How to think of proofs of arguments:

Think of a proof as a *path* from premises to a conclusion, i.e., as a way for getting to the conclusion from the premises. Each line in the proof (each step you take down the path) **must** be allowed by a rule.

This means that in order to offer a proof of an argument you need two skills that we have used before and one new skill: (i) recognizing that a formula might be an instance of different forms, (ii) identifying which forms a formula is an instance of, and (iii) making a plan for the proof (i.e., sketching the path).

1. For each of the following formulas, give at least two statements with metavariables¹ that the formula is an instance of:

(a)
$$p \to ((\neg p \land s) \lor q)$$

(b)
$$(q \to r) \land (r \lor p)$$

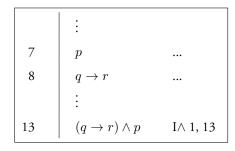
(c)
$$\neg (r \lor t) \lor t$$

2. See the following rules:

¹Greek letters like ϕ , ψ and χ

Now, determine which of the following proofs have a correct instance of the rule in question (i.e., which of the following applications of rules have no mistakes).

(a) I∧



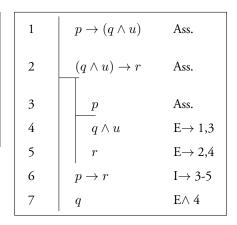
1	$s \to p$	Ass.
2	$u \lor r$	Ass.
22	:	IA 1 2
23	$s \wedge u$	$I \wedge 1, 2$

1	$s \wedge u$	Ass.
	:	
10	$q \vee \neg r$	••••
	:	
16	$(q \vee \neg r) \wedge (s \wedge u)$	I∧ 1, 10

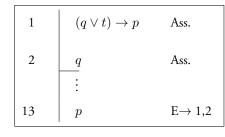
(b) E∧

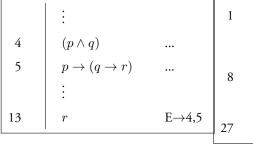
1	$p \wedge q) \wedge r$	Ass.
	:	
	:	
13	r	E∧ 1
14	$p \wedge q$	E∧ 1

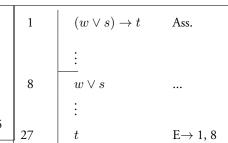
	:	
16	$(p \wedge q) \wedge r$	
17	q	E∧ 16
18	r	E∧ 16
	:	



(c) $E \rightarrow$







(d) IV

	:	
16	$p \wedge q) \wedge r$	
17	$r \vee s$	I∨ 16
	:	

	:	
4	<u>p</u>	Ass.
8	$ \vdots \\ (p \lor q) \lor r $	I∨ 4

1	p	Ass.
2	$q \lor p$	I∨ 1

(e) $I \rightarrow$

	:	
6	$(p \wedge q) \wedge r$	
7	u	Ass.
8	$\mid \mid r$	E∧ 6
9	$u \to r$	$I\rightarrow 7-8$

$ \begin{array}{c ccc} 1 & p & Ass. \\ 2 & q & Ass. \end{array} $	
2	
$2 \mid q \text{Ass.}$	
\overline{q} Rep. 2	
$4 \qquad q \to q \qquad \qquad I \to 2-3$	

(f) E∨

	:	
13	$(r \lor t) \to p$	
14	$s \to p$	•••
15	$u \wedge t$	
16	$(r \lor t) \lor s$	
17	p	EV 13,14,16
18	u	E∧ 15
19	$u \wedge p$	I∧ 17,18
	:	

$r \rightarrow s$	Ass.
$(p \land q) \to s$	Ass.
t	Ass.
$(p \land q) \lor r$	Ass.
s	E∨ 1,2,4
:	
	$(p \land q) \rightarrow s$ t $(p \land q) \lor r$

3. Offer a proof for the following arguments:

(a)
$$s, s \to p, (p \land q) \to r, q \vdash r$$

(b)
$$(p \lor q) \to s, p \vdash s$$

(c)
$$s \to t, r \to t, p \land (s \lor r) \vdash t$$

(d)
$$r \to q, s, (q \land s) \to u \vdash r \to u$$

(e) Hard:
$$s \to r, q \to r, u \to q, (r \lor u) \to p, s \lor u \vdash (w \to r) \land r^2$$

²This is easier with the rule Repetition.