5.7 (R) Avoidance Concept

This section introduces *Platform Independent Avoidance Concept* core functionality (fig. **??**) modules responsible for *path finding* and *navigation* including *data fusion* interface. The sections are organized like follow:

1. *Data Fusion* (sec. 5.7.1) - implementation details of *input interface* responsible for *processing partial known world data* into final visibility, obstacle, intruder, and, constraints ratings.

2. *Avoidance Grid Run* (sec. **??**) (inner avoidance run) - the *best path finding* in one *Avoidance Grid* with *situation assessment* done.

3. *Mission Control Run* (sec . 5.7.3) (outer navigation run) - main navigation and decision making algorithm for *non-cooperative obstacle avoidance*.

4. *Computation Complexity* (sec. 5.7.4) - the *computational feasibility study* and *weak point identification* of our approach.

5. *Safety Margin Calculation* (sec. 5.7.5) - the boundaries of *Safety Margin* and identified *impact factors*.

5.7.1 (R) Data fusion

The data fusion interfaces *Sensor Field* and *Information Sources* from *cell/trajectory properties*. The *Data Fusion Function* is outlined in (**??**).

First there will be an outline of *Partial Ratings* commutation. Then these ratings will be discredited into Boolean values as properties of *Avoidance Grid/Trajectory*. Then these Boolean values will be used for further classification of space into *Free(t), Occupied(t), Restricted(t)* and *Uncertain(t)*.

All mentioned ratings are result of *Filtered Sensor Readings* from *Sensor Field* and *Information Sources* with prior processing. This section will focus on *final fuzzy value calculation* and *discretization*.

Note. All rating values are in *range:* $[0, 1]$ and they were introduced in previous sections.

Visibility: The *sensor reading* of *sensor* if *Sensor field* returns a value of *visibility* for cell space in time of decision t_i.

The *visibility* for cell is given in (eq. 5.1) as minimal visibility calculated from all capable sensors in *Sensor Field*.

$$visibility(cell_{i,j,k}) = \min \begin{Bmatrix} visibility(cell_{i,j,k}, sensor_i) : \\ \forall sensor_i \in SensorField \end{Bmatrix} \tag{5.1}$$

The example of *visibility* calculation for *LiDAR* sensor is given in (sec. **??**).

Note. Sensor reliability for *visibility* is already accounted prior *data fusion*. If not *weighted average* should be used instead.

Detected Obstacle: The *physical obstacles* are detected by *sensors* is *Sensor Field.* Each *sensor* returns *detected obstacle rating* in range $[0, 1]$ reflecting the probability of obstacle occurrence in given cell.

The *maximal value* of *detected obstacle* rating is selected from readings multiplied by *visibility rating* to enforce *visibility bias.*

$$obstacle(cell_{i,j,k}) = \max \left\{ \begin{matrix} obstacle(cell_{i,j,k}, sensor_i) : \\ \forall sensor_i \in SensorField \end{matrix} \right\} \times \dots$$
$$\dots \times visibility(cell_{i,j,k}) \quad (5.2)$$

The example of *detected obstacle rating* calculation for *LiDAR* sensor is given in (sec. **??**).

Map Obstacle: The *Information Sources* are feeding *Avoidance Grid* with partial information of *Map obstacle rating. Map Obstacle Rating* shows the certainty that *charted obstacle* is in given cell. This property is bound to *Information Source* and it has *range* in $[0, 1]$.

The *Map Obstacle Rating* for cell (eq. 5.3) is calculated as product of maximal *Map Obstacle Rating* and *inverse visibility.* This gives *visibility biased* certainty of *Map Obstacle.*

$$map(cell_{i,j,k}) = \max \left\{ \begin{matrix} map(cell_{i,j,k}, source_i) : \\ \forall source_i \in InformationSources \end{matrix} \right\} \times \dots$$
$$\dots \times (1 - visibility(cell_{i,j,k})) \quad (5.3)$$

The example of *Map Obstacle Rating* calculation is given in (sec. **??**).

Intruder: There is a set of *Active Intruders*, each intruder is using its own *parametric intersection model.* This parametric *intersection* model calculates *partial intersection ratings* representing *intersection certainty* ranging in $[0, 1]$. The more *partial intersection rating* is closer to 1 the higher is the probability of aerial collision with that intruder in that cell.

The *geometrical bias* is used for cumulative of multiple intruders, the *intruders are not cooperative*, therefore their occurrence can not be addressed by simple *maximum.* The proposed formula (eq. 5.4) is simply bypassing the intruder rating if there is one intruder. If there is more intruders the geometrical bias is applied.

$$intruder(cell_{i,j,k}) = 1 - \prod_{\forall intruder_i \in Intruders} \left(1 - intersection \begin{pmatrix} cell_{i,j,k}, \\ intruder_i \end{pmatrix} \right) \quad (5.4)$$

The *intruder intersection models* are outlined in (sec. **??**).

Constraint: The *constraints* are coming from various *Information Sources*, the *hierarchical constraint application* is resolved by higher level logic. All *constraints* in this context are considered as *hard*.

The *Constraints rating* (eq. 5.5) is in *range* $[0, 1]$ reflecting certainty of constraint application in cell (usually 1).

$$constraint(cell_{i,j,k}) = \max \left\{ \begin{matrix} constraint(cell_{i,j,k}, source_i) : \\ \forall source_i \in InformationSources \end{matrix} \right\} \tag{5.5}$$

The *Constraint Rating* calculation example for *static* constraints is given in (sec. ??), the example for *moving* constraints is given in (sec. ??).

Note. Weather is already considered in constraints, the weather is handled as soft/hard static/moving constraints.

Threat: The concept of threat is *rating of expected harm* to receive in given segment of space. The threat can be time-bound to *decision time* t_i (time sensitive *intruder intersection models*).

The *harm prioritization* is addressed by higher navigation logic (fig. 5.7). All *sources of harm* are considered as equal. Threat is formalized in *following definition*:

Definition 1. *Threat is considered as any source of harm. The threat is* maximal aggregation *of various harm ratings. Our* threat *for specific cell is defined by (eq. 5.6).*

$$threat(cell_{i,j,k}) = \max \left\{ \begin{matrix} obstacle(cell_{i,j,k}), map(cell_{i,j,k}), \\ intruder(cell_{i,j,k}), constraint(cell_{i,j,k}) \end{matrix} \right\} \tag{5.6}$$

Reachibility: The *Reachibility* for trajectory reflects how safe is *path along.* The *Threat* (def. 1) for each cell has been already assessed. The set of *Passing Cells* is defined in *Trajectory Footprint* (eq. ??).

The *Trajectory Reachibility* is given as product of *Threats* along the trajectory (eq. 5.7). The *Trajectory Reachibility* can be calculated for each *trajectory segment* given as $\{movement_1, \ldots, movement_i\} \subset Buffer$ originating from $state_0$.

$$reachibility(Trajectory) = \prod_{PassingCells}^{\forall cell_{i,j,k} \in} (1 - threat(c_{i,j,k})) \tag{5.7}$$

Note. The *Reachibility* of *trajectory* segment gives the property of *safety* of route from beginning, until last point of segment. There can be a very unsafe trajectory which is very safe from beginning.

The *Reachibility* of *cell* is given by best trajectory segment passing trough the *given cell.* This is given by property, that every trajectory is originating from root $state_0$, which means that one safe route is sufficient to reach space in cell.

The *Trajectory segment* reachability is sufficient, because the overall performance is not interesting, the *local reachability* is sufficient. The cell reachibility is formally defined in (eq. 5.8).

$$\begin{aligned} reachibility(cell_{i,j,k}) = \max\{ & Trajectory.Segment(cell_{i,j,k}).Reachibility : \\ & \forall Trajectory \in PassingTrajectories(celli, j, k)\} \end{aligned} \tag{5.8}$$

Note. Function Trajectory.Segment($cell_{i,j,k}$).Reachibility gives same results for any segment in $cell_{i,j,k}$, because (eq. 5.7) accounts each cell *threat* only once.

Discretization: The *fault tolerant* implementation needs to implement sharp Boolean values of properties mentioned before. The *fuzzy values* are usually threshold to Boolean equivalent. The *operational standards* for *Manned Aviation* [1] demands the fail rate below 10^{-7}, because there is no definition for *UAS* the *minimal fail rate* is expected to be at similar level.

The *fuzzy values* $[0, 1]$ are projected to *Boolean* properties of *cell* and *Trajectory* in following manner (tab. 5.1).

Threshold $= 10^{-7}$

Visibile	$visibility(cell_{i,j,k})$	$\geq$	$(1 - threshold)$
Detected Obstacle	$obstacle(cell_{i,j,k})$	$\geq$	$threshold$
Map Obstacle	$map(cell_{i,j,k})$	$\geq$	$threshold$
Intruder	$intruder(cell_{i,j,k})$	$\geq$	$threshold$
Constraint	$constraint(cell_{i,j,k})$	$\geq$	$threshold$
Reachable Trajectory	$reachibility(trajectory)$	$\geq$	$(1 - threshold)$
Reachable Cell	$reachibility(cell_{i,j,k})$	$\geq$	$(1 - threshold)$

Table 5.1: Changing ratings from fuzzy to Boolean parameters.

The high values of *Visibility* (eq. 5.1) and *Reachability* (eq. 5.8, 5.7) are expected. The low *threshold* for *threaths* values is expected. The error margin is solved by *Sensor Fusion* therefore initial *false positive* cases have low rate. The *Detected Obstacle Rate* (eq. 5.2), *Map Obstacle Rate* (eq. 5.3), *Intruder Rate* (eq. 5.4), and *Constraint Rate* (eq. 5.5) thresholds are considered low.

Space Classification: The *Data Fusion Function* is outlined in (**??**). This classification is resulting into four distinct space sets.

The *Uncertain* space for decision time t_i is a portion of *Avoidance Grid* which *UAS* can not *read* with *Sensor Field*. The *cells* with $\neg Visible$ property. The *Uncertain* space is given by (eq. 5.9).

$$Uncertain(t_i) = \{cell_{i,j,k} : cell_{i,j,k} \in AvoidanceGrid(t_i), cell_{i,j,k}.\neg Visible\} \tag{5.9}$$

The *Occupied* space for decision time t_i is a set of cell which are classified as *Detected Obstacles*. The *Visibility* is not an issue, due the initial damping in (eq. 5.2). The formal definition is the space portion where it is possible to detect *obstacle bodies* or their portions (eq. 5.10).

$$Occupied(t_i) = \left\{ cell_{i,j,k} : \begin{array}{c} cell_{i,j,k} \in AvoidanceGrid(t_i), \\ cell_{i,j,k}.DetectedObstacle \end{array} \right\} \tag{5.10}$$

The *Constrained* space for decision time t_i is *Visible* portion of *Avoidance Grid* where the *Intruder* or *Constraint* is present. The mathematical formulation is given in (eq. 5.11).

$$Constrained(t_i) = \left\{ cell_{i,j,k} : \begin{array}{l} cell_{i,j,k} \in AvoidanceGrid(t_i), \\ cell_{i,j,k}.Visible, \\ cell_{i,j,k}.Constraint \vee cell_{i,j,k}.Intruder \end{array} \right\} \tag{5.11}$$

The *Free* space is the space which is *Visible* and $\neg Obstacle$, $\neg Intruder$, and, $\neg Constrained$. The mathematical definition is simple set subtractions from *Avoidance Grid* (eq. 5.12).

$$\begin{aligned} Free(t_i) = AvoidanceGrid(t_i) - \dots \\ \dots - (Uncertain(t_i) \cup Occupied(t_i) \cup Constrained(t_i)) \end{aligned} \tag{5.12}$$

The *Reachable* space for time t_i, used in *Avoidance* because its free and there is a safe trajectory, is given as a set of cells from *Avoidance Grid* which are *Reachable*. The mathematical definition is given in (eq. 5.13).

$$Reachable(t_i) = \left\{ cell_{i,j,k} : \begin{array}{l} cell_{i,j,k} \in AvoidanceGrid(t_i), \\ cell_{i,j,k}.Reachable \end{array} \right\} \tag{5.13}$$

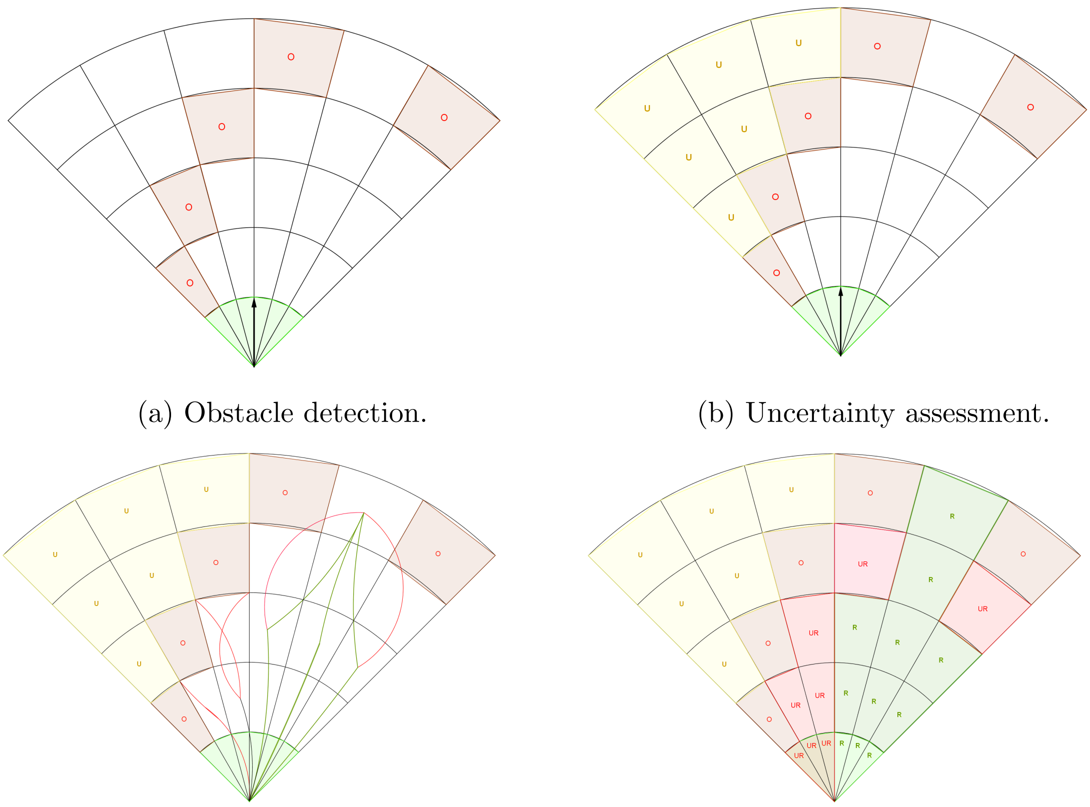
Note. The Reachable Space at decision time t_i: The *Reachable space* is non-empty set and its a subset of *Free(t_i)* space:

$$|Reachable(t_i)| > 0, \quad Reachable(t_i) \subset Free(t) \tag{5.14}$$

5.7.2 (R) Avoidance Grid Run

Main Goal: The main goal of this section is to introduce the trajectory selection process, based on a *situation assessment*, originating from *Data Fusion Procedure* (sec. 5.7.1).

Note. The *rating calculation* is outlined in (sec. 5.7.1). Low cost sensor fusion example usable to feed our data fusion procedure is given in [2]. Semi-optimal concatenation trajectory search like ours can be found in [3].



(a) Obstacle detection. (b) Uncertainty assessment.

(c) Trajectories reachibility evaluation. (d) Cell reachibility evaluation.

Figure 5.1: Significant steps of *Avoidance grid run* (inner loop).

Note. The *Sensor Fusion Procedure* is solving all following steps (sec. 5.7.1). The *main purpose* of *Avoidance Run* is finding best path under certain conditions.

Space Assessment Principle: The *Avoidance Grid* is fed trough *Data Fusion* (sec. 5.7.1). The process of *ratings assessment* (tab. 5.1) is given in (fig. 5.1):

1. *Obstacle detection* (fig. 5.1a) - assessment of *detected obstacles* (eq. 5.2). The red (O) *cells* have Detected obstacle set as *true*. The other threats: *map obstacles* (eq. 5.3), *intruders* (eq. 5.4), *constraints* (eq. 5.5) are false. The red (0) *cells* are representing $Occupied(t_i)$ (eq. 5.10) space in *Avoidance Grid* at decision time t_i.

2. *Uncertainty assessment* (fig. 5.1b) - the uncertain cells are cells which status can not be *assessed*. The *Visibility* (eq. 5.1) is low. The *Uncertain* cells (yellow (U) mark) are equal to $Uncertain(t_i)$ (eq. 5.9) in *Avoidance Grid* in *decision time* t_i. The $Constrained(t_i)$ (eq. 5.10) space is equal to $\varnothing$ in this example.

3. *Trajectory reachibility evaluation* (fig. 5.1c) - the *Reach Set* given as *Trajectory Set* (eq. **??**). is then projected trough *Avoidance Grid* and pruned according to (def. **??**). *Reachable Trajectories* (eq. 5.7) are only those contained in $Free(t_i)$ space (eq. 5.12). The *Reachable Trajectories* are denoted as *green lines*. The *Unreachable* trajectory segments are denoted as *red lines*.

4. *Cell reachibility evaluation* (fig. 5.1d) - the evaluation of *cells* reachibility is going according to (eq. 5.8). The *Reachable cells* are those which *contains* at least one *Reachable Trajectory Segment*.

Finding Best Path: [1] Each $cell_{i,j,k}$ in *Avoidance Grid* at *decision time* t_i has assessed ratings according to *data fusion procedure* (tab. 5.1). The following properties are know prior the *trajectory* selection:

1. *Reachibility* for each $cell_{i,j,k}$ (eq. 5.8).

2. *Reachibility* for each $Trajectory(\circ)$ (eq. 5.7).

3. *Free Space* as non empty set of *cells* in *Avoidance Grid* (eq. 5.12), with *Reachable Space* (eq. 5.13).

4. *Goal Waypoint* $\mathcal{WP}_G$ from *Mission Control Run* (sec. 5.7.3).

The *Algorithm* (alg. 5.1) is based on *shortest path* search. Navigation is trying to reach *goal waypoint*, therefore it tries to shorter distance between *trajectory final cell* and *goal waypoint*. If there is *reachable space* two situations can occur:

1. *Goal waypoint is inside the Avoidance Grid* - the *avoidance cell* is $\text{cell}_{i,j,k}$ containing *goal waypoint* if reachable.

2. *Goal waypoint is outside the Avoidance Grid* - the *avoidance cell* is closest cell considered as *outer cell* to *goal waypoint*.

Note. Outer cell is a $\text{cell}_{i,j,k}$ which has at least one *wall* directly neighbouring with *outer space* ($Universe - KnownWorld(t_i)$). The *outer cell* is selected to prevent navigation to the *trap*.

The *Avoidance Path* selection is simple lowest cost selection of *Trajectory* $\in \text{cell}_{i,j,k}$.

[1] Avoidance Run Function Implementation: `RuleEngine/MissionControl/MissionControl.m::findBestPath(avoidanceGrid)`

Algorithm 5.1: Find best *Path* in *Avoidance Grid*

```
Input  : Cell[] reachable (eq. 5.13), Waypoint goal, AvoidanceGrid(t_i) grid
Output: Trajectory avoidancePath, Error message

# Initialization & Reachibility test;
avoidancePath = ∅;
if reachable == ∅ then
    message = "No path available, empty Reach Set";
    return [avoidancePath,message]
end
avoidanceCell = GetRandomCell(reachable);

# Look for for goal cell;
if goal ∈ grid then
    # Goal is inside Avoidance Grid, Check if reachable;
    avoidanceCell = grid.selectCellXYZ(goal);
    if avoidanceCell.Reachable != true then
        message = "Waypoint not Reachable";
        return [avoidancePath,message]
    end
else
    # Goal is outside Avoidance Grid, look for closest reachable cell_{i,j,k};
    minimalDistance = distance(avoidanceCell,goal);
    for cell_{i,j,k} ∈ reachable do
        if distance(cell_{i,j,k},goal) < minimalDistance then
            if isOuterCell(cell_{i,j,k}) then
                minimalDistance = distance(cell_{i,j,k},goal);
                avoidanceCell = cell_{i,j,k};
            end
        end
    end
end

# Reachable cell was found, Look for cheapest reachable trajectory;
avoidancePath = GetRandomTrajectory(avoidanceCell);
for trajectory ∈ avoidance Cell && trajectory.Reachable == true do
    if trajectory.Cost < avoidancePath.cost then
        avoidancePath = trajectory;
    end
end
message = ∅;
return [avoidancePath,message]
```

Space Assessment Example: For better understanding there is following example of *space assessment* and *Best Path Selection*.

The *UAS* (blue plane) is following *mission plan* in open space. Then there is a detection of an *collision situation* (fig. 5.2). The *Obstacle* is detected in *top-right* Avoidance Grid corner.

The *LiDAR hits* are denoted as red filled circles. The *Avoidance Grid* space is constrained by black dashed line. The *Avoidance Grid* is separated into 5 layers going from top to *bottom*. The *Reach Set* is projected as a set of *Trajectories* with colorization.

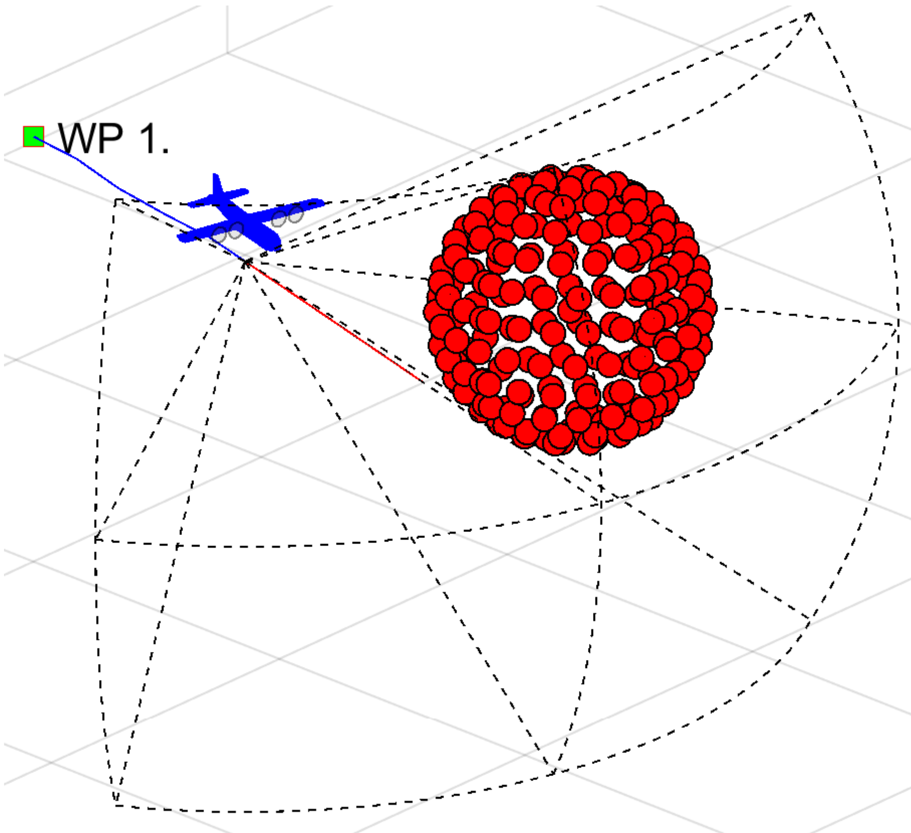


Figure 5.2: Example: The situation to be evaluated by *Avoidance Run*.

Visibility Assessment: The visibility assessment (fig. 5.3) divides the *Avoidance Grid* into two

1. *Visible space* (blue filled cells) is space *trough* which *LiDAR* rays roamed freely until they hit an *Obstacle*.

2. *Uncertain space* (black filled cells) is space where no *LiDAR ray* passed nor hit. Therefore its status is uncertain.

Note. The *detected obstacle cells* are part of *visible space*, because there is certainty about its containment.

The *Reach Set* trajectories are colored based on their visibility, blue for *uncertain* trajectories and *green* for visible trajectories.

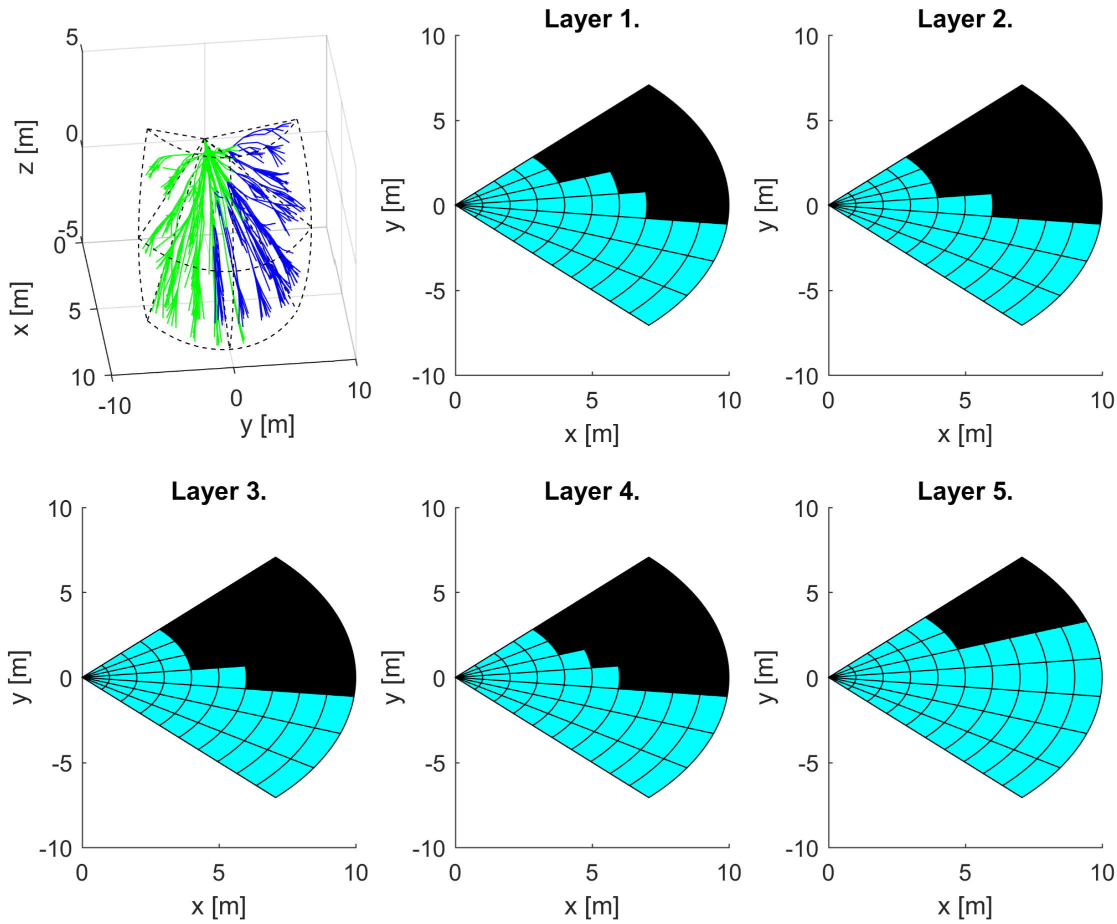


Figure 5.3: Example: The *Visibility* evaluation by *Avoidance Run*.

Reachibility Assessment: For Each trajectory the *Reachibility* is assessed (fig. 5.4). The *Obstacle Space* and *Uncertain Space* are rendering *reachibility*, effectively separating *trajectories* into two categories:

1. *Unreachable Trajectories* (red lines) - there is at least one trajectory segment leading trough *Obstacle* or *Uncertain* space.
2. *Reachable Trajectories* (green lines) - all trajectory segments are lying in *Free* space.

Cells in Avoidance grid are divided in similar matter, depending on count of *reachable trajectories* passing trough them:

1. *Unreachable Cells* (red fill) - there is no trajectory trough *free space* or the *cell* is not in *free space*.
2. *Reachable cells* (green fill) - there is at least one *feasible trajectory* reaching *free cell*.

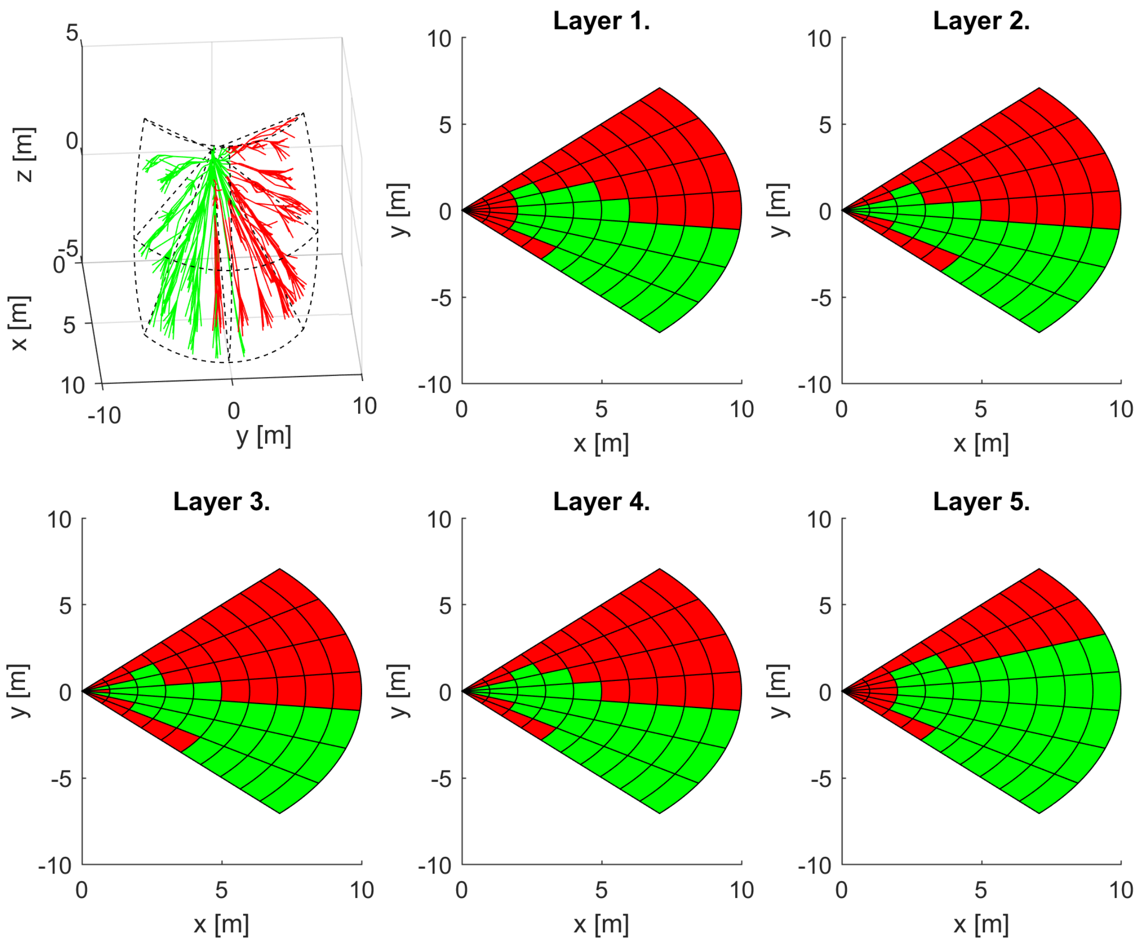


Figure 5.4: Example: The *Reachibility* evaluation by *Avoidance Run*.

Note. The *best avoidance path* is selected form *reachable outer cells* (green fill in fig. 5.4), depending on *goal waypoint* according to (alg. 5.1).

5.7.3 (R) Mission Control Run

Introduction and Motivation: This section will introduce *Navigation Concept* using *Reach Set Approximation*. The *Avoidance Framework Concept* (fig. **??**) defines *Navigation Module* as *sub-system* for long term *trajectory tracking*. The *Avoidance Grid Run* (sec. 5.7.2) is solving the *Path Search* problem inside operation space constrained by *Avoidance Grid* for time t_i.

There is a need to build a trajectory between *Waypoints* which are further away than *distance* of one *Avoidance Grid*. The *UAS* is controlled via *Movement Automaton*. The *Movements* which are in *Movement Buffer* can be replaced with another movements. This feature of *Movement Automaton* is called *Movement Chaining* (eq. **??**).

To join the multiple *Avoidance Grids* paths following terminology needs to be established (fig. 5.5a):

1. *Goal* (Selecting Goal of Navigation) - the point where UAS want to get in global coordinate frame. The selection needs to be defined.

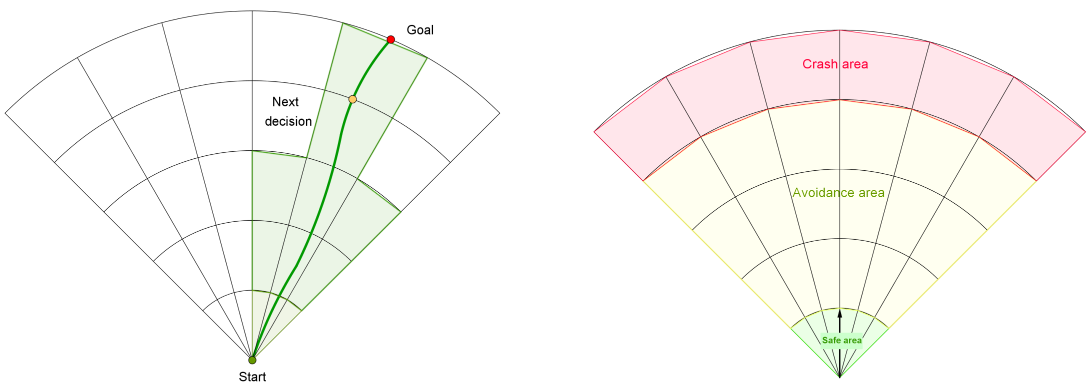
2. *Next Decision* - the point when the next *Avoidance Grid Run* is applied. The outline of events and triggers is required. The *decision* will be made in *next decision time* t_{i+1}.

The *Avoidance Grid* from *UAS* viewpoint can be separated into following zones (fig. 5.5b):

1. *Crash Area* (last layers) - there is no place for safe return and the *border* of *Avoidance Grid* is near. The *Decision Point* needs to lie before this zone.

2. *Avoidance Area* (middle layers) - the area of *Active Avoidance Maneuvering*. The *Reach Set Approximation* performance (sec. **??**) is important in this area.

3. *Safe Zone* (first layers) - there is space for safe return or damage mitigation.



(a) Mission control run example.

(b) Grid Zones.

Figure 5.5: Definitions for *Mission Control Run* (outer loop).

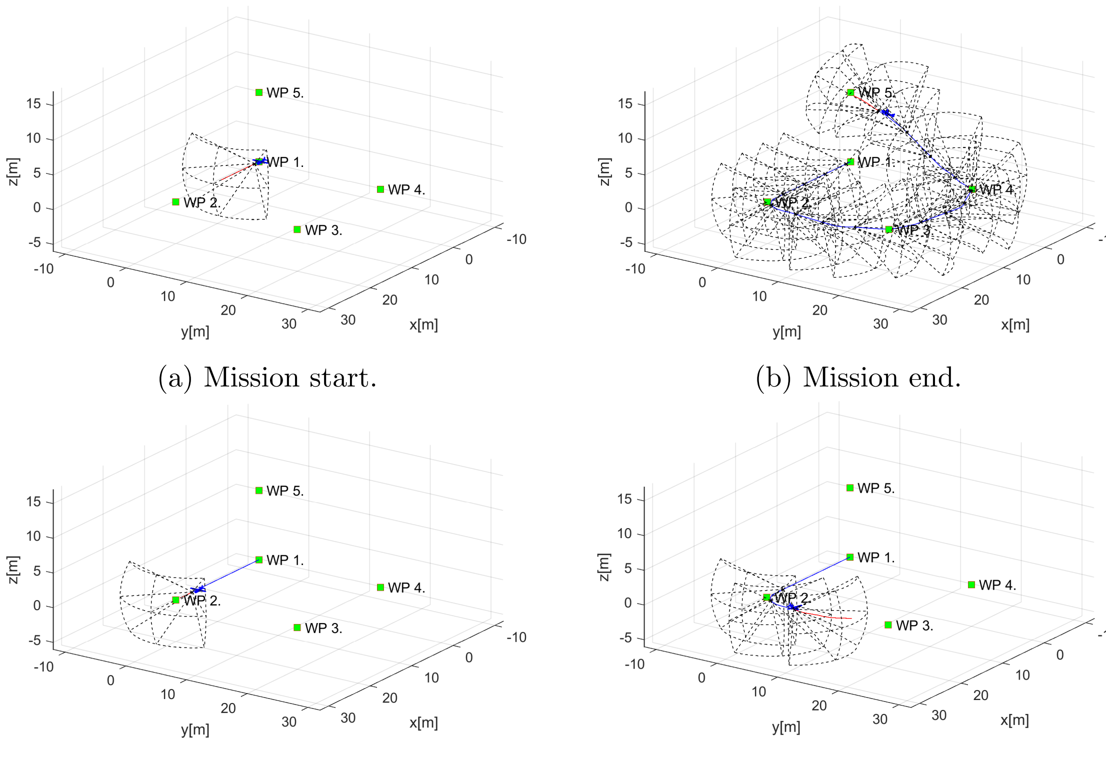
Joining *Avoidance Grid Runs* (fig. 5.6) example portrays *Avoidance Grid Runs* invoked on various *Decision Points* to achieve *Navigation* functionality. The UAS (blue plane) is flying Mission (green numbered waypoints). The *Avoidance Grid* boundary (black dashed line) for each *Decision Point* (UAS position at time t_i). Following example of *Navigation* (fig. 5.7) run is shown:

1. *Mission Start* (fig. 5.6a) - UAS at the start of the mission have one *Avoidance Grid* at its position to determine the *Navigation Path* to *Waypoint 2* (goal waypoint). The planned path (red line) is leading directly to *Avoidance Grid* boundary (black dashed line).

2. *Mission End* (fig. 5.6b) - UAS have reached *last waypoint*. All *Avoidance Grid* boundaries (black dashed line) for all *runs* are drawn along flown trajectory.

3. *Waypoint Reach* (fig. 5.6c) - the *waypoint* is inside *Avoidance Grid*, the navigation path (red line) leads directly to *goal waypoint*. (Excessive *Avoidance Grid* boundaries are removed.)

4. *Next Waypoint* (fig. 5.6d) - the new *Goal Waypoint* is selected, the UAS moves to new goal (invoking *Avoidance Grid Runs* when necessary).



(a) Mission start.

(b) Mission end.

(c) Waypoint reach.

(d) Next waypoint.

Figure 5.6: Joining multiple *Avoidance Grid Runs* for achieve Navigation.

General Concept: [2] The *General Concept* is taken from [4, 5], consisting from following main modules:

1. *Navigation Loop* - module responsible for *Navigation* providing *Goal Waypoint*.
2. *Data Fusion* (background in sec. 5.7.1) - module responsible for *Surveillance Data Feed*.
3. *Situation Assessment* - module responsible for *UAS Safety Evaluation*.
4. *Avoidance Run* (background in sec. 5.7.2) responsible for *Avoidance Path* selection.

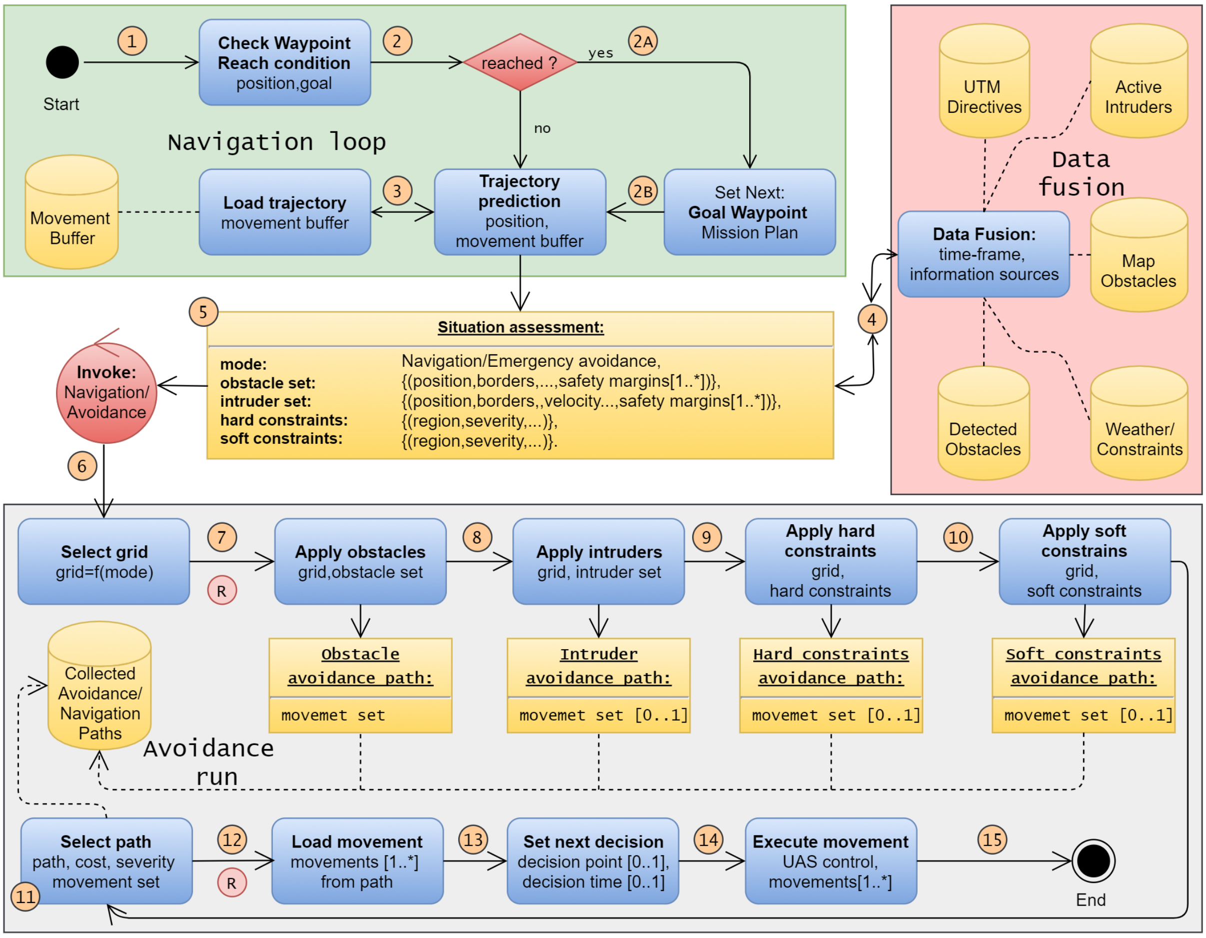


Figure 5.7: Mission control run activity diagram.

The main changes to *Navigation architecture* are given in *Mission Control Run* activity diagram (fig. 5.7):

1. *Situation Assessment* - added event-based mode switching control.
2. *Avoidance Run* - added hierarchical evaluation for *Avoidance Path* selection. Prioritizing threat avoidance according to a type.

[2]Mission Control Run Function Implementation: `RuleEngine/MissionControl/MissionControl.m::runOnce(.)`

The *Operation Mode* is introduced, based on *Situation assessment* and *Triggering Events* one of following modes are selected in *Avoidance Run*:

1. *Navigation Mode* - the *UAS* is navigating trough *Airspace* following *cost effective patterns* and obeying *Airspace Authority* (UTM). The *Navigation Grid* is a instance of *Avoidance Grid* (sec. **??**) with initialized *Navigation Reach Set* (ex. *Harmonic Reach Set Approximation* (sec. **??**)).

2. *Emergency Avoidance Mode* - the *UAS* is *threatened* by obstacle, intruder, hard constraint or *soft constraint*, the *UAS* is navigating trough *Airspace* following *safe avoidance patterns* and *minimizing the impact* of possible damages. The *Avoidance Grid* is term used for *Emergency Avoidance Mode*. The *Avoidance Reach Set Approximation* is initialized in *Avoidance Grid* (ex. *Chaotic Reach Set Approximation* (sec. **??**))

Note. Depending on *Operation Mode* the pair of *Avoidance Grid* and *Reach Set* is selected in *Avoidance Run* part.

The *Navigation Grid* and *Avoidance Grid* shares the space segmentation pattern, therefore the *Data Fusion* (sec. 5.7.1) needs to be evaluated only once for both grids.

Decision Time Frame ($[t_i, t_{i+1}[$): The *Mission Control Run* is executed for *Decision Time Frame* bounded to the *period* of the *UAS executed movement* (fig. **??**).

The *UAS System* (sec. **??**) controlled by *Movement Automaton Implementation* (sec. **??**) *Planned Movements* can be changed at any time. The real impact on control is shown after the *actual movement* is executed.

Note. For our *Movement Automaton Implementation* movements the average *movement duration* is *1/velocity second* (tab. **??**, **??**).

The *Decisions* are made based on *system* state in *current* time-frame started at t_i for *next* time frame starting at t_{i+1}.

Note. Because the *Decision Delay* is crucial in *Avoidance System* it is beneficial to have *short time movements*. On the other hands, the *length and duration of movements* is impacting *Reach Set Complexity*. The proper construction of movement automaton is greatly impacting overall *approach performance*.

Initialization: The *UAS* is going to solve a problem for *Rules of the Air* (eq. **??**). Using control scheme (fig. **??**) with given *Sensors*:

$$Sensors = \{LiDAR, ADS-B\} \tag{5.15}$$

The sensors obstacle assessment into avoidance grid is outlined for static obstacles in (sec. **??**) and for moving obstacles in (sec. **??**.)

The *Data Fusion Procedure* is given as follow:

$$DataFusion = \{RatingBasedDataFusion \quad (sec. 5.7.1)\} \tag{5.16}$$

Then the *UAS system* (sec. **??**) with *Movement Automaton Implementation* (sec. **??**) with empty movement buffer:

$$MovementBuffer = \{\} \tag{5.17}$$

The *Avoidance Grids* for both *Operation Modes* are created with *identical space segmentation*. The *Reach Set Approximations* are loaded based on initial *UAS State* at decision time 0. The *Reach Set Approximation* is always selected based on *UAS System State*. The initial *Operation Mode* is set up as *Navigation*. The initialization is summarized like follow:

$$\begin{aligned} AvoidanceGrid(0) &= \{UAS.position(0), AvoidanceReachSet(UAS.ReachSet)\} \\ NavigationGrid(0) &= \{UAS.position(0), NavigationReachSet(UAS.ReachSet)\} \\ OperationMode &= Navigation \end{aligned} \tag{5.18}$$

The *Mission* is set up as a set of *ordered waypoints*. The *initial goal waypoint* is *first waypoint*. The initialization is summarized like follow:

$$\begin{aligned} Mission &= \{Waypoint_1 \dots Waypoint_n\} \\ GoalWaypoint &= Mission.waypoint_1 \\ LastWaypoint &= Mission.waypoint_n \end{aligned} \tag{5.19}$$

The *actual threats* are set as empty sets for *decision time* $t_i = 0$:

$$obstacles = \{\}, intruders = \{\}, hardConstraints = \{\}, softConstraints = \{\} \tag{5.20}$$

Navigation Loop (1^st^-3^rd^ step): The purpose of *Navigation Loop* is to select proper *Goal Waypoint* from *Mission* (sec. **??**). If *last waypoint* have been reached the *Landing Procedure* will be initiated and *Mission Control Run* Ends.

First start with definition of *waypoint reach condition* (def. 2) and *Unreachable waypoint* (def. 3).

Definition 2. *Waypoint Reach Condition for* current *decision time* t_i *for* UAS *position and current* Goal Waypoint *is satisfied only if:*

$$\begin{aligned} &distance(UAS.position(t_i), GoalWaypoint(t_i)) \\ &\qquad\qquad\qquad\leq \\ &\qquad 2 \times \max\{length(movement) : \forall movement \in MovementSet\} \end{aligned} \tag{5.21}$$

Note. The movements in our solution have *uniform length* of *1 m* (tab. **??**, **??**), therefore the waypoint reach condition is satisfied when *distance to goal waypoint* is lesser than 2 m. The maximal movement length has impact on *navigation/avoidance* precision.

Definition 3. *Unreachable Waypoint. The* Goal Waypoint *is evaluated as unreachable in decision time* t_i *when* Avoidance Grid Run *(alg. 5.1) can not find the* navigation/avoidance path *leading to it.*
Formally: The Avoidance/Navigation Grid *has range defined as* final layer distance*. When the* Goal Waypoint *is in* range *of* Grid*:*

$$Grid(t_i).range \geq distance(UAS.position(t_i), GoalWaypoint(t_i)) \tag{5.22}$$

and following condition is satisfied:

$$\begin{aligned}\forall cell_{i,j,k} \in Grid(t_i) \ \nexists cell_{i,j,k}.Reachable == true \wedge \dots \\ \dots \wedge distance(cell_{i,j,k}, GoalWaypoint(t_i)) \leq \dots \\ \dots \leq 2 \times \max\{length(movement) : \forall movement \in MovementSet\}\end{aligned} \quad (5.23)$$

The Goal Waypoint *is unreachable.*

Then the *Navigation Loop* is invoked every *decision time* t_i, *Mission Control Run* (fig. 5.7), it is described as sequence of following steps:

1st Check Waypoint Reach Condition - the *UAS position* for given *time frame* t_i is checked under condition (eq. 5.21). If condition is met continue with 2nd step otherwise continue with 3rd step.

2nd Set Next Waypoint - until following condition is met:

$$GoalWaypoint == LastWaypoint$$

Set next goal waypoint like follow:

$$GoalWaypoint = Mission.getNextWaypoint()$$

Otherwise enforce *Landing sequence* (Out of Scope).

3rd Trajectory Prediction - the *Movement Buffer* is loaded with planned movements from *Movement Automaton*. The *future trajectory* is predicted according to (eq. **??**):

$$\begin{aligned}&PredictedTrajectory = \\ &\qquad Trajectory(state = UAS.state(t_i), buffer = futureMovements)\end{aligned}$$

The *Predicted Trajectory* is used in 5th step *Situation Assessment*.

Data Fusion (4th step) The *Data Fusion* (sec. 5.7.1) in this context is *Threat Sets* preparation for *Avoidance Run*. It is depending on values of *Boolean values* defined in (tab. 5.1) for *threat* classification.

Note. Avoidance Grid‘s Data fusion (sec. 5.7.1) is run in 7th- 10th step (fig. 5.7).

The *static obstacles* source is from *LiDAR* scan received at least at beginning of current *decision frame* t_i:

$$obstacles = LiDAR.scan(UAS.position(t_i))$$

The *intruders* source are valid *active intruders notifications* received from ADS-B In positioned to *future expected positions* at *decision time* t_{i+1}:

$$intruders = ADS - B.getActiveIntruders(t_{i+1})$$

Note. The *Intruders* needs to be predicted for the next decision time-frame starting at time t_{i+1} Due their mobility.

The *hard/soft constraints* are obtained from *Information Sources* and the area of next decision time t_{i+1} *Avoidance Frame* is used as space parameter in search. The sets of hard and soft constraints are obtained in following manner:

$$hardConstraints = InformationSources.fuse(AvoidanceGrid(t_{i+1}))$$

$$softConstraints = InformationSources.fuse(AvoidanceGrid(t_{i+1}))$$

The results of *Data Fusion* threats set preparation are used in next step.

Invoke Navigation/Avoidance based on Situation Assessment (5^{th}-6^{th} step): The *deciding events* depending on *Trajectory Prediction* (3^{rd} step) and *Data Fusion* (4^{th} step) (fig. 5.7) are following:

1. *General Events* are *triggered* regardless *Operation Mode*. They are considered after *specific mode events* are handled and *Navigation/Avoidance Grid* is selected:

   a. *Empty Movement Buffer* ($MovementBuffer = \varnothing$) - if there is no movement in *Movement buffer* to be executed (from 3^{rd} step: Load Trajectory), the *Avoidance Run* is enforced to run with *Navigation/Avoidance Reach Set Approximation* to generate new path.

   b. *Waypoint Reached* (2^{nd} step) - the *Navigation Loop* run is forced to set goal *Goal Waypoint*. If *last waypoint* from *Mission* (sec. **??**) the *Landing Procedure* is enforced.

   c. *Waypoint Unreachable* - this type of event is very situations based. The *Waypoint Reachibility* (assumption. **??**) has not been relaxed, therefore this event is not properly handled in approach. The *implementation* considers *selecting next waypoint in mission* as a goal waypoint of *first waypoint* if *unreached/unreachable waypoints* are exhausted.

2. *Navigation Mode Events* are triggered if *Operation Mode* is set as *Navigation*:

   a. *Empty Navigation Grid* ($|threats| = 0$) - if *movement buffer* contains at least one *movement*, the *Avoidance Run* is omitted. The *Operation Mode* stays in *Navigation Mode*.

   b. *Collision Case Resolution* ($|ActiveCollisionCases| > 0$) - there is new/active *Collision Case* (sec. **??**), the *Navigation Reach Set Approximation* trajectories will be constrained according to active *Collision Case(s)* requirements. If there exists at least one *Reachable* avoidance path, the *Operation Mode* will remain *Navigation*. If there is no *Reachable* avoidance path, the *Operation Mode* switches to *Emergency Avoidance*.

   c. *Static Obstacle Detection* ($LiDAR.Hits > threshold$) - if *static obstacle set* contains at least one *detected obstacle* (sec. **??**) intersecting with *Navigation grid* the *Operation Mode* will be *switched* to *Emergency Avoidance Mode*.

   d. *Intruder Detection* ($intruders > 0$) - if *active intruders set* contains at least one *intruder* which expected impact area (intersection models (sec. **??**)) *Navigation grid* the *Operation Mode* will be *switched* to *Emergency Avoidance Mode*.

e. *Hard or Soft Constraint Occurrence* ($|hardConstraints| > 0 \vee |softConstraints| > 0$) - if *hard/soft constraint set* contains at least one *constraints* which intersects (static constraints (sec. **??**), moving constraints (sec. **??**)) *Navigation grid* the *Operation Mode* will be *switched* to *Emergency Avoidance Mode.*

3. *Emergency Avoidance Events* are triggered if *Operation Mode* is set as *Emergency Avoidance*:

   a. *Empty Avoidance Grid* ($|threats| = 0$) - if there is no *detectable* threat, the remainder of *avoidance path* is removed from *Movement Buffer.* The *Operation Mode* is switched to *Navigation* and new *navigation path* is selected.

5th **Situation Assessment** - if there is any flag raised by *Event Triggers*, there is an *avoidance situation.*

The *Event Triggers* describe complex *Operation Mode* switching. The simplified principle is following: *If UAS is in Emergency Avoidance Mode Always Invoke Avoidance Run. If UAS is in Navigation Mode Invoke Only if Necessary.*

If there was event trigger continue with 7th step, otherwise wait for *next decision time* t_{i+1}, execute movement and continue with 1st step.

6th **Invoke Navigation/Avoidance** depending on the *Operation Mode* the *Reach Set/Grid* pair is selected. The future $state(t_{i+1})$ in next decision frame t_{i+1} is necessary for Grid/Reach Set initialization. The *next decision frame initial state* is obtained by *prediction*:

$$state(t_{i+1}) = Trajectory(state(t_i), currentMovement)$$

The *Reach Set Approximation* is loaded based on *mode* and $state(t_{i+1})$. The *Grid* is initialized as $Free(t_{i+1})$ (eq. 5.12) for all cells.

Avoidance Run (7th-15th step): The *Avoidance Run* goal is to obtain *Path* represented as *Trajectory*(state(t_{+1},MovementBuffer)) (eq. **??**) from *Navigation/Avoidance Grid* and associated *Navigation/Avoidance Reach Set Approximation.*

If the *Operation Mode* is set as *Navigation Mode* the algorithm continues with 11th step. Otherwise the *Avoidance Grid Space Assessment* is run multiple times to obtain $Reachable(t_{i+1})$ (eq. 5.13). The *Threat Data* obtained from 4th step are used.

7th **Apply Obstacles** - The *Space assessment* (tab. 5.1) for *Avoidance Grid* is calculated with following threat modification:

$$intruders = \varnothing, softConstraints = \varnothing, hardConstraints = \varnothing$$

The *Find Best Path* (alg. 5.1) is applied, the resulting *avoidance path* is labeled as *Obstacle Avoidance Path.*

8th **Apply Intruders** - The *Space assessment* (tab. 5.1) for *Avoidance Grid* is calculated with following threat modification:

$$softConstraints = \varnothing, hardConstraints = \varnothing$$

The *Find Best Path* (alg. 5.1) is applied, the resulting *avoidance path* is labeled as *Intruders Avoidance Path*.

9th **Apply Hard Constraints** - The *Space assessment* (tab. 5.1) for *Avoidance Grid* is calculated with following threat modification:

$$hardConstraints = \varnothing$$

The *Find Best Path* (alg. 5.1) is applied, the resulting *avoidance path* is labeled as *Hard Constraint Avoidance Path*.

10th **Apply Soft Constraints** - The *Space assessment* (tab. 5.1) for *Avoidance Grid* is calculated without any modification.

The *Find Best Path* (alg. 5.1) is applied, the resulting *avoidance path* is labeled as *Soft Constraints Avoidance Path*.

Note. The 7$^{\text{th}}$ to 10$^{\text{th}}$ steps are code-optimized for efficient calculation.

11th **Select Path** - based on *Operation Mode* the *Navigation/Avoidance Path* is selected.

The *Navigation Path* for *Navigation Mode* is selected by standard *Find Best Path* (alg. 5.1) procedure. The *Navigation Reach Set Approximation* can be constrained by *Rule Engine* (fig. **??**).

The *Avoidance Path* for *Emergency Avoidance Mode* is selected from *Collected Avoidance Paths* with following priority:

1. *Soft Constraints Avoidance Path* - if exists continue with 12$^{\text{th}}$ step, if does not exist try to select:
2. *Hard Constraints Avoidance Path* - if exists continue with 12$^{\text{th}}$ step, if does not exist try to select:
3. *Intruders Avoidance Path* - if exists continue with 12$^{\text{th}}$ step, if does not exist try to select:
4. *Obstacle Avoidance Path* - continue with 12$^{\text{th}}$ step.

Note. The *Waypoint Reachibility* (assumption **??**) is weakened to the point that it is necessary for waypoint to be *Reachable* only in static obstacle environment. The *Constrained* and *Occupied* spaces are shrunk in following matter to increase UAS survival chances. There are following relaxations with their conditions:

1. *Soft Constraint Relaxation* - they are breakable by default. This kind of situation is allowed to happen under any circumstances.
2. *Hard Constraints Relaxation* - they can be broken in case of emergency (airspace constraints) or UAS robust build (Weather Constraints). This kind of situation is allowed under very specific conditions depending on *broken constraint* severity.
3. *Intruder Occupied Space Relaxation* - this can be broken if and only if there is guarantee the Intruder dynamic and navigation algorithm allows to avoid *Collision* with UAS. This relaxation should be used as *the last resort*.

12th Load Movements - the *Movement Buffer* is flushed for *future decision times* $t_{i+1}, \ldots, t_{i+k}$. The *Navigation/Avoidance Path* movements are pushed into *Movement Buffer* instead. The *executed movement* for *decision time* t_i remains (because its executed at this time point).

13th Set Next Decision - the *next decision point* is set depending on circumstances:

1. Navigation Mode (no active collision cases) - *Decision Point* is set as point before *UAS* enters into *Crash Zone* (fig. 5.5b) in *Navigation Grid*.
2. *Navigation Mode (at least one active collision case)* - *Decision Point* is set after *next movement execution*. Current decision point $UAS.Position(t_i)$, next decision point $UAS.Position(t_{i+1})$.
3. *Emergency Avoidance Mode (any circumstances)* - *Decision Point* is set after *next movement execution*. Current decision point $UAS.Position(t_i)$, next decision point $UAS.Position(t_{i+1})$.

14th Execute Movement - the *First Movement* from *Movement Buffer* is loaded to be executed in decision time frame $[t_{i+1}, t_{i+2}[$.

15th Finish Avoidance Run - if the *UAS* is flying, continue with 1st step.

5.7.4 (R) Computation Complexity

Introduction: The *Computation Complexity* one mission control run assessment is necessary to identify the strong and weak points of approach. Lets get trough modules to assess notable calculations/algorithms complexity on high abstraction level.

Navigation Loop: I the navigation loop, the *waypoint reach condition* (eq. 5.21) is checked, this is unitary operation with worst complexity $\mathcal{O}(1)$. The selection process of the next *Goal Waypoint* can get trough all waypoints in the mission if they are all unreachable the complexity is $\mathcal{O}(|waypoints|)$.

The *notable steps* complexity is following:

$$\begin{aligned} \texttt{Reach Condition:} \quad & \mathcal{O}(1) \\ \texttt{Select Next Waypoint:} \quad & \mathcal{O}(|waypoints|) \end{aligned}$$

Data Fusion: The *data fusion* is all about *threat selection.*

If *UAS* is in *controlled airspace* it needs to iterate over received *collision Cases* to select *active ones.* The complexity of this step is linear, therefore boundary is given as $\mathcal{O}(|collisionCases|)$.

Thresholding *Detected Obstacles* is done by simple comparison of *LiDAR ray hits* in given $cell_{i,j,k}$ of *Avoidance Grid.*

Any loading of *threats* from *information sources* is depending on clustering. The *Airspace Clustering* is considered as static for our setup. Therefore the *count of active airspace clusters* has main impact on complexity. The *count of information sources* is static and not changing over mission time. Information sources usually implement *Hash search function* with complexity $\mathcal{O} \ln |searchedItemSet|$.

The *computation complexity* boundaries for *Data fusion* in our setup are following:

$$\begin{aligned} \texttt{Select Active Collision Cases:} \quad & \mathcal{O}(|collisionCases|) \\ \texttt{Threshold Detected Obstacles:} \quad & \mathcal{O}(|cells|) \\ \texttt{Load Map Obstacles:} \quad & \mathcal{O}(\ln |activeClusters| \times |informationSources|) \\ \texttt{Load Hard Constraints:} \quad & \mathcal{O}(\ln |activeClusters| \times |informationSources|) \\ \texttt{Load Soft Constraints:} \quad & \mathcal{O}(\ln |activeClusters| \times |informationSources|) \end{aligned}$$

Note. The *real-time clustering* is *hard non-polynomial problem* [6]. Usually all information sources and sensor have *polynomial complexity* of processing. The *controlled airspace clusters* are usually set for very long period of time. Therefore *Obstacle Map*, *Airspace Constraints*, and, *Weather Constraints* can be considered as preprocessed

Situation Assessment: The *Situation Assessment* is evaluating triggering events. The *evaluation* is usually simple existence question without further calculations. The *complexity* of *event evaluation* for our case is $\mathcal{O}(1)$. There are 8 triggers. The count of *triggers* needs to be accounted in complexity boundary:

$$\mathcal{O}(|triggers| \times eventEvaluationComplexity)$$

Note. The *trigger calculation complexity* needs to stay low, because the *triggers* are verified every *Mission Control Run.* The *Avoidance Run* trigger frequency should be very low under normal conditions.

Avoidance Run: The *Avoidance run* is most critical part of *Mission Control Run*, because *Avoidance Path* calculation. The *Navigation Path* calculation is less complex (Rule engine is not accounted), therefore *Emergency Avoidance Mode* is assumed.

The *threat insertion* is realized in 7^{th} to 10^{th} step. The first is *Avoidance Grid* filled with *Static Obstacles*. The *Avoidance Grid* is designed to separate rotary *LiDAR* ray space into hit count even cells. Insertion of *LiDAR* scan into *Avoidance Grid* complexity depends on *total cell count*. The *upper boundary* for *insert obstacles* is given like follow:

$$\texttt{Insert Obstacles:}\quad \mathcal{O}(|cells|)$$

The *intruders intersection model* type impact the insertion complexity. The *linear intersection* (sec. **??**) is going trough maximum of *layers count* cells.

The *body volume intersection model* (sec. **??**) can check the *simple intersection condition* over all *Avoidance Grid* in worst case, therefore complexity for this check is bounded by *count of cells*.

The *Maneuverability Uncertainty Intersection* (sec. **??**) can hit all cells in *Avoidance Grid*. The calculation complexity boundary is exponential depending on *horizontal/vertical* spread in $[rad]$. The *intersection* implementation was done *ad-hoc*. The impact of *intersection application* is visible only when there is more than *4* concurrence intruders (fig. **??**).

The *complexity boundary for* intruder insertion is given like follow:

$$\texttt{Insert Intruders:}\quad \mathcal{O}\left(\sum \begin{bmatrix} |linearIntersections| \times |layers| \\ |bodyvolumeIntersections| \times |cells| \\ |cells|^{horizontalSpread \times verticalSpread} \end{bmatrix}\right)$$

Note. The *intruder intersection* is critical in *non-controlled airspace*. The main complexity gain in *controlled airspace* is from *rule application*. Our *rule complexity* is in worst case depending on *Reach Set* node count and *Active Collision Cases* count.

$$\texttt{Apply Our Rules:}\quad \mathcal{O}(|activeCollisionCases| \times |nodes|)$$

For *Hard/Soft Constraints* The algorithm used for intersection polygons was selected based on study [7], the selected algorithm *Shamos-Hoey* [8]. The *calculation complexity* boundary is given like follow:

$$\texttt{Hard Constraints Intersection:}\quad \mathcal{O}(|cells| \times |hardConstraints| \times \max |constraintPoints|^2)$$

$$\texttt{Soft Constraints Intersection:}\quad \mathcal{O}(|cells| \times |softConstraints| \times \max |constraintPoints|^2)$$

Each *threat* category application in *Mission Control Run* is done after *each intersection* in 7^{th} to 10^{th} step. All ratings (tab. 5.1) expect $Reachibility(cell_{ij,k})$ and $Reachibility(Trajectory)$ are calculated. The *calculation complexity* boundary for one *reachibility rating* is $\mathcal{O}(1)$. (eq. 5.7, 5.8). The *Recalculate Reachibility* operation applied $4\times$ have maximal *complexity* boundary given as follow:

$$\texttt{Recalculate Reachibility:}\quad \mathcal{O}(4 \times (|nodes| + |cells|))$$

Each time at the end of in 7^{th} to 10^{th} step the *Avoidance Path is Selected*. The *Worst Case* (expected) scenario is to *select* four paths for each *treath* application. The algorithm for *best path selection* (alg. 5.1) iterates over all *cells* in avoidance grid and over all *trajectories* passing trough that cell. The complexity boundary for *path selection* is given as follow:

$$\texttt{Select Path:}\quad \mathcal{O}\left(4 \times \left(|cells| + \frac{|nodes|}{|cells|}\right)\right)$$

Conclusion: Overall approach complexity is *low*. If proper *Information Sources* with efficient clustering and *intersection models for intruders* are used, the approach will stay within *non-polynomial complexity*. The average load time for *testing scenarios* is summarized in (tab. **??**).

Note. The calculation of *Reach Set* is eliminated by pre-calculation for *state range* [9].

5.7.5 (R) Safety Margin Calculation

Safety Margin Determination: To determine *safety Margin* the *Rule of Thumb* is used:

$$maximalBodyRadius \leq safetyMargin \leq 2 \times turningRadius \tag{5.24}$$

The *lower boundary* is given by *UAS* construction. because the *UAS* body is considered as *unit ball* with radius given as *maximal body radius.*

The *upper boundary* is optional, The *double of* turning radius is used by the *conservative approach* [10].

Safety Margin Bloating: The *discretization* of *Reach Set*, *Operation Space* and *Decisions* imposes standard *mixed integer* problem in terms of *safety.* This section covers *non-exhaustive* list of possible *Safety Margin Bloats* in our approach.

Own Position Uncertainty Bloat: The *sensor fusion* is precise, but not *exact* in own UAS position determination. The maximal usual disparity needs to be accounted into *Safety Margin.*

Intruder Position Uncertainty Bloat: The *sensor fusion* of Intruder is precise, but not *exact* in own UAS position determination. The maximal usual disparity needs to be accounted into *Safety Margin.*

Weather bloat: The *Weather* impact type may result to increased *safety margin.* Example: UAS is not humidity resistant, the clouds will be avoided from greater distance.

Airspace bloat: The *Airspace* depending on cluster or *country* may require greater separation distances, depending on circumstances. The example can be UAS directive to keep minimal separation from obstacles. The *Safety Margin* is usually overridden by UTM directive value.

UTM Synchronization Bloat: *Both UAS* decision times were *synchronized.* The *intruder* can be offset for *full decision frame.* This is not an assumption, but it shows critical performance. Usually safety margin is bloated for (worst case offset):

$$safetyMarginBloat = \begin{pmatrix} intruderVelocity \times \ldots \\ intruderDecisionFrame \end{pmatrix} [m, ms^{-1}, s] \tag{5.25}$$

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