2021 WSMO Accuracy Round

SMO Team

AR 1: Let $x = \sqrt{69 + \sqrt{69 + \sqrt{69 \ldots}}}$. Find the value of $(2x-1)^2$.

AR 2: When Bob is in precalculus, he gets bored and writes all the permutations in "precal". Since he is not very smart, it takes him 5 seconds to write each permutation. When Bob advances to calculus, he gets bored and writes all the permutations in "calculus". He is smart and can now write each permutation in 2 seconds. Find the positive difference in minutes between the time it takes for him to write the permutations of "precal" and "calculus".

AR 3: $f(x) = x^3 - 8x^2 + 10x - 4$ has complex roots a, b, c. Denote $P(n) = a^n + b^n + c^n$. Find $P(-1)P(0)P(1)$.

AR 4: Bob and his 3 friends are standing in a line of 10 people. Given that Bob is not on either end of the line, then the probability the person in front and behind Bob are both his friends is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

AR 5: Bob flips $x+1$ coins and Bobby flips x coins, where x is a random integer chosen between the range of $[27, 100]$. The expected probability that Bob gets more heads than Bobby is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

AR 6: In quadrilateral $ABCD$, there exists a point O such that $AO = BO = CO = DO$ and $\angle(AOB) + \angle(COD) = 120°$. Let K, L, M, N be the foot of the perpendiculars from A to BD, B to AC, C to BD, and D to AC. If $[ABCD] = 20$, find $([KLMN])^2$.

AR 7: How many ordered triplets of integers (a, b, c) satisfy $a^2 + 2ab + b^2 = c^2 - 6c + 9$ and $-2 \le a, b, c \le 7$?

AR 8: Let $f(x) = x^3 - 3x^2 + 4x + 5$ have complex roots a, b, c. Then,

$$\frac{1}{a^2+b^2} + \frac{1}{a^2+c^2} + \frac{1}{b^2+c^2}$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

AR 9: Given circles ω_1, ω_2 with radius $1, 4$ respectively, they are externally tangent to each other. The diameters of ω_1 , ω_2 are AB, CD respectively, satisfying $AB \parallel CD$ and BD is an external tangent of the circles. The third circle ω_3 passes through A, C and is tangent to BD. The minimum possible value of the radius of ω_3 can be expressed as $\frac{a+b\sqrt{c}}{d}$, where c is a squarefree positive integer and a, b, d are relatively prime positive integers. Find $a+b+c+d$.

AR 10: In tetrahedron T of side length 12, let S_1 be the sphere inscribed in T and let S_2 be the sphere circumscribed around T. Let R be a rectangular prism such that all points on

S_1 lie strictly inside or are touching R and all points on R lie strictly inside or are touching S_2. The minimum possible volume of R is $m\sqrt{n}$, where n is a squarefree positive integer. Find $m+n$.