Warning: in propositional logic, we had a failsafe procedure for determining whether an argument was valid or invalid. We don't have such a failsafe procedure for quantificational logic. So figuring out which arguments are valid and which aren't requires *a bunch of practice*.

Here's a reminder of the new rules:

l	$\forall v \varphi$	
m	$\varphi(\tau/v)$	$\forall$E, l

l	$\varphi(\tau/v)$	
m	$\exists v \varphi$	$\exists$I, l

l	$\varphi(\tau/v)$	
m	$\forall v \varphi$	$\forall$I, l

provided τ does not occur in φ or in any undischarged assumption.

k	$\exists v \varphi$	
l	$\varphi(a/x) \to \psi$	
n	ψ	E$\exists$ k, l

where a occurs neither in ψ nor in $\exists v \varphi$ nor in an undischarged assumption.

1. Build a model to show the following argument is invalid: $\forall x(Px \to Qx), Qa \vdash Pa$.

   Domain: 0

   P_:

   Q_: 0

   a: 0

2. Try (and fail!) to build a model to show the following argument is invalid: $\forall x(Px \to Qx), Pa \vdash Qa$. Then, prove the argument.

   Proof:

1	$\forall x(Px \to Qx)$	Assumption
2	Pa	Assumption
3	$Pa \to Qa$	E$\forall$ 1
4	Qa	E$\to$ 2,3

3. What is the mistake in the following proof?

1	$\forall x Rxa$	Assumption
2	Rba	E$\forall$ 1
3	$\forall y Rby$	I$\forall$ 2 You can't generalize about a, since it is not an undischarged assumption

4. What is the mistake in the following proof?

1	$\forall x(Px \rightarrow Qx)$	Assumption
2	$Pa \rightarrow Qb$	E$\forall$ 1 When you eliminate a $\forall$, you must replace the variable with the *same* constant

5. What is the mistake in the following proof?

1	$\exists x Qxb$	Assumption
2	Qbb	Assumption
3	$\exists x Qxx$	I$\exists$ 2
4	$Qbb \rightarrow \exists x Qxx$	I$\rightarrow$ 2-3
5	$\exists x Qxx$	E$\exists$ 1,4

The antecedent of the conditional that is used to eliminate $\exists$ must have the 'x' replaced with a constant that doesn't appear in the proof.

6. Prove the following arguments:
   (a) $\forall x Px, \forall x(Px \to Qx) \vdash \forall x Qx$

1	$\forall x Px$	Assumption
2	$\forall x(Px \to Qx)$	Assumption
3	Pa	E$\forall$ 1
4	$Pa \to Qa$	E$\forall$ 2
5	Qa	E$\to$ 3,4
6	$\forall x Qx$	I$\forall$ 5

   (b) $\vdash \forall x Rax \to (\exists y \forall x Ryx \wedge Rab)$

1	$\forall x Rax$	Assumption
2	$\exists y \forall x Ryx$	I$\exists$ 1
3	Rab	E$\forall$ 1
4	$\exists y \forall x Ryx \wedge Rab$	I$\wedge$ 1

   (c) $\forall x Px \vee \forall x Qx \vdash \forall y(Py \vee (Qy \vee Ry))$

1	$\forall x Px \vee \forall x Qx$	Assumption
2	$\forall x Px$	Assumption
3	Pa	E$\forall$ 2
4	$Pa \vee (Qa \vee Ra)$	I$\vee$ 3
5	$\forall y(Py \vee (Qy \vee Ry))$	I$\forall$ 4
6	$\forall x Px \to \forall y(Py \vee (Qy \vee Ry))$	I$\to$ 2-5
7	$\forall x Qx$	Assumption
8	Qa	E$\forall$ 7
9	$Qa \vee Ra$	I$\vee$ 8
10	$Pa \vee (Qa \vee Ra)$	I$\vee$ 9
11	$\forall y(Py \vee (Qy \vee Ry))$	I$\forall$ 10
12	$\forall x Qx \to \forall y(Py \vee (Qy \vee Ry))$	I$\to$ 7-11
13	$\forall y(Py \vee (Qy \vee Ry))$	E$\vee$ 1,6, 12

   (d) $\forall x(Px \to (Qx \wedge Rx)), \exists x Px \vdash \exists x Rx$

1	$\forall x(Px \to (Qx \wedge Rx))$	Assumption
2	$\exists x Px$	Assumption
3	$\quad Pa$	Assumption
4	$\quad Pa \to (Qa \wedge Ra)$	E$\forall$ 1
5	$\quad Qa \wedge Ra$	E$\to$ 3,4
6	$\quad Ra$	E$\wedge$ 5
7	$\quad \exists x Rx$	I$\exists$ 6
8	$Pa \to \exists x Rx$	I$\to$ 3-7
9	$\exists x Rx$	E$\exists$ 2,8

(e) **Hardest proof (and only doable after Wednesday):** $\vdash \exists x(Fx \to \forall y Fy)$ Email me!