Assignment 2

All assignment submissions are to be made on LEARN Quiz by 11:59 PM on the assigned due date. No extensions will be given, and assignments will not be accepted after the due date.

Problem Statement: A nonlinear model can describe a process that we want to estimate its states:

$$x = [0: 0.01: 10]$$

$$y = can\ be\ found\ as\ a\ separate\ .mat\ file$$

1. Build a 4th order polynomial model of y = f(x), considering that y contains added white noise, using **ordinary least squares**.

   Then answer the questions below:

   a. **What is the obtained weight for x^4?**
   b. **What is the obtained R^2?**

2. Apply a regularization of norm 2, with $\lambda = 2$: your new cost function will be

   $$J_{L2-reg} = e^T e + \lambda \|w\|^2$$

   This is equivalent to solve ridge regression with $\lambda = 2$.

   a. **What would be $\frac{\|W_{L2-reg}\|}{\|W_{LS}\|}$?**
   where W_{L2-reg} is the weight vector resulted from regularized Least Squares and W_{LS} is the weight vector from Least squares.

3. Build a 4th order polynomial model of y = f(x) this time using **Bisquare** method (robust least squares). For simplicity of the weight matrix calculation use the formula below which is a function of absolute error of the points.
   Hint: do not use distance to the curve as it can become computationally expensive for this case.

   $$q_i = \frac{1}{1 + h|e_i|}$$

   s.t. h: 4x standard deviation of the absolute error $h = 4 \times std(|e|)$
   $e_i = y_{act} - \hat{y}$

   a. **What is the obtained weight for x^4?**
   b. **What is the obtained R^2?**
   Note that for **R^2 of weighted LS or robust LS you can use this alternative formula:**

$$R^2 = 1 - \frac{SSE}{SST}$$

$$SSE = \sum q_i e_i^2$$

$$SST = \sum q_i \left(y_i - \frac{\sum_{i=1}^{N} q_i y_i}{\sum_{j=1}^{N} q_j} \right)^2$$

q_is are the weights to be used for samples, to increase or decrease the importance of samples in weighted LS. e_i s are the error of the bisquare regression, $y_i s$ are samples of actual y data.

4. *What are the reasonable orders of model for these measurements?*