

A Phony Proof



go figure

Private detective mathematician Harriet Hesterton was sitting calmly at her desk one day absently flipping through case files when her phone rang.

"My name is Gus Spurious," said the voice on the other end, "and I have a problem. See, I was playing with a bit of algebra the other day, and I proved something that just looks completely wrong. I might've just made a breakthrough that changes the face of mathematics, but I can't figure out how my result could be true! Detective Hesterton, can you help me solve this case?"

Harriet was familiar with algebra. She knew that the golden rule of algebra is *do unto one side as you do unto the other*: start with two things that are equal, and do the same thing to both, and the results should be equal. She figured Gus had probably made a mistake by breaking this rule; that was pretty common.

"Okay, I'll take the case," she told him, grabbing a pad of paper and a pen. "Now tell me exactly what happened."

Gus began to explain over the phone. "First, I started with some numbers x and y equal to each other."

Harriet wrote:

$$x = y$$

"Then," Gus continued, "I multiplied both sides by -1 ."

$$-x = -y$$

"Then I added $2x$ to both sides."

$$\begin{aligned} 2x - x &= 2x - y \\ x &= 2x - y \end{aligned}$$

"Then I subtracted y from both sides."

$$\begin{aligned} x - y &= 2x - y - y \\ x - y &= 2x - 2y \end{aligned}$$

"Then I factored out the 2 on the right side, using the Distributive Law."

$$x - y = 2(x - y)$$

"Then I divided both sides by $x - y$."

$$\frac{x - y}{x - y} = \frac{2(x - y)}{x - y}$$

$$1 = 2$$

"So I've just proved with algebra that $1 = 2$," Gus concluded. "Something really fishy is going on, but I was careful to follow the golden rule of algebra, so it must be true, right?"

Harriet just shook her head. Something was wrong here. The conclusion just wasn't true.

Can you figure out what's wrong?