A Phony Proof – Solution

go figure

"The problem," Detective Harriet Hesterton told Gus, "happens when you divide both sides of the equation by $x - y$. You started by saying that $x = y$, but that means $x - y = 0$. When you divide both sides by $x - y$, you're really dividing by zero."

"Why is that a problem?" Gus asked, not giving up yet.

"Say you start with the equation $0 = 0$," Harriet explained. "Since anything times zero is zero, you can rewrite that equation however you want, as zero times something equaling zero times another something…"

$$0 \times 2 = 0 \times 102$$

"If we were allowed to divide both sides by an equation by zero, we could say the first something equals the second something. But that's not true.

"Here's another way to look at it. When we say $12 \div 4 = 3$, we're saying that 4 is the number we multiply 3 by to get 12. If we tried to say $12 \div 0 = x$, we'd be saying that 0 is the number we multiply x by to get 12. But 0 times x is always 0."

"Oh…" Gus said, audibly disappointed. "So that means my proof is bogus."

"Exactly," Harriet concluded. "Sorry about that. Case closed."