Unsteady MHD flow of a Viscoelastic fluid past over a permeable boundaries with the external impact of magnetic field

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ABSTRACT- The present work investigates the unsteady flow of a viscoelastic and viscous incompressible fluid passes through an oscillating infinite vertical porous plate with the impact of transverse magnetic field. The obtained governing equations are simplified by Finite Difference Technique. The velocity, temperature and concentration profiles are analyzed graphically by the effect of numerous flow parameters.

Keywords: MHD flow, Viscoelastic fluid, Porous plate, unsteady flow.

2010 Mathematics Subject Classification: 92C10.

I. INTRODUCTION

The study of heat and mass transfer with chemical reaction plays an important role in the field of chemical and hydrometallurgical industries. Raju et al. (2019) studied the flow of Visco-elastic fluid passes through infinite vertical porous plate in the influence of uniform temperature, thermal radiation and specific diffusion. Raghunath et al. (2016) investigated the visco-elastic fluid of unsteady MHD flow past an infinite oscillating porous plate in slip flow regime. Kane et al. (2020) depicts the unsteady fluid flow between two moving parallel porous plates in influence of inclined applied magnetic field. Damseh and Sannak (2010) studied the unsteady free convection flow of viscoelastic and viscous incompressible fluid passes through continuous moving vertical porous plate with the effect of first-order chemical reaction. Sharmin and Alam (2017) depicted MHD flow of Viscoelastic Fluid passes through an Infinite Oscillating Porous Plate with Thermal Diffusion and Heat Source. Uwanta et al. (2011) studied the flow Viscoelastic fluid passes through infinite vertical plate with heat dissipation. Gedik et al. (2012) depicts the unsteady flow of two-phase fluid past in circular pipes in the presence of external electrical and magnetic fields. Reddy et al. (2015) studied the Magneto hydrodynamics free convective fluid flow passes through a semi-infinite vertical porous plate with chemical reaction and heat absorption. Ahmad and Das (2013) investigate the mass transfer of MHD flow past an embedded vertical porous plate in a porous medium in a slip flow regime with chemical reaction and thermal radiation. Biswas et al. (2018) studied Magneto hydrodynamics free convective and heat transfer flow through a vertical porous plate in the presence of chemical reaction. Makinde and Mhone (2005) studied oscillatory MHD flow and heat transfer in a channel filled with porous medium. Fenuga et al. (2018) studied the effects of mixed convection and Navier slip on a chemically reactive heat and mass transfer magneto hydrodynamics fluid flow over a permeable surface with convective boundary conditions. Ram and Mishra (1977) investigated an unsteady MHD flow through porous media. Moniem and Hassanin (2013) depict solution of MHD Flow passes through a vertical porous plate with oscillatory suction. Kumar et al. (2010) studied reliable magnetohydrodynamics steady flow through channels permeable boundaries and solved the governing equation by Finite difference technique. Kumar et al. (2010) studied Perturbation technique to unsteady MHD periodic flow of viscous fluid through a planer channel.

II. FORMULATION OF THE PROBLEM Consider MHD flow of viscoelastic and viscous incompressible fluid passes through an oscillating infinite vertical porous plate. Let u*, v* and w* are the velocity components along x*, y* and z* axis respectively. A magnetic field of uniform strength B_0 is applied along z*- axis. The suction or injection velocity at the plate is assumed to be w_0 and pressure in the fluid is taken as constant. The governing equations of the problem under the boundary layer and Boussinesq approximations can be written as the following form;

$$\frac{\partial w^*}{\partial z^*}= 0 \quad \dots \ (1)$$

$$\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} = \nu \frac{\partial^2 u^*}{\partial z^{*2}} - \frac{K_0}{\rho}\left(\frac{\partial^3 u^*}{\partial z^{*2}\partial t} + w^* \frac{\partial^3 u^*}{\partial z^{*3}}\right) - \frac{\sigma B_0^2}{\rho} u^* - \frac{\nu}{k} u^*$$

$$+g\beta(T^* - T_\infty^*) + g\beta^*(C^* - C_\infty^*) \quad \dots \ (2)$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} = \nu \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{K_0}{\rho}\left(\frac{\partial^3 v^*}{\partial z^{*2}\partial t} + w^* \frac{\partial^3 v^*}{\partial z^{*3}}\right) - \frac{\sigma B_0^2}{\rho} v^* - \frac{\nu}{k} v^* \quad (3)$$

$$\frac{\partial T^*}{\partial t^*} + w^* \frac{\partial T^*}{\partial z^*} = \alpha \frac{\partial^2 T^*}{\partial z^{*2}} - S(T^* - T_\infty^*) \quad \dots (4)$$

$$\frac{\partial C^*}{\partial t^*} + w^* \frac{\partial C^*}{\partial z^*} = D \frac{\partial^2 C^*}{\partial z^{*2}} - K_1(C^* - C_\infty^*) \dots \ (5)$$

Equation (1) implies that

$$w^* = -w_0 \quad \dots \ (6)$$

Where w_0 be positive constant and negative sign is taken as the suction is along the porous plate.

Adding Equations (2) and i(3), taking $q^* = u^* + iv^*$

$$\frac{\partial q^*}{\partial t^*} - w_0 \frac{\partial q^*}{\partial z^*} = \nu \frac{\partial^2 q^*}{\partial z^{*2}} - \frac{K_0}{\rho}\left(\frac{\partial^3 q^*}{\partial z^{*2}\partial t} - w_0 \frac{\partial^3 q^*}{\partial z^{*3}}\right) - \frac{\sigma B_0^2}{\rho} q^* - \frac{\nu}{k} q^*$$

$$+g\beta(T^* - T_\infty^*) + g\beta^*(C^* - C_\infty^*) \quad (7)$$

Corresponding boundary conditions are

$$\left.\begin{array}{ll} q^* = U_0 e^{i\omega t} + L_1 \dfrac{\partial q^*}{\partial z^*}, \ T^* = T_w^*, \ C^* = C_w^* & at \ \ z^* = o \\ q^* \to 0, \ T^* \to T_\infty^*, \ C^* \to C_\infty^* & at \ \ z^* \to \infty \end{array}\right\} \quad \dots (8)$$

Mean free path is $L_1 = (2 - m_1)(L/m_1), L = \mu(\pi/2p\rho)^{1/2}$, where $m_1 is$ coefficient of Maxwell's reflection.

Introducing non dimensional quantities:

$$u = \frac{u^*}{U_0}, v = \frac{v^*}{U_0}, t = \frac{t^* U_0^2}{\nu}, \ z = \frac{z^* U_0}{\nu}, w = \frac{\nu w^*}{U_0^2}$$

$$, w_0 = \frac{w_0^*}{U_0}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}$$

The governing equations reduce to the following equations;

$$\frac{\partial q}{\partial t} - w_0 \frac{\partial q}{\partial z} = \frac{\partial^2 q}{\partial z^2} - R_c\left(\frac{\partial^3 q}{\partial z^2 \partial t} + w_0 \frac{\partial^3 q}{\partial z^3}\right) -$$

$$\left(M^2 + \frac{1}{K}\right) q + G_r\theta + G_m\theta \quad \dots (9)$$

$$P_r\left(\frac{\partial \theta}{\partial t} - w_0 \frac{\partial \theta}{\partial z}\right) = \frac{\partial^2 \theta}{\partial z^2} - P_r . S . \theta \quad \dots .(10)$$

$$S_c\left(\frac{\partial C}{\partial t} - w_0 \frac{\partial C}{\partial z}\right) = \frac{\partial^2 C}{\partial z^2} - K_C . S_C . C \quad (11)$$

Where, Hartmann number $M = \sqrt{\frac{\nu \, \sigma B_0^2}{\rho U_0^2}}$, Permeability parameter $K = \frac{\sigma U_0^2}{\nu^2}$, Rarefaction parameter $R = \frac{L_1 . U_0}{\nu}$, Prandtl number $P_r = \frac{\nu}{\alpha}$, Schmidt number $S_C = \frac{\nu}{D}$, Elastic parameter $R_C = \frac{K_0 U_0^2}{\rho \nu^2}$, Chemical reaction parameter $K_C = \frac{\nu K_1}{U_0^2}$, Heat Source parameter $S = \frac{\nu S^*}{U_0^2}$, Grashof number $G_r = \frac{g\beta(T_w^* - T_\infty^*)\,\nu}{U_0^3}$

Suction / injection velocity $w_0 = \frac{w_0^*}{U_0}$, Mass Grashof number $G_m = \frac{g\beta^*(C_w^* - C_\infty^*)\,\nu}{U_0^3}$.

$Where\ \rho\ is\ the\ fluid\ densit, P^*\ is\ the\ pressure$

$\mu\ is\ the\ fluid\ viscocity, g\ is\ the\ acceleration,$

$\beta\ and\ \beta^* are\ the$

$thermal\ and\ concentration\ expansion\ coefficients,$

$T^* and\ T_\infty^* are\ the\ temperature\ of\ the\ fluid\ inside$

$the\ layer\ and\ the\ \ fluid\ temperature\ is\ free\ stream,$

$v\ is\ the\ Kinematic\ Viscocity,$

$K^*\ is\ the\ permeability\ of\ the\ porous\ medium,$

$\sigma\ is\ the\ electrical\ conductivity\ of\ the\ \ fluid, t^*\ is\ the\ time,$

$C^* and\ C_\infty^*\ \ are\ the\ species\ concentration\ in\ the$

$boundary\ layer\ and\ in\ the\ fluid\ away\ from\ the\ plate,$

$B_0\ is\ the\ magnetic\ induction, D\ is\ the\ mass\ diffusivity$

$and\ \alpha\ is\ the\ thermal\ diffusivity\ of\ the\ fluid.$

III. SOLUTION OF THE PROBLEM

Solve the non-linear partial differential equations (9)-(11) under the conditions (8) by applying Finite Difference Technique. The accuracy is increased by increasing the discretization points at some given points. The equations (9)-(11) can be descritized as following.

$$\frac{\partial q}{\partial t} = \frac{q_{i+1,j} - q_{i,j}}{\Delta t}, \qquad \frac{\partial u}{\partial y} = \frac{u_{i-1,j} - u_{i,j}}{\Delta y},$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2}.$$

IV. RESULTS AND DISCUSSION

The effect of various parameters such as Prandtl number (Pr), Modified Grashof Number (Gm), Grashof number (Gr), Chemical reaction parameter (Kc), Schmidt number (Sc), Elastic parameter (Rc), Magnetic parameter (M) and porosity parameter (K) on Velocity profile has been observed. Figure1-8 depicts that the velocity components u and v decrease with increasing value of Pr, K, Rc and Kc while increase with increasing value of M and Gm. The velocity components u decrease and v increase with increasing value of as Gr and Sc. Figure 9-11 depicts that the increasing values of Pr and w_0 reduce the temperature profile. Figure 12-13 depicts that concentration profile reduces with increasing value of Elastic parameter (Rc) and Chemical reaction parameter (Kc).

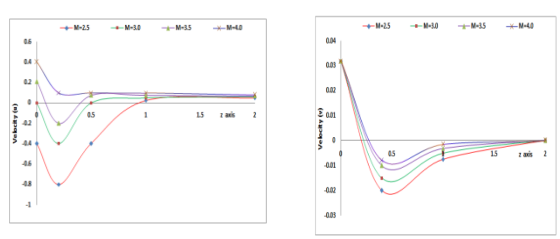
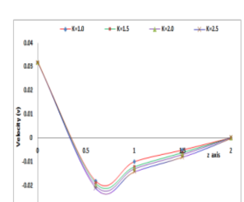


Figure 1: The effect of Magnetic parameter (M) on the velocity Profile.



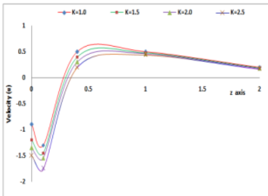


Figure 2: The effect of porosity parameter (K) on the velocity Profile.

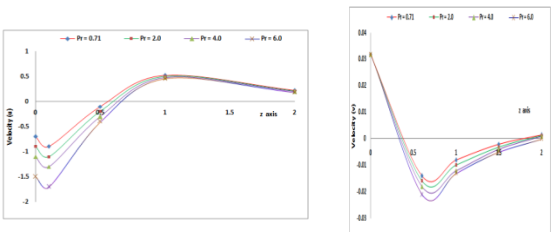


Figure 3: The effect of Prandtl number (Pr) on the velocity Profile.

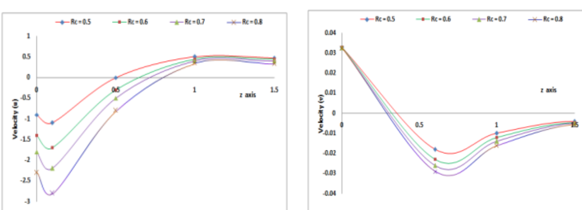


Figure 4: The effect of Elastic parameter (Rc) on the velocity Profile.

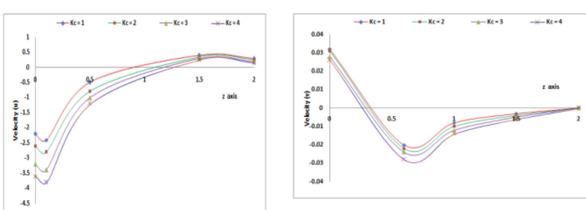


Figure 5: The effect of Chemical reaction parameter (Kc) on the velocity Profile.

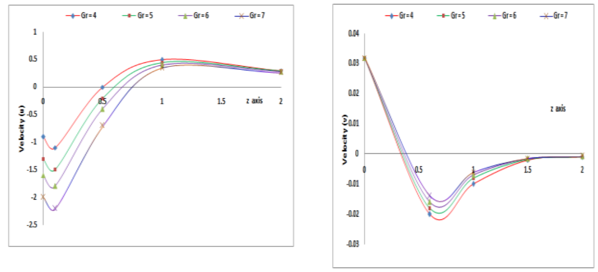


Figure 6: The effect of Grashof number (Gr) on the velocity Profile.

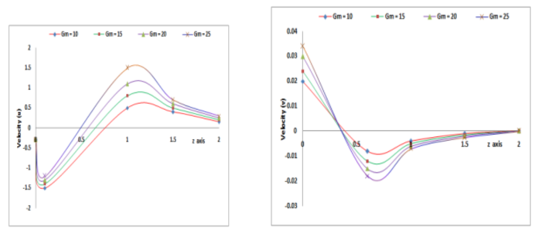


Figure 7: The effect of Grashof Number (Gm) on the velocity Profile.

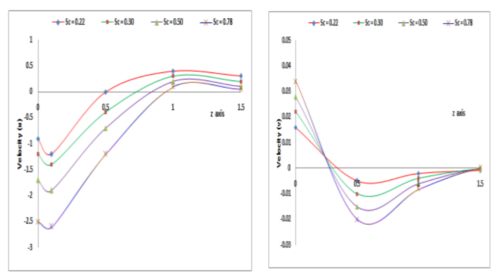


Figure 8: The effect of Schmidt number (Sc) on the velocity Profile.

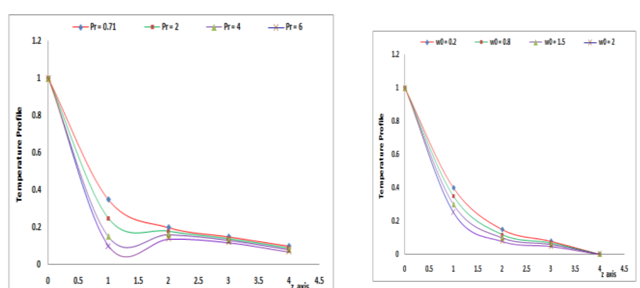


Figure 8: The effect of Prandtl number (Pr) & Suction Velocity w_0 on the temperature profile.

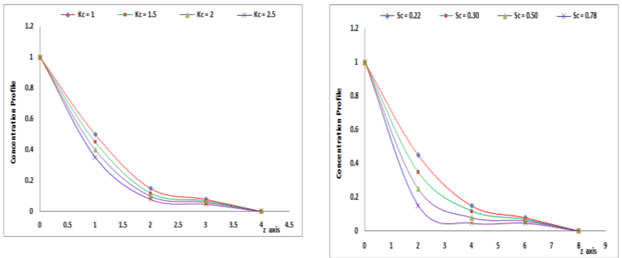


Figure 8: The effect of Chemical reaction parameter (Kc) & Schmidt number (Sc) on the concentration profile.

V. CONCLUSION

The present work, the impact of flow parameters on unsteady MHD flow has been investigated. The velocity profile decreases with increasing value of K, Pr, Rc, Kc while increases with increasing value of Magnetic parameter (M) and Schmidt number (Sc). The temperature profile decreases with increasing value of Pr, S and w_0. The concentration profile reduces with increasing values of Chemical reaction parameter (Kc) and Schmidt number (Sc).

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