2024 SSMO Speed Round

SMO Team

SR 1: Find the sum of the distinct prime factors of $2024^2 - 1$.

SR 2: Gracie's students play with some toys. When 4 or 5 students are present, the toys can be equally distributed to everyone. However, when there are only 3 students, there is one toy leftover after giving everyone the same number of toys. What is the least possible number of toys that Gracie could have?

SR 3: The polynomial $x^3 - 15x^2 + 4x + 4$ has distinct real roots r, s, and t. Find the value of

$$\left|(r^2 + s^2 + t^2)(rst)\right|.$$

SR 4: Sam wants to read the Harry Potter and Warriors books. There are 7 Harry Potter books that must be read in a specific order, and there are 6 Warriors books that also must be read in a specific order; however, he can read the two series at the same time. For example, he could read the first three Harry Potter books, then the first five Warriors books, then the remaining Harry Potter books, and finally the last Warriors book. In how many unique orders can Sam read the books?

SR 5: Let $\triangle ABC$ and $\triangle ADC$ be right triangles, such that $\angle ABC = \angle ADC = 90°$. Given that $\angle ACB = 30°$ and $BC = 3\sqrt{3}$, find the maximum possible length of BD.

SR 6: There are 4 people and 4 houses. Each person independently randomly chooses a house to live in. The expected number of inhabited houses can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 7: Let S denote the set of all ellipses centered at the origin and with axes AB and CD where $A = (-x, 0), B = (x, 0), C = (0, -y)$, and $D = (0, y)$, for $2 \mid x + y$ and $0 \leq x, y \leq 10$. Let T denote the set of similar ellipses centered at the origin and passing through (x, y) for $2 \nmid x + y$ and $0 \leq x, y, \leq 10$. The positive difference between the sum of the areas of all ellipses in T and the sum of the areas of all the ellipses in S can be expressed as $m\pi$. Find m.

SR 8: Bob has two coins; one is fair, and one lands on heads with a probability of $\frac{2}{3}$. Bob chooses a random coin and flips it twice. Alice watches the two coin flips and guesses whether Bob flipped the fair or rigged coin. Given that Alice is a good mathematician and guesses the more likely option (guessing randomly when they are equally likely), the probability she guesses right can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 9: Let a, b, c, and d be positive integers such that $abcd = a + b + c + d$. Find the maximum possible value of a.

SR 10: Let $a_1, a_2, \ldots a_{14}$ be the roots of $(x^7 - x^3 + 2)^2 = 0$. Find the value of $\prod_{i=1}^{14}(a_i^7 + 1)$.