

# Geostatistical Analysis of Machine and Human Coded Data in Space and Time

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Spatial temporal processes are at the heart of many important political questions. How do ideologies of mass publics evolve within and across state boundaries. How does electronic media affect the attitudes and behavior of citizens over time within counties in different states, attitudes and behaviors which through political communication diffuse to and from neighboring counties? How do candidates decide where and when to raise funds ("search for gold") in their constituencies? How do their efforts spawn new contributions over time from locations outside their constituencies? Why and how do civil wars break out in and then spread across governmental jurisdictions dying out in some places but escalating in others?

Political methodology lags behind other disciplines like criminology, meteorology, environmental science, and epidemiology in its analysis of spatial temporal processes. We made real advances in spatial measurement (Monogan and Gill 2015; Cho 2003), causal inference based on geographical designs (Keele and Titiunik 2015) and the study of conflict dynamics (Brandt, et al 2008, 2014). But, many of our spatial analysis ignore or downplay temporal factors; they assess treatment effects at a single point in time. Or, we study spatial processes but in separate or highly aggregated slices of time (Cho and Gimpel 2007). Leading

textbooks on time series in political science and economics (Box Steffensmeier et al. 2014, Enders 2010) say nothing about spatial-temporal processes. Difference in difference causal analysis use two time points to make grand assumptions about time trends in complex political process without any provision for medium and long term dynamics, let alone diffusion across (omitted) units of analysis (Keele and Minozzi 2013). Many conflict studies are macro level investigations in which spatial factors amount to country dyadic exchanges or exchanges between belligerents (Brandt et al. 2008). Subnational conflict studies make a "localization" assumptions—that conflict occurs over time within but not across units of analysis (Silverman 2021).

If they allow for spatial temporal factors in their analysis, political scientists most often place two way fixed effects in their models and, concomitantly, used cluster errors to account for any remaining unit heteroscedasticity. Occasionally, a simple (static) spatial model is used to assess the robustness of the investigator's results. As we explain below, conceptually, two way fixed effects models do not allow for diffusion of the effects of the variables of interest across units and they treat the effect of "time" as identical on all units in each time slice; two way fixed effects set-ups ignore the possibility of individual space-time unit differences in the effects of causal mechanisms.

Some of us have applied and made advances in neighborhood type spatial temporal modeling. These modelers carve up space into discrete units and include in regression models right hand side variables with both spatial lags with *prespecified* connectivity matrices of the dependent variable and also temporal lags of the values of the own unit's past values of the dependent variable. The development of such models for binary dependent variables along with techniques to assess spatial and temporal marginal effects is illustrative (Ward and Gleditsch 2002, Franzese et al. 2015). An innovative application of spatial vector autoregression in the study of subnational conflict also has been applied in international relations (Linke et al. 2012).

With one or two exceptions, what has not been applied and developed in political science,

is the geostatistical approach to spatial temporal analysis. This approach treats the data generating process as continuous in space; it employs a set of techniques for "point process" modeling. Rather prespecifying a connectivity matrix, the geostatistical approach *estimates* the extent of spatial interdependence between units and allows for unit variant temporal evolution of causal mechanisms like diffusion. Out of the geostatistical approach emerges methods for tracking *the distinct effects of causal mechanisms for every individual unit at each point in time*. It is this methodology that has proven so useful recently in other disciplines.<sup>1</sup>

This research note explains a popular geostatistical approach and compares it to the more familiar two way fixed effects and neighborhood methods of modeling spatial temporal processes. We begin by reviewing the ways two familiar methods conceive of spatial temporal processes. The key idea of separability is introduced in this context. Geostatistical modeling based on a stochastic partial differential equation then is introduced. A formal definition of separability and the additional concepts of space-time stationarity and isotropy are discussed briefly here. We then compare the three approaches. We use the modeling of insurgent attacks in Iraq—in particular, Silverman’s recent (2021) two way fixed effects based investigation—as our benchmark. As an added payoff of our analysis, we compare the modeling results for ground truth (army reported SIGACTS), machine coded (ICEWS) and human coded (GED) databases; the rationale for our comparison of the databases is explained elsewhere (Bagozzi et al. 2018; Stundal et al. 2021).

We find....

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<sup>1</sup>Geostatistical modeling is mentioned as a possible approach in one paragraph at the end of the second edition of Ward and Gleditsch (2019). And it has been applied in the form of kriging by Monogan and Gill 2015, Gill 2020, Cho and Gimpel 2007. But these studies are static in character. The spatial-temporal versions of kriging, what is called, co-kriging, to our knowledge, have not been applied by American political methodologists. But see Pavia et al. 2008.

# 1 Geostatistical Modeling Spatial Temporal Processes

Spatial temporal modeling is motivated by theoretical expectations predict and empirical observations which suggest the existence of certain causal mechanisms: as regards space, the possibility of strategic interaction, diffusion, spillover, relocation, and advection and as regards time, persistence, cumulation, and memory. Spatial temporal models aim to capture the *combination* of such mechanisms. The notion that civil conflict in a particular location in one part of country not only spills over to another part of the same or neighboring countries contemporaneously and historically at the same time the conflict persists (is remembered) within the unit in which it originated is illustrative.

Two way fixed effects set ups are among the most common approaches political scientists use to study spatial temporal political processes. Assume we observe  $i = 1 \dots N$  units at  $t = 1 \dots T$  points in time. And we are interested in the effect of a set of covariates,  $X_{kit}$  on variable  $Y_{it}$ . This most simple model is of the form

$$Y_{it} = \alpha_i + \gamma_t + \beta X_{it} + \epsilon_t \quad (1)$$

where  $Y_{it}$  is a  $N \times 1$  vector of dependent variables,  $X_{kit}$  is a  $N \times k$  matrix of covariates,  $\alpha_i$  is the fixed (time invariant) affect of factors at each location  $i$ ,  $\gamma_t$  is the fixed (unit invariant) affect of unknown factors at each respective time point,  $\beta$  is a coefficient to be estimated and  $\epsilon_{it}$  is a  $N \times 1$  error vector which may be heteroscedastic. The fixed effects are separated and added together. There is no provision for interdependence between the values of  $Y_{it}$  of any units, there is no persistence or other kinds of temporal mechanisms governing the evolution of  $Y_{it}$ , and there is no variation in the impact of historical factors across units at each point in time.<sup>2</sup>

A second, familiar model is the so called Spatial Temporal Autoregressive (STAR) Neigh-

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<sup>2</sup>Other simple models are used in political science such as the unit fixed effects model with a lagged outcome variable,  $Y_{it} = \alpha_i + \beta X_{kit} + \rho Y_{i,t-1} + \epsilon_{it}$ . And more complete representations of the effect of history are often employed such as time polynomials. But, again, these alternative set ups ignore the possibility of spatial interdependence and treat the effect of history as unit invariant.

neighborhood model. These are usually expressed as

$$Y_{it} = \rho WY_t + \phi MY_{it} + \beta X_{kit} + \epsilon_{it} \quad (2)$$

As Franzese et al. (2007: 159) explain,  $W$  is the Kroneckert product of a  $T \times T$  identity matrix and an  $N \times N$  prespecified spatial weights matrix ( $I_t \otimes W_N$ ),  $M$  is an  $NT \times NT$  matrix of zeros except for ones on the minor diagonal at coordinates  $(N+1,1)$ ,  $(N+2,2)$  ...  $(NT,NT-N)$ , and  $\phi$  is the unit invariant temporal autoregressive coefficient. Anselin (2006: Section 26.2.1)) argues that this set-up captures diffusion, persistence and the other causal mechanisms enumerated above. The first term on the right represents spatial interdependence while the second term captures unit persistence and related temporal factors. Again, these factors are assumed to be separable and additive.<sup>3</sup>

Neighborhood models may suffer from “inappropriate discretization” (Lindgren and Rue 2015: 3). The prespecified connectivity matrices used in neighborhood models treats spatial dependence as a step function—the same for some subset of units and nonexistent for another subset of units. In reality the spatial dependence of errors at different sites may vary *continuously* in space. The alternative—geostatistical spatial error models—produce *estimates* of both the range of spatial error dependence and of the site specific impact of unobserved factors including measurement errors on the unit of interest.

Geostatistical models analyze point referenced data. These models are based on the idea

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<sup>3</sup>There is a related model that according to Anselin and others is best suited to analyzing spatial connections in the errors. Bagozzi et. al. 2018 and Stundal et al. 2021 argue that conceptually, the spatial error model (SEM) for answering our questions about the validity of human and machine coded event data. SEMs capture model errors for neighboring units that cluster together—“smaller(larger) errors for observation  $i$  ... go together with smaller [larger] errors for [neighbor]  $j$ ” (Ward and Gleditsch 2019: 76). Errors also may be correlated because of the mismatch between the spatial scale of a process and the discrete spatial units of observations (Anselin 2006: 907). These error patterns correspond respectively to what researchers call the remoteness problem. For example, remoteness means that a model’s underestimates of violence in a unit distant from a capital city correlate with underestimates of violence in a neighboring unit which is also distant from the same city. An example, is the spatial probit error model, SPEM. A technique based on the conditional log likelihood and variance-covariance matrix of the model can be used to estimate it (Martinelli and Geniaux 2017). The model provides estimates of a  $\lambda$  parameter which, with a row-standardized connectivity matrix, indicates the average dependence in the errors of a prespecified set of neighbors on the estimation error in a unit of interest.

of a continuous spatial domain. For example, even though terrorist events are observed at specific locations and therefore “inherently discrete” these events are interpreted as realizations of a continuously indexed space-time process of violence perpetrated against civilians and government officials (Python et al. 2017, 2018). Formally, the data are defined by a process indexed in time and space:  $Y(s, t) \equiv [y(s, t), (s, t) \in D \subset R^2 \times R]$ . The spatio-temporal covariance function for the process is written as  $Cov(y(s_i, t)y(s_j, u)) = C(y_{it}, y_{ju})$ . Under an assumption of stationarity (see below) the covariance function can be expressed in terms of a combination of spatial distance,  $\Delta_{ij} = ||s_i - s_j||$ , and temporal lag  $\Lambda_{tu} = |t - u|$ . Thus  $Cov(y_i, y_{ju}) = C(\Delta_{ij}, \Lambda_{tu})$ .<sup>4</sup>

As in neighborhood modeling, several concepts are especially important in geostatistical analysis. First, as in the fixed effects models and STAR setups, spatial and temporal factors often are assumed to be *separable*. This means that covariance function can be written as a sum or product of its spatial and temporal parts, for instance,  $Cov(y_{it}, y_{ju}) = C_1(\delta_{ij})C_2(X_{tu})$ . Gneiting (2002) proposes a test for separability for modeling spatial-temporal data.<sup>5</sup> xxxx is the assumption of *spatial stationarity*. The idea that xxxx is called *isotropy*. Because time is ordered and space is not, isotropy has no meaning in the spatial-temporal context (Harvill 2010: 375).

One geostatistical approach that assumes separability and stationarity is Continuous Domain Bayesian Modeling with Integrated, Nested Laplacian Approximation (INLA).<sup>6</sup> Briefly, this approach “does not build models solely for *discretely observed data* but for approximations of *entire processes* defined over continuous domains” (Lindgren and Rue 2015:3, emphasis in the original). It assumes that the data generating process is a Gaussian field,

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<sup>4</sup>The data, say  $y(s)$ , are a random outcome at a specific location and the spatial index,  $s$ , can vary continuously in a fixed domain;  $s$  is a two-dimensional vector with latitudes and longitudes (three-dimensional if altitudes are considered) This definition of the process is taken from Blangiardo and Cameletti 2015: 235-236.

<sup>5</sup>Harvill (2010: 375) explains that “covariance functions imply that small changes in the locations of observations can lead to large changes in the correlations between certain linear combinations of observations.” Jun and Cook (2020) propose modeling terrorism with nonstationary models.

<sup>6</sup>The following description draws from Blangiardo and Cameletti (2015: Chapter 6) and especially the passage on pps. 234-5 of Python et al. (2017). Extensions that allow for modeling nonstationary processes are reviewed in Lindgren and Rue (2015).

$\xi(s)$ , where  $s$  denotes a finite set of locations,  $(s_1 \dots s_m)$ . As such it suffers from a “big n problem,” analyzing the Gaussian field is costly computationally (Lindgren et al. 2011). Therefore, a particular linear, stochastic partial differential equation is assumed to apply to the Gaussian field:

$$(\kappa^2 - \Delta)^{\frac{\alpha}{2}}(\tau\xi(s)) = W(s), \quad s \in D \quad (3)$$

where  $\Delta$  is a Laplacian,  $\alpha$  is a smoothness parameter such that  $\alpha = \lambda+1$  (for two dimensional processes),  $\kappa > 0$  is a scale parameter,  $\tau$  is a precision parameter, the domain is denoted by  $D$ , and  $W(s)$  is Gaussian spatial white noise. The solution of this equation is a stationary Gaussian field with the Matérn covariance function:

$$Cov(\xi(s_i), \xi(s_j)) = \sigma_{\xi_i}^2 \frac{1}{\Gamma(\lambda)2^{\lambda-1}}(\kappa || s_i - s_j ||)^{\lambda} K_{\lambda}(\kappa || s_i - s_j ||) \quad (4)$$

where  $|| s_i - s_j ||$  denotes the Euclidean distance between locations  $s_i$  and  $s_j$ ,  $\sigma_{\xi_i}^2$  is the marginal variance,  $\Gamma(\lambda) = \lambda!$ ,  $K_{\lambda}$  is the modified Bessel function of the second kind and order  $\lambda > 0$ . The distance at which the spatial correlation becomes negligible (for  $\lambda > .05$ ) is the range,  $r$ . The solution to the SPDE implies that the formula for the marginal variance is  $\sigma^2 = \frac{\Gamma(\lambda)}{\Gamma(\alpha)(4\pi)^{\frac{d}{2}} \kappa^{2\lambda} \tau^2}$  where  $d = 2(\alpha - \lambda)$ . And the formula for the range is  $r = \frac{\sqrt{8\lambda}}{\kappa}$ . In this way, the Gaussian field can be represented (approximated) by a GMRF. A finite element method using basis functions defined on a Constrained Refined Delaunay Triangularization (mesh) over a corresponding shapefile of latitude-longitude event data is used for this purpose.

A hierarchical Bayesian framework can be used to model the data. For dichotomous data like the discrete observation of a human rights violation, three equations are employed:

$$y_i | \eta_i, \theta \sim \text{Bernoulli}(\pi_i), \quad i = 1 \dots m \quad (5)$$

$$\eta_i | \theta = \beta_0 + \sum_{k=1}^{n_{\beta}} \beta_k z_{k,i} + \xi_i, \quad i = 1 \dots m \quad (6)$$

$$\theta = p(\theta) \tag{7}$$

where  $y_i$  is the observation at point  $i$ ,  $m$  is the number of vertices in the Delaunay Triangularization, the second equation is the linear predictor, here  $\eta_i = \text{probit}(\pi)$ , with spatially explicit covariates  $z_{k,i}$ ,  $\xi_i$  the Gaussian field as defined by equations (1) and (2) and approximated by the GMRF at point  $i$ , and equation (1) assigns the hyperparameters  $\theta = (\kappa, \sigma_\xi^2)$ .<sup>7</sup>

INLA is used to estimate the model. INLA performs numerical calculation of posterior densities and in this regard it is more efficient than Markov Chain Monte Carlo methods. Besides estimates of the effects of spatially explicit covariates on the probability of events and of the range of spatial error dependence for each dataset, this geostatistical approach produces useful estimates of the parameters in the GMRF—in particular, the mean and standard deviation of the latent field at each point in the dataset. These estimates tell us about the impact of uncertainty produced by both the scarcity (absence) of data and measurement error at each site. The estimates of the GMRF therefore tell us how human and machine coded data compare in terms of the remoteness problem.<sup>8</sup>

Table 1 summarizes some of the (separable) spatial temporal models used in the study conflict.

## 2 Three Approaches to Spatial-Temporal Modeling Compared

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<sup>7</sup>For comparability to the SPEM, we employ the probit link function in our geostatistical evaluation of the ICEWS and GED data. In the Appendix, we also report substantively comparable results for standard probit model with no spatial error component.

<sup>8</sup>A more complex model assumes space-time separability. It also decomposes the stochastic part of the model into a GMRF and Gaussian white noise both of which are time dependent. The Gaussian white noise component then is interpreted as measurement error. See Python et al. (2018: 7-8). We return to these more complex geostatistical models in the Discussion section.



Neighborhood, Discrete Space Time	Geostatistical, Continuous Space Time
-General Linear Models with Two Way Fixed Effects and Error Clustering Condra and Shapiro 2015 Berman et al. 2015 Weidman and Shapiro 2015 Silverman 2021	-Stochastic Integral Differential Equation Models (square lattice) Zammit-Mangion 2012
-Spatial Temporal Autoregressive Models Weidman and Ward 2010 Franzese et al. 2015	-Stochastic Partial Differential Equation Models (triangular mesh) Python et al. 2016,2018
-Spatial Vector Autoregressions Linke et al. 2012	-State Space Models None?
-Spatial Hierarchical Linear Models Hagan et al. 2016	

Table 1: Types of Separable Space Time Models Used in the Study of Subnational Civil Conflict and Terrorism with Selected Citations

### 3 References

- Anselin, Luke 2006. "Spatial Econometrics" Chapter 26 in *Palgrave Handbook of Econometrics* Volume 1. Econometrics. Basingstone, Palgrave MacMillon, pps. 901-969
- Bagozzi, Benjamin E., Patrick T Brandt, John R. Freeman, A, Kim, A. Palao, and Carly Potz-Nielsen. 2018 "The Prevalence and Severity of Underreporting Bias in Machine and Human Coded Data" *Political Science Research and Methods* 7(3): 641-649.
- Berman Eli, Michael Callen, Joseph H. Felter and Jacob Shapiro. 2011. "Do Working Men Rebel? Insurgency in Afghanistan, Iraq, and the Philippines." *Journal of Conflict Resolution* 55(4): 496-528//
- Biddle, Stephen, Jeffrey A. Friedman, and Jacob N. Shapiro. 2020. "Testing the Surge: Why Did Violence Decline in Iraq in 2007?" *International Security* 37(1): 7-40.//
- Blangiardo, Marta and Michela Cameletti. 2015. *Spatial and Spatial-temporal Bayesian Models with R-INLA* New York, John Wiley and Sons.
- Box-Steffensmeier, Janet, John R. Freeman, Matthew P. Hitt, and Jon C. Pevehouse. 2014. *Time Series Analysis for the Social Sciences* New York. Cambridge U
- Braithwaite, Alex and Shane D. Johnson. 2012 "Space-Time Modeling of Insurgency and Counterinsurgency in Iraq" *Journal of Quantitative Criminology* 28: 31-48.
- Brandt, Patrick T., John R. Freeman and Philip Schrodt. 2014. "Evaluating Forecasts of Political Conflict Dynamics." *International Journal of Forecasting* 30(4): 944-962.

- Brandt, Patrick T., Michael Colaresi, and John R. Freeman 2008. "The Dynamics of Reciprocity, Accountability, and Credibility." *Journal of Conflict Resolution* 52(3): 343-374.
- Cameletti, Michela, Rosaria Ignaccolo, and Stefano Bande. 2011 "Comparing Spatio-temporal models for Particulate Matter in the Piemonte." *Environmentrics* 22: 985-996.
- Center for Army Analysis. Deployed Analyst History Report–Volume 1. Analytic Support to Combat Operations in Iraq 2002-2011. 2012 Fort Belvoir, Virginia.
- Cho, Wemdy K. "Contagion Effects and Ethnic Contribution Networks" *American Journal of Political Science* 47(2): 368-387.
- Cho, Wendy K. and James G. Gimpel. 2003 "Prospecting for Gold" *American Journal of Political Science* 51(2): 255-268.
- Condra, Luke N. and Jacob N. Shapiro. 2012 "Who Takes the Blame? The Strategic Effects of Collateral Damage" *American Journal of Political Science* 56(1): 167-187.
- Donnay, Karsten and Vladimir Filmanov. 2014. "Views to a War: Systematic Differences in Media and Military Reporting of the War in Iraq" *EPJ Data Science* 3 XXX
- Enders, Walter. 2010. *Applied Econometric Time Series* Third Edition. New York, Wiley.
- Franzese, Robert Jr. , Jude C. Hays, and Scott J. Cook. 2015. "Spatial and Spatial-temporal-Autoregressive Probit Models of Interdependent Binary Outcomes." *Political Science Research and Methods* 4(1): 151-173.
- Franzese, Robert Jr. and Jude C. Hays. 2007. "Spatial-Econometric Models of Cross-Section Interdependence in Political Science Panel and Time Series Cross Section Data." *Political Analysis* 15(2): 140-164.
- Gill 2020.XXXX
- Gneiting, Thomas. 2002 "Nonseparable, Stationary Covariance Functions for Space-Time Data" *Journal of the American Statistical Association* 97(458): 590-600
- Hagan, John, Joshua Kaiser, and Anna Hanson. 2016. *American Sociological Review* 81(2): 316-346.
- Hammond, Jesse and Nils B. Weidman. 2014. "Using Machine Coded Even Data for the Micro-level Study of Political Violence" *Reaearch and Politics* July-September: 1-6.
- Imai,Kosuke, and InSong Kim 2020. "On the Use of Two-Way Fixed Effects Regression Models for Causal Inference with Panel Data." *Political Analysis* 29: 405-415
- Imai, Kosuke, and InSong Kim 2019 "When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?" *American Journal of Political Science* 63(2): 467-490.
- Harvill, Jane L. 2010 "Spatio-temporal Processes." [wires.wiley.com/compstats](http://wires.wiley.com/compstats). vol 2: 376-382
- Jun, and Scott Cook 2020.
- Keele, Luke and Rocio Titiunik. 2015. "Geographic Boundaries as Regression Discontinuities." *Political Analysis* 23: 127-155.
- Keele, Luke and William Minozzi. 2013. "How Much is Minnesota Like Wisconsin? Assumptions and Inference in Observational Data." *Political Analysis* 21(2): 193-216
- Linke, Andrew M., Frank W. Witmer, and John O'Loughlin. 2012. "Space Time Granger

- Analysis of the War in Iraq: A Study of Coalition and Insurgent Action-Reaction” *International Interactions* 38: 402-425.
- Lindgren, F. and H. Rue. 2015. ”Bayesian Spatial and Spatial-Temporal Modeling with R-INLA.” *Journal of Statistical Software* 63(19).
- Lindgren, F., H. Rue, and J. Lindstrom. 2011. ”An Explicit Link Between Gaussian fields and Gaussian Markov Random Fields. The Stochastic Differential Equation Approach (with Discussion).” *Journal of the Royal Statistical Society Series B* 73(4): 423-498
- Martinelli, D. and G. Geniaux. 2017 ”Approximate Likelihood Estimation of Spatial Probit Models” *Regional Science and Urban Economics* 64: 30-45.
- Monogan and Gill 2015XXXX
- Pavia, Jose Manuel, Beatriz Larraz, and Jose Maria Montero. 2008. ”Election Forecasts Using Spatiotemporal Models” *Journal of the American Statistical Association* 103(483): 1050-1059
- Python, A., J.B. Illian, C.M. Jones-Todd, and M. Blangiardo. 2018. ”A Bayesian Approach to Modeling Subnational Spatial Dynamics of World Wide Non-state Terrorism, 2010-2016.” *Journal of the Royal Statistical Society Series A* pps. 1-22.
- Python, A., J. Illian, C. Jones-Todd, and M. Blangiardo. 2017. ”Explaining the Lethality of Boko Haram’s Terrorist Attacks in Nigeria, 2009-2014.” In *Bayesian Statistics in Action* R.Argento et al. eds. Springer.
- Schutte, Sebastian and Karsten Donnay. 2014. ”Matched Wake Analysis: Finding Causal Relationships in Spatiotemporal Data” *Political Geography* 41: 1-10.
- Shapiro, Jacob N. and Nils B. Weidman 2015. ”Is the Phone Mightier Than the Sword? Cellphones and Insurgent Violence in Iraq” *International Organization* 69: 247-274
- Silverman, Daniel 2021. ”Too Late to Apologize? Collateral Damange, Post-Harm Compensation, and Insurgent Violence in Iraq.” *International Organization* XX
- Stundal, Logan, Benjamin E. Bagozzi, John R. Freeman, and Jennifer S. Holmes. 2021. ”Human Rights in Space: Statistical Modelss of Machine Coded vs. Human Coded Data.” Unpublished.
- vonBorzykowski, Inkenand Michael Wahman. 2021 ”Systematic Measurement Error in Election Violence Data: Causes and Consequences.” *British Journal of Political Science* XXX
- Ward, Michael D. and Kristian Skrede Gleditsch. 2019 *Spatial Regression Models* Second Edition. Los Angeles California. Sage Publications.
- Ward, Michael D. and Kristian Skrede Gleditsch. 2002. ”Location, Location, Location. A MCMC Approach to Modeling the Spatial Context of War and Peace.” *Political Analysis* 10(3): 244-260.
- Weidman, Nils B. and Michael Ward. 2010. ”Predicting Conflict in Space and Time” 2010. *Journal of Conflict Resolution* 54(6): 883-901
- Zammit-Mangion, Andrew, Michael Dewar, Visakan Kadirkamanathan, and Guido Sanguinetti. 2012. ”Point Process Modeling of the Afgan War Diary”. *PNAS* 109(31): 12414-12419.