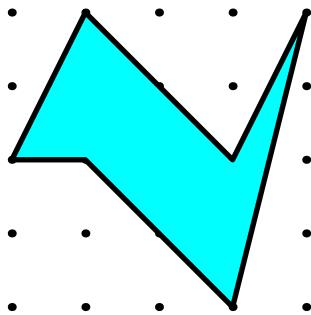


2021 WSMO Speed Round

SMO Team

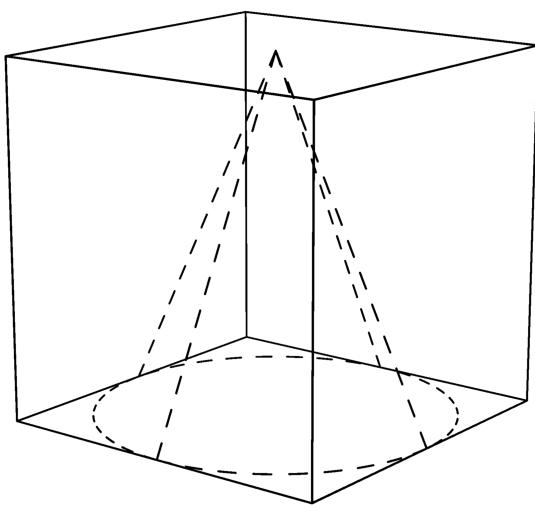
SR 1: Find the number of square units in the area of the shaded region.



SR 2: There are 4 tables and 5 chairs at each table. Each chair seats 2 people. There are 10 people who are seated randomly. Andre and Emily are 2 of them, and are a couple. If the probability that Andre and Emily are in the same chair is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 3: There are 6 pairs of socks for each color of the rainbow (red, orange, yellow, green, blue, indigo, violet) in a sock drawer. How many socks must be drawn from the drawer to guarantee that a pair of red socks have been drawn?

SR 4: A right circular cone is inscribed in a right prism as shown. If the ratio of the volume of the cone to the volume of the prism is $\frac{m}{n}\pi$, where m and n are relatively prime positive integers. Find $m + n$.



SR 5: There exists a rational function $f(x)$ such that for all x in the range $(0, 1)$, $f(x) = \sum_{n=1}^{\infty} nx^n$. The maximum of $f(x)$ over $[6, 9]$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 6: Let ABC be an equilateral triangle of side length 6. Points $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2$, and C_3 are chosen such that A_1, A_2, A_3 divide BC into four equal segments, B_1, B_2, B_3 divide AC into four equal segments, and C_1, C_2, C_3 divide AB into four equal segments. If i, j, k are chosen from the set $\{1, 2, 3\}$ independently and randomly, the expected area of $A_iB_jC_k$ is $\frac{a\sqrt{b}}{c}$, where b is a squarefree positive integer and a and c are relatively prime positive integers. Find $a + b + c$.

SR 7: Let a, b , and c be real numbers such that $a + b + c = 1$ and $e \geq -\frac{1}{3}$, $b \geq -1$ and $c \geq -\frac{5}{3}$. Find the maximum value of $\sqrt{3a+1} + \sqrt{3b+3} + \sqrt{3c+5}$.

SR 8: In regular octagon $ABCDEFGH$ of sidelength 4, quadrilaterals $ACEG$ and $BDFH$ are drawn. Find the square of the area of the overlap of the two quadrilaterals.

SR 9: Suppose that b and c are the roots of the equation $x^2 - \log(16)x + \log(64)$. If $\sqrt{a+b} + \sqrt{a+c} = \sqrt{b+c}$, then 2^a can be expressed as $\frac{\sqrt{m}}{n}$, where m is a squarefree positive integer. Find $m + n$.

SR 10: Consider acute triangle ABC , H is the orthocenter. Extend AH to meet BC at D . The angle bisector of $\angle ABH$ meets the midpoint of AD , M . If $AB = 10, BH = 4$, then the area of ABC is $\frac{a\sqrt{b}}{c}$, where b is a squarefree positive integer and a and c are relatively prime positive integers. Find $a + b + c$.

