Laminar Separation, Reattachment, and Transition of the Flow Over a Downstream-Facing Step

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Results of an experimental investigation of the laminar flow of air over a downstream-facing step are presented. The experiments include visual observations of smoke filaments (in the viscous layer), qualitative velocity fluctuation measurements, and mean velocity profiles. Results are reported over a range of 0.36 — 1.02 cm in step height, 0.61 — 2.44 m/sec in free stream velocity at the step, and 0.16 — 0.51 cm in boundary layer displacement thickness at the step.

Laminar flow to reattachment of a free shear layer is observed for subsonic flow and two criteria for which transition to turbulence at reattachment exists are presented. The laminar reattachment length is not a constant number of step heights as for turbulent flow, but varies with Reynolds number and boundary layer thickness at the step. The shape of the velocity profile at reattachment is found to be similar to the shape of a laminar boundary layer profile at separation and the boundary layer profiles downstream of reattachment are similar to those in a laminar boundary layer developing toward separation except that they are traversed in the reverse sense.

Introduction

Flow separation and reattachment occur in many engineering applications. Some examples of these applications are in flow over airfoils at large angles of attack, in channels whose area suddenly increases, and in wide-angle diffusers. Various techniques such as the Karman-Pohlhausen method are useful in predicting the occurrence of separation, but very little is known about the flow pattern following separation. Modern technology often uses processes operating at low pressures or devices which encounter low pressures when located in space vehicles. Under such conditions, the flow is frequently in the laminar regime. The present investigation was initiated to obtain a better understanding of fluid flow following separation including reattachment and the redevelopment of the boundary layer following reattachment in this regime. The downstream-facing step was selected as an experimental model since the point of separation is fixed. The basic flow situation is shown in Fig. 1. Fluid with a free-stream velocity U flows past a step of height h. The separated shear layer will reattach at some distance x_r, forming a new boundary layer.

Published information on separated flow deals almost exclusively with the turbulent or transitional regime. Seban, Emery, and Levy [1],[2] Seban [2, 3], Abbott and Kline [4], Filetti and Kays [5], and Mueller, Korst, and Chow [6] all report constant reattachment lengths between 5 and 8 step heights for turbulent subsonic flow. Mueller, Korst, and Chow also conclude that the mean velocity profiles in the redeveloping boundary layer constitute a one-parameter family which in terms of the shape parameter is the same as the one parameter family for a boundary layer developing toward separation.

Moore [7] reports "laminar velocity profiles" downstream of a rearward-facing step at a Reynolds number (based on boundary layer displacement thickness) of 338. The reattachment length is 22 step heights. Grove, et al. [8], have observed laminar separated regions downstream of a circular cylinder in a crossflow with a splitter plate.

Cramer [9] postulates a rather idealized model to obtain analytically a correlation for laminar separation bubbles. He assumes incompressible flow over a stepped flat plate and that "the usual laminar boundary layer assumptions are applicable." He further assumes that for a small step height, the air in the bubble is stagnant and the flow downstream of the step before reattachment grows toward the wall in the manner of a spreading laminar jet. He finds

$$\frac{x_r}{\delta_h{}^*} = \frac{\mathrm{Re}_{\delta^*}}{3}\left[\left(\frac{h}{\delta_h{}^*}+1\right)^2 - 1\right] \tag{1}$$

where $\delta_h{}^*$ is the displacement boundary layer thickness at the

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[2] Numbers in brackets designate References at end of paper.

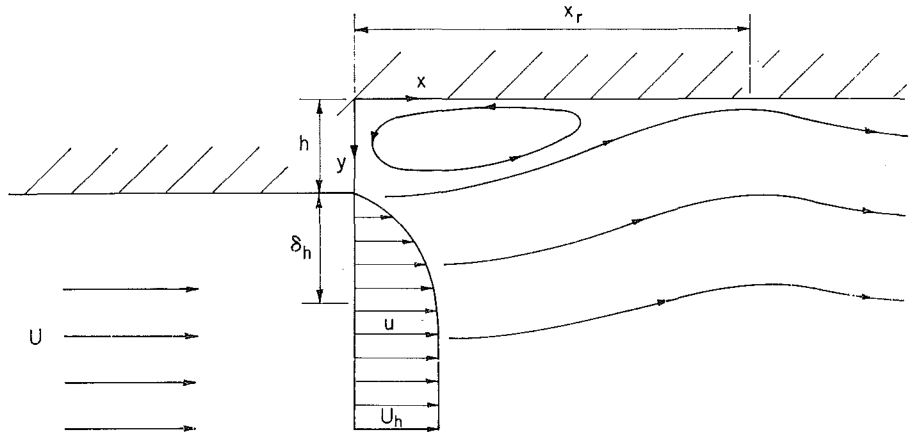

Fig. 1 Basic geometry of investigation

step. He also mentions short "turbulent type" bubbles occurring for $Re_{\delta^*} > 500$ and longer "laminar type" bubbles occurring for $Re_{\delta^*} < 500$ that were observed by Owen and Klanfer [10].

Jacobs and von Doenhoff [11] conducted boundary layer surveys in the transition region of a separating and reattaching flow over a flat plate in an adverse pressure gradient. They find that the entire length of the transition region can be expressed by a constant Reynolds number of 70,000. This length is the distance from the laminar separation point to the first fully developed turbulent boundary layer profile. Since turbulence begins somewhere upstream of this turbulent profile, they assume that the distance from the laminar separation point to the point from which turbulence effectively begins to spread corresponds to a Reynolds number of 50,000. They infer that transition with its accompanying spreading turbulence is responsible for reattachment of the separated shear layer.

Tani and Sato [12] studied the effect of a two-dimensional roughness element on the boundary layer transition on a flat plate. They find the transition differs considerably depending on whether the height of the roughness element d is small or large compared to the thickness of the boundary layer δ_d at the element. For d/δ_d small, they found transition in the reattached boundary layer that is similar to transition in an undisturbed boundary layer. For large d/δ_d, they found transition occurs in the separated layer before reattachment to the plate. They report Blasius type boundary layer profiles and a separated layer that extends over a considerable range for small d/δ_d, but velocity profiles characteristic of a separated layer and rather short reattachment lengths for d/δ_d large. They find transition to turbulence when

$$\frac{U\delta_t}{\nu}\frac{d}{\delta_d} \geq 840 \tag{2}$$

where δ_t is the thickness which the boundary layer would have at the point of transition if the roughness element were not present.

Roshko and Lau [13] report experiments on the shear layer following a downstream facing step at a free-stream velocity of 9 m/sec. Transition occurred before reattachment in all instances. After considering their data along with results of similar experiments found in the literature, they predict that the criterion for the dividing line between laminar and turbulent reattachment is

$$\frac{\delta_h}{h} = 1 \tag{3}$$

Thus it would appear to be impossible to have a completely laminar separated flow in which the shear layer thickness is small compared to the step height.

Sato [14] studied the transition of a laminar "half jet" (flow over an infinite corner) in the velocity range 3–15 m/sec with $70 < Re_\theta < 380$. He points out that the laminar and turbulent mean velocity distributions in the shear layer differ so slightly that it is difficult to find the transition point from the shape of the velocity profile. This is in contrast to an attached boundary layer profile whose shape factor, $H = \delta^*/\theta$, drops sharply at transition. Since the momentum thickness in a separated layer is proportional to the square root of the distance from a virtual origin for laminar flow while it is proportional to the distance

Nomenclature

d = diameter of roughness element
h = step height
H = boundary layer shape factor = δ^*/θ
Re_d = Reynolds number based on free stream velocity and diameter of roughness element
Re_h = Reynolds number based on free stream velocity at the step and characteristic length h = $U_h h/\nu$
Re_{δ^*} = Reynolds number based on free stream velocity at the step and the boundary layer displacement thickness at the step with no step in the test section = $U_h\delta_h^*/\nu$
Re_θ = Reynolds number based on free-stream velocity and boundary layer momentum thickness
u = component of velocity in x-direction in the shear layer
U = free-stream velocity in test section
U_h = free-stream velocity in the plane of the step
x = distance downstream from step
y = distance normal to wall
x_r = reattachment length
δ = boundary layer thickness
δ_h = boundary layer thickness at the step
δ_t = boundary layer thickness at the point of transition
δ^* = boundary layer displacement thickness
δ_h^* = boundary layer displacement thickness at the location of the step with no step in the test section
θ = boundary layer momentum thickness
ν = kinematic viscosity
δ_d = boundary layer thickness at a roughness element of diameter d

for turbulent flow, he defines the transition point as the point where the laminar relation breaks down. Comparing these results with transition found from velocity fluctuation measurements, he concludes that transition in the separated shear layer occurs 40–50 momentum thicknesses downstream of the separation point.

Laminar separation and reattachment have been observed for supersonic flow. Chapman, Kuehn, and Larson [15] report laminar separations that are steady in a supersonic stream and depend only to a relatively small extent on Reynolds number. They also note that the stability of a separated laminar mixing layer increases markedly with Mach number. Laminar reattachment lengths of about 18 step heights and turbulent reattachment lengths of about 6 step heights are reported for a downstream-facing step in a supersonic stream. Sfeir [16] conducted experiments over a backward-facing step for a Mach number of 2.4. He reports laminar reattachment lengths of 7–8 step heights.

Ginoux [17] describes strong, regular, and repeatable three dimensional effects when separation and reattachment are laminar in supersonic flow. These three dimensional effects could not be explained by irregularities either in the air flow upstream of the flow models or in the models themselves. Sfeir [15] also observed three dimensional effects.

Kline [18] in a discussion of a paper by Hekestad describes a periodic fluctuation of two dimensional separated flows which he says is due to part of the oncoming flow passing into the separated zone inside the dividing streamline. The separated zone grows in this way until some of the recirculating flow leaves and the separated zone is reduced in size. The process then repeats itself. Kline argues that an escape path must be provided for the stagnation pressure deficient fluid to leave the separated zone in order for the flow to be truly steady and two-dimensional. He also describes a three-dimensional array of vortex motions that can provide an escape path in low Reynolds number, steady, separated zones.

The aforementioned studies indicate that reattachment of a turbulent shear layer occurs about 5–8 step heights downstream of a step and appears to be essentially independent of Mach number and Reynolds number. Very little information is available concerning a free shear layer that is laminar at both separation and reattachment for subsonic flow.

The present investigation is limited to subsonic flow over a downstream-facing step with the free shear layer laminar until reattachment. Of special interest are the laminar reattachment length, the conditions under which the free shear layer remains laminar up to reattachment, the laminar boundary layer growth following reattachment, and the secondary flows in the separated region downstream of the step. The range of variables is 0.36–1.02 cm in step height, 0.61–2.44 m/sec in free-stream velocity at the step, and 0.16–0.51 cm in boundary layer displacement thickness at the step.

Experimental Apparatus and Test Procedure

All tests are performed in a small air tunnel with a rectangular test section fabricated of Plexiglas walls. This test section is of a constant 10.2 cm width, and is 15.3 cm in height upstream of the step location and 15.3 cm plus the step height downstream of the step. The bottom is flat for all tests except a few in which it is gradually raised an amount equal to two step heights opposite the step location in order to check the effect of tunnel cross-section area on reattachment position. Two lengths of test section upstream of the step can be used, 4.07 cm and 30.5 cm, respectively, in order to obtain different boundary layer thicknesses at the step for a given free-stream velocity. The test section is 20.3 cm long downstream of the step, with a movable top to provide for various step heights. All joints are sealed to preclude infiltration, since the test section is at a pressure slightly below that of the ambient atmosphere.

The entrance to the test section has a short rounded inlet; a 50.8 $\times$ 76.2 cm rectangular section containing flow straighteners consisting of packed plastic drinking straws about 0.64 cm in diameter and 26.7 cm long and six screens ranging from 18 to 60 mesh; and a 1.14 m long contraction section with a 25 to 1 contraction ratio. This provides laminar flow in the test section with a flat velocity profile over the entire cross section except for the boundary layer along the four walls. Cigar-smoke filaments from a smoke generator are introduced at various locations in the entrance section to confirm the laminar nature of the flow. The smoke filaments pass through the test section without dispersion. The measured turbulence intensity in the test section is less than 0.05 percent. The absence of secondary flows without the step is confirmed by these smoke injections as well as by introducing smoke into the boundary layers through numerous holes in the walls of the test section. The flat velocity profiles in the core flow are measured with a hot-wire anemometer calibrated for low air speeds and with a large impact tube connected to a sensitive micromanometer having an optical lever read out. Accuracy of the hot wire anemometer is about ± 0.02 m/sec at 0.30 m/sec, varying monotonically to about ± 0.03 m/sec at 2.44 m/sec. Accuracy of the micromanometer is ± 1.0 percent.

The discharge section is of constant 10.2 $\times$ 17.8 cm cross section, the top being a continuation of the stepped test section upper surface, and the bottom containing a variable smooth transition from the bottom of the test section. Straightening tubes precede the small blower which is driven by a small motor whose speed is controlled by an auto transformer.

The reattachment location x_r for the shear layer is obtained by observing cigar smoke from a smoke generator introduced in minute wisps through individual small holes along the top of the stepped upper surface of the test section. Two rows of 0.12 cm diameter holes spaced 0.64 cm apart, one along the centerline and one along one quarterline (midway between the centerline and the right sidewall) are located in the upper surface and plugged with round toothpicks. One hole at a time is opened and smoke introduced gently by rolling a finger tip over the hole while smoke enters. Thus the last wisp of smoke enters with nearly zero velocity. The fore or aft direction of motion of the smoke is observed and from this the location of shear layer reattachment can be inferred within 0.32 cm for a given air speed U_h. In some instances the tunnel flow rate is varied and the speed at which reattachment occurs at a fixed location x_r is determined directly. Observations of reattachment position made along the center line and quarterline rows of holes generally agree with one another. In a very few instances for large values of x_r the observed location of reattachment is perhaps 0.64 cm larger at the center line than at the quarterline.

Transition of the laminar shear layer at the reattachment location is determined by smoke observations and by traversing the shear layer with a hot-wire anemometer. Transition is determined from the motion of the wisps of smoke introduced into the stepped surface of the test section, and of smoke filaments introduced upstream of the step. Fig. 2(a) shows a typical filament completely laminar; Fig. 2(d) shows a typical filament for which transition occurs far upstream of reattachment.

In addition to smoke observations the shear layer region at and near reattachment can be probed with a hot wire anemometer whose output is displayed on an oscilliscope and recorded on a Honeywell Visicorder strip chart. This provided a more explicit determination of transition to turbulent flow. At transition there is a significant rise in the signal velocity fluctuations.

Mean velocity profiles in the boundary layer and free shear layer are measured with a Flowmeter Corporation hot wire anemometer calibrated at air speeds from about 0.30 – 3.05 m/sec. The position of the hot wire probe is determined within 0.005 cm. Values of the displacement thickness δ^* and momentum thickness θ are obtained by numerical integration using least squares polynomials to fit different sections of the velocity profile.

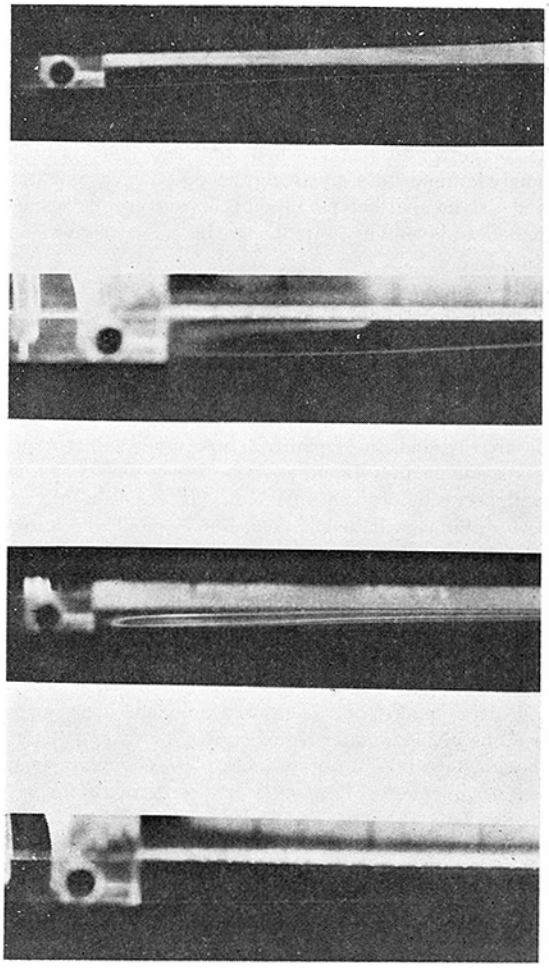

Fig. 2 Photographs of flow patterns

(a) Laminar reattachment
(b) Partial recirculation pattern
(c) Complete recirculation pattern
(d) Transition before reattachment

Results and Discussion

Laminar Reattachment. Photographs of the region downstream of the step are shown in Fig. 2. The streamlines are made visible by injecting smoke into various parts of the test section. Excluding the photograph that shows transition before reattachment (Fig. 2(*d*), the smoke filaments do not disperse indicating that the flow is laminar.

The reattachment length x_r should be a function of the free stream velocity at the step U_h, the step height h, the fluid viscosity ν, and the boundary layer thickness at the step δ_h (in place of δ_h, the displacement thickness with zero step height at the location of the step δ_h^* can be used). Applying dimensional analysis,

$$\frac{x_r}{h} = f\left(\frac{U_h h}{\nu}, \frac{\delta_h^*}{h}\right) \tag{4}$$

Fig. 3 shows x_r/h plotted against $U_h h/\nu$. There is only a slight effect of δ_h^*/h on the results. A least squares straight line fit to the data in Fig. 3 yields

$$\frac{x_r}{h} = 2.13 + 0.021 \ \mathrm{Re}_h, \text{ where } \mathrm{Re}_h = \frac{U_h h}{\nu} \tag{5}$$

Note that the normalized laminar reattachment length is not a constant as in the turbulent case, but increases with Reynolds

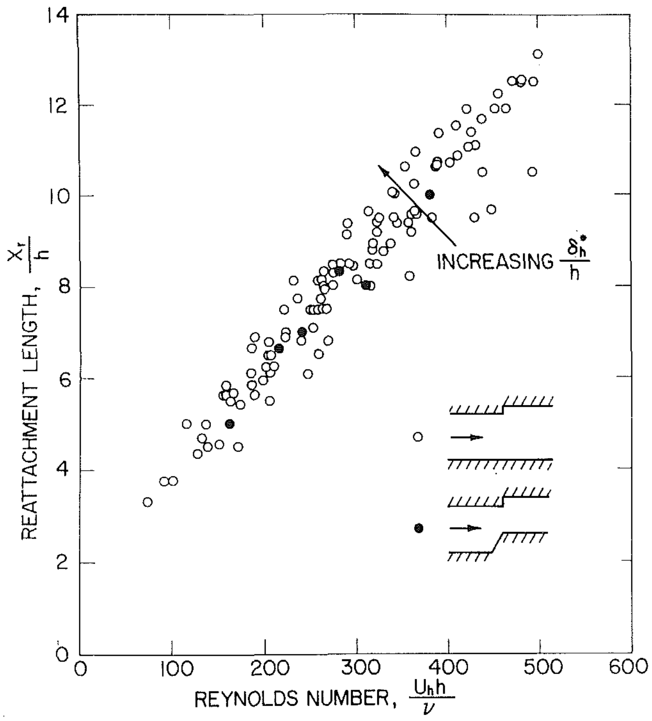

Fig. 3 Laminar reattachment position as a function of Reynolds number based on step height

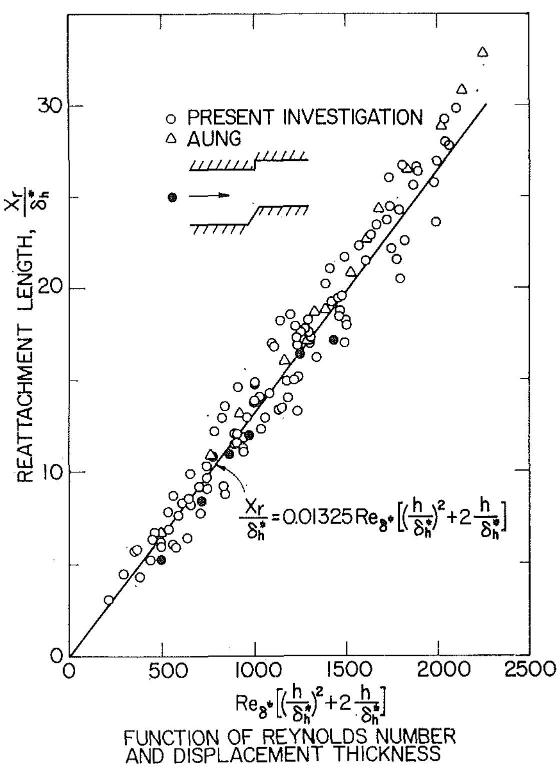

Fig. 4 Laminar reattachment position as a function of Reynolds number and displacement thickness

number. Its maximum value is about 13, nearly twice the turbulent result found by other investigators, but much less than the value of 22 that Moore found.

Of several attempts to correlate the data on a basis of both δ_h^*/h and Re_h, the form of the equation suggested by Cramer [9] is the most successful although the data does not fit this somewhat more complex relation with significantly less scatter than they did the equation obtained from the representation in

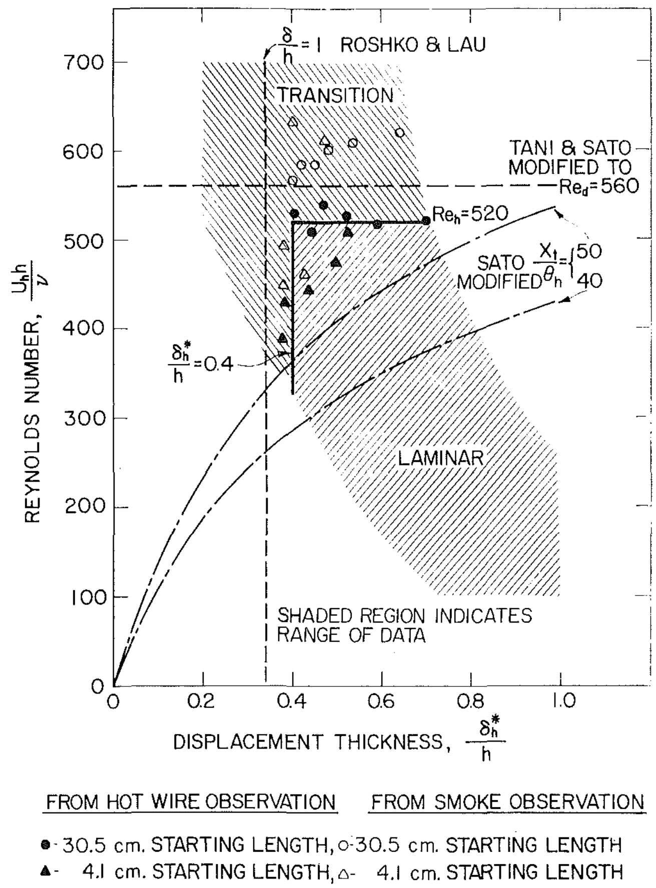

Fig. 5 Reynolds number based on step height versus displacement thickness indicating values at which transition to turbulence occurs at reattachment

Fig. 3. With this representation, Fig. 4, the equation of the least squares straight line passing through the origin is

$$\frac{x_r}{\delta_h^*} = 0.01325 \, \mathrm{Re}_{\delta^*} \left[\left(\frac{h}{\delta_h^*} \right)^2 + 2 \left(\frac{h}{\delta_h^*} \right) \right] \tag{6}$$

It should also be noted that the constant in this equation is much less than the value of $^1/_3$ that Cramer used. This discrepancy is not surprising considering his assumptions. Visual observations using smoke indicate that the free shear layer is deflected toward the tunnel wall considerably. Cramer's assumption that the shear layer grows toward the wall in the manner of a spreading jet that is not deflected would thus give a much greater value of the reattachment length than the experimental results do.

The shaded points in Figs. 3 and 4 are results obtained with the bottom of the test section elevated two step heights ($2h$) opposite the step. The effect of the not-quite uniform free stream throughout the entire test section length due to the slightly greater cross-sectional area downstream of the step is seen to be negligible.

The triangles in Fig. 4 represent data taken by Aung [19]. His data were taken in a wind tunnel of similar design to the one used in the present experiments except that the cross-sectional area of his test section is twice the cross-sectional area of the tunnel used here. Agreement between the two sets of data is good. The pressure gradient in the test section therefore seems to have very little effect on the reattachment length.

Transition. Fig. 5 shows the conditions under which laminar reattachment occurs. The open points correspond to observations of smoke filaments that are injected into the free shear layer. The solid points are determined from the velocity fluctuations sensed by a hot wire probe at the reattachment point. These latter points are probably more accurate since it is easier to distinguish the velocity fluctuations that are recorded by the hot wire apparatus than it is to determine when the smoke filament starts to break down. Each point corresponds to the condition for which the shear layer is no longer completely laminar for a given step height. Once transition has occurred at the reattachment point, it moves upstream along the free shear layer very rapidly as the free stream velocity is increased.

The data shown in Fig. 5 may be interpreted by considering two separate criteria for transition at the reattachment point. First of all the normalized boundary layer displacement thickness at the step, δ_h^*/h, must be greater than some value, approximately 0.4. Secondly, the Reynolds number based on step height must not exceed a certain value, about 520. If both criteria are satisfied, the separated shear layer will remain laminar to reattachment. If either or both are unsatisfied, transition will occur before reattachment. It seems a bit unusual that two criteria must be satisfied, but it should be noted that the shape of the plot of points is the same for both the smoke and hot wire methods of determining transition. The data taken with smoke lie above those taken by the hot wire technique because it is easier to detect the first velocity fluctuations with the hot wire apparatus.

The region in which data are taken is shown as the cross-hatched area in Fig. 5. The reattachment data presented earlier all lie in the laminar region.

Although they did not obtain a completely laminar separation and reattachment for flow over a downstream facing step, Roshko and Lau [13] have predicted that the criterion for laminar reattachment is $\delta/h \geq 1$. For a Blasius velocity profile at the step, this condition can be written $\delta_h^*/h \geq 0.34$. This result shown in Fig. 5 agrees with the first criterion obtained in the present investigation, but does not seem to be sufficient.

From his experimental study of a laminar half jet, Sato [14] found that transition of the laminar free shear layer occurred 40 to 50 momentum thicknesses downstream from the separation point. To apply this criterion to the present study, the equation for the laminar reattachment (equation (6)) can be combined with the equation for the shape factor of the boundary layer with no step in the wind tunnel ($H = \delta_h^*/\theta_h = 2.35$) to give

$$\frac{x_r}{\theta_h} = 0.0311 \, \mathrm{Re}_h \left[\frac{h}{\delta_h^*} + 2 \right] \tag{7}$$

Using Sato's criterion for the transition of a laminar free shear layer, the following equation for transition at the reattachment point results.

$$\mathrm{Re}_h = [1286 \text{ to } 1607] \frac{\dfrac{\delta_h^*}{h}}{\dfrac{\delta_h^*}{h} + 2} \tag{8}$$

The two curves bounding this region are also shown in Fig. 5. It is not surprising that these curves fall below the data from the present study, since the presence of the wall would tend to damp out small disturbances that would be amplified in a separated shear layer that did not reattach.

Although their geometry differs from that considered here, the results of Tani and Sato [12] will be shown to agree with the results of this investigation. Using a fine cylindrical rod with diameter d attached to a flat plate as a two-dimensional roughness element, transition occurs in the separated layer before reattachment if δ_d/d is small. For large δ_d/d, transition occurs in the reattached boundary layer. Qualitatively, this result agrees with that of Roshko and Lau and with the first criterion for transition at reattachment from the present tests. Tani and Sato present their data for transition downstream of but not too close to, the position of the roughness element in the form

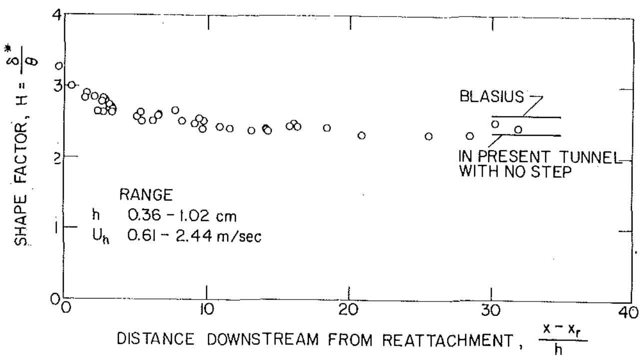

Fig. 6 Shape factor of the redeveloping boundary layer

$$\frac{U_h \delta_t}{\nu} \frac{d}{\delta_d} = 840 \tag{9}$$

where δ_t is the boundary layer thickness at the transition point. As discussed below (cf. Fig. 8) the value of θ/θ_h at the reattachment point is approximately 1.5. Substitution of this value into the above equation (assuming $\delta/\delta_h = \theta/\theta_h$) gives

$$\mathrm{Re}_d = 560 \tag{10}$$

as a criterion for transition at the reattachment point. This line is shown in Fig. 5.

Moore [7] states that one of his runs "displays profiles which are more consistent with the notion that the boundary layer has reattached while still in a laminar state." If one considers the boundary layer in that test to grow as though it were on a flat plate, the values of Re_h and δ_h^*/h are 800 and 0.425, respectively. These values fall outside of the laminar regime described in the present study. It is therefore probable that his run does not represent a steady, laminar reattachment, but falls into the transition regime described in the foregoing. The transition from steady laminar flow is not usually discernible from the mean velocity profile. The authors and Aung [19] have observed long reattachment lengths, of 20 or more step heights, at higher Reynolds numbers than are reported in the present paper, but the observations of both smoke filaments and velocity fluctuations indicate that these are not steady, laminar reattachments.

Boundary Layer Profiles. The shape factor of the redeveloping boundary layer is plotted against the dimensionless distance downstream of the reattachment point in Fig. 6. The shape factor decays to the value of 2.35 (found in the wind tunnel with no step present), reaching this about 15 step heights downstream of reattachment. The shape factor with no step in the wind

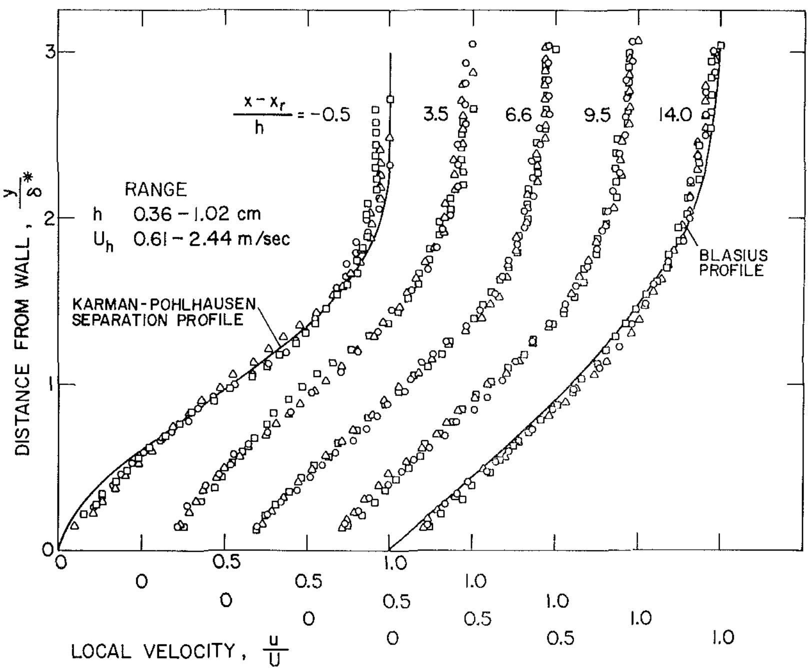

Fig. 7 Velocity profiles in the redeveloping laminar boundary layer

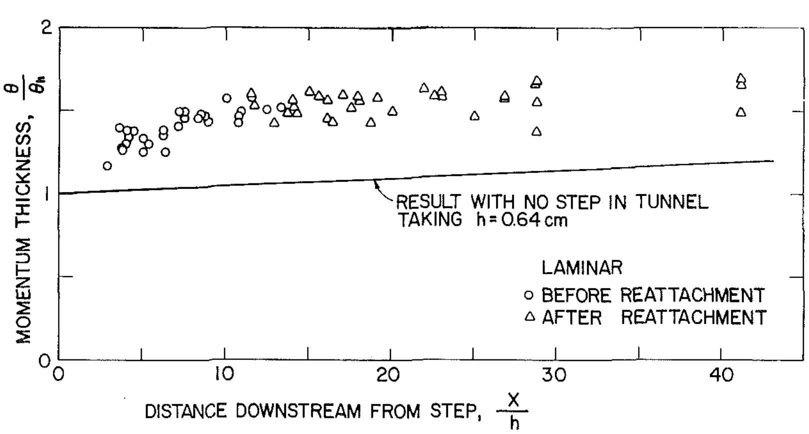

Fig. 8 Laminar momentum thickness downstream from the step

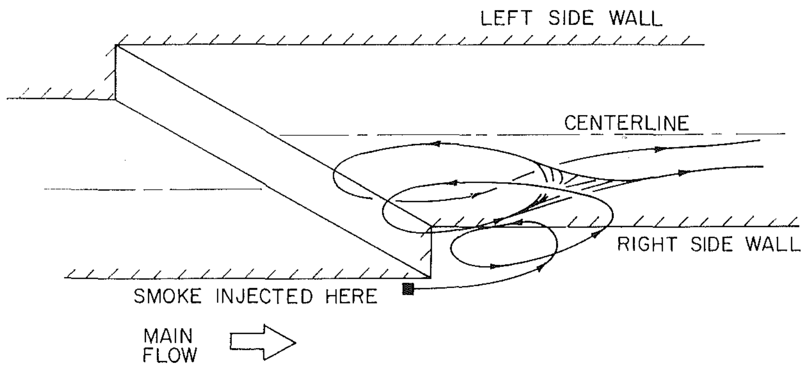

Fig. 9 Secondary flow pattern in the separated region

tunnel is less than the Blasius flat plate value of 2.6 because of a slight pressure gradient in the wind tunnel.

The shape factor at the reattachment point is about 3.1. Since the wall shear stress is zero at reattachment, one might expect the value of the shape factor at reattachment to be the same as at separation. This comparison is difficult to make since the shape factor changes markedly near the reattachment point and several values of the shape factor for a velocity profile at separation have been predicted. The shape factor for a wedge type flow is about 4, while the Karman-Pohlhausen value for a fourth degree polynomial is 3.5. Sandborn and Kline [20] show that the value of the shape factor at separation is a function of the degree polynomial used to approximate the velocity profile. The shape factor decreases from 5.00 to 2.69 as the degree of the polynomial increases from 2 to 100. A better method to compare velocity profile shapes at separation and reattachment would then be to look at the actual profiles themselves rather than the shape factor.

Fig. 7 shows measured velocity profiles at five dimensionless distances from the reattachment point. These profiles represent essentially the complete range of step heights and velocities used in the experiments. The Karman-Pohlhausen separation profile is shown with the measured profiles 0.5 step heights upstream of reattachment. The measurements agree with the curve except very near the wall where free convection from the hot wire probe may have altered the flow pattern. The profile shape downstream of reattachment gradually changes from a separation type profile to a flat plate type profile. The profiles 14 step heights downstream of reattachment agree with the Blasius flat plate velocity profile. It can therefore be concluded, as Mueller, Korst, and Chow [6] did for the turbulent case, that the mean velocity profiles in the redeveloping laminar boundary layer are similar to those for a laminar boundary layer developing toward separation except that they are traversed in the reverse sense.

Fig. 8 shows the growth of the momentum thickness of the laminar shear layer downstream of separation. The momentum thickness of the free shear layer grows to approximately 1.5 times its value at the step before reattaching to the wall. The reattached boundary layer then grows much slower than the free shear layer.

Far downstream of the step, one would expect the boundary layer to grow as a flat plate boundary layer does. It is difficult to make an accurate comparison of this nature in Fig. 8 because the step height is in the denominator of the abscissa. For comparison the line on the figure has been drawn to show the growth of a boundary layer with zero step height in the wind tunnel while assuming a value of $h = 0.64$ cm (the approximate midpoint of the range of experimental step height settings) in order to plot the abscissa. The line and the plot of experimental data increase at approximately the same rate.

The momentum thickness of the laminar free shear layer increases to approximately 1.5 times its value at separation before reattachment occurs. It then grows approximately like the momentum thickness of a flat plate boundary layer which has been artificially thickened. It should be noted that the shape of the velocity profile following reattachment is not that of a flat plate boundary layer, but follows the variation of the shape factor shown in Fig. 6. The displacement thickness thus does not grow like the displacement thickness of a flat plate boundary layer until approximately 15 step heights downstream of reattachment.

Secondary Flow Pattern. With no step in the top of the test section, there is no evidence of secondary flow when fine smoke filaments are introduced into the wind tunnel entrance or into the test section. The absence of diffusion of these filaments indicates laminar flow throughout the test section. Secondary flows are present in the separated region downstream of the step, however. The magnitude of these secondary flows increases with step height.

Smoke filaments introduced into the boundary layer just upstream of the step and near the side walls of the test section are deflected slightly towards the centerline. When smoke is introduced into the separated zone through the holes along a quarterline (midway between the centerline and side wall of the test section beyond the step) upstream of the reattachment location a consistent pattern of secondary flows is observed. This smoke filament flows upstream along the upper tunnel surface as shown in Fig. 2(*b*) and is also deflected toward the tunnel centerline as shown in Fig. 9. It reaches the centerline before arriving at the step where it then flows down along the surface of the step, enters the free shear layer, and follows the free shear layer to the reattachment point, where it appears to leave the separated zone.

The entire secondary flow pattern shown in Fig. 9 can be observed by introducing smoke through the side wall of the test section, just upstream of the step at the upper tunnel surface. This smoke enters the boundary layer in the corner of the test section. Upon reaching the step, it enters the separated region and recirculates in a spiral-like fashion, finally leaving the separated zone in the plane of the centerline. Smoke introduced through the side wall of the test section just upstream of the step, but outside the boundary layer formed in the corner, remains outside of the separated zone. Smoke introduced in the plane of the centerline remains in this plane.

Only the outer loop of the spiral shown in Fig. 9 is smaller than the others. This loop lies in the boundary layer that grows on the side wall of the test section. The other two loops are approximately the same size. This same secondary flow pattern with three loops has also been observed in the larger test section of Aung [19].

Conclusions

1 A subsonic separated flow that is laminar at both separation and reattachment does exist for the geometry of the downstream facing step.

2 The laminar reattachment length x_r cannot be expressed as a fixed number of step heights as for turbulent flow, but is given by the equation

$$\frac{x_r}{\delta_h{}^*} = 0.01325 \ \mathrm{Re}_{\delta^*} \left[\left(\frac{h}{\delta_h{}^*} \right)^2 + 2 \frac{h}{\delta_h{}^*} \right]$$

or to a fair degree of approximation within the test range,

$$\frac{x_r}{h} = 2.13 + 0.021 \ \mathrm{Re}_h$$

3 Two criteria must simultaneously be satisfied for this flow to exist over the range studied.

(*a*) The ratio of the displacement boundary layer thickness at the step to the step height must be greater than 0.4.

(*b*) The Reynolds number based on step height must be less than 520.

4 The mean velocity profiles in the redeveloping laminar boundary layer downstream of reattachment are similar to those in a laminar boundary layer developing toward separation except that they are traversed in the reverse sense. The profile closely approximates a flat plate profile about 15 step heights downstream of reattachment.

5 The momentum thickness of the shear layer increases to about 1.5 times its value at separation before reattachment occurs. It then grows as the momentum thickness of a flat plate boundary layer at that location would. The basic difference between the momentum thickness of the reattached boundary layer and that on a flat plate is that the momentum thickness of the reattached boundary layer has been thickened by an amount $0.5\theta_h$.

6 A secondary flow pattern exists in the separated zone. Fluid from the boundary layer in the corner of the test section just upstream of the step enters the separated zone at the step, recirculates in a spiral fashion to the tunnel center line, and leaves the separated region near the reattachment point in the plane of the centerline.

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