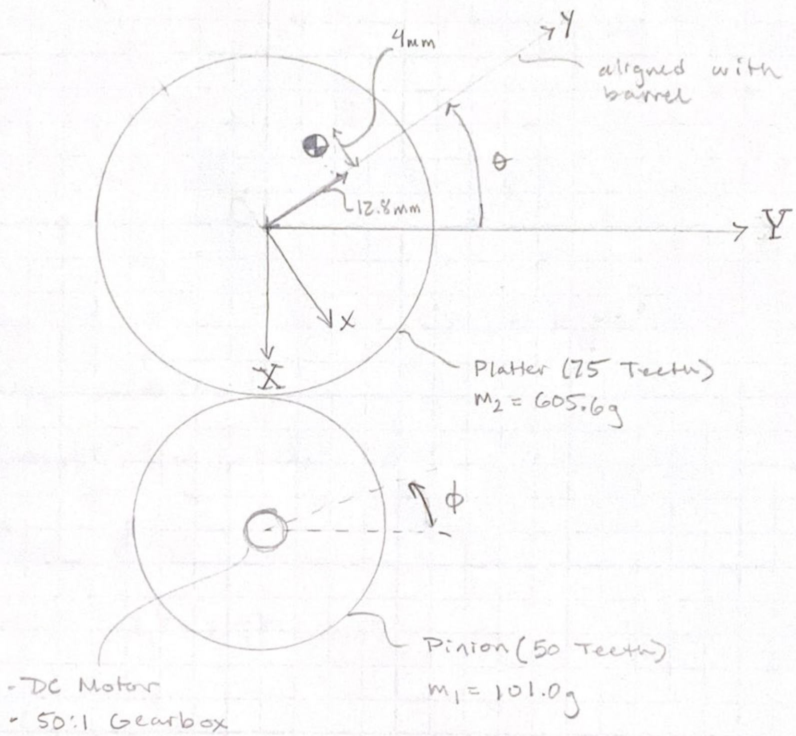
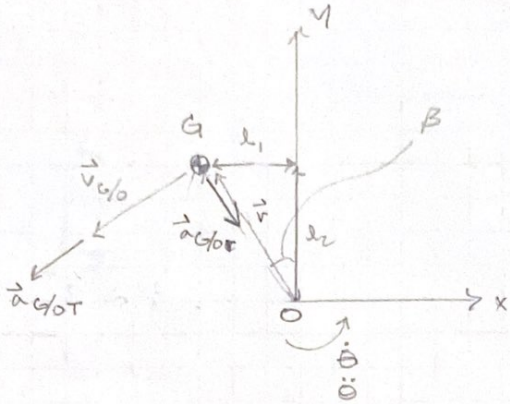

4mm
Y
aligned with barrel
θ
12.8mm
Y
x
X
Platter (75 Teeth)
$m_2 = 605.6g$
ϕ
Pinion (50 Teeth)
$m_1 = 101.0g$
- DC Motor
- 50:1 Gearbox

FOR TURRET LOCAL FRAME:



$\vec{r} = -l_1\hat{\imath} + l_2\hat{\jmath}$ $\qquad \beta = \tan^{-1}\left(\frac{l_1}{l_2}\right)$

$|r| = \sqrt{l_1^2 + l_2^2}$

$\vec{v}_{G/O} = \vec{r} \times \vec{\dot{\theta}} = (-l_1\hat{\imath} + l_2\hat{\jmath}) \times (\dot{\theta}\hat{k})$

$\vec{v}_{G/O} = l_2\dot{\theta}\hat{\imath} + l_1\dot{\theta}\hat{\jmath}$

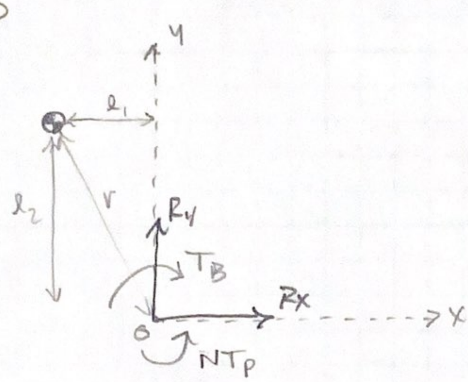
$|\vec{v}_{G/O}| = \dot{\theta}\sqrt{l_1^2 + l_2^2} = \dot{\theta} r$

$\vec{a}_{G/OT} = \vec{r} \times \ddot{\theta}$

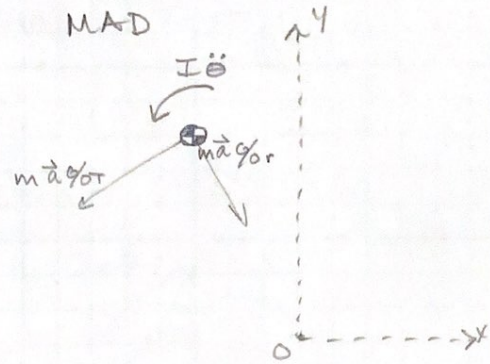
$= (-l_1\hat{\imath} + l_2\hat{\jmath}) \times (\ddot{\theta}\hat{k})$

$\vec{a}_{G/OT} = l_2\ddot{\theta}\hat{\imath} + l_1\ddot{\theta}\hat{\jmath}$

FBD



MAD



$$\left(\sum M_{FBD}\right)_o = \left(\sum M_{MAD}\right)_o$$

$$NT_p\hat{k} - T_B\hat{k} = I\ddot{\theta}\hat{k} + \vec{r}\times\vec{a}_{G/O_T}\, m$$

$$NT_p\hat{k} - \dot{\theta} b\hat{k} = (I_{yy} + mr^2)\ddot{\theta}\hat{k} + (-l_1\hat{i} + l_2\hat{j})\times(ml_2\ddot{\theta}\hat{i} + ml_1\ddot{\theta}\hat{j})$$

$$= (I_{yy} + mr^2)\ddot{\theta}\hat{k} + m(-l_1^2\ddot{\theta}\hat{k} - l_2^2\ddot{\theta}\hat{k})$$

$$NT_p - \dot{\theta} b = (I_{yy} + m(r^2 - l_1^2 - l_2^2))\ddot{\theta}$$

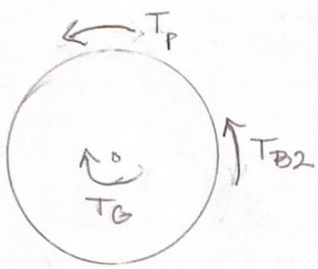
$$NT_p - \dot{\theta} b = (I_{yy} + m(\cancel{r^2 - r^2}))\ddot{\theta}$$

(1) $$\boxed{\ddot{\theta} = -\dot{\theta}\frac{b_1}{I_{yy}} + \frac{N}{I_{yy}}T_p}$$

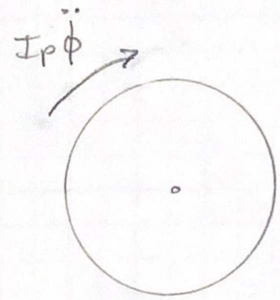
Determine Torque to Platter Through Pinion gear

Pinion gear:

FBD



MAD



$$\Sigma M_{FBD} = \Sigma M_{MAD}$$

$$T_p + T_{B2} - T_G = -I_p\ddot{\phi}$$

$$\boxed{T_p = -b_2\dot{\phi} - I_p\ddot{\phi} + T_G}$$

Assuming Motor Gearbox has no inertia/damping

$$\boxed{\underbrace{T_G}_{\text{Gearbox torque}} = 50\underbrace{T_m}_{\text{Motor Torque}} = 50\,\underline{K_t\, i_m}}$$

Motor Equations:

$$\frac{di_m}{dt} = -\frac{R}{L} i_m - \frac{K_v}{L} \Omega_m + \frac{1}{L} V_m$$

$$V_m = \frac{PWM}{100} \cdot 12V$$

$$\frac{di_m}{dt} = -\frac{R}{L} i_m - \frac{K_v}{L}(N50)\,\dot{\theta} + \frac{1}{L}\frac{12}{100} PWM$$

$$\theta = N\phi \qquad \phi = \frac{1}{N}\theta$$
$$\dot\theta = N\dot\phi \qquad \dot\phi = \frac{1}{N}\dot\theta$$
$$\ddot\theta = N\ddot\phi \qquad \ddot\phi = \frac{1}{N}\ddot\theta$$

$$T_p = -b_2\frac{1}{N}\dot\theta - I_p\frac{1}{N}\ddot\theta + T_G$$

Sub into Eq 1

$$\ddot\theta = -\dot\theta\frac{b}{I_{yy}} + \frac{N}{I_{yy}}\left(-\frac{b_2}{N}\dot\theta - \frac{I_p}{N}\ddot\theta + 50k_t\, i_m\right)$$

$$\ddot\theta = -\dot\theta\frac{b}{I_{yy}} - \frac{b_2}{I_{yy}}\dot\theta - \frac{I_p}{I_{yy}}\ddot\theta + \frac{N50k_t}{I_{yy}} i_m$$

$$\ddot\theta\left(\frac{I_{yy}+I_p}{I_{yy}}\right) = -\dot\theta\left(\frac{b_1+b_2}{I_{yy}}\right) + \frac{N50k_t}{I_{yy}} i_m$$

$$\boxed{\frac{d\dot\theta}{dt} = -\dot\theta\left(\frac{b_1+b_2}{I_{yy}+I_p}\right) + \frac{N50k_t}{I_{yy}+I_p} i_m}$$

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \dot\theta \\ i_m\end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b_1+b_2}{I_{yy}+I_p} & \frac{N50k_t}{I_{yy}+I_p} \\ 0 & -\frac{K_vN50}{L} & -\frac{R}{L}\end{bmatrix}}_{A}\begin{bmatrix}\theta \\ \dot\theta \\ i_m\end{bmatrix} + \underbrace{\begin{bmatrix}0 \\ 0 \\ \frac{12}{100L}\end{bmatrix}}_{B}\begin{bmatrix}PWM\end{bmatrix}$$

Outputs of interest: θ, $\dot{\theta}$, v_M

$$v_M = \frac{PWM}{100} \cdot 12$$

$$\underbrace{\begin{bmatrix} \theta \\ \dot{\theta} \\ v_M \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} \theta \\ \dot{\theta} \\ i_M \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{12}{100} \end{bmatrix}}_{D} \begin{bmatrix} PWM \end{bmatrix}$$