Get Answers Checked

Pen-and-paper exercises - 2D position and orientation

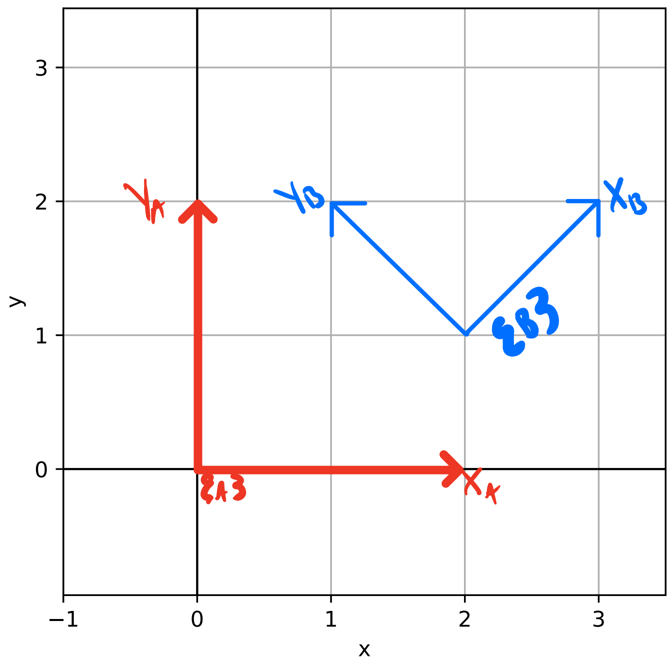
Exercise 1

We have two coordinate frames {A} and {B} defined by the following transformation matrices:

$$T_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.1}$$

$$T_B = \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & 2 \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & 1 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.2}$$

Draw both coordinate frames in the plot below.



$$T_B = \begin{pmatrix} \cos(45) & -\sin(45) & 2 \\ \sin(45) & \cos\theta & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$t_x = 2$

$t_y = 1$

$\theta = 45^\circ$ $\leftarrow$ <u>Rotation</u>!

$\frac{\pi}{4}$

Exercise 2

We start from the default coordinate frame:

$$T_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.1}$$

We will then apply two consecutive transformations, ${}^{A}T_B$ and ${}^{B}T_C$:

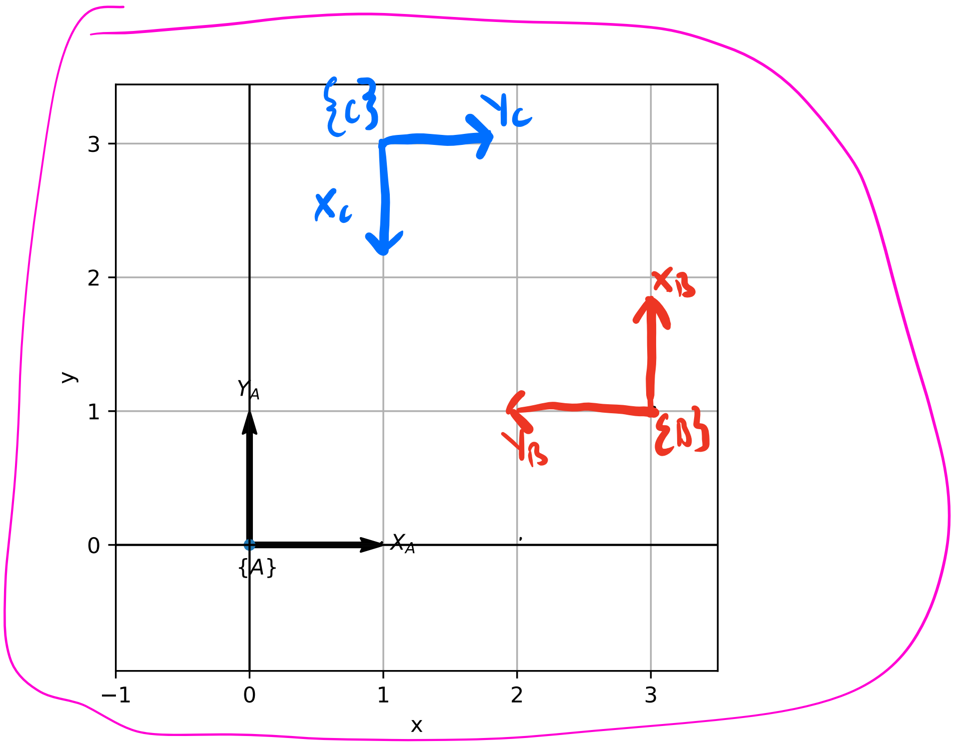
$${}^{A}T_B = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.2}$$

$${}^{B}T_C = \begin{pmatrix} -1 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.3}$$

Draw the coordinate frames {B} and {C} that are defined by:

$$T_B = T_A \cdot {}^{A}T_B \tag{2.4}$$

$$T_C = T_B \cdot {}^{B}T_C \tag{2.5}$$



$T_B \rightarrow 90^\circ$ or $\frac{\pi}{2}$, $t_x = 3$, $t_y = 1$

$T_C \rightarrow 270^\circ$ or $\frac{3\pi}{2}$, $t_x = 1$, $t_y = 3$

$$T_B = T_A \cdot {}^{A}T_B$$

$$T_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad {}^{A}T_B = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Multiplying these transformations

$$T_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 0+0\cdot 1+0\cdot 0, & -1\cdot 1+0\cdot 0+0\cdot 0, & 1\cdot 3+0\cdot 1+0\cdot 1 \\ 0\cdot 0+1\cdot 1+0\cdot 0, & 0\cdot -1+1\cdot 0+0\cdot 0, & 0\cdot 3+1\cdot 1+0\cdot 1 \\ 0\cdot 0+0\cdot 0+1\cdot 0, & 0\cdot -1+0\cdot 0+1\cdot 0, & 0\cdot 3+0\cdot 1, 1\cdot 1 \end{bmatrix}$$

$$T_B = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\cos(\theta) = 0$, $\sin(\theta) = 1$

$\theta = 90°$ or $\frac{\pi}{2}$, $t_x = 3$, $t_y = 1$

$${}^{B}T_C = \begin{pmatrix} -1 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_C = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+0+0, & 0+1+0, & 0-2+3 \\ -1+0+0, & 0+0+0, & 2+0+1 \\ 0+0+0, & 0+0+0, & 0+0+1 \end{bmatrix}$$

$$T_C = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

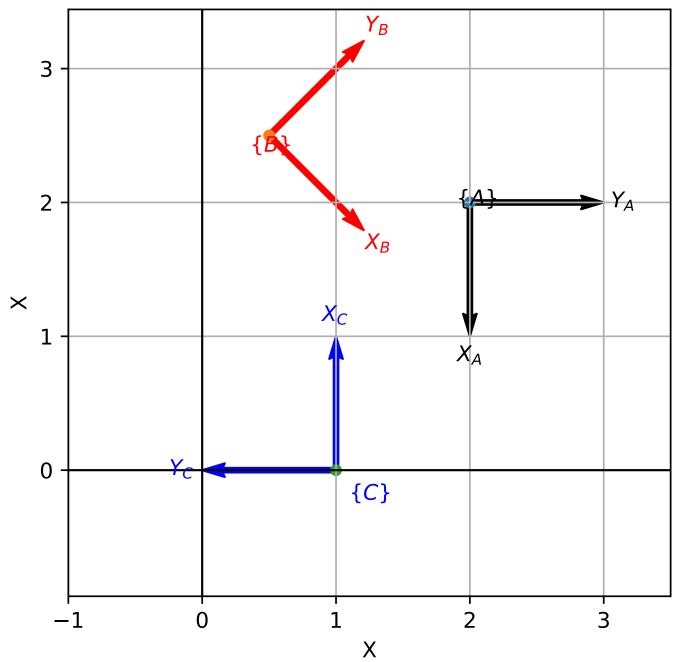
$\cos(\theta) = 0$
$\sin(\theta) = -1$

$t_x = 1$, $t_y = 3$

$\theta = 270°$ or $\frac{3\pi}{2}$

Exercise 3

Given the coordinate frames {A}, {B} and {C}.



$\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}$

Write down the transformation matrix for the three coordinate frames.

$T_A \rightarrow$ $\theta = 270°$ or $\frac{3\pi}{2}$

$t_x = 2, t_y = 2$

$\cos(\theta) = 0$ $\quad \sin(\theta) = -1$

$-\sin(\theta) = 1$

$, T_B$ $\quad \theta = 315°$ or $\frac{7\pi}{4}$

$t_x = .5, \quad t_y = 2.5$

$\cos\theta = \frac{\sqrt{2}}{2}$ $\quad \sin\theta = -\frac{\sqrt{2}}{2}$

$-\sin\theta = \frac{\sqrt{2}}{2}$

$T_c \rightarrow$ $\theta = 90°$ or $\frac{\pi}{2}$

$t_x = 1, t_y = 0$

$\cos\theta = 0$ $\quad \sin\theta = 1$

$-\sin\theta = -1$

$$T_A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}, \quad T_B = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & .5 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2.5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_C = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$