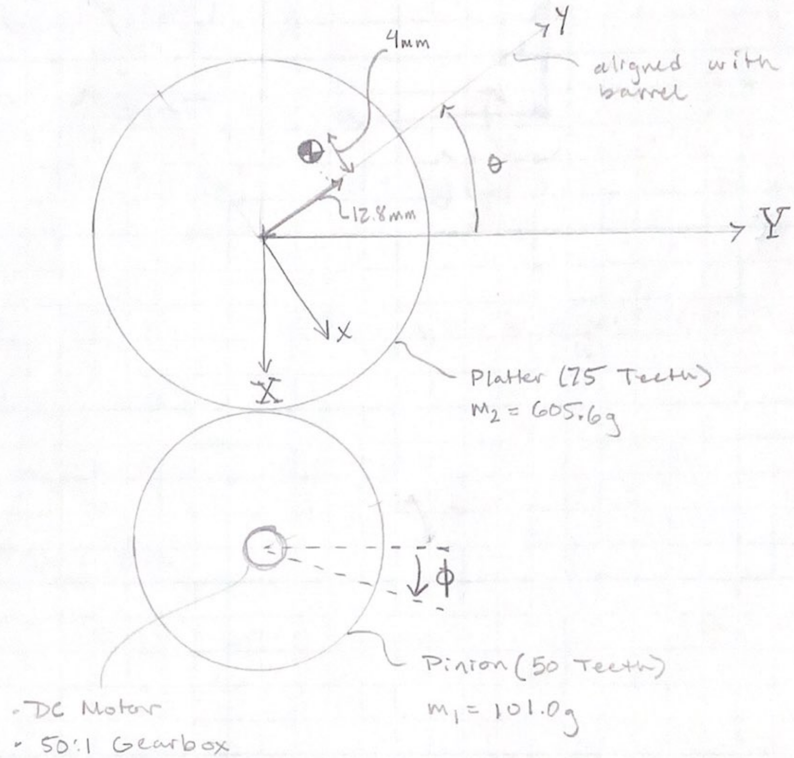
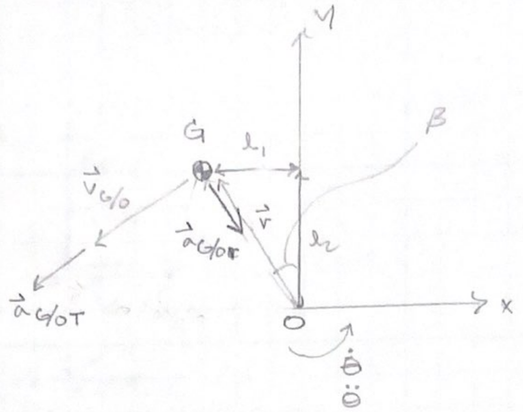


Y
aligned with barrel
4mm
θ
12.8mm
Y
x
X
Platter (75 Teeth)
$m_2 = 605.6g$
ϕ
Pinion (50 Teeth)
$m_1 = 101.0g$
· DC Motor
· 50:1 Gearbox

FOR TURRET LOCAL FRAME:



$\vec{r} = -l_1\hat{\imath} + l_2\hat{\jmath}$ $\quad\quad \beta = \tan^{-1}\left(\frac{l_1}{l_2}\right)$

$|r| = \sqrt{l_1^2 + l_2^2}$

$\vec{V}_{G/O} = \vec{\dot{\theta}} \times \vec{r} = (\dot{\theta}\hat{k}) \times (-l_1\hat{\imath} + l_2\hat{\jmath})$

$\vec{V}_{G/O} = -l_2\dot{\theta}\hat{\imath} + -l_1\dot{\theta}\hat{\jmath}$

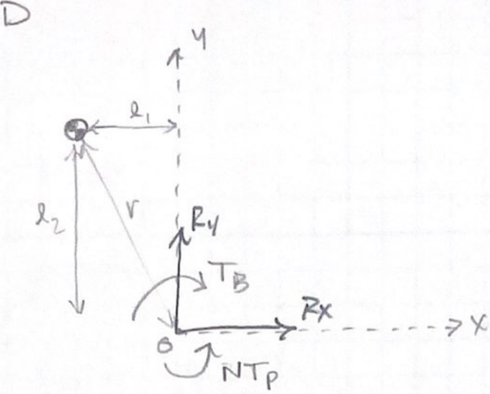
$|\vec{V}_{G/O}| = \dot{\theta}\sqrt{l_1^2 + l_2^2} = \dot{\theta} r$

$\vec{a}_{G/OT} = \vec{\ddot{\theta}} \times \vec{r}$

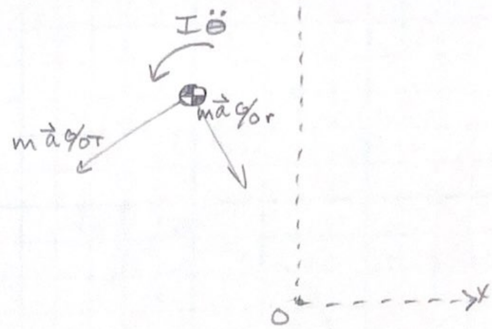
$= (\ddot{\theta}\hat{k}) \times (-l_1\hat{\imath} \times l_2\hat{\jmath})$

$\vec{a}_{G/OT} = -l_2\ddot{\theta}\hat{\imath} - l_1\ddot{\theta}\hat{\jmath}$

FBD



MAD



$$\left(\sum M_{FBD}\right)_O = \left(\sum M_{MAD}\right)_O$$

$$NT_p\hat{k} - T_B\hat{k} = I\ddot{\theta}\hat{k} + \vec{r}\times\vec{a}_{G/O_T}\,m$$

$$NT_p\hat{k} - \dot{\theta}b\hat{k} = (I_{yy})\ddot{\theta}\hat{k} + (-l_1\hat{\imath} + l_2\hat{\jmath})\times(-ml_2\ddot{\theta}\hat{\imath} - l_1 m l_1\ddot{\theta}\hat{\jmath})$$

$$= (I_{yy}\ddot{\theta})\hat{k} + m(l_1^2\ddot{\theta}\hat{k} + l_2^2\ddot{\theta}\hat{k})$$

$$NT_p - \dot{\theta}b = (I_{yy} + m(l_1^2 + l_2^2))\ddot{\theta}$$

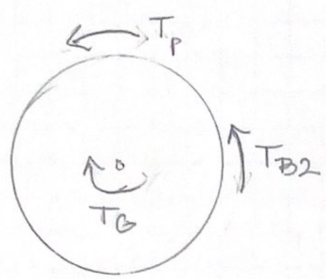
$$NT_p - \dot{\theta}b = (I_{yy} + mr^2)\ddot{\theta}$$

(1) $$\boxed{\ddot{\theta} = -\dot{\theta}\frac{b}{I_{yy}+mr^2} + \frac{N}{I_{yy}+mr^2}T_p}$$

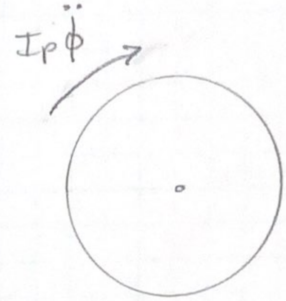
Determine Torque to Platter Through Pinion gear

Pinion gear:

FBD



MAD



$$\Sigma M_{FBD} = \Sigma M_{MAD}$$

$$-T_p - T_{B2} + T_G = I_p\ddot{\phi}$$

$$\boxed{T_p = -b_2\dot{\phi} - I_p\ddot{\phi} + T_G}$$

Assuming Motor Gearbox has no inertia/damping

$$\boxed{\underbrace{T_G}_{\text{Gearbox torque}} = 50\underbrace{T_m}_{\text{Motor Torque}} = 50\,\underbrace{K_t}\,i_m}$$

Motor Equations:

$$\frac{di_m}{dt} = -\frac{R}{L} i_m - \frac{K_v}{L}\Omega_m + \frac{1}{L} V_m$$

Let u represent the PWM % duty cycle

$$V_m = \frac{12V}{100} u$$

$$\boxed{\frac{di_m}{dt} = -\frac{R}{L} i_m - \frac{K_v}{L}(N50)\acute{\theta} + \frac{1}{L}\frac{12}{100} u} \quad (2)$$

Turret Pinion relationship:

$$\theta = N\phi \longrightarrow \phi = \frac{1}{N}\theta$$

$$\dot{\theta} = N\dot{\phi} \qquad \dot{\phi} = \frac{1}{N}\dot{\theta}$$

$$\ddot{\theta} = N\ddot{\phi} \qquad \ddot{\phi} = \frac{1}{N}\ddot{\theta}$$

Sub into pinion EOM:

$$T_P = -b_2\dot{\phi} - I_P\ddot{\phi} + T_G$$

$$\Downarrow$$

$$T_P = -b_2\frac{1}{N}\dot{\theta} - I_P\frac{1}{N}\ddot{\theta} + T_G$$

Sub into eq 1

$$\ddot{\theta} = \frac{-b_1}{I_{yy}+mr^2}\dot{\theta} + \frac{N}{I_{yy}+mr^2}\left(-\frac{b_2}{N}\dot{\theta} - \frac{I_P}{N}\ddot{\theta} + 50K_t i_m\right)$$

$$\ddot{\theta} = -\frac{b_1}{I_{yy}+mr^2} - \frac{b_2}{I_{yy}+mr^2}\dot{\theta} - \frac{I_P}{I_{yy}+mr^2}\ddot{\theta} + \frac{N50K_t}{I_{yy}+mr^2} i_m$$

$$\ddot{\theta}\left(\frac{I_{yy}+mr^2+I_P}{I_{yy}+mr^2}\right) = -\dot{\theta}\left(\frac{b_1+b_2}{I_{yy}+mr^2}\right) + \left(\frac{N50K_t}{I_{yy}+mr^2}\right) i_m$$

$$\boxed{\frac{d}{dt}\dot{\theta} = -\left(\frac{b_1+b_2}{I_{yy}+mr^2+I_P}\right)\dot{\theta} + \left(\frac{N50K_t}{I_{yy}+mr^2+I_P}\right) i_m} \quad 3$$

State Equations:

$$\frac{d}{dt}\theta = \dot{\theta}$$

$$\frac{d}{dt}\dot{\theta} = -\left(\frac{b_1 + b_2}{I_{yy} + mr^2 + I_p}\right)\dot{\theta} + \left(\frac{N50K_t}{I_{yy} + mr^2 + I_p}\right) i_m$$

$$\frac{d}{dt} i_m = -\left(\frac{N50K_v}{L}\right)\dot{\theta} + \left(-\frac{R}{L}\right) i_m + \frac{12}{100L} u$$