MATRIX NOTATION - REVIEW

A matrix consists of a rectangular array of elements represented by a single symbol. A matrix is denoted by [A] and a_{ij} as an individual element of the matrix.

[A] has "m" rows and "n" columns

i.e. [A] is $m \times n$

if m = 1, then

$[B] = [b_1, b_2, \ldots b_n]$ which is called a **row matrix**.

If n = 1, then

$$[C] = \begin{bmatrix} C_1 \\ C_2 \\ -- \\ -- \\ C_m \end{bmatrix} 1$$ which is called a **column matrix.**

If m = n, $\Rightarrow$ square matrix.

Note: Square matrices are particularly important when solving a set of simultaneous equations. For such systems, the number of equations (corresponding to rows) and the number of unknowns (corresponding to columns) must be equal in order for a unique solution to be possible.

<u>Matrix Operations</u>

1. [A] = [B]

   if $a_{ij} = b_{ij}$ for all i and j

2. Addition:

   [C] = [A] + [B]

   $C_{ij} = a_{ij} + b_{ij}$

   Subtraction:

   [D] = [E] - [F]

   $d_{ij} = e_{ij} - f_{ij}$

Note: Addition and subtraction can only be performed on matrices having equal dimensions.

Both addition and substraction are commutative and associative.

- Scalar Multiplication:

$$[B] = g\ [A] = \begin{bmatrix} ga_{11} & ga_{12}\text{---} \\ & \text{---} \\ & \text{---} \end{bmatrix}$$

- Product of Two Matrices:

[C] = [A] [B]

$$c_{ij} = \sum_{k=1}^{n} a_{ik}\, b_{kj}$$

$\therefore$ $[A]_{m\times n}\ [B]_{n\times p} = [C]_{m\times p}$

$[A]^{-1}$ is called the inverse of [A]. It is defined such that $[A]\ [A]^{-1} = [I]$ where [I] is called the identify matrix. $[A]^T$ is called the transpose of [A] and is defined as: if $a_{ij}\ \varepsilon\ [A]$, then $a_{ji}\ \varepsilon\ [A]^T$.

- Linear algebraic equations can be represented by: [A] [X] = [C], then $[X] = [A]^{-1}\ [C]$

The determinant of the coefficient matrices of the sets of equations to be non-zero before unique solution could be obtained. i.e. set of equations need to be linearly independent.

The rank of the matrix is the order of the largest non-zero determinant which can be obtained considering all minors of the matrix.

OR

Rank is the number of linearly independent rows (columns) of the matrix.

Now, consider a set of homogenous linear algebraic equations.

$a_{11}\ X_1 + a_{12}\ X_2 + a_{13}\ X_{13} + \ldots + a_{1n}\ X_n = 0$

$a_{21}\ X_1 + a_{22}\ X_2 + \ldots\ldots\ldots + a_{2n}\ X_n = 0$

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$a_{n1} X_1 + \ldots\ldots\ldots\ldots\ldots + a_{nn} X_n = 0$, or

A system of m linear algebraic equations in n unknowns has a solution if, and only if, the coefficient matrix and the augmented matrix of the set have the same rank.

Any system of homogenous linear algebraic equations always has a trivial solution: i.e. $X_1 = X_2 = - - - X_n = 0$.

Non-trivial solutions will exist for a set of homogenous equations if and only if the rank *r* of the matrix of the set is less than the order *n*.

Note that in obtaining non-trivial solutions for the homogenous system of equations, unique values are not obtained. However, relationships are established between the unknowns. i.e. any combination of X_i which satisfy these relationships constitutes a solution.

Eigen value problems or characteristic-value problems.

$(a_{11} - \lambda) X_1 + a_{12} X_2 + \ldots + a_{1n} X_n = 0$

$a_{21} X_1 + (a_{22} - \lambda) X_2 + \ldots + a_{2n} X_n = 0$

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$a_{n1} X_1 + \ldots\ldots + (a_{nn} - \lambda) X_n = 0$

1. a_{ij} are real

2. X_i are system variables

3. λ is a particular parameter of the system having unknown values or $(A - \lambda I) X = 0$

The column matrix X is called an <u>eigen vector</u> with $X_1, X_2, \ldots, X_n$ as elements.

The values obtained for λ are called <u>eigen values</u>.

<u>How to Obtain Eigen Values:</u>

Note that trivial solution has no meaning in the engineering field. But, for non-trivial solution that one would get by solving:

$$|D| = |A - \lambda I| = 0$$

Solve for $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$.

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