Controllability Analysis of a Quadrotor-like Autonomous Underwater Vehicle

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***Abstract*—A quadrotor-like autonomous underwater vehicle (QLAUV) that is similar with but rather different from quadrotor unmanned aerial vehicles has been reported recently. This paper addresses the local controllability problem of QLAUV, which is base and premise for further localization and navigation studies. By using geometric control theory, the accessibility property of the QLAUV has been firstly analyzed. Then, a nonlinear controllability, the so-called small-time locally controllability (STLC) has been analyzed for the horizontal plane motion, including kinematics and dynamics. Finally, we investigate the conditions that the STLC should be satisfied.**

I. Introduction

The development of autonomous underwater vehicles (AUVs) started in the early 1970s. Since then, due to the dramatic advancements in computing and sensory technology, AUVs have been increasingly used for a variety of tasks, such as scientific, military research and commercial applications [1–3]. Accurate navigation and localization are essential for these applications. In general, taking into account weight, reliability, complexity, and efficiency, AUVs are commonly designed without a lateral actuator, which is a typical underactuated control system. For such a system, navigation and localization are still a challenging research area of increasing interest, partly because it exhibits nonholonomic constraints and is not full feedback linearizable [5, 6]. From a theoretical perspective, it can not be stabilized by smooth static state feedback [7]. In order to investigate the limitation of localization and navigation of AUVs, it is preferable to study the controllability of AUVs that is further beneficial to the controller design.

Controllability is a problem about the ability to drive the system from one point to another with the given class of inputs in finite time. Kalman *et al.* [8] gave an answer for the controllability of linear systems over half a century ago. For a nonlinear system, it is very difficult to analyze the global controllability since it does not satisfy superposition in general [9]; while often is the local controllability analysis. Two major types of approaches have been used to study the local nonlinear controllability. The classical approach to analyze the nonlinear controllability utilizes the local linearization about a given point. In [10], the authors reviewed the first nonlinear local controllability result, which states that if the linearized system at an equilibrium point p is controllable, then the system is locally controllable from p in small time. This "small-time locally controllability property" is abbreviated as STLC.

However, the controllability condition for a linearized system about a given point is not necessary for a nonlinear system, that is, there exists a class of nonlinear systems that are linearly uncontrollable but nonlinear controllable [11]. The alternative is to use geometric control theory to analyze the local nonlinear controllability. In [12], Bloch introduced some definitions and applications about geometric control theory in the background of controllability analyses of nonlinear systems. In [13], the accessibility and controllability of nonlinear systems based on planning algorithms are introduced comprehensively. In [10], a general sufficient condition for local controllability of a nonlinear system at an equilibrium point was proved, and many previous methods were demonstrated to be particular cases of this result. Based on the working in [10], Goodwine proposed a method for assessing the nonlinear controllability of systems with unilateral control inputs in [14].

Some works have been done to study the controllability of aerial and underwater vehicles. In [15], Saied addressed the attitude controllability issue for a multirotor unmanned aerial vehicle (UAV) in case of actuators failures. The author analyzed the STLC of the system attitude dynamics utilizing the nonlinear controllability theory and compared fault tolerant capabilities of a variety of actuators configurations. Hassan investigated the airplane flight dynamics using the linear controllability analysis and geometric control formulation in [16] and [17], respectively. For the underwater vehicles, Pettersen proved that the surface vessel satisfied the STLC in [18]. In [19], Smith validated that underwater vehicles met the broad controllability with the proper vertical actuation and utilizing spatiotemporal variations in water speed and direction.

This paper makes a comprehensive analysis for the nonlinear controllability of a quadrotor-like autonomous underwater vehicle (QLAUV). Similar to quadrotor unmanned aerial vehicles (UAV), the designed QLAUV is equipped with four identical thrusts only to implement motion control [20]. In this paper, the STLC of the QLAUV has been analyzed based on geometric control theory for the horizontal plane motion. The remainder of this paper is organized as follows. In Section II, the kinematic and dynamic models of the QLAUV are presented. The nonlinear controllability analysis based

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on geometric control theory is introduced in Section III. In Section IV, the STLC of the QLAUV for the horizontal plane motion is analyzed. Finally, some concluding remarks are made in Section V.

II. QLAUV NONLINEAR MODELLING

This section introduces primarily the kinematic and dynamic equations of motion for the QLAUV. Section II-A describes the vehicle as a whole. Section II-B gives the kinematic and dynamic equations of the system, complex hydrodynamic effects are also taken into account. Finally, Section II-C investigates the actuation system of the vehicle, related to forces and moments equations.

A. Vehicle description

The QLAUV has been developed recently in our laboratory [20]. It is supported by the Project of the State Key Laboratory of Industrial Control Technology. This QLAUV is about 120 cm in length and 28 cm in diameter, and it is equipped with four identical thrusts. For reducing the negative impact of the hydrodynamic forces, the hull of the vehicle is designed to a cylindrical shape, and the hand and the tail are elliptical, very typical for underwater vehicles. Fig. 1 depict the actual shape of the QLAUV.

As shown in Fig. 1, four thrusts are symmetrically located apart at two sides of the hull, and both the center of the gravity of the QLAUV and the geometric center of the thrusts are designed to be in the same horizontal plane. Nevertheless, there exists a deflection angle between the body of the thrusts and the horizontal plane. Besides, the direction of the heads of the thrusts is opposite: on one side, the heads of the two thrusts are close, on the other side, the heads keep away from each other. Through above thrusts configuration, Attitude control of the QLAUV can be implemented independently by the actuation system of the QLAUV.

B. Vehicle kinematics and dynamics

Before analyzing the kinematics and dynamics of the QLAUV, it is convenient to define a inertial coordinate frame $\{U\} = (x_n, y_n, z_n)$ with origin o_n and a body-fixed coordinate frame $\{B\} = (x_b, y_b, z_b)$ with origin o_b, as depicted in Fig. 2. The detailed body-fixed coordinate frame is defined as:

- x_b: longitudinal axis (directed from aft to fore)
- y_b: transversal axis (directed to starboard)
- z_b: normal axis (directed from top to bottom)

Fig. 1. Quadrotor-like autonomous underwater vehicle

where x_b, y_b and z_b are chosen to coincide with the principal axes of inertia. Assume the center of gravity (CG) of the vehicle is coincident with o_b.

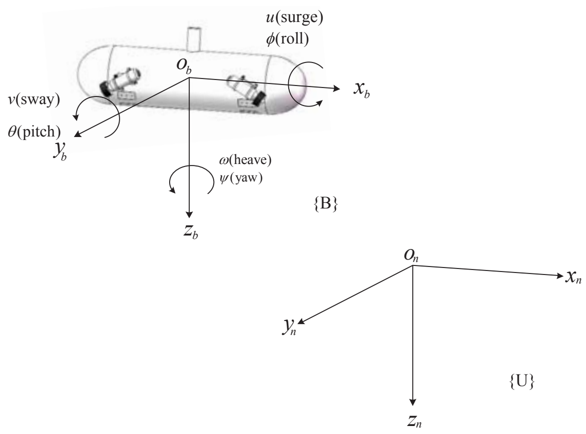

Fig. 2. Inertial frame and body-fixed frame for QLAUV.

Assume the underwater vehicle is naturally buoyant. With the assumption and neglecting hydrostatics produced by gravitational/buoyancy forces and moments due to the existence of the metacentric height, the kinematic and dynamic model of the underwater vehicle moving in six degrees of freedom is expressed as [21, 22]:

$$\begin{aligned} &\dot{\eta} = J(\eta)\nu \\ &(M_{RB} + M_A)\dot{\nu} = -(C_{RB}(\nu) + C_A(\nu))\nu - D(\nu)\nu + \Gamma \end{aligned} \tag{1}$$

where $\eta = [x, y, z, \phi, \theta, \psi]^T$, $\nu = [u, v, w, p, q, r]^T$. ϕ, θ, ψ, p, q, and r denote the roll, pitch and yaw angles and angular velocities; while x, y, z, u, v, w, are the surge, sway and heave displacements and velocities, respectively. The terms $J(\eta)$, M_{RB}, M_A, $C_{RB}(\nu)$, $C_A(\nu)$, $D(\nu)$, and Γ denote the kinematic transformation, rigid-body inertia mass, hydrodynamic added mass, rigid-body Coriolis, hydrodynamic Coriolis, damping and actuation system, respectively, and will be defined in the sequel.

The kinematic transformation $J(\eta)$ is given as:

$$\begin{aligned} J(\eta) &= \begin{bmatrix} J_1(\eta) & 0_{3\times3} \\ 0_{3\times3} & J_2(\eta) \end{bmatrix} \\ J_1(\eta) &= \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\theta & c\psi s\theta c\phi + s\psi s\theta \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\theta & s\psi s\theta c\phi - c\psi s\theta \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \\ J_2(\eta) &= \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix} \end{aligned} \tag{2}$$

where $c(\cdot)$, $s(\cdot)$ and $t(\cdot)$ are reduced forms of $\cos(\cdot)$, $\sin(\cdot)$ and $\tan(\cdot)$, respectively. Assume that the vehicle has three planes of symmetry so that the inertia, added mass and damping matrices are diagonal. In this setting, the inertia mass M_{RB} and added mass M_A are expressed as:

$$\begin{aligned} M_{RB} &= diag(m, m, m, I_x, I_y, I_z) \\ M_A &= -diag(X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}). \end{aligned} \tag{3}$$

The rigid-body Coriolis $C_{RB}(\nu)$ and hydrodynamic Coriolis $C_A(\nu)$ can be derived using Kirchhoff's equation [21], the detailed form is expressed as:

$$\begin{aligned} C_{RB}(\nu) &= \begin{bmatrix} 0 & 0 & 0 & 0 & mw & -mv \\ 0 & 0 & 0 & -mw & 0 & mu \\ 0 & 0 & 0 & mv & -mu & 0 \\ 0 & mw & -mv & 0 & I_z r & -I_y q \\ -mw & 0 & mu & -I_z r & 0 & I_x p \\ mv & -mu & 0 & I_y q & -I_x p & 0 \end{bmatrix} \\ C_A(\nu) &= \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot{w}} w & Y_{\dot{v}} v \\ 0 & 0 & 0 & Z_{\dot{w}} w & 0 & -X_{\dot{u}} u \\ 0 & 0 & 0 & -Y_{\dot{v}} v & X_{\dot{u}} u & 0 \\ 0 & -Z_{\dot{w}} w & Y_{\dot{v}} v & 0 & -N_{\dot{r}} r & M_{\dot{q}} q \\ Z_{\dot{w}} w & 0 & -X_{\dot{u}} u & N_{\dot{r}} r & 0 & -K_{\dot{p}} p \\ -Y_{\dot{v}} v & X_{\dot{u}} u & 0 & -M_{\dot{q}} q & K_{\dot{p}} q & 0 \end{bmatrix}. \end{aligned} \tag{4}$$

Also, the damping matrix D is expressed as:

$$D = diag(X_u, Y_v, Z_w, K_p, M_q, N_r). \tag{5}$$

For simplicity, the nonlinear damping terms of D are neglected. Actuation system Γ will be introduced in the following subsection.

C. Actuation system Γ

Due to the fact that the dynamic model of an AUV is derived in the body-fixed reference frame, the actuation system should be described in the same frame. The configuration of the thrusts is described as Fig. 3, in which thruster i presents the ith thrust, $i = 1, 2, 3, 4$. And i is limited to the same range for the remainder of the paper if not be stated explicitly. Fig. 3 shows that thrust *1* and *2* signed blue are located on the right side of the QLAUV, and thrust *3* and *4* signed red are located on the left.

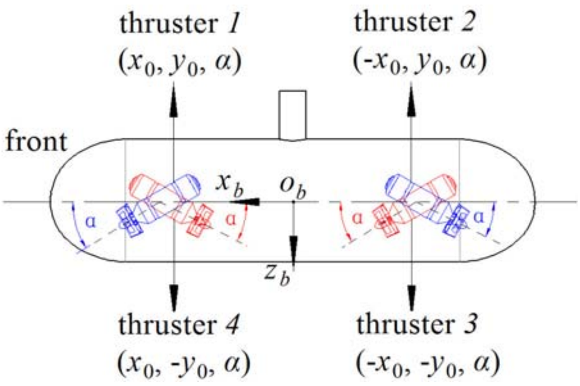

Fig. 3. The configuration of the thrusts in a left view.

As described in subsection II-A, the centers of the four thrusts are located symmetrically in the horizontal $x_b y_b$-plane with respect to the axis x_b and y_b. The positions of the thrusts relative to origin o_b can be expressed by the vector $\boldsymbol{q} = (x_0, y_0, \alpha)$, where x_0, y_0, α denote the longitudinal position, transversal position and deflection angle with respect to the horizontal $x_b y_b$-plane, respectively. Note that the deflection angles of four thrusts are identical. Thus, the configurations of the four thrusts are: (x_0, y_0, α), $(-x_0, y_0, \alpha)$, $(-x_0, -y_0, \alpha)$, and $(x_0, -y_0, \alpha)$, respectively.

According to the geometry of the QLAUV, the actuation system, i.e., the mapping between the thrusts lifts and the total forces and moments is given by

$$\Gamma = BF \tag{6}$$

where $\Gamma = [X, Y, Z, K, M, N]^T$ denote the total forces and moments, $F = [F_1, F_2, F_3, F_4]^T$ are the thrusts lifts, with F_i being the lift produced by the thrust *i*, and the force-distributed matrix B in a parameterized form is

$$B = \begin{bmatrix} \cos\alpha & -\cos\alpha & \cos\alpha & -\cos\alpha \\ 0 & 0 & 0 & 0 \\ \sin\alpha & \sin\alpha & \sin\alpha & \sin\alpha \\ y_0 \sin\alpha & y_0 \sin\alpha & -y_0 \sin\alpha & -y_0 \sin\alpha \\ -x_0 \sin\alpha & x_0 \sin\alpha & x_0 \sin\alpha & -x_0 \sin\alpha \\ -y_0 \cos\alpha & y_0 \cos\alpha & y_0 \cos\alpha & -y_0 \cos\alpha \end{bmatrix}. \tag{7}$$

The elements of B are derived by the geometric analysis of the forces imposed on the QLAUV. For example, as shown in Fig. 4, f_{ix_b}, f_{iy_b} and f_{iz_b} denote the components of F_i with respect to the x_b, y_b and z_b axes, respectively, and o_i denotes the application point of F_i. The amplitude of f_{ix_b} can be presented by $F_i \cos\alpha$. Thus the total force of surge motion X can be computed by:

$$\begin{aligned} X &= f_{1x_b} + f_{2x_b} + f_{3x_b} + f_{4x_b} \\ &= F_1 \cos\alpha - F_2 \cos\alpha + F_3 \cos\alpha - F_4 \cos\alpha \\ &= \begin{bmatrix} \cos\alpha & -\cos\alpha & \cos\alpha & -\cos\alpha \end{bmatrix} F, \end{aligned} \tag{8}$$

which forms the first row of B. Similar processes can be followed in the other directions of motion.

The columns of B imply that each thrust can provide forces or moments in the directions of surge, heave, roll, pitch, and yaw so that the actuation system is coupled. A transformation matrix

$$T = \begin{bmatrix} \frac{1}{4\cos\alpha} & \frac{1}{4\sin\alpha} & \frac{1}{4y_0\sin\alpha} & -\frac{1}{4x_0\sin\alpha} \\ -\frac{1}{4\cos\alpha} & \frac{1}{4\sin\alpha} & \frac{1}{4y_0\sin\alpha} & \frac{1}{4x_0\sin\alpha} \\ \frac{1}{4\cos\alpha} & \frac{1}{4\sin\alpha} & -\frac{1}{4y_0\sin\alpha} & \frac{1}{4x_0\sin\alpha} \\ -\frac{1}{4\cos\alpha} & \frac{1}{4\sin\alpha} & -\frac{1}{4y_0\sin\alpha} & -\frac{1}{4x_0\sin\alpha} \end{bmatrix} \tag{9}$$

is introduced here in order to decouple the actuation system. Define $F' = [F'_1, F'_2, F'_3, F'_4]^T$ satisfying

$$F = TF', \tag{10}$$

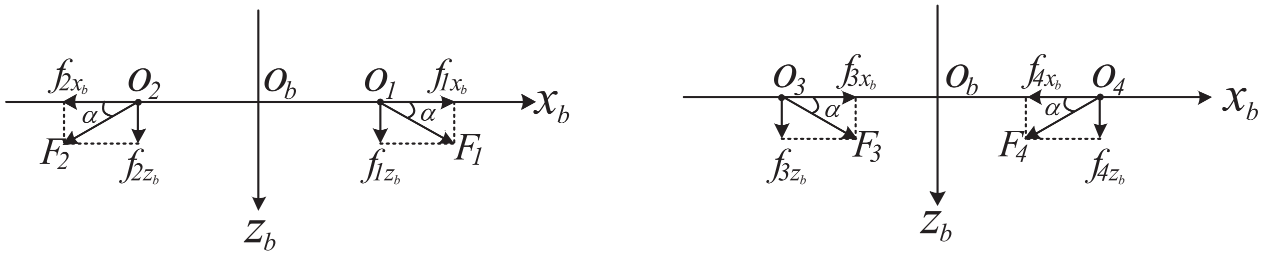

Fig. 4. The decomposition of the thrusts lifts in the plane of projection of $x_b z_b$-plane.

by which the mapping of the actuation system becomes

$$\varGamma = \begin{bmatrix} F_1^{'} \\ 0 \\ F_2^{'} \\ F_3^{'} \\ F_4^{'} \\ \beta F_4^{'} \end{bmatrix}. \tag{11}$$

Now $\beta = (y_0 \cos\alpha)/(x_0 \sin\alpha)$ is the only coupling term in the actuation system. To guarantee the column full rank of the matrix B and nonsingularity of the matrix T, the parameter α should satisfy $\alpha \neq k\pi/2$, $k \in N$. Compared with traditional AUVs deploying the thrust for speed control and the rudder for angle control, the QLAUV can implement more independent motions in different directions. For instance, since yaw angle control is independent of surge control, the QLAUV can implement yaw control at the zero surge speed state.

Applying the above notations, the system (1) can be rewritten into the following affine nonlinear model:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \sum_{i=1}^{4} \boldsymbol{g}_i F_i^{'} \tag{12}$$

where $\boldsymbol{x} \in R^{12}$, $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}_i$, are smooth vector fields, and $F_i^{'}$ denotes the decoupled thrust force. The detailed model is expressed in Appendix.

III. Nonlinear controllability (STLC) analysis

Before studying the STLC of the affine system, some definitions and useful criteria should be outlined. Consider the affine nonlinear control system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \sum_{i=1}^{m} \boldsymbol{g}_i(\boldsymbol{x}) u_i, \tag{13}$$

which is defined on a smooth n-dimensional manifold M, where $\boldsymbol{x} \in M$, $\boldsymbol{f}$ and the $\boldsymbol{g}_i, i = 1, ..., m$, are smooth real analytic vector fields on M. The admissible control input u_i is a time-dependent map from the nonnegative reals to a constraint set $\Omega \subset R$.

Definition 3.1 ([12]): Given $x_0 \in M$, $R(x_0, t)$ is defined to be the set of all $x \in M$ for which there exists an admissible control u to steer the control system from $x(0) = x_0$ to $x(t) = x$. The **reachable set** from x_0 at time T is defined to be

$$R_T(x_0) = \cup_{0<t\leq T} R(x_0, t). \tag{14}$$

Definition 3.2 ([14]): A system is said to be **accessible** from x_0 if for any $T > 0$, the set $R_T(x_0)$ has a nonempty interior; In addition, if x_0 lies in the interior of $R_T(x_0)$, the system is called **small-time locally controllable (STLC)** from x_0.

However, it is difficult to analyze the accessibility and STLC of nonlinear systems by the above definitions. A more practical way is to use geometric control theory. Relative definitions and theorems are introduced as follows.

Definition 3.3 ([12]): The **accessibility algebra** ζ of system (13) is the smallest Lie algebra of vector fields on M that contains the vector fields $\mathbf{f}, \mathbf{g}_1, ..., \mathbf{g}_m$.

Note that the accessibility algebra is just the span of all possible Lie brackets of $\boldsymbol{f}$ and the $\boldsymbol{g}_i$.

Definition 3.4 ([12]): The **accessibility distribution** C of system (13) is defined to be the distribution generated by the vector fields in ζ; i.e., $C(x)$ is the span of the vector fields X in ζ at x.

Furthermore, Bloch gave the following condition for the accessibility property of (13) based on the above definitions in [12].

Theorem 3.1: Consider the system (13), if *dim* $C(x_0) = n$ (i.e., the accessibility algebra spans the tangent space to M at x_0), then for any $T > 0$, the set $R_T(x_0)$ has a nonempty interior; i.e., the system has the accessibility property from x_0.

When the hypotheses of this theorem hold, we say that the ***Lie Algebra Rank Condition (LARC)*** holds at x_0 [23].

For a driftless control-affine system in which the vector filed $\boldsymbol{f}$ equals to a zero vector, LARC is equivalent to STLC [15]. However, with drift, the condition is not sufficient, as some of the vectors involved drift term may have directional constraints, which obstructs the controllability of the system [13]. For the systems with drift, Sussman's General Theorem[10] states that in order to guarantee the STLC, some conditions about good and bad Lie brackets should be satisfied in addition to the LARC.

Theorem 3.2: A system that satisfies the LARC by good Lie brackets terms up to degree i is STLC if all bad Lie brackets of degree $j \leq i$ are neutralized.

Note that a bad Lie bracket is neutralized if it can be presented by a linear combination of good Lie brackets of lower degree. Thus, the bad bracket does not obstruct STLC. Particularly, a bad Lie bracket is not an obstruction to STLC if it equals to zero at the equilibrium x_0.

A method of classifying Lie brackets as *good* or *bad* is as follows [10]: For a given Lie bracket X comprising the drift vector $\boldsymbol{f}$ and control vectors $\boldsymbol{g}_1, ...\boldsymbol{g}_m$, the degree of X with respect to the $\boldsymbol{f}$ and $\boldsymbol{g}_i$ is denoted by $\chi^f(X)$ and $\chi^{g_i}(X)$, respectively, defined by

- $\chi^f(X)$: the number of times that $\boldsymbol{f}$ appears in the bracket X,
- $\chi^{g_i}(X)$: the number of times that $\boldsymbol{g}_i$ appears in the bracket X.

The degree of X is defined by the value of $\chi^f(X) + \sum_{i=1}^m \chi^{g_i}(X)$. If $\chi^f(X) + \sum_{i=1}^m \chi^{g_i}(X)$ is odd and $\chi^{g_i}(X)$ is even (including 0), the bracket X is called a bad bracket; or else, the bracket is called good. For the affine system (13), f, $[g_1, [f, g_1]$, and $[g_2, [f, g_2]$ are bad brackets, however, $[g_1, [f, g_2]$ and $[f, g_1]$ are thought of as good brackets.

IV. Geometric control formulation for AUV kinematics and dynamics

This section mainly analyzes STLC of the QLAUV under the condition of the horizontal plane motion by using geometric control theory, including dynamics and kinodymanics.

A. Controllability of dynamics

Controllability of dynamics mainly focuses on velocities and angular velocities of the vehicle regardless of displacements and attitudes. In the horizontal plane motion, we mainly consider surge speed, sway speed and yaw angular speed as the system dynamics. The 3-DOF dynamics of the QLAUV can be expressed in a control-affine form as in (13), where $\boldsymbol{x} = [u, v, r]^T$ is the state. The detailed form is

$$\begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_6 \end{bmatrix} + \begin{bmatrix} \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} F_1^{'} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} F_2^{'} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} F_3^{'} + \begin{bmatrix} 0 \\ 0 \\ \frac{\beta}{m_6} \end{bmatrix} F_4^{'}. \tag{15}$$

The drift field $\boldsymbol{f}$ is written as $\boldsymbol{f}(\boldsymbol{x}) = [f_{1-2}, f_6]^T$, the explanation of the simplified form f_{1-2} can be found in Appendix. And $\boldsymbol{g}_i$ is the corresponding control input vector with proper orders relative to the input $F_i^{'}$. Similar expressions are used with a slight abuse at the rest of the section. When applying the Lie brackets and Lie Algebra, the accessibility distribution is expressed as:

$$C = span\{\boldsymbol{g}_1, \boldsymbol{g}_4, [\boldsymbol{g}_1, [\boldsymbol{f}, \boldsymbol{g}_4]]\} \tag{16}$$

with det $(C) = (-\beta^2)/(m_1 m_2 m_6^2) \neq 0$ when $\beta \neq 0$, so its dimension is equal to that of the state $\boldsymbol{x}$. The LARC is satisfied by good brackets with maximum degree ≤ 3.

Note that all bad brackets satisfying degree ≤ 3 include

$$\boldsymbol{f}, [\boldsymbol{g}_1, [\boldsymbol{f}, \boldsymbol{g}_1]], [\boldsymbol{g}_2, [\boldsymbol{f}, \boldsymbol{g}_2]], [\boldsymbol{g}_3, [\boldsymbol{f}, \boldsymbol{g}_3]], [\boldsymbol{g}_4, [\boldsymbol{f}, \boldsymbol{g}_4]]. \tag{17}$$

In zero-velocities states, all the brackets abovementioned are equal to $\boldsymbol{0}_{3\times1}$, they can be written as a linear combination of good Lie brackets of lower degree, so-called neutralized. Therefore the system is STLC from zero-velocities states considering u, v, r degrees only. Even if the drift vector f is equal to zero at the equilibrium, the system can not be thought of as a driftless one, by which the system will reduce some dynamics, resulting the Lie bracket $[\boldsymbol{g}_1, [\boldsymbol{f}, \boldsymbol{g}_4]]$ becomes $\boldsymbol{0}_{3\times1}$, for example.

As shown in the section II, the coupling term β is relevant to the deflection α. When the thrusts are vertical to the horizon plane, which means that α is equal to $\pi/2$, the value of β is 0. The rank of the accessibility distribution C is also equal to 0. Thus, the LARC does not hold at the zero-velocities states and the system is not STLC from zero-velocities states in the case.

B. Controllability of Horizon plane kinodynamics

In this part, considering the kinematics and dynamics of the vehicle in the horizontal plane, where $\boldsymbol{x} = [u, v, r, x, y, \psi]^T$, the drift field $\boldsymbol{f} = [f_{1-2}, f_{6-8}, f_{12}]^T$, and the control input fields $\boldsymbol{g}_i$ are:

$$\begin{aligned} \boldsymbol{g}_1 &= [1/m_1, 0, 0, 0, 0, 0]^T \\ \boldsymbol{g}_2 &= \boldsymbol{g}_3 = \boldsymbol{0}_{6\times1} \\ \boldsymbol{g}_4 &= [0, 0, 1/m_6, 0, 0, 0]^T. \end{aligned} \tag{18}$$

The accessibility distribution is expressed as:

$$\begin{aligned} C = span\{&\boldsymbol{g}_1, \boldsymbol{g}_4, [\boldsymbol{f}, \boldsymbol{g}_1], [\boldsymbol{f}, \boldsymbol{g}_4], [\boldsymbol{g}_1, [\boldsymbol{f}, \boldsymbol{g}_1]], \\ &[\boldsymbol{g}_1, [\boldsymbol{f}, [\boldsymbol{f}, \boldsymbol{g}_4]]]\} \end{aligned} \tag{19}$$

with $\det(C) = (-\beta^4 (c\phi)^2 (m_1 - m_2))/(m_1^3 m_2^2 m_6^4)$. The roll angle $\phi \approx 0$ and $c\phi \approx 1$, then $\det(C) \approx \beta^4 (m_2 - m_1)/(m_1^3 m_2^2 m_6^4) \neq 0$ when $m_1 \neq m_2$. Thus, LARC is satisfied with maximum degree ≤ 4. As the degree of the bad brackets is odd, so the corresponding bad brackets satisfying degree ≤ 4 are identical with (17). Simple computation indicates that abovementioned bad brackets are equal to $\boldsymbol{0}_{6\times1}$ in zero-velocities states, so the system in the case is STLC if $m_1 \neq m_2$.

V. Conclusion

In the paper, the small-time locally controllability (STLC) of the QLAUV has been analyzed based on geometric control theory, and a transformation matrix T is technically introduced to simplify greatly the computation of accessibility distribution C. The STLC conditions for the horizontal plane motion have been developed, and the STLC holds for critical horizontal plane motion from zero-velocities states.

The detailed model of the QLAUV

In the part, we present the detailed model of the QLAUV in the affine nonlinear form used in the paper. The model can be expressed in a control-affine system as:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \sum_{i=1}^{4} \boldsymbol{g}_i F_i^{'} \tag{20}$$

where

$$\begin{aligned} \boldsymbol{x} &= [u, v, w, p, q, r, x, y, z, \phi, \theta, \psi]^T \\ \boldsymbol{g}_1 &= [1/m_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \\ \boldsymbol{g}_2 &= [0, 0, 1/m_3, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \\ \boldsymbol{g}_3 &= [0, 0, 0, 1/m_4, 0, 0, 0, 0, 0, 0, 0, 0]^T \\ \boldsymbol{g}_4 &= [0, 0, 0, 0, 1/m_5, \beta/m_6, 0, 0, 0, 0, 0, 0]^T \end{aligned}$$

$$
\boldsymbol{f}(\boldsymbol{x}) = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}]^T
= \begin{bmatrix}
-(X_u u - m_2 vr + m_3 wq)/m_1 \\
-(Y_v v + m_1 ur - m_3 wp)/m_2 \\
-(Z_w w - m_1 uq + m_2 vp)/m_3 \\
-(K_p p - (m_5 - m_6) qr + (m_2 - m_3) vw)/m_4 \\
-(M_q q + (m_4 - m_6) pr - (m_1 - m_3) uw)/m_5 \\
-(N_r r - (m_4 - m_5) pq + (m_1 - m_2) uv)/m_6 \\
w(s\phi s\psi + c\phi c\psi s\theta) - v(c\phi s\psi - c\psi s\phi s\theta) + uc\psi c\theta \\
v(c\phi c\psi + s\phi s\psi s\theta) - w(c\psi s\phi - c\phi s\psi s\theta) + uc\theta s\psi \\
wc\phi c\theta - us\theta + vc\theta s\phi \\
p + rc\phi t\theta + qs\phi t\theta \\
qc\phi - rs\theta \\
rc\phi / c\theta + qs\phi / c\theta
\end{bmatrix}
$$

in which $m_1 = m - X_{\dot{u}}$, $m_2 = m - Y_{\dot{v}}$, $m_3 = m - Z_{\dot{w}}$, $m_4 = I_x - K_{\dot{p}}$, $m_5 = I_y - M_{\dot{q}}$, $m_6 = I_z - N_{\dot{r}}$. For simplicity, the drift field $\boldsymbol{f}(\boldsymbol{x})$ is expressed as a combination form in the paper. For instance, $\boldsymbol{f}(\boldsymbol{x}) = [f_{1-3}, f_{5-6}]^T = [f_1, f_2, f_3, f_5, f_6]^T$.

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