Useful patterns to notice

The way to read these boxes is as follows: when in a proof you have lines of the form of the lines that come before the dots, you will be able to derive (by applying a series of rules that appeal to those previous lines), the line that comes after the dots:

l	$\phi \vee \psi$
m	$\neg\phi$
	$\vdots$
n	ψ

l	$\phi \rightarrow \psi$
m	$\neg\psi$
	$\vdots$
n	$\neg\phi$

l	$\neg(\phi \vee \psi)$
	$\vdots$
m	$\neg\phi \wedge \neg\psi$

l	$\neg(\phi \wedge \psi)$
	$\vdots$
m	$\neg\phi \vee \neg\psi$

l	$\neg\phi \wedge \neg\psi$
	$\vdots$
m	$\neg(\phi \vee \psi)$

l	$\neg\phi \vee \neg\psi$
	$\vdots$
m	$\neg(\phi \wedge \psi)$

Exercises

1. If I am asked to prove an argument of the following form ' ... $\vdash \phi$', should I start my proof by assuming ϕ? Why?
2. Offer a proof of the following argument:
   (a) $(p \rightarrow q) \wedge r, \neg((p \wedge q) \wedge (r \wedge s)), p \vdash \neg s$

Practice Quiz

Offer a proof of the following arguments in 20 minutes:

1. $(p \wedge r) \vee (r \wedge s) \vdash r$
2. $\neg((s \vee p) \vee q) \vdash \neg s \wedge \neg q$
3. $p \rightarrow s, \neg(r \wedge q) \rightarrow \neg s \vdash p \rightarrow (r \wedge q)$

4. $s \to (p \to r), p \wedge \neg r \vdash \neg s$

5. $(p \vee q) \to r, \neg r \vdash \neg q \vee r$