Spider Socks

go figure

Harvey the Spider is putting on his argyle socks and boots in the morning. Naturally, he has 8 socks and 8 boots to put on, and for every one of his 8 legs, he has to put the sock on before the boot.

Let's number Harvey's feet 1 through 8, call the argyle sock for his $i\text{th}$ foot A_i, and call the boot for his $i\text{th}$ foot B_i. Then one valid order Harvey could use for putting on socks and boots is

$$A_1B_1A_2B_2A_3B_3A_4B_4A_5B_5A_6B_6A_7B_7A_8B_8$$

Another valid order is

$$A_4A_3B_4A_7B_3A_1A_5B_5A_6B_1B_6A_8B_7B_8A_2B_2$$

since each A_i comes before its corresponding B_i. But it would be impossible for Harvey to use the order

$$A_4\boldsymbol{B}_3B_4A_7\boldsymbol{A}_3A_1A_5B_5A_6B_1B_6A_8B_7B_8A_2B_2$$

since now B_3 comes before A_3. Harvey also can't use the order

$$B_1A_1B_2A_2B_3A_3B_4A_4B_5A_5B_6A_6B_7A_7B_8A_8$$

since all the sock-boot pairs are backwards.

Harvey is sleepy this morning and not paying attention to his order of socks and boots, so he puts on his socks and boots in random order. What is the probability that he picks a valid order for his socks and boots?

(If you get stuck, you might want to read the techniques in Better Ways to Count: http://gofiguremath.org/better-ways-to-count/.)