

2022 SSMO Relay Round 2

SMO Team

RR 2 Part 1: Let P be a randomly selected point on a circle, and let A be a randomly selected point inside the same circle. A dilation centered at P with a scale factor of 2 sends A to A' . Given that the probability that PA' is less than the length of the diameter of the circle can be expressed as $\frac{a\pi+b\sqrt{c}}{d\pi}$, where a, b, c, d are integers such that a and d are positive, c is squarefree, and $\gcd(a, b, d) = 1$, find the value of $a + b + c + d$.

RR 2 Part 2: Let $T = \text{TNYWR}$. Suppose that the monic quadratic $f(x)$ is tangent to the function $g(x) = |x + 2| - T$ at two points, when graphed on the coordinate plane. Then $|f(1)|$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $10m + n$.

RR 2 Part 3: Let $T = \text{TNYWR}$. Let $a + b = \lfloor \sqrt{T} \rfloor$. If $a^5 + b^5 = 15$, then ab has two possible values. The absolute difference of these values is $\frac{x\sqrt{y}}{z}$, where x, y and z are positive integers, x and z are relatively prime, and y is not divisible by the square of any prime. What is $x + y + z$?