6.3 Space Discretization - Avoidance Grid

Operation Space: The *Operation Space* is a space where UAS can effectively surveillance its surroundings, and it has the capability to act.

A Motivation for Discretization: The UAS surroundings needs to be represented in an *avoidance-friendly manner*, following principles matters:

1. *Discrete representation* - the space around UAS should be segmented into finite and exclusive portions which are considered as one point of the grid. This enables fast situation assessment.

2. *Threat proximity* - a threat in any form is getting more important with decreasing distance to UAS.

3. *LiDAR swipe density* - one LiDAR swipe scans many points; the grid needs to be customized to swipe characteristics.

The *Main Sensor* is *LiDAR* (problems **??**.-**??**). The *effective occupancy computation* needs to be done for all problems; the inspiration is taken from [1]. The *effective occupancy computation* is done in *LiDAR* scan portioned into *polar coordinates grid.* The *operation space* is abstracted as a *grid* where *space portions* are representing the points in the grid.

Note. Each member of the grid is a cell, represented as a point with shared properties, like threat level, visibility.

The *Discrete Situation Evaluation* is executed for a *UAS* local coordinate frame in fixed *time.* The goal is to enable *fast discrete situation assessment.*

LiDAR Swipe: The *point* scanned by *LiDAR*, where the *UAS position* is center of the *local coordinate frame*, and *UAS heading is defining the main axes* is given as:

$$point = [distance, horizontal^\circ, vertical^\circ]. \tag{6.1}$$

Note. For polar/Euclidean transformations and local/global coordinate frames refer to background theory (app. **??**).

The *right side* of UAS $horizontal^\circ \in \,]-\pi, 0[$, the *left-side* of UAS $horizontal^\circ \in [0, \pi]$, the *down-side* of UAS $vertical^\circ \in \,]-\pi, 0[$, the *top side* of UAS $vertical^\circ \in [0, \pi]$

LiDAR Swipe Portioning: The *polar coordinate space* can be portioned into distinctive cells, which contains the portion space. This cell then represents one point in the grid.

The *reason* for this swipe portioning is *LiDAR* scanning density[1], which is extremely dense. The *threat* state in the cell can be assessed with linear complexity.

[1]Example rotary LiDAR Velodyne VL-16 specs: `https://www.cadden.fr/wp-content/uploads/2017/02/Velodyne_VLP-16-Puck.pdf`

The *polar* $\rightarrow$ *euclidean* coordinate frame transformation is not amenable for LiDAR swipe. The *threat* assessment based on *LiDAR swipe* in *planar space portions* has minimal complexity, and it is cost effective. [2].

Cell: To discretize operational space into a grid of points there is a need to define cell space, which bounds the portion of the *local planar coordinate frame.* The point (eq. 6.1) is defined by distance, horizontal° offset angle, and vertical° offset angle. The cell is a closed compact set of such points. The boundary can be defined like follow:

Definition 1. *Cell*
The cell *bounds a portion of space in UAS local polar coordinate frame, defined by boundary ranges:*

1. Distance Range - *starts and ends:* $distance_{start} < distance_{end}$ *in* $\mathbb{R}^+$.
2. Horizontal Range - *starts and ends:* $horizontal^\circ_{start} < horizontal^\circ_{end} \in]-\pi, \pi]$.
3. Vertical Range - *starts and ends: by* $vertical^\circ_{start} < vertical^\circ_{end} \in]-\pi, \pi]$.

The space portion *belonging to the* cell *is given by function as:*

$$
\begin{aligned}
&cell.spacePortion\ldots\\
&\left\{\begin{array}{l} point \in \mathbb{R}^3 \ \ where: \\ \qquad \begin{pmatrix} cell.distance_{start} < & point.distance \leq & cell.distance_{end}, \\ cell.horizontal^\circ_{start} < & point.horizontal^\circ \leq & cell.horizontal^\circ_{end}, \\ cell.vertical^\circ_{start} < & point.vertical^\circ \leq & cell.vertical^\circ_{end} \end{pmatrix} \end{array}\right\}
\end{aligned} \tag{6.2}
$$

To evaluate a static obstacle threat*, it is necessary to know how many LiDAR hits landed in the cell space portion. For one* LiDAR Scan *the* hits set *is given a* set *of* all points *which lands into cell space portion:*

$$cell.LiDARHits = \{point \in LidarScan : point \in cell.spacePortion\} \tag{6.3}$$

Note. The *cell* space portion volume is increasing with the distance. This satisfies the requirement for threat-distance importance. The cell is considered as a point of the grid with common properties abstraction valid for all cell space portion.

Effective Operation Space: The goal is to determine which of the operation space is going to be considered in our avoidance grid. The effective operation space determination according to [3] is influenced by the following factors:

1. *Sensors ranges* - there is no reason to assess the situation over effective *sensor range.*

2. *Information sources* impact - there is no real impact on *effective space boundary*, the information search and intersection algorithms are only of the importance.

3. *UAS maneuverability* - the space where UAS can maneuver, bounded by space-time (reach set boundary).

4. *Computation power* - the situation evaluation and threat assessment capabilities of the onboard computer.

5. *Airworthiness requirements* - the *regulations* can impose some minimal requirements on *effective operation space boundary.*

Let show an example of an *effective operation space* for the UAS (fig. 6.1). The *full LiDAR Swipe* (cyan and red lines) of *UAS* (blue plane) has a *shape* of the conical cylinder.

Note. Under *ideal circumstances*, the *LiDAR swipe* would have a *ball shape*, but in real cases the *UAS body portion* where *LiDAR* is mounted is unused.

The *frontal portion* (red line) is a set of cells where *UAS* can make maneuvers. According to the *previous conditions*, there is no reason to consider a space portion out of the maneuverable area.

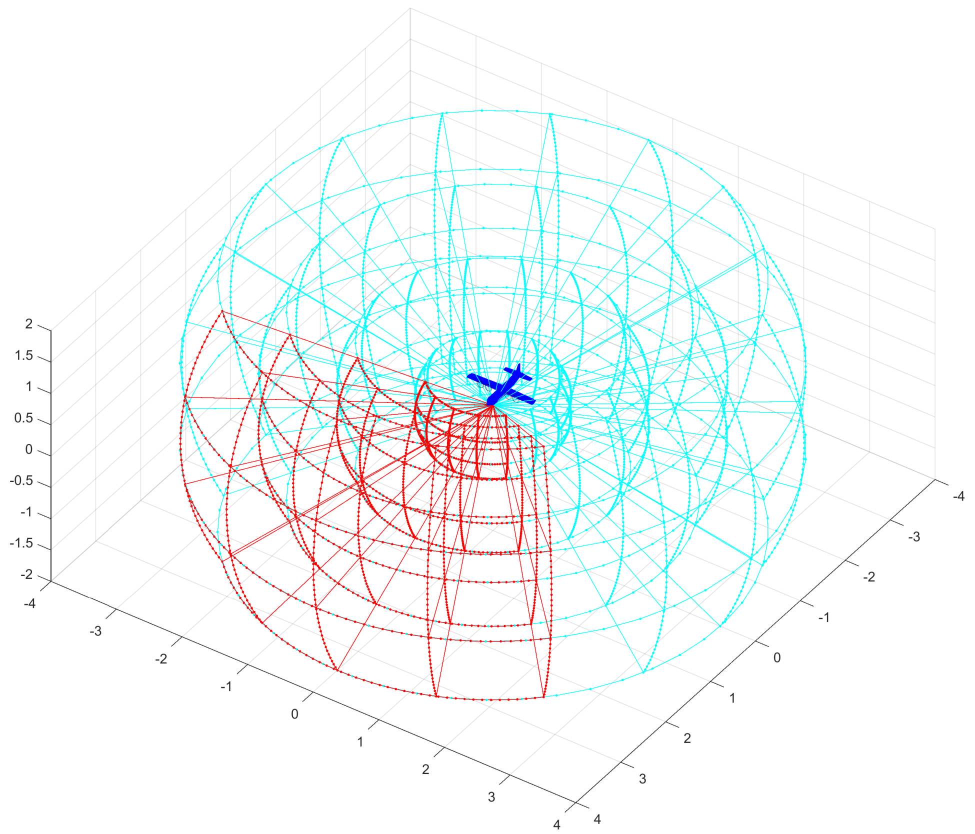

Figure 6.1: Example: The *LiDAR* reading portioning - cells.

Avoidance Grid Definition: The *effective operation space* is going to be portioned into cells. The set of these cells is going to be called *Avoidance Grid*. The idea is to split operational space into cells with even distance, horizontal angle, and vertical angle ranges.

Definition 2. *Avoidance Grid*
The effective space portion *(fig. 6.1 red lines) given by a portion of space in UAS local polar coordinate frame, bounded by:*

1. Distance Range - *in range* $distance_{start} < distance_{end}$ *in* $\mathbb{R}^+$.
2. Horizontal Range - *in range by* $horizontal^\circ_{start} < horizontal^\circ_{end} \in]-\pi, \pi]$.
3. Vertical Range -*in range* $vertical^\circ_{start} < vertical^\circ_{end} \in]-\pi, \pi]$.

The goal is to separate the effective operation space *into cells (def. 1). The idea is to split distance range into multiple distinctive distance ranges with count* $layerCount \in \mathbb{N}^+$. *The ranges for* distance layers *are given as follow:*

$$\begin{aligned} layer^i_{start} &= (i-1) \times \frac{distance_{end} - distance_{start}}{layerCount} \\ layer^i_{end} &= i \times \frac{distance_{end} - distance_{start}}{layerCount} \end{aligned} \quad ; \quad i \in 1 \dots layerCount \tag{6.4}$$

The same separation Layer horizontal/vertical separations defined by $horizontalCount \in \mathbb{N}^+/verticalCount \in \mathbb{N}^+$:

$$\begin{aligned} horizontal^j_{start} &= (j-1) \times \frac{horizontal^\circ_{end} - horizontal^\circ_{start}}{horizontalCount} \\ horizontal^j_{end} &= j \times \frac{horizontal^\circ_{end} - horizontal^\circ_{start}}{horizontalCount} \end{aligned} \quad ; \quad j \in 1 \dots horizontalCount \tag{6.5}$$

$$\begin{aligned} vertical^k_{start} &= (k-1) \times \frac{vertical^\circ_{end} - vertical^\circ_{start}}{verticalCount} \\ vertical^k_{end} &= k \times \frac{vertical^\circ_{end} - vertical^\circ_{start}}{verticalCount} \end{aligned} \quad ; \quad k \in 1 \dots verticalCount \tag{6.6}$$

Then $cell_{i,j,k}$ *space portion by (def. 1) has the following ranges:*

1. Cell Distance Range *(eq. 6.4) depending on layer index* i.
2. Cell Horizontal Angle Range *(eq. 6.5) depending on horizontal angle index* j.
3. Cell Vertical Angle Range *(eq. 6.6) depending on vertical index* k.

Note. The example of *Avoidance Grid Cells* is given in (fig. 6.1 red boundary).

The Avoidance Grid *is the set of cells:*

$$AvoidanceGrid = \left\{ cell_{i,j,k} : \begin{matrix} i \in 1 \dots layerCount \\ j \in 1 \dots horizontalCount \\ k \in 1 \dots verticalCount \end{matrix} \right\} \tag{6.7}$$

Note. For any distinctive cells $cell_{i,j,k}$, $cell_{m,n,o}$ their *space portion intersection* is empty set:

$$\forall cell_{i,j,k}, cell_{m,n,o} : cell_{i,j,k} \cap cell_{m,n,o} = \varnothing, i \neq o \vee j \neq n \vee k \neq o \tag{6.8}$$

Grid Sizing Approach: The sizing approach used in this work is outlined in (app. **??**).

Cell in Avoidance Grid Properties: For each cell $\vec{p} \in \mathbb{R}^3$ in the there are properties to be checked:

1. *Is there visibility to the cell?* - how good is an observation of the cell by Sensor Field.

2. *Is there threat present?* - how sure the data fusion is that there is eminent threat in the cell.

3. *Is the cell reachable?* - if there is any trajectory which can get UAS to that cell without too much threat along the way.

The answers to these questions are given later in *data fusion procedure* outline (tab. **??**).

Bibliography

[1] Florian Homm, Nico Kaempchen, Jeff Ota, and Darius Burschka. Efficient occupancy grid computation on the gpu with lidar and radar for road boundary detection. In *Intelligent Vehicles Symposium (IV), 2010 IEEE*, pages 1006–1013. IEEE, 2010.

[2] Sandeep Gupta, Holger Weinacker, and Barbara Koch. Comparative analysis of clustering-based approaches for 3-d single tree detection using airborne fullwave lidar data. *Remote Sensing*, 2(4):968–989, 2010.

[3] Osmar R Zaïane and Chi-Hoon Lee. Clustering spatial data when facing physical constraints. In *Data Mining, 2002. ICDM 2003. Proceedings. 2002 IEEE International Conference on*, pages 737–740. IEEE, 2002.