Definientia

Revision: May 9, 2016; Rendered: May 25, 2021

Definability in terms of projections, for computing definientia by interpolation. Makes use of scratch_forgetting. Formalized with the *PIE* system.

1 Definientia

The following formula is valid if and only if formula G is definable in terms of predicates S within formula F. Definientia are exactly the interpolants of its antecedent and consequent.

---

definiens(G, F, S)

Defined as

$$\mathit{proj}(S, (F \land G)) \rightarrow \neg \mathit{proj}(S, (F \land \neg G)).$$

The following specification based on literal projection allows to restrict the polarity of the predicates in S:

---

definiens_lit(G, F, S)

Defined as

$$\mathit{projlit}(S, (F \land G)) \rightarrow \neg \mathit{projlit}(S_1, (F \land \neg G)),$$

where

$$S_1 := \text{duals of } S.$$

definiens_lit_lemma is an incomplete version of *definiens_lit* that yields formulas which are more efficient to handle:

definiens_lit_lemma(G, F, S)

Defined as

$$\mathit{lemma\_projlit}(S, (F \land G)) \rightarrow \neg \mathit{lemma\_projlit}(S_1, (F \land \neg G)),$$

where

$$S_1 := \text{duals of } S.$$

Definability of a single predicate in terms of a given set of predicates:

predicate_definiens(P, F, S)

Defined as

$$\mathit{definiens}(P_X, F, S),$$

where

$$\begin{aligned} &N := \text{arity of } P \text{ in } F,\\ &X := \text{a sequence of N fresh symbols},\\ &P_X := P(X). \end{aligned}$$

Definability of a single predicate in terms of all other predicates:

predicate_definiens(P, F)

Defined as

$$\exists P\, (F \land P_X) \rightarrow \neg \exists P\, (F \land \neg P_X),$$

where

$$\begin{aligned} &N := \text{arity of } P \text{ in } F,\\ &X := \text{a sequence of N fresh symbols},\\ &P_X := P(X). \end{aligned}$$

1.1 Definientia: Examples

ex_definiens$_1$

Defined as

$$\begin{aligned} \mathit{definiens}(&\mathsf{pa},\\ &\forall x\, (\mathsf{p}x \leftrightarrow \mathsf{q}x) \land \forall x\, (\mathsf{p}x \leftrightarrow \mathsf{r}x),\\ &[\mathsf{q}]). \end{aligned}$$

Input: *ex_definiens*$_1$.
Result of interpolation:

$$\mathsf{qa}.$$

Index