7.6 Reduced Reach Sets Performance

Constrained Expansion Method (alg. **??**) is creating *Reach Sets* from the *Root Node* as a tree expansion using *Expansion Constraint function* (depending on type).

The *Reach set creation procedure* is creating the following artifacts:

1. *Nodes* - tree Node containing necessary data for discrete Trajectory portion, notably *System State Evolution*, *buffer*, and, *Reachability Rating.*

2. *Trajectories* - leaf *Node* containing *unique buffer* which is not *prefixed* in others Node buffer.

The *Reach Set Computation Time* depends strongly on *Movement Automaton* prediction complexity and Node count. The *Constrained Expansion Method* (alg. **??**) is separating all nodes entering into $cell_{i,j,k}$ into two distinctive groups: *Candidates for expansion* and *Leftover Nodes.*

The *Leftover Nodes* are thrown away every expansion. The *Leftover Nodes* are not expanded in the next *Wave-front* iteration, but they leave a notable *computation* and *memory* footprint.

Note. Average Trajectory Smoothness Rate (def. **??**) is important only in *Navigation Mode*; this aspect has been covered over (sec. **??**, **??**, **??**).

Approach: For the same conditions (*Testing Avoidance Grid*, *UAS initial state*, *Movement Automaton*) compare the performance of *Reach Set Approximations* created by various methods for the following parameters:

1. *Coverage Ratio* - defined in (def. **??**) shows how versatile *Reach Set Approximation* is (up to 100% of complete reach set coverage).

2. *Node count* - count of Nodes in *Reach Set Approximation* counted like:

   a. full - all active nodes existing over computation time,

   b. pruned - active nodes for real-time use.

3. *Count of Trajectories* - count of Trajectories (leaf Nodes) counted like:

   a. full - all active trajectories existing over computation time,

   b. pruned - active trajectories are leading to coating cells of *Avoidance Grid.*

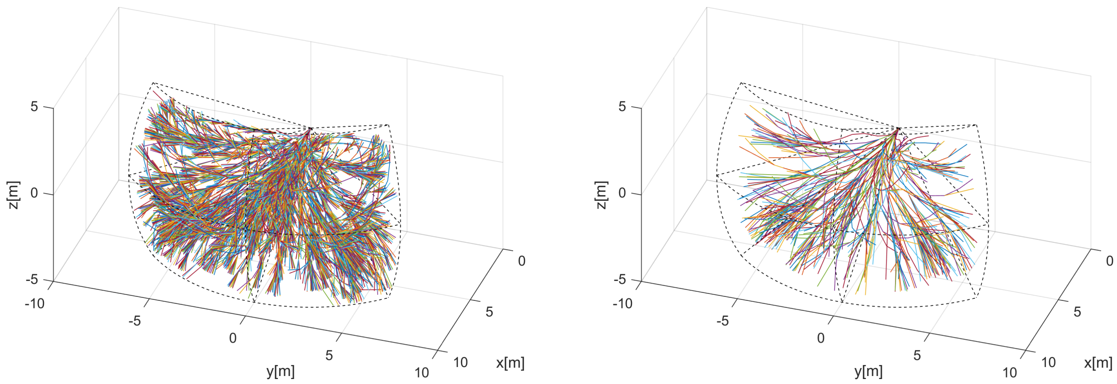
Testing Avoidance Grid with *Distance 10 m*, *Layer count 10*, *Horizontal range* $[-45°, +45°]$, *Horizontal Cell Count 7*, *Vertical range* $[-30°, +30°]$, and *Vertical Cell Count 5.*

Note. The sizing of the *Avoidance Grid* was chosen a small scale because the property of *Coverage Ratio* can be calculated exactly up to some scale, after that it can be only assumed. Various sizes of *Avoidance Grid* was tested in [1].

The UAS is at *Back-side* of *figure* (the initial state is at all *Trajectory Origins*). The *black dashed line* marks *Avoidance Grid* space boundary. Each trajectory has own color and ends at *Front-side* of *Avoidance Grid Boundary.*

Coverage-Maximizing Reach Set (sec. **??**) is used in *Emergency Avoidance Mode* for *Non-Controlled Airspace.* The *full* set of trajectories is given in (fig. 7.1a). The *Pruned* set of trajectories is given in (fig. 7.1b).

Tuning parameters were selected like follow: *Spread Ratio* is 15 (unique footprint trajectories in the cell), and *trajectory footprint length* is 3 (last three unique passing cells).

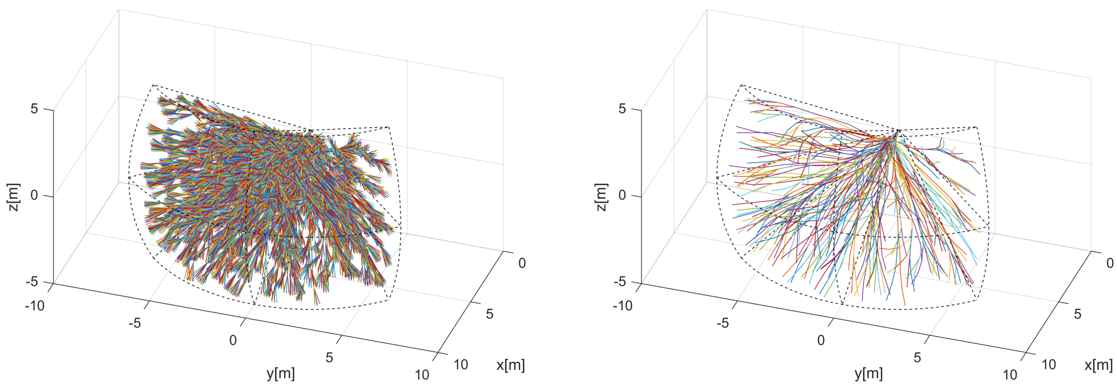


(a) Full. (b) Pruned.

Figure 7.1: Coverage-maximizing reach set computation example.

Turn-Minimizing Reach Set (sec. **??**) is used in *Navigation Mode* for *Non Controlled Airspace.* The *full set* of trajectories is given in (fig. 7.2a). The *Pruned* set of trajectories is given in (fig. 7.2b).

Tuning parameter for *harmonic spread ratio* was set to 9 (which implies low coverage).

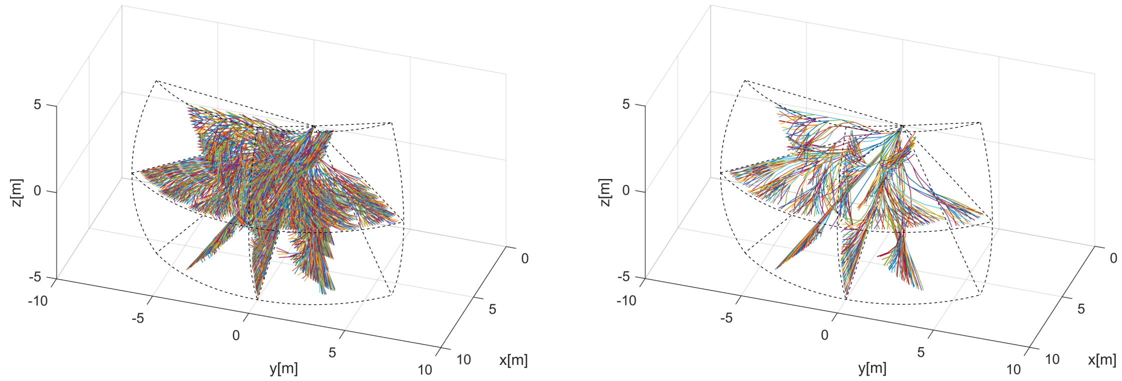


(a) Full. (b) Pruned.

Figure 7.2: Turn-minimizing reach set computation example.

Combined Reach Set (sec. **??**) is combination of *Coverage-Maximizing Reach Set* (fig. 7.1) and *Turn-Minimizing Reach Set* (fig. 7.2). The *tuning parameters* are the same for the respective methods. It is used for both *Emergency Avoidance* and *Navigation.*

ACAS-like Reach Set (sec. ??) is used in *Navigation Mode* for *Controlled Airspace*. The separations used are *Horizontal*, *Vertical*, *Slash*, and, *Backslash*, to give the worst possible nodes and trajectories count. The *full* set of trajectories is given in (fig. 7.3a). The *Pruned* set of trajectories is given in (fig. 7.3b).



(a) Full. (b) Pruned.

Figure 7.3: ACAS-like reach set computation example.

Computation Methods Performance Comparison (tab. 7.1) gives overview of memory consumption and *Coverage Ratio*.

Node count: *Full Node Count* shows how much memory it takes to compute *Reach set*. *Pruned Node Count* shows how much *memory* is needed for storage.

Note. The total size of *full/pruned Reach Set* depends on Node implementation. The Object-oriented prototype implementation in Matlab for example avoidance grid took up to 1 megabyte of system memory. The effective implementation would take up to 100 kilobytes.

Constrained expansion (alg. ??) have different selection rate, depending on the method. The survival rate directly reflects strictness of selection criteria. The rate of *node pruned* is summarized in (eq.7.1)

$$\begin{aligned}&\texttt{Nodes pruned}\\ &\quad CM-RSA &&: 78.93\%\\ &\quad TM-RSA &&: 18.50\%\\ &\quad ACAS-like &&: 79.05\%\end{aligned} \tag{7.1}$$

The *interpretation of results* for each reach set estimation method is like follow:

1. *Coverage-Maximizing* - the main exploration drive is *Coverage Rate*, the *Trajectory* segments are not usually smooth. For our *Movement Automaton*, there is only one *Smooth Movement*: Straight. Other eight are considered *Chaotic Movements*. Impact of this fact is significant because 4/5 of nodes were pruned.

2. *Turn-Minimizing* - the main exploration drive is *Smoothness* of contained *Trajectories.* The *Trajectory segments* which are getting further away from *cell center* are not feasible. If *Smooth Movements* set size is considered, the Smooth/Chaotic movement ratio is 1/8 for our *Movement Automaton* implementation. The low node count was expected in this approach. Another Contributing factor is *Trajectory Footprint Length* for uniqueness selection, which is not a tuning parameter in this method, and it is set to the most strict selection.

3. *ACAS-like* - the main drive is to create set consisting from *multiple 2D separation planes.* The expansion method applies full movement set on the *candidate node.* The *Separation plane movement subset* is determining, which node will be selected for further expansion. The size of separation plane subset to the size of movement set rate is 1 : 3. There are four separation planes: horizontal, vertical, slash and backslash each containing full 2D plane reach set approximation which caused high node prune rate. Nodes used rate should get lower with increasing grid size.

Trajectories count: *Full trajectories count* shows how many *leaf nodes* were existing during the calculation process without pruning. The difference between *full node count* and *full trajectories count* is count of inner tree nodes.

Pruned trajectories count shows how many *leaf nodes* are used in run-time of *avoidance algorithm.* The difference between *pruned node count* and *pruned trajectories count* shows the count of inner nodes in active reach set.

The most of *waste leaf nodes* are removed during *layer pruning*: function *reachSet.purge- SameFootprint()* (alg. **??**). The *Waste trajectories* or *unused leaf nodes count* have significant impact. Because *leaf nodes* are a side product of *Node Expansion procedure* the amount of *pruned trajectories* is around 90 % regardless of the used method. The results are summarized in (eq. 7.2)

$$\begin{array}{ll} \texttt{Trajectories pruned} & \\ \quad CM-RSA & :91.24\% \\ \quad TM-RSA & :88.21\% \\ \quad ACAS-like & :89.43\% \end{array} \tag{7.2}$$

Calculation method	Node count		Trajectories		Coverage ratio	Parameters
	full	pruned	full	pruned		
CM-RSA	6727	1417	4557	399	90%	spread:15
TM-RSA	1724	1405	1528	180	30%	spread:9
combined	-	2405	-	435	95%	CH spread:15 H spread:9 tree comb.
ACAS-like	11294	2366	7437	786	74.95%	Separations: H/V/S/BS Coverage pruning: disabled

Table 7.1: *Reduced reach set* computation methods performance

Coverage ratio: (def. **??**) is showing how much maneuvering versatility of *Reach Set*. *Full Reach Set Approximation* have coverage ratio of 100 %. It is possible to construct *Reference Reach Set* without constrained expansion method which contains all possible *trajectory footprints*. Following observations for *coverage ratio* can be made:

1. *Coverage-maximizing* reach set estimation method by design select *Nodes* which have the high probability of *trajectory footprint* diversification. The high coverage ratio was achieved at values around 90 %.

2. *Turn-Minimizing* reach set estimation method by design selects most smooth trajectories which cause low *trajectory footprint* diversity. The fairly high coverage ratio of 30 % has been achieved.

3. *Combined* reach set estimation method takes two reach set and combines their trajectory trees into a single trajectory tree. It is given that *Coverage ratio* will achieve at least maximal coverage ratio of original reach sets. Harmonic reach set supplemented narrow smooth trajectories which were throw away previously; this increased overall *coverage ratio* to 95 %.

4. *ACAS-like* reach set estimation method contained four separation planes, which caused that it was similar to *Coverage-Maximizing Reach Set Approximation* for given *Avoidance Grid*, concerning of performance. The coverage ratio For 2D plane was 100 %.

Bibliography

[1] Alojz Gomola, Pavel Klang, and Jan Ludvik. Probabilistic approach in data fusion for obstacle avoidance framework based on reach sets. In *Internal publication collection*, pages 1–93. Honeywell, 2017.