$$\vec{\nabla} \cdot (f\vec{A}) = f(\vec{\nabla} \cdot \vec{A}) + \vec{A} \cdot (\vec{\nabla} f)$$

Since

$$\int_V (\vec{\nabla} \cdot \vec{A})\, d\tau = \oint_S \vec{A} \cdot\, d\vec{a}$$

We can insert a $d\tau$ into the first equation to get the relation shown above to appear in the original equation. By doing do, we can replace the integral in that spot with a different form and solve for one of the remaining parts to integrate. Thus,

$$\int_V \vec{\nabla} \cdot (f\vec{A})\, d\tau = \int_V (f(\vec{\nabla} \cdot \vec{A}) + \vec{A} \cdot (\vec{\nabla} f))\, d\tau$$

Using

$$\int_a^b u\, dv = u|_a^b v|_a^b - \int_a^b v\, du$$

We can express the integration by parts equation as

$$\int_V \vec{\nabla} \cdot (f\vec{A})\, d\tau = \int_V f(\vec{\nabla} \cdot \vec{A})\, d\tau + \int_V \vec{A} \cdot (\vec{\nabla} f)\, d\tau$$

$$\oint_S (f\vec{A}) \cdot\, d\vec{a} = \int_V \vec{\nabla} \cdot (f\vec{A})\, d\tau - \int_V \vec{A} \cdot (\vec{\nabla} f)\, d\tau$$