Warning: in propositional logic, we had a failsafe procedure for determining whether an argument was valid or invalid. We don't have such a failsafe procedure for quantificational logic. So figuring out which arguments are valid and which aren't requires *a bunch of practice*.

Here's a reminder of the new rules:

l	$\forall v\varphi$	
m	$\varphi(\tau/v)$	$\forall$E, l

l	$\varphi(\tau/v)$	
m	$\forall v\varphi$	$\forall$I, l

provided τ does not occur in φ or in any undischarged assumption.

l	$\varphi(\tau/v)$	
m	$\exists v\varphi$	$\exists$I, l

k	$\exists v\varphi$	
l	$\varphi(a/x) \to \psi$	
n	ψ	E$\exists$ k, l

where a occurs neither in ψ nor in $\exists v\varphi$ nor in an undischarged assumption.

1. Build a model to show the following argument is invalid: $\forall x(Px \to Qx), Qa \vdash Pa$.
2. Try (and fail!) to build a model for the following argument: $\forall x(Px \to Qx), Pa \vdash Qa$. Then, prove it.
3. What is the mistake in the following proof?

1	$\forall xRxa$	Assumption
2	Rba	E$\forall$ 1
3	$\forall yRby$	I$\forall$ 2

4. What is the mistake in the following proof?

1	$\forall x(Px \to Qx)$	Assumption
2	$Pa \to Qb$	E$\forall$ 1

5. What is the mistake in the following proof?

1	$\exists x Qxb$	Assumption
2	$\quad\mid\ Qbb$	Assumption
3	$\quad\mid\ \exists x Qxx$	I$\exists$ 2
4	$Qbb \to \exists x Qxx$	I$\to$ 2-3
5	$\exists xx Qxx$	E$\exists$ 1,4

6. Prove the following arguments:

   (a) $\forall x Px, \forall x(Px \to Qx) \vdash \forall x Qx$

   (b) $\vdash \forall x Rax \to (\exists y \forall x Ryx \wedge Rab)$

   (c) $\forall x Px \vee \forall x Qx \vdash \forall y(Py \vee (Qy \vee Ry))$

   (d) $\forall x(Px \to (Qx \wedge Rx)), \exists x Px \vdash \exists x Rx$

   (e) **Hardest proof (and only doable after Wednesday):** $\vdash \exists x(Fx \to \forall y Fy)$