Here are five important factorizations, with the standard choice of letters (usually A) for the original product matrix and then for its factors. This book will explain all five.

$$\boxed{\boldsymbol{A = LU} \qquad \boldsymbol{A = QR} \qquad \boldsymbol{S = Q\Lambda Q^{\mathrm{T}}} \qquad \boldsymbol{A = X\Lambda X^{-1}} \qquad \boldsymbol{A = U\Sigma V^{\mathrm{T}}}}$$

At this point we simply list key words and properties for each of these factorizations.

1 $A = LU$ comes from **elimination**. Combinations of rows take A to U and U back to A. The matrix L is lower triangular and U is upper triangular as in equation (4).

2 $A = QR$ comes from **orthogonalizing** the columns $\boldsymbol{a}_1$ to $\boldsymbol{a}_n$ as in "Gram-Schmidt". Q has orthonormal columns ($Q^{\mathrm{T}}Q = I$) and R is upper triangular.

3 $S = Q\Lambda Q^{\mathrm{T}}$ comes from the **eigenvalues** $\lambda_1, \ldots, \lambda_n$ of a symmetric matrix $S = S^{\mathrm{T}}$. Eigenvalues on the diagonal of Λ. **Orthonormal eigenvectors** in the columns of Q.

4 $A = X\Lambda X^{-1}$ is **diagonalization** when A is n by n with n independent eigenvectors. *Eigenvalues* of A on the diagonal of Λ. *Eigenvectors* of A in the columns of X.

5 $A = U\Sigma V^{\mathrm{T}}$ is the **Singular Value Decomposition** of any matrix A (square or not). **Singular values** $\sigma_1, \ldots, \sigma_r$ in Σ. Orthonormal **singular vectors** in U and V.