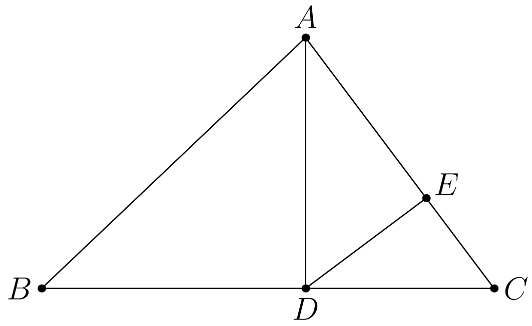
2024 SSMO Relay Round 1

SMO Team

RR 1 Part 1: Let AD be an altitude of triangle ABC and let DE be an altitude of triangle ACD. If $AB = 29$, $CE = 9$, and $DE = 12$, what is the area of triangle ABC?



RR 1 Part 2: Let $T = TNYWR$. A circular necklace is called *interesting* if it has T black beads and T white beads. A move consists of cutting out a segment of consecutive beads and reattaching it in reverse. It is possible to change any *interesting* necklace into any other *interesting* necklace using at most x moves. Find x. (Note: Rotations and reflections of a necklace are considered the same necklace).

RR 1 Part 3: Let $T = TNYWR$. In a circle, there are T people. $T - 2$ of them have red shoes, and two of them have blue shoes. First, they will randomly eliminate somebody from the circle. Then, they will randomly eliminate somebody with red shoes from the circle, and the cycle repeats until there is only one person left. The probability this person has blue shoes can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

2024 SSMO Relay Round 2

SMO Team

RR 2 Part 1: In a regular hexagon $ABCDEF$, let X be a point inside the hexagon such that $XA = XB = 3$. If the area of the hexagon is $6\sqrt{3}$, then XE^2 can be expressed as $a + b\sqrt{c}$, where a, b, c are positive integers with c squarefree. Find $a + b + c$.

RR 2 Part 2: Let $T = TNYWR$. If

$$a = \sum_{n=1}^{N} n(n+1)(n+2),$$

find the last three digits of a.

RR 2 Part 3: Let $T = TNYWR$. A point P is randomly chosen inside the square with vertices $A = (0, 0), B = (0, T), C = (T, T)$, and $D = (T, 0)$. Find the perimeter of the set S containing all points P for which $AP + CP \geq BP + DP$.

2024 SSMO Relay Round 3

SMO Team

RR 3 Part 1: Alice and Bob on the second floor of the building right next to the escalator moving upwards. Alice and Bob each run at 200 steps per minute and the escalator is a total of 225 steps. Alice chooses to use the escalator moving upwards, while Bob chooses to run to the escalator moving downwards, 300 steps away, and then ride the escalator down. If Alice and Bob hit the bottom floor at the exact same time, find the speed of the escalators in terms of steps per minute.

RR 3 Part 2: Let $T = TNYWR$. Find the greatest odd integer n for which $n^2 + (T-1)n$ is a perfect square.

RR 3 Part 3: Let $T = TNYWR$. Riley and Boris are playing a game on a $(T-1) \times (T-1)$ grid of dots. The game alternates turns and starts with Riley. Each turn, a player draws a line connected two different random dots, exactly 1 unit apart. The first person to complete the first unit square loses the game. Given that Riley plays optimally and Boris plays randomly, the probability that Riley wins can be expressed as P. Find the least positive integer a such that aP is an integer.

2024 SSMO Relay Round 4

SMO Team

RR 4 Part 1: Freddy the Frog can jump 1 unit right or up. He is at $(1, 1)$ and wants to get to $(7, 4)$. However, he is scared of points $(3, 1)$ and $(3, 2)$ and will not hop onto those points. How many ways can he reach his destination?

RR 4 Part 2: Let $T = TNYWR$. Regular octagon $OLYMPIAD$ is perfectly inscribed within Circle Q. Circle Q has area $T\pi$. If the area of octagon $OLYMPIAD$ is $a\sqrt{b}$, where a and b are positive integers with b squarefree. Find $a + b$.

RR 4 Part 3: Let $T = TNYWR$. Real numbers a, b, and c satisfy

$$a + b = -ca^3 - abc \qquad = 4b^3 - abc = T$$

Find the value of $abc - c^3$.

2024 SSMO Relay Round 5

SMO Team

RR 5 Part 1: Let the super factorial $!(n)$ be defined on positive integers as $\prod_{i=1}^{n} i!$. Find the largest positive integer k such that that there are exactly k positive integers n such that $!(n)$ has fewer than k trailing zeroes.

RR 5 Part 2: Let $T = TNYWR$. In the game of high and low, the computer chooses two integers without replacement from the set $\{1, 2, 3, \ldots, T\}$. The computer displays the first integer and asks the player if the second integer is higher or lower. Given that the player always plays optimally, the chances of guessing correctly can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

RR 5 Part 3: Let $T = TNYWR$. Let k be the maximum prime factor that divides T. How many values of x satisfy

$$\sin x^2 + \cos x^2 = \sin^2 x + \cos^2 x \quad \text{and} \quad -k \leq x \leq k?$$