

Warning: in propositional logic, we had a failsafe procedure for determining whether an argument was valid or invalid. We don't have such a failsafe procedure for quantificational logic. So figuring out which arguments are valid and which aren't requires *a bunch of practice*.

Here's a reminder of the new rules:

$$\begin{array}{l|l} l & \forall v \varphi \\ m & \varphi(\tau/v) \quad \forall E, l \end{array}$$

$$\begin{array}{l|l} l & \varphi(\tau/v) \\ m & \exists v \varphi \quad \exists I, l \end{array}$$

$$\begin{array}{l|l} l & \varphi(\tau/v) \\ m & \forall v \varphi \quad \forall I, l \end{array}$$

provided τ does not occur in φ or in any undischarged assumption.

$$\begin{array}{l|l} k & \exists v \varphi \\ l & \varphi(a/x) \rightarrow \psi \\ n & \psi \quad \text{E}\exists k, l \end{array}$$

where a occurs neither in ψ nor in $\exists v \varphi$ nor in an undischarged assumption.

1. Build a model to show the following argument is invalid: $\forall x(Px \rightarrow Qx), Qa \vdash Pa$.
2. Try (and fail!) to build a model to show the following argument is invalid: $\forall x(Px \rightarrow Qx), Pa \vdash Qa$. Then, prove the argument.
3. What is the mistake in the following proof?

1	$\forall x Rxa$	Assumption
2	Rba	$E\forall 1$
3	$\forall y Rby$	$I\forall 2$

4. What is the mistake in the following proof?

1	$\forall x(Px \rightarrow Qx)$	Assumption
2	$Pa \rightarrow Qb$	$E\forall$ 1

5. What is the mistake in the following proof?

1	$\exists xQxb$	Assumption
2	Qbb	Assumption
3	$\exists xQxx$	$I\exists$ 2
4	$Qbb \rightarrow \exists xQxx$	$I\rightarrow$ 2-3
5	$\exists xxQxx$	$E\exists$ 1,4

6. Prove the following arguments:

(a) $\forall xPx, \forall x(Px \rightarrow Qx) \vdash \forall xQx$

(b) $\vdash \forall xRax \rightarrow (\exists y\forall xRyx \wedge Rab)$

(c) $\forall xPx \vee \forall xQx \vdash \forall y(Py \vee (Qy \vee Ry))$

(d) $\forall x(Px \rightarrow (Qx \wedge Rx)), \exists xPx \vdash \exists xRx$

(e) **Hardest proof (and only doable after Wednesday):** $\vdash \exists x(Fx \rightarrow \forall yFy)$