## How to think of proofs of arguments:

Think of a proof as a *path* from premises to a conclusion, i.e., as a way for getting to the conclusion from the premises. Each line in the proof (each step you take down the path) **must** be allowed by a rule.

This means that in order to offer a proof of an argument you need two skills that we have used before and one new skill: (i) recognizing that a formula might be an instance of different forms, (ii) identifying which forms a formula is an instance of, and (iii) making a plan for the proof (i.e., sketching the path).

1. For each of the following formulas, give at least two statements with metavariables<sup>1</sup> that the formula is an instance of:

(a) 
$$p \to ((\neg p \land s) \lor q)$$

(b) 
$$(q \to r) \land (r \lor p)$$

(c) 
$$\neg (r \lor t) \lor t$$

2. See the following rules:

<sup>&</sup>lt;sup>1</sup>Greek letters like  $\phi$ ,  $\psi$  and  $\chi$ 

Now, determine which of the following proofs have a correct instance of the rule in question (i.e., which of the following rules have no mistakes).

## (a) I∧

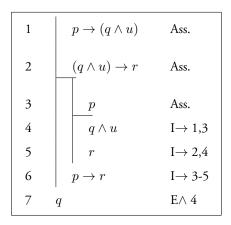
|    | :                      |          |
|----|------------------------|----------|
| 7  | p                      | •••      |
| 8  | $q \rightarrow r$      |          |
|    | :                      |          |
| 13 | $   (q \to r) \land p$ | I∧ 1, 13 |

| 1  | $s \to p$    | Ass.            |
|----|--------------|-----------------|
| 2  | $u \lor r$   | Ass.            |
|    | :            |                 |
| 23 | $s \wedge u$ | $I \wedge 1, 2$ |

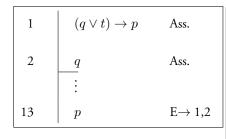
| 1  | $s \wedge u$                            | Ass.     |
|----|---|----------|
|    | :                                       |          |
| 10 | $q \vee \neg r$                         |          |
|    | :                                       |          |
| 16 | $ (q \vee \neg r) \wedge (s \wedge u) $ | I∧ 1, 10 |

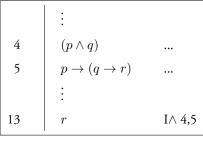
## (b) E∧

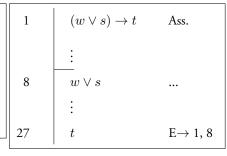
| 1  | $ \qquad \qquad (p \wedge q) \wedge r$ | Ass.           |
|----|--|----------------|
|    | :                                      |                |
|    | <u>:</u>                               |                |
| 13 | r                                      | $E \wedge \ 1$ |
| 14 | $p \wedge q$                           | E∧ 1           |



## (c) $E \rightarrow$







(d) IV

|    | <b>:</b>                |       |
|----|-------------------------|-------|
| 16 | $(p \wedge q) \wedge r$ |       |
| 17 | q                       | E∧ 16 |
|    | :                       |       |

|   | :                   |             |
|---|---------------------|-------------|
| 4 | <i>p</i>            | Ass.        |
|   | :                   |             |
| 8 | $(p \lor q) \lor r$ | $I\!\vee 4$ |

| 1 | p          | Ass. |
|---|------------|------|
| 2 | $q \lor p$ | I∨ 1 |

(e)  $I \rightarrow$ 

|   | :                       |                     |
|---|-------------------------|---------------------|
| 6 | $(p \wedge q) \wedge r$ |                     |
| 7 | u                       | Ass.                |
| 8 | r                       | E∧ 6                |
| 9 | $u \to r$               | $I \rightarrow 7-8$ |
|   | :                       |                     |

|    | :                       |                                |
|----|-------------------------|--------------------------------|
| 11 | $r \rightarrow s$       | •••                            |
| 12 | $(p \wedge q) \wedge r$ | •••                            |
| 13 | u                       | Ass.                           |
| 14 | r                       | E∧ 12                          |
| 15 | s                       | $E\rightarrow 11, 14$          |
| 16 | $u \to s$               | $I{\rightarrow}\ 14\text{-}15$ |
|    | :                       |                                |
|    |                         |                                |

| 1 | p         | Ass.                 |
|---|-----------|----------------------|
| 2 | q         | Ass.                 |
| 3 | q         | Rep. 2               |
| 4 | $q \to q$ | $I{\rightarrow}\ 23$ |

(f) E∨

|    | :                   |             |
|----|---------------------|-------------|
| 13 | $(r \lor t) \to p$  | •••         |
| 14 | $s \to p$           |             |
| 15 | $u \wedge t$        |             |
| 16 | $(r \lor t) \lor s$ |             |
| 17 | p                   | EV 13,14,16 |
| 18 | ig  u               | E∧ 15       |
| 19 | $u \wedge p$        | I∧ 17,18    |
|    | :                   |             |

| 1 | $r \rightarrow s$    | Ass.     |
|---|----------------------|----------|
| 2 | $(p \land q) \to s$  | Ass.     |
| 3 | t                    | Ass.     |
| 4 | $(p \land q) \lor r$ | Ass.     |
| 5 | s                    | E∨ 1,2,4 |
|   | :                    |          |

3. Offer a proof for the following arguments:

(a) 
$$s, s \to p, (p \land q) \to r, q \vdash r$$

(b) 
$$(p \lor q) \to s, p \vdash s$$

(c) 
$$s \to t, r \to t, p \land (s \lor r) \vdash t$$

(d) 
$$r \to q, s, (q \land s) \to u \vdash r \to q$$

(e) Hard: 
$$s \to r, q \to r, u \to q, (r \lor u) \to p, s \lor u \vdash (w \to r) \land r^2$$

<sup>&</sup>lt;sup>2</sup>Requires the rule Repetition.