HANDS ON 03 – INTEGRALES PARAMÉTRICAS

COMPLEMENTOS DE MATEMÁTICAS

CURSO 2023-2024

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- FUNCIONES ÚTILES – REVISAR DOCUMENTACIÓN

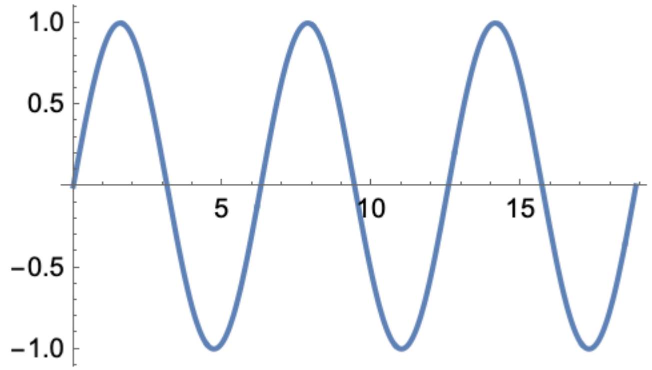
Plot [f, {x, x_{min}, x_{max}}]

generates a plot of f as a function of x from x_{min} to x_{max}.

Plot a function:

In[1]:= **Plot[Sin[x], {x, 0, 6 Pi}]**

Out[1]=



https://reference.wolfram.com/

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Integrate $[f, \{x, x_{min}, x_{max}\}]$
gives the definite integral $\int_{x_{min}}^{x_{max}} f\,dx$.

Compute a definite integral:

In[1]:= **Integrate[1 / (x ^ 3 + 1), {x, 0, 1}]**

Out[1]= $\frac{1}{18}\left(2\sqrt{3}\,\pi + \text{Log}[64]\right)$

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NIntegrate$[f, \{x, x_{min}, x_{max}\}]$
gives a numerical approximation to the integral $\int_{x_{min}}^{x_{max}} f\, dx$.

Compute a numerical integral:

```
In[1]:= NIntegrate[Sin[Sin[x]], {x, 0, 2}]

Out[1]= 1.24706
```

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```
In[10]:= Integrate[Sin[x], {x, 0, 1}]
         integra      seno
Out[10]= 1 – Cos[1]

In[11]:= N[1 – Cos[1]]
         valo... coseno
Out[11]= 0.459698

In[12]:= NIntegrate[Sin[x], {x, 0, 1}]
         integra numé...  seno
Out[12]= 0.459698
```

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f'

represents the derivative of a function *f* of one argument.

Derivative of a defined function:

```
In[1]:= f[x_] := Sin[x] + x^2

In[2]:= f'[x]

Out[2]= 2 x + Cos[x]
```

This is equivalent to $\frac{\partial f(x)}{\partial x}$:

```
In[3]:= D[f[x], x]

Out[3]= 2 x + Cos[x]
```

https://reference.wolfram.com/

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- EJERCICIO
  - REPRESENTAR LA FUNCIÓN $F(a) = \int_0^1 \frac{x^a - 1}{\ln x} dx$ en el intervalo $a \in [0,10]$

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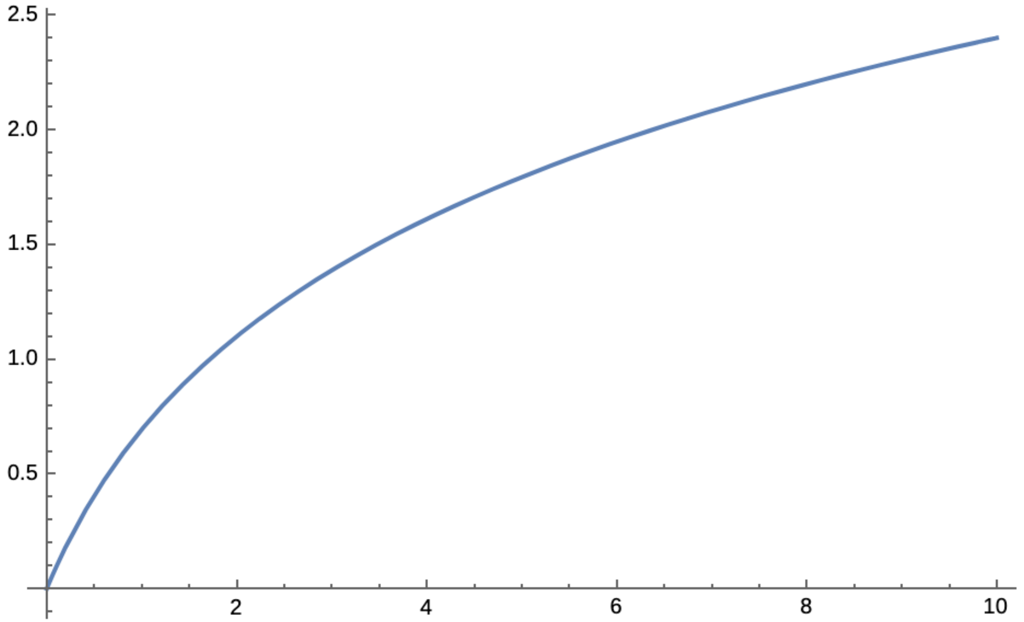
- EJERCICIO
  - REPRESENTAR LA FUNCIÓN $F(a) = \int_0^1 \frac{x^a - 1}{\ln x} dx$ en el intervalo $a \in [0,10]$

In[7]:=
```
Plot[NIntegrate[(x^a-1)/Log[x], {x, 0, 1}], {a, 0, 10}]
```
repr... integra numéricamente logaritmo

Out[7]=



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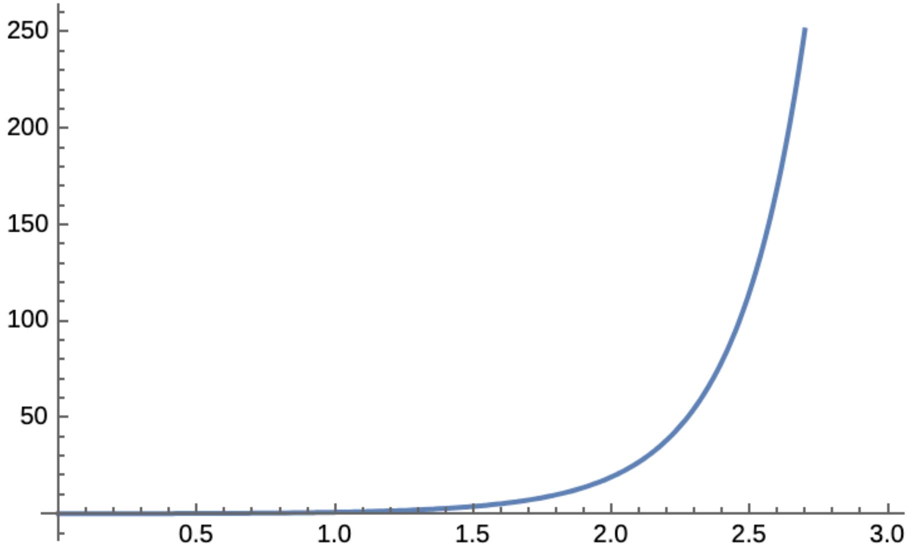
- EJERCICIO
  - REPRESENTAR LA FUNCIÓN $F(a) = \int_0^{a^2} \frac{x^a - 1}{\ln x} dx$ en el intervalo $a \in [0,3]$

WOLFRAM CLOUD

- EJERCICIO
  - REPRESENTAR LA FUNCIÓN $F(a) = \int_0^{a^2} \frac{x^a - 1}{\ln x} dx$ en el intervalo $a \in [0,3]$

In[9]:=
```
Plot[NIntegrate[(x^a - 1) / Log[x], {x, 0, a^2}] , {a, 0, 3}]
```
repr... integra numéricamente logaritmo

Out[9]=



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- EJERCICIO
  - REPRESENTAR LA FUNCIÓN $F(a) = \int_0^\infty -a \exp -ax \, dx$ en el intervalo $a \in [0,10]$

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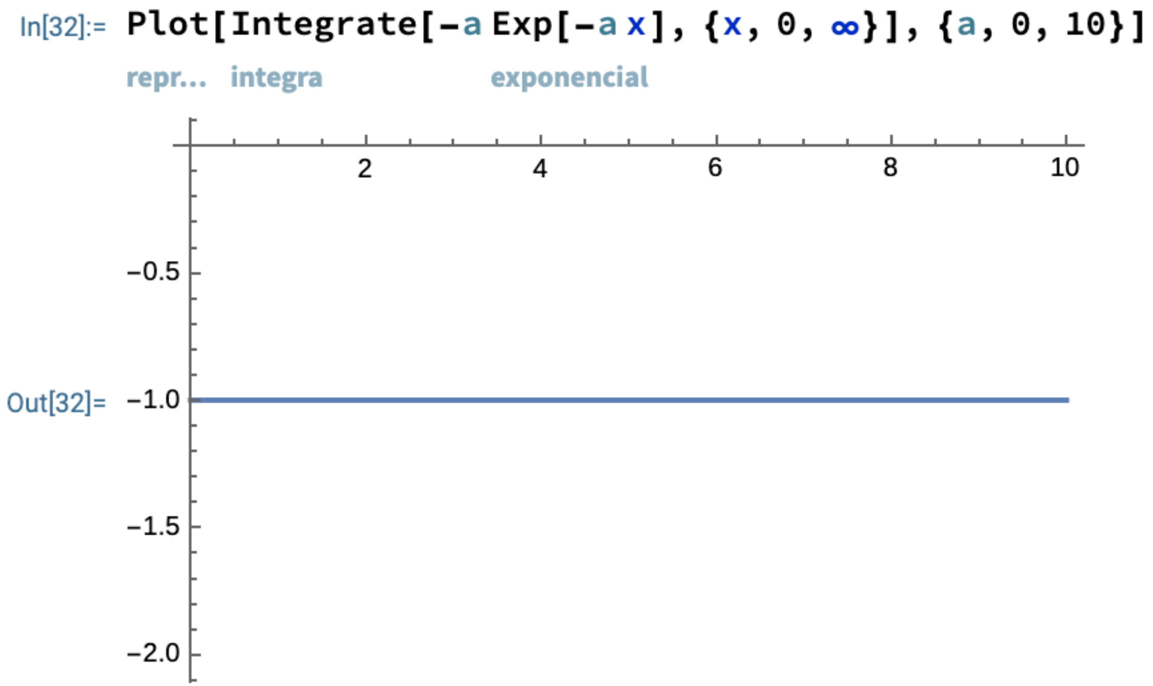
- EJERCICIO
  - REPRESENTAR LA FUNCIÓN $F(a) = \int_0^\infty -a \exp -ax \, dx$ en el intervalo $a \in [0,10]$



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- EJERCICIO
  - REPRESENTAR LA FUNCIÓN $F(a) = \int_0^{\infty} -a \exp -ax \, dx$ en el intervalo $a \in [-10,10]$

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- EJERCICIO
  - REPRESENTAR LA FUNCIÓN $F(a) = \int_0^{\infty} -a \exp -ax \, dx$ en el intervalo $a \in [-10,10]$

```
In[33]:= Plot[Integrate[-a Exp[-a x], {x, 0, ∞}], {a, -10, 10}]
         repr...  integra          exponencial
```

- Integrate : Integral of 9.99959 $e^{9.99959 x}$ does not converge on {0, ∞}.
- NIntegrate : The integrand 9.99959 $e^{9.99959 x}$ has evaluated to Overflow, Indeterminate, or Infinity for all sampling points in the region with boundaries {{0., 5.23043×10^7}}.
- NIntegrate : The integrand 9.99959 $e^{9.99959 x}$ has evaluated to Overflow, Indeterminate, or Infinity for all sampling points in the region with boundaries {{0., 5.23043×10^7}}.
- NIntegrate [Copy to clipboard.] 1828$^{9.99959 x}$ has evaluated to Overflow, Indeterminate, or Infinity for all sampling points in the region with boundaries {{0., 5.23043×10^7}}.
- General : F… inumri will be suppressed during this calculation.
- Integrate : Integral of 9.59143 $e^{9.59143 x}$ does not converge on {0, ∞}.
- Integrate : Integral of 9.18326 $e^{9.18326 x}$ does not converge on {0, ∞}.
- General : Further output of Integrate::idiv will be suppressed during this calculation.

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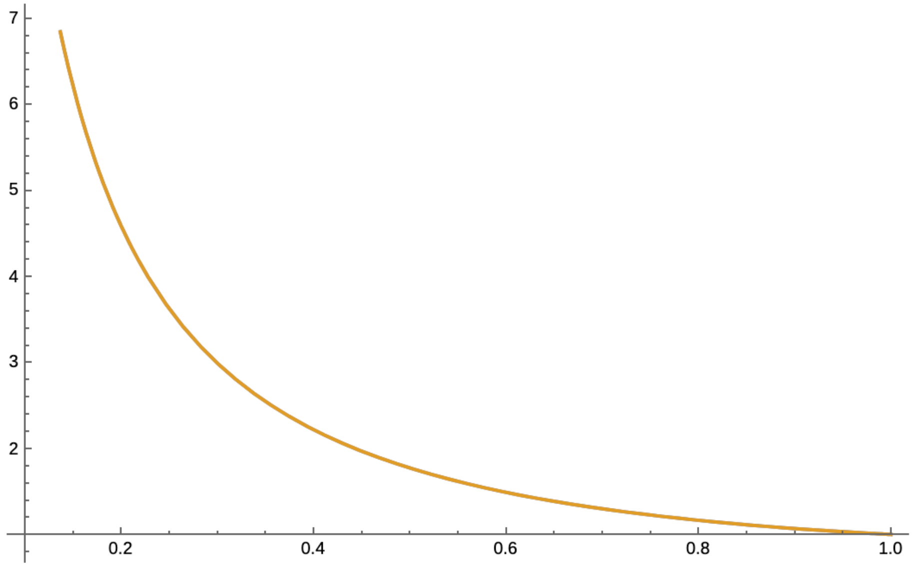
- EJERCICIO
  - REPRESENTAR LA FUNCIÓN GAMMA DE EULER
  - $\Gamma(p) = \int_0^{\infty} x^{p-1} \exp -x \ dx$ en el intervalo $\mathrm{p} \in [0.1,1]$

WOLFRAM CLOUD

- EJERCICIO
  - REPRESENTAR LA FUNCIÓN GAMMA DE EULER
  - $\Gamma(p) = \int_0^{\infty} x^{p-1} \exp -x \; dx$ en el intervalo $p \in [0.1,1]$

```
In[41]:= Plot[{NIntegrate[Exp[-x] x^(p - 1), {x, 0, ∞}], Gamma[p]}, {p, 0.1, 1}]
         repre...  integra numé...  exponencial                        gamma de Euler
```

Out[41]=



WOLFRAM CLOUD

- EJERCICIO
  - COMPROBAR LA VALIDEZ DE LA REGLA DE LEIBNIZ PARA LA DERIVACIÓN BAJO EL SIGNO DE LA INTEGRAL EN EL SIGUIENTE CASO
  - $\int_0^1 \sin(a\,x)dx$

Regla de Leibniz

$$\frac{d}{dx}\int_a^b f(x,t)\,dt = \int_a^b \frac{\partial f(x,t)}{\partial x}dt$$

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- EJERCICIO
  - COMPROBAR LA VALID
    EL SIGNO DE LA INTEG
  - $\int_0^1 \sin(a\,x)dx$

```
In[14]:= D[Integrate[Sin[a x], {x, 0, 1}], a]
         ··· integra      seno

Out[14]= -(1 - Cos[a])/a^2 + Sin[a]/a

In[15]:= Integrate[D[Sin[a x], a], {x, 0, 1}]
         integra      ··· seno

Out[15]= (-1 + Cos[a] + a Sin[a])/a^2

In[16]:= D[Integrate[Sin[a x], {x, 0, 1}], a] - Integrate[D[Sin[a x], a], {x, 0, 1}]
         ··· integra      seno                    integra      ··· seno

Out[16]= -(1 - Cos[a])/a^2 + Sin[a]/a - (-1 + Cos[a] + a Sin[a])/a^2

In[17]:= Simplify[-(1 - Cos[a])/a^2 + Sin[a]/a - (-1 + Cos[a] + a Sin[a])/a^2]
                           coseno      seno             coseno     seno
         simplifica

Out[17]= 0
```

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- EJERCICIO
  - COMPROBAR LA VALIDEZ DE LA REGLA DE LEIBNIZ PARA LA DERIVACIÓN BAJO EL SIGNO DE LA INTEGRAL EN EL SIGUIENTE CASO
  - $\int_0^t \sin(a\,x)dx$

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- EJERCICIO
  - COMPROBAR LA VAL
    EL SIGNO DE LA INTE
  - $\int_0^t \sin(a\,x)dx$

```
In[18]:= D[Integrate[(Sin[x t]), {x, 0, t}], t]
         ··· integra      seno
```

$$\text{Out[18]=}\ -\frac{1-\text{Cos}[t^2]}{t^2}+2\,\text{Sin}[t^2]$$

```
In[19]:= Integrate[D[(Sin[x t]), t], {x, 0, t}]
         integra      d... seno
```

$$\text{Out[19]=}\ \frac{-1+\text{Cos}[t^2]}{t^2}+\text{Sin}[t^2]$$

```
In[20]:= D[Integrate[(Sin[x t]), {x, 0, t}], t] - Integrate[D[(Sin[x t]), t], {x, 0, t}]
         ··· integra      seno                   integra      d... seno
```

$$\text{Out[20]=}\ -\frac{1-\text{Cos}[t^2]}{t^2}-\frac{-1+\text{Cos}[t^2]}{t^2}+\text{Sin}[t^2]$$

$$\text{In[21]:=}\ \text{Simplify}\left[-\frac{1-\underset{\text{coseno}}{\text{Cos}}[t^2]}{t^2}-\frac{-1+\underset{\text{coseno}}{\text{Cos}}[t^2]}{t^2}+\underset{\text{seno}}{\text{Sin}}[t^2]\right]$$

simplifica

$$\text{Out[21]=}\ \text{Sin}[t^2]$$

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- EJERCICIO
  - COMPROBAR LA VAL
    EL SIGNO DE LA INTE
  - $\int_0^t \sin(a\ x)dx$

In[18]:=
```
D[Integrate[(Sin[x t]), {x, 0, t}], t]
```
··· integra seno

Out[18]= $-\frac{1-\text{Cos}[t^2]}{t^2}+2\,\text{Sin}[t^2]$

In[19]:=
```
Integrate[D[(Sin[x t]), t], {x, 0, t}]
```
integra d... seno

Out[19]= $\frac{-1+\text{Cos}[t^2]}{t^2}+\text{Sin}[t^2]$

In[20]:=
```
D[Integrate[(Sin[x t]), {x, 0, t}], t] - Integrate[D[(Sin[x t]), t], {x, 0, t}]
```
··· integra seno integra d... seno

Out[20]= $-\frac{1-\text{Cos}[t^2]}{t^2}-\frac{-1+\text{Cos}[t^2]}{t^2}+\text{Sin}[t^2]$

In[21]:= $\text{Simplify}\left[-\frac{1-\text{Cos}[t^2]}{t^2}-\frac{-1+\text{Cos}[t^2]}{t^2}+\text{Sin}[t^2]\right]$

coseno coseno simplifica seno

Out[21]= $\text{Sin}[t^2]$

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```
In[18]:= D[Integrate[(Sin[x t]), {x, 0, t}], t]
```

Forma General

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f\left(x,t\right)dt = \int_{a(x)}^{b(x)} \frac{\partial f\left(x,t\right)}{\partial x}dt + f\left(x, b\left(x\right)\right)\frac{d\,b\left(x\right)}{dx} - f\left(x, a\left(x\right)\right)\frac{d\,a\left(x\right)}{dx}$$

```
In[20]:= D[Integrate[(Sin[x t]), {x, 0, t}], t] - Integrate[D[(Sin[x t]), t], {x, 0, t}]
         ··· integra        seno                    integra      d... seno
```

$$\text{Out[20]=} -\frac{1-\text{Cos}[t^2]}{t^2} - \frac{-1+\text{Cos}[t^2]}{t^2} + \text{Sin}[t^2]$$

$$\text{In[21]:=} \ \text{Simplify}\left[-\frac{1-\text{Cos}[t^2]}{t^2} - \frac{-1+\text{Cos}[t^2]}{t^2} + \text{Sin}[t^2]\right]$$

coseno coseno

simplifica seno

$$\text{Out[21]=} \ \text{Sin}[t^2]$$

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- EJERCICIO
  - COMPROBAR LA
    EL SIGNO DE LA I
  - $\int_0^t \sin(a\,x)dx$

```
In[18]:= D[Integrate[(Sin[x t]), {x, 0, t}], t]
         ··· integra      seno
```

$$\text{Out[18]=}\ -\frac{1-\mathrm{Cos}[t^2]}{t^2}+2\,\mathrm{Sin}[t^2]$$

```
In[22]:= Integrate[D[(Sin[x t]), t], {x, 0, t}] + Sin[t t] D[t, t]
         integra   d... seno                    seno     deriva
```

$$\text{Out[22]=}\ \frac{-1+\mathrm{Cos}[t^2]}{t^2}+2\,\mathrm{Sin}[t^2]$$

```
In[23]:= D[Integrate[(Sin[x t]), {x, 0, t}], t] - (Integrate[D[(Sin[x t]), t], {x, 0, t}] + Sin[t t] D[t, t])
         ··· integra      seno                     integra   d... seno                    seno     deriva
```

$$\text{Out[23]=}\ -\frac{1-\mathrm{Cos}[t^2]}{t^2}-\frac{-1+\mathrm{Cos}[t^2]}{t^2}$$

$$\text{In[24]:=}\ \mathrm{Simplify}\left[-\frac{1-\mathrm{Cos}[t^2]}{t^2}-\frac{-1+\mathrm{Cos}[t^2]}{t^2}\right]$$

simplifica · coseno · coseno

```
Out[24]= 0
```

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- EJERCICIO
  - COMPROBAR LA
    EL SIGNO DE LA I
  - $\int_0^t \sin(a\,x)dx$


```
In[18]:= D[Integrate[(Sin[x t]), {x, 0, t}], t]
         ··· integra      seno
Out[18]= -(1 - Cos[t^2])/t^2 + 2 Sin[t^2]

In[22]:= Integrate[D[(Sin[x t]), t], {x, 0, t}] + Sin[t t] D[t, t]
         integra   d... seno                      seno     deriva
Out[22]= (-1 + Cos[t^2])/t^2 + 2 Sin[t^2]

In[23]:= D[Integrate[(Sin[x t]), {x, 0, t}], t] - (Integrate[D[(Sin[x t]), t], {x, 0, t}] + Sin[t t] D[t, t])
         ··· integra      seno                      integra   d... seno                      seno     deriva
Out[23]= -(1 - Cos[t^2])/t^2 - (-1 + Cos[t^2])/t^2

In[24]:= Simplify[-(1 - Cos[t^2])/t^2 - (-1 + Cos[t^2])/t^2]
                      coseno              coseno
         simplifica
Out[24]= 0
```

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- EJERCICIO
  - COMPROBAR LA EL SIGNO DE LA I
  - $\int_0^t \sin(a\,x)dx$

In[18]:=
```
D[Integrate[(Sin[x t]), {x, 0, t}], t]
```
… integra seno

Out[18]= $-\frac{1-\text{Cos}[t^2]}{t^2}+2\,\text{Sin}[t^2]$

In[22]:=
```
Integrate[D[(Sin[x t]), t], {x, 0, t}] + Sin[t t] D[t, t]
```
integra d… seno seno deriva

Out[22]= $\frac{-1+\text{Cos}[t^2]}{t^2}+2\,\text{Sin}[t^2]$

In[23]:=
```
D[Integrate[(Sin[x t]), {x, 0, t}], t] - (Integrate[D[(Sin[x t]), t], {x, 0, t}] + Sin[t t] D[t, t])
```
… integra seno integra d… seno seno deriva

Out[23]= $-\frac{1-\text{Cos}[t^2]}{t^2}-\frac{-1+\text{Cos}[t^2]}{t^2}$

In[24]:= $\text{Simplify}\left[-\frac{1-\text{Cos}[t^2]}{t^2}-\frac{-1+\text{Cos}[t^2]}{t^2}\right]$

coseno coseno

simplifica

Out[24]= 0