Quantum Computing Modeling in Scala

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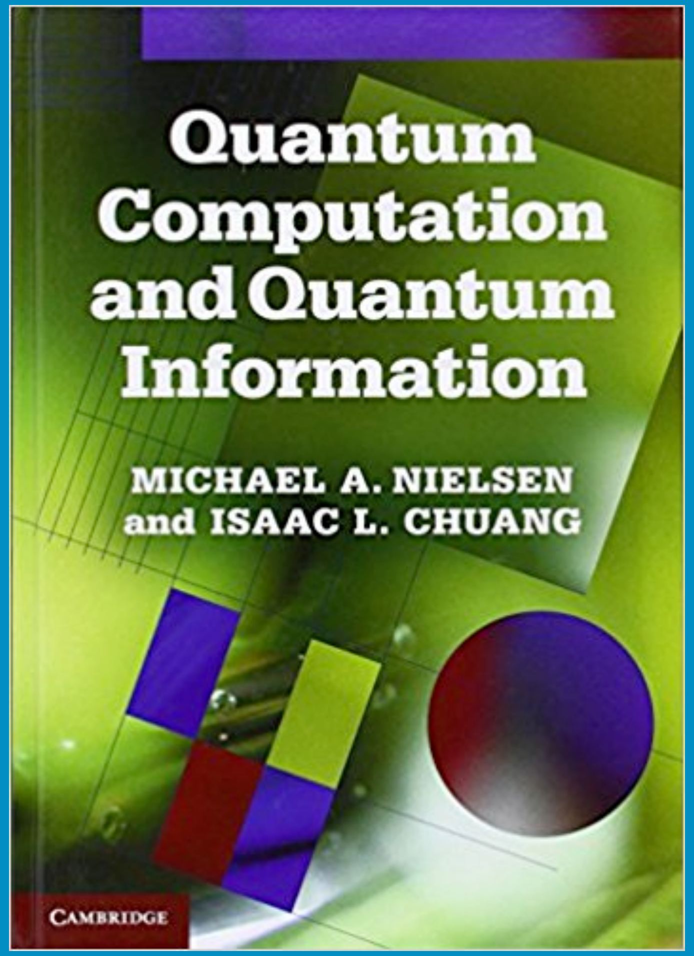


“

The postulates of quantum mechanics were derived after a long process of trial and (mostly) error, which involved a considerable amount of guessing and fumbling by the originators of the theory.

Quantum Postulates

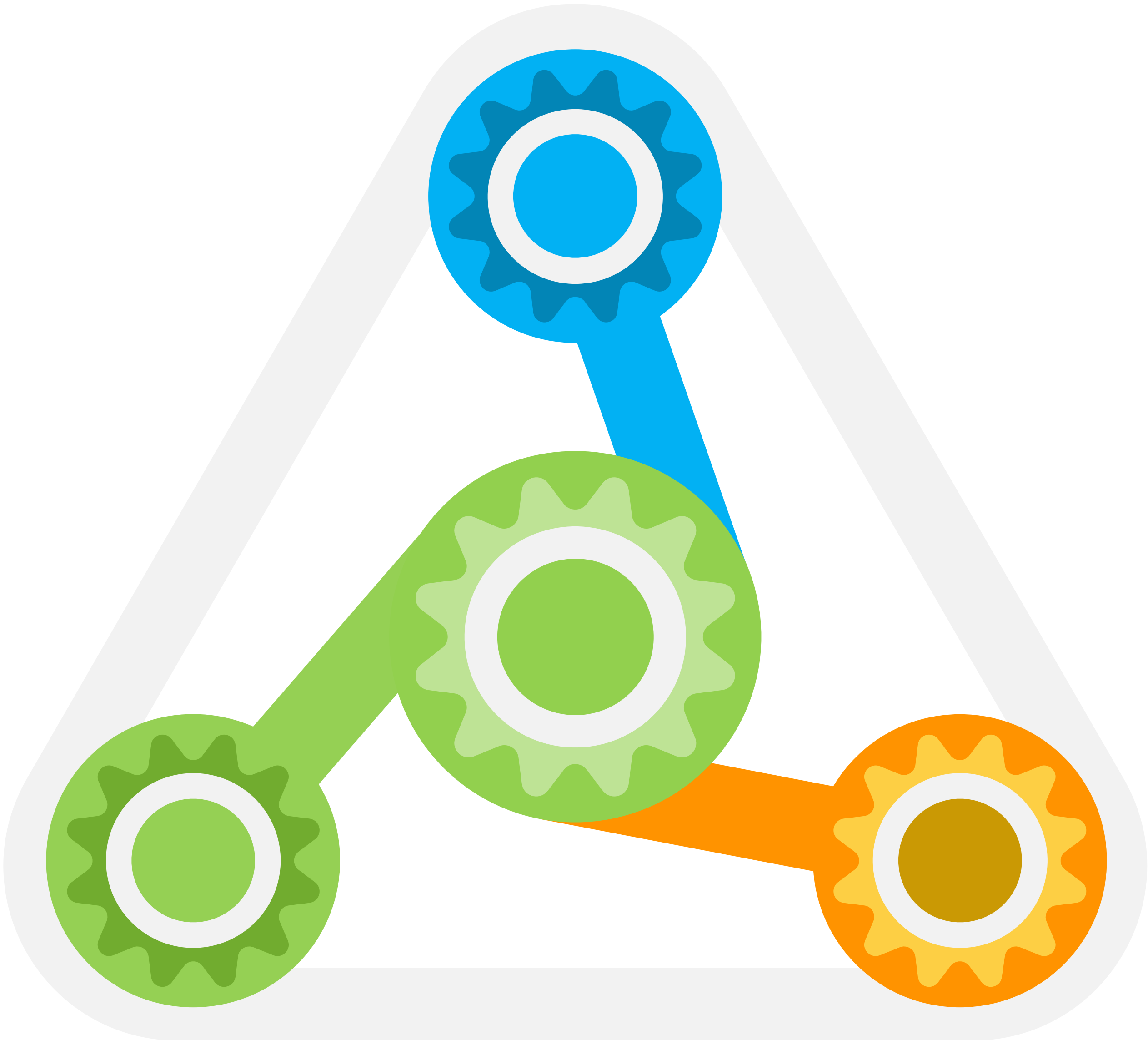
What quantum state is, how it changes, how it is measured and composed

State Space

A quantum system is completely described by its state vector

Evolution

States at two different times are related by a unitary operator



Measurement

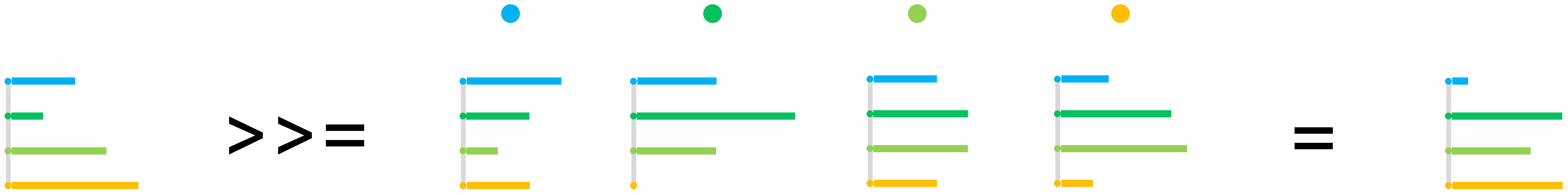
Only certain outcomes may occur in an experiment

Composition

The state space of a composite system is the tensor product of component states

Monadic State Evolution

The system state is defined by labeled values (allocations)



State: `List[(B, V)]`

Change: `B => List[(B, V)]`

State: `List[(B, V)]`

Examples: resource allocation, accounting systems, probabilistic systems, quantum systems

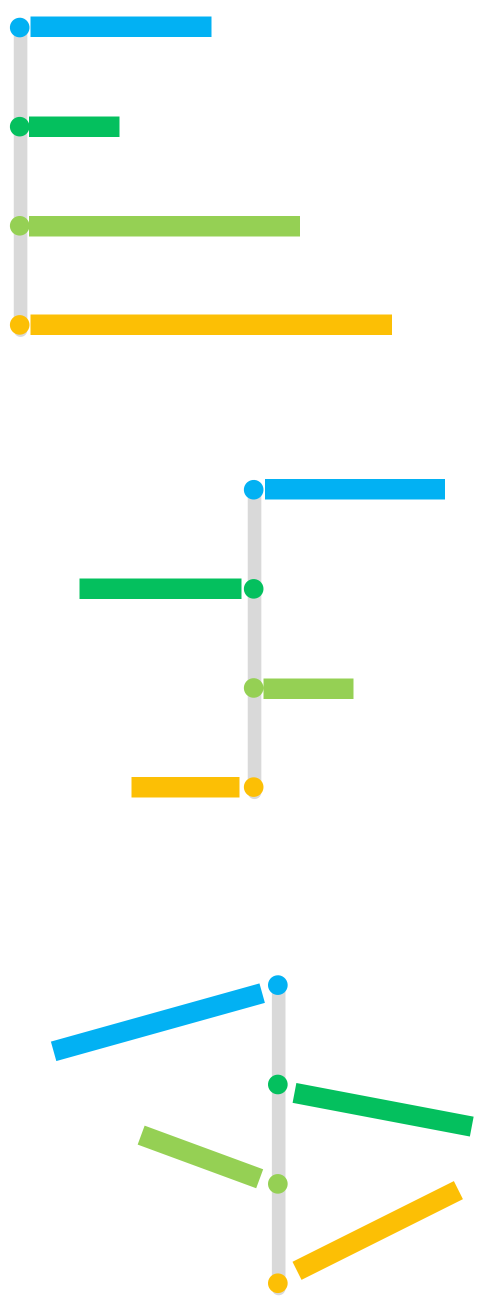
Compare with typical monads:

Container: M[A] >>= Change: A => M[A] = Container: M[A]

State Representation and Evolution

Unified monadic approach to classical, probabilistic and quantum state

```scala
trait UState[+This <: UState[This, B, V], B, V] {
  protected val bins: List[(B, V)]
  protected val m: Monoid[V]

  protected val normalizeStateRule: List[(B, V)] => List[(B, V)] = identity

  protected val combineBinsRule: List[(B, V)] => List[(B, V)] = { bvs =>
    bvs.groupBy(_._1).toList.map {
      case (b, vs) => (b, vs.map(_._2).foldLeft(m.empty)(m.combine))
    }
  }
  protected val distributionRule: ((B, V), List[(B, V)]) => List[(B, V)]

  def create(bins: List[(B, V)]): This

  def normalize(): This = create(normalizeStateRule(bins))

  def flatMap(f: B => List[(B, V)]): This = {
    val updates: List[(B, V)] = bins.flatMap({ case (b, v) => distributionRule((b, v), f(b)) })
    create(normalizeStateRule(combineBinsRule(updates)))
  }

  def >>=(f: B => List[(B, V)]): This = flatMap(f)
}
```



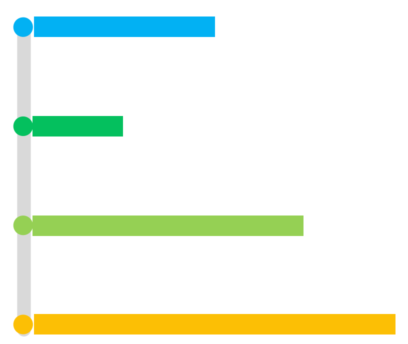
State: `List[(B, V)]`

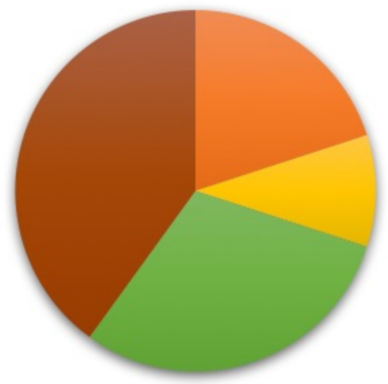
Change: `B => List[(B, V)]`

Portfolio Balancing

Percentage based resource allocation

```scala
val bins: List[(String, Double)] = List("a" -> .2, "b" -> .1, "c" -> .3, "d" -> .4)

val m = RState[String](bins)
val changeA = List("a" -> .25, "b" -> .5, "c" -> .25)
val changeB = List("b" -> 1.0)
val changeC = List("c" -> 1.0)
val changeD = List("d" -> 1.0)

val state = m >>= Map("a" -> changeA, "b" -> changeB, "c" -> changeC, "d" -> changeD)

assert(state.bins.toSet == Set("a" -> .05, "b" -> .2, "c" -> .35, "d" -> .4))
```





Implementation

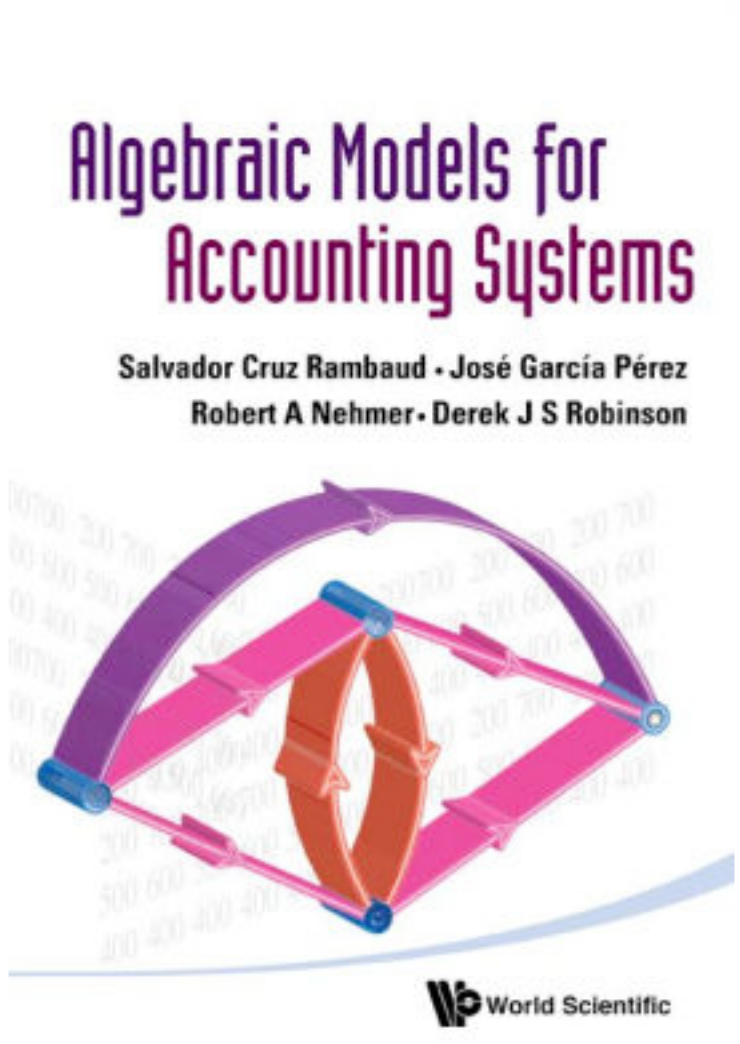
```scala
case class RState[B](bins: List[(B, Double)]) extends UState[RState[B], B, Double] {
  val m = new Monoid[Double] {
    override val empty: Double = 0.0
    override val combine: (Double, Double) => Double = _ + _
  }

  override val distributionRule: ((B, Double), List[(B, Double)]) => List[(B, Double)] = {
    case ((b, v), cs) => cs.map { case (c, u) => (c, u * v) }
  }

  override def create(bins: List[(B, Double)]) = RState(bins)
}
```

Invariant: sum = 1

Account Balances

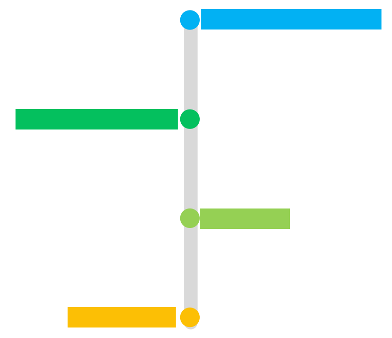
State of a closed accounting system

```scala
val bins: List[(String, Double)] = List("a" -> 2, "b" -> 3, "c" -> 5, "d" -> -8, "e" -> -2)

val z = ZState[String](bins)
val changeA = List("a" -> -1.0, "b" -> 1.0)
val changeB = List("b" -> -2.0, "c" -> 1.0, "d" -> 1.0)

val state = z >>= Map("a" -> changeA, "b" -> changeB, "c" -> Nil, "d" -> Nil, "e" -> Nil)

assert(state.bins.toSet == Set("a" -> 1.0, "b" -> 2.0, "c" -> 6.0, "d" -> -7.0, "e" -> -2.0))
```



Implementation

```scala
case class ZState[B](bins: List[(B, Double)]) extends UState[ZState[B], B, Double] {
  val m = new Monoid[Double] {
    override val empty: Double = 0.0
    override val combine: (Double, Double) => Double = _ + _
  }
  override val distributionRule: ((B, Double), List[(B, Double)]) => List[(B, Double)] = {
    case ((b, v), cs) => List((b -> v)) ++ cs
  }

  override def create(bins: List[(B, Double)]) = ZState(bins)
}
```

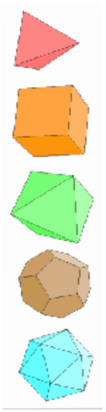


Invariant: sum = 0

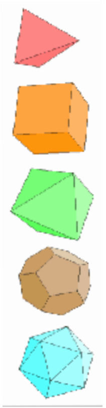
Probabilistic State

Each possible outcome is assigned a probability

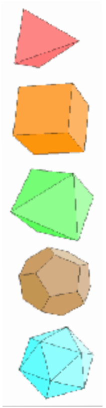
Repeatedly rolling one of the 5 platonic solid dice yields the following sequence: 6, 6, 8, 7, 7, 5, 4.
Guess which die was used?



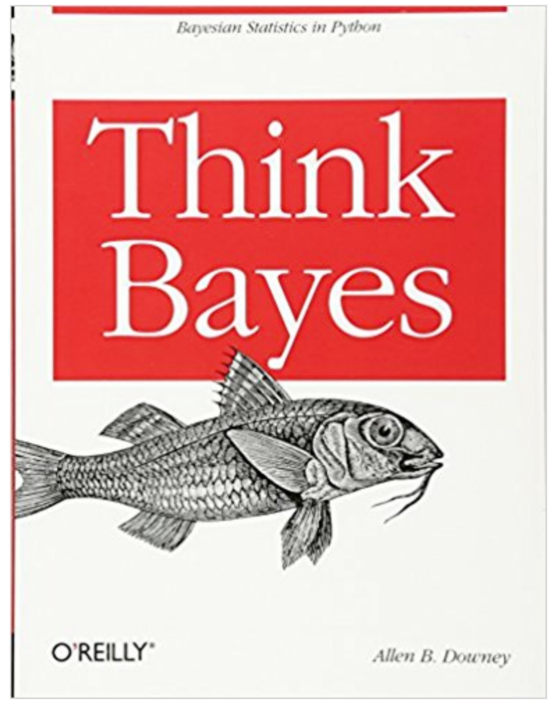
```
Priors:
4   0.2    ##########
6   0.2    ##########
8   0.2    ##########
12  0.2    ##########
20  0.2    ##########
```



```
After a 6 is rolled:
4   0.0
6   0.3921 ###################
8   0.2941 ##############
12  0.1960 #########
20  0.1176 #####
```

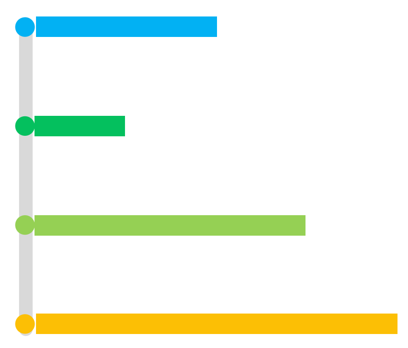


```
After 6, 8, 7, 7, 5, 4 are rolled after the first 6:
4   0.0
6   0.0
8   0.9432 ###############################################
12  0.0552 ##
20  0.0015
```



Probabilistic State

Bayes Theorem



```scala
case class PState[B](bins: List[(B, Double)]) extends UState[PState[B], B, Double] {
  val m = new Monoid[Double] {
    override val empty: Double = 1.0
    override val combine: (Double, Double) => Double = _ * _
  }

  override val distributionRule: ((B, Double), List[(B, Double)]) => List[(B, Double)] = {
    case ((b, v), cs) => cs.map { case (c, u) => (c, u * v) }
    //case ((b, v), cs) => List((b -> v)) ++ cs  // both work
  }

  override val normalizeStateRule = { bvs: List[(B, Double)] =>
    val sum = bvs.map(_._2).foldLeft(0.0)(_ + _)
    if (sum == 1.0) bins else bvs.map {
      case (b, v) => (b, v / sum)

    }
  }

  override def create(bins: List[(B, Double)]) = PState(bins)
}
```

Invariant: normalized sum = 1

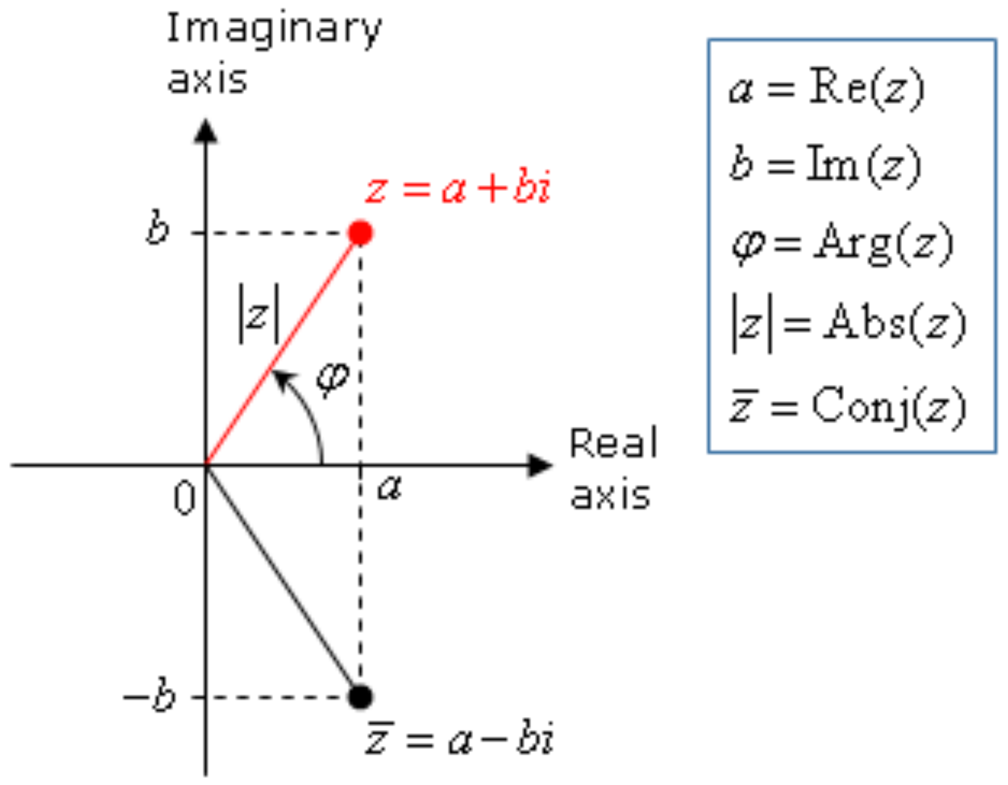
Change: data point likelihoods

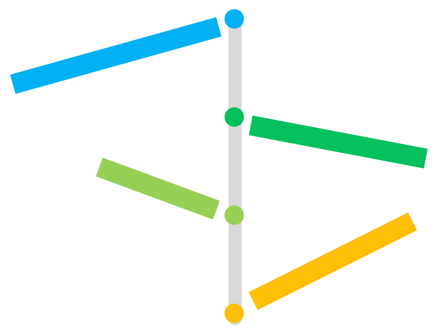
```scala
val change = Map(
  "a" -> List("a" -> 0.2),
  "b" -> List("b" -> 0.7),
  "c" -> List("c" -> 0.0))
```

Quantum State

Complex numbers (2 -dimensional vectors) as values

```scala
case class QState[B](bins: List[(B, Complex)]) extends UState[QState[B], B, Complex] {
  val m = new Monoid[Complex] {
    override val empty: Complex = Complex.zero
    override val combine: (Complex, Complex) => Complex = Complex.plus
  }

  override val distributionRule: ((B, Complex), List[(B, Complex)]) => List[(B, Complex)] = {
    case ((b, v), cs) => cs.map { case (c, u) => (c, u * v) }
  }

  override def create(bins: List[(B, Complex)]) = QState(bins)
}
```



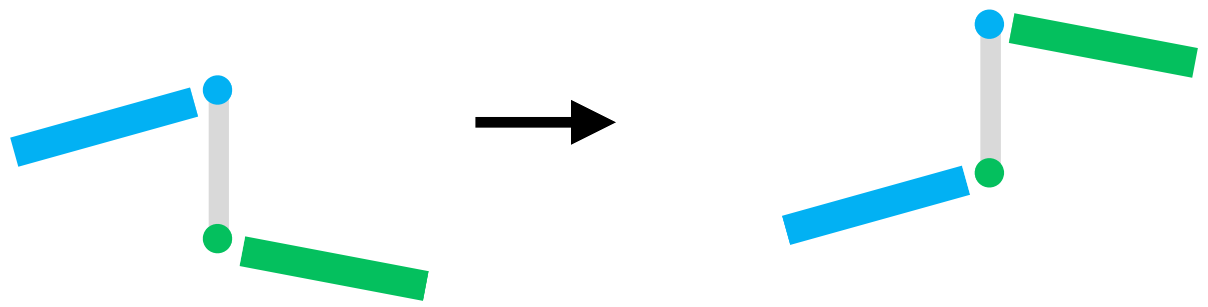


Invariant:
sum of squared magnitudes
= 1

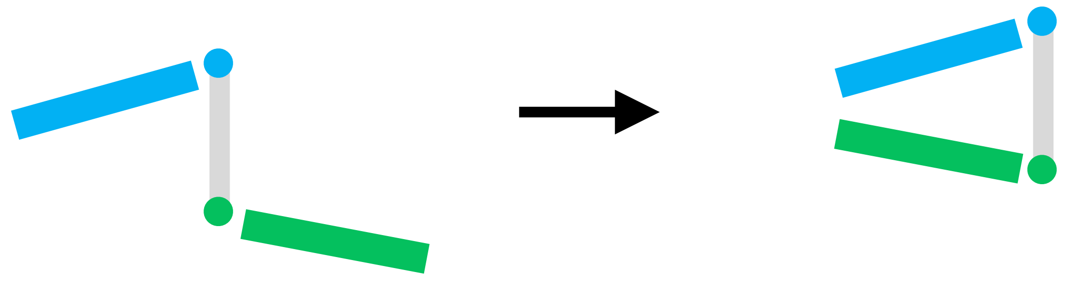
Quantum State Transformations

Standard single qubit transformations

X transformation

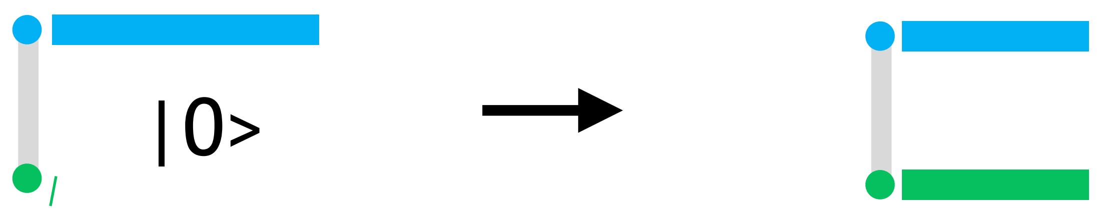


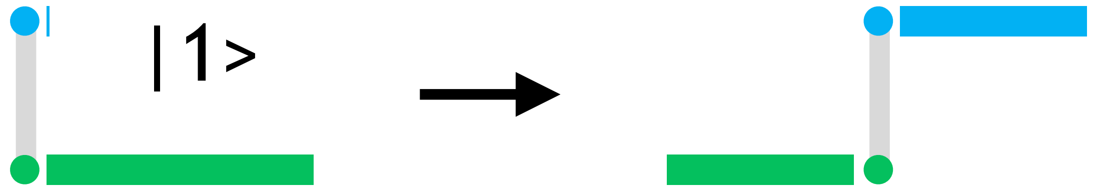
Z transformation



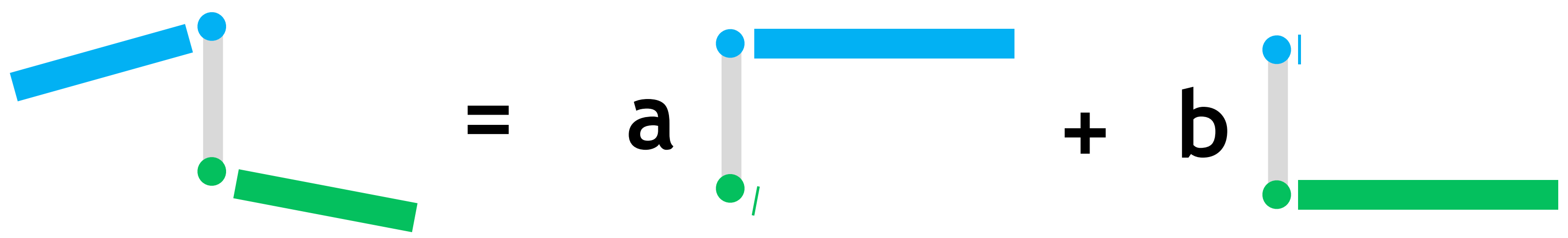
Hadamard transformation

```
val sq = toComplex(1 / math.sqrt(2))
val H = Map(
  "|0>" -> List("|0>" -> sq, "|1>" -> sq),
  "|1>" -> List("|0>" -> sq, "|1>" -> -sq)
)
```



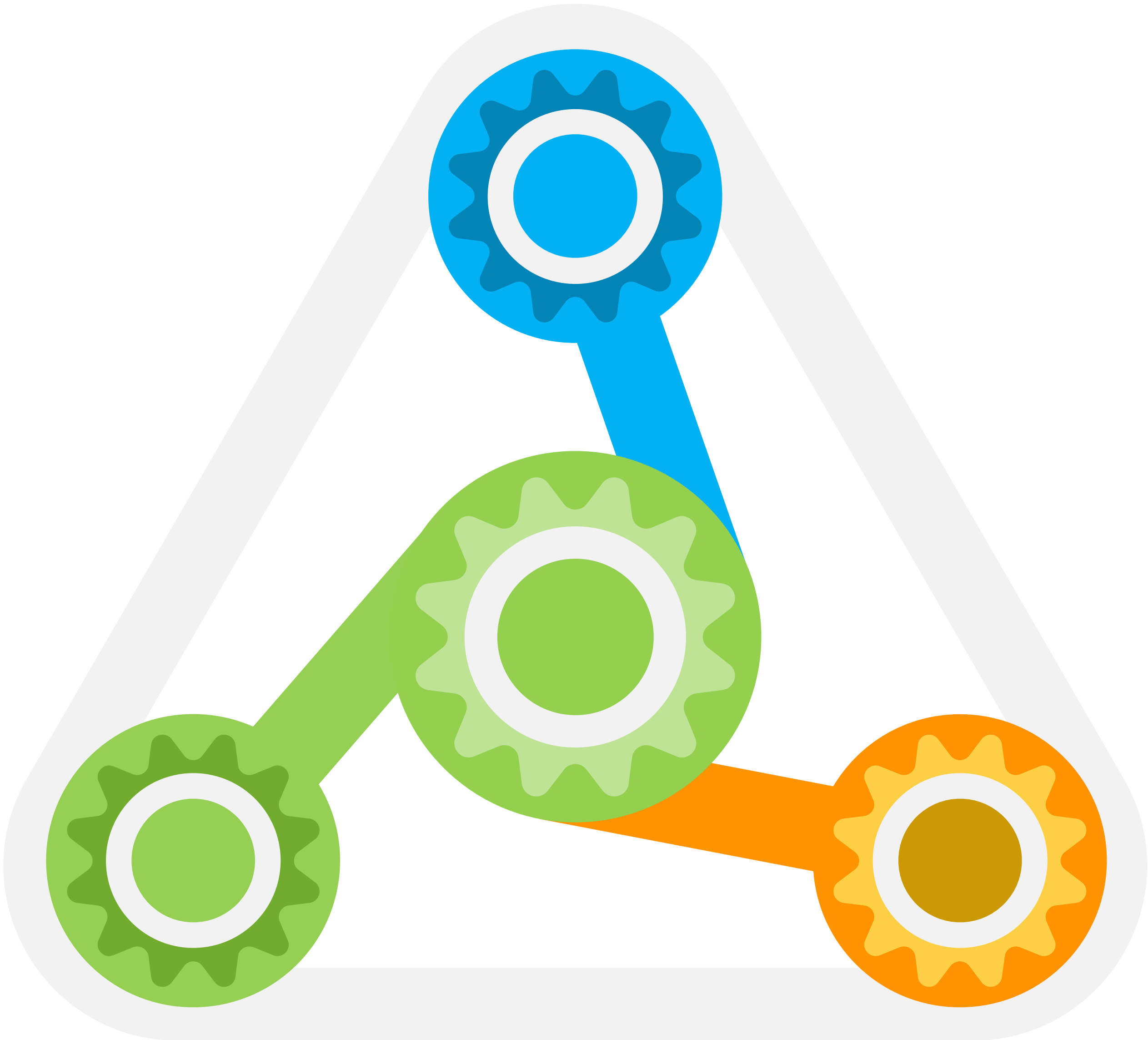


Basis:



Quantum Postulates

What quantum state is, how it changes, how it is measured and composed



State Space

A quantum system is completely described by its state vector

Measurement

Only certain outcomes may occur in an experiment

Evolution

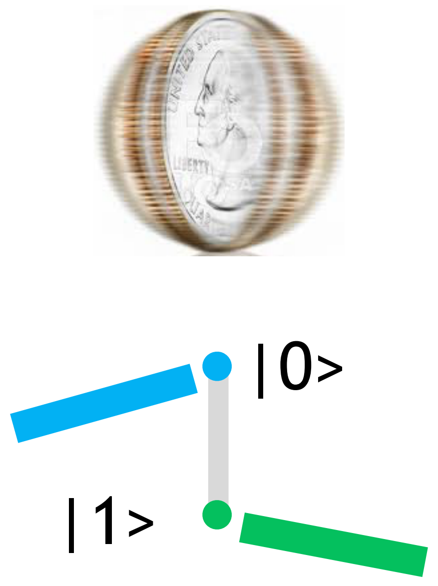
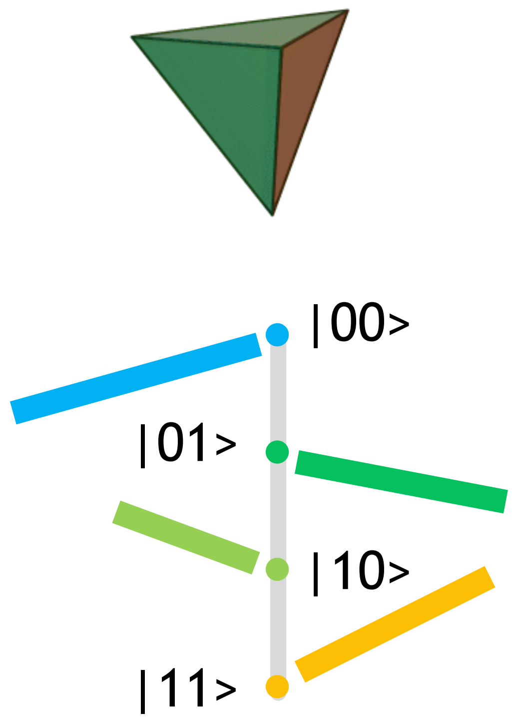
States at two different times are related by a unitary operator

Composition

The state space of a composite system is the tensor product of component states

Composition and Measurement

Qubits, superposition, entanglement

Qubits	Quantum State	Measurement Outcomes
	One amplitude for each possible outcome	The probability of an outcome is the squared magnitude of its associated amplitude
		$|0\rangle$  $|1\rangle$ 
		$|00\rangle$  $|01\rangle$  $|10\rangle$  $|11\rangle$ 

Calculating Fibonacci Numbers

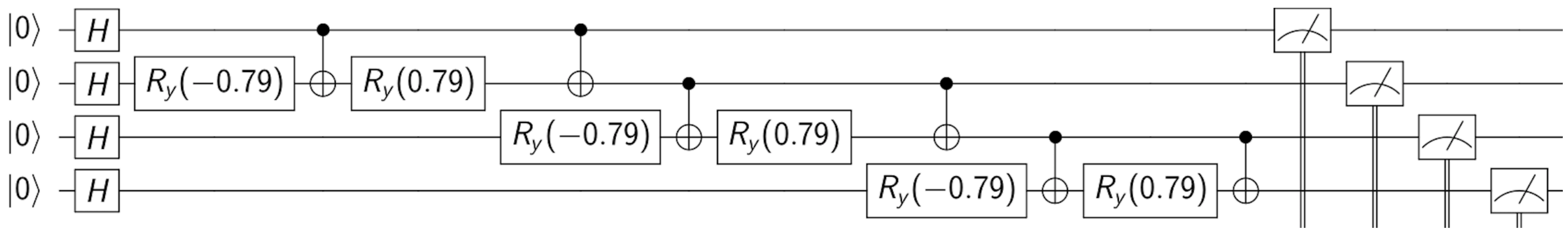
Counting binary words with no consecutive ones

```
def fib(n: Int): QState[Word] = {
  var state = pure(Word.fromInt(0, n))
  for (i <- 0 until n) state = state >>= wire(i, H)
  for (i <- 0 until n - 1)  state = state >>= controlled(i, i + 1, ZERO)
  state
}
```

```
F(1) = 2
F(2) = 3
F(3) = 5
F(4) = 8
F(5) = 13
F(6) = 21
F(7) = 34
F(8) = 55
F(9) = 89
F(10) = 144
F(11) = 233
F(12) = 377
F(13) = 610
F(14) = 987
F(15) = 1597
```

Circuit implementation:

```
for (i <- 0 until n - 1)  state = state >>= wire(i + 1, Ry(-math.Pi/4)) >>=
  controlled(i, i + 1, X) >>= wire(i + 1, Ry(math.Pi/4)) >>= controlled(i, i + 1, X)
```



Thank You

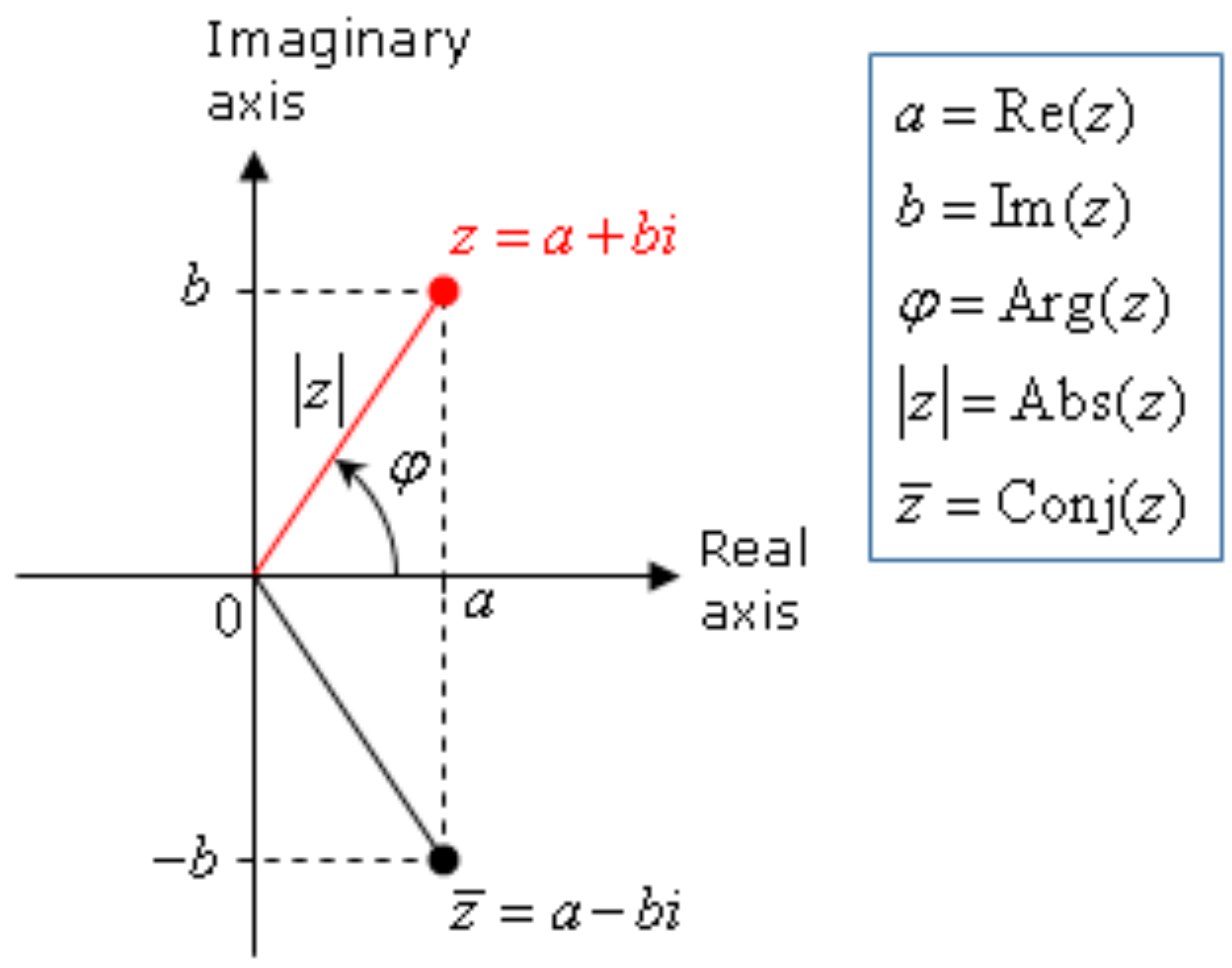
Credits

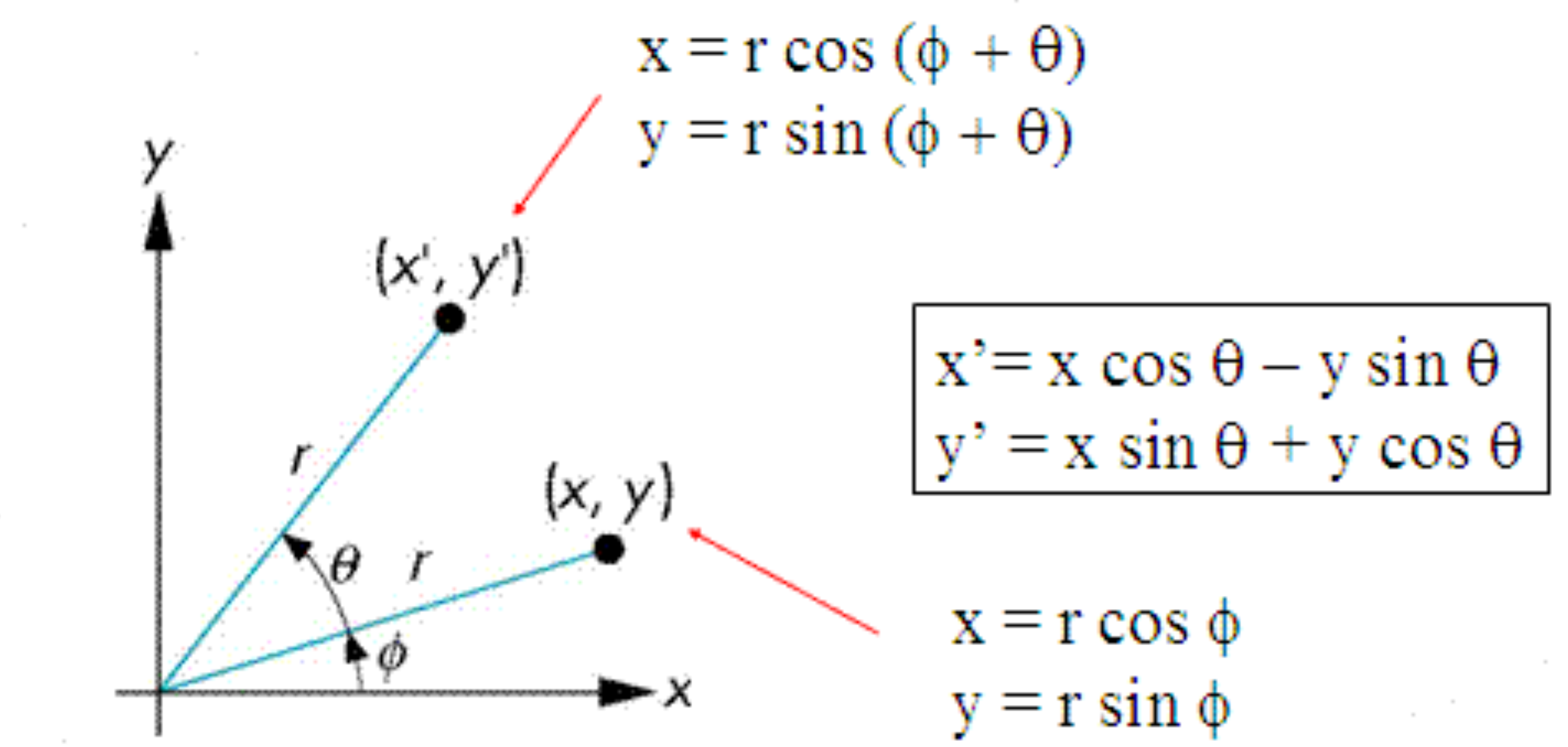
https://github.com/jliszka/quantum-probability-monad
https://sigfpe.wordpress.com/2007/03/04/monads-vector-spaces-and-quantum-mechanics-pt-ii/

Appendix

Complex Numbers

Complex numbers (2 -dimensional vectors) as values

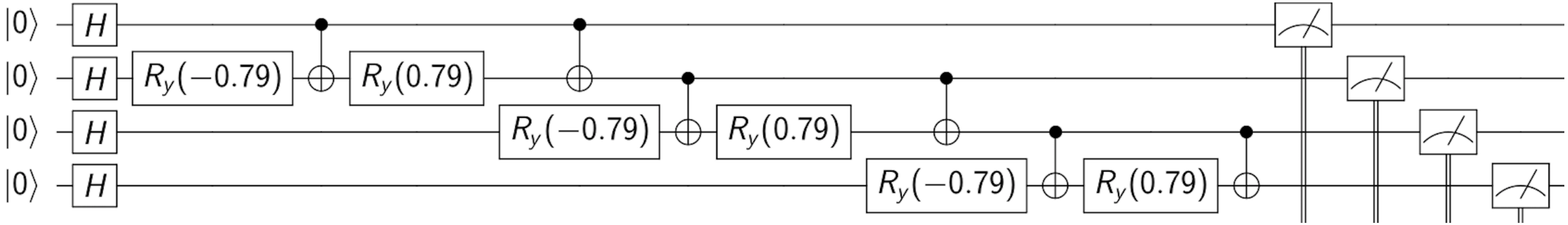




http://www.thefouriertransform.com/math/complexmath.php

Circuits and Gates

Composing and measuring qbits




```scala
"Ry(pi/2)Z" should "equal H" in forAll { state:  QState[Std] =>

  val y: QState[Std] = state >>= Z >>= Ry(math.Pi/2)
  val h: QState[Std] = state >>= H

  assert(y(S0).toString == h(S0).toString)
  assert(y(S1).toString == h(S1).toString)
}
```

```scala
"Ry(theta)" should "mix the amplitudes of |0> and |1> (like vector rotation)"
  val theta = ts._1
  val state = ts._2

  val y: QState[Std] = Ry(theta)(state)

  // same formula as 2-dimensional vector rotation (but with half angle)
  val t0 = state(S0) * math.cos(theta/2) - state(S1) * math.sin(theta/2)
  val t1 = state(S0) * math.sin(theta/2) + state(S1) * math.cos(theta/2)

  assert(y(S0) == t0)
  assert(y(S1) == t1)
```