case with the benchmark solution of Roy et al. (2015) on Table 4 and Table 5. (**20 Points)**

Properties of fluid in the cavity:

$$
\begin{aligned}
&Water\ @\ 20°C \\
&\rho = 998.3 kg / m^3 \\
&k = 0.609 W \\
&\mu = 1.002 \times 10^{-3} N \cdot s / m^2 \\
&C_p = 4.183 kJ / kg \cdot K
\end{aligned}
$$

Definition of Residuals:

$$R_U = \frac{\sum_{node} \left| a_e u_e - \sum a_{nb} u_{nb} - b_u - A_e \left(P_P - P_E \right) \right|}{\sum_{node} \left| a_e u_e \right|}$$

$$R_V = \frac{\sum_{node} \left| a_n v_n - \sum a_{nb} v_{nb} - b_v - A_n \left(P_P - P_N \right) \right|}{\sum_{node} \left| a_n v_n \right|}$$

$$R_P = \frac{\sum_{node} \left| \left(\rho_w u_w - \rho_e u_e \right) dy + \left(\rho_s u_s - \rho_n u_n \right) dx \right|}{\rho u_{ref} L_{ref}}$$

$$R_T = \frac{\sum_{node} \left| a_P T_P - \sum a_{nb} T_{nb} - b_T \right|}{\sum_{node} \left| a_P T_P \right|}$$

Notes:

i. For this homework, you do not need to calculate the temperature residual, since you will not solve energy equation.

ii. For Reynolds number and calculation for R_P, use cavity height as characteristic length L_{ref} and top velocity U_t as reference velocity u_{ref}.

iii. For the second part, nondimensionalize centerline velocities by dividing your result by u_{ref}.

Reference

Pratanu Roy, N. K. Anand, Diego Donzis, A Parallel Multigrid Finite-Volume Solver on Collocated Grid for Incompressible Navier-Stokes Equations, *Numerical Heat Transfer, Par B: Fundamentals,* **67(5)**, 2015