How to think of proofs of arguments:

Think of a proof as a *path* from premises to a conclusion, i.e., as a way for getting to the conclusion from the premises. Each line in the proof (each step you take down the path) **must** be allowed by a rule.

This means that in order to offer a proof of an argument you need two skills that we have used before and one new skill: (i) recognizing that a formula might be an instance of different forms, (ii) identifying which forms a formula is an instance of, and (iii) making a plan for the proof (i.e., sketching the path).

1. For each of the following formulas, give at least two statements with metavariables¹ that the formula is an instance of:

(a)
$$p \to ((\neg p \land s) \lor q)$$
 e.g., $\phi \to \psi$ and $\phi \to ((\neg \phi \land \chi) \lor \psi)$

(b)
$$(q \to r) \land (r \lor p)$$
 e.g., $\phi \land \psi$ and $(\phi \to \psi) \land (\psi \lor \chi)$

(c)
$$\neg (r \lor t) \lor t$$
 e.g., $\phi \lor \psi$ and $\neg \phi \lor \psi$

2. See the following rules:

¹Greek letters like ϕ , ψ and χ

Now, determine which of the following proofs have a correct instance of the rule in question (i.e., which of the following applications of rules have no mistakes).

X

X

X

(a) I∧ **X**

	:	
7	p	
8	$q \rightarrow r$	
	:	
13		I∧ 1, 13

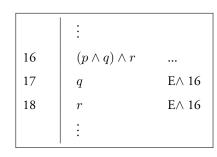
1	$s \to p$	Ass.
2	$u \lor r$	Ass.
23	$\begin{array}{c} \vdots \\ s \wedge u \end{array}$	I∧ 1, 2

1	$s \wedge u$	Ass.
	:	
10	$q \vee \neg r$	
	:	
16	$(q \vee \neg r) \wedge (s \wedge u)$	I∧ 1, 10

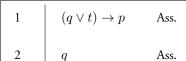
X

(b) E∧ **√**

1	$(p \land q) \land r$	Ass.
	<u> </u>	
	<u>:</u>	
13	r	E∧ 1
14	$p \wedge q$	E∧ 1



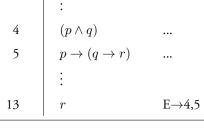
1	$p \to (q \land u)$	Ass.
2	$\boxed{ (q \land u) \to r}$	Ass.
3	p	Ass.
4	$q \wedge u$	$E\rightarrow$ 1,3
5	r	$E\rightarrow 2,4$
6	$p \rightarrow r$	$I\rightarrow 3-5$
7	q	$E \! \wedge 4$

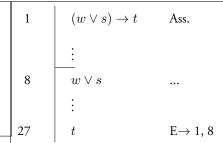


X

(c) $E \rightarrow$

	<u>q</u>	ASS.
	:	
13	p	$E\rightarrow 1,2$





(d) IV X

 :

 $\begin{array}{c|cccc} & & & \\ & & p & & \\ \hline & \vdots & & \\ & & (p \lor q) \lor r & & \text{I} \lor 4 \end{array}$

X

 \checkmark

 \checkmark

1	p	Ass.
2	$q \lor p$	$I \vee 1$

(e) $I \rightarrow \checkmark$

 $\begin{array}{c|cccc} & \vdots & & & \vdots \\ 6 & & (p \wedge q) \wedge r & & \dots \\ 7 & & u & & Ass. \\ 8 & & r & & E \wedge 6 \\ 9 & & u \rightarrow r & & I \rightarrow 7-8 \\ & \vdots & & & \vdots \\ \end{array}$

| ;

11 $r \rightarrow s$... 12 $(p \wedge q) \wedge r$... 13 uAss. 14 E∧ 12 r15 $E\rightarrow 11, 14$ s $I{\rightarrow}\,14\text{-}15$ 16 $u \to s$

1 p Ass.

 $\begin{array}{c|cccc} 2 & & q & \text{Ass.} \\ \hline 3 & & q & \text{Rep. 2} \\ \hline 4 & q \rightarrow q & & \text{I} \rightarrow 2\text{-}3 \\ \end{array}$

(f) E∨ 13 $(r \lor t) \to p$ 14 $s \to p$ 15 $u \wedge t$ ••• 16 $(r \lor t) \lor s$ EV 13,14,16 17 pE∧ 15 18 u19 I∧ 17,18 $u \wedge p$

 \checkmark

 $\begin{array}{c|cccc} 1 & r \rightarrow s & \text{Ass.} \\ 2 & (p \land q) \rightarrow s & \text{Ass.} \\ 3 & t & \text{Ass.} \\ 4 & (p \land q) \lor r & \text{Ass.} \\ \hline 5 & s & \text{E} \lor 1,2,4 \\ \vdots & \vdots & & \\ \end{array}$

3. Offer a proof for the following arguments:

(a)
$$s, s \to p, (p \land q) \to r, q \vdash r$$

	I.	
1	s	Assumption
2	$s \to p$	Assumption
3	$(p \land q) \to r$	Assumption
4	q	Assumption
5	p	$E \rightarrow 1,2$
6	$p \wedge q$	I∧ 4,5
7	r	$E\rightarrow 3.6$

(b)
$$(p \lor q) \to s, p \vdash s$$

$$\begin{array}{c|cccc} 1 & & & & & \\ & & & & \\ 2 & & & p & & \\ \hline & & & \\ 3 & & p \lor q & & \\ 4 & & s & & \\ \hline & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ &$$

(c)
$$s \to t, r \to t, p \land (s \lor r) \vdash t$$

(d) $r \to q, s, (q \land s) \to u \vdash r \to u$

	I	
1	r o q	Assumption
2	s	Assumption
3	$(q \land s) \to u$	Assumption
4	r	Assumption
5	q	$E\rightarrow 1,4$
6	$q \wedge s$	I∧ 2,5
7	$oxed{u}$	$E\rightarrow 3,6$
8	$r \rightarrow u$	I→ 3-7

(e) Hard:
$$s \to r, q \to r, u \to q, (r \lor u) \to p, s \lor u \vdash (w \to r) \land r^2$$

	I .	
1	$s \rightarrow r$	Assumption
2	q o r	Assumption
3	$u \rightarrow q$	Assumption
4	$(r \vee u) \to p$	Assumption
5	$s \lor u$	Assumption
6	u	Assumption
7	q	E→ 3,6
8	$oxed{r}$	E→ 2,7
9	u o r	I→ 6-8
10	r	E∨ 1,5,9
11	w	Assumption
12	$oxed{r}$	Repetition 10
13	w o r	$I \rightarrow 11-12$
14	$(w \to r) \wedge r$	I∧ 10, 13

²This is easier with the rule Repetition.