

# 2022 SSMO Relay Round 4

SMO Team

**RR 4 Part 1:** On any given day, there is a 70% chance that a robot will find a new organism, a 20% chance it will find an already discovered organism, and a 10% chance that it will find nothing. Given that it has found a new organism, there is a 90% chance it will correctly determine that it is a new organism, and given that it has found an already discovered organism, there is a 75% chance that it will correctly determine that it has already been discovered. The expected number of days that the robot will take to report that it has found a new organism (regardless of whether it actually has) can be expressed as  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

**RR 4 Part 2:** The roots of  $f(x) = x^3 + 5x + 8$  are  $r_1, r_2, r_3$ . Let  $g_n(x)$  be a polynomial with roots  $r_1 + n, r_2 + n, r_3 + n$ . If

$$S = \sum_{n=1}^T (-1)^n g_n(5),$$

find the remainder when  $S$  is divided by 1000.

**RR 4 Part 3:** Let  $T = \text{TNYWR}$ . If  $f(1) = 1$ ,  $f(2) = 12$ , and

$$f(n+2) = 12f(n+1) - 20f(n)$$

for all positive integers  $n$ , find the remainder when  $f(T)$  is divided by 1000.

