How to think of proofs of arguments:

Think of a proof as a *path* from premises to a conclusion, i.e., as a way for getting to the conclusion from the premises. Each line in the proof (each step you take down the path) **must** be allowed by a rule.

This means that in order to offer a proof of an argument you need two skills that we have used before and one new skill: (i) recognizing that a formula might be an instance of different forms, (ii) identifying which forms a formula is an instance of, and (iii) making a plan for the proof (i.e., sketching the path).

1. For each of the following formulas, give at least two statements with metavariables¹ that the formula is an instance of:

(a)
$$p \to ((\neg p \land s) \lor q)$$

(b)
$$(q \to r) \land (r \lor p)$$

(c)
$$\neg (r \lor t) \lor t$$

2. See the following rules:

¹Greek letters like ϕ , ψ and χ

Now, determine which of the following proofs have a correct instance of the rule in question (i.e., which of the following applications of rules have no mistakes).

(a) I∧

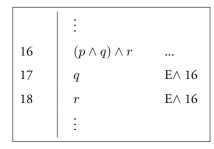
	:	
7	p	
8	$q \rightarrow r$	
	:	
13	$ \qquad \qquad (q \to r) \land p $	I∧ 1, 13

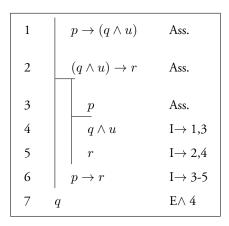
1	$s \to p$	Ass.
2	$u \lor r$	Ass.
	:	
23	$s \wedge u$	I∧ 1, 2

1	$s \wedge u$	Ass.
	÷	
10	$q \vee \neg r$	••••
	:	
16	$ \qquad \qquad (q \vee \neg r) \wedge (s \wedge u) $	I∧ 1, 10

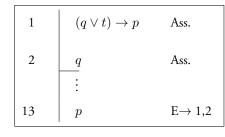
(b) E∧

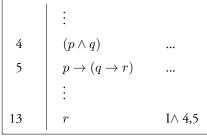
1	$p \wedge q) \wedge r$	Ass.
	:	
	:	
13	r	E∧ 1
14	$p \wedge q$	E∧ 1

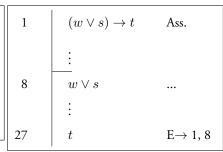




(c) $E \rightarrow$







(d) IV

	:	
16	$(p \wedge q) \wedge r$	
17	q	E∧ 16
	:	

	:	
4	<i>p</i>	Ass.
	:	
8	$(p \lor q) \lor r$	$I\!\vee 4$

1	p	Ass.
2	$q \lor p$	I∨ 1

(e) $I \rightarrow$

	:	
6	$(p \wedge q) \wedge r$	
7	u	Ass.
8	r	E∧ 6
9	$u \to r$	$I \rightarrow 7-8$
	:	

	:	
11	$r \rightarrow s$	•••
12	$(p \wedge q) \wedge r$	•••
13	u	Ass.
14	r	E∧ 12
15	s	$E\rightarrow 11, 14$
16	$u \to s$	$I{\rightarrow}\ 14\text{-}15$
	:	

1	p	Ass.
2	q	Ass.
3	q	Rep. 2
4	$q \to q$	$I{\rightarrow}\ 23$

(f) E∨

	:	
13	$(r \lor t) \to p$	•••
14	$s \to p$	
15	$u \wedge t$	
16	$(r \lor t) \lor s$	
17	p	EV 13,14,16
18	ig u	E∧ 15
19	$u \wedge p$	I∧ 17,18
	:	

1	$r \rightarrow s$	Ass.
2	$(p \land q) \to s$	Ass.
3	t	Ass.
4	$(p \land q) \lor r$	Ass.
5	s	E∨ 1,2,4
	:	

3. Offer a proof for the following arguments:

(a)
$$s, s \to p, (p \land q) \to r, q \vdash r$$

(b)
$$(p \lor q) \to s, p \vdash s$$

(c)
$$s \to t, r \to t, p \land (s \lor r) \vdash t$$

(d)
$$r \to q, s, (q \land s) \to u \vdash r \to q$$

(e) Hard:
$$s \to r, q \to r, u \to q, (r \lor u) \to p, s \lor u \vdash (w \to r) \land r^2$$

²Requires the rule Repetition.