Useful patterns to notice

The way to read these boxes is as follows: when in a proof you have lines of the form of the lines that come before the dots, you will be able to derive (by applying a series of rules that appeal to those previous lines), the line that comes after the dots:

l	$\phi \lor \psi$
m	$\neg\phi$
	$\vdots$
n	ψ

l	$\phi \to \psi$
m	$\neg\psi$
	$\vdots$
n	$\neg\phi$

l	$\neg(\phi \lor \psi)$
	$\vdots$
m	$\neg\phi \land \neg\psi$

l	$\neg(\phi \land \psi)$
	$\vdots$
m	$\neg\phi \lor \neg\psi$

l	$\neg\phi \land \neg\psi$
	$\vdots$
m	$\neg(\phi \lor \psi)$

l	$\neg\phi \lor \neg\psi$
	$\vdots$
m	$\neg(\phi \land \psi)$

Exercises

1. If I am asked to prove an argument of the following form ‘ ... $\vdash \phi$’, should I start my proof by assuming ϕ? Why? I shouldn’t, because when I assume ϕ the only things I *can* end up with are $\neg\phi$ (via I$\to$) or a conditional of the form ‘$\phi \to$ ...’, but what I wanted to obtain was ϕ!

2. Offer a proof of the following argument:

(a) $(p \to q) \wedge r, \neg((p \wedge q) \wedge (r \wedge s)), p \vdash \neg s$

1	$(p \to q) \wedge r$	Assumption
2	$\neg((p \wedge q) \wedge (r \wedge s))$	Assumption
3	p	Assumption
4	$\quad s$	Assumption
5	$\quad r$	E$\wedge$ 1
6	$\quad p \to q$	E$\wedge$ 1
7	$\quad r \wedge s$	I$\wedge$ 4, 5
8	$\quad\quad (p \wedge q)$	Assumption
9	$\quad\quad (p \wedge q) \wedge (r \wedge s)$	I$\wedge$ 7, 8
10	$\quad\quad \bot$	E$\neg$ 2, 9
11	$\quad \neg(p \wedge q)$	I$\neg$ 8-10
12	$\quad q$	E$\to$ 3, 5
13	$\quad\quad p$	Assumption
14	$\quad\quad p \wedge q$	I$\wedge$ 12,13
15	$\quad\quad \bot$	E$\neg$ 11, 14
16	$\quad \neg p$	I$\neg$ 13-15
17	$\quad \bot$	E$\neg$ 3, 16
18	$\neg s$	I$\neg$ 4-17

Practice Quiz

Offer a proof of the following arguments in 20 minutes:

1. $(p \wedge r) \vee (r \wedge s) \vdash r$

1	$(p \wedge r) \vee (r \wedge s)$	Assumption
2	$\quad p \wedge r$	Assumption
3	$\quad r$	E$\wedge$ 2
4	$(p \wedge r) \to r$	I$\to$ 2-3
5	$\quad r \wedge s$	Assumption
6	$\quad r$	E$\wedge$ 5
7	$(r \wedge s) \to r$	I$\to$ 5-6
8	r	E$\vee$ 1,4, 7

2. $\neg((s \vee p) \vee q) \vdash \neg s \wedge \neg q$

1	$\neg((s \vee p) \vee q)$	Assumption
2	$\quad s$	Assumption
3	$\quad s \vee p$	I$\vee$ 2
4	$\quad (s \vee p) \vee q$	I$\vee$ 3
5	$\quad \bot$	E$\neg$ 1,4
6	$\neg s$	I$\neg$ 2-5
7	$\quad q$	Assumption
8	$\quad (s \vee p) \vee q$	I$\vee$ 7
9	$\quad \bot$	E$\neg$ 1,8
10	$\neg q$	I$\neg$ 7-9
11	$\neg s \wedge \neg q$	I$\wedge$ 6,10

3. $p \to s, \neg(r \wedge q) \to \neg s \vdash p \to (r \wedge q)$

1	$p \to s$	Assumption
2	$\neg(r \wedge q) \to \neg s$	Assumption
3	$\quad p$	Assumption
4	$\quad s$	E$\to$ 1,3
5	$\quad\quad \neg(r \wedge q)$	Assumption
6	$\quad\quad \neg s$	E$\to$ 2, 5
7	$\quad\quad \bot$	E$\neg$ 4,6
8	$\quad \neg\neg(r \wedge q)$	I$\neg$ 5-7
9	$\quad r \wedge q$	$\neg\neg$ 8
10	$p \to (r \wedge q)$	I$\to$ 3-9

4. $s \to (p \to r), p \wedge \neg r \vdash \neg s$

1	$s \to (p \to r)$	Assumption
2	$p \wedge \neg r$	Assumption
3	$\quad s$	Assumption
4	$\quad p \to r$	E$\to$ 1, 3
5	$\quad p$	E$\wedge$ 2
6	$\quad r$	E$\to$ 5
7	$\quad \neg r$	E$\wedge$ 2
8	$\quad \bot$	E$\neg$ 6,7
9	$\neg s$	I$\neg$ 3-8

5. $(p \vee q) \to r, \neg r \vdash \neg q \vee r$

1	$(p \vee q) \to r$	Assumption
2	$\neg r$	Assumption
3	$\quad q$	Assumption
4	$\quad p \vee q$	I$\vee$ 3
5	$\quad r$	E$\to$ 1,4
6	$\quad \bot$	E$\neg$ 2,5
7	$\neg q$	I$\neg$ 3-6
8	$\neg q \vee r$	I$\vee$ 7