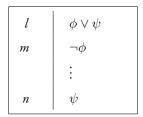
Useful patterns to notice

The way to read these boxes is as follows: when in a proof you have lines of the form of the lines that come before the dots, you will be able to derive (by applying a series of rules that appeal to those previous lines), the line that comes after the dots:



$$\begin{array}{c|cccc}
l & \phi \to \psi \\
m & \neg \psi \\
\vdots \\
n & \neg \phi
\end{array}$$

$$\begin{array}{c|c} l & \neg(\phi \lor \psi) \\ \vdots & \\ m & \neg\phi \land \neg\psi \end{array}$$

$$\begin{array}{c|c} l & \neg(\phi \land \psi) \\ \vdots \\ m & \neg\phi \lor \neg\psi \end{array}$$

$$\begin{array}{|c|c|c|c|}
l & \neg \phi \land \neg \psi \\
\vdots & \vdots \\
m & \neg (\phi \lor \psi)
\end{array}$$

$$\begin{array}{c|c} l & \neg \phi \lor \neg \psi \\ \vdots & \\ m & \neg (\phi \land \psi) \end{array}$$

Exercises

- 1. If I am asked to prove an argument of the following form '... $\vdash \phi$ ', should I start my proof by assuming ϕ ? Why? I shouldn't, because when I assume ϕ the only things I can end up with are $\neg \phi$ (via I \rightarrow) or a conditional of the form ' $\phi \rightarrow$...', but what I wanted to obtain was ϕ !
- 2. Offer a proof of the following argument:

(a)
$$(p \to q) \land r, \neg ((p \land q) \land (r \land s)), p \vdash \neg s$$

1	$(p \to q) \land r$	Assumption
2	$\neg((p \land q) \land (r \land s))$	Assumption
3	p	Assumption
4	s	Assumption
5	r	E∧ 1
6	$p \rightarrow q$	E∧ 1
7	$r \wedge s$	I∧ 4, 5
8	$(p \wedge q)$	Assumption
9	$(p \wedge q) \wedge (r \wedge s)$	I∧ 7, 8
10		E¬ 2, 9
11	$\neg(p \land q)$	I¬ 8-10
12	q	$E\rightarrow 3, 5$
13	p	Assumption
14	$p \wedge q$	I∧ 12,13
15		E¬ 11, 14
16	$\neg p$	I¬ 13-15
17		E¬ 3, 16
18	$\neg s$	I¬ 4-17

Practice Quiz

Offer a proof of the following arguments in 20 minutes:

1.
$$(p \wedge r) \vee (r \wedge s) \vdash r$$

1	$(p \wedge r) \vee (r \wedge s)$	Assumption
2	$p \wedge r$	Assumption
3	r	E∧ 2
4	$(p \wedge r) \to r$	$I{\rightarrow}\ 23$
5	$r \wedge s$	Assumption
6	r	E∧ 5
7	$(r \wedge s) \rightarrow r$	$I\rightarrow 5-6$
8	r	E∨ 1,4, 7

2.
$$\neg((s \lor p) \lor q) \vdash \neg s \land \neg q$$

1	$\neg((s\vee p)\vee q)$	Assumption
2	s	Assumption
3	$s \lor p$	I∨ 2
4	$(s \lor p) \lor q$	I∨ 3
5		E¬ 1,4
6	$\neg s$	I¬ 2-5
7	q	Assumption
8	$(s \vee p) \vee q$	I∨ 7
9		E¬ 1,8
10	$\neg q$	I¬ 7-9
11	$\neg s \wedge \neg q$	I∧ 6,10

3. $p \to s, \neg(r \land q) \to \neg s \vdash p \to (r \land q)$

1	$p \rightarrow s$	Assumption
2	$\neg(r \land q) \to \neg s$	Assumption
3	p	Assumption
4	S	$E\rightarrow 1,3$
5	$\neg (r \land q)$	Assumption
6	$\neg s$	$E\rightarrow 2, 5$
7		E¬ 4,6
8	$\neg\neg(r \land q)$	I¬ 5-7
9	$r \wedge q$	¬¬ 8
10	$p o (r \wedge q)$	$I\rightarrow 3-9$

4.
$$s \to (p \to r), p \land \neg r \vdash \neg s$$

1	$s \to (p \to r)$	Assumption
2	$p \wedge \neg r$	Assumption
3	s	Assumption
4	$p \rightarrow r$	$E\rightarrow 1, 3$
5	p	E∧ 2
6	r	$E \rightarrow 5$
7	$\neg r$	E∧ 2
8		E¬ 6,7
9	$\neg s$	I¬ 3-8

5. $(p \lor q) \to r, \neg r \vdash \neg q \lor r$

1	$ (p \vee q) \to r$	Assumption
2	eg r	Assumption
3	q	Assumption
4	$p \lor q$	I∨ 3
5	r	$E\rightarrow 1,4$
6	上	E¬ 2,5
7	$\neg q$	I¬ 3-6
8	$\neg q \lor r$	I∨ 7