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PH336 FINAL LAB REPORT

MODELING THE DYNAMICS OF A PROPULSIVE VEHICLE IN A QUADRATIC FLUID

INTRODUCTION

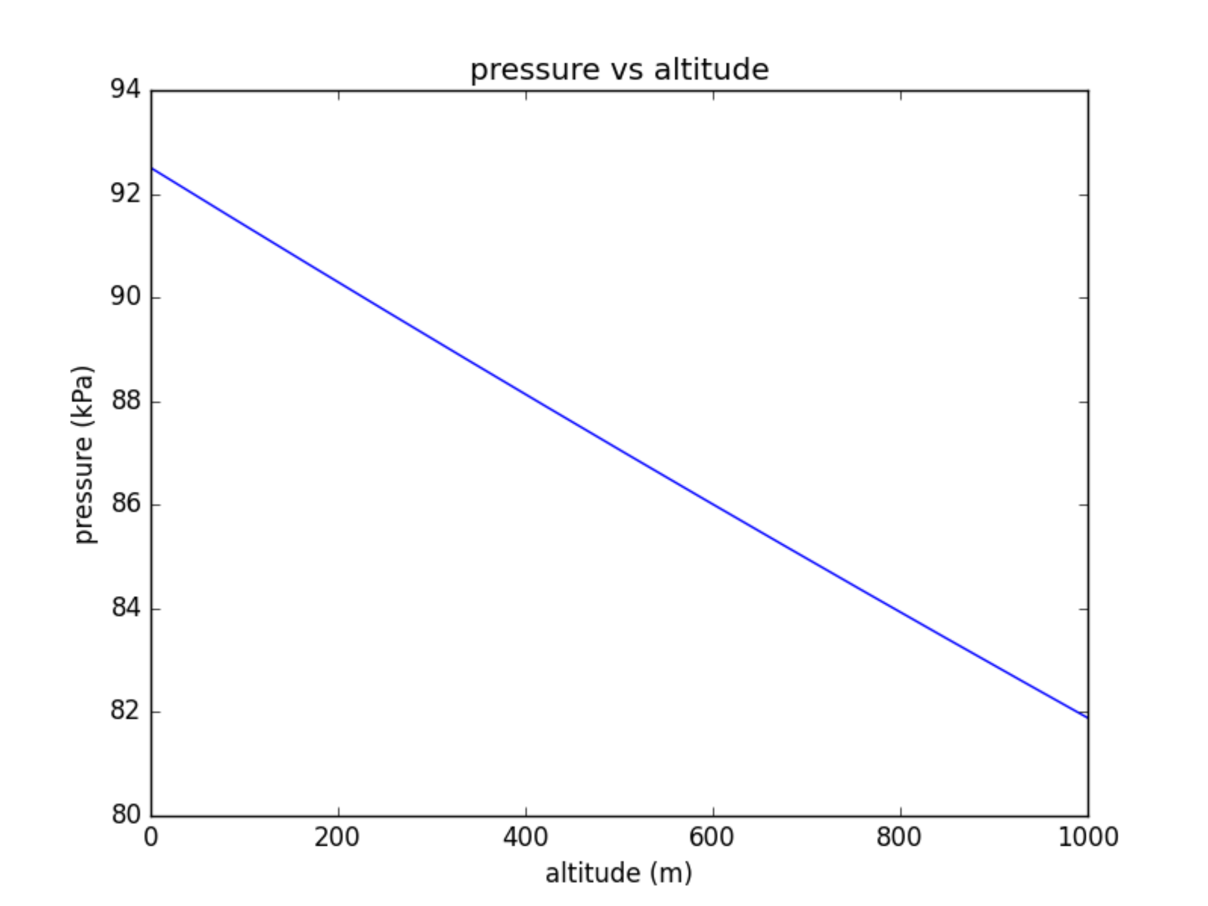
- Model a propulsive variable-mass 3 DoF system with Python
- Measure model rocket behavior for model comparison and cross-validation
- Estimate landing zone radial distance from launch site, maximum altitude, and total time in air
- α-level significance of 0.05 in statistical correlation between the model and the data
- Monte-Carlo approach to estimate parameters and uncertainties

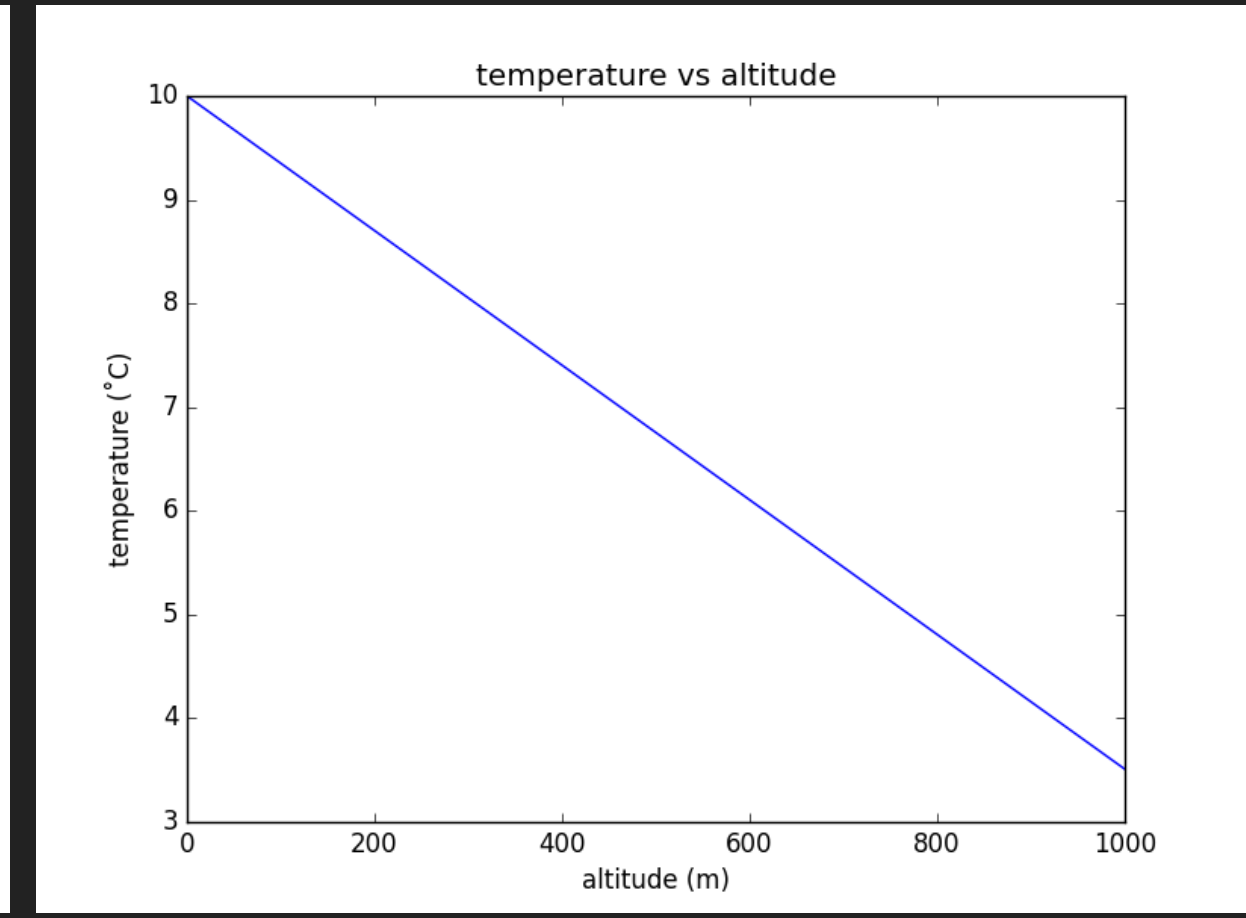
DYNAMICS ANALYSIS – WORLD FRAME OF REFERENCE

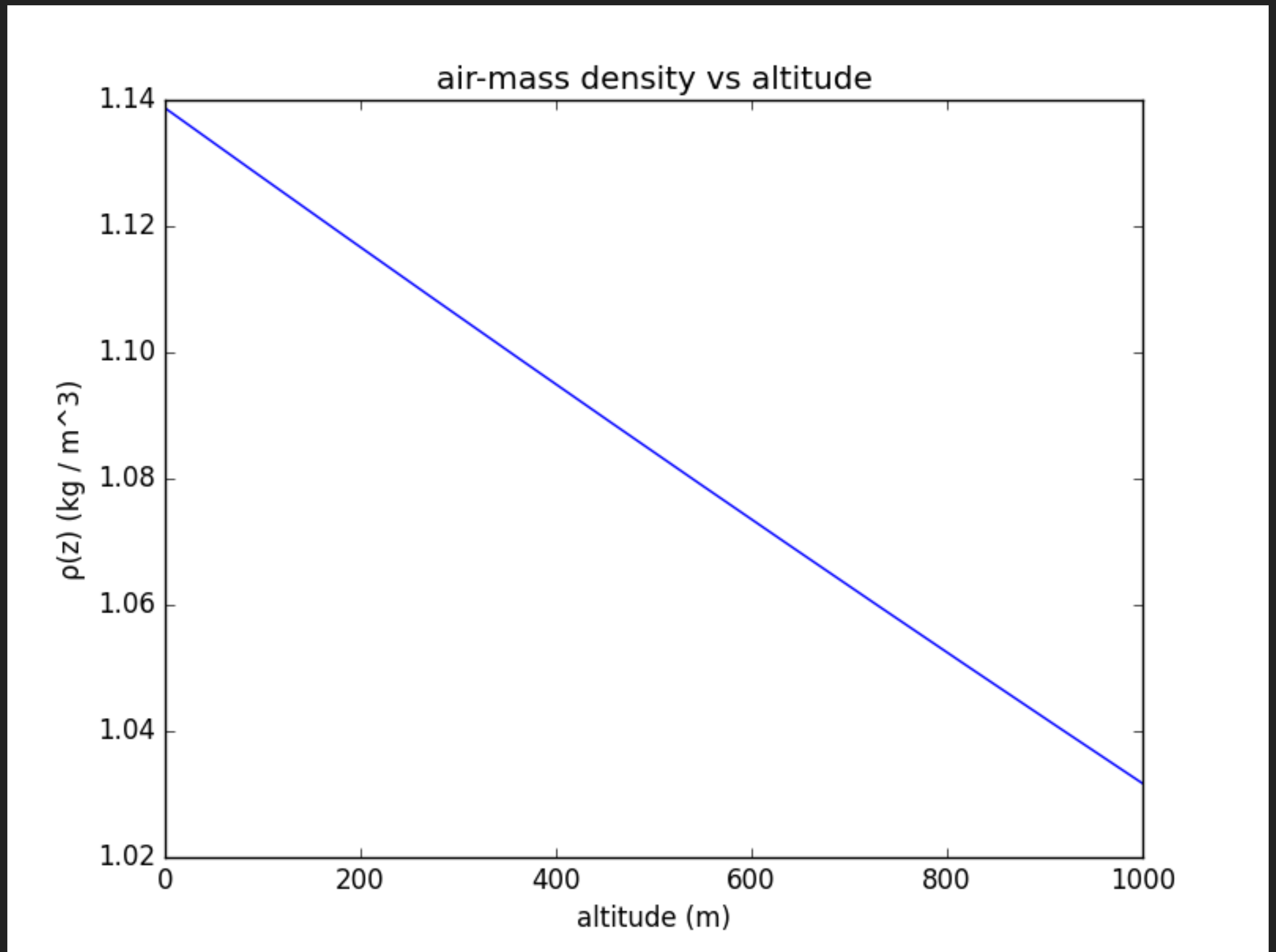
- Newton's second law:
  - $\overrightarrow{F} = m\vec{a} = \sum \overrightarrow{F}_i$
  - $\overrightarrow{F}_D = -\frac{1}{2}\rho C_d A\, \dot{r}^2\, \hat{r}$ where $\dot{\vec{r}} \equiv \dot{\vec{s}} - \vec{v}_{\text{wind}}$, and $\dot{\vec{s}}$ is the time derivative of the rocket's position vector
  - $\overrightarrow{F}_g \approx -g\,\hat{z}$
- Newton's third law:
  - $\overrightarrow{F}_{\text{A on B}} = -\overrightarrow{F}_{\text{B on A}} \Rightarrow \overrightarrow{T}(t)_{\text{motor on rocket}} \approx T(t)\,\hat{z}$

ATMOSPHERIC MODEL

- Troposphere region for $h < 11{,}000$ meters
- Thermodynamic parameters:
  - $T(z) \approx T_{\text{ambient}} - 0.00649 \cdot z\ ^\circ\text{C}$
  - $p(z) \approx 101.29 \cdot \left[\dfrac{T(z) + 273.15}{288.08}\right]^{5.256}$ kPa
  - $\rho(z) \approx \dfrac{p(z)}{0.2869 \cdot (T(z) + 273.15)} \dfrac{\text{kg}}{\text{m}^3}$

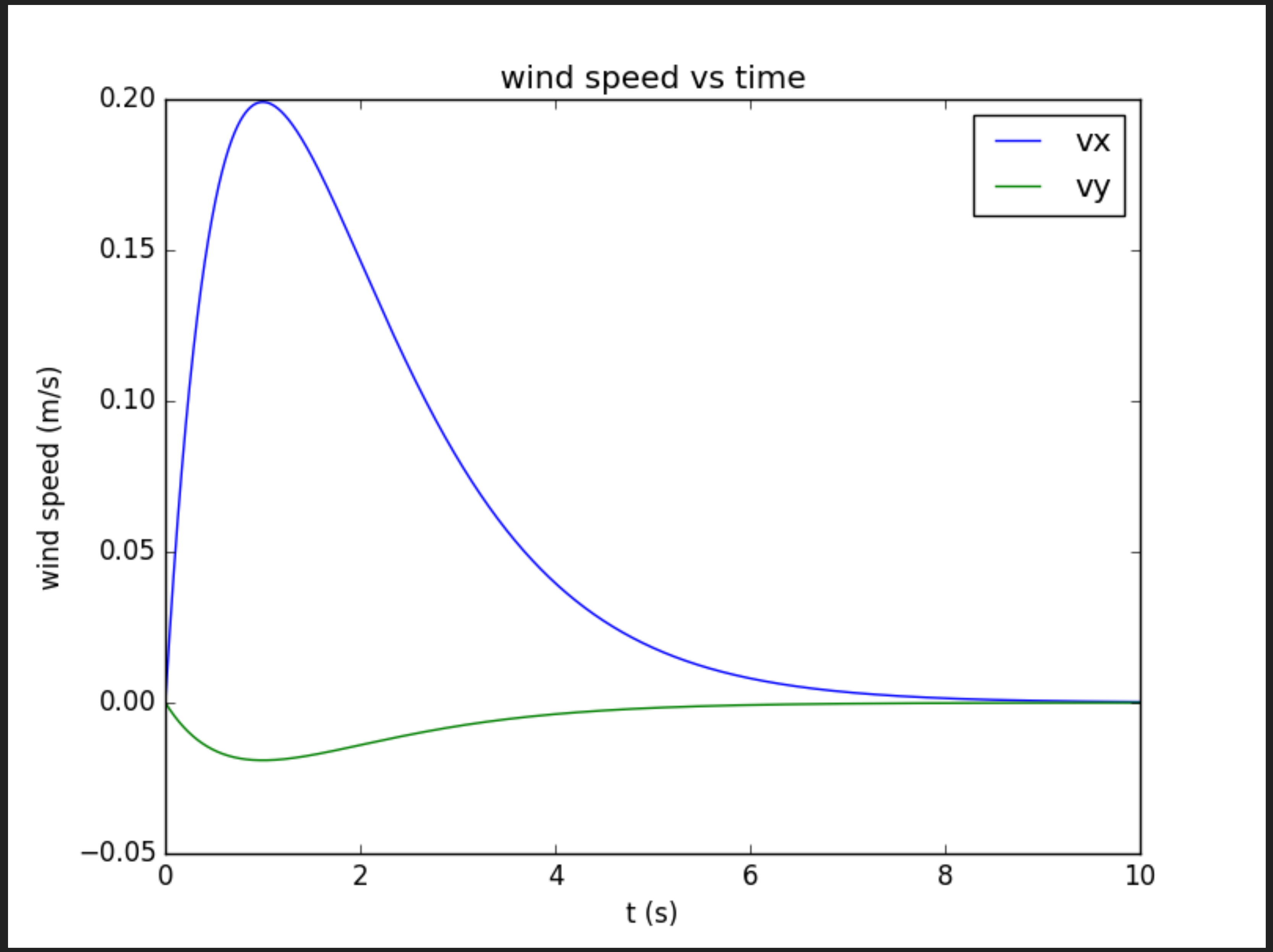






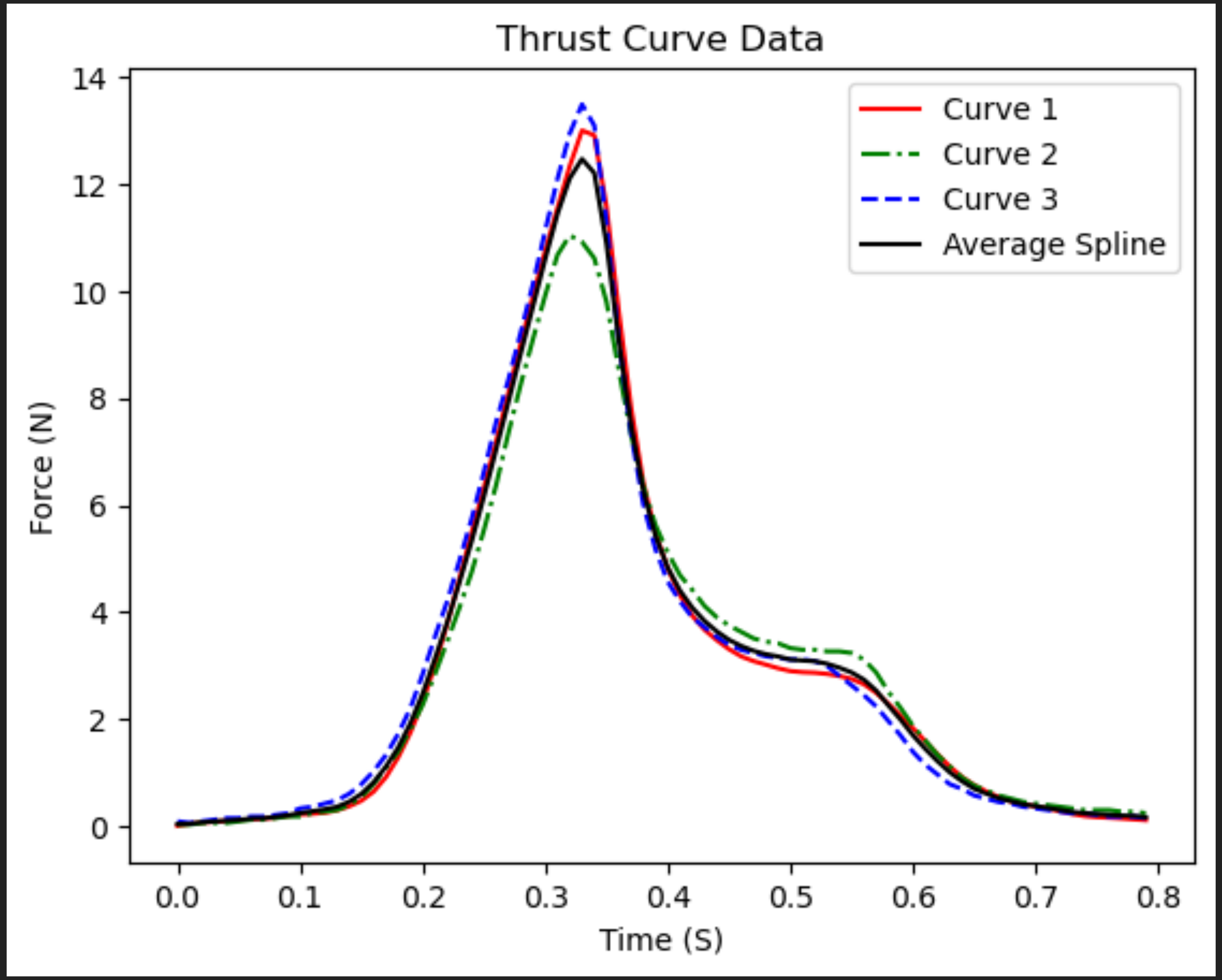
WIND MODEL

- Wind behavior modeled by $v(t) = \alpha \cdot (t - t_0)e^{-r(t-t0)}$ where $\alpha = \frac{v_{\text{max}}}{r} \cdot e^{r^2}$ and r is the wind gust decay rate
- Conditional probability at time t where t is close to t_0:
  - Select U ~ Uniform(0, 1)
  - Compute $R = e^{-f(t-t_0)/\beta}$ where f is the mean gust frequency and $\beta = \frac{1}{2\ln 2}$ ensures 50% probability of starting next gust
  - Compare $U > R$. If
    - True: Set $t_0 = t$
    - False: Move to next time step
- Wind speed is discontinuous due to the random process, but are smoothed by averaging via RK4 method



THRUST MODEL

- Empirical modeling
- Measure force vs time on test stand
- Cubic spline fits
- Estimate σ_T (uncertainty in thrust force) with standard deviation of 3 spline fits
- Cubic spline on mean of thrust curves, sample thrust uncertainties from normal with $\mu = \sigma_T$ and $\sigma^2 = \dfrac{1}{\sqrt{N}}$ (depends on sample size, ours is $N = 3$)

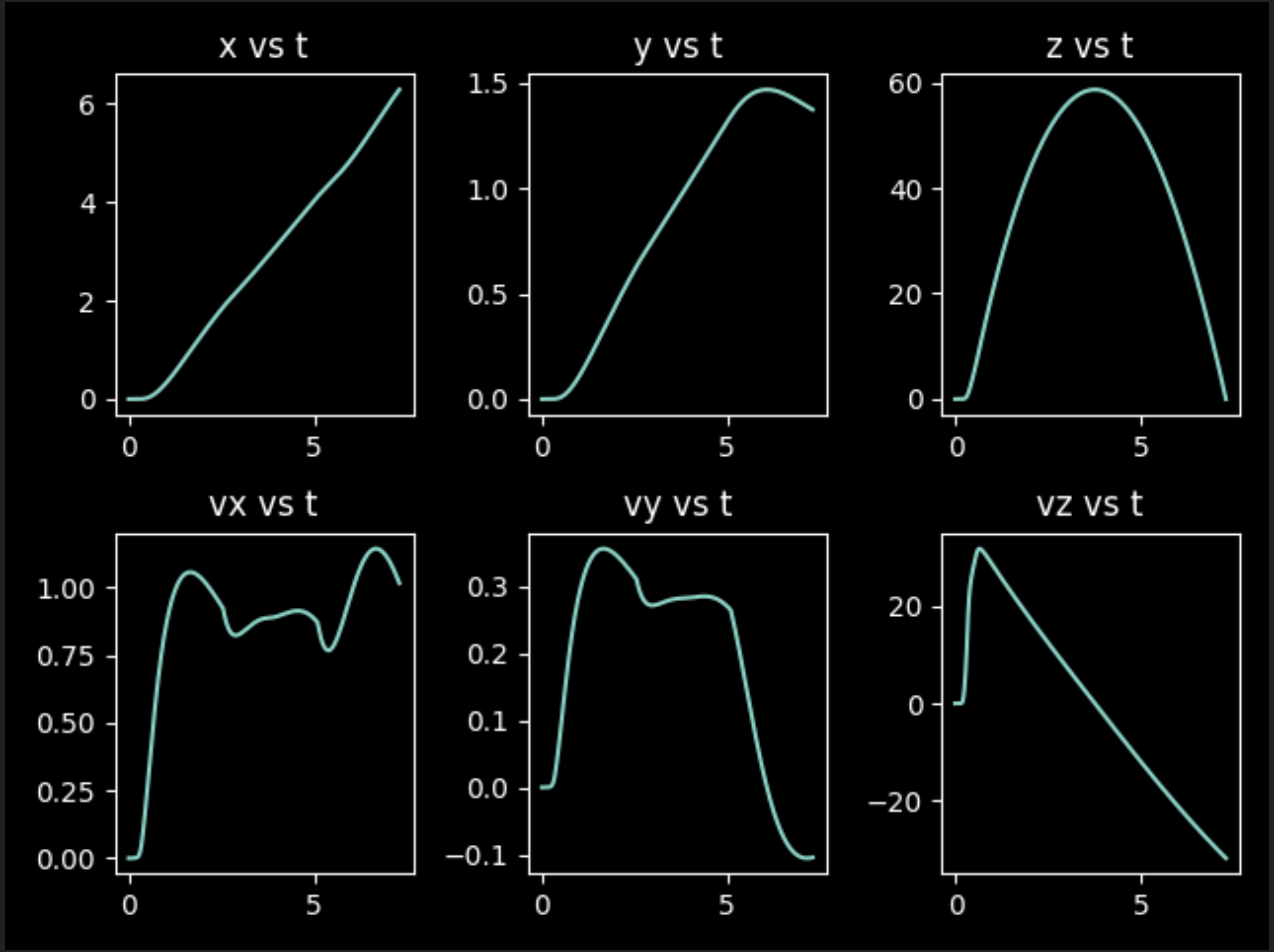


SIMULATION CONDITIONS DEMO

- $N = 50$
- $v_{\text{wind}} \approx 1.3\,\text{m / s}$
- $\sigma_v \approx 0.13\,\text{m / s}$
- $\hat{v}_{\text{wind}} \approx \langle 1, 0, 0 \rangle$
- $\hat{\sigma}_v \approx 0.1$
- $m_0 = (0.0171 + 0.0481)\,\text{kg} = 0.0652\,\text{kg}$
- $\dfrac{dm}{dt} = \dot{m} \approx \dfrac{0.0041\,\text{kg}}{0.7\,\text{s}} \qquad 0 < t \leq 0.7\,\text{s}$

SIMULATION RESULTS DEMO

- $t = 6.011 \pm 2.741\,\mathrm{s}$
- $z_{\mathrm{max}} = 48.097 \pm 22.534\,\mathrm{m}$
- $r = 5.348 \pm 2.506\,\mathrm{m}$



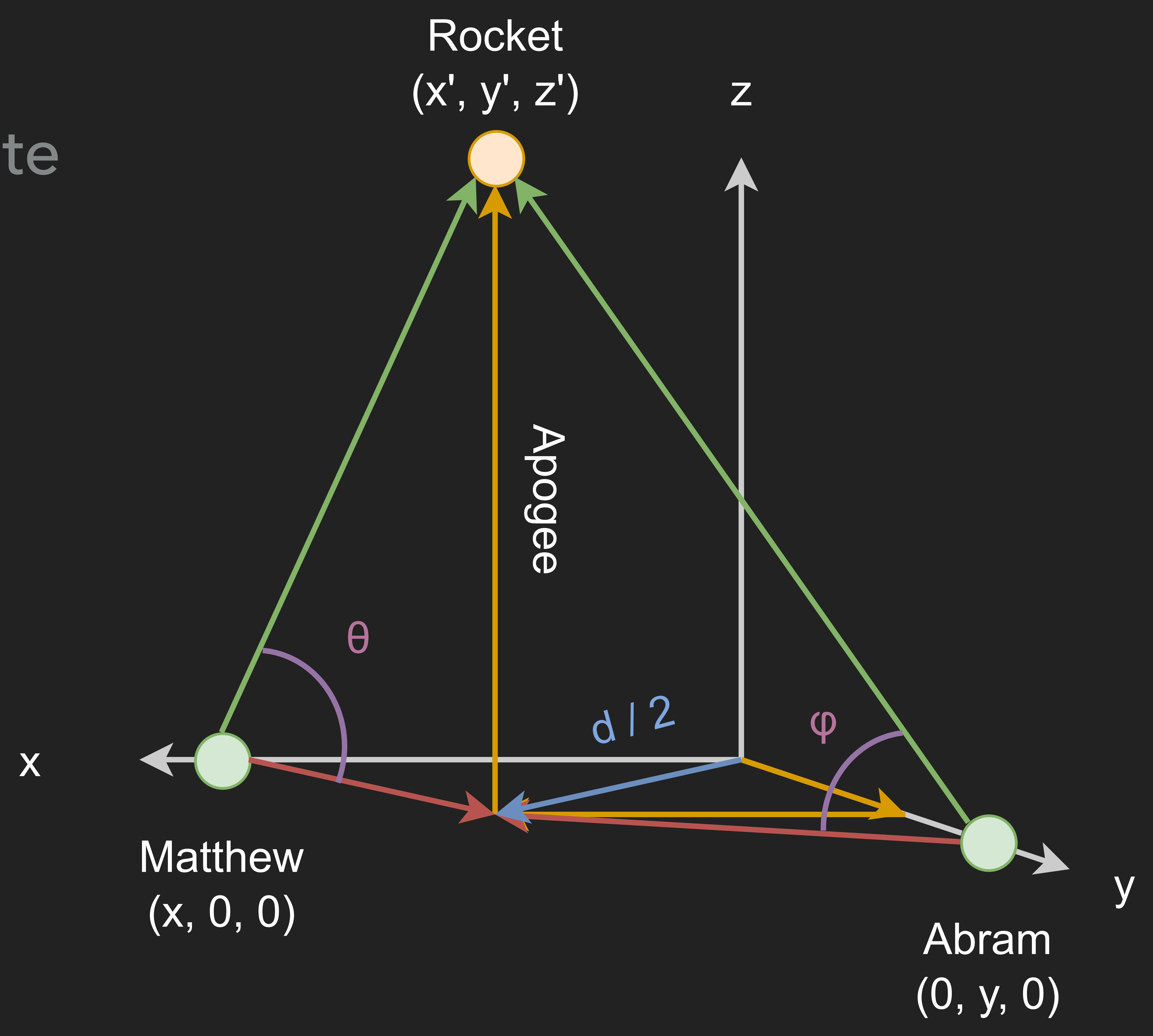
MEASUREMENT PROCEDURES

- Swivel protractors with friction locks estimate θ and ϕ in degrees
- Pedometers estimate x and y
- Estimate $z' = h_1 \tan\theta = h_2 \tan\phi$ using

$$h_1 = \sqrt{(x - x')^2 + (y')^2},$$

$$h_2 = \sqrt{(x')^2 + (y - y')^2}, \text{ and}$$

$$\frac{d}{2} = \sqrt{(x')^2 + (y')^2}$$



RESULTS #1

Chi Squared Test					
	Experimental		Simulated		Chi Squared
	Mean	Standard Dev	Mean	Standard Dev	
Flight Time (s):	7.2	0.76	7.204	0.01	2.7696E-05
Apex Height (m):	48.61	5.59	57.109	0.16	2.309704352
Radial Distance (m):	8.9	4.27	6.072	0.015	0.438629363
					2.748361411
		P Val:	0.253046831		

RESULTS #2

Chi Squared Test					
	Experimental		Simulated		Chi Squared
	Mean	Standard Dev	Mean	Standard Dev	
Flight Time (s):	7.37	0.24	7.14	0.01	0.9168111
Apex Height (m):	39.18	5.37	56.09	0.14	9.9093133
Radial Distance (m)	21.75	13.44	33.59	0.07	0.776056
					11.60218
		P Val:	0.003024256		

UNCERTAINTY PROPAGATION

- $y = d \cdot \tan\theta$
- $\delta y \approx \sqrt{(d \cdot \sec^2(\theta)\,\delta\theta)^2 + (\tan(\theta)\delta d)^2}$
- $\bar{y} = \dfrac{\dfrac{y_1}{\sigma_1^2} + \dfrac{y_2}{\sigma_2^2}}{\dfrac{1}{\sigma_1^2} + \dfrac{1}{\sigma_2^2}}$
- $\sigma^2 = \left(\dfrac{1}{\sigma_1^2} + \dfrac{1}{\sigma_2^2}\right)^{-1}$

CONCLUSION

- Large uncertainties in simulation results reflect overestimated sensitivities in the environment
- Measure wind gust frequency to higher precision and accuracy
- Account for varying aerodynamic interactions influenced by attitude deltas
- Level up simulator to 6 DoF for non-zero moment arm dynamics
- Pre-generate wind environment at start of simulation using stochastic methods for selecting velocity vectors, using nonlinear fit on random variables to enforce smoothness