1. Determine whether the following arguments in propositional logic are valid:

   (a) $p \vee (q \rightarrow r) \vDash p$

   (b) $p \rightarrow (s \wedge q), r \rightarrow (q \vee \neg s), p \vDash \neg r$

2. Consider the following translation key:

   p: Having a belief is having a disposition to act in some ways.

   q: Non-living objects can be disposed to act in some ways.

   r: Non-living objects can have beliefs.

   s: Non-living objects represent the world being some way or another.

   Now, translate the following arguments to propositional logic and determine whether they are valid:

   (a) Neither can non-living objects represent the world being some way or another, nor is having a belief having a disposition to act in some ways only if non-living objects can have beliefs. In conclusion, non-living objects cannot represent the world being some way or another unless non-living objects can have beliefs.

   Goal: to translate an argument that is hard to parse, even if it seems bad.

   (b) If having a belief is having a disposition to act in some ways, and if non-living objects can be disposed to act in some ways, then non-living objects can have beliefs. But non-living objects can have beliefs only if they represent the world being some way or another, and non-living objects do not represent the world being some way or another.

   Goal: to translate an argument that is easier to parse and seems good.

Therefore, it is false that non-living objects can be disposed to act in some ways.

3. Fill out the missing rules in the following proof:

1	$p \wedge q$	Assumption
2	r	Assumption
3	p	____ 1
4	$p \wedge r$	____ 2, 3

4. Fill out the missing lines in the following proof:

1	$p \to q$	Assumption
2	$p \to r$	Assumption
3	p	Assumption
4	q	$\to E$ ____
5	r	$\to E$ ____
6	$q \wedge r$	$\wedge I$____

5. **Extra:** Offer a proof of the following arguments:

   (a) $(p \wedge q) \wedge (r \wedge s) \vdash r \wedge s$

   (b) $(p \wedge r) \wedge (s \wedge q) \vdash r \wedge s$

   (c) $p \rightarrow r, s \wedge p \vdash r \wedge s$

A *derivation* or *proof* is a finite sequence of formulas each of which is either an assumption or the outcome of an application of a rule of inference to prior formulas. The last formula in the sequence is the conclusion of the proof.

Here's a reminder of the rules need for these derivations:

$$\begin{array}{r|ll} l & \varphi \wedge \psi & \\ n & \varphi & \text{E}\wedge\ \text{l} \end{array}$$

$$\begin{array}{r|ll} l & \varphi \wedge \psi & \\ m & \psi & \text{E}\wedge\ \text{l} \end{array}$$

$$\begin{array}{r|ll} l & \phi & \\ m & \psi & \\ n & \phi \wedge \psi & \text{I}\wedge\ \text{l, m} \end{array}$$

$$\begin{array}{r|ll} l & \phi \rightarrow \psi & \\ m & \phi & \\ n & \psi & \text{E}\rightarrow\ \text{l, m} \end{array}$$