Math 312

Group Project

Due 4/19/2022

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1. **Write a report for your Monte Carlo simulation. You should write a short report to introduce the one sample t-test, your simulation plan and discuss your findings from the simulation results.**

The t-test is a method of hypothesis testing for means. It compares the sampled mean μ to the theoretical mean μ_0. Thus, the null hypothesis H_0 is $\mu = \mu_0$ and the alternative hypothesis H_1 is $\mu \neq \mu_0$. To simulate each distribution, we set μ_0 equal to the expected mean of the distribution and sample μ from a random sample.

For chi-squared, the mean μ_0 is equal to the degrees of freedom, in this case 1. In the uniform distribution, the mean is equal to the mean of the minimum and maximum, in this case of 0 and 2, so the mean is 1. In the exponential distribution, the mean is equal to 1 over the rate, which in this case is 1.

In each simulation, repeated m=10000 times, we generate a random sample from the distribution of size n=20 using R functions rchisq(), runif(), or rexp(). Then, we use the t.test() function in R to compare the sample with the null hypothesis H_0: $\mu = \mu_0$. The alternative hypothesis is H_1: $\mu \neq \mu_0$ meaning we use “two.sided” as an argument to the function. We record the p-value for each test in a vector.

A Type I error means that in testing, we reject the null hypothesis H_0 when in reality H_0 is true. If the p-value is less than alpha, we reject the null hypothesis. To find p-hat, we take the mean of the vector for values where the p-value is less than alpha. In other words, the probability of a Type I error is the probability we reject H_0 given that H_0 is true. The results are recorded below.

Distribution	Chi-squared	Uniform	Exponential
Type 1 Error rate	0.157	0.1001	0.128

The uniform distribution had the closest Type I error rate to alpha. It had a rate for Type I error of 0.1001 which is extremely close to $\alpha = 0.10$. This is a result of the uniform distribution being evenly distributed. Furthermore, the variance of the uniform distribution is the smallest of the three distributions at 0.33.

2. **Explain the task allocations in your group.**

Ken: found the mean of each distribution, wrote code for the simulation, recorded results

Logan: found the Type I error rate given the vector of p-values, finished and printed the report.

Appendix

GitHub: https://github.com/loganizer405/MATH312Project

Chi-squared Distribution

```r
#Set seed
set.seed(420)

#Set our n to 20 for X1...X20 and alpha to 0.1
n <- 20
alpha <- 0.10
df <- 1
#Mean of a chisq distribution is equal to the degrees of freedom
mu0 <- df

#Set m to 10000 replicates
m <- 10000

p <- numeric(m) #To store p value

#Run our simulation
for(j in 1:m){
  x <- rchisq(n, df)
  ttest <- t.test(x, alternative = "two.sided", mu = mu0)
  p[j] <- ttest$p.value
}

#Print our p.hat
(p.hat <- mean(p < alpha))
```

```
## [1] 0.157
```

Uniform Distribution

```r
#Set seed
set.seed(420)

#Set our n to 20
n <- 20
alpha <- 0.10

#Set m to 10000 replicates
```

```r
m <- 10000

min <- 0
max <- 2
#Mean of a uniform distribution is equal to (a + b)/2
mu0 <- (min+max)/2

p <- numeric(m) #To store p value

#Run our simulation
for(j in 1:m){
  x <- runif(n, min, max)
  ttest <- t.test(x, alternative = "two.sided", mu = mu0)
  p[j] <- ttest$p.value
}

#Print our p.hat
(p.hat <- mean(p < alpha))
```

```
## [1] 0.1001
```

Exponential Distribution

```r
#Set seed
set.seed(420)

#Set our n to 20
n <- 20
alpha <- 0.10

#Set m to 10000 replicates
m <- 10000

rate <- 1
#Mean of a exponential distribution is equal to 1/rate
mu0 <- 1/rate

p <- numeric(m) #To store p value

#Run our simulation
for(j in 1:m){
  x <- rexp(n, rate)
  ttest <- t.test(x, alternative = "two.sided", mu = mu0)
  p[j] <- ttest$p.value
}

#Print our p.hat
(p.hat <- mean(p < alpha))
```

```
## [1] 0.128
```