HANDS ON 02 – CURVAS

COMPLEMENTOS DE MATEMÁTICAS

CURSO 2023-2024

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WOLFRAM CLOUD

- FUNCIONES ÚTILES – REVISAR DOCUMENTACIÓN

ParametricPlot[$\{f_x, f_y\}$, $\{u, u_{min}, u_{max}\}$]
generates a parametric plot of a curve with x and y coordinates f_x and f_y as a function of u.

ParametricPlot3D[$\{f_x, f_y, f_z\}$, $\{u, u_{min}, u_{max}\}$]
produces a three-dimensional space curve parametrized by a variable u which runs from u_{min} to u_{max}.

ArcLength[reg]
gives the length of the one-dimensional region reg.

https://reference.wolfram.com/

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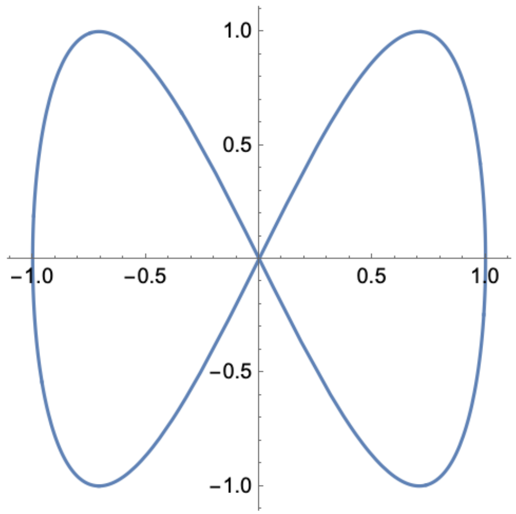
- EJEMPLOS

ParametricPlot[$\{f_x, f_y\}$, $\{u, u_{min}, u_{max}\}$]
generates a parametric plot of a curve with x and y coordinates f_x and f_y as a function of u.

Plot a parametric curve:

In[1]:= **ParametricPlot[{Sin[u], Sin[2 u]}, {u, 0, 2 Pi}]**

Out[1]=



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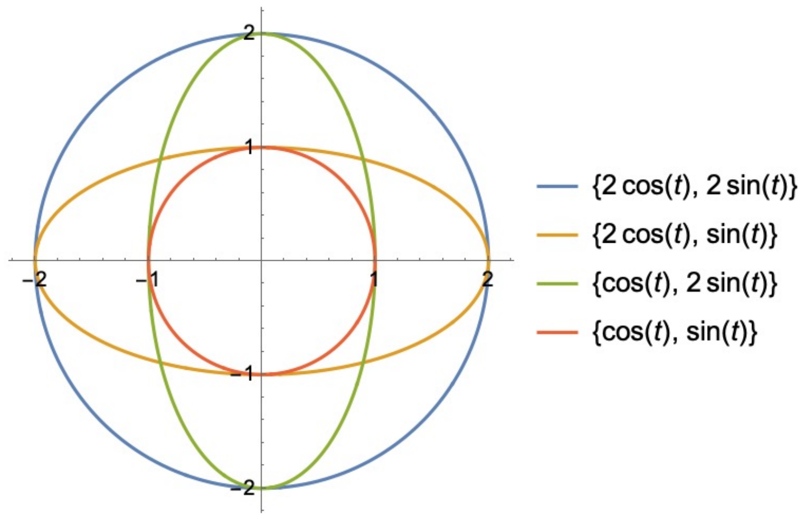
generates a parametric plot of a curve with x and y coordinates f_x and f_y as a function of u.

- EJEMPLOS

Plot several parametric curves with a legend:

```
In[1]:= ParametricPlot[
  {{2 Cos[t], 2 Sin[t]}, {2 Cos[t], Sin[t]}, {Cos[t], 2 Sin[t]}, {Cos[t], Sin[t]}},
  {t, 0, 2 Pi}, PlotLegends → "Expressions"]
```

Out[1]=



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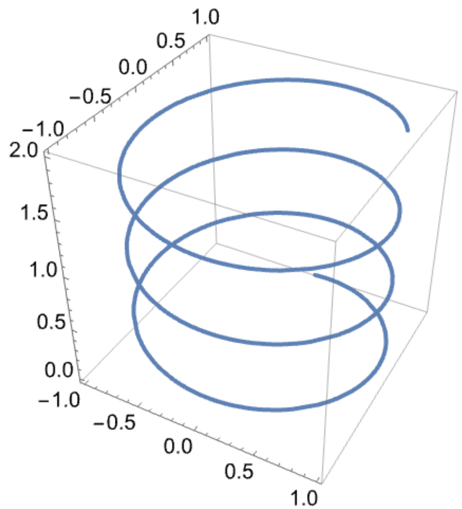
ParametricPlot3D [$\{f_x, f_y, f_z\}$, $\{u, u_{min}, u_{max}\}$]
produces a three-dimensional space curve parametrized by a variable u which runs from u_{min} to u_{max}.

Plot a parametric space curve:

In[1]:= **ParametricPlot3D[{Sin[u], Cos[u], u / 10}, {u, 0, 20}]**

Out[1]=



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ArcLength [*reg*]
gives the length of the one-dimensional region *reg*.

Circumference of a parameterized unit circle:

In[1]:= **ArcLength[{Sin[*θ*], Cos[*θ*]}, {*θ*, 0, 2 Pi}]**

Out[1]= 2π

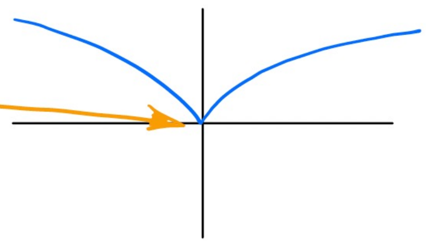
EJEMPLO 1

Ejemplo de curva <u>no</u> regular

$\bar{r}(t) = (t^3, t^2)$

$\bar{r}'(t) = (3t^2, 2t) \Rightarrow \bar{r}'(0) = (0,0)$ X

$\left.\begin{array}{l} x = t^3 \\ y = t^2 \end{array}\right\} \; y = x^{2/3}$



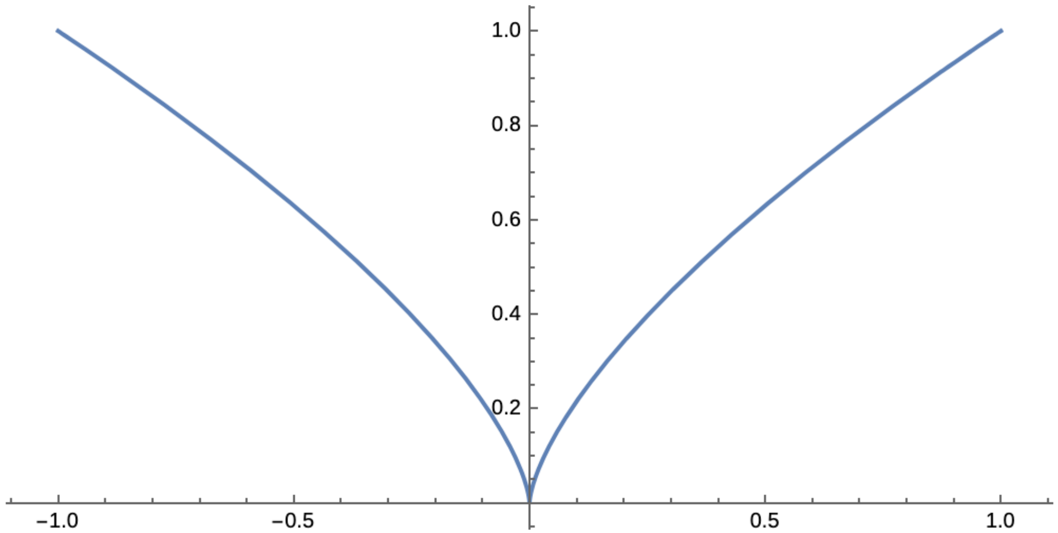
Que la curva sea regular está relacionado con que $\exists$ la recta tangente.

EJEMPLO 1

In[10]:= `ParametricPlot[ {t^3, t^2}, {t, -1, 1}]`

gráfico paramétrico

Out[10]=



EJEMPLO 2

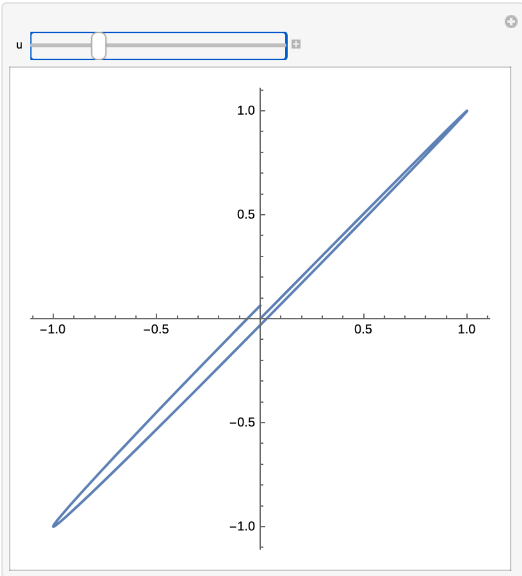
- Representar la curva $r(t) = (\sin(t)\,, \sin(nt))$ con $t \in [0{,}2\pi]$ y $n \in [0{,}4]$ (usar manipulate)

EJEMPLO 2

- Representar la curva $r(t) = (\sin(t)\,, \sin(nt))$ con $t \in [0,2\pi]$ y $n \in [0,4]$

In[16]:=
```
Manipulate[ParametricPlot[ {{Sin[t], Sin[u t]}}, {t, 0, 2 π}], {u, 0, 4}]
manipula   gráfico paramétrico   seno   seno
```

Out[16]=



EJEMPLO 3

Ejemplo: Hallar la recta tangente a la curva C

$$C : \bar{r}(t) = (\cos(t), \sin(t), t) \quad , \quad t \in [0, 2\pi] \text{ en } t = \frac{\pi}{2}$$

$$\bar{r}(t) = (\cos t, \sin t, t)$$
$$\bar{r}(\pi/2) = (0, 1, \pi/2)$$
$$\bar{r}'(t) = (-\sin t, \cos t, 1)$$
$$\bar{r}'(\pi/2) = (-1, 0, 1)$$

De modo que $\bar{r}_t(t) = \bar{r}'(t_0)(t - t_0) + \bar{r}(t_0)$

$$\bar{r}_t(t) = (-1, 0, 1)(t - \pi/2) + (0, 1, \pi/2) = (\pi/2 - t, 1, t)$$

EJEMPLO 3

Ejemplo: Hallar la recta tangente a la curva C

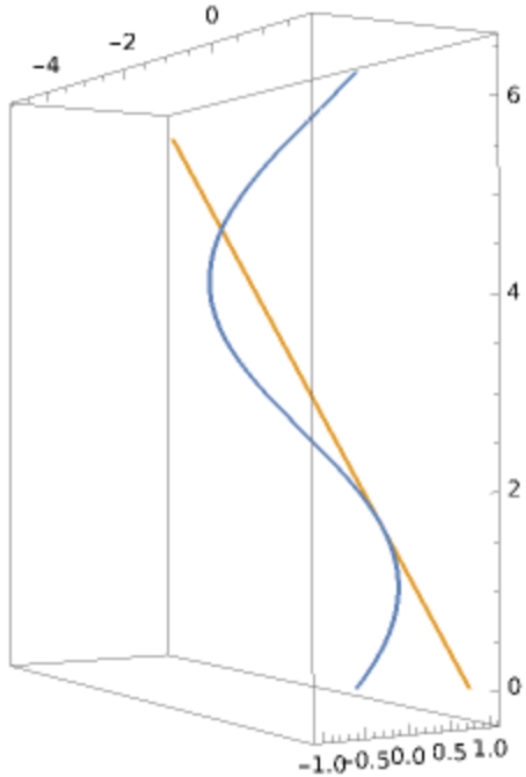
$$C : \vec{r}(t) = (\cos(t), \sin(t), t) \quad , \quad t \in [0, 2\pi] \text{ en } t = \frac{\pi}{2}$$

In[3]:=
```
D[{Cos[t], Sin[t], t}, t] /. t → π / 2
```
d... coseno seno

Out[3]=
```
{-1, 0, 1}
```

In[8]:=
```
ParametricPlot3D[{{Cos[t], Sin[t], t}, {π / 2 - t, 1, t}}, {t, 0, 2 π}]
```
gráfico paramétrico 3D coseno seno

Out[8]=



EJEMPLO 4

EJERCICIO

$\bar{r}(t) = (t, \sin t, \cos t) \quad 0 \leq t \leq 2\pi$. Calcular la longitud de la curva y reparametrizar con la longitud de arco.

$$\bar{r}'(t) = (1, \cos t, -\sin t) \longrightarrow \|\bar{r}'(t)\| = \sqrt{1 + \cos^2 t + \sin^2 t} = \sqrt{2}$$

entonces

$$s(t) = \int_0^t \sqrt{2}\, du = \sqrt{2}\, t \longrightarrow t = \frac{\sqrt{2}}{2} s$$

$$L = \int_0^{2\pi} \sqrt{2}\, du = s(L) = 2\sqrt{2}\pi \quad // \quad \bar{r}(s) = \left(\frac{\sqrt{2}}{2} s, \sin\left(\frac{\sqrt{2}}{2} s\right), \cos\left(\frac{\sqrt{2}}{2} s\right) \right)$$

EJEMPLO 4

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In[20]:= `ArcLength[{t, Sin[t], Cos[t]}, {t, 0, 2 π}]`

longitud de arco seno coseno

Out[20]= $2\sqrt{2}\pi$

EJERCICIO

1 Calcular los vectores del triedro de Frenet, la curvatura y la torsión de la curva $\bar{r}(t) = (1+2\sin t,\ 2\cos t,\ 4\cos t + 8\cos t \cdot \sin t)$ con $t = [0, 2\pi]$ en $(3,0,0)$ (Basado en VII.3)

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ArcLength [reg]
gives the length of the one-dimensional region reg.

FrenetSerretSystem [$\{x_1, \ldots, x_n\}$, t]
gives the generalized curvatures and Frenet–Serret basis for the parametric curve $x_i[t]$.

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- EJEMPLOS

The curvature, tangent, and normal for a circle in two dimensions:

```
In[1]:= FrenetSerretSystem[{Cos[t], Sin[t]}, t]

Out[1]= {{1}, {{-Sin[t], Cos[t]}, {-Cos[t], -Sin[t]}}}
```

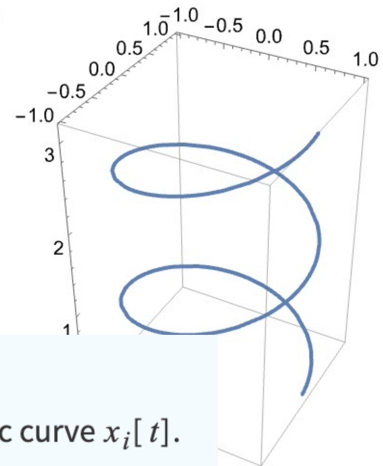
The curvature, torsion, and associated basis for a helix expressed in cylindrical coordinat

```
In[1]:= FrenetSerretSystem[{1, t, t/4}, t, "Cylindrical"]
```

Out[1]= $\left\{\left\{\frac{16}{17}, \frac{4}{17}\right\}, \left\{\left\{0, \frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}}\right\}, \{-1, 0, 0\}, \left\{0, -\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}}\right\}\right\}\right\}$

```
In[2]:= ParametricPlot3D[
          CoordinateTransform["Cylindrical" → "Cartesian", {1, t, t/4}] // Evaluate,
          {t, 0,  4 Pi}, PlotStyle → Thick]
```

Out[2]=



FrenetSerretSystem [$\{x_1, \ldots, x_n\}$, t]

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