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EME165 - Heat Transfer
24 Nov. 2023
Final Project

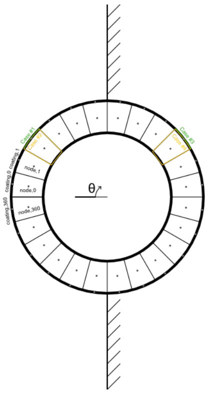

Figure 1: Nodal Schematic with numbering and breakdown of the special cases (in green for coating and yellow for cylinder), the number of nodes is simplified down from actual number used for visibility.

Knowns

- $r_i = 15\text{mm}$
- $t = 3\text{mm}$
- $k = 80 \frac{W}{mK}$
- $Q(\theta) = 100 \cdot 10^4 \cos(\theta)$
- $T_\infty = 40°C$
- $h_\infty = 25 \frac{W}{m^2 K}$
- $\alpha = 0.95$
- $\varepsilon = 0.75$
- $T_{\text{surr}} = 40°C$
- $R'_{\text{tc}} = 10^{-4} \frac{mK}{W}$
- $T_{\text{fluid}} = 600°C$
- $h_{\text{fluid}} = 1000 \frac{W}{m^2 K}$

Find

- $T_{\text{coating}}(\theta)$
- $T_{\text{node}}(\theta)$

Assumptions

1) Steady State
2) No conduction in the coating
3) No radial conduction on the cylinder
4) Perfect insulation on the 1/2 of the cylinder
5) Gray surface to long wave radiation

General Equations:

$$r_o = r_i + t = 18 \text{ mm}$$

$$r_{\text{avg}} = \frac{r_o + r_i}{2}$$

$$\Delta\theta = \frac{2\pi}{N}$$

$$A_s = 2\pi r L \cdot \frac{\Delta\theta}{2\pi} = r\Delta\theta L$$

$$A_{\text{c/s}} = (r_0 - r_i)L$$

$$q'_{\text{tc}} = \frac{T_{\text{coating}} - T_{\text{node}}}{R'_{\text{tc}}}$$

$$q'_{\text{conv},o} = h_\infty r_o \Delta\theta \left(T_{\text{coating}} - T_\infty\right)$$

$$q'_{\text{conv},i} = h_{\text{fluid}} r_i \Delta\theta (T_{\text{node}} - T_{\text{fluid}})$$

$$q'_{\text{rad, solar}} = \alpha r_o \Delta\theta Q(\theta)$$

$$q'_{\text{rad, net}} = \varepsilon\sigma\left(T^4_{\text{coating}} - T^4_{\text{surr}}\right) r_o \Delta\theta$$

$$q'_{\text{cond, node+1}} = \frac{k(r_o - r_i)}{\Delta\theta} \cdot (T_{\text{node+1}} - T_{\text{node}})$$

$$q'_{\text{cond, node-1}} = \frac{k(r_o - r_i)}{\Delta\theta} \cdot (T_{\text{node-1}} - T_{\text{node}})$$

Let:

$$L_{\text{arc},o} = r_o \Delta\theta$$

$$R_{\text{inv}} = \frac{1}{R'_{\text{tc}}}$$

$$m_1 = \frac{1}{R_{\text{inv}} + h_\infty L_{\text{arc},o}}$$

$$m_2 = \alpha L_{\text{arc},o}$$

$$m_3 = h_\infty L_{\text{arc},o} T_\infty$$

$$m_4 = \varepsilon\sigma L_{\text{arc},o}$$

$$m_5 = {T_{\text{surr}}}^4$$

$$n_1 = \frac{k(r_o - r_i)}{\Delta\theta \cdot r_{\text{avg}}}$$

$$n_2 = h_{\text{fluid}} r_i \Delta\theta$$

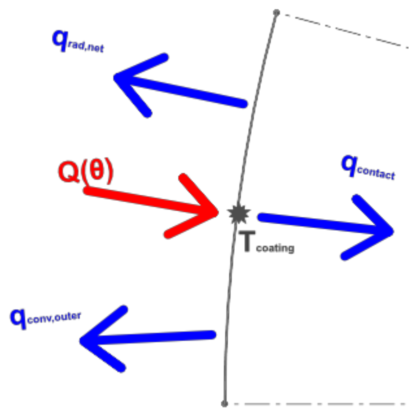

Figure 2: Control Surface of exposed coating

$$\dot{E}_{\text{in}} + \dot{E}_{\text{gen}} = \dot{E}_{\text{out}} + \frac{dE_{\text{st}}}{dt}$$

$$\dot{E}_{\text{gen}} = 0 \quad ; \quad \frac{dE_{\text{st}}}{dt} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$q'_{\text{rad, solar}} = q'_{\text{rad,net}} + q'_{\text{conv},o} + q'_{\text{contact}}$$

$$0 = q'_{\text{rad, solar}} - q'_{\text{rad, net}} - q'_{\text{conv},o} - q'_{\text{contact}}$$

$$0 = \alpha L_{\text{arc},o} Q(\theta) - \varepsilon\sigma L_{\text{arc},o}\left({T_{\text{coating}}}^4 - {T_{\text{surr}}}^4\right) - h_\infty L_{\text{arc},o}\left(T_{\text{coating}} - T_{\text{infinity}}\right) - \frac{T_{\text{coating}} - T_{\text{node}}}{R'_{\text{tc}}}$$

$$0 = \alpha L_{\text{arc},o} Q(\theta) - \varepsilon\sigma L_{\text{arc},o}\left({T_{\text{coating}}}^4 - {T_{\text{surr}}}^4\right) - h_\infty L_{\text{arc},o} T_{\text{coating}} + h_\infty L_{\text{arc},o} T_{\text{infinity}} - \frac{T_{\text{coating}}}{R'_{\text{tc}}} + \frac{T_{\text{node}}}{R'_{\text{tc}}}$$

$$h_\infty L_{\text{arc},o} T_{\text{coating}} + \frac{T_{\text{coating}}}{R'_{\text{tc}}} = \alpha L_{\text{arc},o} Q(\theta) - \varepsilon\sigma L_{\text{arc},o}\left({T_{\text{coating}}}^4 - {T_{\text{surr}}}^4\right) + h_\infty L_{\text{arc},o} T_{\text{infinity}} + \frac{T_{\text{node}}}{R'_{\text{tc}}}$$

$$T_{\text{coating}}\left(R_{\text{inv}} + h_\infty L_{\text{arc},o}\right) = \alpha L_{\text{arc},o} Q(\theta) + \varepsilon\sigma L_{\text{arc},o}\left({T_{\text{surr}}}^4 - {T_{\text{coating}}}^4\right) + h_\infty L_{\text{arc},o} T_{\text{infinity}} + \frac{T_{\text{node}}}{R'_{\text{tc}}}$$

$$T_{\text{coating}} = \frac{1}{R_{\text{inv}} + h_\infty L_{\text{arc},o}}\left[\alpha L_{\text{arc},o} Q(\theta) + \varepsilon\sigma L_{\text{arc},o}\left({T_{\text{surr}}}^4 - {T_{\text{coating}}}^4\right) + h_\infty L_{\text{arc},o} T_{\text{infinity}} + \frac{T_{\text{node}}}{R'_{\text{tc}}}\right]$$

Apply self defined constants:

$$T_{\text{coating}} = m_1\left[m_2 Q(\theta) + R_{\text{inv}} T_{\text{node}} + m_3 + m_4\left(m_5 - {T_{\text{coating}}}^4\right)\right]$$

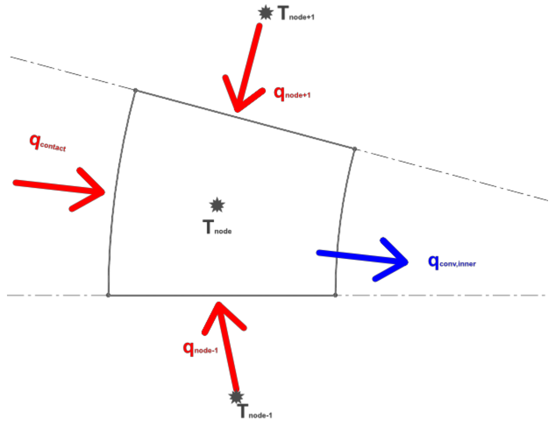

Figure 3: Control Volume of an outer facing node

$$\dot{E}_{\text{in}} + \dot{E}_{\text{gen}} = \dot{E}_{\text{out}} + \frac{dE_{\text{st}}}{dt}$$

$$\dot{E}_{\text{gen}} = 0 \quad ; \quad \frac{dE_{\text{st}}}{dt} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$q'_{\text{tc}} + q'_{\text{cond, node+1}} + q'_{\text{cond, node-1}} = q'_{\text{conv, fluid}}$$

$$0 = q'_{\text{tc}} + q'_{\text{cond, node+1}} + q'_{\text{cond, node-1}} - q'_{\text{conv, fluid}}$$

$$0 = \frac{T_{\text{coating}} - T_{\text{node}}}{R'_{\text{tc}}} + \frac{k(r_o - r_i)}{\Delta\theta \cdot r_{\text{avg}}}(T_{\text{node+1}} - T_{\text{node}}) + \frac{k(r_o - r_i)}{\Delta\theta \cdot r_{\text{avg}}}(T_{\text{node-1}} - T_{\text{node}}) - h_{\text{fluid}} r_i \Delta\theta (T_{\text{node}} - T_{\text{fluid}})$$

Apply self defined constants:

$$0 = R_{\text{inv}} T_{\text{coating}} - R_{\text{inv}} T_{\text{node}} + n_1 T_{\text{node+1}} - n_1 T_{\text{node}} + n_1 T_{\text{node-1}} - n_1 T_{\text{node}} - n_2 T_{\text{node}} + n_2 T_{\text{fluid}}$$

$$R_{\text{inv}} T_{\text{node}} + 2n_1 T_{\text{node}} + n_2 T_{\text{node}} = R_{\text{inv}} T_{\text{coating}} + n_1 (T_{\text{node+1}} + T_{\text{node-1}}) + n_2 T_{\text{fluid}}$$

$$T_{\text{node}}(R_{\text{inv}} + 2n_1 + n_2) = R_{\text{inv}} T_{\text{coating}} + n_1 (T_{\text{node+1}} + T_{\text{node-1}}) + n_2 T_{\text{fluid}}$$

$$T_{\text{node}} = \frac{1}{R_{\text{inv}} + 2n_1 + n_2} \left[R_{\text{inv}} T_{\text{coating}} + n_1 (T_{\text{node+1}} + T_{\text{node-1}}) + n_2 T_{\text{fluid}} \right]$$

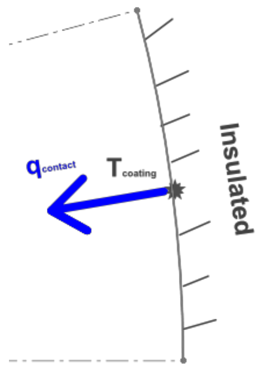

Figure 4: Control Surface of coating on the insulated side

$$\dot{E}_{\text{in}} + \dot{E}_{\text{gen}} = \dot{E}_{\text{out}} + \frac{dE_{\text{st}}}{dt}$$

$$\dot{E}_{\text{in}} = 0 \quad ; \quad \dot{E}_{\text{gen}} = 0 \quad ; \quad \frac{dE_{\text{st}}}{dt} = 0$$

$$0 = \dot{E}_{\text{out}}$$

$$0 = q'_{\text{tc}}$$

$$0 = \frac{T_{\text{coating}} - T_{\text{node}}}{R'_{\text{tc}}}$$

$$0 = T_{\text{coating}} - T_{\text{node}}$$

$$T_{\text{coating}} = T_{\text{node}}$$

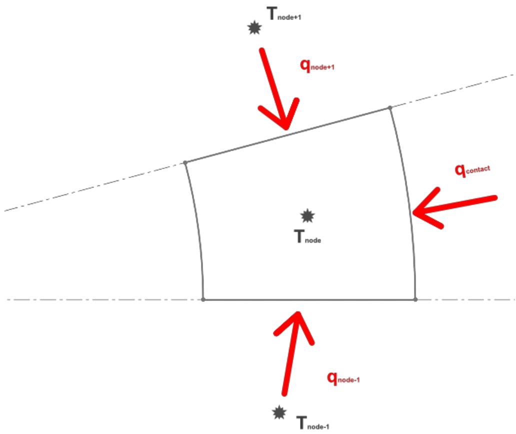

Figure 5: Control Volume of an inner facing node

$$\dot{E}_{\text{in}} + \dot{E}_{\text{gen}} = \dot{E}_{\text{out}} + \frac{dE_{\text{st}}}{dt}$$

$$\dot{E}_{\text{gen}} = 0 \quad ; \quad \frac{dE_{\text{st}}}{dt} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$q'_{\text{tc}} + q'_{\text{cond, node+1}} + q'_{\text{cond, node-1}} = q'_{\text{conv, fluid}}$$

$$0 = q'_{\text{tc}} + q'_{\text{cond, node+1}} + q'_{\text{cond, node-1}} - q'_{\text{conv, fluid}}$$

$$0 = 0 + \frac{k(r_o - r_i)}{\Delta\theta \cdot r_{\text{avg}}}(T_{\text{node+1}} - T_{\text{node}}) + \frac{k(r_o - r_i)}{\Delta\theta \cdot r_{\text{avg}}}(T_{\text{node-1}} - T_{\text{node}}) - h_{\text{fluid}} r_i \Delta\theta (T_{\text{node}} - T_{\text{fluid}})$$

Apply self defined constants:

$$0 = n_1 T_{\text{node+1}} - n_1 T_{\text{node}} + n_1 T_{\text{node-1}} - n_1 T_{\text{node}} - n_2 T_{\text{node}} + n_2 T_{\text{fluid}}$$

$$2n_1 T_{\text{node}} + n_2 T_{\text{node}} = n_1 (T_{\text{node+1}} + T_{\text{node-1}}) + n_2 T_{\text{fluid}}$$

$$T_{\text{node}}(2n_1 + n_2) = n_1 (T_{\text{node+1}} + T_{\text{node-1}}) + n_2 T_{\text{fluid}}$$

$$T_{\text{node}} = \frac{1}{2n_1 + n_2}\left[n_1 (T_{\text{node+1}} + T_{\text{node-1}}) + n_2 T_{\text{fluid}}\right]$$

Final Equations:

For $0 \leq \theta < \frac{\pi}{2}$ **:**

$$T_{\text{coating}} = m_1[m_2 Q(\theta) + R_{\text{inv}} T_{\text{node}} + m_3 + m_4\left(m_5 - T_{\text{coating}}{}^4\right)$$

$$T_{\text{node}} = \frac{1}{R_{\text{inv}} + 2n_1 + n_2}\left[R_{\text{inv}} T_{\text{coating}} + n_1\left(T_{\text{node+1}} + T_{\text{node-1}}\right) + n_2 T_{\text{fluid}}\right]$$

For $\frac{3\pi}{2} \leq \theta < 2\pi$ **:**

$$T_{\text{coating}} = T_{\text{node}}$$

$$T_{\text{node}} = \frac{1}{2n_1 + n_2}\left[n_1\left(T_{\text{node+1}} + T_{\text{node-1}}\right) + n_2 T_{\text{fluid}}\right]$$

In Part A, the emissivity is ignored which means $\varepsilon = 0 \therefore m_2 = 2$, giving the simplified exception of:

$$T_{\text{coating}} = m_1[m_2 Q(\theta) + R_{\text{inv}} T_{\text{node}} + m_3]$$

Results

Part A

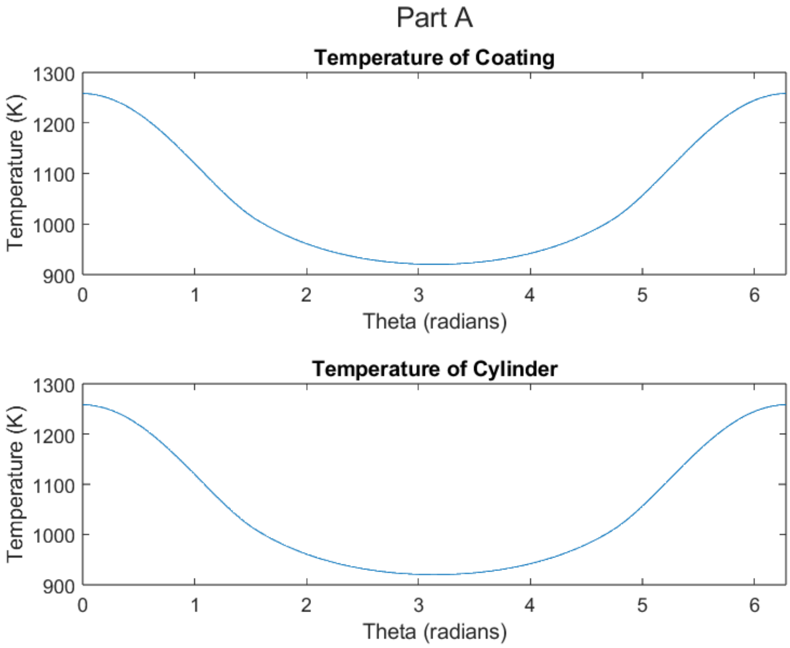

Figure 6: Plot of θ vs $T[K]$, with the temperature of the coating in red and the temperature of the pipe in blue.

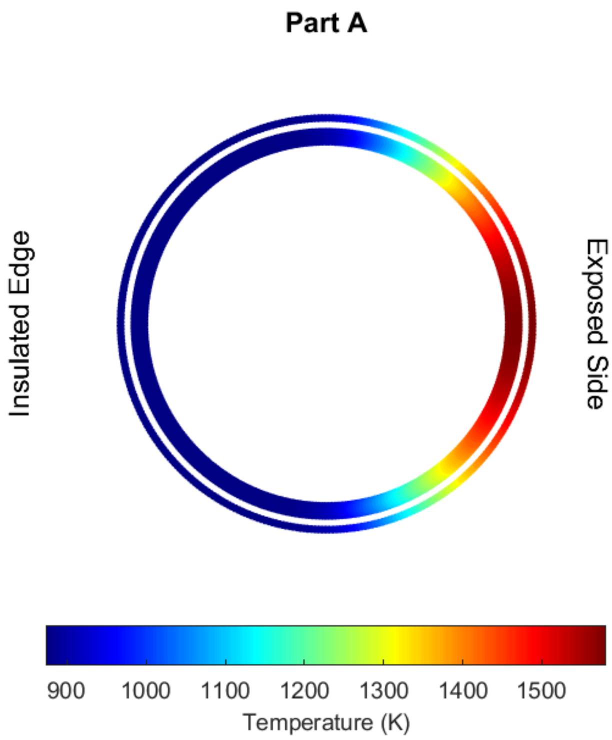

Figure 7: Graph of heat distribution around the pipe and coating

Part B

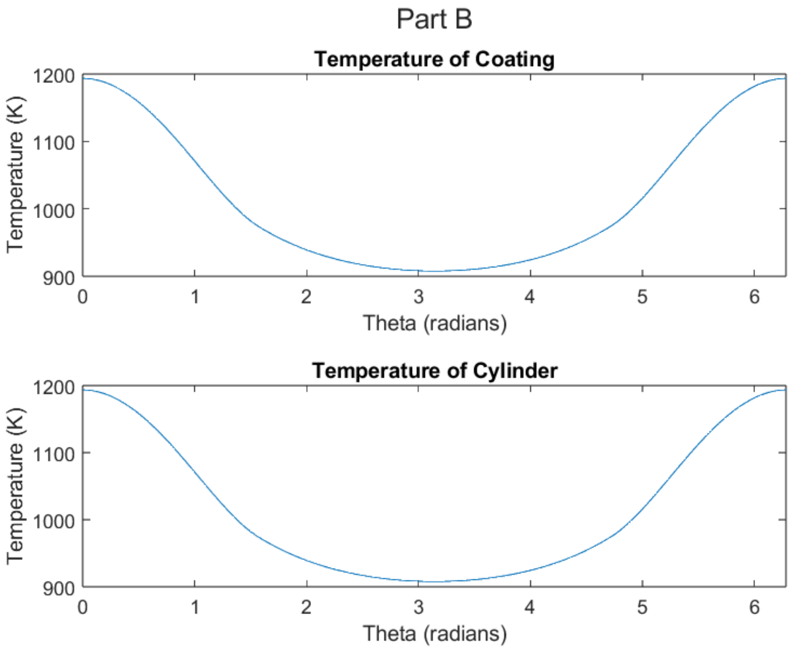

Figure 8: Plot of θ vs $T[K]$, with the temperature of the coating in red and the temperature of the pipe in blue.

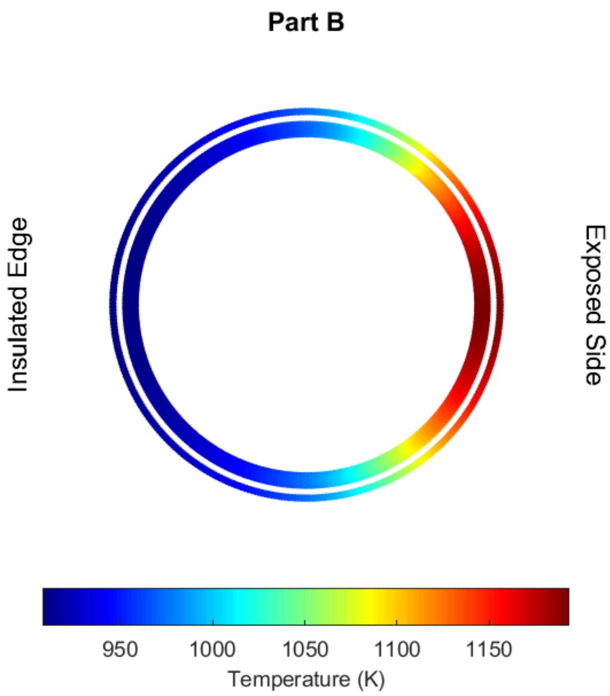

Figure 9: Graph of heat distribution around the pipe and coating

Analysis

Both Parts A and Part B have similar temperature distribution profiles, with Part B having a 50K downshift in temperature. This makes sense because the emissivity was not a function of theta and would just subtract from the heat added by the solar radiation. Both parts reach the temperature of the fluid on the insulated side.