

2022 SSMO Relay Round 4

SMO Team

RR 4 Part 1: On any given day, there is a 70% chance that a robot will find a new organism, a 20% chance it will find an already discovered organism, and a 10% chance that it will find nothing. Given that it has found a new organism, there is a 90% chance it will correctly determine that it is a new organism, and given that it has found an already discovered organism, there is a 75% chance that it will correctly determine that it has already been discovered. The expected number of days that the robot will take to report that it has found a new organism (regardless of whether it actually has) can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

RR 4 Part 2: The roots of $f(x) = x^3 + 5x + 8$ are r_1, r_2, r_3 . Let $g_n(x)$ be a polynomial with roots $r_1 + n, r_2 + n, r_3 + n$. If

$$S = \sum_{n=1}^T (-1)^n g_n(5),$$

find the remainder when S is divided by 1000.

RR 4 Part 3: Let $T = \text{TNYWR}$. If $f(1) = 1$, $f(2) = 12$, and

$$f(n+2) = 12f(n+1) - 20f(n)$$

for all positive integers n , find the remainder when $f(T)$ is divided by 1000.