1. Determine whether the following arguments in propositional logic are valid:

   (a) $p \vee (q \to r) \vDash p$

   (b) $p \to (s \wedge q), r \to (q \vee \neg s), p \vDash \neg r$

2. Consider the following translation key:

   p: Having a belief is having a disposition to act in some ways.

   q: Non-living objects can be disposed to act in some ways.

   r: Non-living objects can have beliefs.

   s: Non-living objects represent the world being some way or another.

   Now, translate the following arguments to propositional logic and determine whether they are valid:

   (a) Neither can non-living objects represent the world as being some way or another, nor is having a belief having a disposition to act in some ways only if non-living objects can have beliefs. In conclusion, non-living objects cannot represent the world as being some way or another unless non-living objects can have beliefs.

   $\neg s \wedge \neg(p \to r) \vDash \neg r \to \neg s$

   Valid

   Goal: to translate an argument that is hard to parse, even if it seems bad.

   (b) If having a belief is having a disposition to act in some ways, and if non-living objects can be disposed to act in some ways, then non-living objects can have beliefs. But non-living objects can have beliefs

   Goal: to translate an argument that is easier to parse and seems good.

only if they represent the world as being some way or another, and non-living objects do not represent the world as being some way or another. Therefore, non-living objects cannot be disposed to act in some ways.

$(p \wedge q) \rightarrow r, (r \rightarrow s) \wedge \neg s \vDash \neg q$

Invalid

3. Fill out the missing rules in the following proof:

1	$p \wedge q$	Assumption
2	r	Assumption
3	p	$\wedge$E 1
4	$p \wedge r$	$\wedge$I 2, 3

4. Fill out the missing lines in the following proof:

1	$p \to q$	Assumption
2	$p \to r$	Assumption
3	p	Assumption
4	q	$\to E$ 1,3
5	r	$\to E$ 2,3
6	$q \wedge r$	$\wedge I$ 4,5

5. **Extra:** Offer a proof of the following arguments:

   (a) $(p \wedge q) \wedge (r \wedge s) \vdash r \wedge s$

1	$(p \wedge q) \wedge (r \wedge s)$	Assumption
2	$r \wedge s$	$\wedge$E 1

A *derivation* or *proof* is a finite sequence of formulas each of which is either an assumption or the outcome of an application of a rule of inference to prior formulas. The last formula in the sequence is the conclusion of the proof.

Here's a reminder of the rules needed for these derivations:

l	$\varphi \wedge \psi$	
n	φ	E$\wedge$ l

l	$\varphi \wedge \psi$	
m	ψ	E$\wedge$ l

l	ϕ	
m	ψ	
n	$\phi \wedge \psi$	I$\wedge$ l, m

l	$\phi \to \psi$	
m	ϕ	
n	ψ	E$\to$ l, m

(b) $(p \wedge r) \wedge (s \wedge q) \vdash r \wedge s$

1	$(p \wedge r) \wedge (s \wedge q)$	Assumption
2	$p \wedge r$	$\wedge$E 1
3	$s \wedge q$	$\wedge$E 1
4	r	$\wedge$E 2
5	s	$\wedge$E 3
6	$r \wedge s$	$\wedge$I 4,5

(c) $p \to r, s \wedge p \vdash r \wedge s$

1	$p \to r$	Assumption
2	$s \wedge p$	Assumption
3	p	$\wedge$E 1
4	r	$\to$E 1,3