

Warning: in propositional logic, we had a failsafe procedure for determining whether an argument was valid or invalid. We don't have such a failsafe procedure for quantificational logic. So figuring out which arguments are valid and which aren't requires *a bunch of practice*.

Here's a reminder of the new rules:

$$\begin{array}{c|c} l & \forall v\varphi \\ m & \varphi(\tau/v) \end{array} \quad \forall E, l$$

$$\begin{array}{c|c} l & \varphi(\tau/v) \\ m & \exists v\varphi \end{array} \quad \exists I, l$$

$$\begin{array}{c|c} l & \varphi(\tau/v) \\ m & \forall v\varphi \end{array} \quad \forall I, l$$

provided τ does not occur in φ or in any undischarged assumption.

$$\begin{array}{c|c} k & \exists v\varphi \\ l & \varphi(a/x) \rightarrow \psi \\ n & \psi \end{array} \quad E\exists k, l$$

where a occurs neither in ψ nor in $\exists v\varphi$ nor in an undischarged assumption.

1. Build a model to show the following argument is invalid: $\forall x(Px \rightarrow Qx), Qa \vdash Pa$.
2. Try (and fail!) to build a model to show the following argument is invalid: $\forall x(Px \rightarrow Qx), Pa \vdash Qa$. Then, prove the argument.
3. What is the mistake in the following proof?

$$\begin{array}{c|c} 1 & \forall xRxa \quad \text{Assumption} \\ \hline 2 & Rba \quad E\forall 1 \\ \hline 3 & \forall yRby \quad I\forall 2 \end{array}$$

4. What is the mistake in the following proof?

1	$\forall x(Px \rightarrow Qx)$	Assumption
2	$Pa \rightarrow Qb$	$E\forall 1$

5. What is the mistake in the following proof?

1	$\exists xQxb$	Assumption
2	Qbb	Assumption
3	$\exists xQxx$	$I\exists 2$
4	$Qbb \rightarrow \exists xQxx$	$I\rightarrow 2-3$
5	$\exists xxQxx$	$E\exists 1,4$

6. Prove the following arguments:

(a) $\forall xPx, \forall x(Px \rightarrow Qx) \vdash \forall xQx$

(b) $\vdash \forall xRax \rightarrow (\exists y\forall xRyx \wedge Rab)$

(c) $\forall xPx \vee \forall xQx \vdash \forall y(Py \vee (Qy \vee Ry))$

(d) $\forall x(Px \rightarrow (Qx \wedge Rx)), \exists xPx \vdash \exists xRx$

(e) **Hardest proof (and only doable after Wednesday):** $\vdash \exists x(Fx \rightarrow \forall yFy)$