2024 SSMO Accuracy Round

SMO Team

AR 1: Let a time of day be three-full if exactly three of its digits are 3s when displayed on a 12-hour clock in the $hh:mm:ss$ format. How many seconds of the day are three-full?

AR 2: Equilateral triangle N is inscribed within circle O. A smaller equilateral triangle P is inscribed within N such that the vertices of P lie on the midpoints of N. The ratio of the areas between O and P can be expressed as $\frac{a\pi\sqrt{b}}{c}$, where b is a squarefree positive integer and a and c are relatively prime positive integers. Find $a + b + c$.

AR 3: Three distinct random integers a, b, and c are selected so that $1 \le a, b, c \le 10$. The probability that $5 | a! + b! + c!$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

AR 4: Right triangle ABC has a right angle at C and hypotenuse 1. Let points D and E lie on AC such that $\angle BDC = \angle BEC = 45°$. A, D, C, and E are collinear in that order. Given that $EA = 13DA$, the area of $\triangle ABC$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

AR 5: Let $ABCD$ be a convex quadrilateral such that $\angle ABC = 120°$ and $\angle ADC = 60°$. If $AB = BC = CD = 5$, the area of $ABCD$ can be expressed as $a + b\sqrt{c}$, where a, b, c, are positive integers with c squarefree. Find $a + b + c$.

AR 6: Three six-sided dice are rolled. Then, the product of the three numbers on the top faces in calculated. The probability that the product is not a perfect square can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

AR 7: Find the value of $a + b + c + d$ given

$$a^2 + d^2 = b^2 + c^2 = 361,$$
$$ac + bd = 247, \text{ and}$$
$$ab + cd = 570.$$

AR 8: $ABCD$ is a convex cyclic quadrilateral with $AB = 2, BC = 5, CD = 10$, and $AD = 11$. Let W, Y, X, and Z be the midpoints of sides AB, BC, CD, and DA respectively. The value of $|(WX^2 - YZ^2)|$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

AR 9: In the game of S-set, there are 3^{12} unique cards, each card containing a 12 traits of variants A, B, and C. A full house in the game of Sset is defined to be a hand of cards in which for each trait, all cards in the hand either share the same variant or have all different variants. The number of hands that can be considered a full house in the game of set can be expressed as $a^b + c^d + e^f$, where a, b, c, d, e, and f are positive integers and $a + c + e$ is minimized. Find $a + b + c + d + e + f$.

AR 10: Bobby is spinning a rigged wheel with three sections labeled A, B, and C. Two integers $0 \leq x, y \leq 50$ are chosen randomly and independently, such that there is a $x\%$ chance the wheel lands on A and a $y\%$ chance the wheel lands on B. Given that the wheel lands on C the first time, the probability that it will land on C if Bobby spins it again can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.