

FACULDADE DE ENGENHARIA DA UNIVERSIDADE DO PORTO

# Obstacle Avoidance Framework based on Reach Sets

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I declare that I carried out this research plan independently, and only with the cited sources, literature and other professional sources.

In Porto on .....

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## (C)Nomenclature

This chapter summarize used symbols (tab. 3), acronyms (tab. 1), terminology (tab. 4) and, organizations (tab. 2) mentioned in work.

Acronym	Meaning
UAV	Unmanned Aerial Vehicle
UAS	Unmanned Autonomous System(including naval vehicles)
RPAS	Remotely Piloted Aerial System(lesser degree of autonomy)
OPA	Optionally Piloted Aircraft
RPV	Remotely Piloted Vehicle
PIC	Pilot-in-Command
LOS	Line Of Sight
VLOS	Visual Line Of Sight
BLOS	Behind Line Of Sight
SAA	Sense And Avoid
DAA	Detect And Avoid
MAC	Mid-Air Collision
OAC	Off-Air Collision
ABSAA	Airborne Sense and Avoid
GBSAA	Ground Based Sense and Avoid
POA	Preemptive Obstacle Avoidance
ROA	Reactive Obstacle Avoidance
TCAS	Traffic Alert and Collision Avoidance System
ACAS X	Airborne Collision Avoidance System X
ACAS XU	Airborne Collision Avoidance System X for UAS
CD&R	Collision Detection and Resolution
RTK	Real-Time Kinematik
GPS	Global Positioning System
IMU	Internal Measurement Unit
LiDAR	Light Detection and Ranging
ADS-B	Automatic Dependent Surveillance – Broadcast
GSE	Ground Support Equipment
ATC	Air Traffic Control
ATO	Air Traffic Organization
C2	Control and Communications
MASPS	Minimum Aviation System Performance Standard
MOPS	Minimum Operational Performance Standard
RVSM	Reduced Vertical Separation Minimum
RHSM	Reduced Horizontal Separation Minimum
SM	Safety Margin

Table 1: List of Acronyms

Acronym	Organization name
ICAO	International Civil Aviation Organization (UN)
ITU	International Telecommunication Union (UN)
EASA	European Aviation Safety Agency (EU)
JARUS	Joint Authorities for Regulation of Unmanned Systems (EU)
FAA	Federal Aviation Administration (USA)
FCC	Federal Communications Commission (USA)
LSTS	Laboratório de Sistemas e Tecnologia Subaquática (PT)
FEUP	Faculdade de Engenharia da Universidade do Porto (PT)
ITK	Institutt for teknisk kybernetikk NTNU (NO)
NTNU	Norges teknisk-naturvitenskapelige universitet (NO)
IST	Instituto Superior Técnico - Universidade de Lisboa (PT)
HWI	Honeywell International (CZ/USA)

Table 2: List of Organizations

Symbol	Explanation
$A, B, C, D, \dots$	Capital letters are used for matrices
$A(\dots), B(\dots), \dots$	Functional matrices, (...) denotes parameters
$f(\dots), g(\dots), \dots$	Vector or scalar functions (...) denotes parameters
$\vec{f}(\dots), \vec{g}(\dots), \dots$	Explicit vector functions, when equation contains both types of scalar and vector functions
$t, x, y, z, \dots$	Vectors or scalar coefficients
$\vec{x}, \vec{o}, \vec{g}, \dots$	Explicit vectors, when function contains both types of scalar and vector parameters.
$\theta, \varphi$	Greek letters denoting angles in radians

Table 3: List of symbols

Terminology	Definition
Air Traffic Control	A service operated by appropriate authority to promote the safe, orderly, and expeditious flow of air traffic
Aircraft	A device that is used or intended to be used for flight in the air
Airspace	Any portion of the atmosphere sustaining aircraft flight and which has defined boundaries and specified dimensions. Airspace may be classified as to the specific types of flight allowed, rules of operation, and restrictions in accordance with International Civil Aviation Organization standards or State regulation
Civil Aircraft	Aircraft other than public aircraft.
Collision Avoidance	The Sense and Avoid system function where the UAS takes appropriate action to prevent an intruder from penetrating the collision volume. Action is expected to be initiated within a relatively short time horizon before closest point of approach. The collision avoidance function engages when all other modes of separation fail.

Terminology	Definition
Communication Link	The voice or data relay of instructions or information between the UAS pilot and the air traffic controller and other NAS users.
Control Station	The equipment used to maintain control, communicate with, guide, or otherwise pilot an unmanned aircraft.
Crewmember (UAS)	In addition to the crewmembers identified in 14 CFR Part 1, a UAS flightcrew member includes pilots, sensor/payload operators, and visual observers, but may include other persons as appropriate or required to ensure safe operation of the aircraft.
Data Link	A ground-to-air communications system which transmits information via digital coded pulses.
Detect and Avoid	Term used instead of Sense and Avoid in the Terms of Reference for RTCA Special Committee 228. This new term has not been defined by RTCA and may be considered to have the same definition as Sense and Avoid when used in this document.
ICAO	International Civil Aviation Organization is a specialized agency of the United Nations whose objective is to develop the principles and techniques of international air navigation and to foster planning and development of international civil air transport.
Manned Aircraft	Aircraft piloted by a human onboard.
Model Aircraft	An unmanned aircraft that is capable of sustained flight in the atmosphere; flown within visual line-of-sight of the person operating the aircraft and flown for hobby or recreational purposes.
Optionally Piloted Aircraft	An aircraft that is integrated with UAS technology and still retains the capability of being flown by an onboard pilot using conventional control methods.
Pilot-in-Command	Pilot-in-command means the person who: <ol style="list-style-type: none"> <li>has final authority and responsibility for the operation and safety of the flight;</li> <li>has been designated as pilot-in-command before or during the flight; and</li> <li>holds the appropriate category, class, and type rating, if appropriate, for the conduct of the flight.</li> </ol>
Public Aircraft	An aircraft operated by a governmental entity (including federal, state, or local governments, and the U.S. Department of Defense and its military branches) for certain purposes
RTCA	RTCA, Inc. is a private, not-for-profit corporation that develops consensus-based recommendations regarding communications, navigation, surveillance, and air traffic management system issues. RTCA functions as a Federal Advisory Committee. Its recommendations are used by the FAA as the basis for policy, program, and regulatory decisions and by the private sector as the basis for development, investment and other business decisions ( <a href="http://www.rtca.org">www.rtca.org</a> )
See and Avoid	When weather conditions permit, pilots operating instrument flight rules or visual flight rules are required to observe and maneuver to avoid another aircraft.

Terminology	Definition
Self-Separation	Sense and Avoid system function where the UAS maneuvers within a sufficient time-frame to remain well clear of other airborne traffic.
Sense and Avoid	The capability of a UAS to remain well clear from and avoid collisions with other airborne traffic. Sense and Avoid provides the functions of self-separation and collision avoidance to establish an analogous capability to “see and avoid” required by manned aircraft.
Unmanned Aircraft	<ol style="list-style-type: none"> <li>1. A device used or intended to be used for flight in the air that has no onboard pilot. This devise excludes missiles, weapons, or exploding warheads, but includes all classes of airplanes, helicopters, airships, and powered-lift aircraft without an onboard pilot.</li> <li>2. An aircraft that is operated without the possibility of direct human intervention from within or on the aircraft.</li> </ol>
Unmanned Aircraft System	<p>An unmanned aircraft and its associated elements related to safe operations, which may include control stations (ground, ship, or air-based), control links, support equipment, payloads, flight termination systems, and launch/recovery equipment.</p> <p>An unmanned aircraft and associated elements (including communications links and the components that control the unmanned aircraft) that are required for the pilot-in-command to operate safely and efficiently in the national airspace system.</p>
Visual Line of Sight	Unaided (corrective lenses and/or sunglasses exempted) visual contact between a pilot-incommand or a visual observer and a UAS sufficient to maintain safe operational control of the aircraft, know its location, and be able to scan the airspace in which it is operating to see and avoid other air traffic or objects aloft or on the ground.

Table 4: Terminology

*Note. Acronyms* (tab. 1) and *Terminology* (tab. 4) are in compliance with *ICAO*, *FAA*, and, *EASA* definitions, refer to [10] for more detailed information.

# Chapter 1

## (W) Introduction

*To be done here:*

- Introduce UAV problem of integration into non segregated airspace.
- Open JARUS/NASA requirements and discuss legal framework a little
- Introduce Honeywell interest into topic.

### 1.1 (W) Related Work

*To be done here:*

- Ramasy work, Sabatiny work on LiDAR and obstacle avoidance, introduce movement automaton etc...
- Lattice search related work, the problem of lattice search above 4th dimension
- Reach set approximation related work

### 1.2 (W) Goals

*To be done here:*

- Propose abstract obstacle avoidance framework able to avoid obstacles in real time and guaranteeing safe path independent of controlled platform
- Define guarantee of safety margin concept.
- Define requirements for avoidance set.

### 1.3 (W) Assumptions

*To be done here:*

- Guarantee feasible safe trajectory in open world space at low altitude of the flight. Manage information resources about real obstacles, weather obstacles, ATM restrictions.
- create previously mentioned points as assumptions, xxx is accessible to feasible extent, etc ...

## **1.4 (W) Overview**

*To be done here:*

- Chapter overview
- Notation of key concepts

## **1.5 (W) Contributions**

*To be done here:*

- Notable contributions
- List of concepts and articles with references

# Chapter 2

## (W) Collision avoidance

The context of Collision Avoidance is introduced in table 2.1, the structure was taken from Gardi [9] and modified to reflect actual state of art.

<i>Function</i>	<i>Equipment/Task</i>
Communication	Telecommunication datalinks, Controller Pilot Data Link-Control (CPDLC), Voice Communication
Navigation	Navigation sensors including GNSS, INS, etc. providing 3D/4D navigation capabilities.
Surveillance	Cooperative Systems (TCAS, ACAS, etc.) Non-cooperative Sensors (LiDAR, Cameras, etc.)
Situation Awareness	Early Warning Systems, CDTI Display
Autonomous Decision Making	Strategic, Tactical, Emergency Flight Planning, Intelligent Collision Detection, Conflict Resolution and Prevention, Weather/Terrain/Constraints Avoidance

Table 2.1: Collision Avoidance Systems Context Overview [9].

*Communication* overview elaboration on capability, reliability, security, architecture have been summarized by Johansen et. al. in [11].

*Navigation* overview is given by Nex [12] *Waypoint planning in 3D environment* is elaborated in [13]. *Waypoint Tracking and Test Environments* are thoughtfully discussed in [14, 15, 16, 17].

*Surveillance* cooperative surveillance is covered by TCAS and ACAS systems, interesting aspect of these systems are *Resolution Advisories* [18] for TCAS [19]. For ACAS. The visualization of surroundings has been introduced in [20]. *LiDAR* based *SAA* system have been introduced by Sabatini [21] further enhanced by Ramasay [22]. Other *Non-Cooperative* sensors and their feasibility have been outlined in Ramasay work [23].

*Situation awareness* is implemented mainly as *human-based* systems, *Early Warning System* has been proposed by Lee [24] and adaptive version by Miller [25]. Effects of *CDTI Display* visualization and human decision impact have been examined by Thomas [26]. *Self Separation* aspect have been examined by Williams [27].

*Autonomous Decision Making -TBD:* This will be added later, because its main part...

*Note.* The purpose following sections is to introduce current state in:

1. Introduce *Airspace Classification* in 2.1, from current ICAO/FAA/EASA viewpoint, emphasizing common viewpoint among regulations.
2. Provide background for *manned aircraft operation rules* in 2.2, restriction imposed by Visual(2.2.1)/Instrumental(2.2.2) Flight rules.
3. Define "*Well Clear*" (sec.2.3) state of aircraft in airspace. Introducing the roles of *Air Traffic Control* (2.3.1) and current CAS systems TCAS (2.3.2) and ACAS-X (2.3.3).

The purpose of following sections is to introduce ATM extension for UAS systems:

1. *UAS Traffic Management* functionality is analysed in 2.4, two major movements EU USPACE (2.4.1) and US NASA UTM (2.4.2).
2. *Event Based Avoidance* (2.5) defines basic event based control invoked by *UTM*, two major categories are analyzed in *MACP* (2.5.1) section and *Weather Impact* (2.5.2)
3. *UTM Functional decomposition* is given in section 2.6.

## **2.1 (W) Airspace classification**

- Airspace classes,
- Controlled airspace (Risk analysis, Actors)
- Uncontrolled airspace (Risk analysis, Actors)
- Aircraft categorization
- MOPS for different airspace

## **2.2 (W) Aircraft operational rules**

- Introduction to rules and classification and regulations.
- Analysis of impact.

### **2.2.1 (W) Visual Flight Rules (VFR)**

- Rules of the Air
- Visual separation

### **2.2.2 (W) Instrumental Flight Rules (IFR)**

- Instrumental separation
- Alert/Notice definition
- Prioritization IFR/VFR

## 2.3 (W) Remaining "Well Clear"

- Well clear definition/Near miss definition
- Well clear zones (Crash margin (near miss), warning margin, alert margin)

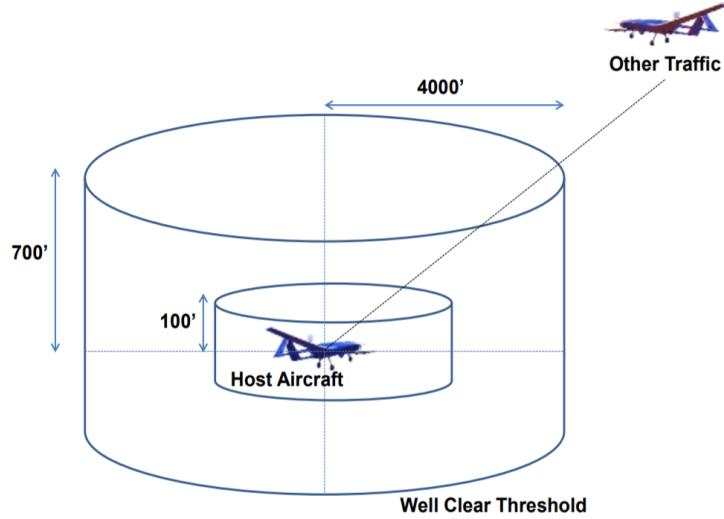


Figure 2.1: Well Clear Threshold [1, 2].

### 2.3.1 (W) Air Traffic Control(ATC)

- Keeping Aircraft  $\leftrightarrow$  UTM  $\leftrightarrow$  Aircraft well clear
- ATC situations - Climb/Descent/Leveled flight (presentation), put emphasis on horizontal/vertical separation
- warning/notice/resolution notation and definitions
- standard ATM functionality
- Standard procedures convergence/divergence
- Standard procedures for maneuver restrictions

Real-time *Airspace Management* approach have been presented in [28]. Following *Dynamic Airspace Management* [3].

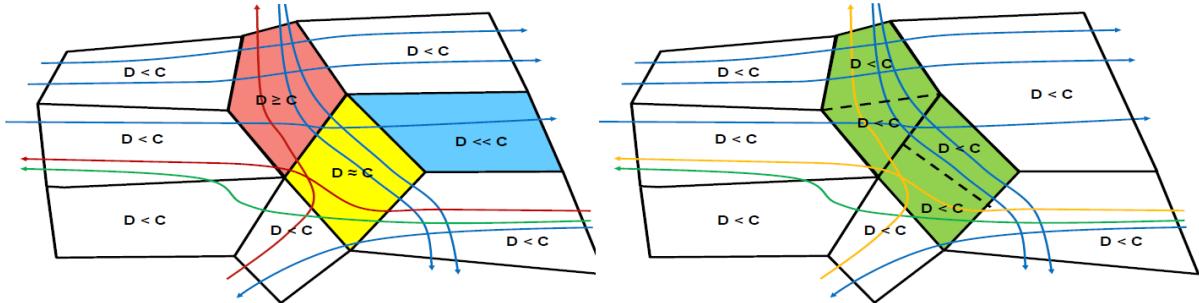


Figure 2.2: Example of DAM flight rerouting to homogenize traffic density [3].

### **2.3.2 (W) Traffic Collision Avoidance System (TCAS)**

- Keeping Aircraft ↔ Aircraft well clear,
- Traffic advisories and alerts messaging,
- Basic principle dissemination,

### **2.3.3 (W) Airborne Collision Avoidance System X (ACAS-X)**

*ACAS-X<sub>U</sub>* concept have been implemented as *Deep Neural Network Solver* in [29]

- ACAS principles
- ACAS XA addition TCAS

## **2.4 (W) UAS Traffic Management (UTM)**

- Introduce the role of future UTM system
- Discuss crucial requirements for UTM systems in sense of DAA topic
- Strongly refer to DAA UTM CONOPS (Appendix B) already sent to Joao
- focus on controlled airspace
- outline challenges for G class airspaces

### **2.4.1 (W) U-Space (EU)**

- Introduce U-Space project in terms of EUROControl, EASA, ICAO, JARUS intention and involvement
- Outline supported DAA functionality for Phase 1 - Phase 4
- Refer to USPACE leaflet, already sent to Joao

### **2.4.2 (W) NASA UTM (US)**

- Focus on DAA aspect and level flight in controlled airspace
- Refer to materials at: <https://utm.arc.nasa.gov/index.shtml>

## **2.5 (W) Event Based Avoidance**

- Introduce the context for Event based avoidance:
  - Preventive DAA - before flight preparation
  - Event-based DAA - reaction window in minutes, usually cooperative via notifications and other methods

- Reactive DAA - you know the drill boys ..., reaction window in 10th of seconds  
...
- Introduce the process of notification and Event Resolution
- Outline hierarchy in U(A)TM controlled space
- Outline expected minimal set of data sources

### 2.5.1 (W) Mid-Air Collision Prevention

- Identify and dispute shortcomings of Aircraft classification in ICAO
- List of MAC events
- For each event describe trigger and resolution
- cite barrel [30]

### 2.5.2 (W) Weather Impact

- do not use term "climate change"
- critical events are getting more localized and their magnitude is increased
- List of Weather event
- Discuss weather mitigation

*Weather Impact* on transportation in general has been introduced by Koetse in study [31].

*Weather-based* preemptive planning have been introduced into manned aviation in 2015 [32]. *-TBD:This one shows global model impact*

*Severe Weather Condition Detection Capabilities* for current level of standard aviation equipment have been reviewed in [33] *-TBD:This one shows intermediate weather detection*

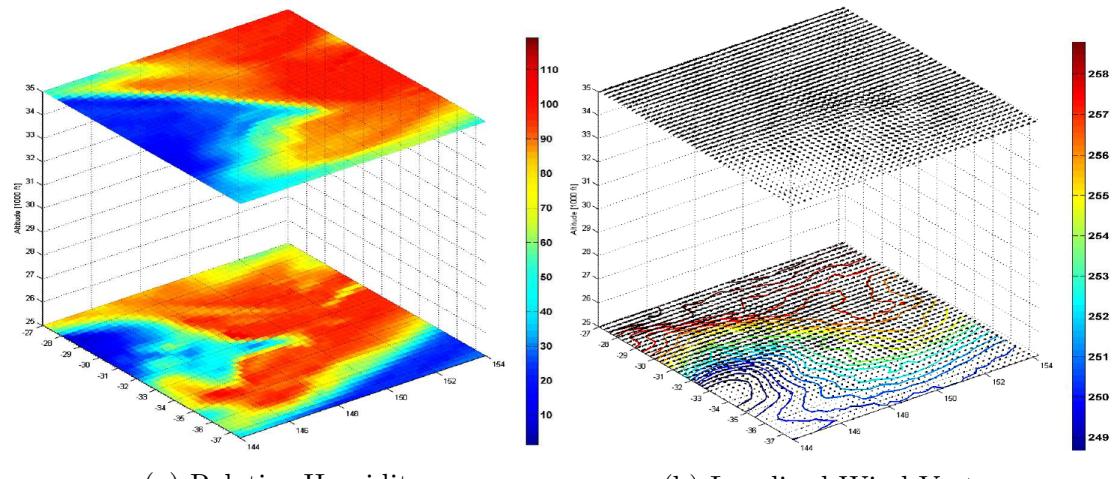


Figure 2.3: Localized Weather Model [4].

*Icing risk* in localized environment have been predicted based on numerical model [34]. It is shown that icing can be prevented by *planned* and *reactive* avoidance.

*Weather Models* can be extracted from *Climate and weather model archive at the National Oceanic and Atmospheric Administration* [5].

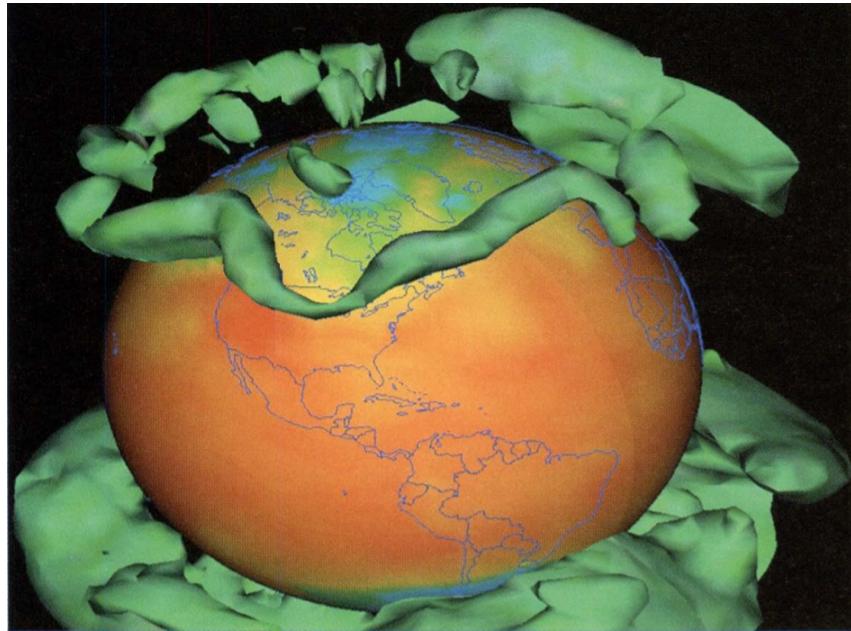


Figure 2.4: Example of upper troposphere winds [5].

## 2.6 (W) UTM Functional Decomposition

- general thinking, approach
- notification run
- resolution run

### 2.6.1 (W) Information Exchange Principles

- Information Exchange schematics in general, describe data flow and actors

#### Collision Case

- what collision case contains and how is calculated
- define conditions and role prioritization
- define blank spaces in VFR procedures

#### Weather Case

- This section needs to be discussed with subject matter expert.

## **2.6.2 (W) Resolution Advisories**

- just describe the resolution format from ATM
- discuss manned/unmanned interaction and reaction delays, reaction uncertainty, etc ...

**Movement Restriction**

**Convergence/Divergence**

# Chapter 3

## (W) Background Theory

*To be done here:*

- State of art introduction from *Research plan*.

### 3.1 (W) UAS Motion Model

*To be done here:*

- Continious time model.
- Discrete time model.
- Properties like controllability, observability, invariance to a set of points.
- Taken mostly from *Research plan, Article, and MPC report*

### 3.2 Reach Sets

Informally, the *Reach Set* of a UAS system described by a differential equation is the *set of all states that can be reached from an initial state within a given time interval*.

**For following definitions** consider *nonlinear UAS system* described in (sec. 3.1).

**Definition 1** (Reach set starting at a given point). *Suppose the initial position and time ( $state_0, time_0$ ) are given. The reach set  $ReachSet[\tau, time_0, state_0]$  of nonlinear system at time  $\tau \geq time_0$ , starting at  $(state_0, time_0)$  is given by:*

$$ReachSet[\tau, time_0, state_0] = \bigcup \{ state(\tau) : input(s) \in Inputs(s), s \in (time_0, \tau] \} \quad (3.1)$$

**Reach set starting at given set** can be used to determine reach set in case of *hybrid system* input control switch and it is defined as follow:

**Definition 2.** *set starting at a given set] The reach set at time  $\tau > t_0$  starting from set  $States_0$  is defined as:*

$$ReachSet[\tau, time_0, States_0] = \bigcup \{ ReachSet[\tau, time_0, state_0] : state_0 \in States_0 \} \quad (3.2)$$

**Reach set for adversarial behavior** can be used to calculate possible escape routes from pursuer and it is defined as follow:

**Definition 3** (Reach set under adversarial behavior). *Consider now the case of adversarial behavior([35, 36]). where  $input(t)$  is our control and  $adversary(t)$  is adversary control which is independent of  $input(t)$ , let  $differentialControl(t) = input(t) - \sup_{state \in state(t)} adversary(t)$ , which represents worst possible input change in given state and time, then reach set for system is represented as:*

$$ReachSet \begin{bmatrix} \tau, \\ time_0, \\ state_0 \end{bmatrix} = \bigcup \left\{ state(\tau) : \begin{array}{l} differentialControl(s) \in DifferentialControlSet(s), s \in (time_0, \tau] \\ DifferentialControlSet(s) \end{array} \right\} \quad (3.3)$$

**Reach set under constraints** are usable to define state constrained systems in terms of dynamics and technical capabilities.

**Definition 4** (Reach set under state constraints). *Suppose the initial position and time ( $state_0, time_0$ ) and state constraints are given  $state(t) \in \mathbb{A} \subset \mathbb{R}^n, \dot{x}(t) \in \mathbb{B} \subset \mathbb{R}^n$ . The reach set  $ReachSet[\tau, time_0, state_0]$  of nonlinear UAS system at time  $\tau \geq time_0$ , starting at position and time ( $state_0, time_0$ ) is given by:*

$$ReachSet \begin{bmatrix} \tau, \\ time_0, \\ state_0 \end{bmatrix} = \bigcup \left\{ state(\tau) : \begin{array}{l} \forall s \in (time_0, \tau], state(s) \in \mathbb{A}, \\ state(s) \in \mathbb{B}, \\ \exists input(s) \in Inputs(s) \end{array} \right\} \quad (3.4)$$

### 3.2.1 Computation of Reach Sets

Several techniques for reachability analysis of systems have been proposed. They can be (roughly) classified into two kinds:

1. Purely symbolic methods based on:
  - a. the existence of analytic solutions of the differential equations and
  - b. the representation of the state space in a decidable theory of the real numbers.
2. Methods that combine
  - a. numeric integration of the differential equations
  - b. symbolic representations of approximations of state space typically using (unions of) polyhedra or ellipsoids.

These techniques provide the algorithmic foundations for the tools that are available for computer-aided verification of hybrid systems ([37], [38], [39]).

The set-valued Lebesgue integral provides a conceptual tool for the direct computation of the reach set. In what follows we describe techniques from dynamic optimization which are used to compute reach sets for dynamic systems.

The relation between dynamic optimization and reachability was first observed in [40]. A typical problem of optimal control can be formulated as follows:

$$\max \left( \int_{initialTime}^{finalTime} cost(time, state(time), control(time)) dt + \dots \right) + FinalCost(state(finalTime)) \quad (3.5)$$

For nonlinear system:

$$state(t) = f(t, state(t), control(t)), control(t) \in ControlSet(t) \subset \mathbb{R}^p \quad (3.6)$$

Where *cost* is given as cost function of time, state and input and *FinalCost* represents cost functional. There are two main techniques to solve this problem:

1. The maximum principle
2. Dynamic programming. The maximum principle gives necessary conditions of optimality. Dynamic programming may be used to derive sufficient conditions of optimality.

A good reference on the maximum principle is [41]. A less known reference with detailed geometric interpretations is [42]. A good reference on dynamic programming is [43].

### 3.3 (W) Space Segmentation

*To be done here:*

- Planar space segmentation.
- local vs global space segmentation.
- benefits of uneven space segmentation.

### 3.4 (R/W) Movement Automaton

*Movement Automaton* is basic interface approach for discretization of *trajectory evolution* or *control input* for any *continuous or discrete system model*.

*Main function* of *Movement Automaton* is for system given by equation  $state = f(time, state, input)$  with initial state  $state_0$  to generate *reference trajectory*  $\hat{state}(t)$  or *control signal*  $input(t)$ .

Using *Movement Automaton* as *Control Proxy* will provide us with *discrete command chain* interface. This will reduce the *non deterministic* element from *Evasive trajectory generation*, by reducing infinite maneuver set to finite *movement set*.

*Non determinism* of *Avoidance Maneuver* have been discussed as an issue in following works:

1. Newton gradient method for evasive car maneuvers [44].
2. Non-holistic methods for trajectory generation [45].
3. Stochastic approach to elliptic trajectories generation [46].

*Examples of Movement Automaton Implementation as Control Element* can be mentioned as follows:

1. Control of traffic flow [47].
2. Complex air traffic collision situation resolution system [6, 48].
3. SAA/DAA capable avoidance system [49].

### 3.4.1 Hybrid Automaton

First the notion of *hybrid automaton* [37, 50, 51] needs to be introduced:

**Definition 5.** *Hybrid automaton (3.7) is given as structure:*

$$\text{HybridAutomaton}(\text{States}, \text{SystemState}, \text{VectorField}, \text{DiscreteTransition}, \text{ResetMap}) \quad (3.7)$$

*States ( $Q$ ) is given as set of discrete states, for every time  $t \in \text{Domain}$  hybrid automaton stays in exactly one of states.*

*SystemState ( $x$ ), is given in domain  $x \in \mathbb{R}^n, n \in \mathbb{N}^+$ , representing the trajectory evolution.*

*VectorField ( $f$ ) (3.8) is bounded to single State  $\in \text{States}$  and represents local SystemState evolution, when given automaton State is Active.*

$$\text{VectorField} : \text{State} \times \text{SystemState} \rightarrow \text{SystemState} \quad (3.8)$$

*DiscreteTransition ( $\varphi$ ) (eq. 3.9) indicates changes of states in automaton, the changes are triggered by satisfying specific condition given by State and SystemState.*

$$\text{DiscreteTransition} : \text{State} \times \text{SystemState} \rightarrow \text{State} \quad (3.9)$$

*ResetMap ( $\rho$ ) (eq. 3.10) defines changes of State to some default value, this change is triggered by specific automaton State and SystemState.*

$$\text{ResetMap} : \text{State} \times \text{SystemState} \rightarrow \text{SystemState} \quad (3.10)$$

### 3.4.2 Building Blocks

**Definition 6.** *Movement Primitive:*

*States from Hybrid automaton can be taken as Movements in Movement Automaton. MovementPrimitive (eq. 3.11) is describing the Movement behaviour as transfer function VectorField enriched with parameters.*

$$\begin{aligned} \text{MovementPrimitive} &(\text{vectorField}, \text{minimalDuration}, \text{parameters}) \\ \text{VectorField} &: \text{SystemState} \times \text{parameters} \rightarrow \text{SystemState} \end{aligned} \quad (3.11)$$

**Example:** Let say that *UAS* system is given as  $\text{position} = \text{velocity}$ , then let us have two *MovementPrimitives*:

1. *Stay* -  $\text{minimalTime} = 1\text{s}$ ,  $\text{parameters} = \{\}$ ,  $\text{VectorField} : \text{position} = 0$ .
2. *Move* -  $\text{minimalTime} = 1\text{s}$ ,  $\text{parameters} = \{\text{velocity}\}$ ,  $\text{VectorField} : \text{position} = \text{velocity}$ .

**Trajectory from Movement Primitives:** The UAS should *Move* for 5s with velocity 10m/s, then *Stay* for 10s, then move for 7s with velocity 4m/s, with initial position  $position_0 = 0$  and initial time  $t_0 = 1$ . The standard approach is to derive transfer function  $position = \Theta(\dots)$

$$position(t) = \Theta(\dots) \begin{cases} t \in [0, 5] & : 10 \times t + position(0) \\ t \in (5, 15] & : 0 \times (t - 5) + position(5) \\ t \in (15, 22] & : 4 \times (t - 15) + position(15) \end{cases} \quad (3.12)$$

The *example* given by (eq. 3.12) is fairly primitive, but imagine UAS system given by nonlinear dynamics  $\dot{x} = f(x, u, t)$  [52]. Then defining transfer function for given command chain can be impossible.

**Definition 7. Movement Transition:**

System state *can be different than intended movement application*, the notion of *Transition* is therefore introduced as stabilizing element in movement chaining (eq. 3.13).

$$Transition : MovementPrimitive \times SystemState \rightarrow MovementPrimitive \quad (3.13)$$

**Trajectory with Transitions:** Introducing two transitions *Transition(Move, Stay)* and *Transition(Stay, Move)* reflecting periods when vehicle stop moving or speed-up to desired velocity. The transfer function (eq. 3.12) can be rewritten as combination of *MovementPrimitives* (eq. 3.11) and *Transitions* (eq. 3.13):

$$\begin{aligned} & Transition(Stay, Move), Move(5s, 10m/s), \\ & Transition(Move, Stay), Stay(10s), \\ & Transition(Stay, Move), Move(7s, 4m/s) \end{aligned} \quad (3.14)$$

*Note.* There are two types of *MovementPrimitives*:

1. *Stationary* - when system state is considered neutral and they are considered as entry point for automaton.
2. *Dynamic* - when the system state is considered evolving and they needs to be terminated with *stationary* transition.

**Movement Mapping Example:** Transition/MovementPrimitive pairs (eq. 3.13) can be mapped into movements (eq. 3.15).

$$\begin{aligned} & Move(5s, 10m/s) : Transition(Stay, Move), Move(5s, 10m/s), \\ & Stay(10s) : Transition(Move, Stay), Stay(10s), \\ & Move(7s, 4m/s) : Transition(Stay, Move), Move(7s, 4m/s) \end{aligned} \quad (3.15)$$

**Definition 8. Movement:**

*Movement can consist from multiple Transitions (eq. 3.13) and one MovementPrimitive (eq. 3.11), the duration of MovementPrimitive can be shortened by Transitions duration. Movement is defined as follows:*

$$Movement \left( \begin{array}{l} initialState, \\ initialTime[0..1], \\ duration, \\ parameters[0..1] \end{array} \right) = Chain \left( \begin{array}{l} InitialTransition(\dots)[0..*], \\ MovementPrimitive \left( \begin{array}{l} transitionState, \\ remainingDuration, \\ parameters \end{array} \right) \\ LeaveTransition(\dots)[0..*], \end{array} \right) \quad (3.16)$$

Chain function *connects multiple* initial Transitions which are applied at initial State at initialTime. Then own MovementPrimitive (eq. 3.11) is invoked with transitionnsState. Transitions state is state changed by Initial Transitions. After Movement Primitive there can be Leave Transitions Movement

**Minimal Movement Time:** Given by (eq. 3.17) for movement is given as sum of MovementPrimitive (eq. 3.11) minimal time, and Transition (eq. 3.13) in/out combined minimal time.

$$minimalTime(Movement) = \frac{minimalTime(MovementPrimitive)}{\max_{in/out}\{time(Transition)\}} + \quad (3.17)$$

**Movement Chaining:** Movements can be *chained* and applied to initial system state to generate system trajectory. Example of trajectory is given by (eq. 3.12). Movements are reversibly obtained by participation such trajectory into Movement primitives and Transitions. Then sample Trajectory for  $n \in \mathbb{N}^+$  movements looks like (eq. 3.18).

$$\begin{aligned} Trajectory(t_0) &= State(t_0) \\ Trajectory(t_0, t_1] &= Movement_1(Trajectory(t_0), t_0, duration_1, parameters_1) \\ Trajectory(t_1, t_2] &= Movement_2(Trajectory(t_1), t_1, duration_2, parameters_2) \\ Trajectory(t_2, t_3] &= Movement_3(Trajectory(t_2), t_2, duration_3, parameters_3) \\ &\vdots \\ Trajectory(t_{n-1}, t_n] &= Movement_n(Trajectory(t_{n-1}), t_{n-1}, duration_n, parameters_n) \end{aligned} \quad (3.18)$$

Given Trajectory at time  $t_0$  is given as initial State of System. For time interval  $(t_0, t_1]$ , which length is equal to  $duration_1$ , the State is given by Movement<sub>1</sub> with parameters<sub>1</sub> and base time  $t_0$ . This behaviour continues for movements  $2, \dots, n$ .

**Definition 9. Movement Buffer:**

Movements can be chained into Buffer with assumption of continuous movement execution. Continuous movement executions each movement in chain (eq. 3.18) is executed in time interval  $\tau_i = (t_{i-1}, t_i]$  where  $i$  is movement order and  $\forall Movement_i$  starting time is  $t_0$  or  $t_{i-1}$  from previous movement. With given assumption Buffer is given as (eq. 3.19) with parameters  $t_{i-1}, t_i$  omitted, due  $t_0$  and  $duration_i$  dependency.

$$Buffer = \{Movement_i(duration_i, parameters_i)\} i \in \mathbb{N}^+ \quad (3.19)$$

**Definition 10. Movement Automaton Trajectory:**

Let say system State  $\in \mathbb{R}^n$  which Trajectory is defined by movement chaining (eq. 3.18),

applied on some initial time  $t_0 \in \mathbb{R}^+$  and final time  $t_f = t_0 + \sum_{i=1}^I duration_i$ , with movements contained in Buffer (eq. 3.19) is given as Trajectory (eq. 3.20).

$$Trajectory(t_0, State(t_0), Buffer) \text{ or } Trajectory(State_0, Buffer) \text{ if } t_0 = 0 \quad (3.20)$$

Note. The space dimension of *Trajectories* is  $\mathbb{R}^{n+1}$  if the space dimension of state *Space* is  $R^n$ , because *Trajectory space* contains evolution of *Space* in time interval  $T[t_0, t_f]$ .

The transformation from *transfer function* (eq. 3.12) to *trajectory* (eq. 3.20) is natural, only set of *Movement primitives* (eq. 3.11) and set of *Transitions* (eq. 3.13) is required.

**State Projection:** *Trajectory* (eq. 3.20) is naturally evolution of space over time, then there exists *StateProjection* function (eq. 3.21) which returns *State* for specific *Time*.

$$StateProjection : Trajectory \times Time \rightarrow State(Time) \quad (3.21)$$

### 3.4.3 (W) Definition and Properties

**Definition 11.** *Movement Automaton* is given as follow:

$$InitialState : \in \mathbb{R}^h, h \in \mathbb{N}^+ \quad (3.22)$$

$$System : \dot{State} = f(Time, State, Input) \text{ or vectorField} \quad (3.23)$$

$$Primitives = \left\{ MovementPrimitive_i \begin{pmatrix} vectorField, \\ minimalDuration, \\ parameters \end{pmatrix} \right\} i \in \mathbb{N}^+ \quad (3.24)$$

$$Transitions = \left\{ Transition_j \begin{pmatrix} MovementPrimitive_l, \\ MovementPrimitive_k \end{pmatrix}_{k \neq l} \right\} j \in \mathbb{N}^+ \quad (3.25)$$

$$Movements = \left\{ Movement_m \begin{bmatrix} Transition_o[0..*], \\ MovementPrimitive_p \\ Transition_r[0..*], \end{bmatrix}_{o \neq r} \right\} m \in \mathbb{N}^+ \quad (3.26)$$

$$Buffer = \{Movement_s(duration_s, parameters_s)\} s \in \mathbb{N}^+ \quad (3.27)$$

$$Executed = \{Movement_s(duration_s, parameters_t)\} t \in \mathbb{N}^+ \quad (3.28)$$

$$Builder : Movement \times MovementPrimitive \rightarrow Movement \quad (3.29)$$

$$Trajectory : InitialState \times Movement^u \rightarrow State \times Time, u \in \mathbb{N}^+ \quad (3.30)$$

$$StateProjection : Trajectory \times Time \rightarrow State(Time) \quad (3.31)$$

System (eq. 3.23) is given in form of differential equations  $\dot{x} = f(t, x, u)$  or other transformable equivalent, with initial state (eq. 3.22).

Movements (eq. 3.18) are defined as sequence of necessary initial transitions (eq. 3.25), movement primitive (eq. 3.24), and, leave transitions (3.25).

Buffer contains a set of movement primitives (eq. 3.24) to be executed in order to achieve desired goal. Builder (eq. 3.29) assures that first movement primitive (eq. 3.11) from Buffer (eq. 3.27) is transformed into next movement (eq. 3.26) based on current movement (eq. 3.26).

*System trajectory (eq. 3.30) is defined in (eq. 3.20). State projection (eqs. 3.21,3.31) is giving State variable for time  $t \in [t_0, t_{max}]$  where  $t_{max}$  is given by:*

$$t_{max} = t_0 + \sum_{i=1,u} Buffer.Movement(i).movementDuration \quad (3.32)$$

TODO important properties of movement automaton,

# Chapter 4

## (R) Problem Statement

An *UAV* equipped with several types of sensors is tasked to fly several types of *Missions* in a 3-dimensional space. There is a *Terrain map*, an *Object map*, and a *Weather* forecast for target region that are known a priori. The map of the *Terrain* may not be up to date and *Uncharted obstacles* may affect flight safety. The *UAV* has to comply to a set of *Flight Rules* specifying *Flight Constraints*. The performance of the *UAV*, including sensor performance, is affected by the *Weather*.

Several difficulties must be faced. First, the design space of is large and complex. Second, *Trajectory calculation* Third, *Navigation*.

### 4.1 Basic Definitions

A few definitions are needed.

#### 4.1.1 World

The world of interest consists of:

*Space* has a Global Reference Frame with three axes  $X^+, Y^+, Z^+$ .

$$Space \subseteq \mathbb{R}^3 : \text{main axes: } X^+, Y^+, Z^+, \text{ center}[x_0, y_0, z_0] \quad (4.1)$$

*Object* (4.2) is generic subset of *Space* which has *Boundary*.

$$Object \subset Space : \forall point \in Object, point \text{ is solid} \quad (4.2)$$

The dynamic model of the *UAV* is given by a nonlinear first order state-space model:

$$\dot{x} = f(t, x, u) \quad (4.3)$$

Where  $x \in \mathbb{R}^{6+n}, n \in \mathbb{N}^{0+}$  is *system state* containing minimal information about vehicle position  $[x, y, z]$  and orientation  $[\theta, \varphi, \rho]$  (roll, pitch yaw), and  $u(t)$  is *control signal* belonging to  $R^k, k \in \mathbb{N}^+$ , bounded by control set  $u(t) \in U$ .

The map of the terrain is given by *TerrainMap* mapping  $(x, y)$  to the terrain elevation

$$TerrainMap : (x, y) \rightarrow z \quad (4.4)$$

*Weather* (4.5) can impact the performance of the *UAV*. *wind* can decrease flying capabilities and maneuverability, *visibility* can impair the performance of sensors such

as LiDAR and cameras, *humidity* can be serious danger to electronic equipment and in combination low *temperature* it can cause icing on vehicle, and *air pressure* can impact ranging sensor and flight performance.

$$Weather : (position \in Space) \times time \rightarrow \begin{bmatrix} \text{wind } (w_x, w_y, w_z) \\ [m/s, m/s, m/s] \\ \text{visibility } v [m] \\ \text{humidity } h [0 - 100\%] \\ \text{air pressure } a [hPa] \\ \text{temperature: } \tau [C] \end{bmatrix} \quad (4.5)$$

#### 4.1.2 Mission

*Mission* (4.6) which UAV should fly is given as set of *ordered, feasible in terms of vehicle dynamic* (4.3), *waypoints* in subspace of *Space* (4.1).

$$Mission = \left\{ waypoint_1, waypoint_2, \dots, waypoint_m : \begin{array}{l} \forall_{i=1 \dots m} waypoint_i \in Space \end{array} \right\}, \quad m \in \mathbb{N}^+, m \geq 2 \quad (4.6)$$

*Waypoint Passing* (4.7) function maps system (4.3) trajectory *projection in Space* and *Mission* waypoints (4.6) to a vector of *passing times*.

$$WaypointPassing : TrajectoryProjection \times Mission \rightarrow Time^m \quad (4.7)$$

*Note.* The *Mission* (4.6) is considered as successfully completed if and only if  $\forall$  waypoints are reached and in given order (check output of 4.7).

#### 4.1.3 Flight constraints

Flight constraints arise from aviation rules and from ATM.

There are  $H$  sets of hard flight constraints expressed as subsets of *Space*:

$$HFlightConstraints = \{HFlightConstraint_1, \dots, HFlightConstraint_H\} \quad (4.8)$$

The union of all  $HFlightConstraint_i$  is the set  $HFlightConstraint$ . Example: turning radius of UAV, climb rate etc.

There are  $S$  sets of soft flight constraints expressed as subsets of *Space*:

$$SFlightConstraints = \{SFlightConstraint_1, \dots, SFlightConstraint_S\} \quad (4.9)$$

The union of all  $SFlightConstraint_i$  is the set  $SFlightConstraint$ . Example: sharp maneuvers, restricted flight areas etc.

The *Weather* may impose either soft or hard flight constraints.

#### 4.1.4 Partition of Space

In what follows it is convenient at each time  $t$  to partition the *Space* into four disjoint subsets *Free*( $t$ ), *Restricted*( $t$ ), *Occupied*( $t$ ) and *Uncertain*( $t$ ).

There is a *SpaceClassification* function (4.10)

$$\text{SpaceClassification} : y \in \text{Space} \mapsto s \in \{\text{Free}, \text{Restricted}, \text{Occupied}, \text{Uncertain}\} \quad (4.10)$$

*Note.* *SpaceClassification* (4.10) is *total* function mapping each element of *Space*. Other followup *classifications* are considered as *total* functions.

There is set of *Space* Hard constraints, where one *HardConstraint* (4.11) is given as:

$$\text{HardConstraint} = \{\text{point} \in \text{Space} : \text{SpaceClassification}(\text{point}) = \text{Occupied}\} \quad (4.11)$$

There is set of *Space* Soft constraints, where one *SoftConstraint* (4.12) is given as:

$$\text{SoftConstraint} = \{\text{point} \in \text{Space} : \text{SpaceClassification}(\text{point}) = \text{Restricted}\} \quad (4.12)$$

#### 4.1.5 Information source

**Definition 12.** *Information source* (4.13) represents *a priori information* for each point in *Space* at time *prior mission execution* mapping it into one of distinctive categories Free, Occupied, Restricted, and Uncertain.

$$\text{InformationSource} = \text{SpaceClassification}(\text{PriorTime}) \quad (4.13)$$

**Definition 13.** *Landmark* is partition of *Space* (4.1) which is notable in context of society or law given properties. *Landmark* can be: notable buildings, notable natural structures or crucial infrastructure. *Landmark* is part of terrain map, but its special status can induce additional properties.

For obstacle avoidance problem following *Information sources* are available:

1. *Terrain map* - map of terrain with notable landmarks.
2. *Object map* - map of notable structures with *protection zones* considered as *occupied* or *restricted* space.
3. *Fly zones restriction map* - map of restricted flight areas considered as *restricted* constraints.

#### 4.1.6 Single Observation by single sensor classification

*Vehicle* is equipped with single *Sensor*, which returns *Space* classification for given *observation position* and observation time.

Observation (4.14) at given UAV *position, time*, and for given *sensor* sorts points from *sensor reading* into following distinguish sets (4.14):

1.  $Free_O(position, sensor, time)$  - observable space by sensor and considered as *Free* by sensor reading,
2.  $Occupied_O(position, sensor, time)$  - observable space by sensor and considered as *Occupied* by sensor reading,
3.  $Uncertain_O(position, sensor, time)$  - all other points.

$$Observation(position, sensor, time) \rightarrow \begin{cases} Free_O(position, sensor, time) \\ Occupied_O(position, sensor, time) \\ Uncertain_O(position, sensor, time) \end{cases} \quad (4.14)$$

#### 4.1.7 Multiple observations by single sensor classification

Let add pairs of *observation position* and *observation time* in manner  $\{(position_1, t_1), (position_2, t_2), \dots (position_k, t_k)\}$ ,  $k \in \mathbb{N}^+$  that point position is independent and time is ordered in fashion  $t_1 < t_2 < \dots < t_k$ .

*Free space* from multiple sensor *reading* over multiple *positions* is inclusive space, because we are obtaining additional information regarding to space reachability by sensor reading independent on sensor space and orientation limitation. Therefore the union of single instances of observations are used to represent *Combined free space*  $Free_O(sensor)$  (4.15).

$$Free_O(sensor) = \bigcup_{i=\{1, \dots, k\}} Free_O(sensor, position_i, time_i) \quad (4.15)$$

*Occupied space* (4.16) from multiple sensor *reading* over multiple *positions* is inclusive space, because of increased information, therefore it has similar handling to *Free space*.

$$Occupied_O(sensor) = \bigcup_{i=\{1, \dots, k\}} Occupied_O(sensor, position_i, time_i) \quad (4.16)$$

*Uncertain space* (4.17) from multiple sensor *reading* over multiple *positions* is exclusive space, because of decrease in uncertainty.

$$Uncertain_O(sensor) = \bigcap_{i=\{1, \dots, k\}} Uncertain_O(sensor, position_i, time_i) \quad (4.17)$$

#### 4.1.8 Sensor fusion

The observation at fixed time  $t_{fix}$  can be made by multiple sensors  $sensor_1, sensor_2, \dots, sensor_k$ ,  $k \in \mathbb{N}^k$ , for  $k$  sensors execute observations  $Observation_1(sensor_1, position_1, t_{fix})$ ,  $Observation_2(sensor_2, position_2, t_{fix})$ ,  $\dots$ ,  $Observation_k(sensor_k, position_k, t_{fix})$ .

*Sensor Fusion* (4.18) function with *data fusion parameters* is introduced which combines *Observations* for each *sensor* and *Weather*, then uniquely maps each point into four distinguish sets:  $Free(t_{fix})$ ,  $Occupied(t_{fix})$ ,  $Restricted(t_{fix})$ , and  $Uncertain(t_{fix})$  (special case of (4.10)).

$$SensorFusion : \left[ \begin{array}{l} Observation_1(sensor_1, position_1, t_{fix}) \times \\ Observation_2(sensor_2, position_2, t_{fix}) \times \\ \times \dots \times \\ Observation_k(sensor_k, position_k, t_{fix}) \times \\ Weather(\dots) \\ fixed\ time\ t_{fix} \times \\ sensorFusionParameters \end{array} \right] \rightarrow \left\{ \begin{array}{l} Free(t_{fix}) \\ Occupied(t_{fix}) \\ Restricted(t_{fix}) \\ Uncertain(t_{fix}) \end{array} \right\} \quad (4.18)$$

#### 4.1.9 Data fusion

Multiple *Information Sources*:  $InformationSource_1, InformationSource_2, \dots, InformationSource_l$ ,  $l \in \mathbb{N}$ , where  $l$  is count of information sources needs to be fused with *Sensor Fusion* function (4.18) outcome at *fixed time*  $t_{fix}$ .

*Data fusion* function (4.19) combines the classification from various *Information Sources* with classification from *Sensor Fusion* function (4.18) for vehicle position at *fixed time*  $t_{fix}$ , under given *Data Fusion Parameters*.

$$DataFusion : \left[ \begin{array}{l} InformationSource_1 \times \\ InformationSource_2 \times \\ \times \dots \times \\ InformationSource_l \times \\ SensorFusion(\dots) \times \\ Weather(\dots) \\ position \times \\ fixed\ time\ t_{fix} \times \\ dataFusionParameters \end{array} \right] \rightarrow \left\{ \begin{array}{l} Free(t_{fix}) \\ Occupied(t_{fix}) \\ Restricted(t_{fix}) \\ Uncertain(t_{fix}) \end{array} \right\} \quad (4.19)$$

Each *point* in *Space* is uniquely classified into one of sets  $Free(t_{fix}), Occupied(t_{fix}), Restricted(t_{fix}), Uncertain(t_{fix})$  (special case (4.10)).

*Note.* Moreover *Data Fusion* function is covering the case of *Multiple information sources, combined with multiple sensor readings over multiple times*, including *Weather* as *sensor impact factor* and *information source*.

#### 4.1.10 Known world

*Known world* (4.20) for some *fixed time*  $t_{fix}$  is given as joint set of points which belongs to one of *Data Fusion* (4.19) output sets  $Free(t_{fix})$  or  $Occupied(t_{fix})$  or  $Restricted(t_{fix})$ . The *Known World* is compact set with existing boundary.

$$KnownWorld(t_{fix}) = Free(t_{fix}) \cup Occupied(t_{fix}) \cup Restricted(t_{fix}) \quad (4.20)$$

#### 4.1.11 Safety margin

Let say that *mission* was executed in time interval  $t \in [missionStart, missionEnd]$  in *Known world* (4.20). For every *position*, extracted from *UAV model state*  $x(t)$ , keeps distance from any point in *Occupied*( $t$ ) with greater or equal to *safetyMargin* ( $s_m$ ) (4.21).

$\forall t \in [missionStart, missionEnd] :$

$$distance(x(t), Occupied(t), t) \geq safetyMargin \quad (4.21)$$

## 4.2 Basic obstacle avoidance problem

Given:

1. Initial system state  $x_0$ , for UAV model (4.3).
2. Mission (4.6) to be executed.
3. Space (4.1), with existing objects (4.2).
4. Sensor system  $\{sensor_1, sensor_2, \dots, sensor_k\}$  with existing sensor fusion function (4.18).
5. Weather information (4.5) during the flight is available.
6. Information sources  $informationSource_1, informationSource_2, \dots, informationSource_l$  containing hard (4.11) and soft space constraints (4.12), with existing data fusion function (4.19).
7. Hard (4.8) and Soft 4.9 flight constraints given by ATM and rules of the air.

Generate *Control input signal*  $u(t)$  to complete mission (4.6) with satisfaction of following conditions:

1. Waypoint passing function condition (4.7).
2. Set safety margin  $s_m$  to *Occupied* space in *KnownWorld*( $time$ ) (4.20) is not breached at any time (4.21).

## 4.3 Initial assumptions

*Initial assumptions* are following:

**Assumption 1.** *Filtered sensor readings are available.*

*SensorObservation* (4.14) for given position, time returns classification of Space which is corresponding with real situation.

**Assumption 2.** *There are no moving obstacles.*

*The initial Space Classification Function* (4.10) is static for all observation times  $t \in (-\infty, \infty)$ . Moreover there are no intruders or adversaries present.

**Assumption 3.** *The movement takes place in the unrestricted airspace .*

**Assumption 4.** *Mission consists of set of reachable waypoints.*

*For specific UAV system (4.3) and Mission (4.6), there exists control signal  $u(t)$  which satisfies Waypoint passing (4.7) criterion and SafetyMargin (4.21) condition.*

**Assumption 5.** *The UAV is moving with constant velocity.*

*For given UAV system (4.3) there is a subset of state  $velocity(t) \subset x(t)$  which contains velocity parameters. Then there exist transformation function  $LinearVelocity(\circ)$  which maps  $velocity(t)$  to linear velocity  $\in \mathbb{R}^1$ . For time  $t$  in missionStart and missionEnd in Mission (4.6) constraint (4.22) with some  $constantVelocity \in \mathbb{R}^+$  holds.*

$$\forall t \in \begin{bmatrix} missionStart, \\ missionEnd \end{bmatrix} : LinearVelocity(velocity(t)) = constantVelocity \quad (4.22)$$

*Note. Initial assumptions 1., 2., 3., 4, and 5. will be relaxed in Incremental problem definition.*

## 4.4 Avoidance requirements

**SAA systems** have following conflicting performance criteria:

1. *Energy efficiency* - minimize energy consumption and flight time.
2. *Trajectory tracking* - stick to the proclaimed trajectory in a mission plan.
3. *Safety* - avoid harm sources during the mission execution.

*Note. Energy efficiency and Trajectory Tracking are an optional criteria, while the safety is mandatory.*

### 4.4.1 Energy efficiency

**Energy efficiency** can be measured by *cost function* (eq. 4.23), consisting from the *cost of flew trajectory* (eq. 4.24) and the *expected reach cost* (eq. 4.25) portions. There are optimalizaiton techniques based on *Reach sets* [53]. The inputs for the *cost function* are:

1. *Time* - current mission time.
2. *Initial state* - UAS state at the beginning of a *mission*.
3. *Applied movements* - list of already executed movements.
4. *Future movements* - list of movements to be applied in future.
5. *Current state* - UAS state at *Time*, with current position and orientation included.
6. *Waypoint* - current goal waypoint.

$$Cost(t, \dots) = costTrajectoryFlew(t, \dots) + expectedReachCost(t, waypoint, \dots) \quad (4.23)$$

**Cost of flew trajectory** (eq. 4.24) from *initial state* to *current state* is calculated as a sum of an energy consumed for each movement with following components:

1. *Direct cost* - cost of consumed energy to execute movement.
2. *Horizontal cost* - portion of direct cost which was used for horizontal steering multiplied by *horizontal penalization*.
3. *Vertical cost* - portion of direct cost which was used for ascending/descending multiplied by *vertical penalization*.

$$costTrajectoryFlew \left( \begin{array}{l} time, \\ initialState, \\ appliedMovements \end{array} \right) = \sum_{\substack{movement \in \\ appliedMovements}} \left( \begin{array}{l} duration.directCost + \\ horizontal(cost, penalization) + \\ vertical(cost, penalization) \end{array} \right) \quad (4.24)$$

**Expected reach cost** (eq. 4.24) is calculated for a *planned trajectory* portion and a *direct waypoint distance* to the *latest future UAS position*. *Cost of planned trajectory* is calculated by same formula as a *cost of flew trajectory* (eq. 4.24), the initial state is replaced with a *current state* and *executed movements* are replaced with *planned movements*.

$$expectedReachCost \left( \begin{array}{l} time, \\ currentState, \\ futureMovements, \\ waypoint \end{array} \right) = \left( \begin{array}{l} distance(futureState, waypoint) + \\ costTrajectoryFlew \left( \begin{array}{l} currentState, \\ futureMovements \end{array} \right) \end{array} \right) \quad (4.25)$$

*Note.* The tuning parameters of cost function are: *Horizontal penalization*  $\in [0, \infty]$  and *Vertical penalization*  $\in [0, \infty]$ , which are used to enhance the outcome of *cost function*.

Following setup of tuning parameters are used in our simulations:

1.  $horizontalPenalization \leq verticalPenalization < \infty$  - in an *uncontrolled airspace*, all kind of maneuvers are allowed. Horizontal maneuvers are cheaper for a plane UAS.
2.  $horizontalPenalization < verticalPenalization = \infty$  - in a *controlled airspace* any kind of horizontal maneuvering must be allowed by UTM.

The tuning parameters are set up  $verticalPenalization \leq horizontalPenalization < \infty$  for *copter* UAS in an *uncontrolled airspace*

#### 4.4.2 Trajectory tracking

**Trajectory Tracking** is crucial parameter for *controlled airspace* and is expected to be important in *upcoming UTM systems*. There is *mission plan* which is compared with *real-time airspace situation* obtained from *UAS Position notifications*. The optimization based on *Reach Set* is given in [54].

**Motivation:** Situation awareness for modern DAA systems is depending on planned trajectory tracking. The main conflict is between *navigaiton precision* and *situation evaluation*. If the planned trajectory is defined for *continuous domain* it takes a lot of effort to calculate collision points.

The discrete domain of *Movement Automaton* (def. 11) can be used as *tool for situation awareness*. The main idea is to use *Movement automaton as predictor for trajectory intersection* [6, 48].

**Movement Automaton trajectory tracking:** There is a *reference trajectory* which is used as comparison by *aviation authority* (ATM,UTM) given as:

$$\text{ReferenceTrajectory} = \{(point_1, time_1), (point_2, time_2), \dots, \dots (point_n, time_n)\} \quad n \in \mathbb{N}^+, point_k \in \mathbb{R}^3 \quad (4.26)$$

*Reference Trajectory* (eq. 4.26) is given as set of *points* in *Global Coordinate System* for given *UAS*, *operational time-frame* and other authority depending properties.

The *movement automaton* is executing *Trajectory(initialState, buffer)* where *buffer* is set of *Movements*. The *buffer* is changing according to following pattern during *mission* time frame:

$$\begin{aligned} & \text{Mission Start:} \\ & \quad buffer = \{\text{plannedMovements}\}, planned = m \\ & \text{During Mission:} \\ & \quad buffer = \{\text{executedMovements}, \text{plannedMovements}\}, \\ & \quad \quad \quad \text{executed} = n, planned = o \\ & \text{Mission End:} \\ & \quad buffer = \{\text{executedMovements}\}, executed = p; \\ & \text{Movement count constraints:} \\ & \quad m, n, o, p \in \mathbb{N}^+, n + o = m, m \leq p \end{aligned} \quad (4.27)$$

At the beginning of the mission (eq. 4.27) the *buffer* is filled with  $m$  movements, the *Trajectory* generated from this *buffer* and *initial state* is *predicted*.

During *mission execution phase*, *buffer* contains *executed movements* and *planned movements*. *Trajectory* created from *initial state* and this *buffer* can be split into:

1. *Executed part* - trajectory portion generated and executed from *executed movements*
2. *Predicted part* - trajectory portion generated as future reference from *planned movements*

After *mission execution* there is only *executed movements*. The trajectory generated from *initial state* and *buffer* is *Executed Trajectory*.

*Note.* The part of the trajectory bounded to the past, the part of the trajectory lies in the future. The strong point of *Movement Automaton* is its ability to work as *predictor* and *trajectory memory* at the same time.

By selecting proper time series  $t_1 \dots t_n$  one can compare future or past segments of trajectory (eq. 4.27) with reference (eq. 4.26)

**Reference Trajectory Deviation** for reference trajectory given by (eq. 4.26) and *Trajectory segment* (Executed/Predicted) (4.27) with existing *State projection function* (eq. 3.21) for *time series* is given as:

$$\text{Deviation} \left( \begin{array}{c} \text{timeSeries}, \\ \text{Trajectory}, \\ \text{Reference} \end{array} \right) = \sum_{\substack{\text{time}_i \in \\ \text{timeSeries}}} \left( \frac{\text{StateProjection}(\text{Trajectory}, \text{time}_i)}{\text{Reference}(\text{time}_i)} - \right)^2 \quad (4.28)$$

*Reference Trajectory Deviation* (eq. 4.28) is designed as discrete *Mean Square Error* function, where the *timeSeries* is set of *times* from *reference trajectory* ( $\text{point}_i, \text{time}_i$ ) pair. The *state projection*.

**Trajectory tracking** is defined as *dual minimization problem* where the *primary objective* is depending on the *airspace type*:

1. *Reference trajectory deviation* (eq. 4.28) in *Controlled Airspace*.
2. *Cost of Trajectory flew* (eq. 4.23) in *Non-controlled Airspace*.

*Trajectory tracking* can be defined as optimization problem (eq. 4.29).

$$\text{Minimize: } \text{costOfTrajectoryFlew} \quad (4.23)$$

$$\text{Minimize: } \text{referenceTrajectoryDeviation} \quad (4.28)$$

Subject to:

$$UAS \text{ Dynamics} \quad (3.23)$$

$$(3.22)$$

$$\begin{aligned} & MovementAutomatonControl && \vdots \\ & && (3.31) \end{aligned} \quad (4.29)$$

$$Mission \quad (4.6)$$

$$KnownWorld(t) \quad (4.20)$$

$$SafetyMargin(t) \quad (4.21)$$

$$HardFlightConstraints \quad (4.8)$$

$$SoftFlightConstraints \quad (4.9)$$

$$HardSpaceConstraints \quad (4.11)$$

$$SoftSpaceConstraints \quad (4.12)$$

The *reference trajectory* is given by *mission* set of *waypoints*. The *UAS* dynamics with specific *Movement Automaton* goal is to fly in *Known World* to keep *Safety Margin* form *Obstacle Space*. The *Obstacle space* is result of *Data fusion* procedure (sec. 4.1.9) combining the *sensor reading*, information sources and constraints.

**Feasible Trajectory** for *tracking problem* (eq. 4.29) is a trajectory which in addition to *basic obstacle problem* (sec. 4.2) keeps deviation from *reference trajectory* under certain threshold:

$$\text{Deviation}(\text{timeSeries}, \text{Trajectory}, \text{Reference}) \leq \text{performanceMargin} \in \mathbb{R}^+ \quad (4.30)$$

Feasible trajectory condition (eq. 4.30) is used as *margin* for airworthiness,s and *Deviation* is used as performance indicator further in this work.

### 4.4.3 Safety

*Safety* is very broad term there are following incidents which can occur and will be discussed in (sec. 6.7.5) *Safety margin* is broad term describing minimal distance to the center of intruder/adversary, surface of obstacle, boundary of protected area.

#### Controlled airspace safety

Safety for *controlled airspace* in given *flight level* is given as list of incidents:

1. *Soft constrained zone breach* - UAS fly to *soft constraint body* or *hard constraint protection zone*, these incidents can happen, and have least avoidance priority.
2. *Hard constrained zone breach* - UAS fly to *hard constraint protection zone*, typical geo-fencing, restricted airspace breaches.
3. *Well clear breach* - UAS fly to *well clear barrel* without impacting other aircraft, via the wake turbulence or other induced physical phenomenons and vice-versa. This type of breaches are allowed in case of inevitable *near miss situations* or *Clash incidents*
4. *Near miss situation*- UAS fly to *near miss cone/barrel*, inducing wake turbulence, or other kind of flight disturbance. This incidents are allowed in very low rate (near  $1 : 10^6$ ).
5. *Clash incident* - UAS body impacts other aircraft hull/propulsion/steering systems and components. This kind of incidents are very severe and it should never happen.

*Note.* It is assumed that flight level in controlled airspace is free of terrain, static ground obstacles, the climb/descent maneuvers are not covered in this work and they are topic for multiple dissertation thesis. For more information refer to ICAO document 4444.

*Relation* for breach of *safety margin* and *body margin* for each object is given in (tab. 4.1):

Violation of:	Safety Margin	Body Margin
Soft constraint	none	Soft constraint zone breach
Hard constraint	Soft constrained zone breach	Hard constrained zone breach
Intruder	Well clear breach, Near Miss situation	Clash incident

Table 4.1: Controlled airspace margins violations incidents.

#### Uncontrolled airspace safety

Safety for *uncontrolled airspace* is applied in *F/G* class of airspace, which is given as airspace between the *ground level* and *other airspace prevalence*.

*Note.* Clarification of controlled/uncontrolled airspace

- *Class F* airspace is given as space between the ground level or water surface and it is constrained up to the 500 feet above ground level.
- *Class C* airspace or *Controlled airspace* is considered starting at first flight level, which is given by Air traffic control zone starting at least at 300 feet from highest ATC zone ground point. It is measured based on Above Sea Level altitude. *This is not a problem in Portugal, because of terrain diversity, but its a huge problem in Netherlands.*
- *Class A* airspace starts at ground level and covers majority of airport infrastructure - this is not a problem, because its modeled as hard constraint, which are unbreakable in non controlled airspace.
- *This is material for chapter 2 airspace classification.* I put it here for explanation.

Safety for *uncontrolled airspace* is given as list of incidents:

1. *Soft constrained zone breach* - UAS fly into *soft constrained zone* or *hard constraint protection zone*, it is allowed to happen on very low rate.
2. *Hard constrained zone breach* - UAS fly into *hard constrained zone*, only airports and critical infrastructure are considered as hard constraints, it is not allowed to happen.
3. *Intruder near miss* - UAS fly into *other aircraft near miss zone*, it is allowed to happen on very low rate in case of other intermediate threats with higher priority.
4. *Intruder clash* - UAS has a contact with other man-made aircraft, it is not allowed to happen.
5. *Adversary clash* - UAS has a contact with other flying object who did not intentionally avoided UAS. (*Bird strike, Differential games,etc..* is out of scope of this thesis).
6. *Structure harm* - UAS fly close to structure and its propulsion can damage/harm structure.
7. *Structure crash* - UAS fly into natural/man-made ground structure (building, tree, human).
8. *Ground harm* - UAS fly close to the ground and its propulsion reflection can impact Ground or UAS.
9. *Ground collision* - UAS collides with ground.

*Relation* for breach of *safety margin* and *body margin* for each object is given in (tab. 4.2):

Violation of:	Safety Margin	Body Margin
---------------	---------------	-------------

Soft constraint	none	Soft constraint zone breach
Hard constraint	Soft constrained zone breach	Hard constrained zone breach
Intruder	Intruder near miss	Intruder clash
Adversary	none	Adversary clash
Structure	Structure harm	Structure crash
Ground	Ground harm	Ground crash

Table 4.2: Controlled airspace margins violations incidents.

## 4.5 (W) Navigation requirements

*Navigation requirements* are not main part of this work, they are used to show variability of the approach for *DAA* requirements:

1. *Constrained space navigation* - support dynamic constraint environment including:
  - a. *Static obstacles*,
  - b. *Moving obstacles*,
  - c. *Hard constraints*,
  - d. *Soft constraints*
2. *Contextual behaviour* - change navigation and decision behaviour based on context:
  - a. *Airspace type* - controlled/uncontrolled,
  - b. *Navigation mode* - Cooperative/Emergency Avoidance
3. *Determinism* - same result for same dataset in finite time. *Threat prioritization* - threats prioritization based on *context*, (tab. 4.1) and (tab. 4.2).
4. *Rule compliance* - compliance with given set of rules based on context (focus on rules of the air).

Requirement	Evaluation metrics
Constrained space navigation	Mission scenario does not have direct path between waypoints, additional borderline cases.
Contextual behaviour	Avoidance system changes behaviour based on the mission and vehicle context.
Determinism	Multiple runs of same non-borderline scenario returns same avoidance paths.
Rule compliance	Rules applied are in compliance with aviation standardization.

Table 4.3: Navigation requirements evaluation metrics.

## 4.6 (W) Incremental problem definition

*UAV system* defined by model (??) is executing *Mission* (??) in *KnownWorld* (??)  $\subset$  (??) *Space*. Solving avoidance problem defined in section ??.

*Constraints for all problems:*

Bla bla



$$\forall \text{Trajectory}(\text{initialState}, \text{Buffer}), \forall \text{HflightConstraints} : \quad (4.31)$$

**Problem 1.** *Basic problem is to navigate trough KnownWorld under assumption that every waypoint in Mission is reachable.*

$$\begin{aligned} \text{KnownWorld} &:= \text{Space}, \forall \text{point} \in \text{KnownWorld} = \text{Free} \cup \text{Occupied} \\ \text{Mission} &:= \forall \text{waypoint} \in \text{Mission} \text{ are reachable} \\ \text{HflightConstraints} &:= \{\text{vehicle dynamic}\} \end{aligned} \quad (4.32)$$

Challenges for problem 1. :

1. Control concept - *method for vehicle control using abstract control representaiton.*
2. Navigation framework - *techniques and constraints to execute mission with emphasis on SafetyMargin(??) to Occupied (??) set.*

**Problem 2.** *Known world evolution, in addition to Basic problem (pr. 1) The KnownWorld is fed trough SensorFusion function which is joining LiDAR scanning into Free(t), Occupied(t), and, Unknown(t) sets in real time t.*

$$\begin{aligned} \text{KnownWorld} &:= \text{SensorFusion}(t) \forall \text{point} \in \text{KnownWorld}(t) \\ &= \text{Free}(t) \cup \text{Occupied}(t) \cup \text{Unknown}(t) \\ \text{Mission} &:= \forall \text{waypoint} \in \text{Mission} \text{ are reachable} \\ \text{Sensors} &:= \{\text{LiDAR}\} \\ \text{SensorFusion} &:= \{\text{Simple joint sets (??)}\} \\ \text{HflightConstraints} &:= \{\text{vehicle dynamic}\} \end{aligned} \quad (4.33)$$

Challenges for problem 2. :

1. Evolving sets representation - *continuous LiDAR scanning are imposing problem of abstract data representation requiring introduction of some sort of Grid representation.*
2. Evolving navigation - *imposes problem of path re-planning including Decision point determination, such as point when next decision should be made to guarantee return path in Known world.*

**Problem 3.** *Intruder detection in addition to Known world evolution (pr.2) the ADS-B sensor is introduced into Sensors array, this imposes HardConstraint of Flight corridor for detected intruders impacting the evolution of Free(t) and Occupied(t) sets significantly.*

$$\begin{aligned}
KnownWorld &:= SensorFusion(t) \forall point \in KnownWorld(t) \\
&= Free(t) \cup Occupied(t) \cup Unknown(t) \\
Mission &:= \forall waypoint \in Mission \text{ are reachable} \\
Sensors &:= \{LiDAR, ADS - B\} \\
SensorFusion &:= \{\text{Advanced joint sets}\} \\
HFlightConstraints &:= \{\text{vehicle dynamic}\} \\
HardConstraints &:= \{\text{intruder corridors}\}
\end{aligned} \tag{4.34}$$

Challenges for problem 3. :

1. Moving obstacles - *continuous ADS-B updates reveals adversaries which can significantly impact the situation along planned path.*
2. Advanced joint sets - *the Known world is impacted two sources of information, therefore SensorFusion needs to be adapted to feed existing unified grid with relevant data. The information priority sensor fusion needs to be introduced.*

**Problem 4.** *Terrain map, in addition to Intruder Detection (pr. 3) TerrainMap information source is introduced. This change impacts KnowWord by changing the final set distribution to DatFusion. For DataFusion simple data fusion from one information source is used. The terrain from TerrainMap extends HardConstraints.*

$$\begin{aligned}
KnownWorld &:= DataFusion(t) \forall point \in KnownWorld(t) \\
&= Free(t) \cup Occupied(t) \cup Unknown(t) \\
Mission &:= \forall waypoint \in Mission \text{ are reachable} \\
Sensors &:= \{LiDAR, ADS - B\} \\
SensorFusion &:= \{\text{Advanced joint sets}\} \\
InformationSources &:= \{\text{TerrainMap}\} \\
DataFusion &:= \{\text{Simple data fusion}\} \\
HFlightConstraints &:= \{\text{vehicle dynamic}\} \\
HardConstraints &:= \{\text{intruder corridors, terrain}\}
\end{aligned} \tag{4.35}$$

Challenges for problem 4. :

1. Data fusion - *introduce post-SensorFusion processing Free(t), Occupied(t), Unknown(t) sets.*
2. Terrain map clustering - *fetching only relevant TerrainMap space portion to KnownWorld.*

**Problem 5.** *Static restrictions, in addition to TerrainMap problem (pr. 4) the InformationSources are expanded by static restriction sources:*

1. ObstacleDatabase - *database containing notable landmarks, buildings, structures, with well defined protection zones.*
2. FlightRestrictions - *database containing ATM flight restrictions in non-segregated airspace for UAV relevant airspace categories.*

*This change impacts DataFusion by splitting Free(t) set into Free(t) and Restricted(t) disjoint sets. In addition SoftConstraints are introduced which contain restricted areas from relevant information sources.*

$$\begin{aligned}
KnownWorld &:= DataFusion(t) \forall point \in KnownWorld(t) \\
&= Free(t) \cup Occupied(t) \cup Unknown(t) \cup Restricted(t) \\
Mission &:= \forall waypoint \in Mission \text{ are reachable} \\
Sensors &:= \{LiDAR, ADS - B\} \\
SensorFusion &:= \{\text{Advanced joint sets}\} \\
InformationSources &:= \{\text{TerrainMap, ObstacleDatabase, FlightRestriction}\} \\
DataFusion &:= \{\text{Advanced data fusion}\} \\
HFlightConstraints &:= \{\text{vehicle dynamic}\} \\
HardConstraints &:= \{\text{intruder corridors, terrain, obstacles}\} \\
Softconstraints &:= \{\text{protection zones}\}
\end{aligned} \tag{4.36}$$

Challenges for problem 5. :

1. Restricted zones - Obstacle map introduces protection zones around notable landmarks, there is hierarchy given by law, other source of restrictions is Flight restrictions, which also has hierarchy.
2. Navigation improvement - Extension to problem is given by virtual obstacles form Restricted zones. These restrictions can be broken under certain condition, the rule engine must consider implementation of legal measurements to allow such navigation.

**Problem 6.** Dynamic restrictions in addition to Static restrictions (pr. 5), the Weather as information source is introduced. Soft constraints are extended by medium level dangerous zones from weather map. Hard constraints are expanded by protection zones where the weather conditions are harsh. Overall Weather constraints are dynamic and changing position and shape over mission time. Modern weather systems can provide streamline overview of weather situation.

$$\begin{aligned}
KnownWorld &:= DataFusion(t) \forall point \in KnownWorld(t) \\
&= Free(t) \cup Occupied(t) \cup Unknown(t) \cup Restricted(t) \\
Mission &:= \forall waypoint \in Mission \text{ are reachable} \\
Sensors &:= \{LiDAR, ADS - B\} \\
SensorFusion &:= \{\text{Advanced joint sets}\} \\
InformationSources &:= \{\text{TerrainMap, ObstacleDatabase, FlightRestriction, Weather}\} \\
DataFusion &:= \{\text{Advanced data fusion}\} \\
HFlightConstraints &:= \{\text{vehicle dynamic}\} \\
HardConstraints &:= \{\text{intruder corridors, terrain, obstacles, protection zones}\} \\
Softconstraints &:= \{\text{protection zones}\}
\end{aligned} \tag{4.37}$$

Challenges for problem 6. :

1. Changing Restricted zones - *Weather is changing with time, that means the weather restriction zones are changing their position, shape and severity over time.*
2. Sensor impact - *Weather severely impacts functionality of some sensor, at this point scalable avoidance method to address this fact needs to be introduced*
3. Navigation algorithm - *Restricted areas from Weather are changing their severity. This imposes dynamic changes to maneuverability and virtual obstacles priority.*

**Problem 7.** *Rules of the air, in addition to Dynamic restrictions (pr. 6), Rules of the air framework introduction, inducing new SFlightConstraints including air-spaces and rules of air impact on control mechanism.*

$$\begin{aligned}
 KnownWorld &:= DataFusion(t) \forall point \in KnownWorld(t) \\
 &= Free(t) \cup Occupied(t) \cup Unknown(t) \cup Restricted(t) \\
 Mission &:= \forall waypoint \in Mission \text{ are reachable} \\
 Sensors &:= \{LiDAR, ADS - B\} \\
 SensorFusion &:= \{\text{Advanced joint sets}\} \\
 InformationSources &:= \{\text{TerrainMap}, \text{ObstacleDatabase}, \\
 &\quad \text{FlightRestriction}, \text{Weather}\} \\
 DataFusion &:= \{\text{Advanced data fusion}\} \\
 HFlightConstraints &:= \{\text{vehicle dynamic}\} \\
 SFlightConstraints &:= \{\text{airspaces, rules of the air}\} \\
 HardConstraints &:= \{\text{intruder corridors, terrain, obstacles, protection zones}\} \\
 Softconstraints &:= \{\text{protection zones}\}
 \end{aligned} \tag{4.38}$$

Challenges for problem 7. :

1. Soft constraints on control - *Control mechanism which follows constraints on control to execute maneuvers is biased by implementation of situation based maneuvers.*

# Chapter 5

## (R) State of art

This chapter is introducing notable works and concepts of fellow *researchers* in the field of *control theory*, *software engineering* and other essential fields used in *Detect and Avoid* systems.

**Conceptual scheme:** The overall concept of *Detect and Avoid Framework* (fig. 5.1) is taking architecture from LSTS tool chain [55, 56]. The UAS part is based on *LSTS Dune* and it can be easily integrated in future.

1. *Continuous control* - is not solved in this work, its kept in scheme for reference.
2. *Discrete control* - it bridges event based *Detect and Avoid* core functionality with *Continuous control*. Its covered by *Movement Automaton* (sec. 5.1).
3. *Event based control* - covers major functionalists:
  - a. *Sensor (Data) fusion* - the main feed of information, implementation of *sensor fusion* (sec. 4.1.8) and *data fusion* (sec. 4.1.9) contributing the avoidance events, introduced in (sec. 5.2).
  - b. *Mission plan* - feeding actual goal and objectives to *Navigation Algorithm* (sec. 5.3) and obeying *UTM directives* (sec. 5.6).
  - c. *Avoidance Grid* - using mainly *Approximation of Reachable Space* (sec. 5.4) in *Avoidance Maneuver Estimation*.
  - d. *Rule engine* - enforcing UTM directives (sec. 5.6).

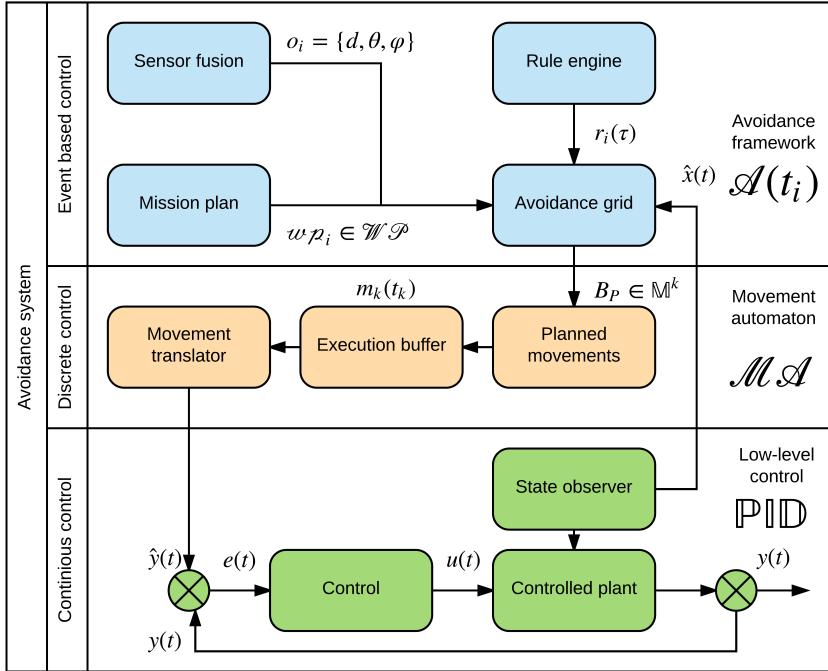


Figure 5.1: Obstacle avoidance based on Reach sets concept.

## 5.1 (R) Movement Automaton

**Idea:** The key idea is to create *interface* between *controlled plant* (UAV) and *Avoidance Algorithm* to ensure *Concept Reusability* at maximum degree. The concept is following:

1. *Interface consumes* discrete command chain and guides UAS along *desired trajectory*.
2. *Interface* can be used to *predict trajectory* based on *initial state* and future command chaining.

Frazolli provided the concept of *Movement Automaton* (def. 11) a specialized type of *Hybrid Automaton* (eq. 3.7), the concept is taken from his works [6, 48]. Other aspects and similarities are discussed over this chapter.

**Architecture:** The Movement Automaton can be seen as a consistent hierarchical abstraction of the continuous dynamics, in the sense outlined in [57]: *Any sequence of movement primitives generated by the Movement Automaton results by construction in a trajectory which is executable by the full continuous system. We will give a deeper meaning to hierarchical consistency.* The implementation of our movement automaton is given in (fig.5.1).

**Optimal Path Generation:** If the maneuvers are instantaneous (i.e. the UAS can transition instantaneously between two different trim trajectories), Reduction of stronger results obtained by Dubins [58] and Reeds [59] concerning optimal paths for kinematic cars on the plane (see also [60]).

**Controllability:** The systems controlled by Movement Automaton (as in [61]), is controllable according to our definition, even though it is not small-time controllable [62].

**Other Properties:** The other properties of movement automaton, like *Stability*, *R robustness* and other important control properties are proven in [6].

**Example:** The *example* is given in (fig. 5.2). The *States* (Barrels) are connected by *Transitions* (green arrows).

*Hover* is neutral and *initial state*, in this place the UAS stays on place and maintains altitude.

*Forward flight* is when *UAS* is flying in frontal direction with constant speed. The speed-up and slow-down is incorporated in *Transition* between *Hover* and *Forward flight* states and its takes some time to execute. *Transitions* between Turning states and *Flight forward* state are almost instant.

*Steady turn left/right* is when *UAS* is flying in frontal direction and starts steady turning left or right.

*Note.* UAS in (fig. 5.2) ignores the vertical maneuvering and it is expected to fly on horizontal plane.

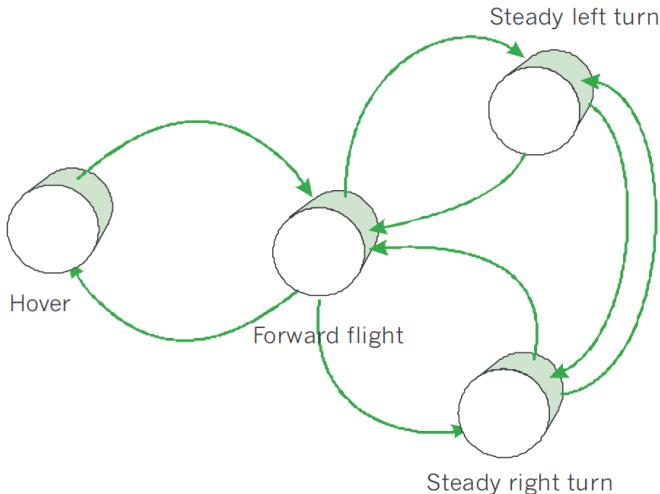


Figure 5.2: Movement Automaton for Copter UAS [6].

**Used concepts:** The movement automaton is essential part of this work. Supporting the idea, that *Obstacle avoidance framework* can be platform independent.

The *Movement Automaton* is used as is (sec. 3.4.3). The implementation is described in (sec. 6.3). The testing configuration was given in (tab. 7.2).

## 5.2 (R) Sensor (Data) Fusion - Input Interface

**Idea:** There is *Need for abstract representation* of *operational space*, which is independent of used sensors, technologies, information sources. The universal obstacle avoidance system should have *portability property*. Our previous work *Obstacle avoidance framework based on reach sets* [49] have introduced similar concept of *control interface*.

The original concept was using cell status interpretation, which was hardwired to LiDAR technology. The new demand is to incorporate concepts of *visibility*, *reachability* and *obstacle probability*. The base methodst for *Statistical Sensor Fusion* were outlined in [63].

**Key Concept:** *Data fusion interface* (sec. 4.1.9) - interface to fuse sense data from various online, offline, cooperative, non-cooperative sources into single scalable space and trajectory evaluation procedure.

**Related work:** UAS specific sensor fusion has been proposed by Ramsay in [23]. *Next generation avoidance concept* [64] is introducing concept of higher level sensor fusion called *data fusion*.

The uncertainty and properties in *Remotely Piloted Systems* have been discussed in [65]. The work provided concept of various performance ratings like visibility and obstacle rating, more details have been given in [66]. This ratings were modeled only for operator decision making [67], results are usable for automated decision making and space assessment.

*Probabilistic trajectory assessment* has been firstly proposed in [68] where trajectory was tracking and predicting *safety properties* along.

*Game theory* viewpoint is firstly used in [69]. Probabilistic path planning using safety zones similar to cell classification of this work have been used in [70].

Probabilistic path search similar to our reach set representation using rapidly exploring path trees have been used in [71, 72]. Relationship between classic grid search and probabilistic lattice search have been established in [73]. A probabilistic approach for trajectory estimation via reduced lattice search is known from 1986 from work of Gessel [74] lattice paths were enumerated via movement sequences and similar technique is used in our reach set estimation method using movement automaton. Pruning methods comparison and complexity can be found in [75].

Overall concepts of probabilistic sets have been given by Hirota in [76]. Free flight safety rating similar to our reachability concept have been presented in [77].

### Shortcomings:

1. *Hierarchical calculation* - there is need to calculate *avoidance trajectory* for incremental constraint applications. For example:
  - a. Calculate *Minimal escape path* covering physical obstacles and intruders.
  - b. Apply next level of constraints, like airspace restrictions and some virtual constraints. Then calculate path if exists, continue.
  - c. Apply nice to have constraints, like non lethal weather, recalculate path.
2. *Source Reliability Evaluation* - reliability evaluation is empirical process usually done by hand. The result aggregation is not standardized. There can be multiple sources of same rating, for example visibility, which needs to be aggregated into one.
3. *Ambiguous rating definition* - There is multiple definitions especially for *Reachability rating* in works [71, 72, 74].

**Improvements in Our Work:** *Hierarchical calculation* is addressed in *Mission Control run* (sec: 6.7.4) where threats are hierarchically applied based on *severity*.

*Source reliability evaluation* is addressed in *Static Obstacles* (sec. 6.4) and *Moving Obstacles* 6.5). The main rating for *Detected obstacle*, *Map Obstacle* and *Visibility* of space are established there.

*Clear rating definition* - the *Reachability* of space portion and *Safety* rating for trajectory are established in *Avoidance Grid Run* (sec. 6.7.3)

## 5.3 (R) Navigation Algorithms

**Idea:** The basic idea is to provide hierarchical *navigation frame* with *some optimal path search capabilities*.

**Standard Navigation:** The standard navigation is given as *expected cost optimization problem* for *future cost function* (eq. 4.25). The key concept of navigation algorithm was fully taken from [78]. The decision was made based on navigation survey [79]. The *descent* for landing is out of scope in this work, can be found in [80]. The navigation principle is roughly described in (sec. 6.7.4).

**Maze Solving Capabilities:** The *maze solving capability* is usable in *controlled airspace* where 2D maze solving algorithms are applicable. The notable implementation was for *micro mouse robot* based on right hand rule [81]. Flood fill algorithm is partially usable for 3D environment [82]. The application of *maze solving* was given in case study [83].

**Hybrid Automaton Path Planning:** A hybrid automaton path planning based on *A\** algorithm was given by Richards in [84]. The key idea was to use *hybrid automaton* (eq. 3.7) as a reference generator. This idea was taken and formulated as *Movement Automaton Predictor mode*.

The similar idea where *potential fields* were used as *intruder model* and path was re-planned based on events is given in [85].

**Mode Switch:** The *Mode Switch Control* idea has been presented in [86]. There were definition of behavioural switch between:

1. *Navigation Mode* - navigation control and behaviour was used.
2. *Task Specific Mode* - mode specific for tasks, authors were using modes for search and rescue.

This concept will be reused, the *Task specific mode* will be *Emergency Avoidance Mode* in Our case. The triggering events and switch conditions will be defined in (sec. 6.7.4).

**Used concepts:** The *Following concepts* were used in navigation loop:

1. *Standard navigation* taken from [78] minor implementation changes using offline optimization. The purpose of navigation loop is to bring us closest to the waypoint, if its reachable. Navigation example (sec. 7.4.3).

2. *Maze solving capabilities* partially taken as secondary functionality based on [82]. The purpose is the *looping prevention*. The example was given in (sec. 7.3.3) .
3. *Mode switch* partially taken as main feature from [86], the triggering events were identified and defined by author and can be found over (chapter 6).

## 5.4 (R) Reach Set Estimation

**Idea:** The basic idea for *Discrete Reach set Estimation method* is taken from [7]. The focus of their work is to generate paths that are kinematically feasible. Path following controllers in order to find techniques to stabilize the system around these paths [87, 88].

Lattice-based planners have been deployed with great success on several robotic platforms [89, 90, 91, 92, 93]. However, a problem with lattice-based approaches is the exponential complexity in the dimension of the state space which can limit the use for more complicated models.

The optimization problem was solved real-time by *Avocado solver* [94].

**Example:** The *example of movement lattice* is given in (fig. 5.3). Truck (black rectangle) is towing Trailer (red rectangle). The *state* have only one *reach set impacting variable - trailer displacement*. When trailer displacement is  $0^\circ$  (fig. 5.3a) the lattice representation of *Reach Set* is different than in case of small left/right tilt (fig. 5.3b).

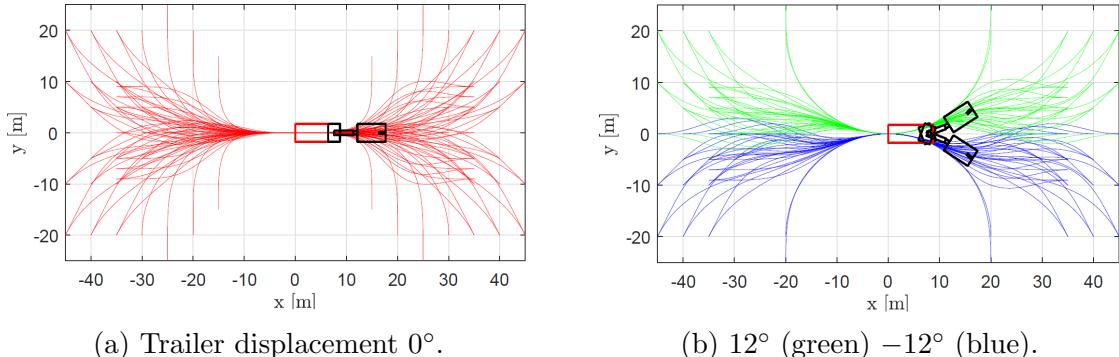


Figure 5.3: *Movement set primitives* for Lattice Based Movement Planning. [7].

**Benefits:** Presented method of *Lattice Search* is de-facto *Reduced Reach Set Approximation* in open space for *Truck-Trailer* system.

The idea of *Movement primitives* is very close to *Movement Automaton* (def. 11), which can be used as *control interface* effectively.

The *Constraints* of obstacle set in *Known world* (sec. 4.1.10) are supported to some degree.

**Shortcomings:** There are following shortcomings which were identified in this approach:

1. *Limited system dimension* - given method works in *real time* only if dimension of *system state space* does not exceed  $4^{th}$  rank.

2. *Real time optimization* - real time optimization is main cause of *limited system dimension*. If the decision time can be discrete (which movement automaton enforces) then offline optimization can be used.
3. *Continuous space disparity* - the example (fig. 5.3) shows there are member variables of *State Space* which significantly impacts the shape of lattice (reach set estimation). This is not a problem in real-time environment. The discretization of *Time domain* raises this as a shortcoming.
4. *Trajectory Tracking* - approach generates *Continuous Domain* reference signal. For *Discrete Domain* it is necessary to address this issue.

**Improvements in Our Work:** *Limited system dimension* - the discretization due the higher system dimension and increased maneuver complexity goes hand-in-hand with *pre-calculation* of the *Reach Set*. This shortcoming is addressed in (sec. 6.6.2).

*Real time optimization* - replaced by *Discrete offline optimization problem*. The *general cost function* is given in (eq. 4.25). The optimization problem solved in this work is defined in (eq. 4.29).

*Continuous space disparity* - The *pre-calculated reach set estimation* can be valid with small *marginal error* for some region in *system state space*. The dynamic method for state space segmentation can be used [95]. This aspect is not addressed in this work, because it is strongly depending on the system behind movement automaton.

*Trajectory Tracking* - The *movement automaton* (def. 11) in Control Mode can be used to track reference trajectory in form of *Movement Buffer*(def. 9). Other option is to use *thick waypoint trajectory tracking for UAV* like in [96] or [97]. The work will use only *Movement Automaton* as controller/predictor.

## 5.5 (R) Testing Framework

**Idea:** *Reuse LSTS toolchain architecture* for DAA testing framework.

**LSTS Toolchain:** Software architecture used in modern unmanned aerial vehicles must be system independent and scalable. Writing own control software for unmanned aerial vehicle and ground station is unthinkable in current state of art. Most notable framework for unmanned aerial vehicle development is LSTS tool chain from University of Porto [98]. This tool chain is widespread in other universities and multiple independent applications are based on it [99].

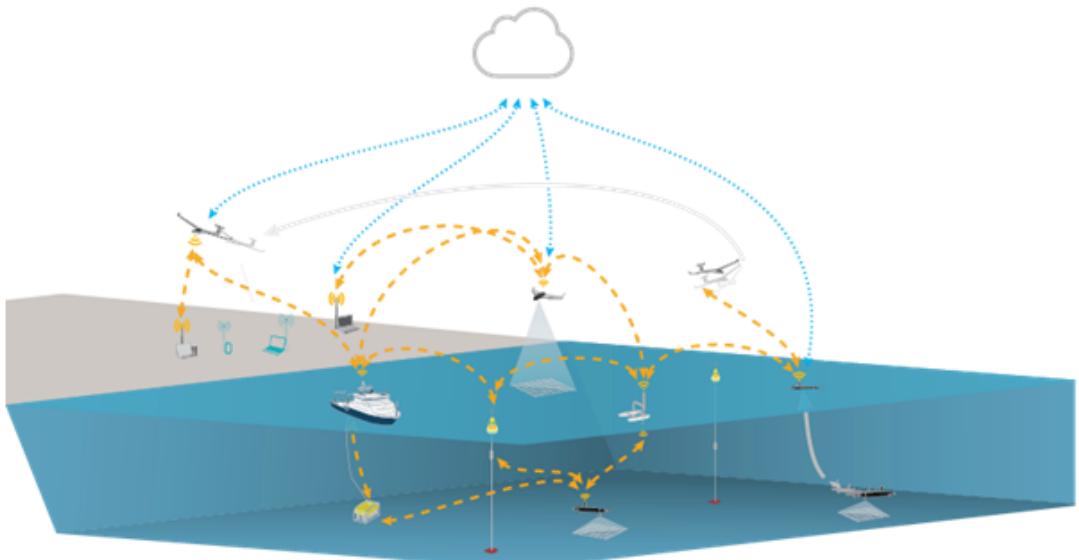


Figure 5.4: Example of LSTS toolchain deployment in real environment [8]

Example of software architecture implementation is shown in figure 5.4 LSTS HUB (cloud iconography) is collecting all important data from currently executing missions. Data are transferred via REST API (dotted blue lines) to HUB. Commanded vehicles can be unmanned copters, planes, ships, submarines or floating sensors compounds. Each vehicle has installed DUNE which is responsible for vehicle command and ground control station communication. Deployment, range of command and status messages can vary. Ground station can be implemented on personal computer (any platform) in NEPTUS environment or mobile platform (Android OS) in ACCU environment. Ground station environments are customizable and open source. Layout of ground station can be customized to need of current mission via plugins or console configuration. Vehicles and ground stations are communicating via IMC protocol (orange dashed lines). Communication channel is platform independent.

*Glued* is a Debian-based open source operating system which was initially released in 2010. Over the years it has become fairly widespread and been provided continuous updates. Meanwhile, a notable development community has emerged, where advice and support can be received as more applications are investigated. Some of the merits of the operating system are its reliability and customizability. Operation system is developed for unmanned autonomous vehicles. It is widely used in air, sea and land vehicles. Customization allows the operating system to be tailored to the specific usage. For this purpose, this includes stripping off functionality which is not required, thus releasing resources to be focused on the essential tasks. *Glued* is the operating system favored by the Beaglebone development community, and thus there exist considerable amounts of helpful documentation for this set-up.

*DUNE: Unified Navigational Environment* is an on-board software solution for unmanned vehicles. Multiple applications already run on this software. Almost any extension can be added to this to this environment. The software solution provides means for interacting with the connected components as well as control, navigation, supervision and plan execution. It is both CPU architecture independent and OS independent. It is written in C++ and developed by LSTS: Underwater Systems and Technology Laboratory.

*Neptus* [8, 100, 101] is a command and control software operated from a ground station.

It is designed to operate well together with Dune and was also developed by LSTS. Neptus provides tools for remotely monitoring UAVs and assigning plans and commands in real-time missions, supporting multiple connections dynamically. Furthermore, it provides possibilities for both simulating missions and reviewing previously performed operations. This is presented in a customizable interface equipped with map layers and control panels. It is written in Java and available for both Windows and Linux systems.

The *Inter-Module Communication* [102] (IMC) protocol was developed by LSTS to provide reliable communication between the systems. The protocol is message-oriented, such that messages can be sent and received from a bus which connects independently run threads or systems. Thus it functions as a method of communication between tasks internally in Dune, and can also be passed to and from other vehicles or computers running Dune or Neptus. IMC is platform independent at multiple messages have been already developed and supported by both DUNE and Neptus (around 400 status/command messages).

*HUB* is communication hub for data dissemination and situation awareness. This module is responsible for complex mission execution, when cooperation of multiple pilots/vehicles is required. This module can be imagined as airport tower center, which is monitoring all flights in airport (operation site) area. This tool implements REST API for communication between various ground stations and operating vehicles.

**Movement Automaton Control** Marconi used *hybrid automaton* with forced *State Switch* via buffer [103]. The key concept is that *Automata* state switch is forced as *external source command*. Our *Movement Automaton* implementation know only *forced state switch* like in [48].

**Used concepts:** Most of the architecture was re-used in our approach, the concept of *Rule engine* (sec. 6.9) was introduced to cover missing *UTM* related functionality. The implementation in Matlab was influenced by Alessandetti works [104, 105]. The other aircraft dynamic and control related concepts were taken from [106].

## 5.6 (R) UTM Services

**Idea:** Take the Airbus UTM concept [107] combine it with *EUROCONTROL* concept [108] to obtain legal framework. *Provide conflict resolution functionality* for *Controlled Airspace*:

1. *Collision Detection* - define minimal required functionality for collision detection.
2. *Collision Resolution* - implement *Rules of the Air* [109].

The *implementation* of *UTM services* described in (sec. 2.4) is given in (sec. 6.8).

**UTM Operation Modes:** *defined in* [107] are following:

1. *Free route* (fig. 5.5a) is when aircraft can fly any path, so long as their planned path is coordinated with and de-conflicted from the paths of other aircraft by a traffic manager and approved based on calculated risk. Free routing is being introduced worldwide, such as free route airspace. This allows commercial flights to freely plan their route through participating sectors during cruise. There is less freedom for an aircraft in this situation than in basic flight, since its request may be rejected.

2. *Corridors* (fig. 5.5b) are defined volumes in space, useful for managing airspace in high demand or to manage traffic flow and separation. Coordination is necessary to ensure safety in this airspace. A corridor may take on many different shapes. Aircraft are often guided inside corridors using predetermined routes analogous to approach procedures used worldwide today.
3. *Fixed route* (fig. 5.5c) are used to ensure safety when there is high traffic density or in any location where structure is required to ensure safe operations. These routes could be constructed or modified dynamically based on calculated risk. The most restrictive version is a predetermined path, where the only variable is when an aircraft is at a specific point in the path.

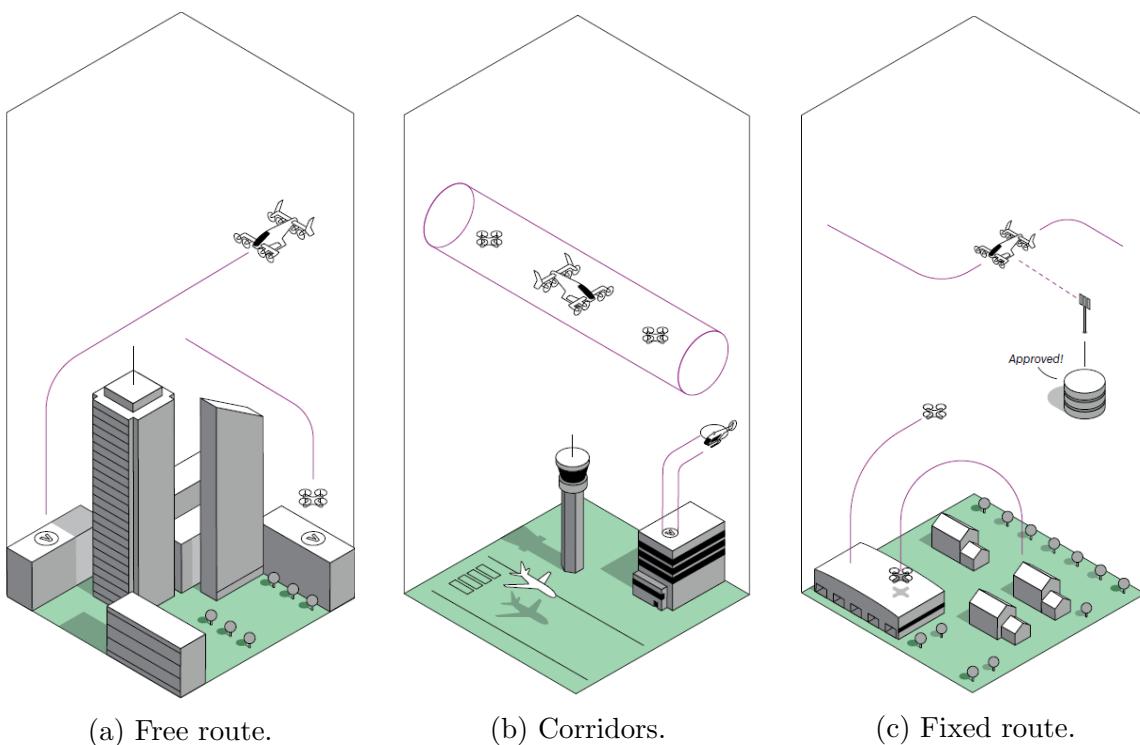


Figure 5.5: UTM Operation Modes.

**Used Concepts:** The implementation of our UTM services is focused on *Free route mode* (fig. 5.5a). The *Corridors* and *Fixed routes* are just additional *space/time constraints*.

# Chapter 6

## (W) Approach

### 6.1 (W) Overview

- Clarification where are we at the *Obstacle Avoidance*.
- Assumption that *Preemptive avoidance* could fail.
- *Basic idea* is to cover *Reactive avoidance* by creating discretization of *Avoidance space, Reach set, Obstacle space, Intruder space ...*
- *Navigation concept* can be reused in *Event based avoidance* to address *Weather, Rules of the air, Geo-fencing*.
- mention discretization, general algorithm, UTM, ...
- Aircraft conflict prediction mentioned in [110].

Preemptive Avoidance	Event Avoidance	Reactive Avoidance
<ul style="list-style-type: none"><li>• Well defined Waypoints</li><li>• Free Space guarantee</li><li>• Preflight preparation</li><li>• Legal compliance</li></ul>	<p><u>Targets of avoidance:</u></p> <ul style="list-style-type: none"><li>• Cooperative intruders (Rules of the air)</li><li>• Bad weather</li><li>• Geofencing</li></ul>	<p><u>Targets of avoidance:</u></p> <ul style="list-style-type: none"><li>• Non-Cooperative intruders</li><li>• Terrain</li><li>• Other physical obstacles</li></ul>

Figure 6.1: Avoidance levels based on reaction time.

### 6.2 (W) Space Discretization - Avoidance Grid

UAS operates in some space which is given:

- Sensor ranges
- UAS maneuverability
- Computation power
- Airworthiness requirements (IFR/VFR)
- intended reaction times for event-based/emergency situations

In this section discuss:

- Proposed clustering method near thick, far loose , [111].
- Planar grid vs Euclidian homogenous grid
- Trajectory intersection (mixed-integer) problem
- Avoidance grid scaling based on previously mentioned factors
- Space (cell) properties:
  - Visibility - how is space visible
  - Occupancy - is space occupied/free ?
  - Reachibility - is there safe way to given point ?

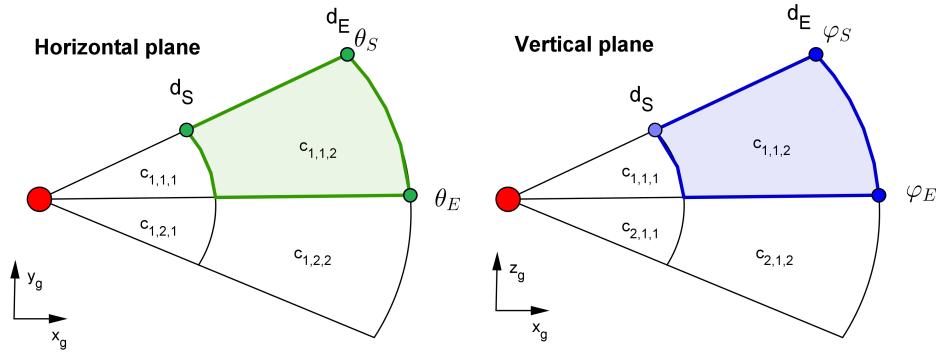


Figure 6.2: *Cell of Avoidance Grid with Boundaries.*

## 6.3 (W) UAS Model and Control

Model and Movement Automaton initialization there.

## 6.4 (W) Static Obstacles and Constraints

Fast overview of obstacles and space constraints which position is static relative to mission time-frame:

- Static obstacles
- Geo-fencing areas (constraints)
- Long term bad weather areas (constraints)
- *Constraint based path search and obstacle modeling* is summarized in [112].

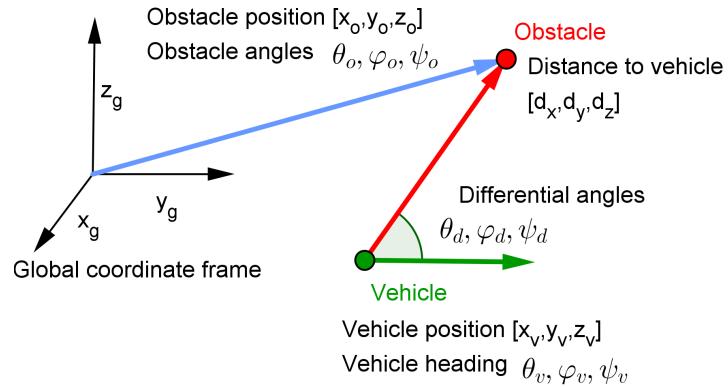


Figure 6.3: Obstacle, UAS local, and UAS global coordinate frame relations.

#### 6.4.1 (W) Detected obstacles

Define certainty of obstacles (*Sensor fusion*) (reuse some content from Linkoping work)

### 6.4.2 (W) Map obstacles

Define certainty of obstacles from map (*Data fusion*), (reuse some content from Linkoping work)

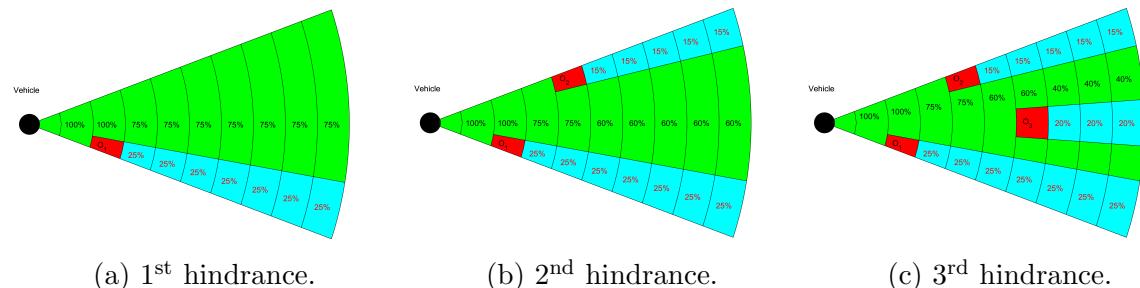


Figure 6.4: Obstacle hindrance impact on visibility in *Avoidance Grid Slice*.

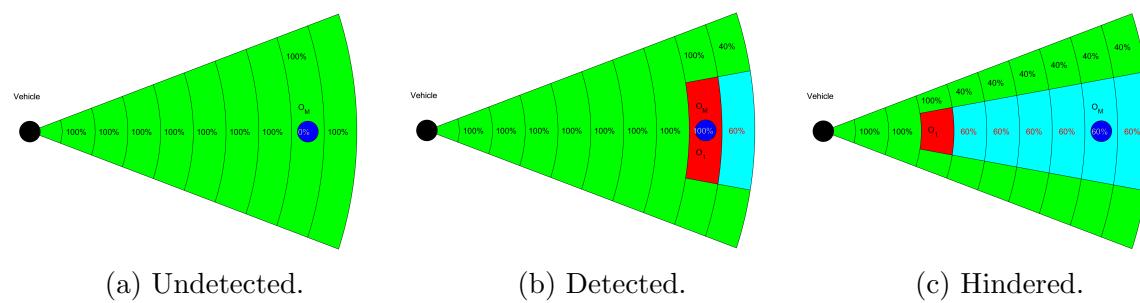


Figure 6.5: Map obstacle states after *Data fusion*.

### 6.4.3 (W) Virtual constraints

Introduction of virtual constraints, their representation as convex polygon areas with altitude boundaries (new text)

## 6.5 (W) Moving obstacles and intruders

Fast overview of obstacles and space constraints which position is dynamic relative to mission or avoidance time frame:

- Non-cooperative intruders
- Short term bad weather areas (constraints)
- The models are based on *analytical geometry* for *cones and ellipsoids* taken from [113].
- The cell boundary was approximated as set of points, the intersection objects were discretized.
- Closest point problem [114] was solved by application of method [115]
- Moving obstacles were addressed in [116].

### 6.5.1 (W) Intruder behaviour prediction

Prediction of impact area in *avoidance grid* based on available parameters:

- Intruder/Constraint position
- Intruder/Constraint velocity/heading
- Intruder movement certainty (Horizontal/Vertical maneuverability)

### 6.5.2 (W) Linear intersection model

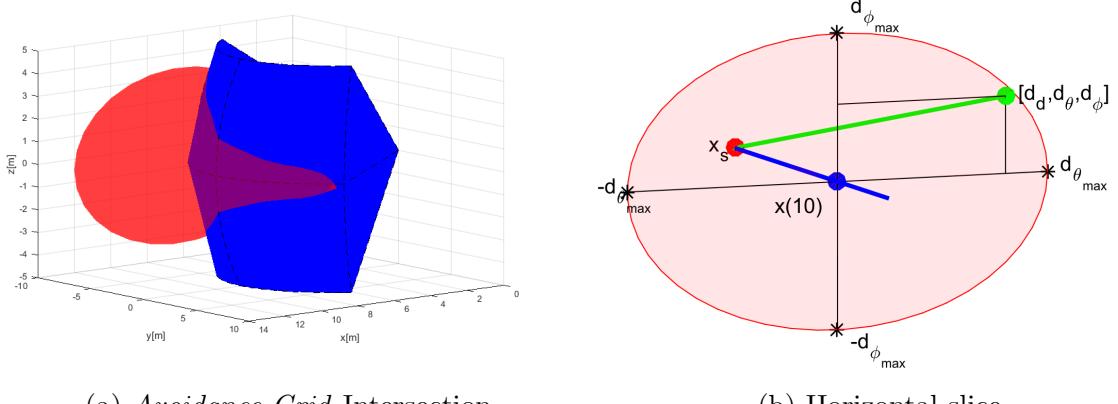
Prediction of *impact area*, based on intruder position/velocity, usable for *small body volume relative to average cell size*.

### 6.5.3 (W) Body-volume intersection

Prediction of *impact area* based on intruder position/velocity/estimated body volume, mandatory for *large scale objects* and *Short term bad weather areas*.

### 6.5.4 (W) Maneuverability uncertainty intersection

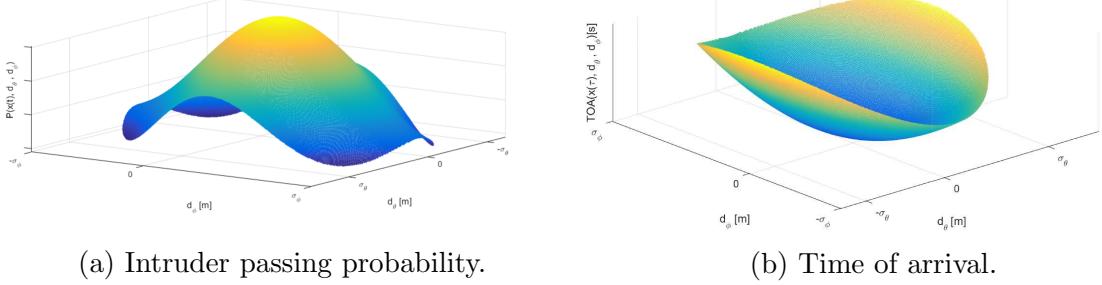
Prediction of *impact area* based on intruder position/velocity/estimated maneuverability, usable for *expected adversarial behaviour*. Intersection polytope volume calculation [117]



(a) *Avoidance Grid* Intersection.

(b) Horizontal slice.

Figure 6.6: Elliptical cone represented maneuverability approximation.



(a) Intruder passing probability.

(b) Time of arrival.

Figure 6.7: Properties for one elliptic cone slice.

## 6.6 (R) Reach Set Approximation

**Motivation:** *Reach set* is strong tool for *Obstacle Avoidance* because it contains all possible *avoidance maneuvers* in set. The current implementation have following flaws:

1. *Realistic approximation* - *nonlinear systems* or *heavily constrained systems* can not be approximated well by *continuous-time Reach Sets*.
2. *Non Deterministic calculations* - continuous-time *Reach Set* contains infinite possibilities for *avoidance maneuvers*, the SAA system demands conflict resolution in finite time.
3. *Property binding* - binding related properties seems problematic, because *continuous-time reach sets* does not have unique identifier of maneuver, trajectory nor segment.

**Proposed Solution Features:** Our Reach set Estimation method will provide following features:

1. *System Control Interface* - implemented via *Movement Automaton*, requiring only *discrete command chain* to approximate system behaviour.
2. *Deterministic Calculation* - finite number of elements in *Reach set* will enable *scalable* calculation.

3. *Property binding* - approximation of Reach set as a set of trajectories, each trajectory can be split into finite number of segments. Each element will have unique identifier enabling both-side property binding.
4. *Behaviour encoding* - some specific behaviour, like horizontal/vertical separation, or maneuver shape can be encoded into *Reach Set*.

**Discretization of Reach set:** There is a need for a discrete finite *Reach Set approximation* to enable *Avoidance Strategy Evaluation* in finite time. Replacing *Continuous Control Set Inputs*( $t$ ) by *Movement Automaton* is feasible:

**Definition 14** (Reach set Approximation by Movement Automaton). A trajectory (*def. 10*) for system state  $= f(\text{time}, \text{state}, \text{input})$  under control of the movement automaton  $\mathcal{MA}$  is given as execution of movement buffer (*def. 9*) with initial state of system  $\text{state}_0$ . Therefore notation  $\text{Trajectory}(\text{state}_0, \text{buffer})$  is used.

**The Complete Reach Set** (6.1) for system with initial state  $\text{state}_0$  with existing control strategy  $\text{control}(\text{time}) \in \text{Controls}(\text{time})$ . for time  $\tau > \text{time}_0$ .

$$\text{ReachSet}(\tau, \text{time}_0, \text{state}_0) = \bigcup \{\text{state}(s) : \text{control}(s) \in \text{Controls}(s), s \in (\text{time}_0, \tau]\} \quad (6.1)$$

**The Reach Set Approximation by Movement Automaton** (6.2) of the system under the control of the movement automation  $\mathcal{MA}$  consist from the set of trajectories  $\text{Trajectory}(\text{state}_0, \text{Buffer})$ , which are executed in constrained time  $\tau > \text{time}_0$ .

$$\text{ReachSet}(\tau, \text{time}_0, \text{state}_0) = \left\{ \text{Trajectory}(\text{state}_0, \text{buffer}) : \begin{array}{c} \text{duration(buffer)} \\ \leq \\ (\text{time}_0 - \tau) \end{array} \right\} \quad (6.2)$$

Note. *Reach Set Approximation* (*def. 14*) is subset of *Full Reach Set* (*def. 2*) in continuous space  $\mathbb{R}^n$  it inherits all important properties, like *Invariance* [118].

*Discretization of Reach Set* have been achieved leaving us with *finite count of Trajectories*, instead of *Infinite subspace or  $\mathbb{R}^N$*

**Approximated Reach Set Containment:** The *Approximated Reach Set* introduced in (*def. 14*) is constrained only by *future expansion time*  $\tau$ . UAS makes space assessment in *Avoidance Grid*. There is no point to consider Trajectories outside of *Avoidance Grid*

**Definition 15** (Contained Aproximated Reach Set). For pair  $(\text{state}_0, \text{AvoidanceGrid}_0)$  at time  $\text{time}_0$  and prediction horizon  $\tau = \infty$  there is Contained Reduced Reach Set:

$$\text{ReachSet} \begin{pmatrix} \text{time}_0, \\ \text{state}_0, \\ \text{AvoidanceGrid}_0 \end{pmatrix} = \left\{ \text{Trajectory}(\dots) : \begin{array}{l} \in \\ \text{ReachSet}(6.2) \end{array} : \begin{array}{l} \forall \text{segment} \in \text{AvoidanceGrid}_0, \\ \text{segment} \in \text{Trajectory}(\dots) \end{array} \right\} \quad (6.3)$$

**Properties:** Container Aproximated Reach Set *contains only trajectories where all segments belongs to Avoidance Grid, there are following functions:*

1. Membership function *for any Trajectory in Constrained Reduced Reach set returns Ordered Set of Passing Cells.*
2. Cost function *for any Trajectory Portion in Constrained Reduced Reach Set return Cost of Execution*

**Passing cell:** Cell of Avoidance Grid which has some intersection with Trajectory.

*Note.* Contained Reduced Reach Set (eq. 6.3) which is contained in Avoidance Grid and have an *Membership Function* enable Property transition between Reach set and Avoidance grid.

*Example:* Visibility from cells along Trajectory can be gathered to calculate Trajectory's feasibility.

**Reach Set Pruning:** There is a need to implement *Set Difference* between Reach Set and Constraint Set. Constraint Set can be *Obstacle Set* from Known World (sec. 4.1.10) and other different constraints.

**Reach Set Trajectory Tree:** (6.4) Any Reach Set where *Control Strategy Constraint* is implemented as *Movement Automaton*, with defined *Movements* set and for singe initial  $state_0$ . The Reach Set is given as discrete tree with root  $Trajectory(state_0, \emptyset)$ .

$$ReachSet(state_0, \dots) = \left\{ Trajectory(state_0, buffer) : \begin{array}{l} buffer \in Movements^i, \\ i \in \{1, \dots, k\} \end{array} \right\} \quad (6.4)$$

For each *Trajectory Segment*, there exists *intersection function* which evaluates as true if there exists at least one point in *Segment* which belongs to *Constraint Set*. Formally:

$$intersection(segment, Set) : \begin{cases} \exists point \in segment, & : true \\ point \in Set & \\ Otherwise & : false \end{cases} \quad (6.5)$$

**Definition 16** (Pruned Reach Set). For Reach set represented as Trajectory Tree (eq. 6.4) and some constraint set (Set) where exist intersection function (eq. 6.5). The Pruned Reach set is given as follows:

$$Prune(ReachSet, Set) = \left\{ Trajectory(\dots) : \begin{array}{l} \forall segment \in Trajectory, \\ \neg intersection(segment, Set) \end{array} \right\} \quad (6.6)$$

*Note.* Pruning(def. 16) [119] is applicable multiple times for various Constraints Set.

Example of *Approximated Reach set Calculation* (def. 14), *Reach Set Containment* (def. 15), and, *Pruning* is given in [49].

### 6.6.1 (R) Reach Set Performance Criteria

**Motivation:** The need to Make *Reach Set* scalable approach. This may be a problem due the *Expansion rate*. *Reach set* represented as a *Trajectory Tree* (eq. 6.4) for Avoidance Grid with *layerCount* and Movement automaton with *movementCount*, the *Node count* is given as:

$$1 + \left( \sum_{i \in \{1 \dots layerCount\}} (movementCount)^i \right) \quad (6.7)$$

This scaling is not feasible for *Avoidance Grid* with many layers ( $< 10$ ) or *Movement Set* with many movements ( $< 9$ ). There is need for *Reduced Reach set calculation*.

**Core Performance Criteria:** The scaling factor (eq. 6.7) shows that there are going to be many trajectories. The main point is that not every trajectory in *Reach Set* are giving us *maneuverability advantage*. Our expectations lies in following *Performance Requirements*:

1. *Reach set* must *Cover* maximum of the *possible unique maneuvers* in *Avoidance Grid*.
2. *Trajectories* in *Reach Set* should be smoothest possible to prevent cargo damage / UAS wear.

**Trajectory footprint:** Discrete space of *Avoidance Grid* is organized in cells. *Cell* is minimal space portion accessible by *property binding*. There is need to know if two trajectories contribution to *Maneuverability* in this environment.

Each trajectory passes trough space in *Avoidance Grid*. If there exists a method to extract unique identifier for each *trajectory passed cells*, we can compare two trajectories *Coverage* in *Avoidance Grid*.

**Definition 17** (Trajectory footprint). *For Trajectory from Reach set (def. 15) defined for Avoidance Grid has membership function. Membership Function returns ordered set of passing cells:*

$$\text{footprint}_{\text{AvoidanceGrid}} \left( \begin{array}{c} \text{Trajectory}, \\ \text{AvoidanceGrid} \end{array} \right) = \left\{ \begin{array}{l} \text{cell} \in \text{AvoidanceGrid} : \\ \quad \text{isMember}(\text{trajectory}, \text{cell}) \end{array} \right\} \quad (6.8)$$

Then we can define equality function for *Trajectory*<sub>1</sub> and *Trajectory*<sub>2</sub>, as comparison of their footprints in common *Avoidance Grid* as follow:

$$\text{isEqual}_{\text{AvoidanceGrid}} \left( \begin{array}{c} \text{Trajectory}_1, \\ \text{Trajectory}_2, \\ \text{AvoidanceGrid} \end{array} \right) : \left\{ \begin{array}{ll} \left( \begin{array}{c} \text{footprint}(\text{Trajectory}_1, \dots) \\ = \\ \text{footprint}(\text{Trajectory}_2, \dots) \end{array} \right) & : \text{true} \\ \text{Otherwise} & : \text{false} \end{array} \right. \quad (6.9)$$

*Note.* Depending on *Movement Automaton's* movement set and *Avoidance Grid* parameters, there can be multiple *trajectories* which are equal.

**Coverage set:** Now it is possible to create set of unique *trajectory footprints* due to *footprint function* (eq. 6.8). Similarly there is a possibility to create *Reach set skeleton* containing unique trajectories, by using *equality function* (eq. 6.9). *Coverage set* is sufficient for now.

**Definition 18** (Coverage Set). Coverage set (6.10) is defined for Avoidance Grid and Reach Set pair as set of unique Trajectory footprints:

$$\text{CoverageSet} \left( \begin{array}{c} \text{AvoidanceGrid}, \\ \text{ReachSet} \end{array} \right) = \left\{ \text{footprint} \left( \begin{array}{c} \text{Trajectory}, \\ \text{AvoidanceGrid} \end{array} \right) : \forall \text{Trajectory} \in \text{ReachSet} \right\} \quad (6.10)$$

**Coverage set properties:** Trajectory footprint (eq. 6.8) is not *bijection*, neither *injection* for  $\text{ReachSet} \rightarrow \text{CoverageSet}$ . This implies following properties:

1. Equal *Reach Sets* in same *Avoidance Grid* have equal *Coverage Sets*.
2. Equal *Coverage Sets* does not imply *Reach Set* equality.
3. For two Coverage Sets there is a possibility to compare their member count to create coverage ratio.

The second *Property* gives us a preposition that there is a possibility of *Reach Set Reduction* without loosing *Coverage*.

**Definition 19** (Coverage Ratio). Coverage Ratio is a ratio of Coverage Set Member Count between two Reach Sets. *Reach set* with lesser count of unique Trajectories is considered as Reduced Reach Set. *Reach set* with greater Count of unique Trajectories is considered as Reference Reach Set.

$$\begin{aligned} \text{referenceCoverage} &= |\text{CoverageSet}(\text{ReferenceReachSet}, \text{AvoidanceGrid})| \\ \text{reducedCoverage} &= |\text{CoverageSet}(\text{ReducedReachSet}, \text{AvoidanceGrid})| \\ \text{CoverageRatio} &= \frac{\text{reducedCoverage}}{\text{referenceCoverage}} \in [0, 1] \end{aligned} \quad (6.11)$$

*Note.* Reference Reach Set is usually *Full Reach Set* containing all possible trajectories in space contained by *Avoidance Grid*.

In case *Full Reach Set* can not be computed, Avoidance Grid is too large, most complex *Reach Set* is used as *Reference Reach Set*.

**Trajectory smoothness:** Trajectory other than straight line have some changes in *UAS* heading.

The goal is to minimize *Maneuvering* of UAS, because:

1. Every Heading Change needs to be reported to *UTM*.
2. Sharp Maneuvering can damage cargo/wear UAS.
3. Often course changes makes *Intruder prediction* harder for other Civil General Aviation.

For this purpose *Smoothness Metric* needs to be applied for *Reach Set* or *Trajectory*. In case of *Movement Automaton Control* two distinguish *Movement Sets* can be introduced: *Smooth* nad *Chaotic* movements set with following properties:

$$\begin{aligned} MovementSet &= SmoothMovements \cup ChaoticMovements \\ SmoothMovements \cap ChaoticMovements &= \emptyset \\ |SmoothMovements| > 0, \quad |ChaoticMovements| > 0 \end{aligned} \quad (6.12)$$

Then *Smoothnes clasifier* for *Trajectory(initialState, buffer)* can be defined as *isSmooth* and *Smooth Movement Counter* function as *smoothCount* like follow:

$$\begin{aligned} isSmooth(movement) &= \begin{cases} movement \in SmoothMovements & : 1 \\ movement \in ChaoticMovements & : 0 \end{cases} \\ smoothCount(Trajectory(\dots, buffer)) &= \sum_{\forall movement \in Buffer} isSmooth(movement), \end{aligned} \quad (6.13)$$

**Definition 20** (Smoothness Rating for Trajectory). Smoothness for trajectory generated by Movement Automaton for some Initial State with some Movement Buffer, under assumption of Smooth and Chaotic Movement Set split (eq. 6.12), with existing classification and counter functionals (eq. 6.13) is given as follows:

$$Smoothness(Trajectory(\dots, buffer)) = \frac{isSmooth(Trajectory)}{movementCount(Trajectory)} \in [0, 1] \quad (6.14)$$

For Trajectory with  $buffer = \emptyset$  Smoothness is given as 1.

### 6.6.2 (R) Constrained Trajectory Expansion

**Motivation:** Purpose of Navigation is to move forward to *Goal Waypoint* in *Mission*. Structure of *Avoidance Grid* is designed to enable *forward* and *turning* maneuvers. The *Avoidance Grid* is organized in *Layers* characteristic by same distance from *Avoidance Grid Origin*.

Survey of motion planning algorithm was given in [120]. The ideal candidate for propagation algorithm is *Wave-front* algorithm propagating *Trajectory tree* trough Layers. Due the *Avoidance Grid* onion like layers, there is possibility to implement turn maneuver trough layers iterative and effectively .

**Rapid Exploration Tree** (fig. 6.8) was selected, because it enables *Movement Automaton Utilization* and *Property Binding*. Similar approach was used in rapid exploration tree for space exploration [121].

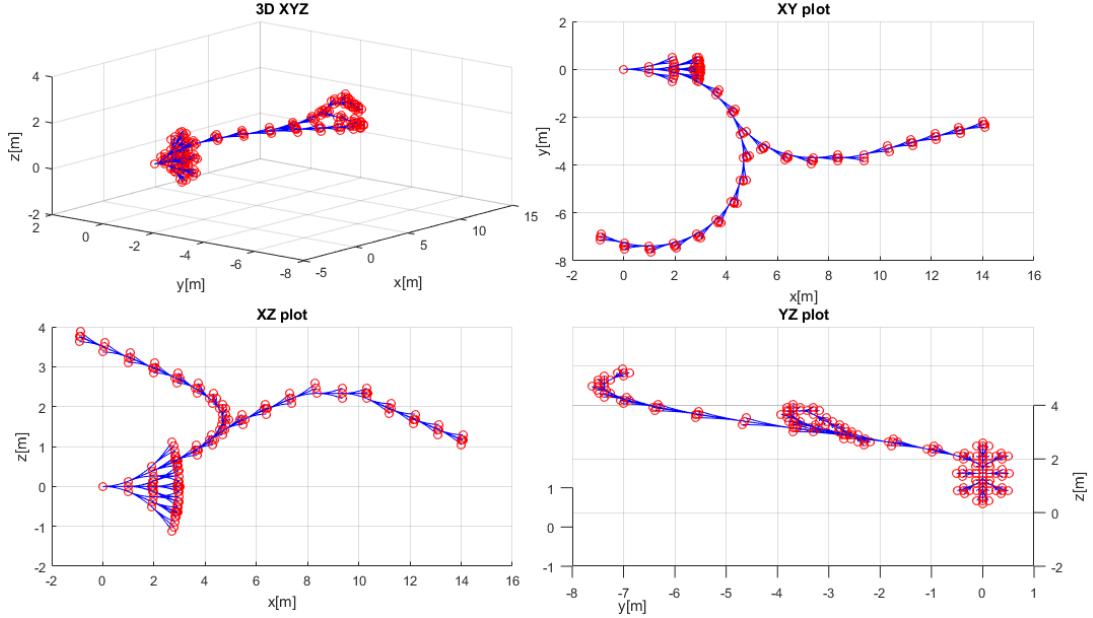


Figure 6.8: *Rapid Exploration tree as result of Constrained trajectory expansion.*

**The example** (fig. 6.8) shows a *Rapid Exploration Tree* in *Free Space* containing *Waypoint Navigation Path* and *Turn Away Path*. Both paths are starting in same *Root Node* (red circle) which was expanded with simple *Movement Automaton* (bunch of nodes originating from one node are showing way of expansion). The connection (blue line) between two nodes (red circles) represents *Executed Movement*.

**Rapid Exploration Tree Node** will contain following information:

1. *Initial state* - root entry point, used in state evolution calculation.
2. *Trajectory (state evolution)* - trajectory passing trough *state space* in local coordinate frame of *Avoidance Grid*.
3. *Buffer* (applied movements) - ordered list of *executed movements* applied on *initial state* to obtain *state evolution*.
4. *Cost* - calculated for *state evolution* based on *predefined cost function*.
5. *Footprint* - ordered set of *passing cells* in *Avoidance Grid*.
6. *Parent Node Reference* - tree reference for parent node, not in case of *root node*.
7. *Other Bounded Properties* - value list of other properties, depending on *Expansion Constraints* and *Reachability* evaluation algorithm.

**Wave-front propagation of Rapid Exploration Tree** is given in (alg. 6.1).

The *Avoidance Grid* have UAS with *position*  $\in$  *Initial State* at the *origin*. The *Grid Layer* is a column ordered set of cells with same *Mean distance* from origin. *Grid Layers* are indexed from origin starting with 1, there is maximum of  $i \geq 1$  layers.

**Step: Initialization** contains base structure preparation like follows:

1. *Avoidance Grid* - Space containing *Reach set* (def. 15).
2. *Movement Automaton* - Used as *Predictor*, consuming *buffer* containing *Movements* to generate *Trajectory(initialState, buffer)*.
3. *Reach Set* - tree consisting from *Wave-frontNodes* representing the end point of *Trajectory(initialState, buffer)* where each *Edge* represents *one Movement application*. The root is set as node containing *Initial State*.

Function *initializeReachSet(root, stack, grid, automaton)* will take the root and enforces *full wavefront propagation* to *First Layer*.

**Step: Wave-front Propagation** is forced propagation of trajectories from layer  $i$  to layer  $i + 1$ . The process goes as follows:

1. *Selection of Feasible candidates* - function  $[candidates, leftovers] = ExpansionConstraints.select(stack)$  for working layer, row and cell selects *feasible trajectory nodes* ordered by *Cost function*. The *Example of Cost Function* can be *Trajectory Smoothness* (def. 20).
2. *Expansion of Candidates* - for each *candidate* function *candidate.expandNode(automaton)* is invoked. This function will expand *Candidate Node structure* by appending *Full Trajectory Tree Evolution* until each *Leaf Trajectory* reaches *Next Layer*. Simply put *Parent Node Node(initialState, buffer, cost, footprint)* buffer is appended by movements until the next layer is reached.
3. *Leftovers purge* - function *reachSet.purge(leftovers)* removes unexpanded *Nodes* leading to cell, effectively removing trajectories which does not lead to *next layer*.
4. *Append Reach Set* - function *reachSet.append(leafs)* puts newly created *Nodes (Trees)* into *Reach Set* structure. The *Wave-front Propagation* for one cell is finished.

**Step: After Layer Propagation Purge** is covered by function *reachSet.purgeSameFootprint()* which takes trajectories with same footprint and keeps some of them based on *Selection criteria*, more in (sec. 6.6.4, 6.6.5). *Pruning methods over Large Decision*

Trees are *fast* and *viable* [122].

---

**Algorithm 6.1:** *Wave-front propagation of Rapid Exploration Tree to form Reach Set.*

---

**Input :** Node(initialState,buffer=∅,cost=0,footprint=∅), AvoidanceGrid,  
ExpansionConstraints, MovementAutomaton(movementSet)

**Output:** ReachSet(AvoidanceGrid)

```
# Initialization Sequence;
grid=AvoidanceGrid, automaton=MovementAutomaton, root = Node;
reachSet = initializeReachSet(root,stack,grid,automaton);

# Main Expansion trough, layers (i), rows (j), cells(k);
for layer(1...i) in grid do
    for row(1...j) in layer do
        for cell(1...k) in row do
            # apply selection criteria ;
            [candidates,leftovers] = ExpansionConstraints.select(stack);

            # collect expansions ;
            leafs = [];
            for candidate in Candidates do
                | leafs= [leafs, candidate.expandNode(automaton)];
            end
            reachSet.purge(leftovers);
            reachSet.append(leafs);
        end
    end
    reachSet.purgeSameFootprint();
end
```

---

*Note.* *Reach Set* is usually computed *Prior the Flight* for some *Initial State* in *Local Coordinate Frame* in *right had coordinate frame* with  $X^+$  used as *main axis*.

### 6.6.3 (W) Avoidance Grid scaling

- Supplement to avoidance grid scaling, based on movement set properties
- Formula for Theta and Varphi parameters adjustment

### 6.6.4 (R) Chaotic Reach set

**Motivation:** Design of calculation method for *Reach Set Approximation* guarantying high *Maneuverability*.

**Background:** There is *Coverage Ratio* property of *Reach Set* (def. 19). It has been shown that creating *Reach Set* via *greedy approach* is not feasible due the *Scaling Factor. Contracted Expansion* (sec. 6.6.2) is enabling to apply selection criteria while building *Reach Set* in given *Cell*.

The *Cell*  $cell_{i,j,k}$  has a center and walls from UAS viewpoint: front wall , back wall (for  $layer > 1$ ), top wall, left wall, right wall, bottom wall. It is expected that trajectory

leading close to one cell walls will continue to different cell, increasing chance to obtain more *Unique Footprints*.

**Expansion Constraint Function Implementation** (alg. 6.2) is based on simple principle: *Select candidate Nodes which are closest to outer walls of Cell, with unique footprint.*

**Tuning Parameters** : *Proximity to Cell outer wall* gives good chances to break into other rows or columns in *Avoidance Grid*. *Unique footprint* guarantees future *Unique Footprint* after appending Trajectory by *Movement application*.

1. *Considered Footprint Length* - how much last cells in footprint should be considered in unique path track, minimal value 1, default value 3, maximal value  $\infty$ . If you want to generate non redundant trajectories use  $\infty$ , it will consider full footprint.
2. *Spread Limit* - upper limit of candidates which are going to be select for further expansion, minimal value 1, default value *Count of unique Moves in Movement set*, maximal value  $\infty$ . If more than default values is selected the algorithm will generate *redundant trajectories*. If less is selected then some trajectories are omitted and *Coverage Rate* decreases sharply.

**Step: Initialization** initialization of *candidate* array (return value), *leftovers* array (return Value). Node array *passing* is populated with *Nodes* which represents *end node of Trajectory* and the tip of *trajectory is constrained in cell<sub>i,j,k</sub>*.

**Step: Evaluate best trajectories with unique Footprints** following steps are executed:

1. *Best Performance Map* is created with *footprint* as key set element to ensure footprint uniqueness.
2. *Wall distance* for *test node* is calculated as a closest trajectory portion distance to *top, bottom, left, right* wall of cell *cell<sub>i,j,k</sub>*
3. *Footprint* for *test node* is created with maximal length given by *Footprint Length* tuning parameter.
4. *Existence and Performance Test* is executed to ensure that best performing node is selected. If there is not key entry in the *Best Performance Map*, then new entry for *Test Node* is created. If there is key entry, the performance of *Old Node* and *Test Node* is compared and better is stored.

**Step: Select candidates** is executed on *Best Performance Map* records using *Wall distance* as pivot parameter, ordering by closest proximity and limited by *Search Limit*

tuning parameter. The *Leftovers* are difference set between *Passing Nodes* and *Candidate Nodes*.

---

**Algorithm 6.2:** Expansion Constraint function for *Chaotic Reach Set Approximation*

---

```

Input           : Node[] stack, Cell celli,j,k
Tuning Parameters: int+ footprintLength, int+ spreadLimit
Output          : Node[] candidates, Node[] leftovers

# Initialize structures;
Node[] candidates = [], Node[] leftovers= [];
Node[] passing = celli,j,k.getFinishingTrajectories(stack);

# Select best performing trajectories with unique footprint;
Map<Footprint,Node> bestPerformanceMap;
for Node test in passing do
    wallDistance= test.minimalDistanceToWall(celli,j,k];
    footPrint = test.getFootprint(lastCells = footprintLength);
    if bestPerformanceMap.contains(footPrint) then
        old = bestPerformanceMap.getByKey(footprint);
        oldPerformance= old.minimalDistanceToWall(celli,j,k);
        if oldPerformance > wallDistance then
            | bestPerformanceMap.setByKey(footprint,test);
        end
    else
        | bestPerformanceMap.setByKey(footprint,test);
    end
end

# Select best performing nodes up to spreadLimit count;
candidates = bestPerformanceMap.select(count =
    spreadLimit).orderBy('wallDistance', 'Ascending');
leftovers = passing - candidates;
return [candidates, leftovers]

```

---

**Example:** for *Avoidance Grid* with *Distance 10 m*, *Layer count 10*, *Horizontal range*  $[-45^\circ, +45^\circ]$ , *Horizontal Cell Count 7*, *Vertical range*  $[-30^\circ, +30^\circ]$ , and *Vertical Cell Count 5*. Is given in (fig. 6.9). The UAS is at *Back-side* of *Figure* (initial state is at all *Trajectory Origins*). The *black dashed line* marks *Avoidance Grid space boundary*. Each trajectory has its own color and ends at *Front-side* of *Avoidance Grid Boundary*.

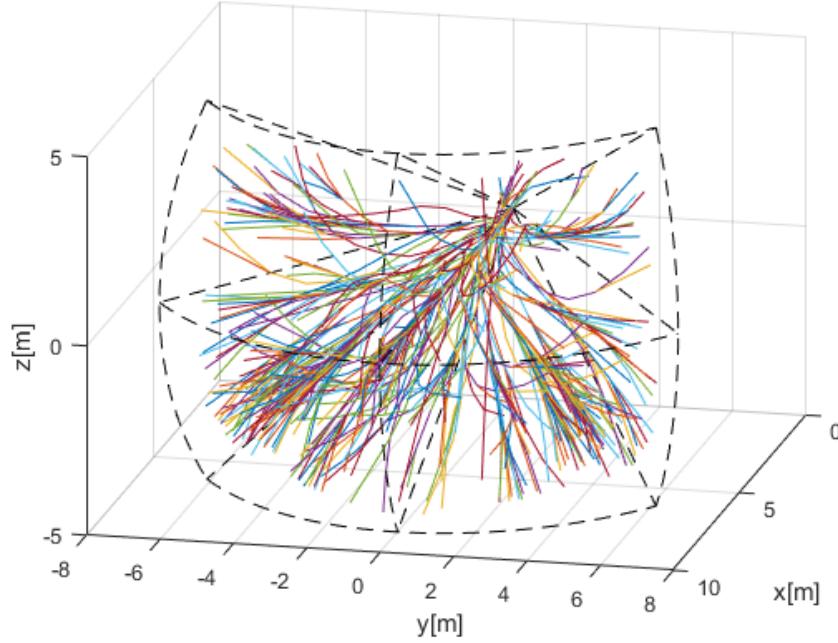


Figure 6.9: *Chaotic reach set approximation.*

**Pros and Cons:** It can be seen from example (fig. 6.9) that *Chaotic Reach Set Approximation Method* (alg. 6.2) generates a lot of *turning* and *shaky trajectories*.

*High Coverage Ratio* ( $\sim 0.9$ ) is provided, while keeping *medium node count*. The calculation complexity scales linearly with grid size. The *upper limit of trajectories* is given as follow:

$$\begin{aligned} \text{countTrajectories}(\text{ReachSet}) &\leq \text{layerCellCount} \times \text{spreadLimit} \\ &\quad \times \text{size}(\text{Movements}) \end{aligned} \quad (6.15)$$

The *upper limit of nodes* is given as follow:

$$\begin{aligned} \text{countNodes}(\text{ReachSet}) &\leq \text{layerCount} \times \text{layerCellCount} \\ &\quad \times \text{size}(\text{Movements}) \times \text{spreadLimit} \end{aligned} \quad (6.16)$$

*Absence of Smooth Trajectories* disqualifies *Chaotic Reach Set Approximation* to be used for *Navigation*. This type of reach set is feasible for *Avoidance*, because it contains variety of maneuvers.

### 6.6.5 (R) Harmonic Reach set

**Motivation:** Imagine having an *Avoidance Grid* like (fig. 6.2). There is a need of *Reach Set Approximation* which will have *Smooth Trajectories* (def. 20) going nearby *cell centers*.

**Background:** The *Smoothness Rating for Trajectory* (def. 20) uses two distinct sets *Smooth Movements* and *Chaotic Movements* (eq. 6.12) which are defined for our *Movement Automaton* (sec. 6.3) like follow:

$$\begin{aligned} \text{Smooth Movements} &= \{\text{Straight}\} \\ \text{Chaotic Movements} &= \text{Movements} - \text{Smooth Movements} \end{aligned} \quad (6.17)$$

*Smooth Movements* contains only *Straight* movement, because others are considered as extreme turning movements. *Smooth Movements* should contain only direct flight movements or slight heading correction. *Chaotic Movements* set is supplement of *Movement Automaton's Movement Set*.

The *Avoidance Grid* (fig. 6.2) cell centers for fixed indexes  $j_{fix}, k_{fix}$  are linearly aligned with *initial state*. That means that cell centers of cells  $cell_{1,j_{fix},k_{fix}}, \dots, cell_{i,j_{fix},k_{fix}}$ , where  $i$  is count of *layers* lies on one line. If the trajectory can achieve *cell center* on some *layer* only minor trajectory corrections are required to stay on given line. This type of trajectory gives us following advantages:

1. *Minimal steering at beginning* - the minimal steering is advantageous in *Controlled Airspace* because is diminishing the amount of communication to *UTM Service*.
2. *Additional safe space in Linear segment* - once the *center of cell* is reached, *Trajectory* sticks to line between cell centers. Each point on this line has *maximal distance* to outer walls of cell. This gives us extra space given as minimum of distance between *UAS position* and *Outer cell walls*.

**Expansion Constraint Function Implementation** (alg. 6.3) is based on simple principle: *Select candidate Nodes which are closest to Cell center, with unique footprint*.

*Note.* *Cell center* can be closely reached by *smooth movement* from previous cell or *chaotic movement* from neighbouring cell from current or previous layer. These trajectories are usually equivalent in *Smoothness*.

**Tuning Parameter:** *Proximity to Cell Center* gives a good chance to keep trajectory smooth or *smooth after one correction maneuver*. It has been mentioned that *Cell Center* can be reached by various trajectories. In this method full footprint length is always considered, therefore only one tuning parameter can be offered:

1. *Spread Limit* - upper limit of candidates which are going to be selected for further expansion, minimal value 1, default value *Count of unique Moves in Movement set*, maximal value  $\infty$ . If maximal value  $\infty$  is selected, algorithm will generate skeleton of *Reach Set* with full Coverage and with the smoothest *Trajectories*.

**Step: Initialization** sets candidate *Nodes* as empty set, leftover *Nodes* as empty set. and selects all *Nodes* from *Stack* which represents *Finishing Trajectories* in working cell  $cell_{i,j,k}$ .

**Step: Evaluate smoothest trajectories with unique Footprints** is implemented as *multi-criteria filtration*.

*First criterion* is *distance to Cell Center* which is penalized by trajectory *smoothness rate* implemented in method *Node.getPerformance(Cell cell<sub>i,j,k</sub>)* defined as follow.

$$getPerformance(Node, Cell) = \frac{distance(Node.Trajectory, Cell.Center)}{SmoothnessRate(Node.Trajectory)} \quad (6.18)$$

Distance of *Trajectory* is *enumerator*, because its considered as *base value* and is defined in interval  $[0, maximalWallDistance]$ . The *Smoothness Rate* is in denominator, because it is a penalization coefficient defined in interval  $[0, 1]$ .

*Second criterion* is *trajectory uniqueness* This is provided by *Best Performance Map*, where best performing *Node* belongs to one unique *trajectory footprint*. The implementation is identical to *chaotic reach set expansion* (alg. 6.2).

**Step: Select candidates** is executed on *Best Performance Map* records using *Penalized Cell Center Distance* as pivot parameter, ordered in ascending order and limited by *Spread Limit* tuning parameter. The *Leftovers* are difference set between *Passing Nodes* and *Candidate Nodes*.

---

**Algorithm 6.3:** Expansion Constraint function for *Harmonic Reach Set Approximation*

---

```

Input           : Node[] stack, Cell celli,j,k
Tuning Parameters: int+ spreadLimit
Output          : Node[] candidates, Node[] leftovers

# Initialize structures;
Node[] candidates = [], Node[] leftovers= [];
Node[] passing = celli,j,k.getFinishingTrajectories(stack);

# Select unique smoothest trajectories;
Map<Buffer,Node> bestPerformanceMap;
for Node test  $\in$  passing do
    centerDistance= test.getPerformance(celli,j,k);
    footPrint = test.getFootprint();
    if bestPerformanceMap.contains(footPrint) then
        old = bestPerformanceMap.getByKey(footprint);
        oldPerformance= old.getPerformance(celli,j,k);
        if oldPerformance > centerDistance then
            | bestPerformanceMap.setByKey(footprint,test);
        end
    else
        | bestPerformanceMap.setByKey(footprint,test);
    end
end

# Select best performing nodes up to spreadLimit count;
candidates = bestPerformanceMap.select(count =
    spreadLimit).orderBy('cellCenterDistance','Ascending');
leftovers = passing - candidates;
return [candidates, leftovers]

```

---

**Example:** for *Avoidance Grid* with *Distance 10 m*, *Layer count 10*, *Horizontal range*  $[-45^\circ, +45^\circ]$ , *Horizontal Cell Count 7*, *Vertical range*  $[-30^\circ, +30^\circ]$ , and *Vertical Cell Count 5*. Is given in (fig. 6.10). The UAS is at *Back-side* of *Figure* (initial state is at all *Trajectory Origins*). The *black dashed line* marks *Avoidance Grid space boundary*. Each trajectory has its own color and ends at *Front-side* of *Avoidance Grid Boundary*. The *Spread Limit* in this case was set to 9 which is *Size of the Movement Set*.

*Note.* Please note *Trajectories* are organized in bundles going around *Cell Centers smoothly*. Most of the steering maneuvers are executed at the *beginning* of *Avoidance Grid*.

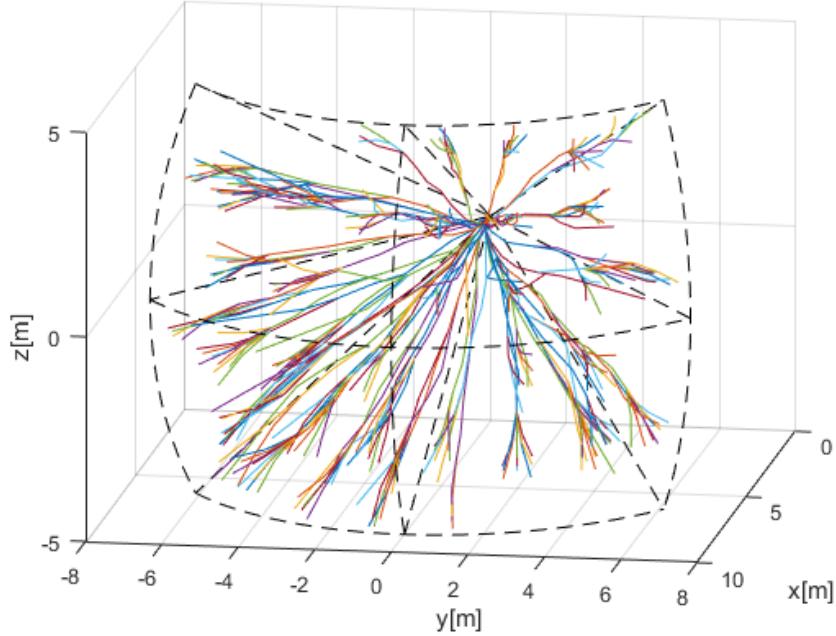


Figure 6.10: *Harmonic reach set approximation*.

**Pros and Cons:** It can bee seen from example (fig. 6.10) that *Harmonic Reach Set Approximation Method* (alg. 6.3) generates *smooth evenly spread trajectories*.

High smoothness ratio ( $\geq 0.9$ ) is provided, while keeping low node count for UAS systems. The calculation complexity scales linearly with grid size. The upper limit of trajectories is given as follow:

$$\begin{aligned} \text{countTrajectories}(\text{ReachSet}) &\leq \text{layerCellCount} \times \text{spreadLimit} \\ &\times \text{size}(\text{Movements}) \end{aligned} \quad (6.19)$$

The *upper limit of nodes* is given as follow:

$$\text{countNodes}(\text{ReachSet}) \leq \text{layerCount} \times \text{layerCellCount} \times \text{spreadLimit} \quad (6.20)$$

Absence of *High Coverage Ratio* disqualifies *Harmonic Reach Set Approximation* to be used for *Emergency Avoidance*. This type of *Reach Set* is feasible for *Open Space Navigation* or *Controlled Airspace Navigation*. Its low turning rate in contained *Trajectories* are desired for such tasks.

### 6.6.6 (R) Combined Reach set

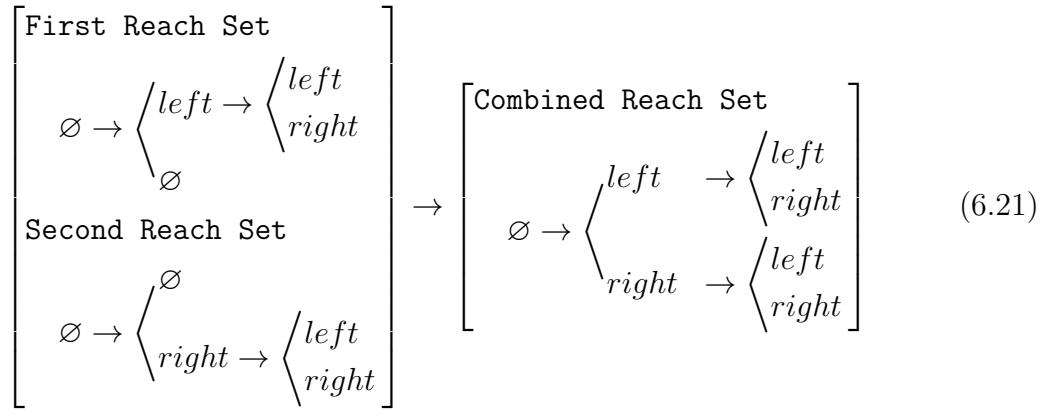
**Motivation:** Harmonic reach set (sec. 6.6.5) is *efficient* for *Navigation in Controlled Airspace*. Chaotic reach set (sec. 6.6.4) is good for *Emergency avoidance*. The need to differentiation between *Navigation* and *Emergency Avoidance* mode is necessary in

*Controlled Airspace*. but not in *Non-controlled Airspace*. The combination of *Harmonic* and *Chaotic* reach sets is obvious solution.

*Automatic mode switch* can be provided by combination of *Navigation Reach Set* and *Avoidance Reach Set* with elevated cost function. Overall having a method to merge multiple trees would be beneficial.

**Background:** If two *Reach Set Approximation* were calculated for same *Avoidance Grid* and *Initial State*, using same *Movement Automaton* and *UAS model* are possible to merge.

The *Reach Set Approximation* is tree with *Root Node* in *initail state* with movement buffer =  $\emptyset$ . The *movement buffer* in each node can be used as *route trace* during merging procedure. The example two reach set merge can be given as follow, where only *latest* applied movement is taken into account.



*First Reach Set* contains two trajectories given by buffers  $\{left, left\}$  and  $\{left, right\}$ . *Second Reach Set* contains two trajectories given by buffers  $\{right, left\}$  and  $\{right, right\}$ . The *Combined Reach Set* contains all four trajectories.

*Note.* The combined tree [123] does not need to have combined amount of original *Reach Sets* trajectories. There can be *Duplicity* which means that any bounded property like *Cost* must be *calculated* again.

**Combined Reach Set Calculation Function** (alg. 6.4) is implemented as function *NodecombinedReachSet(...)* which takes root Node with *initial State*, *Avoidance Grid* and respective parameters for each calculation method. *Harmonic spread* for *Harmonic Reach set calculation* and *Chaotic Spread, Footprint Length* for *Chaotic Reach set calculation*.

*Separate Reach Sets* are calculated using *Wave-front propagation* (alg. 6.1) using respective *Constrained Expansion* functions for *Harmonic* (alg. 6.3) and *Chaotic* (alg. 6.2) reach sets.

*Combined Reach Set* is created using *Node mergeTree(...)* function. Because different cost function or *Bounded Parameters Calculation* may be applied on *Original Reach Sets*.

*Cost* for each node needs to be recalculated due to original reach sets disparity. Function *combined.applyCostFunction()* will recalculate the new cost for each node.

The Goal is to have penalization for *Chaotic behaviour*, implementation of *Automatic Mode Switch* can be done like follows:

1. *Calculate Normal Cost* for Node  $Cost(Node)$  for associated trajectory:  $Cost(Node.Trajectory)$ .

2. Calculate Penalization for Chaotic Behaviour, calculate *Smoothness Rating for Trajectory* (def. 20) in interval  $[0, 1]$ , introduce penalization with base 100%.

The final  $\text{Cost}(\text{Node})$  function is applied on each *Combined Reach Set Node* and look like follows:

$$\begin{aligned}\text{Cost}(\text{Node}) = & \text{Cost}(\text{Node.Trajectory}) \times \dots \\ & \dots \times (1 + (1 - \text{SmoothnessRate}(\text{Node.Trajectory})))\end{aligned}\quad (6.22)$$

**Merge Tree Function**  $\text{mergeTree}(\dots)$  implements *Outer Join* operation on two trees. Example was given in (eq. 6.21). Function is applied on *root Node* iterating over *Movements in Movement Set*, because *Movement is pivot*.

---

**Algorithm 6.4:** Reach Set Merge Function and Combined Reach Set calculation

---

```
# Tree merge function;
Node mergeTree(Node firstNode, Node secondNode)
    # Try to copy reference node or return null;
    Node referenceNode = firstNode?:secondNode? return null;
    Node merged = new Node(referenceNode);
    merged.leafs= [];

    # Try to fetch movement nodes if exist in any sub tree;
    for movement ∈ Movements do
        firstLeaf = firstNode.getLeafFor(movement);
        secondLeaf = secondNode.getLeafFor(movement);
        newLeaf = mergeTree(firstLeaf,secondLeaf);
        if newLeaf ~= null then
            | merged.leafs.append(newLeaf);
        end
    end
    return merged

# Combined Reach Set calculation function;
Node combinedReachSet(Node root, AvoidanceGrid grid,int+ chaoticSpread,
int+ harmonicSpread, int+ footprintLength)
    Node chaotic = chaoticReachSet(root,grid, footprintLength,chaoticSpread);
    Node harmonic = harmonicReachSet(root,grid, harmonicSpread);
    Node combined = mergeTree(chaotic,harmonic);
    combined.applyCostFunction();
    return combined
```

---

**Example:** for *Avoidance Grid* with *Distance 10 m*, *Layer count 10*, *Horizontal range  $[-45^\circ, +45^\circ]$* , *Horizontal Cell Count 7*, *Vertical range  $[-30^\circ, +30^\circ]$* , and *Vertical Cell Count 5*. Is given in (fig. 6.11). The UAS is at *Back-side of Figure* (initial state is at all *Trajectory Origins*). The *black dashed line* marks *Avoidance Grid space boundary*. Each trajectory has its own color and ends at *Front-side of Avoidance Grid Boundary*. The *Chaotic Spread* was set to 8, *Footprint Length* to 3 and *Harmonic Spread* to 1.

*Note.* Notice there are typical trajectories from both *Harmonic* (fig. 6.10) and *Chaotic* (fig. 6.9) *Reach Set Approximations*.

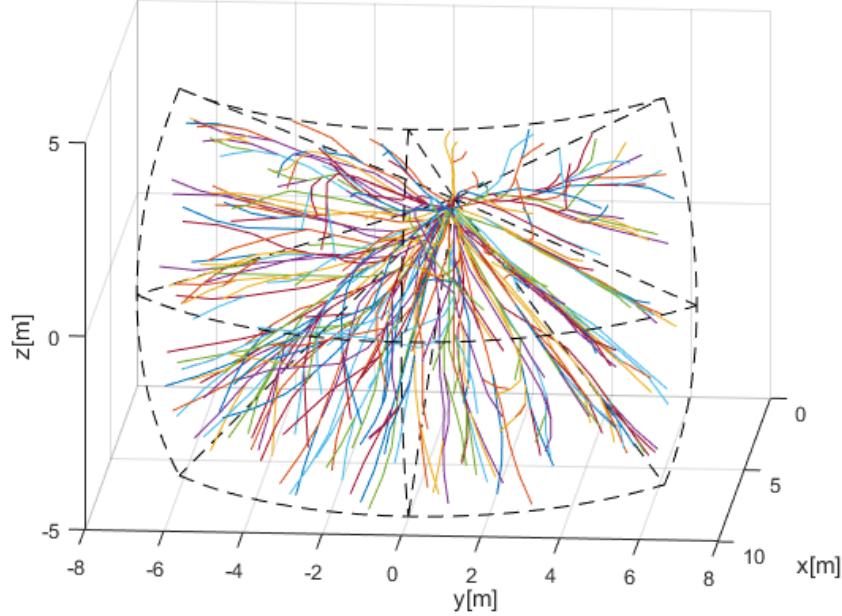


Figure 6.11: *Combined reach set approximation.*

**Pros and Cons:** It can bee seen from example (fig. 6.11) that *Combined Reach Set Approximation* (alg. 6.4) contains both types of maneuvers. *Cheaper Smooth* for navigation and *More Expensive Chaotic* for *Emergency Avoidance*. The upper limit of trajectories is given as follow:

$$\begin{aligned} \text{countTrajectories}(\text{ReachSet}) &\leq \text{countTrajectories}(\text{Chaotic}) \\ &+ \text{countTrajectories}(\text{Harmonic}) \end{aligned} \quad (6.23)$$

The *upper limit of nodes* is given as follow:

$$\text{countNodes}(\text{ReachSet}) \leq \text{countNodes}(\text{Chaotic}) + \text{countNodes}(\text{Harmonic}) \quad (6.24)$$

*Harmonic Reach Set* is ideal for *Non-controlled Airspace* missions, because it contains *Automatic Mode Switch* between *Navigation* and *Emergency Avoidance*.

### 6.6.7 (R) ACAS-X like Reach set

**Motivation:** The implementation of *ACAS-Xu* behavior in DAA system will be mandatory for *National Airspace System Integration* in United spaces [124].

Implementation of *ACAS-Xu* like behaviour increase usability of approach, if it can be achieved without major concept changes.

**Background:** The *ACAS-Xu* system on operational level has been described in [19]. The *Policy for Collision Avoidance* proposal has been given in [125].

Some behavioural patterns can be encoded into *Reach Set*. *ACAS-Xu* navigation part is basically *Look-up table of Maneuvers for Allowed Separations*.

The *Evasive Maneuver* selection process in *ACAS-Xu* is similar to our approach: *Select most energy efficient maneuver in compliance with space-time constraints*. *ACAS-Xu*

intruder model is similar to our *Body Volume Intersection Model* (sec. 6.5.3). The ACAS-Xu defines following base separations:

1. *Horizontal* - movements on *Horizontal Plane* in *Global Coordinate System*.
2. *Vertical* - movements on *Vertical Plane* in *Global Coordinate System*.

There are allowed custom separations which can be used, for further experimentation:

1. *Slash* - movement on  $+45^\circ$  *Tilted Plane to Horizontal Plane* in *Global Coordinate System*.
2. *Backslash* - movement on  $-45^\circ$  *Tilted Plane to Horizontal Plane* in *Global Coordinate System*.

For given *Movement Automaton* implementation (sec. 6.3) the separations are given as follow:

$$\begin{aligned} \text{Horizontal} &= \{\text{Straight}, \text{Left}, \text{Right}\} \\ \text{Vertical} &= \{\text{Straight}, \text{Up}, \text{Down}\} \\ \text{Slash} &= \{\text{Straight}, \text{UpLeft}, \text{DownRight}\} \\ \text{Backslash} &= \{\text{Straight}, \text{UpRight}, \text{DownLeft}\} \end{aligned} \tag{6.25}$$

For each  $\text{Node}(\dots, \text{buffer})$  and each *separation* there is a evaluation function *isSeparation* which decides, if *Trajectory* defined by node buffer is made up only from *Separation* movements. The function *isSeparation*(...) is defined like:

$$\text{isSeparation}(\text{buffer}, \text{separation}) = \begin{cases} \forall \text{movement} \in \text{buffer}, & : \text{true} \\ \text{movement} \in \text{separation} & \\ \text{otherwise} & : \text{false} \end{cases} \tag{6.26}$$

Following *Separation Modes* can be defined with given *separations*:

1. *Horizontal* (ACAS-X defined mode) containing *horizontal* separation.
2. *Vertical* (ACAS-X defined mode) containing *vertical* separation.
3. *Horizontal-Vertical* (ACAS-X defined mode) containing *horizontal*, *vertical* separations.
4. *Full* (custom defined mode) containing all *Separation Modes*.

*Note.* Every separation modes generates 2D trajectories set on *Respective plane*. There is no need for *Tuning parameters* for further *Expansion Constraint*.

**Expansion Constraint Function Implementation** (alg. 6.5) is based on simple principle: *Select only candidate Nodes which Trajectories have at least one desired Separation Mode*.

**Step: Initialization** sets candidate *Nodes* as empty set, leftover *Nodes* as empty set, and, select all nodes form *stack* which represents *Finishing Trajectories* in working  $\text{cell}_{i,j,k}$ ,

**Step: Candidate Selection Process** is evaluated for each *test Node* from *passing Node Set*.

For each *applicable separation*, given as input parameter *separations*, The test function *isSeparation* (eq. 6.26) is applied:

1. If *test Node* trajectory belongs to at least one allowed separation it is added to candidates set.
2. Else is added to *Leftovers*.

*Note.* *Separation sets* (eq. 6.25) are not *exclusive sets* in *Movement Automaton* domain. One *Trajectory* contained by Node can belong to multiple *Separations*.

---

**Algorithm 6.5:** Expansion Constraint function for *ACAS-like Reach Set Approximation*

---

```

Input : Node[] stack, Cell celli,j,k, Separation[] separations
Tuning Parameters: None : Ø
Output : Node[] candidates, Node[] leftovers

# Initialize structures;
Node[] candidates = [], Node[] leftovers= [];
Node[] passing = celli,j,k.getFinishingTrajectories(stack);

# Select nodes containing trajectories with usable separations;
for Node test ∈ passing do
    for separation ∈ separations do
        # Get separations for Node;
        Separations[] nodeSeparations = test.getSeparations();
        # If trajectory given by buffer is on Separation plane;
        if isIn(isSeparation(test.buffer,separation)(6.26) then
            | candidates.append(test);
        end
    end
    # If there was no applicable separation, throw Node away;
    if test ∉ candidates then
        | leftovers.append(test);
    end
end

# Return results;
return [candidates, leftovers]

```

---

**Example:** for *Avoidance Grid* with *Distance 10 m*, *Layer count 10*, *Horizontal range*  $[-45^\circ, +45^\circ]$ , *Horizontal Cell Count 7*, *Vertical range*  $[-30^\circ, +30^\circ]$ , and *Vertical Cell Count 5*. Is given in (fig. 6.12). The UAS is at *Back-side of Figure* (initial state is at all *Trajectory Origins*). The *black dashed line* marks *Avoidance Grid space boundary*. Each trajectory has its own color and ends at *Front-side of Avoidance Grid Boundary*.

*Full separation mode* given in (fig. 6.12a). *Horizontal-Vertical separation mode*, used in original *ACAS-Xu* testing [19], given in (fig. 6.12b). *Horizontal separation mode* given in (fig. 6.12c) is usually used by planes. *Vertical separation mode* given in (fig. 6.12d) is usually used by copters.

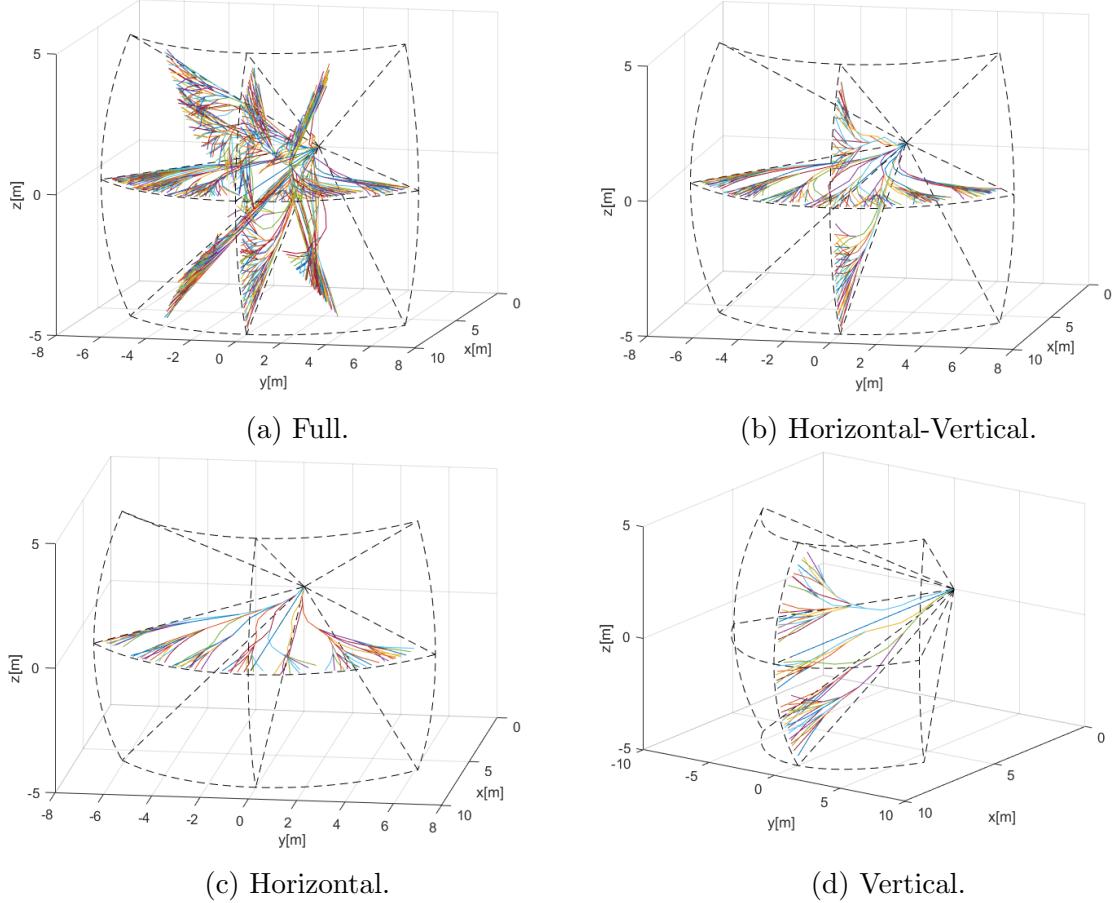


Figure 6.12: ACAS-X imitation *reach set* approximation for various *separation modes*.

**Pros and Cons:** It can be seen from examples (fig. 6.12) that *ACAS-like Reach Set Approximation Method* (alg. 6.5) generates full reach set for 2D plane located in 3D space.

The *Reach Set* contains trajectories with *high coverage ratio* and *high smoothness rating* for selected 2D separation plane. Overall performance compared to full 3D reach sets (sec. 6.6.4, 6.6.5 6.6.6) is poor.

The *node* and *trajectory* count boundary was not implemented. It is common knowledge that *2D* avoidance sets does not require scaling [19]. Otherwise trajectory footprint mechanism like in *Harmonic Reach Set Approximation* (alg. 6.3) can be introduced.

This reach set implements *Planar-Separation* as native feature, it can be used for both *navigation* and *avoidance* tasks in *Controlled Airspace*. For *Non-controlled Airspace* there are far more superior *Combined Reach Set* (sec. 6.6.6).

## 6.7 (W) Avoidance concept

### 6.7.1 (W) Overview

Emergency avoidance algorithm overview:

- Block scheme with used components, to show context and data flow
- Decision time introduction,
- Outer/inner loop distinction ...

- Platform independence

### **6.7.2 (W) UAS model and control**

- UAS mathematical model requirements outline
- Movement set requirements and constraints
- Control interface notion

#### **Used model**

Used model in framework

#### **Movement automaton predictor**

Used movement automaton for given model

### **6.7.3 (W) Avoidance grid run**

- Avoidance grid and data fusion procedure (sensor/information source)
- Obstacle rating calculation - visibility, map, constraints, detected obstacles, long term weather
- Intruder rating calculation - intruder intersection models, dynamic constraints, short term weather
- Reachability rating calculation for trajectory
- Reachability rating for cell calculation
- Feasible avoidance cells selection process
- Waver procedure - when something goes wrong
- Low cost sensor fusion [126].
- Concat trajectory search semioptimal like ours [127].

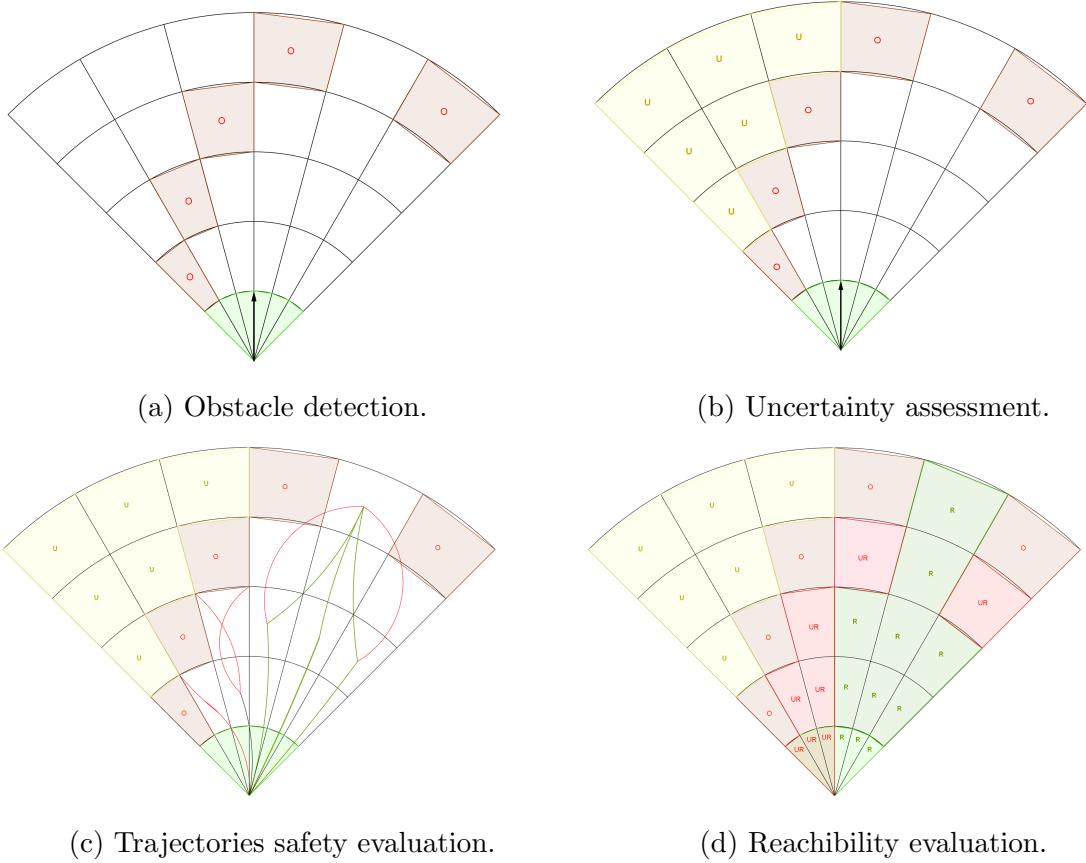


Figure 6.13: Significant steps of *Avoidance grid run* (inner loop).

#### 6.7.4 (W) Mission control run

Cover algorithm outer run in multiple decision frames (As bonus mark decision points for rule engine *decision points*):

- Waypoint reach condition
- Waypoint waver condition (when to give up on chasing unreachable dream)
- Trajectory selection criteria in inner avoidance run
- Complete mission control run, including decision invocation conditions
- Basically MissionControl.runMissionOnce() function in detail
- The navigation concept taken from [128, 129].

Decision invocation conditions:

- Empty movement Buffer (go to Emergency mode)
- New static obstacle/constraint detection (go to Emergency mode)
- Active moving obstacle (Depending on mode)
- In Cooperative Navigation mode - Avoidance grid not empty

- In Emergency avoidance mode - Avoidance grid is empty

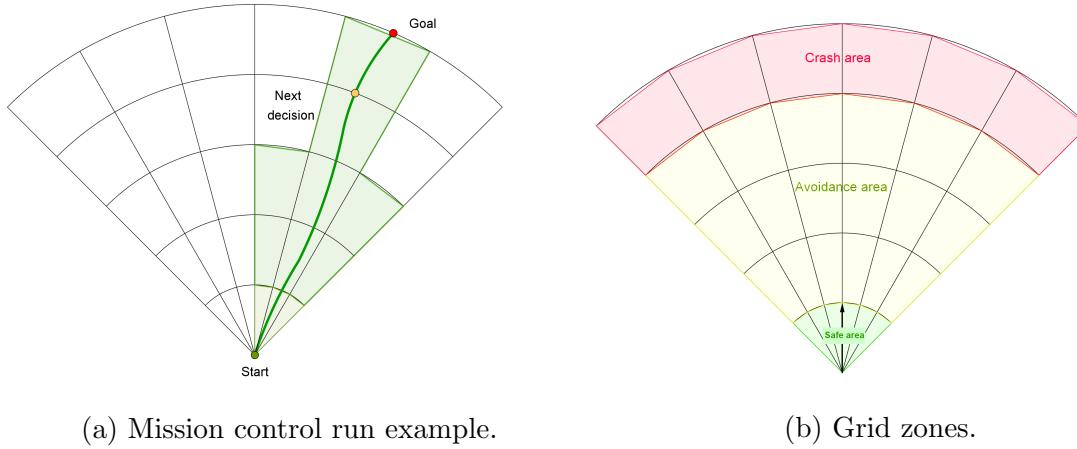


Figure 6.14: Definitions for *Mission Control Run* (outer loop).

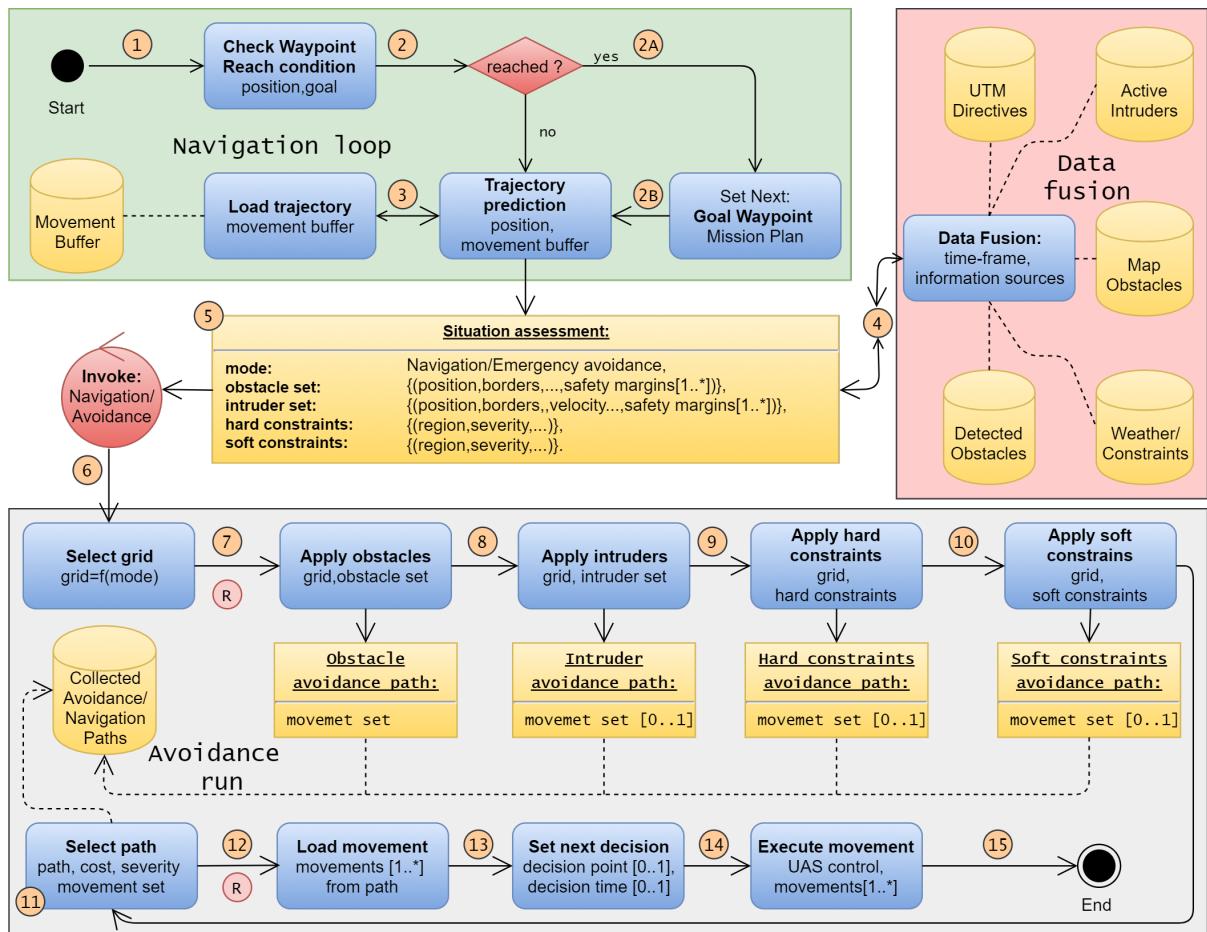


Figure 6.15: Mission control run activity diagram.

### 6.7.5 (W) Safety margin calculation

How to assess safety margin and what impacts it:

- Own uncertainty
- Intruder uncertainty
- Weather
- Airspace
- UTM corrections

Note *Both UAS* decision times were *synchronized*, this is not an assumption, but it shows critical performance. Usually safety margin is bloated for:

$$safetyMarginBloat = \begin{pmatrix} intruderVelocity \times \dots \\ intruderDecisionFrame \end{pmatrix} [m, ms^{-1}, s] \quad (6.27)$$

## 6.8 (W) UAS Traffic Management

Introduction to UTM problematic and base frame required for rules of the air implementation:

- standard centralized, space clustered architecture known from ATM.
- EU plan outline for UTM functionality.
- position notification, weather notification, collision case brief mention.
- What is role of authority and why UTM system is necessary.
- Necessity for integration into national airspace [130].

### 6.8.1 (R) Architecture

**UTM concept** is based on *asynchronous event-based control* [131]. *Event* in *controlled airspace* is handled in form of *cases* [132]. There are following *event sources*:

1. *Weather Information Service* (from [133]) - used to create *weather case* (tab. 6.3).
2. *Position Notification from UAS systems* (tab. 6.1) - used to create *collision cases* (new functionality) (tab. 6.2).

**Decision frame** (eq. 6.28). The *UTM* is operating in discrete decision frames which are starting on current *decision time* and ending at next *decision time*:

$$decisionFrame_i = [decisionTime_i, decisionTime_{i+1}[ , \quad i \in 1, \dots, k, k \in \mathbb{N}^+ \quad (6.28)$$

**Event based airspace control** is collecting events in previous  $decisionFrame_{i-1}$  and issuing commands in current  $decisionFrame_i$ . There are following phases during the *UTM frame* cycle:

1. *Planning* - the detection phase, when the hazardous situations are assessed.
2. *Fulfillment* - the monitoring phase, when the state of affairs for directives and mandates is full filled by controlled UAS systems.
3. *Acknowledgement* - the closing phase, when UTM assess and acknowledges the performance of controlled UAS systems.

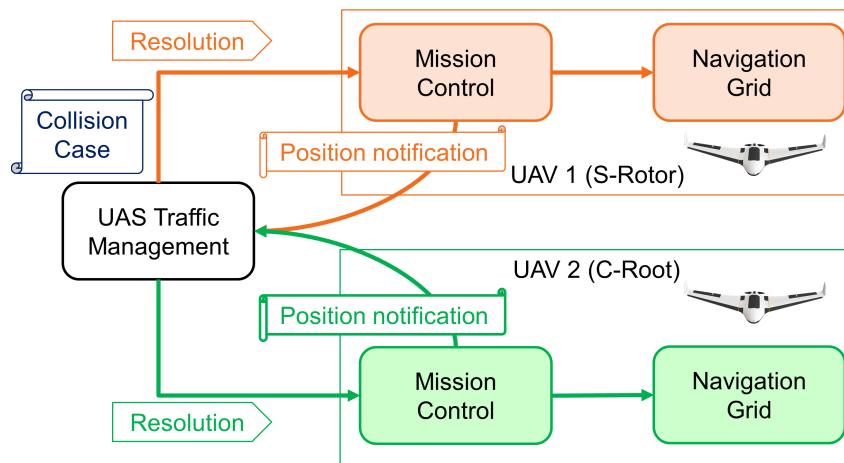


Figure 6.16: UAS Traffic (UTM) Management architecture overview.

**Architecture:** (fig. 6.16). There are multiple UAS systems equipped with standard *Mission Control* and *Navigation* procedures.

Depending on the *airspace cluster* decision time frame they are sending *periodical position notifications* (tab. 6.1).

The *UAS Traffic Management* (UTM) collects the event data from *Weather Information Service* and *Position Notifications* calculating respective *cases*.

If there is an *active collision/weather case* the *UTM* will send *resolutions* to respective airspace attendants.

### 6.8.2 (R) Cooperative Conflict Resolution

**Idea:** There is a *final decision maker* (absolute authority) in conflict resolution. This authority is *UTM* or *air traffic attendant* with higher priority. The future *UTM system* is such authority. The approach to mixed conflict resolution is mentioned in [134], based on navigation [135]. This is similar to our approach.

*Note. Open Issue:* Decentralized model with UTM as approver of directives is possible, but that is topic for own research.

**Goal:** UAS is obligated to follow up committed mission plan with given precision. There is one to five percent allowed deviations for ATM mission plans. Similar rates are achievable according to [134]. This requirement is given by [136] ICAO 4444 document for ATM operations.

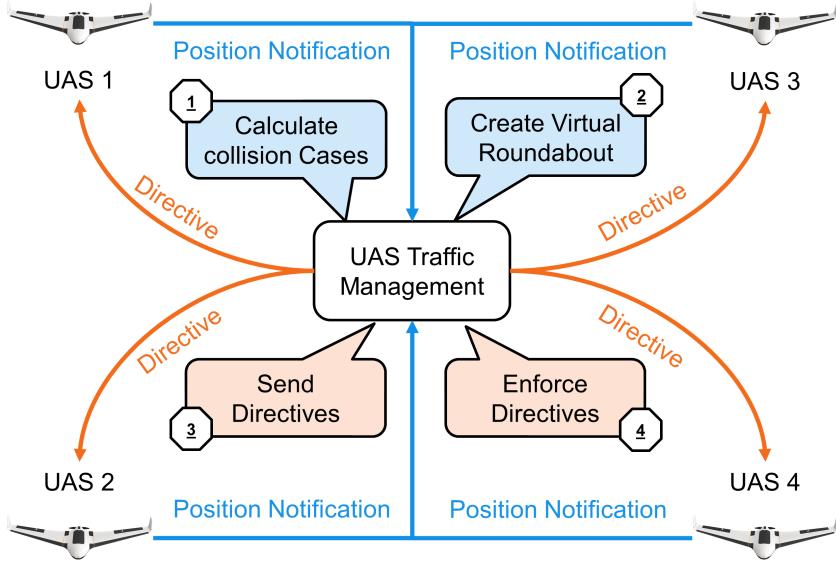


Figure 6.17: Cooperative conflict resolution via UTM authority.

**Cooperative conflict resolution** (fig. 6.17) shows functional diagram of one *UTM time-frame* there are following actors:

1. *Unmanned Autonomous System* (UAS) equipped with necessary navigation and communication modules, providing the unique *identification number*.
2. *UAS Traffic Management* (UTM) posing as central authority for given *airspace cluster*.

The following steps are executed during *Cooperative conflict resolution*:

1.  $UAS_* \rightarrow UTM$  *Send position notification* - each *UAS* is notifying the authority (UTM)
2.  $\circlearrowleft UTM$  *Calculate collision Cases* - UTM gathers data and predicts possible collisions then it tries to link them and manage the situation.
3.  $\circlearrowleft UTM$  *Create virtual Roundabout* - active collision cases are aggregated into virtual roundabout.
4.  $UTM \rightarrow UAS_*$  *Send directives* - UTM sends commands to UAS systems whom needs to change their planned trajectories.
5.  $UTM \rightarrow UAS_*$  *Enforce directives* - UTM is periodically checking constraints imposed in previous *decision frames*.

### 6.8.3 (R) Non-Cooperative Conflict Resolution

**Idea:** There is *main UAS(1)* which is flying in open *non-controlled airspace*. There are other *UAS* operating in its vicinity. It is expected that they are claiming their *planned trajectories*. The *Main UAS(1)* detects the collision with other *UAS(2-4)*.

There is no *final decision maker* nor *supervising authority*, all communication participants have similar level of rights.

*Note.* There is assumption that other airspace users are behaving like intruders, without intent to destroy or harm. The *adversarial behaviour* is not accounted. The response from *intruder* is not mandatory in *non-controlled* airspace.

**Goal:** Provide *mutual avoidance mechanism* in *non-controlled* airspace. Considering the equal standpoint of all airspace attendants.

**Conflict resolution:** The conflict resolution depends on current mode and *handshake* between airspace attendants. The non-cooperative behaviour have been implemented like follows:

1. *Navigation mode* - every *airspace attendant* is calculating own *collision cases* and checking the behaviour of the other (virtual UTM).
2. *Emergency avoidance mode* - is depending on communication mode:
  - a *Response mode* - claiming separation methods and using avoidance mechanism (Avoidance grid with intruder model in our case).
  - b *Blind mode* - every conflict side picks own strategy respecting given *rules of the air*.

*Note. Intruder Intersection model selection:* UAS based on Event detects possible collision for some reason UTM directive is out of question, then try to claim separation (body volume intruder model (sec. 6.5.3)), If separation fails, go full survival mode (uncertain intruder model (sec. 6.5.3)).

**Special cases in manned aviation:** There are IFALPA reports which can give us overview of *enforced non-cooperative* mode causes in *controlled airspace*:

1. *VFR disabled* - flying in fog or thick clouds can render pilot vision, similar to UAS cameras/LiDAR.
2. *IFR equipment broke* - the sensor malfunction is more likely to happen due the lesser redundancy in UAS systems.
3. *C2C Link disabled* - communication loss is more likely to happen, due the lesser redundancy.
4. *ATM failure* - the ground control module of UTM can also fail.

*Note.* Traffic management related fails are lesser than 0.001 cases per one flight (according to IFALPA [137]).

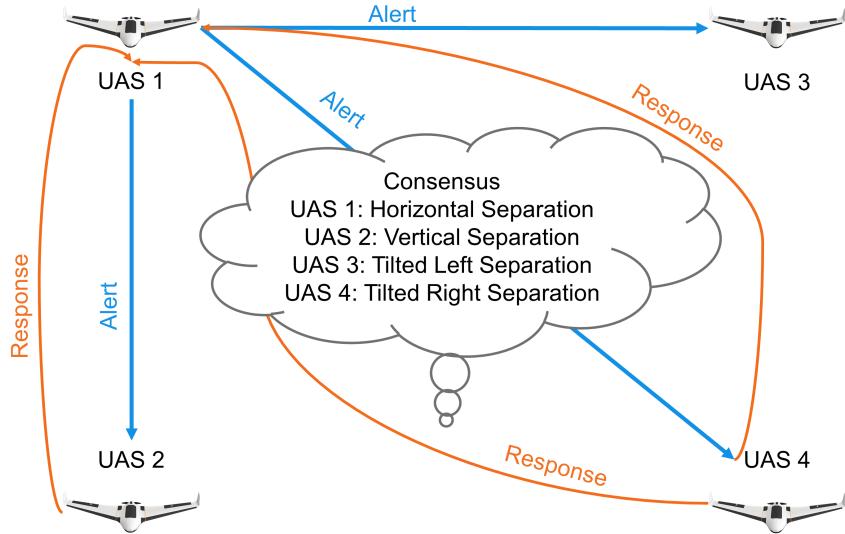


Figure 6.18: Non-cooperative conflict resolution via UAS claims.

**Response mode scenario example:** The *main UAS(1)* is going to collide with other *UAS(2-4)*:

1. *UAS(1)* → *UAS(2 – 4)* sends position and heading notification.
2. Ⓢ *UAS(2 – 4)* calculates possible collision.
3. *UAS(2 – 4)* → *UAS(1)* sends response to the *main UAS(1)* with claimed separation mode.
4. Ⓢ *UAS(1)* acknowledges proposed *separation modes*.
5. Ⓢ *UAS(1 – 4)* avoids each other using claimed separation mode, because every *UAS* achieved *consensus*.

#### 6.8.4 (W) Handling Head on Approach

- Describe VFR manuever in details
- Emphatize on IFR required parameters - angle of approach range, safety margin
- introduce virtual roundabout idea - minimizes wake turbulence and enables thick attendants flow on virtual airline in selected flight level (this is actually one of Boeing concepts for next gen ATM).
- Refer to conditions and triggers from collision case resolution algorithm.
- I prefer structure-process flow, due to my software viewpoint bias, this section can be shifted before 6.8.8

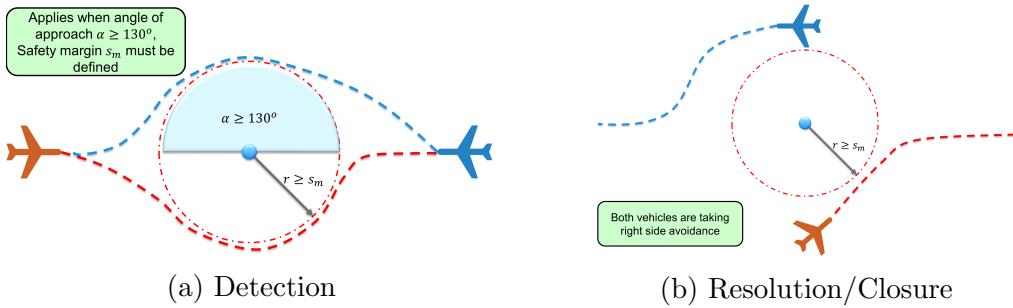


Figure 6.19: Head on approach detection/resolution/Closure

### 6.8.5 (W) Handling Converging Maneuver

- Same as Head on approach for most part,
- Introduce tilt right concept from highways, emphasize the wake turbulence impact based on angle of approach,
- lesser angle of approach means stronger wake turbulence effect and need to increase or adjust safety margin
- Wake turbulence is represented as droplet at the back of the plane and can be calculated based on wake turbulence cone from collision case

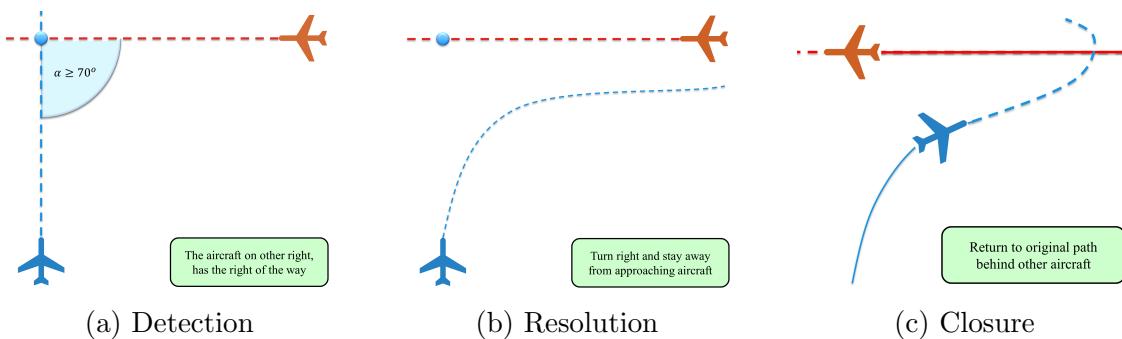


Figure 6.20: Converging maneuver detection/resolution/Closure

### 6.8.6 (W) Handling Overtake Maneuver

- Emphasize on overtake fragility, there is different speed in controlled airspace,
- Fact - most of traffic attendants at same flight level have same cruising speed, de facto lower flight levels are for slower turbo-prop planes and higher altitudes are for jet planes. It is stated that this principle will persist even when UAS will be integrated [138, 139, 140] via multiple air-traffic models.
- Overtake is borderline emergency maneuver, most of conflicts should be solved via virtual round-abounds

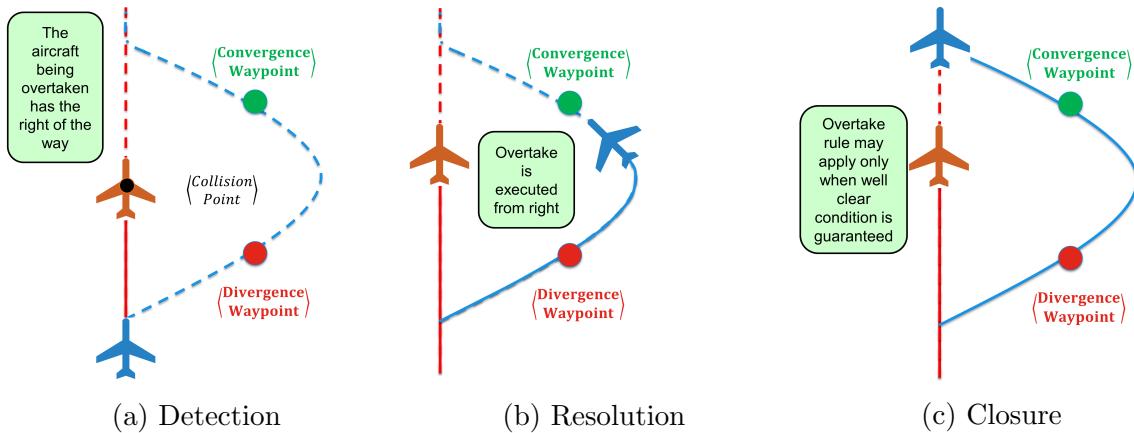


Figure 6.21: Overtake maneuver detection/resolution/Closure

### 6.8.7 (W) Position Notification

Process of creation for data structure from table 6.1.

Position	
latitude	based on GPS/IMU sensor fusion.
longitude	based on GPS/IMU sensor fusion.
altitude	barometric altitude <i>Above Mean Sea Level (AMSL)</i> .
Heading	
orientation	orientation in standard North-East coordinate frame.
velocity	relative UAS velocity.
Flight Levels	
main	flight level, where UAS mass center belongs
passing	flight level, during climb/ascend, or when distance of UAS mass center to flight level boundary $\leq 250ft$ .
Registration	
registration ID	is unique registration number <i>to be issued</i> by local aviation authority for UTM communications purposes.
flight code	or mission code is unique identification number for approved mission plan which is going to be flew by UAS.
UAS name	optional UAS identifier to increase human recognition.
Categorization	
craft category	ICAO main category, based on vehicle type.
maneuverability	secondary categorization specifying size class, horizontal/vertical turning radius, minimal and maximal cruising speed.
Safety margins	
universal	minimal safety margin for any avoidance situation
head on	minimal distance from other similar maneuverability class aircraft in case of head on approach.
converging	minimal distance from other similar maneuverability class aircraft in case of head of converging maneuver.
overtake	minimal distance from other similar maneuverability class aircraft in case of overtake maneuver.
wake angle	for wake turbulence cone.

wake radius | for wake turbulence cone.

Table 6.1: Time-stamped *position notification* structure.

### 6.8.8 (R) Collision Case

**Collision Case Purpose:** There is a need for detection and tracking of possible *controlled airspace traffic attendants* collisions. The presented *collision case structure* (tab. 6.2) is minimalist reflection of *ATM* requirements. Following aspects of *collision case* life cycle are explained in this section:

1. *Base terminology* - the definition of *enforcement procedure* and difference between *Resolution* and *Mandate* from UTM authority. The *severity issue* is open.
2. *Calculation of single case for single decision frame* - step by step calculation and threat evaluation. Prequel to *life cycle*.
3. *Life cycle* gives outlook how collision case data are handled through longer period of time, notably: *Opening*, *collision point handling*, *safety margin handling*, and, *Closure*.
4. *Merge procedure for multiple cases in single cluster* - the naive *merge procedure* to solve *multiple collision cases* via *virtual roundabout*.

**Resolution/Mandate Enforcement:** *Enforcement procedure* is consisting from *Threat detection phase* and *Mitigation phase*. The *migration phase* is a time interval when *UTM* decision is enforced. The decision the UTM is enforcing is delivered in form of *Resolutions* and *Mandates*.

*Resolution* is an order from the *UTM* authority which is followed by subjected UAS. The *subjected UAS* can determine own behaviour to some extent. When there is emerging threat or other destructive event, like new non-cooperative adversary, the UAS is allowed to break *resolution*.

*Mandate* is an order from the *UTM* authority which can not be broken at any cost. The example of the *mandate*: UAS is flying in airspace, the passenger in distress needs it to safely land. The UAS must obey mandate even at event of own destruction.

**Threat Severity Evaluation:** The threat severity evaluation is omitted partially, all threats are considered as equal. All commands from *UTM authority* will be considered as *resolutions*.

**Calculation procedure:** Collision case is calculated for two *Registered UAS systems* in *Unified UTM time-frame*. The *unified UTM time-frame* is a short period of time in future when the anticipated situations are predicted.

**1<sup>st</sup>** The *position* and *orientation* is adjusted according to *mission plan*. Our implementation uses *Movement Automaton* as a predictor:

$$\begin{aligned} \text{adjustedPosition} &= \text{Position}(\text{Trajectory}(\text{notifiedState}, \text{futureMovements})) \\ \text{adjustedOrientation} &= \text{Orientation}(\text{Trajectory}(\text{notifiedState}, \text{futureMovements})) \end{aligned} \quad (6.29)$$

For other cases standard linear prediction can be used:

$$\begin{aligned} \text{adjustedPosition} &= \text{notificationPosition} \times \text{notificationVelocity} \times \text{timeDifference} \\ \text{adjustedOrientation} &= \text{notificationOrientation} \end{aligned} \quad (6.30)$$

**2<sup>nd</sup>** The *maneuverability*, *craft category*, *registration ID* are taken from *position notification*.

**3<sup>rd</sup>** *Collision case check procedure* goes like follows:

1. *Operation space checks* - the controlled airspace and flight level must match for proceeding.
2. *Maneuverability/Category check* - the maneuverability and UAS category must match. If there is mismatch then right of the way is forced to vehicle with higher priority.

**4<sup>th</sup>** *Linear Intersection test* is designed to calculate *closest distance* and *time* of *linear trajectory projections*, First for given *velocity* and *position* for UAS1 and UAS2 the helper variables are calculated:

$$\begin{aligned} A &= \|\text{velocity}_1\|^2 \\ B &= 2 * (\text{velocity}_1^T \times \text{position}_1 - \text{velocity}_2^T \times \text{position}_2) \\ C &= 2 \times \text{velocity}_1^T \times \text{velocity}_2 \\ D &= 2 * (\text{velocity}_2^T \times \text{position}_2 - \text{velocity}_2^T \times \text{position}_1); \\ E &= \|\text{velocity}_2\|^2; \\ F &= \|\text{position}_1\|^2 + \|\text{position}_2\|^2; \end{aligned} \quad (6.31)$$

Then the projection parameters can be calculated:

$$\begin{aligned} \text{time} &= \frac{-B - D}{2 \times A - 2 \times C + 2 \times E} \\ \text{destination}_i &= \text{position}_i + \text{velocity}_i \times \text{time}, \quad i \in \{1, 2\} \\ \text{collisionPoint} &= \frac{\text{destination}_1 + \text{destination}_2}{2} \\ \text{collisionDistance} &= \|\text{destination}_1 - \text{destination}_2\| \end{aligned} \quad (6.32)$$

If  $\text{time} < 0$  the trajectories are diverging from each other (because the closest points already occurred). The procedure ends, *collision flag* is not raised.

If  $\text{time} > \text{timeMargin}$  the trajectories will get close to each other, but in further future and changes are anticipated. The procedure ends, *collision flag* is not raised.

If  $0 \leq \text{time} \leq \text{timeMargin}$  the trajectories are converging to each other and distance needs to be checked. If  $\text{distance} \leq \text{collisionMargin}$  then *collision flag* is raised and *collision point* is set.

*Note.* *Collision Margin* is some number which is determined based on aircraft category and maneuverability. Our work defines collision margin as follow:

$$\text{collisionMargin} = \forall \text{situation} : \max \left\{ \begin{array}{l} \text{safetyMargin}(\text{situation}, \text{UAS1}) \\ + \text{safetyMargin}(\text{situation}, \text{UAS2}) \end{array} \right\} \quad (6.33)$$

Where *safety margin* for every possible situation is evaluated for both *UAS*.

**5<sup>th</sup>** The *trajectory intersection* is *Movement Automaton* specific collision detection method. Its based on the assumption that *UTM* have following information from *mission plan*:

1. *UAS state* - not only *position*, *orientation*, and, *velocity* vectors, but other mathematical model parameters mandatory for *movement automaton*.
2. *Movement Automaton* - movement automaton for our UAS system, so UTM can use it in predictor mode.
3. *Future Movements set* - up to reasonable prediction horizon *timeMargin*.

The *Movement Automaton* can be used as trajectory prediction for system initial state and future movements. The prediction function (eq. 6.34).

$$\text{Prediction} : \text{UAS} \times \text{state} \times \text{futureMovements} \rightarrow [x, y, z, t] \in \mathbb{R}^4 \quad (6.34)$$

*Note.* Then prediction for UAS1 is *Prediction*<sub>1</sub> and for UAS 2 *Prediction*<sub>2</sub>, the predictions are synchronized meaning that time at position *i* is equal in both discrete trajectory matrices.

The *collision distance* for predictor (eq. 6.34) is given as minimal distance of projected synchronized trajectories for UAS1 and UAS2. In our discrete environment the *collision distance* is given as (eq. 6.35).

$$\text{collisionDistance} = \min \left\{ \| \text{point}_1 - \text{point}_2 \| : \forall \begin{pmatrix} \text{point}_1 \in \text{Prediction}_1, \\ \text{point}_2 \in \text{Prediction}_2, \\ t_1 \sim t_2 \end{pmatrix} \right\} \quad (6.35)$$

If *collisionDistance*  $\leq$  *collisionMargin* condition is met, *collision flag* is set.

The collision point is then calculated as mean of *UAS positions* in prediction at time when distance is minimal. The final collision point is arithmetic mean of two positions (eq. 6.36).

$$\text{collisionPoint} = \frac{\text{point}_1 - \text{point}_2}{2} : \begin{pmatrix} \text{point}_1 \in \text{Prediction}_1, \\ \text{point}_2 \in \text{Prediction}_2, \\ t_1 \sim t_2 \text{ at minimal distance} \end{pmatrix} \quad (6.36)$$

*Note.* Collision point is overwritten by trajectory intersection (specific) method, the *linear intersection* is considered *general collision detection method*. The collision detection method in future UTM system needs to be determined. The *Trajectory intersection* method presented in this work is one of the possible candidates.

**6<sup>th</sup>** *Role determination* phase is invoked if and only if previous conditions are met and *collision flag* with *collision point* exists.

There is *adjusted position* of each UAS used as verticals and *collision point* used as center. First step is normalization of adjusted position around collision point for both UAS:

$$normalized_i = adjustedPosition_i - collisionPoint, \quad i \in \{1, 2\} \quad (6.37)$$

Then the right hand coordinate system internal angle calculation method is used:

$$angleOfApproach = \left| \text{atan2} \left( \begin{array}{c} normalized_1 \times normalized_2, \\ normalized_1 \circ normalized_2 \end{array} \right) \right| \quad (6.38)$$

Based on *angle of approach* the *scenario type* is decided like follows:

1.  $130^\circ \leq angleOfApproach \leq 180^\circ$  - the scenario type is set as *Head On Approach* (sec.6.8.4)
2.  $70^\circ \leq angleOfApproach < 130^\circ$  - the scenario type is set as *Converging Maneuver* (sec.6.8.5)
3.  $0^\circ \leq angleOfApproach < 70^\circ$  and *different speed* - - the scenario type is set as *Overtake Maneuver* (sec.6.8.6)

Based on *relative position* and *scenario type*, the *avoidance role* like follows:

1. *Head On Approach* enforces following:
  - a. The *avoidance role* us set as *RoundAbounting* for both UAS.
  - b. None of the *UAS* does not have the *Right Of the Way*.
2. *Converging Maneuver* enforces following:
  - a. *UAS* without free right side have role set as *Converging*.
  - b. *UAS* with free right side have the *Right Of the Way*.
3. *Overtake Maneuver* enforces following:
  - a. *Slower UAS* has *Overtaken* role with *Right Of the Way*.
  - b. *Faster UAS* has *Overtaking* without *Right Of the Way*.
  - c. *Faster UAS* mission plan is altered with *divergence and convergence waypoints*.

**7<sup>th</sup>** *Safety Margin Calculation* Is invoked when the collision case is *Active*. The *Active Collision Case* in this time-frame means that *Collision Flag* is raised. The *avoidance role* determines *safety margin calculation*.

If *Head On Approach* is case type of *Head collision case* then *safety margin* is calculated as maximum of sum of *default* margins or *head on* margins:

$$safetyMargin = \max \left\{ \begin{array}{l} default(UAS1) + default(UAS2), \\ headOn(UAS_1) + headOn(UAS_2) \end{array} \right\} \quad (6.39)$$

If *Converging Maneuver* is case type of *Head collision case* then *safety margin* is calculated based on *avoiding UAS* as maximum of opposing UAS *default margin* and *avoiding converging margin*:

$$safetyMargin = \begin{cases} uas1.role = Converging : \max \left\{ default(UAS2), converging(UAS1) \right\} \\ uas1.role = Converging : \max \left\{ default(UAS1), converging(UAS2) \right\} \end{cases} \quad (6.40)$$

If *Overtake maneuver* is case type of *Head collision case* then *safety margin* is calculated as maximum of *default*, *overtaking*, *overtaken* margins of both UAS:

$$safetyMargin = \max \left\{ default(UAS1), default(UAS2), overtaken(UAS1), overtaking(UAS2), overtaking(UAS1), overtaken(UAS2) \right\} \quad (6.41)$$

**Collision Case Chaining** is procedure when multiple active collision cases for different *time-frame* are chained and creates the time ordered series of *collision cases*. There are two notable instances in the *chain*:

1. *Head Collision Case* - Collision case when the first danger was detected. The notable parameters are *collision point* and UAS *avoidance roles*, because these are enforced by *Rule engine* (sec. 6.9). The *head collision case* is first in chain.
2. *Tail Collision Case* - Collision case when the *collision danger* was not detected. The *tail collision case* is last in the chain.

*Note.* The *Chaining of collision cases* is rather primitive and sensitive for errors/noise.

The *Consistency of Avoidance Manuever* is ensured by enforcing *head collision case* parameters.

**Collision Cases Merge** also known as *Collision Point Adjustment Procedure* purpose it to *merge* multiple collision cases into one general collision case. The clustering is used to identify *airspace congestion events* [141]. Example of *airspace clustering* is given in [142].

The main idea is to *encapsulate multiple collision cases* into one virtual roundabout to ease *traffic load* [143]. The potential risk on *turbo roundabouts* have been outlined in [144].

There are *active collision cases* in focused *cluster* in *controlled airspace*. The multiple collision cases can pop up in different *start times* and they can be active for different *time period*.

The *Collision point* is replaced with *roundabout center* point (eq. 6.42). The *roundabout center* is calculated as weighted average of *active collision cases* collision points. The *weight*  $\in [0, 1]$  depending on severity rating of collision case.

$$roundaboutCenter = \frac{\sum_{\in Cluster}^{\forall collisionCase} collisionCase.collisionPoint \times weight}{|collisionCase \in Cluster|} \quad (6.42)$$

*Note.* The weight in (eq. 6.42) is set to 1 for all time, the weight calculation needs to be determined in future works.

The *smallest circle problem* defined and solved in [30, 145] is used to determine safety margin in our approach. The *naive approach* determining *roundabout safety margin* is to take the maximum of all open case *safety margins* including default ones (eq. 6.43).

$$safetyMargin = \max \left\{ \begin{array}{l} case.UAS_i.roundaboutSafetyMargin, \\ case.UAS_i.defaultSafetyMargin \end{array} \right\},$$

$$\forall case \in Cluster, \quad UAS_i \in \{1, 2\} \quad (6.43)$$

*Note.* The *naive approach* to *roundabout* safety margin is bloated, and do not respect original collision points. The minimal circle problem is minimal roundabout design. The issue if there is feasible dynamic for all roundabout attendees is not addressed.

The parameters of single *Collision Case* are given by (tab. 6.2):

<b>Data for both attendants</b>	
adjusted position	predicted from previous <i>position notifications</i> (6.1) data at time of <i>UTM decision frame</i> start.
adjusted orientation	predicted from previous <i>position notifications</i> (6.1), <i>mission plan</i> , and <i>expected velocity</i> .
velocity	proclaimed velocity for given <i>UTM decision time frame</i> .
registration ID	is unique registration number issued by local aviation authority from <i>position notifications</i> (6.1).
craft category	from <i>position notifications</i> (6.1).
maneuverability	from <i>position notifications</i> (6.1).
mission plan	is acquired from <i>allowed mission registers</i> where it has been registered prior UAS flight
safety margins	list of all safety margins, derived based on craft categorization or overridden by <i>position notifications</i> (6.1).
avoidance role	is given based on situation evaluation.
trajectory prediction	simulated based on <i>position notification</i> (6.1) and <i>mission plan</i> .
<b>Collision case calculated data</b>	
linear intersection	is predicted on attendants <i>position</i> , <i>heading</i> , <i>velocity</i> , based on <i>maneuverability</i> certain thresholds are applied to determine safety properties.
trajectory intersection	is predicted on attendants <i>position</i> , <i>velocity</i> , <i>heading</i> , and <i>related mission plans</i> , based on <i>maneuverability</i> certain thresholds are applied to determine safety properties.
collision point	is created if there is risk of medium/short time period collision, if head collision case has not been closed, collision point is inherited.
adj. collision point	is created if there exists at least one active collision case in nearby surroundings of this case collision point (cluster).
angle of approach( $\alpha$ )	is calculated based on attendants <i>velocity</i> and <i>position</i> , range is $[0^\circ, 180^\circ]$ , it determines <i>primary avoidance roles</i> .
safety margin	is calculated based on <i>avoidance roles</i> , <i>maneuverability</i> , collision indicators, and <i>angle of approach</i> .

margin adjustment	is calculated based on <i>linked collision cases, estimation errors and weather</i> .
linked cases	contains list of collision cases which are active and can have impact on this <i>collision case</i> .
head case	is reference to collision case in time frame when it was first opened.
<b>Collision case indicators</b>	
linear intersection	indicates if there was safety breach on linear trajectories estimation with risk of direct collision.
trajectory intersection	indicates if there was breach on trajectory estimation, with risk of direct collision.
well clear breach	indicates if <i>linear projection</i> or <i>trajectory projection</i> breaches <i>well clear barrel</i> in <i>controlled airspace</i> .
active case	indicates if case is still open.

Table 6.2: Collision case structure for given decision time-frame.

### 6.8.9 (W) Weather Case

Introduce the weather constraint structure given in table 6.3, discuss following topics:

- The convex polygon boundary - most of navigation uses S curve algorithm
- The difference between static/dynamic (moving) boundary
- Elaborate weather rules system - very easy, reuse divergence/convergence waypoint system from overtaking, other solutions are not quite deterministic.

<b>Constrained area</b>	
center position	is given as geometrical <i>center point of boundary</i> .
boundary	is represented as <i>convex polygon</i> on latitude-longitude plane.
start altitude	is lower boundary barometric altitude given at above mean sea level, where given weather factor have significant impact.
end altitude	is upper boundary barometric altitude given at above mean sea level, where given weather factor have significant impact.
<b>Dynamic parameters</b>	
type(s)	lists weather events occurring in <i>constrained area</i> .
severity list	is recorded for each plane <i>category</i>
start	indicates when weather constraint was established.
expected end	of weather constraint.
velocity	indicates if weather phenomenon is moving.
<b>Miscellaneous</b>	
previous impacted	reference to <i>weather constraint</i> decision time-frame data. list of possibly impacted attendees (planes whom obtained divergence order or warning from UTM).

Table 6.3: Static/Dynamic weather constraint for given decision time-frame.

## 6.9 (W) Rule Engine

This section is follow up of *UTM section*, because it is realization of *UTM directives* on *UAS* side.

- Mention aspect oriented programming principle of joint - decision point
- Emphasize configurable process (template programming) - different rules are valid for different airspace, countries, ... they needs to be dynamically woven into the existing hard coded processes.
- Introduce architecture and then selected set of rules for implementation.
- Example of Maritime Rules implementation [146].

### 6.9.1 (W) Architecture

- Context - data structures which are accessed/modified by rule body
- Rule Engine - program module to inject and run rules
- Decision point - points in hard wired process where new behaviour can be connected usually with pre created rule context, but that can be dynamic, depending on DRM
- Rule engine is configurable via *Rule table* which says which rules and under what condition can be fired up.
- Rules are chain-able, from one rule body other rule can be fired up.

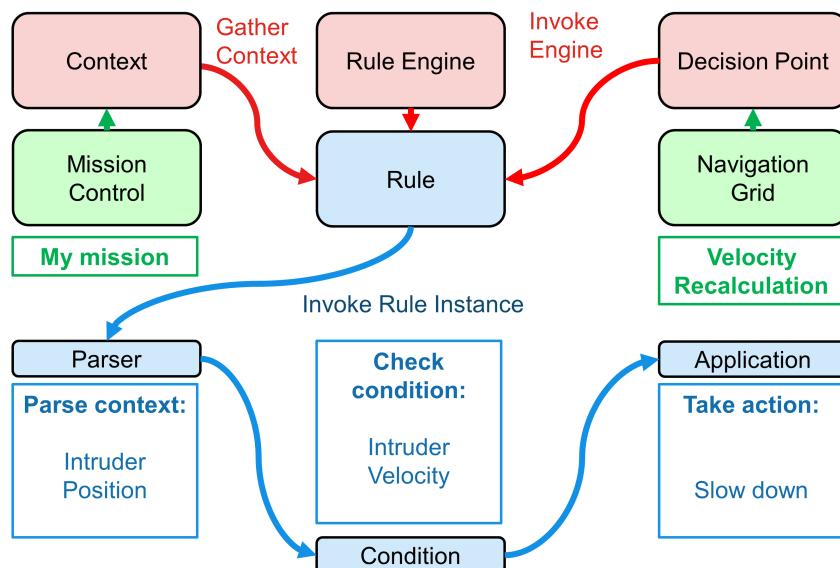


Figure 6.22: Rule engine components overview.

### 6.9.2 (W/R) Rule Implementation

- Make references to *decision points* in avoidance algorithm and navigation loop
- define context state based on *decision points*.
- Short roadmap to rules,

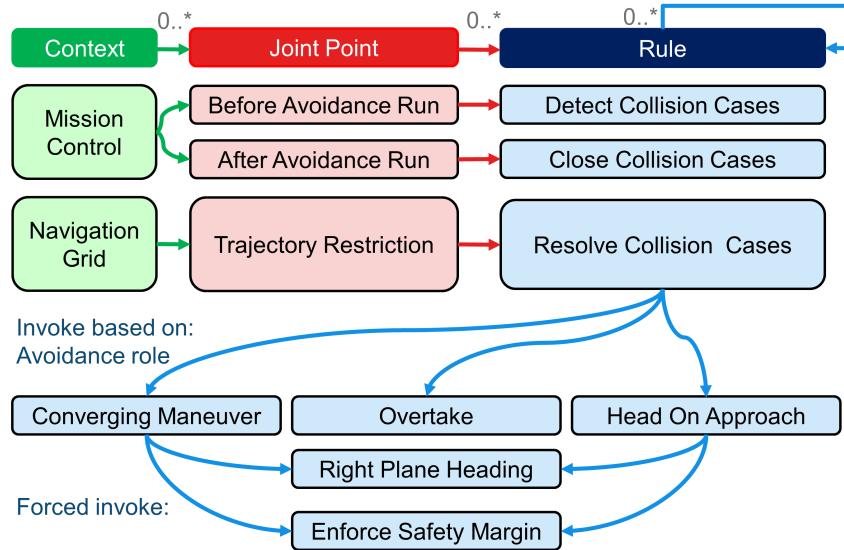


Figure 6.23: Rule engine initialization with Rules of the air.

#### Rule: Detect Collision Cases

This rule is activated each *UAS avoidance run*. *UTM* sent out all related *collision cases* (6.44) based on our *UAS identifier*. Creation of *collision case* is given in sec. 6.8.8 based on air traffic periodical *position notifications* (sec 6.8.7).

$$UTM \times timeFrame \rightarrow UTMCollisionCases \quad (6.44)$$

If there are available *position notifications* (sec 6.8.7) from surrounding air-traffic, UAS will calculate own *collision cases* (6.45).

$$uasStatus \times positionNotification \times utmTimeFrame \rightarrow UASCollisionCases \quad (6.45)$$

Then UAS merges *own collision cases* with *UTM collision cases*, if there exist following disparities UAS will take action:

1.  $distance(ownCollisionPoint, utmCollisionPoint) \geq threshold$ , send UTM notification, use *utmCollisionPoint*
2.  $utmMargin \geq ownMargin$ , use safety margin from UTM.
3.  $utmAvoidanceRole == active, ownAvoidanceRole == inactive$ , use UTM avoidance role.
4.  $utmCollisionCase == active, ownCollisionCase == uncertain$ , use UTM provided collision case, not all *position notifications* are available.

5.  $utmCollisionCase == inactive$ ,  $ownCollisionCase == active$ , notify UTM with new collision case, ignore collision case until UTM approves.

*Note.* *Avoidance role* is classified as *inactive* if and only if UAS has *right of the way*, it is classified as *active* otherwise.

*Safety margin* determined by UTM has priority, because not all calculations factors are available for UAS.

*Collision Case* unknown to UTM are ignored, due to safety reasons (false data spoofing), collision case is activated after UTM confirmation. If there is real intruder not confirmed by UTM it is handled via *non-cooperative* or *emergency* avoidance procedure

*Selection process* of active *collision cases* is based on UAS *avoidance role* in each *collision case*.

- If the *avoidance roles* are following: *Head On Approach*, *Converging Maneuver*, or *Overtake* in all *collision cases* UAS system will stay in cooperative mode.
- If there exists at least one *collision case* with *own avoidance role* or *intruder avoidance role* set as *emergencing*, the UAS will notify UTM and ask for *diversion order*, meanwhile it sets itself into *Emergency avoidance* mode.
- If there exist multiple *Overtake avoidance roles* or combination of *Overtake avoidance role* and *Other active role*, the UAS will decrease its cruising speed like follows:

$$UASSpeed = \max \left\{ \begin{array}{l} \text{minimalUASCruisingSpeed}, \\ \min \{intruderSpeed\} \quad \forall \text{activeCollisionCases} \end{array} \right\} \quad (6.46)$$

During *slow-down* UAS switches to *emergency avoidance mode* and asks for *divergence order* from UTM.

*Ordering of collision cases* starts if and only if the *UAS* is in *cooperative avoidance mode*. The cases are ordered for processing based on severity rating which is calculated based on:

1. *Safety Margin* - the greater safety margins are prioritized.
2. *Intruder vehicle class* - the more dangerous intruders are prioritized.
3. *Collision point distance* - closer collision points are prioritized.
4. *UAS avoidance role* - *Head on Approach* is favored upon *Converging maneuver*, due to direct collision severity.

*Rule engine invocation* for each *active collision case* is then applied on *descending severity sorted* list.

The rule is summarized in table 6.4.

---

*Invocation:* Every *Decision point* in *UAS main loop*

*Objective:*

1. Fetch UTM *Collision cases* for given decision time frame.
2. Create/update own *own collision cases* based on received *Position notifications* from surrounding *Intruders*.
3. Merge *Collision cases* based on *UTM priority order*.
4. Select active *collision cases* based on following conditions:
  - a. *Active participation* in *collision case* where *avoidance role*  $\neq$  *Right of the way*.
  - b. *Collision point* is int front of UAS.
  - c. *Emergency mode detection* there exist at least one non-cooperative participant.
5. Order *collision cases* based on *severity*.
6. If there is at least one *active collision case* enforce rule *Resolve collision case* (tab 6.5) for each *active collision case*.

<i>Context</i>	<i>Condition</i>	<i>Application</i>
UAS Mission control, Before Avoidance Run, UTM/UAS collision cases	Clean <i>avoidance grid</i> , No emergency	Active collision case selec- tion, Prioritization

Table 6.4: Detect collision cases rule definition.

### Rule: Resolve Collision Case

*Active collision cases* are processed one by one. All collision cases are applied to *Navigation grid*. *Navigation grid* contains all possible *trajectories* in form of *Reach set*. All *trajectories* are *reachable* at the beginning of UAS *avoidance frame*. Each application of *collision case resolution* rule disables some subset of feasible *trajectories*. For this reason are *active collision cases* sorted by severity.

It is assumed that UAS is in *cooperative avoidance mode*. If previous application of this rule forced UAS into *emergency mode* the rule is not applied to save system resources. *Emergency mode* is invoked if *rule application* disable all *trajectories* in *Navigation grid*. If there is at least one *feasible trajectory* in *avoidance grid* follow-up rule is invoked based on UAS *avoidance role*.

The rule is summarized in table 6.5.

*Invocation:* This rule is invoked if exists at least one *active collision case* in given *navigation grid time-frame*, moreover *avoidance grid* must be empty and *cooperative avoidance mode* is enforced.

*Objective:* Based on *active collision case* and *UTM directives* enforce behaviour based on *own avoidance role*:

1. *Head on approach* - rule 6.7.
2. *Converging maneuver* - rule 6.8.
3. *Overtake* - rule 6.9.
4. *Emergency mode* - switch from *active avoidance mode* to *emergency mode*.

Context	Condition	Application
UAS mission control, Trajectory restriction, Collision cases,	Active merged collision case, Resolution mandate from UTM	Enforce Rules of Air or Enforce emergency

Table 6.5: Resolve collision case rule definition.

### Rule: Close Collision Cases

*Collection of rule results* detected by rule 6.4 and *resolved* by rule 6.5 is done via *context of rule engine*. For each *time-frame* and each  $\text{trajectory} \in \text{NavigationGrid}$ , there exists rule engine *context query* (6.47) which returns *trajectory status* and *list of applied rules on trajectory*.

$$\text{Context}(\text{trajectory}, \text{timeFrame}) \rightarrow \{\text{State : Enabled/Disabled}, \text{Rule}(s)\} \quad (6.47)$$

*Calculation of possible trajectories* in *navigation grid* is using *collected rule results* (6.47). If the *trajectory state* and linked *rule reason* is sufficient, the *trajectory* is disabled for given *time frame*. *Standard navigation algorithm* (TBD - reference to section with outer navigation loop) is used to select *feasible trajectory*.

*Rules of the air* and their application in *General Aviation* cases is consistent. Increasing traffic density can impose new layers of rules, which may cause the *soft deadlock* in *maneuverability*. In this case *Navigation grid* will have all *possible trajectories* exhausted. Following procedure is executed:

1. UAS switch into *Non-cooperative avoidance mode* or *Emergency avoidance mode* depending on situation severity (One conflict can be handled with *vertical separation* of conflicting aircrafts).
2. UAS broadcasts *warning message* to all nearby aircrafts, and *separation message(s)* to conflicting aircraft. *Separation message* contains *expected collision point* and *preferred separation type*. Each conflicting aircraft then reacts and sends *action notification* to UTM.

- If UAS switches into *emergency mode*, non cooperative avoidance using *avoidance grid* is induced. Each relevant intruder is projected as *timed body volume intruder* (TBD reference to intruder probabilistic model), where *safety margin* is used as *body radius*.

*UAS* notifies *UTM* with *course change*, *planned avoidance trajectory*, *avoidance mode*. *UTM* approves planned changes or sends *plan corrections* (out of scope). The rule summary is given in table 6.6

<p><i>Invocation:</i> There exists at least one <i>active collision case</i> which had impact on <i>Navigation grid</i>.</p> <p><i>Objective:</i> Ensure that multiple <i>avoidance rules</i> application gives feasible <i>avoidance strategy</i>, enter into <i>emergency avoidance mode</i> otherwise. Following steps are executed:</p> <ol style="list-style-type: none"> <li>1. <i>Collect rules</i> applied on <i>navigation grid</i> from <i>active collision cases</i>.</li> <li>2. <i>Calculate possible trajectories for avoidance</i>, there may be none.</li> <li>3. If there is no <i>feasible route</i>, for each <i>intruder</i> from related <i>collision cases</i>:             <ol style="list-style-type: none"> <li>a. Issue <i>warning message</i> containing <i>expected collision point</i> and <i>preferred separation type</i>.</li> <li>b. Create appropriate <i>intruder object</i> for <i>avoidance grid</i>.</li> <li>c. Calculate <i>evasive maneuver</i> based on <i>expected separation type</i>.</li> </ol> </li> <li>4. <i>Notify UTM</i> with <i>collision case resolution</i> for each <i>active collision case</i>. <i>Notify UTM</i> with <i>planned trajectory</i> and <i>avoidance mode</i></li> </ol>
---

Context	Condition	Application
UAS Mission control, After avoidance run, Collision resolutions	At least one trajectory in Navigation grid, Emergency check	Force <i>Emergency mode</i> OR Close Collision Case

Table 6.6: Close collision case rule definition.

### Rule: Head on Approach

Rule (6.7) is invoked based on *angle of approach* range condition, defined *collision case* section 6.8.8. The handling of *head on* avoidance is given in section 6.8.4.

*Virtual round-abound* for UAS and intruder is created by UTM. The center of virtual round-abound and *corrections for participants margins* are determined based on:

1. *Collision case center* - contributes to round-abound center median point.
2. *UAS and intruder maneuverability* - determines *attendants avoidance mode* and *maximal avoidance margins*.
3. *Surrounding air-traffic* - contributes to round-abound center median point, determines ideal *ideal avoidance margins* due to *wake turbulence prevention*.

*Virtual round-about center* is calculated as *corrected median* (6.48) taking *cluster of collision cases* and calculates median of their collision points corrected by *weather* and *wake turbulence* factor.

$$\begin{aligned} \text{correctedMedian} = & \sum_{c_i \in \text{collisionCases}} (c_i.\text{center} + \text{correction}) / \text{count}(\text{collisionCases}) + \\ & + \text{correction}(\text{Weather}) + \text{correction}(\text{WakeTurbulence}) \end{aligned} \quad (6.48)$$

*Corrected margin* needs to be calculated for each *participating aircraft*, because of the *virtual roundabout* center correction (6.48). Each *round-about participant* is ordered based on importance (lowest maneuverability first). Then for each *round-about participant* obtains *corrected margin* (6.49) calculated from *collision case safety margin*, corrections based on other *more important vehicles*, *weather*, *wake turbulence*.

$$\text{correctedMargin} = \min \left[ \text{caseMargin} + \text{correction} \begin{pmatrix} \text{ImportantVehicles}, \\ \text{Weather}, \\ \text{WakeTurbulence} \end{pmatrix} \right] \quad (6.49)$$

*Invocation:* When *UAS avoidance role* is *Head on* avoidance and *avoidance grid* is empty.

*Objective:* Ensure that *UAS body* does not enter into *intruder's well clear zone*.

1. Prevent *left-side leading* maneuvers (rule 6.10).
2. Prevent head on *safety margin* breach(rule 6.11).
3. Return to original course, when *navigation grid* is clear.
4. Prevent *wake turbulence* (by safety margin correction).
5. Enforce *Round-about* behaviour (by clustering collision cases).

Context	Condition	Application
UAS Navigation Grid, Collision Point, Avoidance role	None	Run rules referenced in objective listing.

Table 6.7: Head on Approach rule definition.

### Rule: Converging Maneuver

Rule is invoked based on *angle of approach* range defined in *collision case calculation* (sec. 6.8.8). Behaviour enforced to this rule is equal to rule 6.7 except the *intruder* stays on his original path. UAS behaviour is described in section 6.8.5. The *rule summary* is given by table 6.8.

*Invocation:* When *UAS avoidance role* is *Converging* and *avoidance grid* is empty.

*Objective:* Ensure that *UAS body* does not enter into *intruder's well clear zone*.

1. Prevent *left-side leading* maneuvers (rule 6.10).
2. Prevent head on *safety margin* breach(rule 6.11).
3. Return to original course, when *navigation grid* is clear.
4. Prevent *wake turbulence* encounter (by safety margin correction).

<i>Context</i>	<i>Condition</i>	<i>Application</i>
UAS Navigation grid, Collision point, Avoidance role	None	Run rules from objective.

Table 6.8: Converging maneuver rule definition.

### Rule: Overtake

During overtaking maneuver there is our *UAS* and *Intruder* cruising at same *flight level*. *Angle of approach* ( $\alpha$ ) is lesser than  $70^\circ$ . *UAS* absolute velocity is much greater than *overtaken* absolute velocity.

It is assumed that during *overtake* maneuver *overtaken* intruder will keep constant heading and velocity. If this assumption is broken *UAS* system will invoke *Emergency avoidance* procedure. *UTM* will calculate such *divergence* and *convergence* waypoints that *overtake safety condition* (6.50) is satisfied.

$$distance(uasPosition, overtakenPosition) \geq utmMargin, \forall t \in maneuverTime \quad (6.50)$$

Where *utmMargin* is calculated based on *Collision case* resolution. *Main idea* is to calculate *Safe offset* for *Overtake maneuver*, lets have:

$$velocityDifference = \|uasVelocity - overtakenVelocity\| \quad [ms^{-1}, ms^{-1}, ms^{-1}] \quad (6.51)$$

*Decision distance* (6.52) is given as distance when *UTM mandate* takes effectiveness, its assumed that *UTM* knows *utm decision frame* [s]:

$$decisionDistance = velocityDifference \times uasDecisionFrame \quad [m, ms^{-1}, s] \quad (6.52)$$

*Overtake middle distance*(6.53) is length of hypotenuse for triangle where *positional difference* and *utm margin* for overtaking are cathetus:

$$overtakeMiddle = \sqrt{\|uasPosition - collisionPoint\|_2^2 + safetyMargin^2} \quad [m, \vec{m}, \vec{m}, m] \quad (6.53)$$

*Safe offset* (6.54) is considered as combination of *overtake middle distance* (6.53), *decision distance* and uas *waypoint reach margin*.

$$safeOffset = \begin{aligned} & overtakeMiddle \\ & + decisionDistance \quad [m, m, m] \\ & + waypointReachMargin \end{aligned} \quad (6.54)$$

*Note.* *Waypoint reach margin* [m] is property of own *UAS navigation algorithm*. It represents maximal distance of vehicle position and waypoint at time when waypoint is considered reached.

*Local coordinate frame:* UAS and Overtaken are in Local coordinate frame heading in  $X^+$  axis direction ( $X^+$  front of aircrafts,  $X^-$  back of vehicles,  $Y^-$  right side,  $Y^+$  left side,  $flightLevel \rightarrow Z = 0$ ), Collision Point is considered as  $\vec{0}$ ,

*Divergence point* (6.55) in local coordinates is given as right offset of (*UTM margin*) and *decision distance*:

$$divergence = \begin{bmatrix} 0 \\ -decisionDistance - utmMargin \\ 0 \end{bmatrix} \quad [\vec{m}, m, m] \quad (6.55)$$

*Convergence point* (6.56) in local coordinates is given frontal *safe offset* (6.54) and right offset of *UTM margin* and *decision distance*:

$$divergence = \begin{bmatrix} safeOffset \\ -decisionDistance - utmMargin \\ 0 \end{bmatrix} \quad [\vec{m}, m, m] \quad (6.56)$$

*Convergence* (6.57) and *Divergence* (6.58) waypoint in global coordinate frame is obtained via transformation function  $R_{XYZ}$  as follow:

$$divergenceWaypoint = collisionPoint \\ + R_{XYZ}(overtakenOrientation, divergence) \quad (6.57)$$

$$convergenceWaypoint = collisionPoint \\ + R_{XYZ}(overtakenOrientation, convergence) \quad (6.58)$$

*Overtake rule* is summarized in table 6.9.

*Invocation:* Invoked by rule *Collision Case Resolution* (rule 6.5)

*Divergence Waypoint* (6.57): waypoint to diverge from original UAS path to ensure Intruder safety, with unchanged intruder velocity and heading.

*Convergence Waypoint* (6.58): waypoint when convergence to original UAS path is enabled, within unchanged intruder velocity and heading.

*Objective:*

1. Calculate *Divergence Waypoint* and *Convergence Waypoint*.
2. Enforce Divergence/Convergence waypoint during avoidance.

Context	Condition	Application
UAS Navigation Grid, Collision Point, Avoidance Role	$UASVelocity >> IntruderVelocity$	Calculate & Enforce: • Divergence waypoint, • Convergence waypoint

Table 6.9: Overtake rule definition.

### Rule: Right plane heading

There is need to check if *trajectory* is heading to *right side* from *collision point*. For this purpose we need to define *separation plane in 3D environment*. *Separation plane* will be defined according to Samuelson *hyperplane separation theorem* [147].

*Separation plane* (6.59) is defined by three points in *global coordination frame*:

1. *UAS Position* which is fixed to given *time-frame*.
2. *Collision point* which is not equal to *uas position* by definition.
3. *Gravitational acceleration* vector fitted to *UAS position* and orthogonal to vector  $(uasPosition \rightarrow collisionPoint)$ .

The properties of these three points guarantees that  $scale.usasPosition \neq scale.collisionPoint \neq scale.gravitationalAcceleration$  for any linear  $scale \neq 0$ .

$$SeparationPlane = Plane \left( uasPosition, collisionPoint, loc2glob(uasPosition, gravitationalAcceleration) \right) \quad (6.59)$$

*Separation plane* (6.59) in *right-hand coordinate frame* where  $center = uasPosition$   $X^+$  is given by vector  $\vec{x^+}(uasPosition, collisionPoint)$  and  $Z^-$  is given by vector  $\vec{z^-}(uasPosition, gravitationalAcceleration)$ . Then *right subspace* can be defined as all points where  $y \leq 0$  and *left subspace* as all points where  $y > 0$ .

*Reach set* contains *trajectories*, minimal dataset for trajectory is time-series of *position* and *heading* regardless *underlying nonlinear model*. Let us have *transformation function* which can map UAS *position* and *heading* into *separation plane coordinate frame*.

The *first condition* (6.60) says that each trajectory *point* must lie within *right subspace*.

$$\forall position \in trajectory, \quad position \in rightSubspace \quad (6.60)$$

The *second condition* (6.61) needs to be applied for each *decision point*, when *trajectory* can be re-planned. It must be ensured that in time of reaching *decision point* vehicle is not heading into *left subspace* with given *turning time horizon*. The *minimal information* contains heading (velocity) vector. Checking if linear projection from *position* point with *heading* in given time-frame  $[0, horizon]$  is sufficient.

$$\forall t \in [0, horizon], \quad (position + velocity * t) \in rightSubspace \quad (6.61)$$

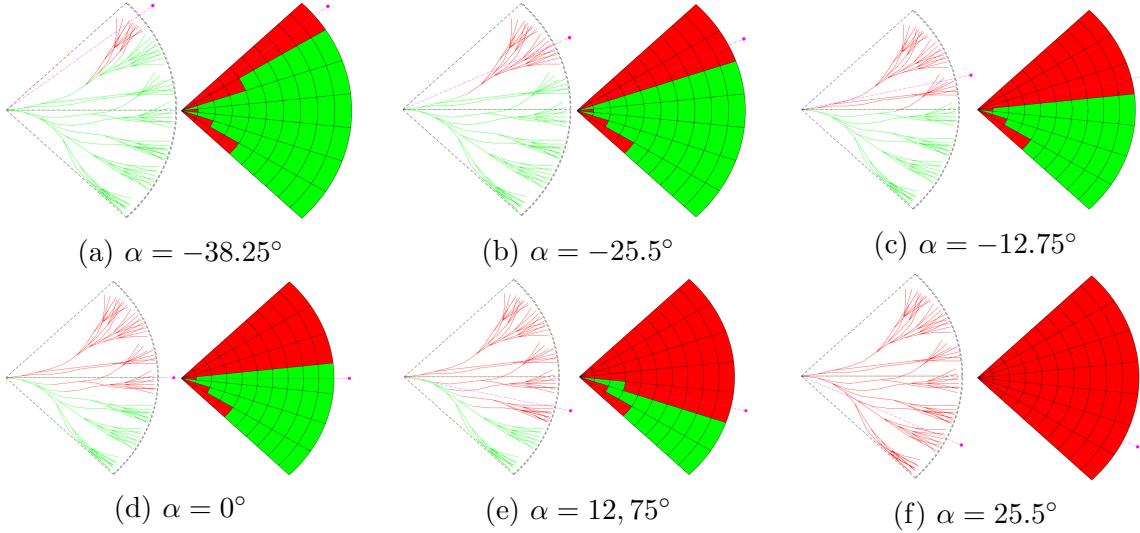


Figure 6.24: Right plane heading rule evaluation for various angles of approach  $\alpha$ .

Figure 6.24. shows *enabled* (green line) and *disabled* (red lines) *trajectories* (left sub-figure). These trajectories are divided according to *separation line* (magenta dashed line), given by *vehicle position* and *collision point* (magenta circle). *Space segmentation* (right subfigure) show *reachable* (green fill) *unreachable* (red fill) space. Situation is shown for various *collision point angles of approach*  $\alpha$ .

Rule for right plane heading check is summarized in table 6.10.

*Invocation:* Invoked by other *maneuver rules*.

*Objective:* Disable all *trajectories* in *Navigation grid's reach set* which are:

1. *Heading into collision zone*
2. *Leading into collision zone*

Context	Condition	Application
UAS Navigation Grid, Collision point (LOC)	There are feasible trajectories in Navigation Grid.	Disable trajectories in Navigation Grid.

Table 6.10: Right plane heading rule definition.

### Rule: Enforce safety margin

Rule 6.10. checks right plane heading for single mass point along *trajectories*. Rule needs to account *body mass* of *intruder* and UAS, other factors like safe distance, regulations etc. All mentioned factors are included in *safety margin*. *Safety margin* is applied as *radius ball* around *collision point*.

*Collision point* can be mapped from *global coordinate frame* to *reach set coordinate frame*, based on UAS *position* and *orientation* in *decision time*. Then comparison of distance between *collision point* and every *trajectory decision point* is trivial.

*Trajectory feasibility condition* for *non-controlled airspace* (6.62) is given as follow:

$$\forall \text{position} \in \text{trajectory}, \quad \text{distance}(\text{position}, \text{collisionpoint}) \geq \text{safetyMargin} \quad (6.62)$$

*Controlled airspace* must maintain *well clear condition*. To enforce protective barrel around *collision point* one must compare *global coordinates*. *Trajectory feasibility condition* for *controlled airspace* (6.63) is given as follow:

$$\begin{aligned} \forall \text{position} \in \text{trajectory}, \\ XY \text{distance}(\text{position}, \text{collisionPoint}) &\geq \text{safetyMargin} \\ \text{flightLevelStart} &\geq Z(\text{position}) \geq \text{flightLevelEnd} \end{aligned} \quad (6.63)$$

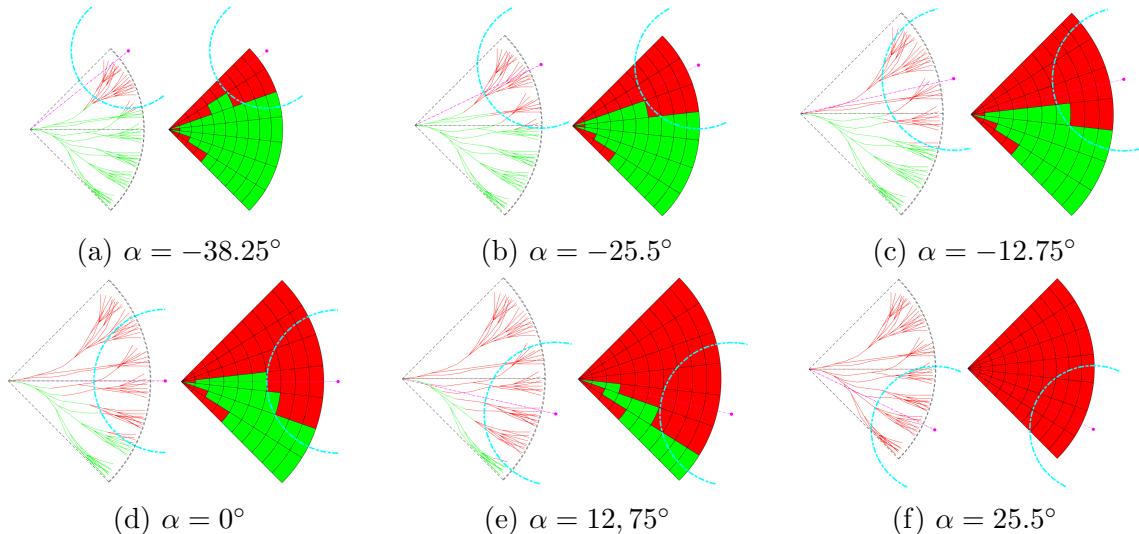


Figure 6.25: Enforce safety margin rule evaluation for various angles of approach  $\alpha$ .

Figure 6.25. shows *enabled* (green line) and *disabled* (red lines) *trajectories* (left sub-figure). These trajectories are divided according to *separation line* (magenta dashed line), given by *vehicle position* and *collision point* (magenta circle). More trajectories are disabled due to *safety margin* (teal dashed line) around the *collision point*. *Space segmentation* (right subfigure) show *reachable* (green fill) *unreachable* (red fill) space. Situation is shown for various *collision point angles of approach*  $\alpha$ .

Rule for safety margin check is summarized in table 6.11.

*Invocation:* Invoked by other *maneuver rules*.

*Objective:* Based on type of airspace, for given *collision point* and *safety margin* disable trajectories in:

1. Ball radius for *non-controlled* airspace (6.62).
2. Well clear barrel *controlled* airspace (6.63).

<i>Context</i>	<i>Condition</i>	<i>Application</i>
UAS Navigation Grid Collision point Safety Margin	There are feasible trajectories for condition application.	Disable trajectories in Navigation Grid.

Table 6.11: Enforce safety margin rule definition.

# Chapter 7

## Simulations

The chapters presents set of simulations developed according to a test plan (sec. 7.1). Test configuration (sec. 7.2) targets at exercising and evaluating proposed framework.

### 7.1 Test Plan

The *Avoidance requirements* are given in (sec. 4.4), namely:

1. *Safety Margin Enforcement* (sec. 4.4.3) - keep UAS safe depending on situation.
2. *Path Tracking* (sec. 4.4.2) - track mission given by set of *waypoints* in the manner of *Energy Efficiency* (sec. 4.4.1).

These are given as nominal behaviour (sec. 6.7.3), further enhanced by rule-based behaviour (sec. 6.9.2).

The *Navigation requirements*, out of this scope, are given in (sec. 4.5). These are satisfied by *Mission Control Run* (sec. 6.7.4).

**Testing Scope** covers following major functional areas, incorporated into *Avoidance Framework*:

1. *Static obstacle and Ground Avoidance* - avoid direct collision with static objects.
2. *Constrained Area Avoidance* - avoid areas which are constrained by authority, contains geo-fencing, airspace restrictions.
3. *Non cooperative intruder avoidance* - avoid intruders which are not cooperating. This is useful against birds, piloted UAS and other flying objects.
4. *Cooperative intruder avoidance* - avoid intruders according to coordinated and agreed procedures, mainly enforced by UTM.
5. *Navigation* - navigate through complicated environment.

### 7.1.1 Testing approach

The purpose of this section is to show complex scenarios, not unit testing of framework functionality. The focus is on *border line* cases for typical situations in *expected environment*. The *mode switch* between *Navigation* and *Emergency Avoidance*.

The *Tests* are designed to focus on particular functionality in specific *operational environment* with main *obstacle/weather/intruder feature* with environment induced *constraints*. There is also *UTM* factor and *Navigation penalty*.

**Operational Environment** is classified according to:

1. *Operation space* - important for *Low Altitude Operations*, the difficulty of *Avoidance Maneuvers* is proportionally increasing with *Obstacle density*. There are following main categories
  - a. *Rural environment* - the relief and man-made structures are sparsely spread around the *operation space*, the UAS is operating on *very low altitude* ( $\leq 50$  feet).
  - b. *Urban environment* - the concentration of the man made structures is much higher and they are more incorporated into land relief pattern, the UAS is operating on *very low altitude*.
  - c. *Open air* - the concentration of ground structures is very low, the concentration of *cooperative* and *non-cooperative intruders* is increased, the UAS is operating in altitude ranging from *50 feet* to *space border*. This bring us to:
2. *Airspace category* - when *Operation Space* pattern is categorized as *Open air* and depending on *altitude above mean sea level*. The UTM is *designed authority* for controlled airspace in current *F/G class airspace*.
  - a. *Controlled* - Open air where authority is present. The cases when *Authority* is not enforced due the UTM malfunction, *C2 link loss* or other cause are not considered.
  - b. *Non-Controlled* - Open air operation space where is no central arbiter to determine or enforce traffic attendants behaviour.

**Static obstacles:** Static obstacles with various features detectable by main *LiDAR* sensor. The main purpose is to show avoidance capabilities combined with heavy restrictions imposed by *soft* and *hard* constraints. The original purpose of our approach was to provide robust framework for static obstacle avoidance. Three tests with increasing obstacle density and navigation complexity are delivered.

**Weather** impose *soft* and *hard* space constraint, which can be moving or static. The *soft constraint avoidance* is covered by *hard constraint avoidance*. The *static constrained area* is covered by *static obstacle avoidance* capability due the *Data fusion procedure* [148]. The only case which is not covered is *Moving constrained area*, small constraints can be covered by intruder models. The ideal candidate is *storm*, because it covers quite large area, the clouds are constantly moving and severity is changing with time.

**Non-cooperative Intruders:** There are some intruders with some degree of authority, size and *severity*. There were three test cases for non-cooperative intruders. Non-cooperative Intruders can be categorized as follow based on behaviour:

1. *Chaotic* intruders usually have tendency to behave unpredictable, for example bird or *UAS in distress*, for this type of intruders *Maneuver Uncertainty Intersection Model* is used (sec. 6.5.4).
2. *Harmonic* intruder usually follow long straight paths, for example UAS converging to waypoint, for this type of intruder *Body Volume Intersection Model* is used. (sec. 6.5.3).

**Cooperative Intruders:** There are cooperative intruders which are obeying authority (UTM) or follow *common consensus*. The work focus on *UTM* authority implementation in four test cases. These test cases are reflecting the traffic management situations essential for successful UTM collision management

**Constraints:** Constraints depends mainly on *operational environment*. There is special type of *constraints* which has not been evaluated in previous works - *UTM Constraints*. The standard set of constraints were taken into account for our test cases:

1. *Very low altitude constraints* (Rural, Urban environment) are: geo-fencing zones, ground (hard constraints), non-controlled airspace altitudes (soft constraints).
2. *Non-controlled airspace constraints* are: geo-fencing zones (hard constraints), restricted airspace (hard constraint), weather (soft/hard constraint), controlled airspace (hard constraint), very low altitude border (soft constraints).
3. *Controlled airspace constraints* are: restricted airspace (hard constraint), weather (soft/hard constraint), non-controlled airspace boundary (hard constraints), UTM Directives (hard constraints).

**UTM:** The *UAS Traffic Management* service should be implemented in *controlled airspace* by 2035. It is necessary to study impact of UTM services on the *Detect and Avoid* systems like ours.

The most basic service is *Identity provider* which should be implemented by 2020.

Then there is *location services*, which are necessary for coordinated collision avoidance, these were implemented in our solution up to necessary level for *Rules Of the Air* implementation.

*Mission tracking* is service tracking deviations from *declared mission plan* and *actual execution*. This statistics were used in all tests to track deviations from reference trajectory.

*Directives* for *Traffic management* and *Collision prevention* are implemented as functional life cycle of *Position notification* (sec. 6.8.7), *Collision Case* (sec. 6.8.8) for UTM. The directive handling is implemented as *Rule engine* (sec. 6.9.2) on UAS side.

**Navigation:** Navigation algorithm is depending on *Navigation mode*. UAS is usually in *Navigation mode* most of the time, despite this fact, UAS was forced into *Emergency Avoidance Mode* most of time in test cases. The navigation complexity have been divided into following categories:

1. *Open space* - UAS has visibility to goal waypoint most of the time, there are no traps.
2. *Hidden waypoint* - UAS does not have visibility to goal waypoint, most of the time, there are irregular traps sometime.
3. *Maze solving* - UAS line of sight for goal waypoint is hindered by multiple obstacles, there are irregular traps often.
4. *Rule following* - UAS navigation capabilities are constrained by rule enforcement.

### 7.1.2 Test Cases Summary

*Test cases* are summarized in (tab. 7.1).

Test Case Name	Operational Environment	Air Traffic Attendants	Weather	UTM	Navigation	Scenario
Building Avoidance	Rural: 4 × buildings 4 × types	-	-	-	Open space	Fly mission around four buildings
Slalom	Rural: 14 × buildings 3 × types	-	-	-	Hidden waypoint	Navigate to hidden waypoint
Maze	Urban: 30 × buildings 4 × types	-	-	-	Maze structure	Solve maze with multiple curves
Storm	Rural: 0 × buildings	-	Storm	-	Open Space	Avoid Approaching Storm
Emergency Converging	Non-controlled Airspace	Non-cooperative UAS (1x)	-	-	Open Space	Converging situation resolution w. o. UTM
Emergency Head on	Non-controlled Airspace	Non-cooperative UAS (1x)	-	-	Open Space	Head on situation resolution w. o. UTM
Emergency Multiple	Non-controlled Airspace	Non-cooperative UAS (3x)	-	-	Open Space	Multi collision case resolution w. o. UTM
Rule-based Converging	Controlled Airspace	Cooperative UAS(1x)	-	Full	Follow Rules	Converging situation resolution with UTM
Rule-based Head on	Controlled Airspace	Cooperative UAS(1x)	-	Full	Follow Rules	Head on situation resolution with UTM
Rule-based Multiple	Controlled Airspace	Cooperative UAS(3x)	-	Full	Follow Rules	Multi collision case resolution with UTM
Rule-based Overtake	Controlled Airspace	Cooperative UAS (1x)	-	Full	Follow Rules	Overtake by UAS different speed ratio

Table 7.1: Test Cases Summary.

## 7.2 Testing configuration

All *simulations* are run with configuration described in this *section*. The UAS used for the purposes is given by *model and control* (sec. 6.7.2).

*UAS parameters*: An *UAS system* (tab. 7.3) is modeled after small scale toy model with: maximal body radius 30 cm, maximal speed 4 m.s<sup>-1</sup>, weight 450 g., maximal flight duration 20 min, maximal turning rate 15 deg.s<sup>-1</sup>. The *body margin* is set to 0.3m, the *near miss radius* is double of *body margin*, thus 0.6 m, the *well clear radius* is set to 5 m. Margins can be set to any value if they are complaint with condition (7.1).

$$0 < \text{bodyMargin} \leq \text{nearMissRadius} \leq \text{wellClearRadius} \leq \text{gridDistance} \quad (7.1)$$

*Note.* Safety margin is broad term used to describe *minimal distance* between UAS and *adversarial object*. The *Safety margin* is:

1. *near miss radius* in case of *non-controlled airspace* or *emergency avoidance mode*.
2. *well clear radius* in case of *controlled airspace* and *navigation mode*.

*Decision time:* Decision time can be set by the user to any positive non-zero value (7.2). The *Decision time* is equal 1 s and *Decision frames* are synchronized.

$$\maxAlgorithmCalculationTime \leq decisionTome \leq \infty \quad (7.2)$$

*Speed:* For all movements constant speed  $1 \text{ m.s}^{-1}$  is used. Speed can be changed to any value in given boundary (7.3).

$$0 \leq speed \leq \min \left( \begin{array}{l} 0.5 \times (navigationGrid.distance / decisionFrame) \\ 0.5 \times (avoidanceGrid.distance / decisionFrame) \end{array} \right) \quad (7.3)$$

*Movement automaton:* The *movement set* is given in (tab. 7.2). The *movement set* contains horizontal, vertical, and, combined movements.

*Grids:* Used *Navigation grid parameters* are given in (tab. 7.4). Selected *Navigation Reach set* is *ACAS-like* with enabled horizontal/vertical separation. Used *Avoidance grid parameters* are given in (tab. 7.5). Selected *Avoidance Reach set* is *combined* because of high *coverage ratio*.

User can define own grid parameters according to the *space discretization rules* (sec. 6.2) and chose own *reach set type* according to preference (sec. 6.6).

Movement	Roll	Pitch	Yaw
Straight	$0^\circ$	$0^\circ$	$0^\circ$
Left	$0^\circ$	$15^\circ$	$0^\circ$
Right	$0^\circ$	$-15^\circ$	$0^\circ$
Up	$0^\circ$	$0^\circ$	$-15^\circ$
Down	$0^\circ$	$0^\circ$	$15^\circ$
UpLeft	$0^\circ$	$15^\circ$	$-15^\circ$
UpRight	$0^\circ$	$-15^\circ$	$-15^\circ$
DownLeft	$0^\circ$	$15^\circ$	$15^\circ$
DownRight	$0^\circ$	$-15^\circ$	$15^\circ$

Table 7.2: Movement orientations.

UAS parameters	
speed	$1 \text{ ms}^{-1}$
horizontal turning r.	$3.82 \text{ m}$
vertical turning r.	$3.82 \text{ m}$
body radius	$0.3 \text{ m}$
near miss r.	$0.6 \text{ m}$
well clear r.	$5 \text{ m}$

Table 7.3: UAS parameters.

Navigation Grid

type	ACAS-like
distance range	$0 - 10 \text{ m}$
layer step	$1 \text{ m}$
horizontal range	$\pm 45^\circ$
horizontal cells	7
vertical range	$\pm 30^\circ$
vertical cells	5

Table 7.4: Navigation Space parameters.

Avoidance Grid

type	combined
distance range	$0 - 10 \text{ m}$
layer step	$1 \text{ m}$
horizontal range	$\pm 45^\circ$
horizontal cells	7
vertical range	$\pm 30^\circ$
vertical cells	5

Table 7.5: Avoidance Space parameters.

Coloring		
Airc.	Executed	Planned
UAS 1	blue	red
UAS 2	cyan	magenta
UAS 3	green	yellow
UAS 4	black	green

Table 7.6: *UAS* coloring.

## 7.3 Non-cooperative test cases

The *main* goal of this section is to show operative capabilities for *non-cooperative avoidance mode* in *emergency* and *solo situations*.

Test avoidance capabilities against *static obstacles*, *non-cooperative intruders*, *moving hard constraints* are covered.

Coverage of the *soft constraints*, *map obstacles* and *detected obstacles* are implicitly covered due the properties of *safety* and *body* margins (tab. 4.2).

1. *Building Avoidance* (sec. 7.3.1) covers *static obstacles* explicitly and *map obstacles*, *hard constraints*, *ground avoidance* implicitly.
2. *Slalom* (sec. 7.3.2) covers *open space navigation capabilities*, showing the determinism of the *avoidance loop run*, in addition to *building avoidance*.
3. *Maze* (sec. 7.3.3) covers *closed space navigation capabilities*, showing the higher level navigation properties of primitive *right-side* 2D maze solver. The main point is to show possibility to enrich the *Navigation loop algorithm* (fig. 6.15).
4. *Storm* (sec. 7.3.4) covers *hard moving constraints avoidance* explicitly and *hard static constraints*, *soft static constraints*, *soft moving constraints* implicitly.
5. *Emergency converging* scenario (sec. 7.3.5) covers *non cooperative intruder with right of the way* avoidance capability.
6. *Emergency head on* scenario covers (sec. 7.3.6) *non cooperative intruder without right of the way* avoidance capability
7. *Emergency mixed* scenario (sec. 7.3.7) covers *multiple intruders with/without right of the way* avoidance capability.

### 7.3.1 Building avoidance

**Scenario:** The *UAS* is flying the mission given by (tab. 7.7) in the *open space environment*. There exists a map of obstacles with defined *safety and body margins*. *Reference trajectory* (direct interconnection of waypoints) is going through partially known space with some charted obstacles.

Position		Waypoints			
$[x, y, z]$	$[\theta, \varpi, \psi]$	$\mathcal{WP}_1$	$\mathcal{WP}_2$	$\mathcal{WP}_3$	$\mathcal{WP}_4$
$[0, 0, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[100, 0, 0]^T$	$[100, 100, 0]^T$	$[0, 100, 0]^T$	$[0, 0, 0]^T$

Table 7.7: Mission setup for *Building avoidance* scenario.

**Obstacle set:** Obstacles are discovered during a flight by *UAS LiDAR sensor*, the set of obstacle is defined in (tab. 7.8).

Obstacle			Body Margin			Safety Margin
id	position	type	min.	max.	avg.	
1	$[0, 0, 0]^T$	polygonal	14	20	16	5
2	$[100, 50, 0]^T$	hospital	12	18	14	7
3	$[50, 100, 0]^T$	unusual	10	20	15	8
4	$[0, 50, 0]^T$	square	18	20	19	4

Table 7.8: *Obstacle set* for *Building avoidance* scenario.

**Main Goal:** Show *static obstacle avoidance capability* in *open space environment*, using *LiDAR scanning* and *obstacle map* as the *information sources*.

#### Acceptance criteria:

1. Proper *algorithm mode switch*:
  - a *Avoidance mode* is active when the *UAS* is in close proximity of obstacle (*distance* (*obstacleCenter*, *UASPosition*)  $\leq 20m$ ).
  - b *Navigation mode* is active when the *UAS* is further away from any obstacle (*UAS* is actively converging to *goal waypoint*);
2. *Minimal safety margin distance*  $\geq 0m$
3. *Reach each waypoint* (tab. 7.7) in given order.

**Testing Setup:** The *standard test setup* defined in (tab. 7.2, 7.3, 7.4, 7.5, 7.6) is used with following parameter override:

1. *Avoidance grid - type - ACAS-like* with enabled *Horizontal maneuvers*

*Note.* Enforced *safety margin* does not exceed the *avoidance grid range* (10 m). The concept of *Static obstacle avoidance* is in detail discussed in the *progress report* [148].

**Simulation Run:** Notable moments from the *simulation run* (fig. 7.1) are following:

1. 1<sup>st</sup> building avoidance. (fig. 7.1a) - UAS avoids the building from left side, because overall trajectory cost is cheaper. The first building is convex obstacle.
2. 2<sup>nd</sup> building avoidance. (fig. 7.1b) - UAS avoids the building from right side, while avoiding active non convex portion of the building.
3. 3<sup>rd</sup> building avoidance. (fig. 7.1c) - UAS avoids the building from right side, missing both traps from it.
4. 4<sup>th</sup> building avoidance. (fig. 7.1d) - UAS avoids the building from right side. This building is also convex obstacle.

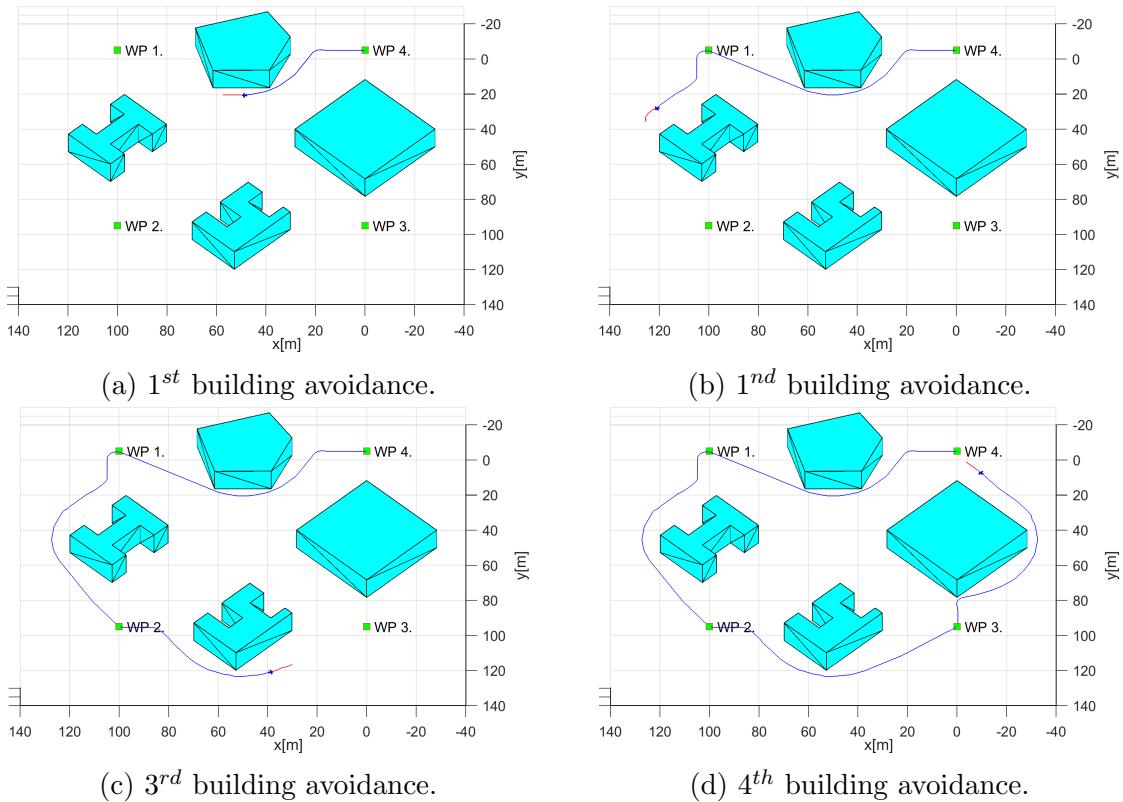


Figure 7.1: Test scenario for *Building avoidance* (static ground obstacles).

**Safety Margin Performance:** The *UAS* center distance to nearest obstacle (blue) does not break a *safety margin* of closest obstacle (yellow) nor *body margin* of closest obstacle (red) as it can be seen in (fig. 7.2). *Acceptance condition for algorithm mode switch* can be shown by *UAS active avoidance of obstacles*.

*Note.* The *body* and *safety margins* are changing depending on *UAS position and orientation*, is changing reflecting (tab. 7.8) margins.

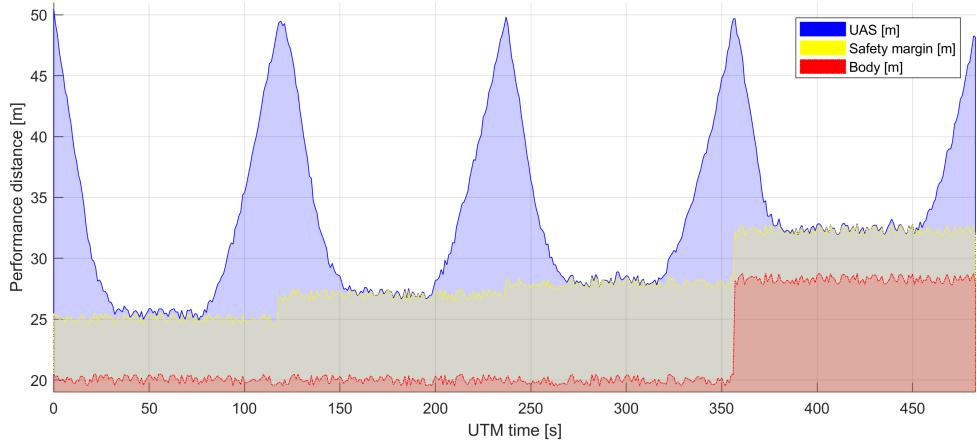


Figure 7.2: *Building avoidance safety margin performance.*

**Safety Margin Distances:** Minimal distance to *obstacle safety coating* is 0.69 m. The *minimal distance to obstacle body* is 4.69 m which is more than sufficient for testing UAS type. *Safety margin acceptance criteria* have been achieved, because minimal distance is greater than zero. The minimal *body margin distance* is 4.69 m for obstacle no. 4 (tab. 7.8).

Parameter	UAS 1	
safety margin distance	min	0.69
	max	24.98
body margin distance	min	4.69
	max	29.98

Table 7.9: *Building avoidance safety and body margin distances.*

**Path Tracking Performance:** Reference path (green dashed line) is given as direct interconnection between waypoints (green numbered square). The real trajectory (blue solid line) is split into its XYZ components. *All mission waypoints* (fig. 7.3) have been reached in given order. There are some deviations on  $X - Y$  horizontal axes, while the UAS was in the *avoidance mode*.

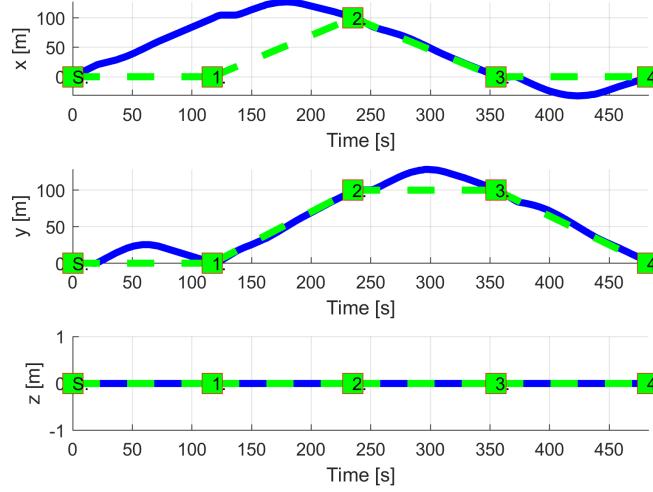


Figure 7.3: *Building avoidance* path tracking.

**Path Tracking Deviations:** Deviations (tab. 7.10) from *reference trajectory* are in expected ranges considering *mission plan* (tab. 7.7) and *obstacle properties* (tab. 7.8).

Param.	UAS 1			
	$WP_1$	$WP$	$WP_3$	$WP_4$
$\max  x $	104	86	5.34	32.52
$\max  y $	25.39	6.59	28.2	4.55
$\max  z $	0	0	0	0
$\max dist.$	107.05	86.2	28.7	32.84

Table 7.10: Path tracking for properties *Building avoidance*.

### 7.3.2 Slalom

**Scenario:** The *UAS* is flying the mission given by (tab. 7.11) in the *open-space environment*. A Operational space is more clustered than in case of *Building Avoidance* (sec. 7.3.1). There map of notable *buildings* with defined *safety and body margins* imposing additional flight constraints. The *UAS* is flying through partially known space with some charted obstacles.

The *goal waypoint* is hidden behind the sensors line of sight. There is multiple cost equivalent trajectories to reach the goal.

Position		$WP_1$
$[x, y, z]$	$[\theta, \varpi, \psi]$	
$[25, 5, 0]^T$	$[0^\circ, 0^\circ, 90^\circ]^T$	$[35, 75, 0]^T$

Table 7.11: Mission setup for *Slalom* scenario.

**Obstacle set:** Obstacles are discovered during a flight by *UAS LiDAR Sensor*. The set of obstacles is defined in (tab. 7.12) Some obstacles does not have *Line of Sight* during a flight, which causes additional constraints during *avoidance trajectory selection* process.

Obstacle		Body Margin			Safety Margin
position	type	min.	max.	avg.	
multiple (4)	hospital	[0.5, 1]	[2.2, 3.1]	[1.5, 3]	[1, 3]
multiple (7)	unusual	[0.3, 1]	[2.3, 3.5]	[2, 3]	[1, 4]
multiple (3)	square	[3, 4]	[4, 5]	[4, 5]	[1, 4]

Table 7.12: *Obstacle set* for *Slalom* scenario.

**Main goal:** Show *static obstacle avoidance* in *clustered environment* with *shorter decision frames* due the obstacle density. Show *hidden waypoint navigation capability* and *Behind Line of Sight* impact on decision making.

**Acceptance Criteria** are given as follow:

1. *Hidden waypoint reach* - the UAS will safely reach *goal waypoint*.
2. *Minimal safety margin distance*  $\geq 0$ .
3. *Hindered space* is accounted into decision making (BLOS impact).

**Testing setup:** The *standard test setup* defined in (tab. 7.2. 7.3. 7.4. 7.5. 7.6) is used with following parameter override:

1. *Avoidance grid - type - ACAS-like* with enabled *Horizontal maneuvers*

*Note.* The *vertical separation* was disabled, because *UAS* will just increase its altitude to reach *goal waypoint*.

**Simulation run:** Notable moments from this *simulation run* (fig. 7.4) are following:

1. *Open space obstacle* (fig. 7.4a) - avoidance of open space obstacle, while tracking *hidden waypoint*. This is standard navigation procedure, the middle building in front of *goal waypoint* is hidden by building in front of UAS.
2. *Hidden waypoint navigation* is shown in three stages start (fig. 7.4b), middle (fig. 7.4c), and end phase (fig. 7.4d). The *hidden goal waypoint* has been reached and first acceptance criteria was fullfilled. The *Decision points* of navigational loop are placed in very high density around this area. The avoided building had following traps which were avoided:
  - a. Trap (fig. 7.4b) on the left side of *UAS* was avoided, because there was no turning point inside of space.
  - b. Trap (fig. 7.4c) on the left side of UAS was avoided, because it was not wide enough to be considered as trajectory space.

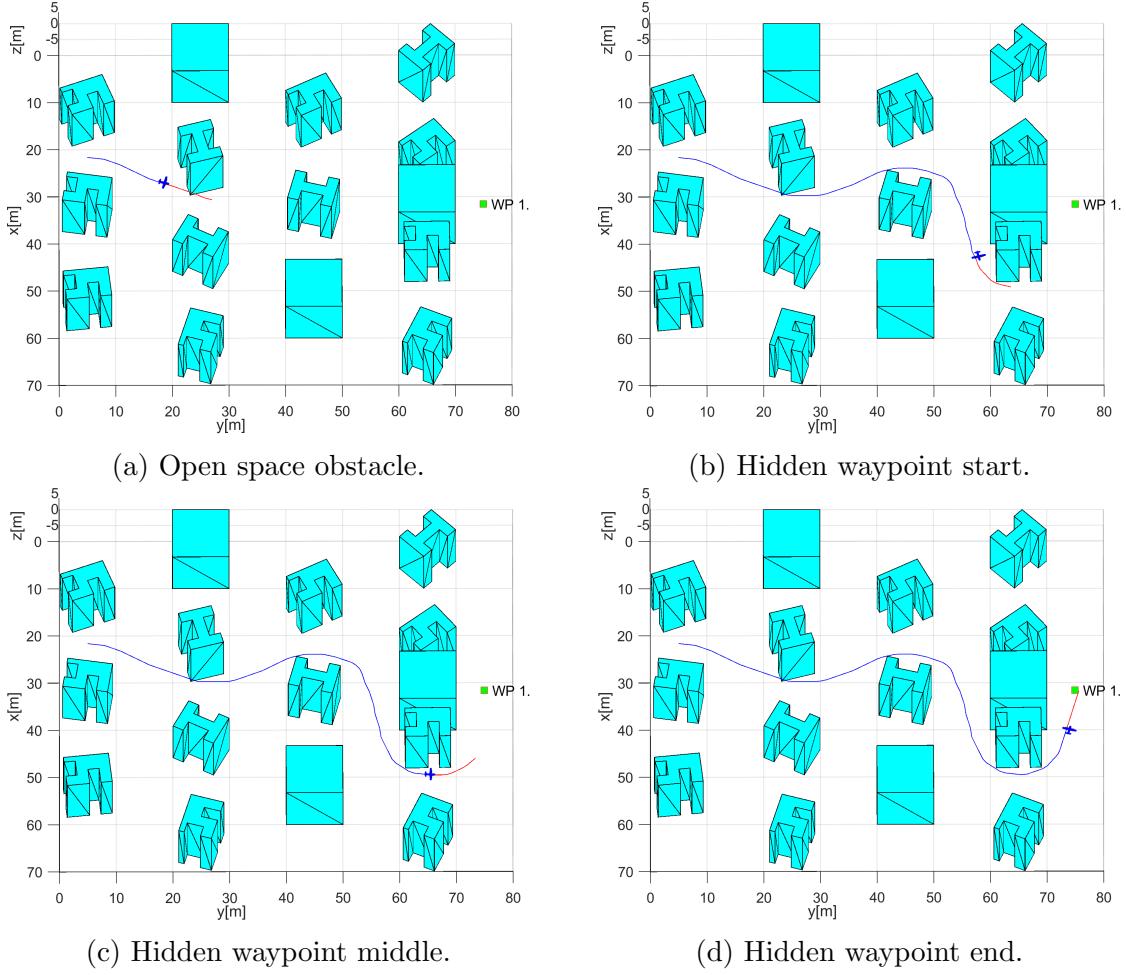


Figure 7.4: Test scenario for *Slalom* with *hidden waypoint*.

**Safety Margin Performance:** The *UAS* (blue fill) does not break a *safety margin* (yellow fill) nor *body margin* (red fill) as you can seen in (fig. 7.5). Hindered space is accounted into decision making, because the distance to closest obstacle will never breach *safety margin* (yellow fill). If it was not, the *UAS* will break *safety* or *body margin*.

*Body* and *Safety margin* are changing values depending on the *nearest obstacle* and *mutual position of obstacle and UAS*. The ranges of *body* and *safety margins* are reflected in (tab. 7.12).

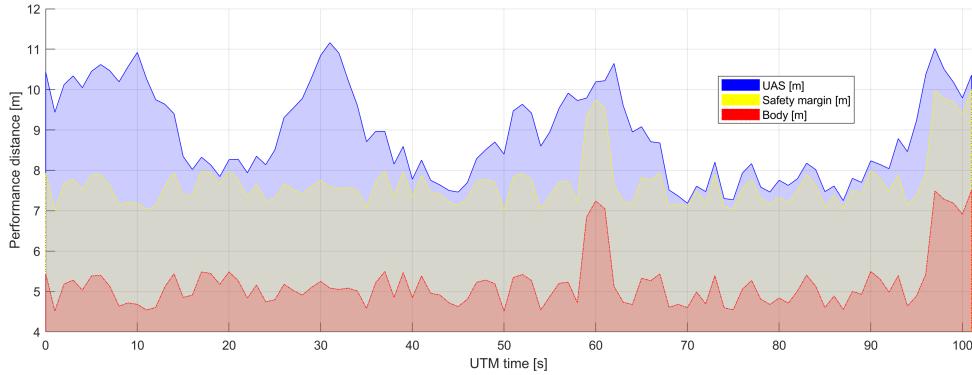


Figure 7.5: *Slalom* safety margin performance.

**Safety Margin Distance:** The boundary of *safety* and *body* margin is given in (tab. 7.13). The minimal distance of *UAS border*(blue line) to *safety margin boundary* (yellow line fig. 7.5) is 0.0856 m which can be considered as marginal 0. The minimal *body margin* distance is 2.5856 m which reflects *safety margin* 2.5 m at that moment. The condition  $safetyMarginDistance \geq 0$  holds.

The difference between minimal and maximal *safety margin distance* is  $\sim 3$  m which indicates that mission environment is tightly packed with obstacles.

Parameter		UAS 1
safety margin distance	min	0.0856
	max	3.7391
body margin distance	min	2.5856
	max	6.2391

Table 7.13: *Slalom* safety and body margin distances.

**Path tracking performance:** Path tracking is given in (fig. 7.6). The line between Starting position (green square, marked S) and goal waypoint (green square marked 1) is reference trajectory (green dashed line). The flew trajectory (blue solid line) is showing evolution over mission time (Time [s]) in global coordinate frame split into three axes (x[m], y[m], z[m]). The UAS was all time in *Emergency Avoidance Mode* due the vicinity of dangerous obstacles.

The *UAS* reached final navigation waypoint, which fulfills acceptance criteria. The UAS has taken a significant detour (x[m] evolution) due to hidden *waypoint*.

The test has been run multiple times to check if *Right-Up* preference for avoidance is always selected. *Small noise* (0.5-1m) was added to obstacle positions. The algorithm always chose similar deterministic path. The higher noise levels were not possible due the obstacle original size (tab. 7.12).

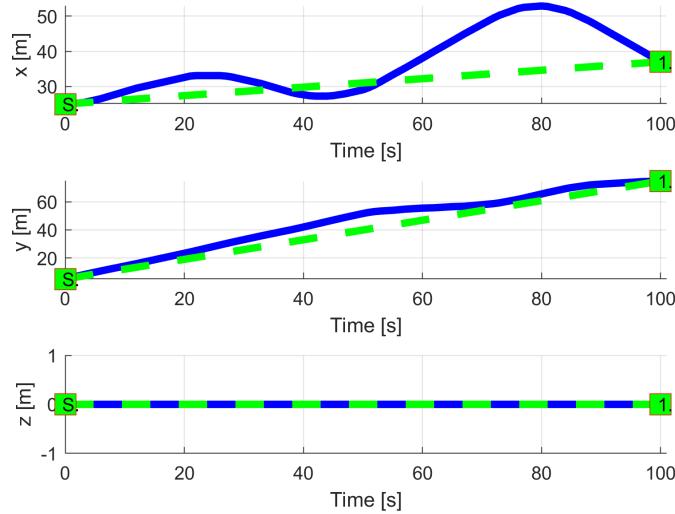


Figure 7.6: *Slalom* path tracking.

**Path Tracking Deviations:** Deviations given in (tab. 7.14) from *reference trajectory* (fig. 7.6) are in expected ranges considering *mission plan* (tab. 7.11) and *obstacle properties* (7.12).

Param.	UAS 1
	$\mathcal{WP}_1$
$\max  x $	17.90
$\max  y $	12.41
$\max  z $	0
$\max dist.$	20.06

Table 7.14: Path tracking properties for *Slalom* scenario.

### 7.3.3 Maze

**Scenario:** The UAS is flying a mission given by (tab. 7.15) in *closed space* constrained by ground from bottom, airspace constraint from top and building from sides. The maneuverable space is *maze-like* with *hidden goal waypoint*.

There exists a *Obstacle map* with defined *safety* and *body margins*. *Reference trajectories* (direct interconnection of initial position and *goal waypoint*) is going through *partially known space* with some charted obstacles.

Position		$\mathcal{WP}_1$
$[x, y, z]$	$[\theta, \varpi, \psi]$	
$[15, 15, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[15, 75, 0]^T$

Table 7.15: Mission setup for *Maze* scenario.

**Obstacle set:** *Obstacles* are discovered during a flight by *UAS LiDAR* sensor. The *Obstacle set* is defined in (tab. 7.16). the obstacles are placed in *virtual grid* with *cell size*  $10 \times 10m$ . There are following obstacles:

1.  $5 \times$  *Hospital building* - H-shaped, with two open traps, with minimal body margin in range  $0.5 - 1m$ , with maximal body margin in range  $2.2 - 3.1m$  and variable *safety margin* in range  $1 - 3m$ .
2.  $12 \times$  *Unusual trap building* - square shaped building with two traps on neighbouring side, with minimal body margin in range  $0.3 - 1m$ , with maximal body margin in range  $2.3 - 3.5m$  and variable *safety margin* in range  $1 - 4m$ .
3.  $6 \times$  *Square building* - square shaped building with minimal body margin in range  $3 - 4m$ , with maximal body margin in range  $4 - 5m$  and variable *safety margin* in range  $1 - 4m$ .
4.  $7 \times$  *U-shaped Trap* - thin walled U shaped trap designed to catch incoming flying objects, with minimal body margin in range  $2 - 4m$ , maximal body margin in range  $3 - 5m$  and various *safety margin* in range  $1 - 2m$ .

The purpose of these *Obstacles* except *Square building* type is to create false positive path diversions. These diversions are designed to take *UAS* into unsolvable situation. *Avoidance* of traps is possible due *Reach set properties*, because many scenarios for avoidance can be evaluated at once.

Obstacle		Body Margin			Safety Margin
position	type	min.	max.	avg.	
multiple (5)	hospital	[0.5, 1]	[2.2, 3.1]	[1.5, 3]	[1, 3]
multiple (12)	unusual	[0.3, 1]	[2.3, 3.5]	[2, 3]	[1, 4]
multiple (6)	square	[3, 4]	[4, 5]	[4, 5]	[1, 4]
multiple (7)	trap	[2, 4]	[3, 5]	[2, 4]	[1, 2]

Table 7.16: *Obstacle set* for *Maze* scenario.

**Main Goal:** Demonstrate static obstacle avoidance in closed space navigation. Focus on determinism of *avoidance run*. Demonstrate the possibilities of primitives *right-hand* maze solver incorporated into *Navigation-loop*.

#### Acceptance Criteria:

1. *Do not break top/bottom boundaries* - the *UAS* Z coordinate should not leave range  $-5$  to  $5m$ . The boundary break occurs when there is no feasible horizontal path and UAS needs to climb up to resolve situation.
2. Minimal safety margin distance  $\geq 0m$ .
3. *Reach hidden goal waypoint* by solving simple maze (tab. 7.15).

**Testing Setup:** The *standard test setup* defined in (tab. 7.2, 7.3, 7.4, 7.5, 7.6) is used with following parameter override:

1. *Avoidance grid - type - ACAS-like* with enabled *Horizontal maneuvers*

**Simulation Run:** Notable moments from the simulation run (fig. 7.7) are following:

1. *The Maze* consist from heavy constrained turns:  $1^{st}$  turn (fig. 7.7a),  $2^{nd}$  turn (fig. 7.7b), and  $3^{rd}$  turn (fig. 7.7c). The hidden waypoint reach is given by (fig. 7.7d).
2. UAS is constantly in *Emergency Avoidance mode*, because there is always a presence of obstacle,
3. *Navigation path* is located in slim corridor with width only 3-6 meters. Mutual distance of obstacles is 20 meters and combined margins takes 14-17 meters.
4. *Maze scenario* was very close to urban environment in terms of obstacle density and computation complexity.
5. *Avoidance run* computation complexity scaled linearly with count of active obstacles in Field of View.
6. *Hidden Goal Waypoint* have been reached as shown in (fig. 7.7d). This satisfy *reach hidden waypoint* acceptance criterion.

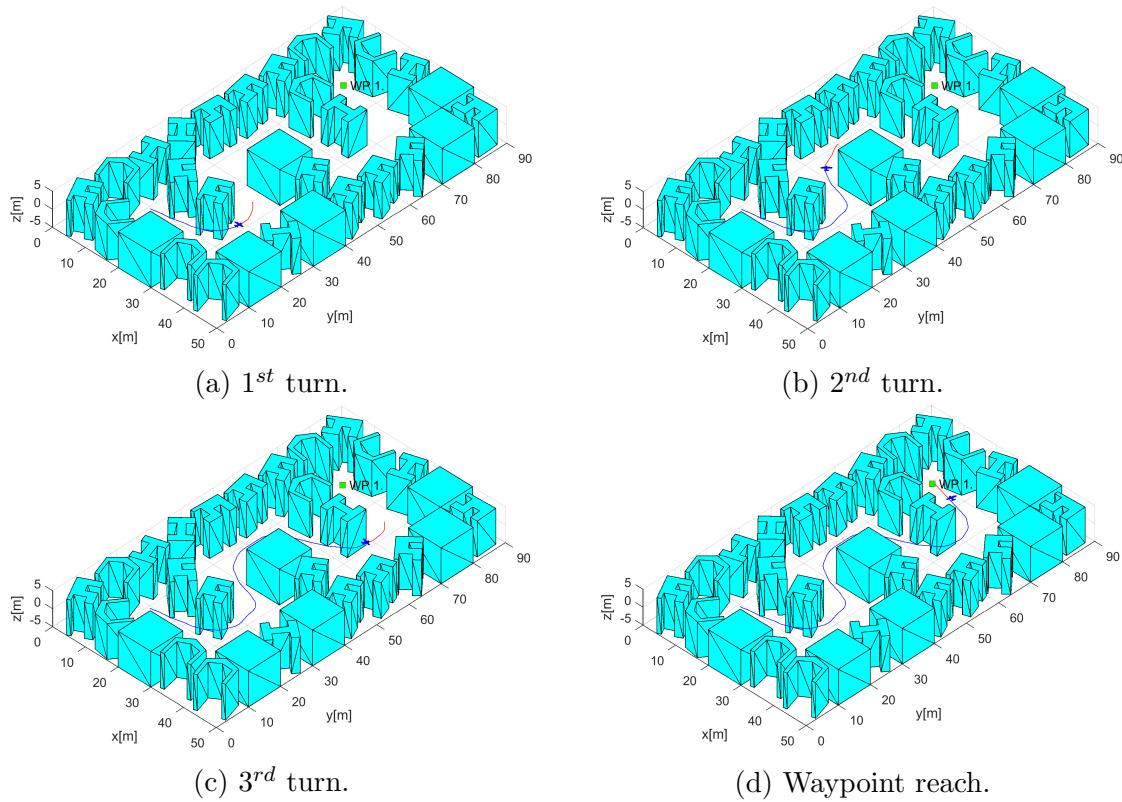


Figure 7.7: Test scenario for *Maze*.

**Safety Margin Performance:** The *safety margin performance* over time (x-axis, sec) given in meters distance (y-axis, m) is given in (fig. 7.8).

The *UAS* center distance to nearest obstacle (blue line) does not break any *Safety Margin* (yellow line) of closest obstacle. *Body Margin* of closest obstacle (red line) has not been break, because it always lies below of *Safety Margin* (yellow).

For UTM time period 37 to 68 sec there is a *margin spike* due avoidance of bloated *Rectangle buildings* (fig. 7.7b) during 2<sup>nd</sup> turn. The *acceptance criterion* for *Safety Margin* is satisfied.

*Note.* The *body* and *safety margin* are changing depending on *UAS position* and *orientation*. The changes are reflected in (tab. 7.17).

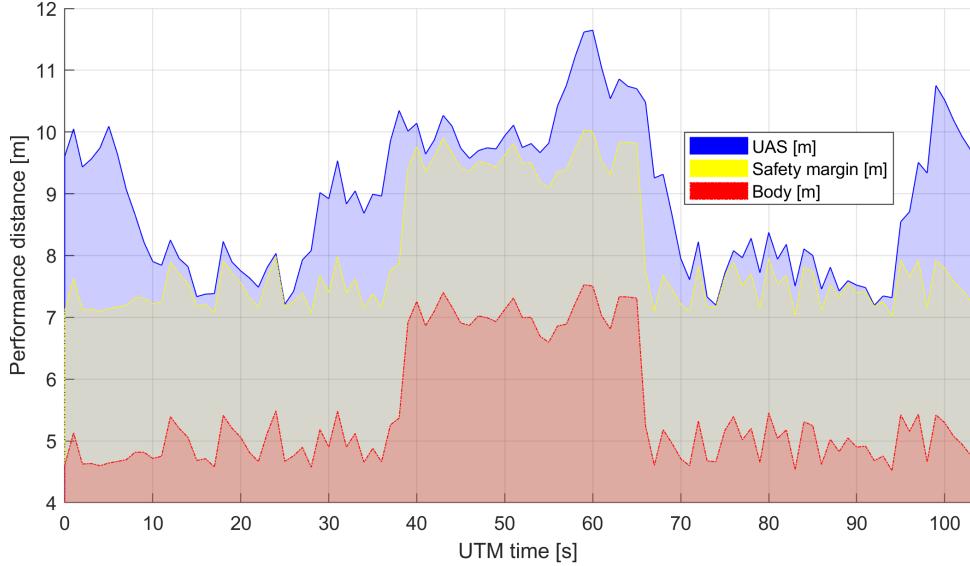


Figure 7.8: *Maze* safety margin performance.

**Safety Margin distances:** The minimal and maximal values for *UAS body closest point to safety margin* distance based on performance (fig. 7.8) is summarized in (tab. 7.17).

The *minimal distance to safety margin* is  $0.0131\text{m}$  which can be taken as  $\sim 0\text{m}$  due the numerical error. The *maximal distance to safety margin* is  $2.9513\text{m}$  which is  $5 \times \text{UAS radius}$ . The safety margin distance is  $\leq 3\text{m}$  which means the scenario is tightly packed with obstacles. The *UAS* never left *Emergency Avoidance Mode* because condition:  $\text{safetyMarginDistance} \geq \text{avoidanceGridLength}$  was never satisfied.

The *minimal body* distance is  $5.0131\text{m}$ , while the *maximal body* distance is  $8.7117\text{m}$ . The difference between minimal and maximal body distance is  $\sim 4\text{m}$  which also indicates obstacle packed scenario.

Parameter	UAS 1	
safety margin distance	min	0.0131
	max	2.9513
body margin distance	min	5.0131
	max	8.7117

Table 7.17: *Maze* safety and body margin distances.

**Path Tracking Performance:** Reference path (green dashed) line is given as direct interconnection of *initial position* (green square with S marker) and *hidden waypoint* (green square with 1 marker). The *UTM Reference Time* is given on x-axis. The evolution of real trajectory (blue solid line) for each axis is given as follow:

1. *X-axis path tracking* - reflects the maneuvering in the curves of the maze.
2. *Y-axis path tracking* - shows horizontal progress to the *hidden goal waypoint*. The expected linear tracking is not achievement due the maneuvering delays on X-axis.
3. *Z-axis path tracking* - shows perfect linear tracking of reference trajectory. The *altitude acceptance criterion*:  $-5\text{m} \leq \text{altitude} \leq 5\text{m}$  have been fulfilled.

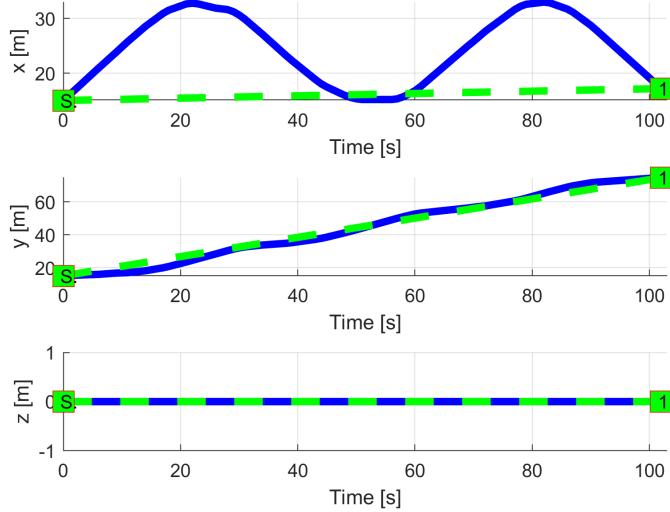


Figure 7.9: *Maze* path tracking.

**Path Tracking Deviations:** Deviations (tab. 7.18) from *reference trajectory* are in expected ranges considering *mission plan* (tab. 7.15) and *obstacle properties* (tab. 7.16).

Param.	UAS 1 $\mathcal{WP}_1$
$\max  x $	27.32
$\max  y $	2.41
$\max  z $	0
$\max dist.$	28.06

Table 7.18: Path tracking properties for *Maze* scenario.

### 7.3.4 Storm

**Scenario:** Small UAS is flying in open space in uncontrolled airspace ( $\leq 500$  feet AGL (Above Ground Level)). A *Weather Service* notices UAS about *Dangerous Weather zone* (virtual constraint s. 6.4.3) which is moving in UAS direction. The *UAS* is executing mission given by (tab. 7.19).

Position		$\mathcal{WP}_1$
$[x, y, z]$	$[\theta, \varpi, \psi]$	
$[0, 0, 0]^T$	$[0^\circ, 0^\circ, 90^\circ]^T$	$[0, 60, 0]^T$

Table 7.19: Mission setup for *Storm* scenario.

**Constraints:** The *storm* is modeled as a *virtual constraint* with parameters given in (tab. 7.20). A constraint is modeled as a *convex polygon* for *horizontal boundary* and altitude for the *vertical boundary*.

The *Storm* is moving through an *operational region* with linear velocity  $0.5ms^{-1}$ . The *storms center* was first detected at *decision frame* 0 at position  $[0, 50, 0]^T$ .

Constraint			Body Margin			Safety Margin
i. position	velocity	type	min.	max.	avg.	
$[0, 50, 0]^T$	$[0, -0.5, 0]$	polygon	9	10	9.5	5

Table 7.20: *Constraint set* for *Storm* scenario.

**Assumption:** Every *avoidable moving constraint* is usually slower than an *Approaching UAS*, or its radius is smaller than turning radius of an *Approaching UAS*.

*Note.* Manned aviation receives permit to operate in a *controlled airspace* only if it has capability outmaneuver every known threat in requested airspace.

The *Constrained space portion* is usually very large, therefore in majority of cases the assumption  $uasSpeed >> constraintSpeed$  holds.

**Main Goal:** Show dynamic moving constraint avoidance capability in an *uncontrolled airspace*.

#### Acceptance criteria:

1. *Hard constraint avoidance* - the *UAS* must not cross the body margin:  $distance(stormCenter, UAS) \geq bodyMargin$ .
2. *Soft constraint avoidance* - the *UAS* can not cross the safety margin to get into close proximity of *Storms surrounding area*:  $distance(stormCenter, UAS) \geq safetyMargin$ .

**Testing setup:** The *standard test setup* defined in (tab. 7.2, 7.3, 7.4, 7.5, 7.6) is used with following parameter override:

1. *Avoidance grid - type* - ACAS-like with enabled *Horizontal maneuvers*.

**Simulation run:** Notable moments from a *simulation run* (fig. 7.10) are following:

1. *Detection* (fig. 7.10a) - the *Storm* (magenta polygon) is detected prior the engagement (retrieved from associated weather service). The *UAS* (blue) stays in *Navigation mode*. *Trajectories* in *Navigation grid* are constrained by rule *Enforce safety margin* (tab. 6.11). The *Planned trajectory* (red) changes to avoid *Storm*.
2. *Avoidance start* (fig. 7.10b) - when *optimal avoidance distance* is reached by UAS, the *navigation reach set* is constrained, forcing UAS to perform evasive maneuver.
3. *Avoidance end* (fig. 7.10c) - navigation space is no longer constrained when the *minimal safe distance/heading* is achieved.
4. *Waypoint reached* (fig. 7.10d) - standard waypoint navigation procedure was used in this case.

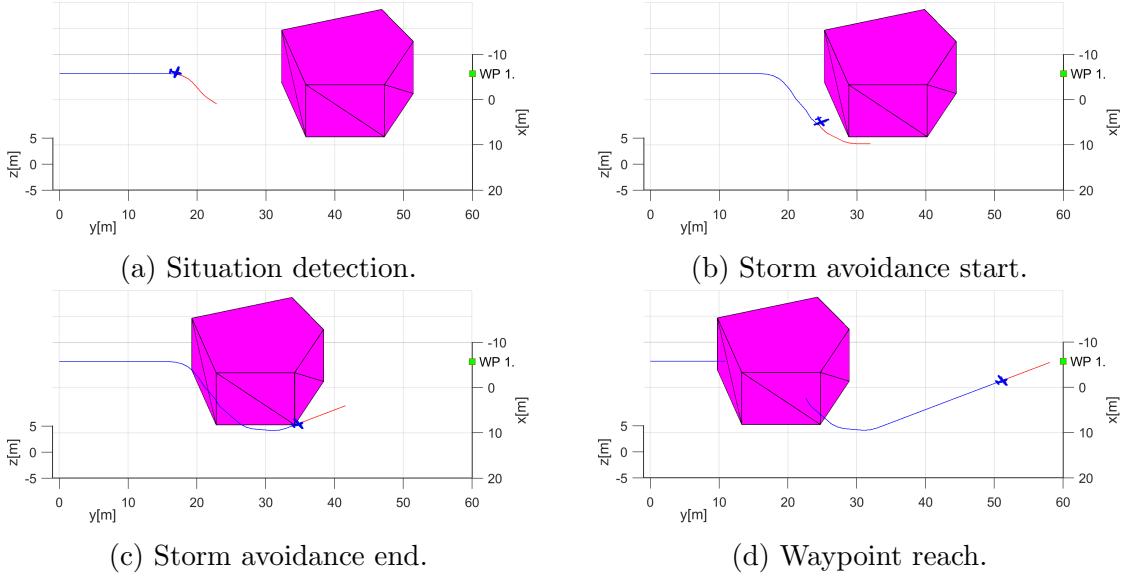


Figure 7.10: Test scenario for *Storm* (Dynamic hard constraint).

**Safety Margin Performance:** The *body margin* (red line) and *safety margin* (yellow line) and *UAS distance to storm center* (blue line) evolution over *UTM time* are given in (fig. 7.11) The *body* and *safety margin* was changing according to mutual position of the *storm* and the *UAS* (see tab. 7.20).

The acceptance criteria for the *hard constraint avoidance* and *soft constraint avoidance* seems to be fulfilled.

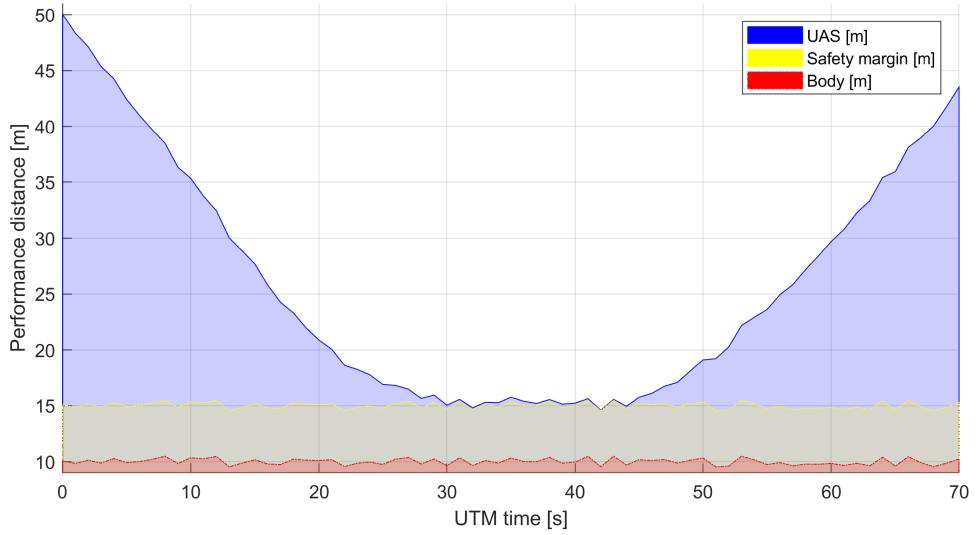


Figure 7.11: *Storm* safety margin performance.

**Safety Margin Distance** A *hard constraint* of *body margin* was not breached, because the  $distance(UAS(t), stormBody(t))$  was all time greater than 0. Thus the *UAS* stayed well clear from *Storm*. The summary (tab. 7.21) shows that the *minimal body margin distance* was 5.0335 m, which proves *avoidance of hard constraint*.

A *soft constraint* of *safety margin* (protective coating around storm body) was not breached, because the  $distance(UAS(t), stormBody(t)) - safetyMargin(t)$  was all time

greater than  $\theta$ . The summary (tab. 7.21) show that the *minimal safety margin distance* was  $0.0355\text{ m}$ , which proves *avoidance of soft constraints*.

Parameter	UAS 1	
safety margin distance	min	0.0355
	max	34.9934
body margin distance	min	5.0355
	max	39.9934

Table 7.21: *Storm* safety and body margin distances.

**Path Tracking Performance:** The *path tracking* (blue solid line) of *reference trajectory* (green dashed line) between *starting waypoint* (green square marked "S") and *final waypoint* (green square marked "1") is portrayed in (fig. 7.12). The *UAS* executes *horizontal right-side avoidance* of the *Storm* as is preferred.

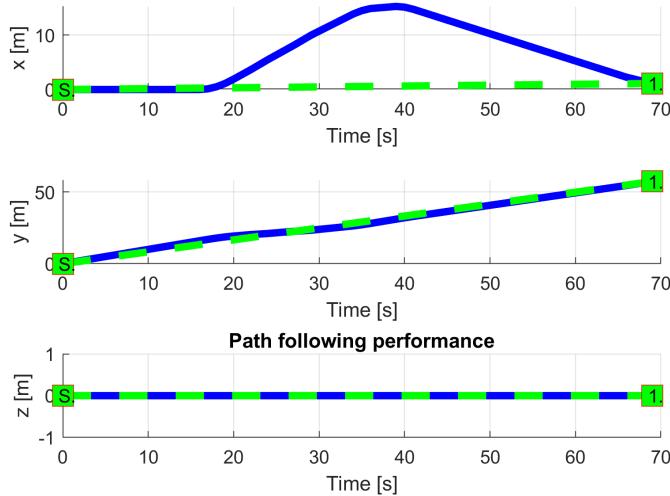


Figure 7.12: *Storm* path tracking.

**Path Tracking Deviations:** *Deviations* (tab. 7.22) are in expected ranges considering the mission plan (tab. 7.19) and *body* and *safety* margins (tab. 7.20).

Param.	UAS 1	$\mathcal{WP}_1$
$\max  x $	15.26	
$\max  y $	1.32	
$\max  z $	0	
$\max dist.$	15.76	

Table 7.22: Path tracking properties for *Storm* scenario.

### 7.3.5 Emergency Converging

**Scenario:** Two *UAS* are flying in an *uncontrolled airspace* (altitude  $\leq 500$  ft. Above the Ground Level) with missions defined in (tab. 7.23). Both *UAS* are in the *Navigation*

*mode* with active *ADSB-In/Out*, receiving position notification from each other. Cruising altitude is sufficient for horizontal separation (50-100 ft. Above the Ground Level). *Horizontal separation* is preferred separation type for both *UAS*.

UAS	Position		$\mathcal{WP}_1$
	$[x, y, z]$	$[\theta, \varpi, \psi]$	
1	$[0, 20, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[40, 20, 0]^T$
2	$[20, 0, 0]^T$	$[0^\circ, 0^\circ, 90^\circ]^T$	$[20, 40, 0]^T$

Table 7.23: Mission setup for *Emergency converging* scenario.

*Note.* *Collision point* is expected at  $\mathcal{C} = [20, 20, 0]^T$ . The *angle of approach* is  $90^\circ$  which classifies situation as *Converging maneuver* (fig. 6.21).

**Main Goal:** Show two *non-cooperative* UAS avoidance capability for *Converging maneuver* scenario in *uncontrolled airspace*.

#### Acceptance criteria:

1. *Proper mode invocation* - when an intruder intersects the UAS with *Right of the Way* navigation grid, both UAS will switch into *Emergency Avoidance Mode*.
2. *Minimal safety margin distance*  $\geq 0m$ .
3. *Each UAS* will reach own goal waypoint (tab. 7.23).

**Testing setup:** The *standard test setup* for each UAS defined in (tab 7.2, 7.3, 7.4, 7.5, 7.6) is used with following without parameter override.

*Intruder intersection* model has been chosen depending on UAS (tab. 7.24). Each UAS is equipped with *ADS-B In/Out* sensor obtaining/distributing following information:

1. *Position* - in operational section coordinate frame.
2. *Velocity* - vector representation in given coordinate frame.
3. *Class size* - class body radius based on UAS propulsion and size.
4. *Safety margin set* - set of safety margins for different collision cases.

*Avoidance parameters* for *Emergency converging scenario* are given in (tab. 7.24). Each UAS has same speed set to  $1ms^{-1}$ . Second UAS has the *Right of The Way*.

*Safety margin* is considered as sum of both participants *near miss margins*. In this case default safety margin is considered as  $1.2 m$ .

UAS	Parameters			Margins		Separation
	velocity	intruder model	ROW	body	safety	
1	1	body + spread	false	0.3	0.6	horizontal
2	1	body + spread	true	0.3	0.6	horizontal

Table 7.24: Avoidance parameters for *Emergency converging* scenario.

*Note.* Both UAS are using body (sec. 6.5.3) and spread (sec 6.5.4) intersection models, reflecting both body volume and maneuverability of intruder. Both UAS have preferred separation mode as *horizontal*, typical for planes.

**Simulation Run:** Notable moments from the simulation run (fig. 7.13) are following

1. *Detection* (fig. 7.13a) - Intruder (UAS2 cyan) is approaching (UAS 1 blue) from right side, Intruder (UAS2 cyan) has the right of the way, because  $70^\circ \leq \text{angleOfApproach} < 130^\circ$ . *Intruder intersection model* (for UAS 2) is created and propagated in *avoidance grid* (for UAS 1).
2. *Start Converging* (fig. 7.13b) - when *UAS 2 (cyan) parametric intruder intersection model* disables *trajectories*, converging maneuver for UAS 1 (blue) starts.
3. *Near miss case* (fig. 7.13c) - UAS 1 (blue) to UAS 2 (cyan) closest distance. The safety margin for *near miss* has not been breached. The safety margin for *well clear* in uncontrolled airspace is invalid.
4. *Waypoint reached* (fig. 7.13d) - the intruder intersection model for *UAS 2* (cyan) is removed from UAS 1 (blue) *avoidance grid* after *converging maneuver competition*, standard navigation procedure is applied afterwards.
5. Note that *UAS 2* (cyan) has *Right of the way* in (tab. 7.24).
6. Note that *UAS 1* (blue) used only horizontal separation (priority) in (fig. 7.15a).

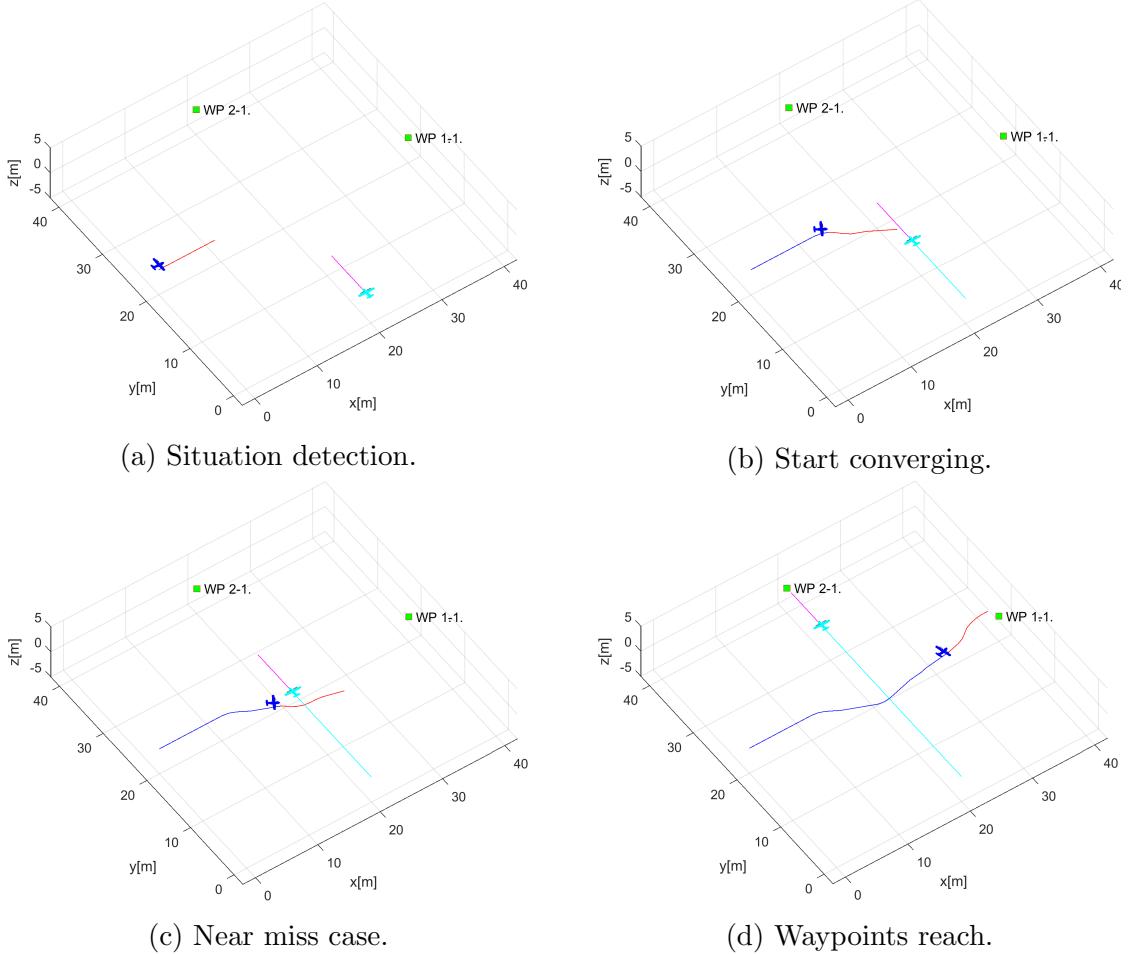


Figure 7.13: Test scenario for *Emergency converging* (Intruder avoidance).

**Safety Margin Performance:** There is need to compare mutual distance between both UAS (y-axis [m]) and its evolution over UTM time (x-axis [s]). The *mutual distance* of *UAS 1* to *UAS 2* is given by *blue line*. The *Safety margin* value is denoted by *red line* at *constant value* of *1.2m*.

The *Proper avoidance Invocation* is shown when UAS systems are getting closer to each other and they start their separation phase (Emergency Avoidance Mode). The *Mutual distance* (blue line) does not cross *safety margin* (red line).

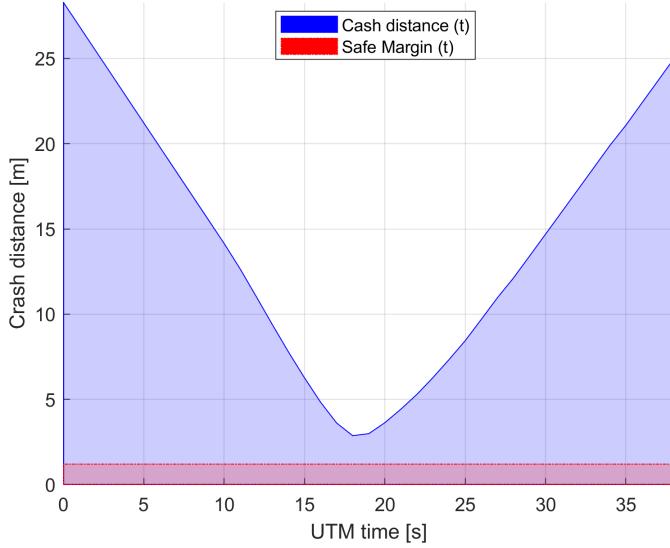


Figure 7.14: *Emergency converging safety margin performance.*

**Safety Margin Distance:** Minimal and Maximal mutual distance to safety margin is summarized in (tab. 7.25). The closest to collision are UAS systems when distance to safety margin is  $1.6676\text{m}$ .

The *minimal safety margin distance*  $\geq 0$  which means that the *safety acceptance criterion* is full filled.

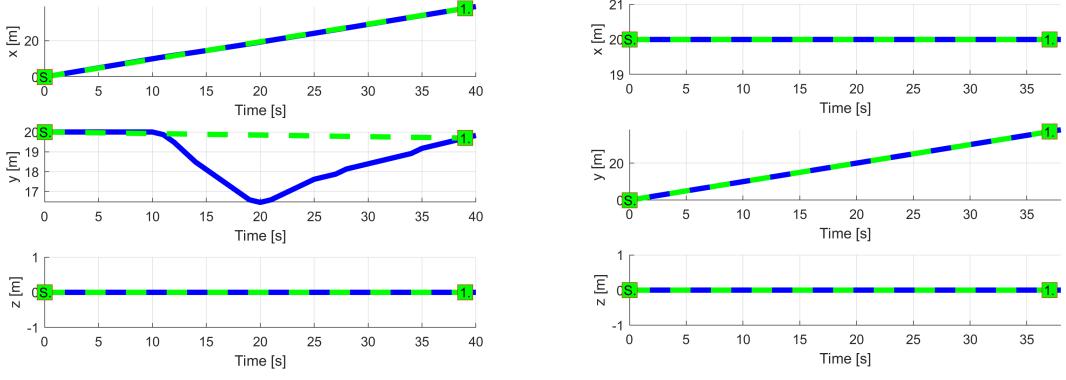
UAS:		1-2
safety margin distance	$\min$	1.6676
	$\max$	27.0843

Table 7.25: *Emergency converging safety margin distances.*

**Path Tracking Performance:** All waypoints (green numbered squares) for both UAS have been reached (fig. 7.15). *Reference trajectories* (green dashed lines), between initial position (green square marked S) and goal waypoint (green square marked 1) are split into three XYZ values with respective figures. The tracked value is on y-axis [m] and time on x-axis [s]. The blue lines represents real parameter evolution over time.

Following observations can be made from path tracking (fig. 7.15) and preferred separations (tab. 7.24):

1. UAS 1 (fig. 7.15a) is using *horizontal separation* (y-axis). The UAS diverges from reference trajectory to minimal necessary time.
2. UAS 2 (fig. 7.15b) has the right of the way and is not using any active avoidance mechanism.



(a) UAS 1.

(b) UAS 2.

Figure 7.15: *Trajectory tracking for Emergency converging test case.*

**Path Following Deviations:** *Deviations* (tab. 7.26) are in expected ranges considering the *mission plans* (tab. 7.23) and *separation safety margin* (tab. 7.24).

Param.	UAS 1	UAS 2
	$\mathcal{WP}_1$	$\mathcal{WP}_1$
$\max  x $	0	0
$\max  y $	3.25	0
$\max  z $	0	0
$\max dist.$	3.25	0

Table 7.26: Path tracking properties for *Emergency converging* scenario.

### 7.3.6 Emergency Head on

**Scenario:** Two *UAS* systems are flying in an *uncontrolled airspace* ( $\text{altitude} \leq 500 \text{ ft}$ . Above the Ground Level) with missions defined in (tab. 7.27). Both *UAS* are in the *Navigation mode* with active *ADSB-In/Out*, receiving position notifications from each other. Cruising altitude is sufficient for horizontal separation (50-100 ft. Above Ground Level). *Horizontal separation* is preferred mode for both *UAS*.

UAS	Position		$\mathcal{WP}_1$
	$[x, y, z]$	$[\theta, \varpi, \psi]$	
1	$[0, 20, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[40, 20, 0]^T$
2	$[40, 20, 0]^T$	$[0^\circ, 0^\circ, 180^\circ]^T$	$[0, 20, 0]^T$

Table 7.27: Mission setup for *Emergency head on* scenario.

*Note.* *Collision point* is expected at  $\mathcal{C} = [20, 20, 0]^T$ . The *angle of approach* is  $180^\circ$  which classifies situation as *Head on maneuver* (fig. 6.19).

**Main Goal:** Show two *non-cooperative* *UAS* avoidance for *Head-on approach scenario* in *uncontrolled* airspace.

### Acceptance criteria:

1. *Proper mode invocation* - when an intruder intersects the opposing *UAS* Navigation grid, bot intruder and *UAS* will switch to *Emergency Avoidance Mode*. None of the *UAS* have *Right Of the Way*.
2. *Minimal Safety Margin distance*  $\geq 0m$ . That means the mutual distance of both *UAS centers* does not go below given *safety margin*.
3. *Both UAS* will reach own goal waypoint (tab. 7.27).

**Testing setup:** The *standard test setup* for each *UAS* defined in (tab 7.2, 7.3, 7.4, 7.5, 7.6) is used with following without parameter override.

*Intruder intersection* model has been chosen depending on *UAS* (tab. 7.28). Each *UAS* is equipped with *ADS-B In/Out* sensor obtaining/distributing following information:

1. *Position* - in operational section coordinate frame.
2. *Velocity* - vector representation in given coordinate frame.
3. *Class size* - class body radius based on *UAS* propulsion and size.
4. *Safety margin set* - set of safety margins for different collision cases.

*Avoidance parameters* for *Emergency head-on scenario* are given in (tab. 7.28). Each *UAS* has same speed set to  $1ms^{-1}$ . None of them have the *Right of The Way*.

*Safety margin* is considered as sum of both participants *near miss margins*. In this case default safety margin is considered as  $1.2 m$ .

UAS	Parameters			Margins		Separation
	velocity	intruder model	ROW	body	safety	
1	1	body (timed)	false	0.3	0.6	horizontal
2	1	body (timed)	false	0.3	0.6	horizontal

Table 7.28: Avoidance parameters for *Emergency head on* scenario.

*Note.* Both *UAS* are using body (sec. 6.5.3) intersection model, reflecting both body volume along expected trajectory. Both *UAS* have preference for *horizontal* separation mode, typical for planes.

**Simulation Run:** Notable moments from the simulation run (fig. 7.16) are following:

1. *Situation detection* (fig. 7.13a) - *UAS 1* (blue) is approaching *UAS 2* (cyan) with  $130^\circ \leq angleOfApproach \leq 180^\circ$ , this is considered head on approach. Head on approach give the *right of the way* neither to *UAS 1* nor *UAS 2*. *Intruder intersection model* for opposite *UAS* is created in respective *avoidance grids*. *Head on emergency avoidance* starts independently in each *UAS* without intruders coordination. First *avoidance maneuver* is invoked when the *intruder intersection model* constraints any trajectory in the *avoidance grid*. When this happens *Navigation mode* switch to the *Emergency avoidance mode*.

2. *Before near miss* (fig. 7.16b) - both *UAS* are in *emergency avoidance mode*, sticking to right side avoidance maneuver.
3. *Near miss case* (fig. 7.16c) - UAS 1 to UAS 2 closest distance. The safety margin for *near miss* has not been breached. The safety margin for *well clear* in uncontrolled airspace is invalid. Both UAS are using also *Horizontal separation* to avoid each other, *Emergency avoidance mode* is switched to the *Navigation mode* when risk of an *aerial clash* is voided.
4. *After near miss* (fig. 7.16d) - both *UAS* are tracking back to respective waypoint, correcting *altitude* (Z-axis in (fig. 7.18)) first.
5. Note *Collision point* was expected at  $\mathcal{C} = [20, 20, 0]^T$
6. Note *Both UAS* used *horizontal* (primary), *vertical* (secondary) separation (fig 7.18).
7. Note *Both UAS* decision times were *synchronized*, this is not an assumption, but it shows critical performance. Usually safety margin is bloated for (eq.6.27).

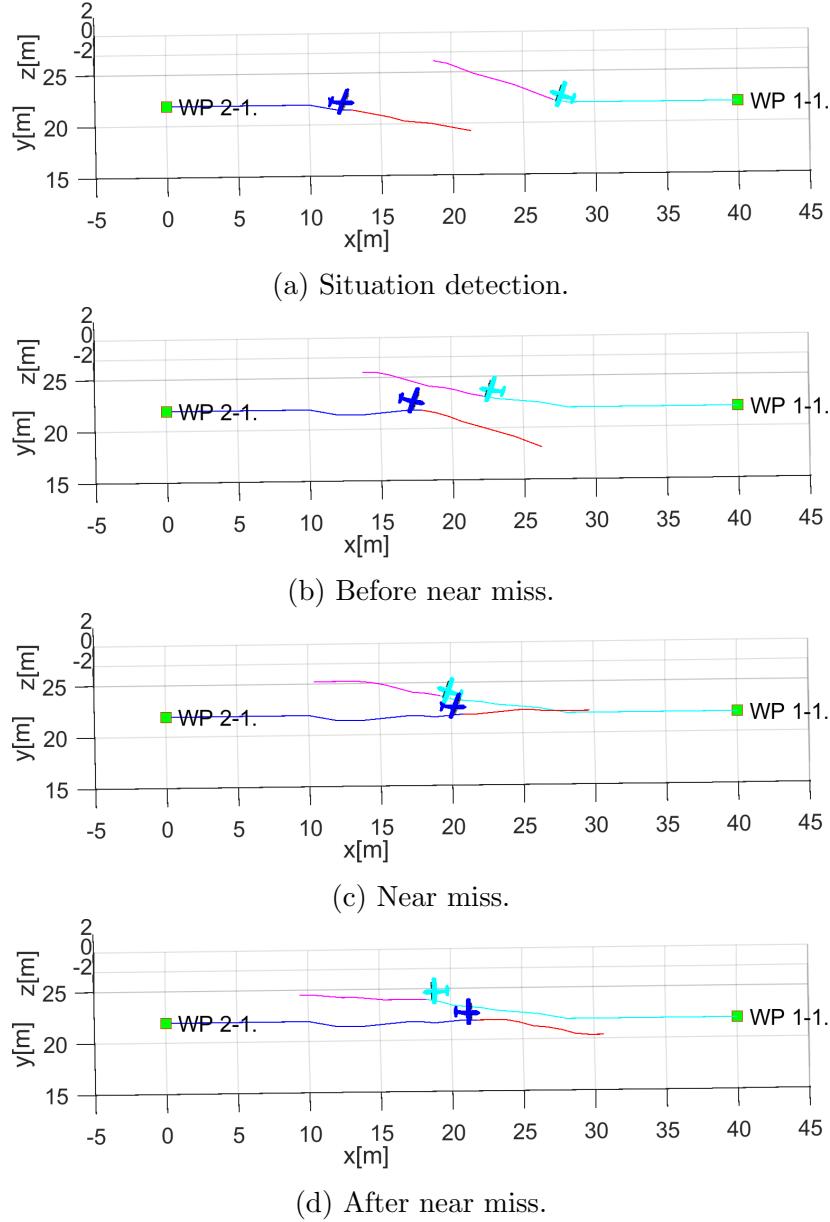


Figure 7.16: Test scenario for *Emergency head on approach* (Intruder avoidance).

**Safety Margin Performance:** There is need to compare mutual distance between both UAS (y-axis [m]) and its evolution over synchronized *UTM time* (x-axis [s].) The *mutual distance* between bodies of *UAS 1*, *UAS 2* (blue line) compared to *Safety Margin* (red line) is given in (fig. 7.17). The *Safety Margin* value was constant for all time at value  $1.2m$  which is double of *Near Miss Margin* for *UAS 1* *UAS 2*.

The proper *Avoidance Invocation* is shown when *UAS* systems are getting closer to each other and they starts their *separation phase* (Emergency Avoidance Mode switch). The mutual distance (blue line) does not cross *safety margin* (red line).

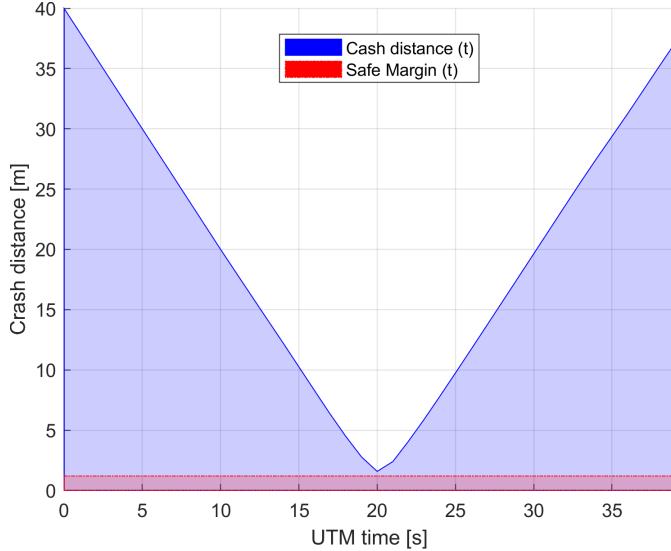


Figure 7.17: *Emergency head on safety margin performance.*

**Safety Margin Distance:** Minimal and Maximal mutual distance to safety margin is summarized in (tab. 7.29). The closest to collision are UAS systems when *distance to safety margin* is 0.3824m.

The *minimal safety margin distance*  $\geq 0$  which means that the *safety acceptance criterion* is full filled.

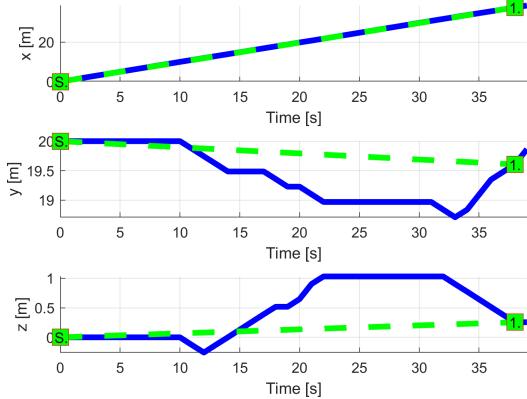
UAS:			1-2
safety margin distance	min		0.3824
	max		38.8000

Table 7.29: *Emergency head on safety margin distances.*

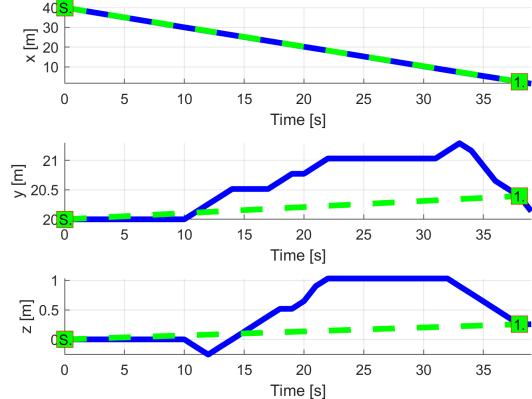
**Path Tracking Performance** All waypoints (green numbered squares) for both UAS have been reached (fig. 7.18). *Reference trajectories* (green dashed lines), between initial position (green square marked S) and goal waypoint (green square marked 1) are split into three XYZ values with respective figures. The tracked value is on y-axis [m] and time on x-axis [s]. The blue lines represents real parameter evolution over time.

Following observations can be made from path tracking (fig.7.18) and preferred separations (tab. 7.28):

1. UAS 1 (fig. 7.18a) is using horizontal separation going to the right (y-axis) and a little bit up (z-axis).
2. UAS 2 (fig. 7.18b) is using horizontal separation going to the right (left in GCS, y-axis) and little bit up (z-axis).



(a) UAS 1.



(b) UAS 2.

Figure 7.18: Trajectory tracking for *Emergency head on* test case.

**Path Following Deviations:** *Deviations* (tab. 7.30) are in expected ranges considering the *mission plans* (tab. 7.27) and *separation safety margins* (tab. 7.28).

Param.	UAS 1	UAS 2
	$W\mathcal{P}_1$	$W\mathcal{P}_1$
$\max  x $	0.05	0.06
$\max  y $	1.37	1.48
$\max  z $	1.03	1.05
$\max dist.$	1.39	1.52

Table 7.30: Path tracking properties for *Emergency head on* scenario.

### 7.3.7 Emergency Mixed Head on with Converging

**Scenario:** Four UAS are flying in an *uncontrolled airspace* (altitude  $\leq 500$  ft. Above the Ground Level) missions defined in (tab. 7.31). All UAS are in the *Navigation mode* with active *ADS-B In*, receiving *position notifications* from each other. Cruising altitude is sufficient for *horizontal separation* (50-100 ft. Above the Ground Level).

UAS	Position		$W\mathcal{P}_1$
	$[x, y, z]$	$[\theta, \varpi, \psi]$	
1	$[0, 20, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[45, 20, 0]^T$
2	$[40, 20, 0]^T$	$[0^\circ, 0^\circ, 180^\circ]^T$	$[-5, 20, 0]^T$
3	$[20, 0, 0]^T$	$[0^\circ, 0^\circ, 90^\circ]^T$	$[20, 45, 0]^T$
4	$[20, 40, 0]^T$	$[0^\circ, 0^\circ, -90^\circ]^T$	$[45, 20, 0]^T$

Table 7.31: Mission setup for *Emergency mixed* scenario.

*Note.* Collision point is expected at  $\mathcal{C} = [20, 20, 0]^T$

**Main Goal:** Show *multiple non-cooperative intruders avoidance capability* in *uncontrolled* airspace.

### Acceptance criteria:

1. Proper avoidance mode invocation - when an *intruder intersection model* impact the *Avoidance Grid*, UAS system will switch to an *Emergency avoidance mode*.
2. Minimal safety margin distance  $\geq 0m$ .
3. Each *UAS* will reach own goal waypoint (tab. 7.31).

**Testing setup:** The *standard test setup* for each UAS defined in (tab 7.2, 7.3, 7.4, 7.5, 7.6) is used with following without parameter override.

*Intruder intersection* model has been chosen depending on UAS (tab. 7.32). Each UAS is equipped with *ADS-B In/Out* sensor obtaining/distributing following information:

1. *Position* - in operational section coordinate frame.
2. *Velocity* - vector representation in given coordinate frame.
3. *Class size* - class body radius based on UAS propulsion and size.
4. *Safety margin set* - set of safety margins for different collision cases.

*Avoidance parameters* for *Emergency mixed scenario* are given in (tab. 7.32). Each UAS has different *intruder model* and separation combination. Each UAS has same speed set to  $1ms^{-1}$ . None of UAS has the *Right of The Way*.

*Safety margin* is considered as sum of both participants *near miss margins*. In this case default safety margin is considered as  $1.2 m$ .

UAS	Parameters			Margins		Separation
	velocity	intruder model	ROW	body	safety	
1	1	body + spread	false	0.3	0.6	horizontal
2	1	body (timed)	false	0.3	0.6	vertical
3	1	body (timed)	false	0.3	0.6	horizontal
2	1	body + spread	false	0.3	0.6	vertical

Table 7.32: Avoidance parameters for *Emergency mixed* scenario.

*Note.* Each *UAS* use different intruder intersection models and primary *separations* (defined in tab. 7.32). UAS reactions are based on primary *Separation* mode, intruders intersection models this is reflected on major axial deviations in (fig. 7.21) and summarized in *path tracking* deviation (tab. 7.34).

**Simulation Run:** Notable moments from the simulation run (fig. 7.19) are following:

1. *Situation detection* (fig. 7.19a) - UAS 1 (blue) is detecting UAS 2 (cyan), UAS 3 (green), and UAS 4 (black) as possible intruders. There are multiple converging and head on approaches depending on mutual positions (UAS and *angle of approach*). There exist at least one *converging case* where each UAS has *Right of the way*. Each UAS creates intruder intersection models depending on the intruder configuration (tab. 7.32). Each UAS enters into the *Emergency avoidance mode* independently, when at least one trajectory is constrained in the *avoidance grid*.

2. *Before near miss* (fig. 7.19b) - all UAS are in *emergency avoidance mode*, using various *separation modes* and *intruder intersection models*. Each UAS is performing its own avoidance maneuver, constantly checking other intruders. If same separation and same intruder model was used, there will be virtual roundabout.
3. *After near miss* (fig. 7.19c) - all UAS avoided each other which is covered in *safety margin performance* (fig. 7.20) and (tab. 7.33).
4. *Situation resolution* (fig. 7.19d) - all UAS returns to *Navigation mode* correcting *altitude* first and continuing to assigned waypoints.

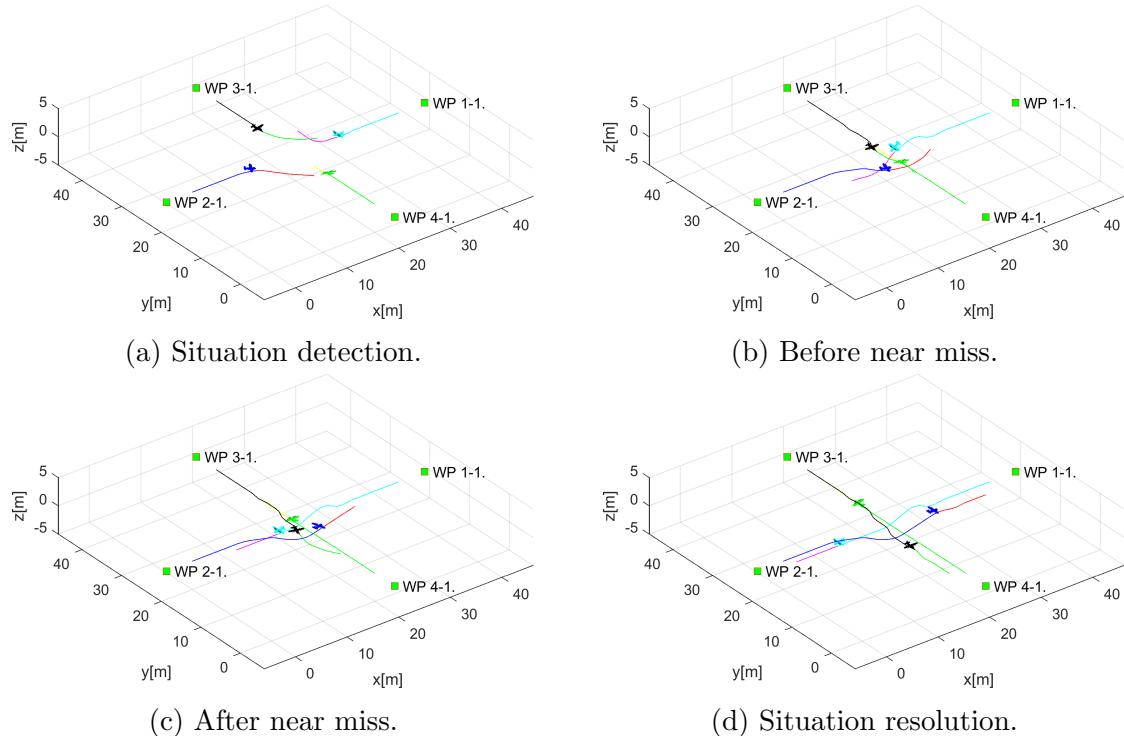


Figure 7.19: Test scenario for *Emergency mixed* situation with *self-separation mode*.

**Safety Margin Performance:** There is need to compare mutual distance between each UAS. The graph (fig. 7.20) shows six figures for each mutual distance (blue line) in this scenario. The *Safety Margin* (red line) ( $1.2\text{ m}$ ) was not breached in any case.

The *Proper avoidance invocation* is shown when UAS systems are getting closer to each other and then they start separation phase (Emergency avoidance mode).

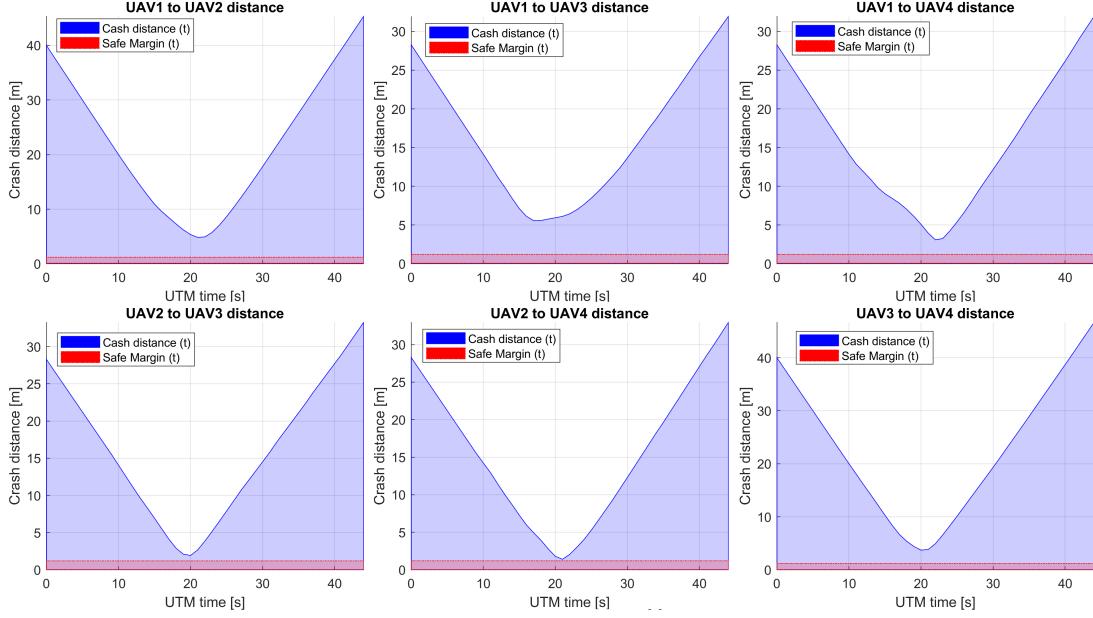


Figure 7.20: *Emergency mixed* situation safety margin performance.

**Safety Margin Distances:** Minimal and Maximal mutual distance to safety margin is summarized in (tab. 7.33). There is no detected breach for any combination.

The *closest to collision* is UAS pair 2 – 4 with mutual safety margin only 0.2019 m. On the other side is UAS pair 1 – 3 with mutual safety margin 4.3721 m.

The *minimal safety margin distance*  $\geq 0$  which means that the *safety condition* is fulfilled.

UAS:	Safety margin distance		
	min	max	breach
1-2	3.6231	44.0831	false
1-3	4.3721	30.7300	false
1-4	1.8959	30.7331	false
2-3	0.7331	32.0266	false
2-4	0.2019	31.7282	false
3-4	2.5171	45.4257	false

Table 7.33: *Emergency mixed* safety margin distances.

**Path Tracking Performance:** All waypoints (Green numbered squares) for all UAS have been reached (fig. 7.21). *Reference trajectories* (green dashed line) have been tracked by *UAS real path* (blue solid line) almost all time.

Following observations can be made from *path tracking* (fig. 7.21) and *preferred separations* (tab. 7.32):

1. UAS 1 (fig. 7.21a) is using *horizontal separation* (y axis right) having *preferred horizontal separation*.
2. UAS 2 (fig. 7.21b) is using vertical separation (z axis up-down), having preferred vertical separation.

3. UAS 3 (fig. 7.21d) is using horizontal/vertical separation (x right, z down), having preferred horizontal separation. This UAS has used other than preferred separation type.
4. UAS 4 (fig. 7.21c) is using horizontal separation (x-axis right/left), having preferred vertical separation. This UAS has used opposite separation type, than preferred.

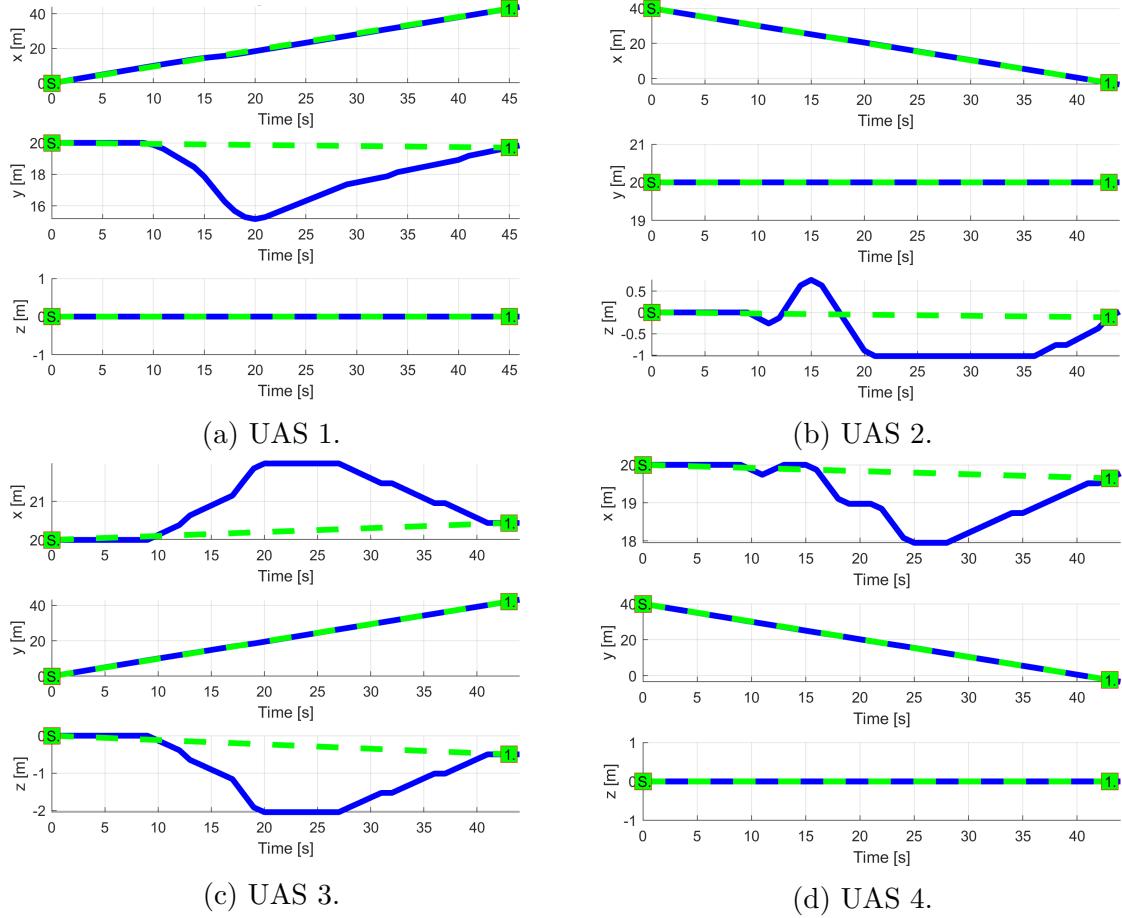


Figure 7.21: Trajectory tracking for *Emergency mixed* situation test case.

**Path Tracking Deviations:** *Deviations* (tab. 7.34) are in expected ranges considering the *mission plans* (tab. 7.31) and *separation safety margins* (tab. 7.32).

Param.	UAS 1	UAS 2	UAS 3	UAS 4
	$\mathcal{WP}_1$	$\mathcal{WP}_1$	$\mathcal{WP}_1$	$\mathcal{WP}_1$
$\max  x $	0	0	1.98	2.05
$\max  y $	4.84	0	0	0
$\max  z $	0	1.23	2.43	0
$\max dist.$	4.84	1.23	3.45	2.05

Table 7.34: Path tracking properties for *Emergency mixed* scenario.

## 7.4 Cooperative test cases

The *main goal* of this section is to show the operational capabilities of *approach* under *UTM supervision*. The minimal UTM functionality set (sec. 6.8) has been implemented, including *position notifications mechanism*, *collision case calculation*, *resolution enforcement* components.

Test cases covers *well clear breach prevention*, *situation based avoidance*, and *rules of the air enforcement*.

Coverage of *near miss situations*, *clash incidents* is given implicitly by *safety* and *body margins* (tab. 4.1).

1. *Rule based converging* (sec. 7.4.1) covers *well clear breach* and *converging rule of the air*, showing determinism and *UTM resolution execution*.
2. *Rule based head on* (sec. 7.4.2) covers *well clear breach* and *head on rule of the air*, showing determinism and *UTM resolution execution*.
3. *Rule based mixed head on with converging* (sec. 7.4.3) covers *well clear breach* and *head on and converging rules of the air*. The main focus is on *virtual roundabout* concept, when multiple collision cases are clustered into one avoidance maneuver.
4. *Rule based overtake* (sec. 7.4.4) covers *well clear breach* during *overtake* by faster UAS.

### 7.4.1 Rule based Converging

**Scenario:** Two *UAS* are approaching an *airway intersection* at *same time* in *controlled airspace* (over 500 feet Above the Ground Level). The mutual position of *UAS* can be classified as *Side approach*. Following *collision hazards* are present:

1. *Active Converging Collision Hazard* - There is an *UAS* approaching from the *right side*, which give him *Right of the Way* and invokes need to actively avoid *Intruder*.
2. *Passive Converging Collision Hazard* - There is an *UAS* approaching from the *left side*, which gave us *Right of the Way* and imposes an obligation of *active avoidance* on other *UAS*.

*Collision Hazards* must be addressed by *UTM* service in the following manner:

1. *Each UAS* in particular *Controlled Space* periodically sends synchronized *Position Notification* messages (tab. 6.1).
2. *UTM* service receives *Position Notifications* and manages *Collision Case* (tab. 6.2) in *Controlled Space*.
3. *UTM* detects *Converging Collision Case* with *Collision Point* in vicinity.
4. *UTM* service Sends *Mandate* to *UAS* without *Right of the Way* and implements *Normative Directive* on all *UAS* in area.

*Mission parameters* for both UAS systems are defined in (tab. 7.35).

UAS	Position		$\mathcal{WP}_1$
	$[x, y, z]$	$[\theta, \varpi, \psi]$	
1	$[0, 20, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[40, 20, 0]^T$
2	$[20, 0, 0]^T$	$[0^\circ, 0^\circ, 90^\circ]^T$	$[20, 40, 0]^T$

Table 7.35: Mission setup for *Rule based converging* scenario.

**Assumptions:** Following assumptions are valid for this test:

1. *Controlled Airspace Airworthiness* - UAS system is equipped with necessary controlled airspace equipment like ADS-B In/Out, Radar, Transponder, etc. Moreover airworthy *UAS* has capability to precisely follow *UTM directives* (max. 5 % deviation).
2. *C2 (Command & control) Link Established* - necessary for (*UAS* ↔ *UAS*) and (*UAS* ↔ *UTM*) communication. If *C2* link is lost the *UAS* will enter into *Emergency avoidance mode*.
3. *Decision frame synchronization with UTM* - necessary in discrete C2 environment otherwise *safety margins* needs to be *bloated*.
4. *Both UAS have identical cruising speed* - simplification impacting *UTM* service implementation. *Obstacle Avoidance Framework* can comprehend various intruders speed, with proper *UAS* directives.

**Main Goal:** Show possibility of *Converging situation resolution* with *forced safety margin* by *UAS Traffic Management* system. The *Obstacle Avoidance Framework based on Reach Sets* is used as *Navigation Module*.

**Acceptance Criteria:** Following criteria must be met:

1. *Well Clear Condition valid for both UAS* - Both *UAS* must have *minimal required distance* from *other UAS* for all *Converging Maneuver* enforcement time.
2. *Fulfillment of UTM Directives* - Both UAS must stay in *Navigation mode* for all *Converging Maneuver* enforcement time. *UAS without Right Of the Way* must stay away for necessary time, before returning to *Original Navigation waypoint*  $\mathcal{WP}_1$  following.

**Testing Setup:** The *standard test setup* for each UAS defined in (tab. 7.2, 7.3, 7.4, 7.5, 7.6) is used with following parameter override:

1. *Navigation grid - type* - *ACAS-like* with enabled *Horizontal maneuvers*

This *configuration* is based on assumption that every UAS is in *controlled airspace* in *FL450* (flight level 45000 feet Above Sea Level), without permission for *climb or descent maneuver*. *Rule engine* is initialized in standard *Rules of the air* configuration (fig. 6.23).

There is *UTM* service for given *airspace cluster* calculating *collision cases* (tab. 6.2) based on incoming *UAS position notifications* (tab. 6.1).

**Simulation Run:** Notable moments from *simulation run* (fig. 7.22) are following:

1. *Collision Case creation* (fig. 7.22a) following events happens in this step:
  - a. Two *UAS* are approaching *airway intersection*: UAS 1 (blue) from left and UAS 2 (cyan) from bottom.
  - b. They are going to *collide* at point  $\mathcal{C} = [20, 20, 0]^T$  of *Flight Level* (elevation is 45,000 feet Above Mean Seal Level).
  - c. UTM service notices future *Collision Situation* and creates *Collision Case*.
  - d. *Converging Directive* for 8 m from *Collision point* is issued for UAS 1 (blue), because UAS 2 (cyan) has *Right Of the Way*.
  - e. *Keep Velocity/Heading Directive* is issued for UAS 2 (cyan) to ensure avoidance maneuver success.
  - f. UAS 1 (blue) corrects its heading according to *UTM* directive.
  - g. UAS 2 (cyan) stays on claimed course and if its necessary adjust its speed.
2. *Well clear before* (fig. 7.22b) UAS 1 (blue) checks the *Collision Point* distance and keeps safe distance given by safety margin. UAS 2 (cyan) checks if there is no intruder in *Avoidance Grid* and if not, stays in *Navigation Mode*.
3. *Well clear after* (fig. 7.22c) UAS 2 (cyan) is *after Collision Point*, it can start negotiations of new speed and heading with UTM. UAS 1 (blue) is still enforced to follow *Converging Maneuver* directive, until the outer boundary of *Collision Zone* is reached.
4. *Waypoints reach* (fig. 7.22d) UAS 1 (blue) leaves outer boundary of *Collision zone*. Leaving *Converging Maneuver Directive*. UTM closes *Collision Case*.

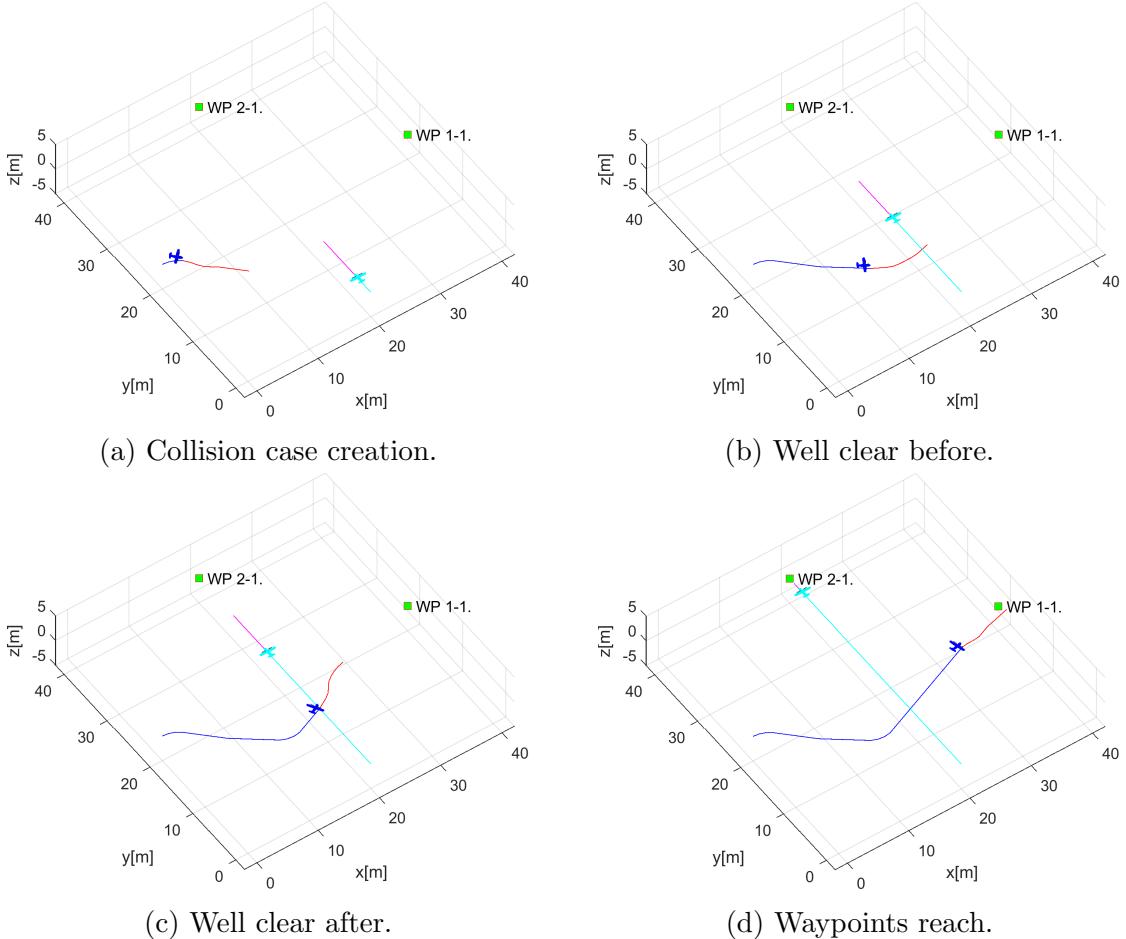


Figure 7.22: Test scenario for *Rule based converging*.

**Collision Case Calculation:** For test scenario in (fig. 7.22) where UAS 1 (blue) is converging to avoid UAS 2 (cyan) the *Collision Case* (tab. 7.36) have been calculated.

The *Collision point* is at [20, 20, 0] in *Flight Level FL450* coordinate frame.

The *angle of approach* was evaluated as  $90^\circ$  which indicates *converging maneuver* in range  $70^\circ \leq \text{angleOfApproach} < 130^\circ$ .

The *mutual position* of UAS 1 (blue) and UAS 2(cyan) is giving the roles: *Right Of the Way* for UAS 2 (cyan) and *Converging* for UAS 1 (blue).

The *safety margin* for *Well Clear* was determined as 3m for UAS 1 and 5m for UAS 2. (Note: Well Clear Margin is usually much greater than Near Miss margin). The *Combined Case* margin which was enforced was 8m. The mutual distance can not go below this threshold.

Collision Case						Margins	
id	UAS	role	collision point	angle of approach	type	safety	case
1-2	1	Converging	[20, 20, 0] <sup>T</sup>	$90^\circ$	Converging	3	8
	2	Right o. W.				5	

Table 7.36: Collision case for *Rule-based converging* scenario.

**Safety Margin Performance:** The safety margins (well clear) (fig. 7.23) in controlled airspace are much greater than in non-controlled airspace (near miss) (fig. 7.14)

The enforced rule was (rule 6.8) with parameters: Collision Point  $[20, 20, 0]^T$  and *Safety Margin* 8m as given by Collision Case (tab. 7.36).

The mutual *UAS distance* (blue line) does not go over *Safety Margin* (red line), which means UAS 1 well clear margin of 3m and UAS 2 well clear margin of 5m are not broken by any means (fig. 7.23).

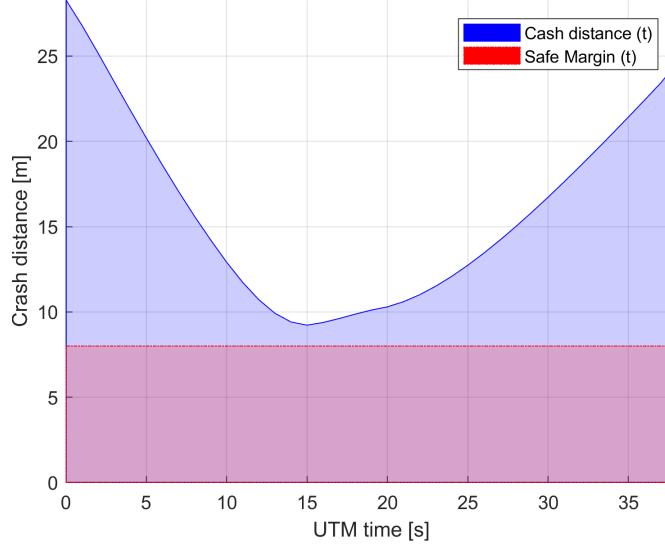


Figure 7.23: *Rule based converging safety margin performance.*

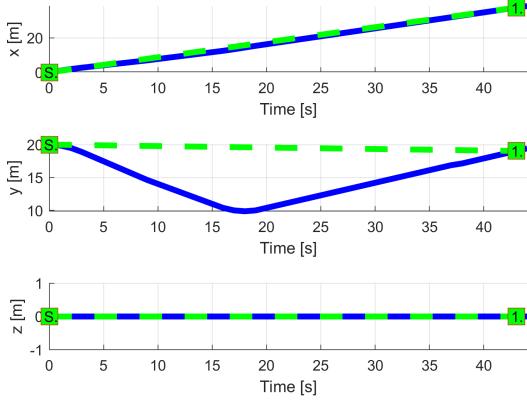
**Safety Margin Distance:** *Safety margin distance* (tab. 7.37) represent the proximity on UAS mutual distance to *breach of well clear condition*. The *well clear breach condition* was not achieved. The *minimal safety margin distance* was 1.2240m. The *maximal safety margin distance* was 20.2843m which represents distance in time of *Collision Case Creation*.

UAS:	Safety margin distance		
	min	max	breach
1-2	1.2240	20.2843	false

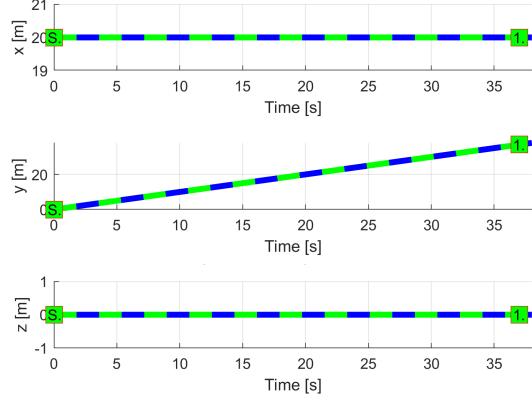
Table 7.37: *Rule based converging safety margin distances.*

**Path Tracking Performance:** *Path tracking* is displayed in (fig. 7.24). The *UAS* trajectory is divided into *X, Y, Z axis tracking over UTM Time*. The *Reference Trajectory* (green dashed line) interconnect starting position of UAS (green square marked S) and goal waypoint (green square marked 1). The *Executed Trajectory* (blue solid line) reflects real UAS trajectory.

1. UAS 1. (fig, 7.24a) do steady right side *converging maneuver* (y-axis).
2. UAS 2. (fig. 7.24b) follows the reference trajectory precisely, because it has *Right Of the Way*.



(a) UAS 1.



(b) UAS 2.

Figure 7.24: *Trajectory tracking for Rule based converging test case.*

**Path Tracking Deviations:** Deviations (tab. 7.38) are in *expected ranges*, considering the *mission plans* (tab. 7.35) and *Collision Case* safety margin of 8m.

The minimal deviation distance was expected at value of *safety margin* (8m). The maximal deviation was 10.22m which is acceptable due the space discretization, UAS dynamic, and, *dynamic decision time*.

Param.		
	UAS 1	UAS 2
	$WP_1$	$WP_1$
max $ x $	0	0
max $ y $	10.22	0
max $ z $	0	0
max dist.	10.22	0

Table 7.38: Path tracking properties for *Rule based converging* scenario.

#### 7.4.2 Rule based Head on

**Scenario:** Two *UAS* are going on same *airway* in same *flight level* in opposite direction in *controlled airspace* (over 500 feet Above the Ground Level). The *mutual position* of UAS can be classified as *Side Approach*. Following *collision hazard* is present:

1. *Head on Collision Hazard* - There is an *UAS* approaching from opposite direction which invokes need to actively avoid *Collision Point*.

*Head on Collision Hazard* must be addressed by *UTM* service in the following manner:

1. *Each UAS* in particular *Controlled Space* periodically sends synchronized *Position Notification* messages (tab. 6.1).
2. *UTM* service receives *Position Notifications* and manages *Collision Cases* (tab. 6.2) in *Controlled Space*.
3. *UTM* detects single *Head on Collision Cases* with *Collision Point* in vicinity.
4. *UTM* service creates *Virtual Roundabout* and implements *Normative Directive* on both *UAS*.

*Mission parameters* for four UAS systems are defined in (tab. 7.39).

UAS	Position		$\mathcal{WP}_1$
	$[x, y, z]$	$[\theta, \varpi, \psi]$	
1	$[0, 20, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[45, 20, 0]^T$
2	$[40, 20, 0]^T$	$[0^\circ, 0^\circ, 180^\circ]^T$	$[-5, 20, 0]^T$

Table 7.39: Mission setup for *Rule based head on* scenario.

**Assumptions:** Following assumptions are valid for this test:

1. *Controlled Airspace Airworthiness* - UAS system is equipped with necessary controlled airspace equipment like ADS-B In/Out, Radar, Transponder, etc. Moreover airworthy *UAS* has capability to precisely follow *UTM directives* (max. 5 % deviation).
2. *C2 (Command & control) Link Established* - necessary for  $(\text{UAS} \leftrightarrow \text{UAS})$  and  $(\text{UAS} \leftrightarrow \text{UTM})$  communication. If *C2* link is lost the *UAS* will enter into *Emergency avoidance mode*.
3. *Decision frame synchronization with UTM* - necessary in discrete *C2* environment otherwise *safety margins* needs to be *bloated*.
4. *Both UAS have identical cruising speed* - simplification impacting *UTM* service implementation. *Obstacle Avoidance Framework* can comprehend various intruders speed, with proper *UAS* directives.

**Main Goal:** Show possibility of *Head on situation resolution* with *forced safety margin* by *UAS Traffic Management* system. The *Obstacle Avoidance Framework based on Reach Sets* is used as *Navigation Module*.

**Acceptance Criteria:** Following criteria must be met:

1. *Well Clear Condition valid for both UAS* - Both *UAS* must have *minimal required distance* from other *UAS* for all *Virtual Roundabout enforcement time*.
2. *Fulfillment of UTM Directives* - Both *UAS* must stay in *Navigation mode* for all *Virtual Roundabout enforcement time*. Both *UAS* must stay on *Virtual Roundabout* for necessary time, before leaving for *Original Navigation waypoint*  $\mathcal{WP}_1$ .

**Testing Setup:** The *standard test setup* for each *UAS* defined in (tab. 7.2, 7.3, 7.4, 7.5, 7.6) is used with following parameter override:

1. *Navigation grid - type - ACAS-like* with enabled *Horizontal maneuvers*

This *configuration* is based on assumption that both *UAS* is in *controlled airspace* in *FL450* (flight level 45000 feet Above Sea Level), without permission for *climb or descent maneuver*. *Rule engine* is initialized in standard *Rules of the air* configuration (fig. 6.23).

There is *UTM* service for given *airspace cluster* calculating *collision cases* (tab. 6.2) based on incoming *UAS position notifications* (tab. 6.1).

**Simulation Run:** Notable moments from the *simulation run* (fig. 7.25) are following:

1. *Collision Case creation* (fig. 7.25a) following events happens in this step:
  - a. Two UAS are on same airway approaching each other from opposite direction, UAS 1 (blue) from the left, UAS 2 (cyan) from the right.
  - b. They are going to *collide* at point  $\mathcal{C} = [20, 20, 0]^T$  of *Flight Level* (Elevation is 45,000 feet Above Mean Sea Level).
  - c. UTM service notices future *Collision Situation* and creates *Collision Case*.
  - d. *Virtual Roundabout* is created at *collision point* with radius 10m. UTM issues directive for both UAS to avoid collision point from different sides.
  - e. UAS 1 (blue) receives directive to avoid *Collision Point* from *right side* (Down side in GCS). UAS 2 (cyan) receives directive to avoid *Collision Point* from *right side* (Up side in GCS).
  - f. Both UAS enters into *Virtual Roundabout*.
2. *Well clear before* (fig. 7.25b) UAS 1 (blue) is keeping *enforced safety margin* (10 m) from *collision point* and *UAS 2 position*. The *Virtual Roundabout* is enforced until the (*Collision point*) is reached by both UAS. Both UAS stays in *Navigation Mode*.
3. *Well clear after* (fig. 7.25c) UTM notices that *Collision point level* has been reached by both UAS. UTM renounce *Directives* and enables a return to *Original Waypoint*  $WP_1$ . Both UAS starts to converging to *Original waypoint* (because possible collision was averted).
4. *Waypoint reach* (fig. 7.25d) Both UAS reaches respective goal points.

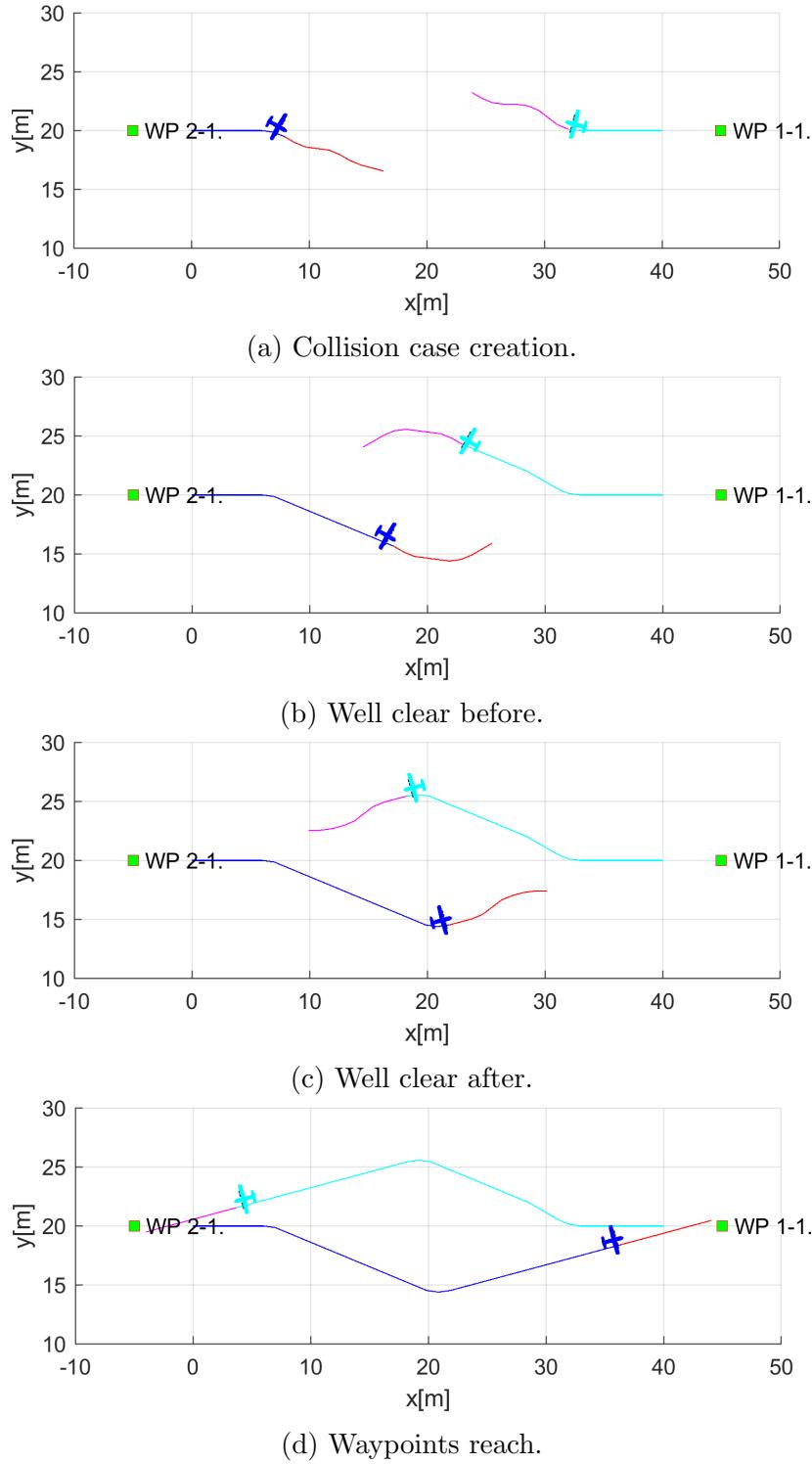


Figure 7.25: Test scenario for *Rule based head on approach* (virtual roundabout).

**Collision Case Calculation:** For test scenario in (fig. 7.25) where UAS 1 (blue) have head on collision with UAS 2 (cyan), *Collision Case* have been calculated (tab. 7.40).

The *Collision point* is at  $[20, 20, 0]^T$  in Flight Level *FL450* coordinate frame.

The *angle of approach* was evaluated as  $180^\circ$  which indicates *Head on Approach* due the  $130^\circ \leq \text{angle of Approach} \leq 180^\circ$  condition.

The *mutual position* of UAS 1 (blue) and UAS 2 (cyan) is giving the roles of *Round-*

about to both UAS.

The *safety margin* for *Well Clear* was determined as 5m for UAS 1 and UAS 2. The combined *Case Margin* is 10 m, which is sum of both. The *mutual distance* can not go below this threshold.

Collision Case						Margins	
id	UAS	role	collision point	angle of approach	type	safety	case
1-2	1	Roundabout	$[20, 20, 0]^T$	180°	Head on	5	10
	2	Roundabout				5	

Table 7.40: Collision case for *Rule-based head on* scenario.

**Safety Margin Performance:** The safety margin(*well clear*) (fig. 7.26) in controlled airspace are much rather than in non-controlled airspace (*near miss*) (fig. 7.17).

The enforced rule war (rule 6.7) with parameters: Collision Point  $[20, 20, 0]^T$  and *Safety Margin* 10m as given by Collision Case (tab. 7.40).

The mutual *UAS* distance (blue line) does not go over *Safety Margin* (red line) which means both UAS well clear margins are not broken by any means (fig. 7.25).

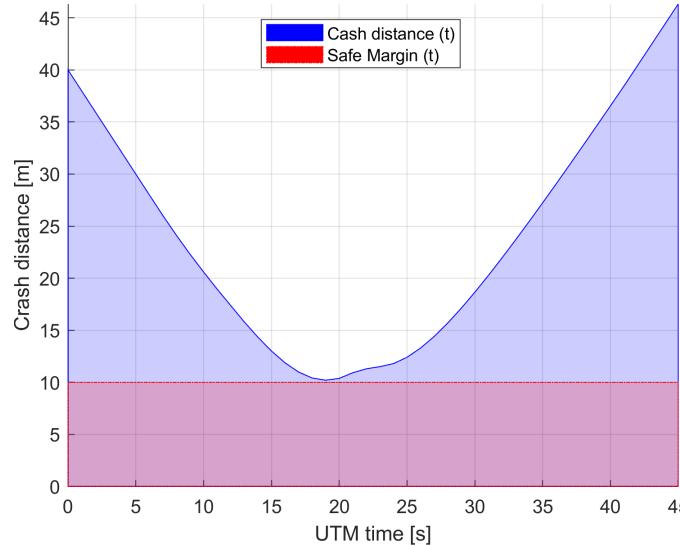


Figure 7.26: *Rule based head on* safety margin performance.

**Safety Margin Distance** (tab. 7.41) represents the proximity on UAS mutual distance to breach *well clear condition*. The *well clear breach condition* was achieved. The *minimal safety margin distance* was 0.2084m. The *maximal safety margin distance* was 36.3253m which represents distance at *Collision Case* closing.

UAS:	Safety margin distance		
	min	max	breach
1-2	0.2084	36.3253	false

Table 7.41: *Rule based head on* safety margin distances.

**Path Tracking Performance:** *Path tracking* is displayed in (fig. 7.27). The *UAS* trajectory is divided into *X, Y, Z axis tracking over UTM Time*. The *Reference Trajectory* (green dashed line) interconnect starting position of *UAS* (green square marked S) and goal waypoint (green square marked 1). The *Executed Trajectory* (blue solid line) reflects real *UAS* trajectory.

1. UAS 1. (fig. 7.27a) do steady right side *roundabout maneuver* (y-axis).
2. UAS 2. (fig. 7.27b) do steady right side *roundabout maneuver* (y-axis).

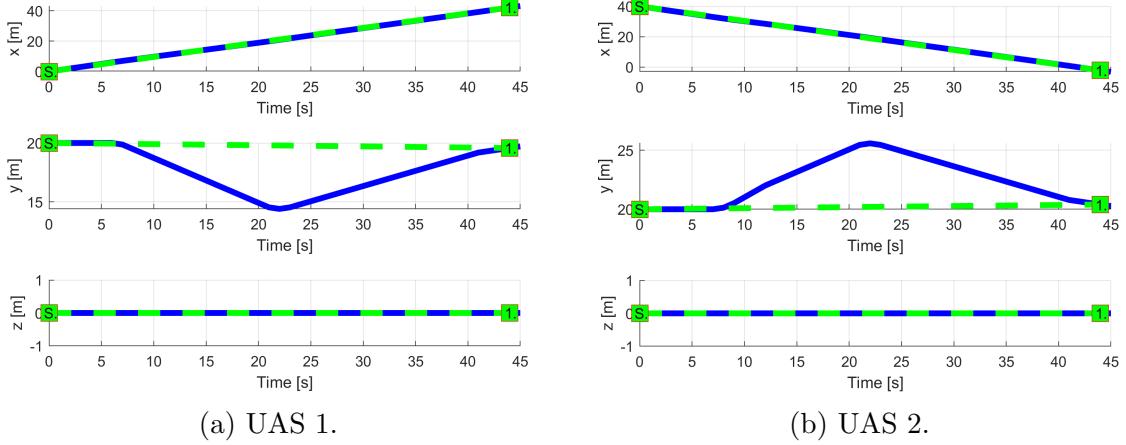


Figure 7.27: *Trajectory tracking for Rule based head on test case.*

**Path Tracking Deviations:** Deviations (tab. 7.42) are in *expected ranges*, considering the *mission plans* (tab. 7.39) and *Collision Case* safety margin of 10m.

Param.		
	UAS 1	UAS 2
	$WP_1$	$WP_1$
$\max  x $	0	0
$\max  y $	5.40	5.40
$\max  z $	0	0
$\max dist.$	5.40	5.40

Table 7.42: Path tracking properties for *Rule based head on scenario*.

### 7.4.3 Rule based Mixed Head on with Converging

**Scenario:** Four *UAS* are approaching an airway *intersection* at *same time* from *opposite direction* in *controlled airspace* (over 500 feet Above Ground Level). Each *UAS* have following *Collision Hazards*:

1. *Head on Collision Hazard* - There is an *UAS* approaching from opposite direction which invokes need to actively avoid *Collision Point*.
2. *Active Converging Collision Hazard* - There is an *UAS* approaching from the *right side*, which give him *Right of the Way* and invokes need to actively avoid *Intruder*.

3. *Passive Converging Collision Hazard* - There is an *UAS* approaching from the *left side*, which gave us *Right of the Way* and imposes an obligation of *active avoidance* on other *UAS*.

*Note.* Presented scenario is *the worst possible situation* in current *manned aviation ATM*. *Mentioned Collision Hazards* must be addressed by *UTM* service in the following manner:

1. *Each UAS* in particular *Controlled Space* periodically sends synchronized *Position Notification* messages (tab. 6.1).
2. *UTM* service receives *Position Notifications* and manages *Collision Cases* (tab. 6.2) in *Controlled Space*.
3. *UTM* detects multiple *Collision Cases* with *Collision Points* in vicinity.
4. *UTM* service creates *Virtual Roundabout* and implements *Normative Directive* on all *UAS* in area.

*Mission parameters* for four *UAS* systems are defined in (tab. 7.43).

UAS	Position		$\mathcal{WP}_1$
	$[x, y, z]$	$[\theta, \varpi, \psi]$	
1	$[0, 20, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[45, 20, 0]^T$
2	$[40, 20, 0]^T$	$[0^\circ, 0^\circ, 180^\circ]^T$	$[-5, 20, 0]^T$
3	$[20, 0, 0]^T$	$[0^\circ, 0^\circ, 90^\circ]^T$	$[20, 45, 0]^T$
4	$[20, 40, 0]^T$	$[0^\circ, 0^\circ, -90^\circ]^T$	$[45, 20, 0]^T$

Table 7.43: Mission setup for *Rule based mixed* scenario.

**Assumptions:** Following assumptions are valid for this test:

1. *Controlled Airspace Airworthiness* - *UAS* system is equipped with necessary controlled airspace equipment like ADS-B In/Out, Radar, Transponder, etc. Moreover airworthy *UAS* has capability to precisely follow *UTM directives* (max. 5 % deviation).
2. *C2 (Command & control) Link Established* - necessary for (*UAS* ↔ *UAS*) and (*UAS* ↔ *UTM*) communication. If *C2* link is lost the *UAS* will enter into *Emergency avoidance mode*.
3. *Decision frame synchronization with UTM* - necessary in discrete C2 environment otherwise *safety margins* needs to be *bloated*.
4. *Every UAS have identical cruising speed* - simplification impacting *UTM* service implementation. *Obstacle Avoidance Framework* can comprehend various intruders speed, with proper *UAS* directives.

**Main Goal:** Show possibility of *Virtual Roundabout* invoked by *UTM* directives where *Obstacle Avoidance Framework based on Reach Sets* is used as a *Navigation Module*.

**Acceptance Criteria:** Following criteria must be met:

1. *Well Clear Condition valid for every UAS* - Each *UAS* must have *minimal required distance* from *other UAS* for all *Virtual Roundabout enforcement time*.
2. *Fulfillment of UTM Directives* - Each *UAS* must stay in *Navigation mode* for all *Virtual Roundabout enforcement time*. Each *UAS* must stay on *Virtual Roundabout* for necessary time, before leaving for *Original Navigation waypoint*  $\mathcal{WP}_1$ .

**Testing Setup:** The *standard test setup* for each *UAS* defined in (tab. 7.2, 7.3, 7.4, 7.5, 7.6) is used with following parameter override:

1. *Navigation grid - type - ACAS-like* with enabled *Horizontal maneuvers*

This *configuration* is based on assumption that every *UAS* is in *controlled airspace* in *FL450* (flight level 45000 feet Above Sea Level), without permission for *climb or descent maneuver*. *Rule engine* is initialized in standard *Rules of the air configuration* (fig. 6.23).

There is *UTM service* for given *airspace cluster* calculating *collision cases* (tab. 6.2) based on incoming *UAS position notifications* (tab. 6.1).

**Simulation Run:** Notable moments from the *simulation run* (fig. 7.28) are following:

1. *Collision cases created* (fig. 7.28a) following events happens in this step:
  - a. Four *UAS* are approaching airways intersection: *UAS 1* (blue) from left, *UAS 2* (cyan) from right, *UAS 3* (green) from bottom, *UAS 4* (black) from top.
  - b. They are going to collide at point  $[20, 20, 0]^T$  of *Flight level* (elevation is 45, 000 feet Above Mean Sea Level).
  - c. *UTM service* notices future *Collision Situations* and creates *Collision Cases*.
  - d. There are many *Collision Cases* in near vicinity. The *Virtual Roundabout* is created with *Safety margin 15 m*.
  - e. The *UTM service* then sends a new *Roundabout Directives* to involved *UAS* systems.
  - f. Each *UAS* starts *Roundabout Entry Maneuver* by correcting own *Heading* and *Speed* (if its necessary).
2. *Roundabout entry* (fig. 7.28b) - Each *UAS* enters into *Virtual Roundabout* while sending *Roundabout Entrance Notification* to *UTM service*.
3. *Roundabout leave* (fig. 7.28c) following events happens in this step:
  - a. Each *UAS* when is going to approach level of *Original Goal Waypoint* sends *Roundabout Leave Request*.
  - b. *UTM system* will check if there is *Sufficient Free Space* to leave *Virtual Roundabout*.
  - c. The *UTM Service* then issues *Virtual Roundabout Leave Approval*.
  - d. Each *UAS* will correct own heading and speed in range of received permit.

4. *Situation resolution* (fig. 7.28d) - Each *UAS* is heading away from *Roundabout Center*, there is no active user of *Virtual Roundabout*. *UTM* will remove *Virtual Roundabout* and closes underlying *Collision Cases*. Each *UAS* will reach respective *Original Goal Waypoint*.

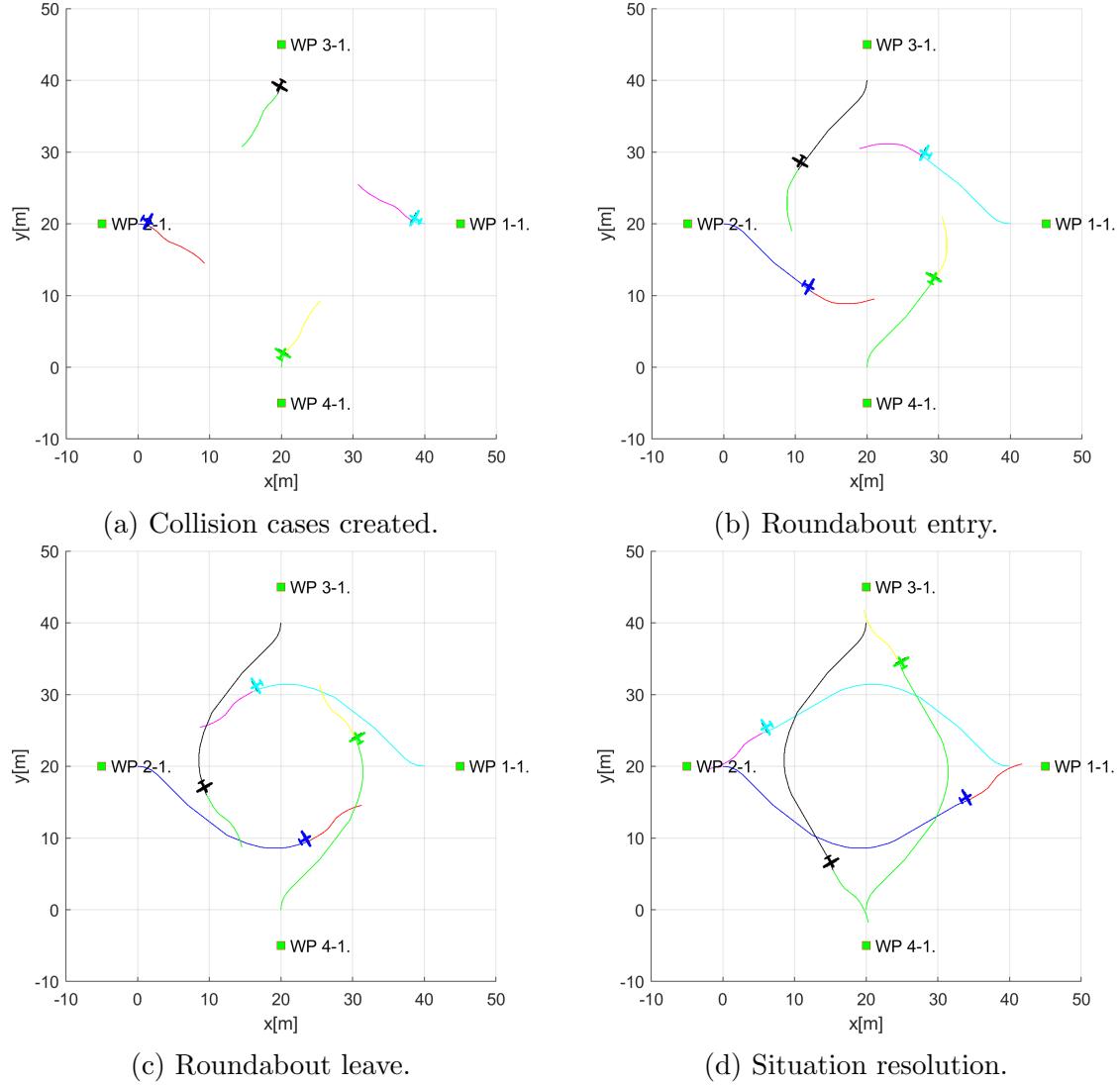


Figure 7.28: Test scenario for *Rule based mixed situation with self-separation mode*.

**Collision Cases Calculation:** The set of original *Collision cases* is given in (tab. 7.44).

Each *UAS* has one *Head on*, *Converging passive*, *Converging active* collision hazard. For example: *UAS 1* have *head on* with *UAS 2*, *converging passive* with *UAS 4*, *converging active* with *UAS 3*. For *UAS 2-4* check *role* in respective *Collision Cases*.

*Note.* *Collision case* calculated by *UTM* are symmetric, which means that collision case for *UAS X, UAS Y* is identical to collision case calculated for *UAS Y, UAS X, X ≠ Y*.

*Safety margin* representing *Well Clear Margin* for single *UAS* in *Collision Case* ranges 5 – 8 m. *Case margin* representing minimal mutual distance between two *UAS systems* to remain well clear, ranges 12 – 15 m.

*Merged Collision Case* is oversimplified for demonstration purposes. *Merge Case Procedure* is out of scope of this work due to its extent. Every *Collision Case* share same *Collision Point*  $[20, 20, 0]^T$  in flight level coordinate frame. *Merged Collision Case* type was set as *Roundabout*, due the number of collision case *attendants* is greater than 2. Each *UAS role* has been set as *Roundabout*. Enforced *safety margin* is equal to 15 m, which is maximum of all *single collision case combined margins*.

Collision Case						Margins	
id	UAS	role	collision point	angle of approach	type	safety	case
1-2	1	Roundabout	$[20, 20, 0]^T$	$180^\circ$	Head on	8	15
	2	Roundabout				7	
1-3	1	Converging	$[20, 20, 0]^T$	$90^\circ$	Converging	8	15
	3	Right o.W.				5	
1-4	1	Right o.W.	$[20, 20, 0]^T$	$90^\circ$	Converging	8	15
	4	Converging				5	
2-3	2	Right o.W.	$[20, 20, 0]^T$	$90^\circ$	Converging	7	12
	3	Converging				5	
2-4	2	Converging	$[20, 20, 0]^T$	$90^\circ$	Converging	7	12
	4	Right o.W.				5	
3-4	3	Roundabout	$[20, 20, 0]^T$	$180^\circ$	Head on	7	14
	4	Roundabout				7	
Merged cases						Safety Margin	
id	UAS	role	collision point	type			
1-2- -3-4	1	Roundabout	$[20, 20, 0]^T$	Roundabout		15	
	2	Roundabout					
	3	Roundabout					
	4	Roundabout					

Table 7.44: Collision cases for *Rule-based mixed scenario*.

**Safety Margin Performance:** *Merged Collision Case Safety Margin* is 15 m and it is valid for all *UAS mutual distances*. The simple condition for *Remain Well Clear* is:

$$\text{crashDistance}(UAS_X, UAS_Y, t) \geq 15m, X \neq Y \in \{1, 2, 3, 4\}, t \in \text{utmTime}$$

*Safety Margin Performance* is given in (fig. 7.29). The mutual distance (Crash Distance [m]) between two UAS is denoted as *blue line*. The enforced safety margin for *Remain Well Clear* condition is denoted as *red line*.

*Note.* *Evolution of mutual crash distance* is symmetric. In any case the mutual distance goes under *safety margin*. *Acceptance criterion* for *Well Clear condition* is fulfilled.

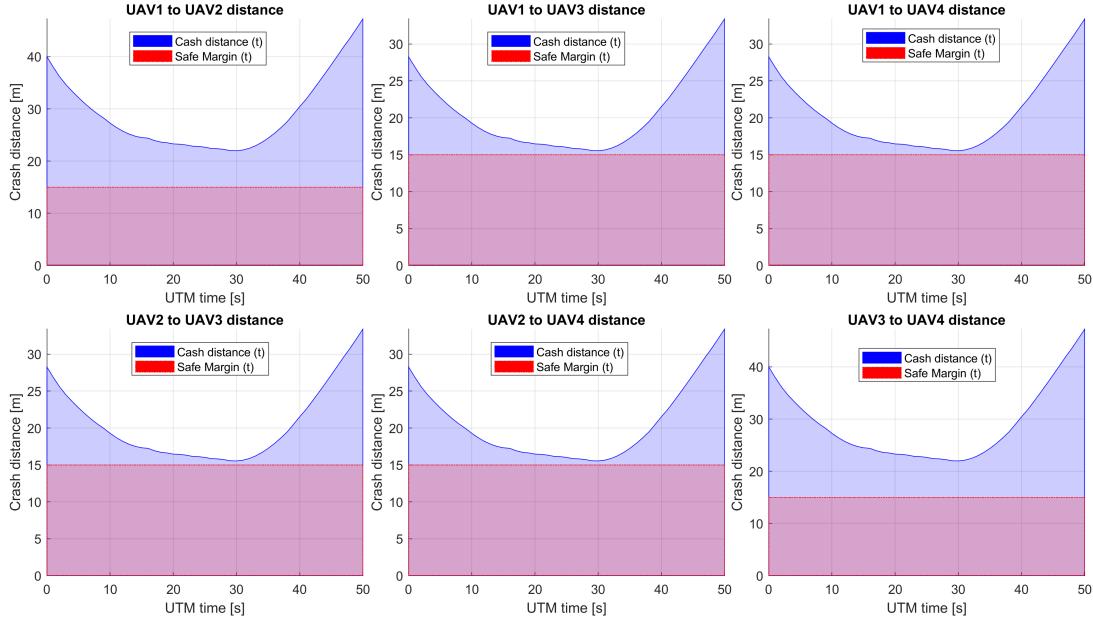


Figure 7.29: *Rule based mixed situation safety margin performance.*

**Safety Margin Distance:** *Safety margin distances* (tab. 7.45) represents the proximity of *UAS mutual distance to breach well clear condition*. The *breach condition* was not fulfilled in any combination.

The *minimal safety margin distance* was 0.5438 m between all four *UAS* systems. The *maximal safety margin distance* ranges between 18 - 32 m which reflects approach and leave maneuver portions of *virtual roundabout*.

UAS:	Safety margin distance		
	min	max	breach
1-2	6.9823	32.2369	false
1-3	0.5438	18.4015	false
1-4	0.5438	18.4015	false
2-3	0.5438	18.4015	false
2-4	0.5438	18.4015	false
3-4	6.9823	32.2369	false

Table 7.45: *Rule based mixed safety margin distances.*

**Path Tracking Performance:** Path tracking is displayed in (fig. 7.30). The *UAS trajectory* is divided into *X, Y, Z axis tracking over UTM Time*. The *Reference Trajectory* (green dashed line) is represented as interconnection between *Start Waypoint* (green square marked S) and *Goal Waypoint*  $WP_1$  (green square marked 1). The *Executed trajectory* (blue solid line) reflects real *UAS* movement.

1. *UAS 1* (fig. 7.30a) is using bottom portion of *Virtual Roundabout* (-Y values), sticking to the boundary of the *Virtual Roundabout*.
2. *UAS 2* (fig. 7.30b) is using upper portion of the *Virtual Roundabout*. (+Y values), sticking to the boundary of the *Virtual Roundabout*.

3. *UAS 3* (fig. 7.30c) is using right portion of the *Virtual Roundabout*. (+X values), sticking to the boundary of the *Virtual Roundabout*.
4. *UAS 4* (fig. 7.30d) is using left portion of the *Virtual Roundabout*. (-X values), sticking to the boundary of the *Virtual Roundabout*.

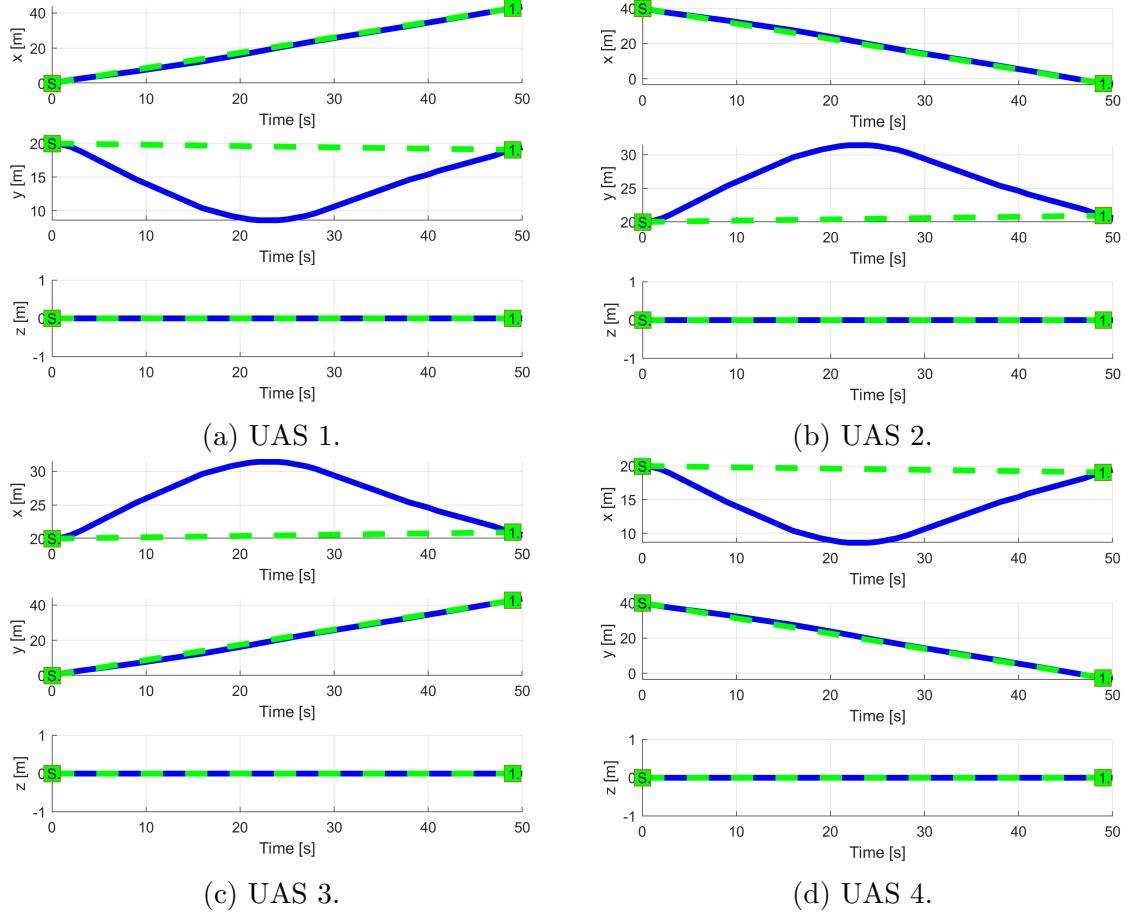


Figure 7.30: Trajectory tracking for *Rule based mixed* situation test case.

**Path Tracking Deviations:** *Deviations* (tab. 7.46) are in expected ranges, considering the mission plans (tab. 7.43) and *Merged Case Safety Margin* (15 m).

Param.	UAS 1	UAS 2	UAS 3	UAS 4
	$\mathcal{WP}_1$	$\mathcal{WP}_1$	$\mathcal{WP}_1$	$\mathcal{WP}_1$
$\max  x $	0	0	11.40	11.40
$\max  y $	11.40	11.40	0	0
$\max  z $	0	0	0	0
$\max dist.$	11.40	11.40	11.40	11.40

Table 7.46: Path tracking properties for *Rule based mixed* scenario.

#### 7.4.4 Rule based Overtake

**Scenario:** Two UAS are flying in the *controlled airspace* (over 500 feet Above Ground Level) on the *airway* (in same direction). *Slower UAS* is in front of *Faster UAS*. There is

possibility of a *collision* or a *near miss incident* or a *well clear breach*. The *Faster UAS* (Overtaking) must contact *UTM* service and ask for *overtake permission*. Scenario steps:

1. *Faster UAS* (Overtaking) notices *UTM* service about *Slower UAS* (Overtaken). (This step is Optional.)
2. *UTM* service issues *Directives* to all *UAS* in area.
3. *Overtake Directive* is received by *Faster UAS* (Overtaking) and *Slower UAS* (Overtaken).
4. *Faster UAS* (Overtaking) mission plan is altered to reflect *Overtake directive*, *Divergence Waypoint* and *Convergence Waypoint* are added.
5. *Faster UAS* (Overtaking) safely overtakes *Slower UAS* (Overtaken) without breaking *Well clear* condition.

*Mission parameters* for both *UAS* systems are defined in (tab. 7.47).

UAS	Position		$WP_1$
	$[x, y, z]$	$[\theta, \varpi, \psi]$	
1	$[-40, 20, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[110, 20, 0]^T$
2	$[-20, 20, 0]^T$	$[0^\circ, 0^\circ, 0^\circ]^T$	$[80, 20, 0]^T$

Table 7.47: Mission setup for all *Rule based overtaking* scenarios.

**Assumptions:** Following assumptions are valid for this test:

1. *Controlled Airspace Airworthiness* - *UAS* system is equipped with necessary controlled airspace equipment like ADS-B In/Out, Radar, Transponder, etc. Moreover airworthy *UAS* has capability to precisely follow *UTM directives* (max. 5 % deviation).
2. *C2 (Command & control) Link Established* - necessary for (*UAS* ↔ *UAS*) and (*UAS* ↔ *UTM*) communication. If *C2* link is lost the *UAS* will enter into *Emergency avoidance mode*.
3. *Decision frame synchronization with UTM* - necessary in discrete *C2* environment otherwise *safety margins* needs to be *bloated*.

**Main Goal:** Show possibility of *Overtake Maneuver* invoked by the *UTM Directive* (event based flight constraint).

**Acceptance Criteria:** Following criteria must be met:

1. *Proper passing of Divergence/Convergence Waypoint* - minimal distance of *UAS trajectory* to *Divergence/Convergence waypoint* must be below passing threshold. Waypoints needs to be passed in given order (Divergence 1<sup>st</sup>, Convergence 2<sup>nd</sup>).
2. *Slower UAS (Overtaken) keeps Right of the Way* - the *UAS* with lesser maneuverability does not stand a chance in avoidance situation, it needs to keep its *Right of the Way*.

3. Both UAS does not breach Well Clear (safety) Margin - mutual distance does not get trough calculated Safety Margin.

**Testing Setup:** The standard test setup for each UAS defined in (tab. 7.2, 7.3, 7.4, 7.5, 7.6) is used with following parameter override:

1. Navigation grid - type - ACAS-like with enabled Horizontal maneuvers

This configuration is based on assumption that every UAS is in controlled airspace in FL450 (flight level 45000 feet Above Sea Level), without permission for climb or descent maneuver. Rule engine is initialized in standard Rules of the air configuration (fig. 6.23).

There is UTM service for given airspace cluster calculating collision cases (tab. 6.2) based on incoming UAS position notifications (tab. 6.1).

**Simulation Run:** Notable moments from the simulation run (fig. 7.31) are following:

1. Collision case creation (fig. 7.31a) - Faster UAS (blue) receives UTM Directive to invoke Overtake Rule (tab. 6.9). Slower UAS (magenta) receives UTM Directive to keep Right of the Way and warning that is going to be Overtaken. Faster UAS (blue) creates two virtual waypoints:

- a. Divergence waypoint at position  $[0, 14, 0]^T$ .
- b. Convergence waypoint at position  $[24, 14, 0]^T$ .

Faster UAS then sets Divergence waypoint as Goal waypoint and It starts overtaking maneuver while checking mutual distance.

2. Divergence waypoint reach (fig. 7.31b) - Faster UAS (blue) successfully reached Divergence Waypoint, setting Convergence Waypoint as new Goal waypoint.
3. Convergence waypoint reach (fig. 7.31c) - Faster UAS (blue) successfully reached Convergence Waypoint, setting Original Goal Waypoint as new Goal waypoint. The UTM service is notified from Faster UAS (blue) that Overtaken Maneuver have been completed. UTM acknowledges maneuver competition and It sends notification to Slower UAS (magenta) that Overtake Maneuver is finished. Slower UAS (magenta) was successfully overtaken.
4. Original waypoint reach (fig. 7.31d) - Faster UAS (blue) successfully reached Original Waypoint, Starting landing Sequence.

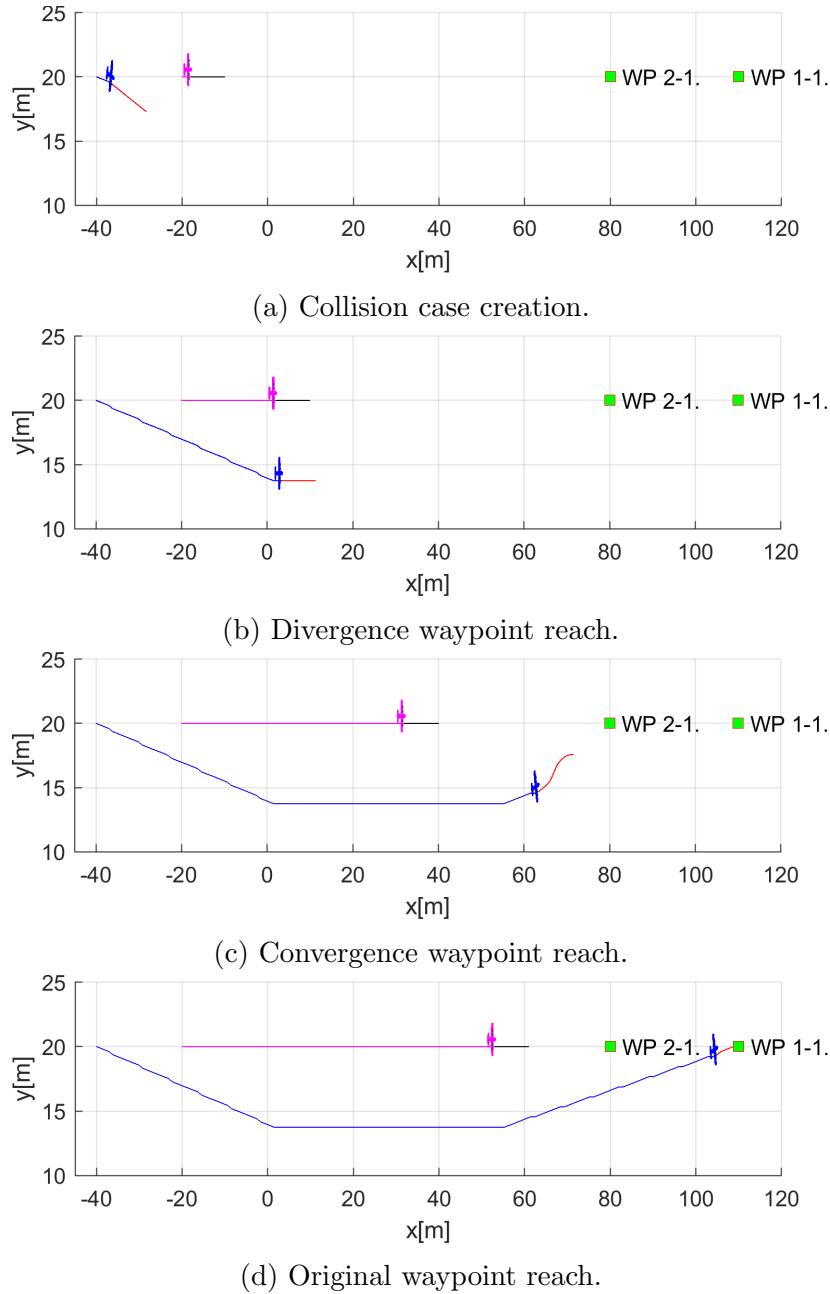


Figure 7.31: Test scenario for *Rule based Overtake* (double speed of overtaking aircraft).

**Collision Case Calculation:** The *Collision Case* (tab. 7.48) was calculated according to *Collision Calculation process* (sec. 6.8.8). *Faster UAS* (1) has *Overtaking* role and *Slower UAS* has *Right of the Way*. *Collision Point* is direct type at  $[0.20.0]^T$ . *Collision case type* was set based on *angle of approach*  $0^\circ$  as *Overtake*. The *Safety Margin* was set as 5 m.

Collision Case						Margins	
id	UAS	role	collision point	angle of approach	type	safety	case
1-2	1	Overtaking	$[0, 20, 0]^T$	$0^\circ$	Overtake	5	5
	2	Right o.W.				5	

Table 7.48: Collision case for *Rule-based Overtake* scenario 2x speed.

**Overtake Speed: Divergence/Convergence Waypoints** *Divergence waypoints* have been calculated according to (eq. 6.55), and, *Convergence Waypoints* have been calculated according to (eq. 6.56). Following *Speed Differences* were taken into account (Faster/Slower UAS speed ratio):  $2x$ ,  $3x$ ,  $4x$ . Following observations can be made:

1. *Distance between Divergence and Convergence waypoint* is decreasing with increasing *speed difference*.
2. *Divergence waypoint* is moving *back/right* in *UAS Local Coordinate Frame* with Increasing *speed difference*.
3. *Convergence waypoint* is moving like *Divergence waypoint* but little bit faster.

Speed diff.	Divergence		Convergence		Final waypoint
	waypoint	difference	waypoint	difference	
2x	$[0, 14, 0]^T$		$[24, 14, 0]^T$		$[110, 20, 0]^T$
		$[-10, -1, 0]^T$		$[-8, -1, 0]^T$	
3x	$[-10, 13, 0]^T$		$[16, 13, 0]^T$		$[110, 20, 0]^T$
		$[-3.4, -1, 0]^T$		$[-1.3, -1, 0]^T$	
4x	$[-13.4, 12, 0]^T$		$[14.7, 12, 0]^T$		$[110, 20, 0]^T$

Table 7.49: Convergence and divergence waypoints for various speed differences.

**Overtake Speed: Impact on Trajectory** Overtake *speed difference* is visible in (fig. 7.32). The *Slower vehicle trajectory*(cyan) is following *standard mission waypoints*. The *Faster vehicle trajectory* for  $2x$  (blue),  $3x$  (green),  $4x$  (black) are following *Divergence/Convergence waypoints* from (tab. 7.49).

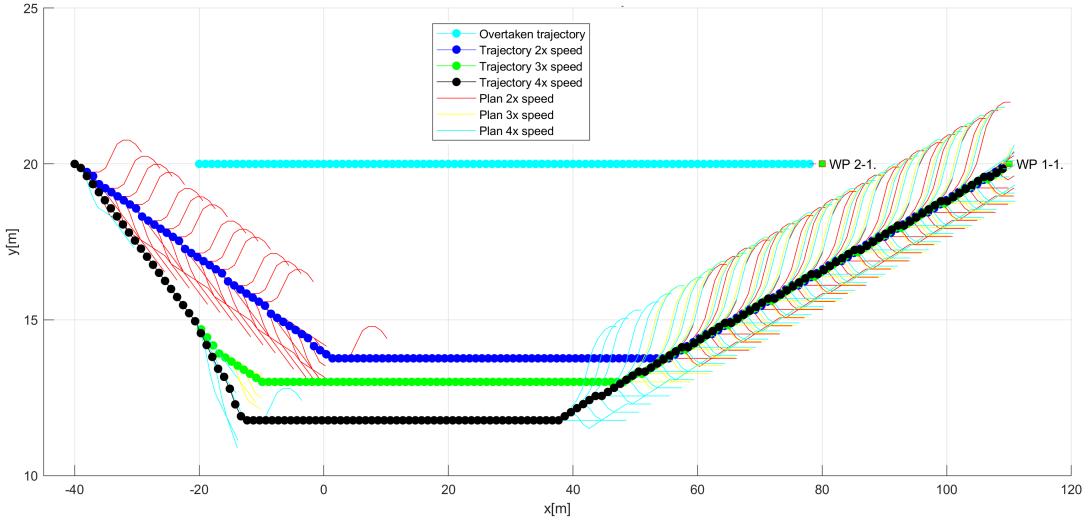


Figure 7.32: *Rule based overtake* trajectories for different speed.

**Overtake Speed: Impact on Safety Margin Performance** *Safety margin* (red line) is set to 5 m. It is obvious that *Faster UAS* will take down *Slower UAS* if there was not for an *Overtake maneuver*. The distance of *Faster UAS* to *Slower UAS* evolution is depending on *Speed difference*. *Inflection point* (closest point of two UAS) is reached sooner with *Higher speed*. *Safety margin performance* was measured for the *UTM performance time* in interval [0, 35] s and *Speed difference* of 2x (blue), 3x (green), 4x (black).

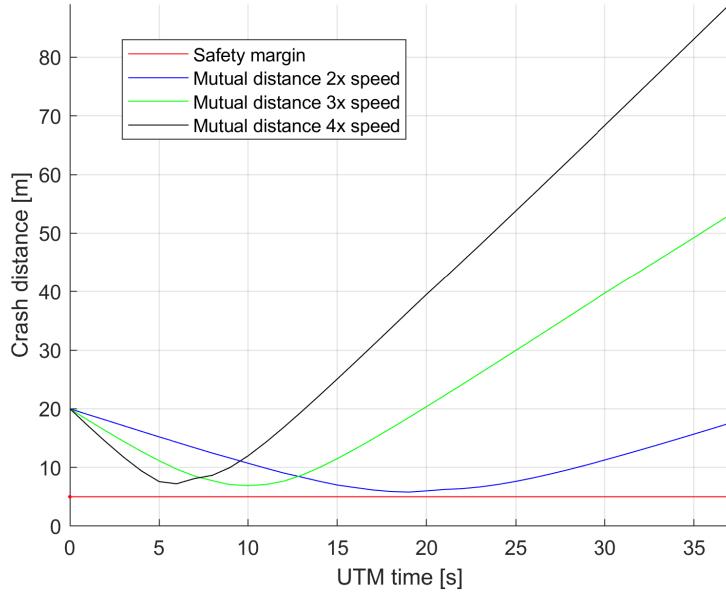


Figure 7.33: *Rule based overtake* safety margin performance for different speed.

**Overtake Speed: Impact on Safety Margin Distance** There is summary table (tab. 7.50) for measurement of minimal and maximal values for *Safety Margin Performance* over *UTM time* (fig.7.33). The minimal *Overtake Distance to Safety Margin* in 0.7991 m for 2x *Speed Difference*. The minimal *Overtake closest point reach time* is 7 s for 4x *Speed Difference*.

For each *Speed difference* (2x, 3x, 4x), the *Well Clear Margin* (*Safety Margin*) was not reached by the *Faster UAS Body boundary*.

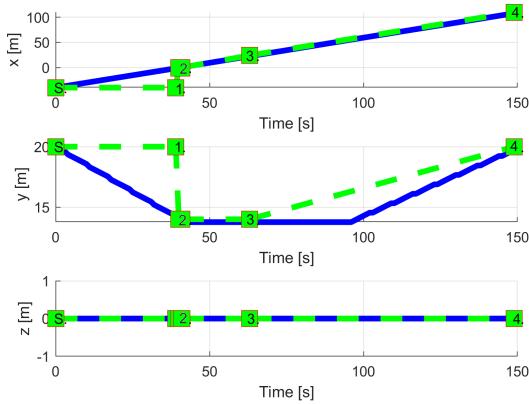
Speed diff.	Minimal		Maximal		Breach
	distance	time	distance	time	
2x	0.7991	20	48.8508	76	false
3x	1.9180	11	73.5336	51	false
4x	2.2154	7	84.0721	38	false

Table 7.50: *Rule based overtake* safety margin distances and times.

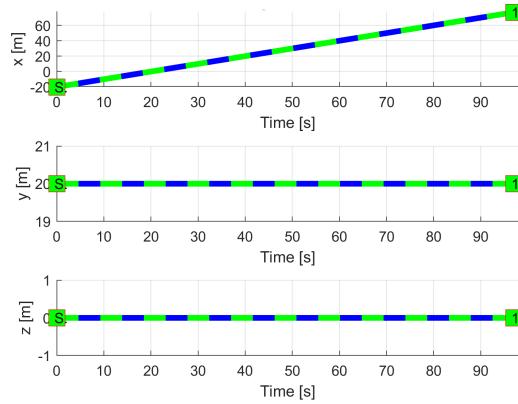
**Path Tracking Performance: 2x Speed** Performance was only evaluated for case when *Faster/Slower UAS speed ratio* is  $2x$ . All waypoints are marked as green numbered *squares* with number. Initial waypoint is marked as green square with  $S$ . Reference trajectory is annotated as *green dashed line*. *Executed trajectory is annotated as blue solid line*.

Following observations can be made from path tracking (fig. 7.34):

1. *UAS 2 has the Right of the Way* (fig. 7.34b) - *reference trajectory* and *executed trajectory* are identical.
2. *UAS 1 is Overtaking* (fig. 7.34a) - the following waypoints are marked on reference trajectory:
  - a. *Collision Point (WP 1.)* - this is not used for navigation, its marking of *Collision Point*.
  - b. *Divergence waypoint (WP 2.)* - there will *Faster UAS* navigate to avoid *Collision*.
  - c. *Convergence waypoint (WP 3.)* - there will *Faster UAS* navigate to gain *Safe Return Distance*.
  - d. *Original Goal Waypoint (WP 4.)* - there will *Faster UAS* continue until *original goal* is reached.



(a) UAS 1.



(b) UAS 2.

Figure 7.34: Trajectory tracking for *Rule based overtake double speed* situation test case.

**Path Tracking Deviations: 2x Speed** Path tracking deviations (tab. 7.51) are interesting for an *Overtake Maneuver* performance.

*Maximal deviation distance* is for important waypoints: Divergence ( $WP_2$ .), Convergence ( $WP_3$ .) and Original Goal Waypoint ( $WP_4$ .), equal to 0 m. This is *desired effect* for *Overtake maneuver*.

*Collision point* ( $WP_1$ .) is avoided at minimal distance 5.7991 m (tab. 7.50) and maximal distance 24.5 m (tab. 7.51).

Other *Speed Difference Ratios* yields similar results.

Param.	UAS 1				UAS 2
	$WP_1$	$WP_2$	$WP_3$	$WP_4$	$WP_1$
	col.	div.	conv.	orig.	nav.
max $ x $	20	0	0	0	0
max $ y $	6	0	4	5	0
max $ z $	0	0	0	0	0
max dist.	24.5	0	4	5	0

Table 7.51: Path tracking properties for *Rule overtake 2x speed* scenario.

## 7.5 Reduced Reach Sets Performance

*Constrained Expansion Method* (alg. 6.1) is creating *Reach Sets* from *Root Node* as a tree expansion using *Expansion Constraint function* (depending on type).

The *Reach set creation procedure* is creating following artifacts:

1. *Nodes* - tree *Node* containing necessary data for discrete Trajectory portion, notably *System State Evolution*, *buffer*, and, *Reachability Rating*.
2. *Trajectories* - leaf *Node* containing *unique buffer* which is not *prefix* in others *Node* buffer.

The *Reach Set Computation Time* is depending strongly on *Movement Automaton* prediction complexity and Node count. The *Constrained Expansion Method* (alg. 6.1) is separating all nodes entering into  $cell_{i,j,k}$  into two distinctive groups: *Candidates for expansion* and *Leftover Nodes*.

The *Leftover Nodes* are thrown away every expansion. The *Leftover Nodes* are not expanded in next *Wave-front* iteration, but they leave a notable *computation* and *memory* footprint.

*Note.* *Average Trajectory Smoothness Rate* (def. 20) is important only in *Navigation Mode*, this aspect has been covered over (sec. 6.6.5, 6.6.6, 6.6.7).

**Approach:** For exactly same conditions (*Testing Avoidance Grid*, *UAS initial state*, *Movement Automaton*) compare performance of *Reach Set Approximations* created by various methods for following parameters:

1. *Coverage Ratio* - defined in (def. 19) shows how versatile *Reach Set Approximation* is (up to 100% of complete reach set coverage).
2. *Node count* - count of Nodes in *Reach Set Approximation* counted like:
  - a. full - all active nodes existing over computation time,

- b. pruned - active nodes for real time use.
3. *Count of Trajectories* - count of Trajectories (leaf Nodes) counted like:
- a. full - all active trajectories existing over computation time,
  - b. pruned - active trajectories leading to coating cells of *Avoidance Grid*.

**Testing Avoidance Grid** with *Distance* 10 m, *Layer count* 10, *Horizontal range*  $[-45^\circ, +45^\circ]$ , *Horizontal Cell Count* 7, *Vertical range*  $[-30^\circ, +30^\circ]$ , and *Vertical Cell Count* 5.

*Note.* The sizing of *Avoidance Grid* was chosen small scale, because property of *Coverage Ratio* can be calculated exactly up to some scale, after that it can be only assumed. Various sizes of *Avoidance Grid* was tested in [148].

The UAS is at *Back-side* of *Figure* (initial state is at all *Trajectory Origins*). The *black dashed line* marks *Avoidance Grid* space boundary. Each trajectory has its own color and ends at *Front-side* of *Avoidance Grid Boundary*.

**Chaotic Reach Set** (sec. 6.6.4) is used in *Emergency Avoidance Mode* for *Non-Controlled Airspace*. The *full* set of trajectories is given in (fig. 7.35a). The *Pruned* set of trajectories is given in (fig. 7.35b).

*Tuning parameters* were selected like follow: *Spread Ratio* is 15 (unique footprint trajectories in cell) and *trajectory footprint length* is 3 (last three unique passing cells).

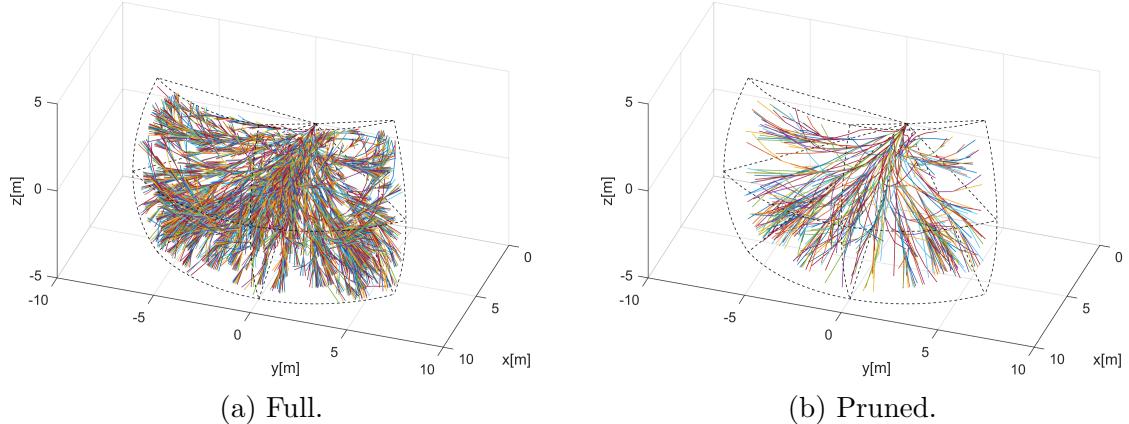


Figure 7.35: Chaotic reach set computation example.

**Harmonic Reach Set** (sec. 6.6.5) is used in *Navigation Mode* for *Non Controlled Airspace*. The *full set* of trajectories is given in (fig. 7.36a). The *Pruned* set of trajectories is given in (fig. 7.36b).

*Tuning parameter* for *harmonic spread ratio* was set to 9 (which implies high coverage).

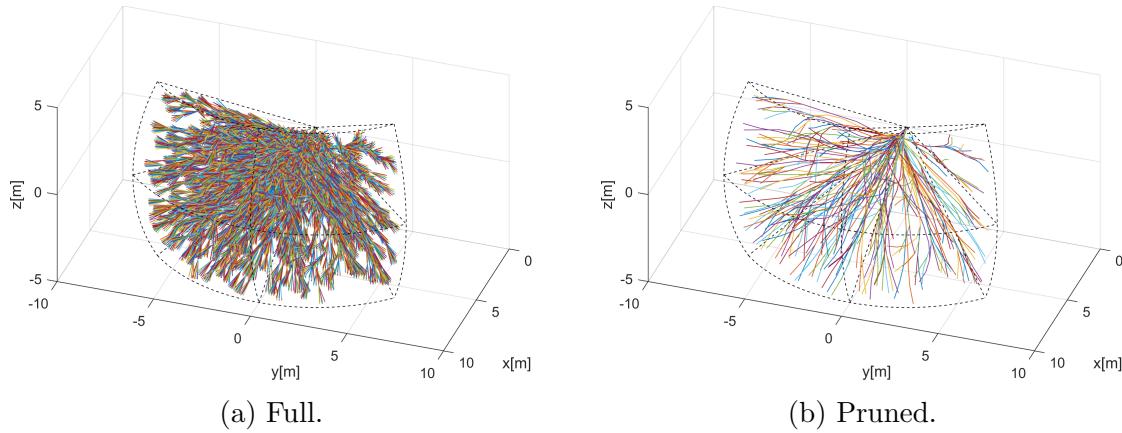


Figure 7.36: Harmonic reach set computation example.

**Combined Reach Set** (sec. 6.6.6) is combination of *Chaotic Reach Set* (fig. 7.35) and *Harmonic Reach Set* (fig. 7.36). The *tuning parameters* are same for respective methods. It is used for both *Emergency Avoidance* and *Navigation*.

**ACAS-like Reach Set** (sec. 6.6.7) is used in *Navigation Mode* for *Controlled Airspace*. The separations used are: *Horizontal*, *Vertical*, *Slash*, and, *Backslash*, to give worst possible nodes and trajectories count. The *full* set of trajectories is given in (fig. 7.37a). The *Pruned* set of trajectories is given in (fig. 7.37b).

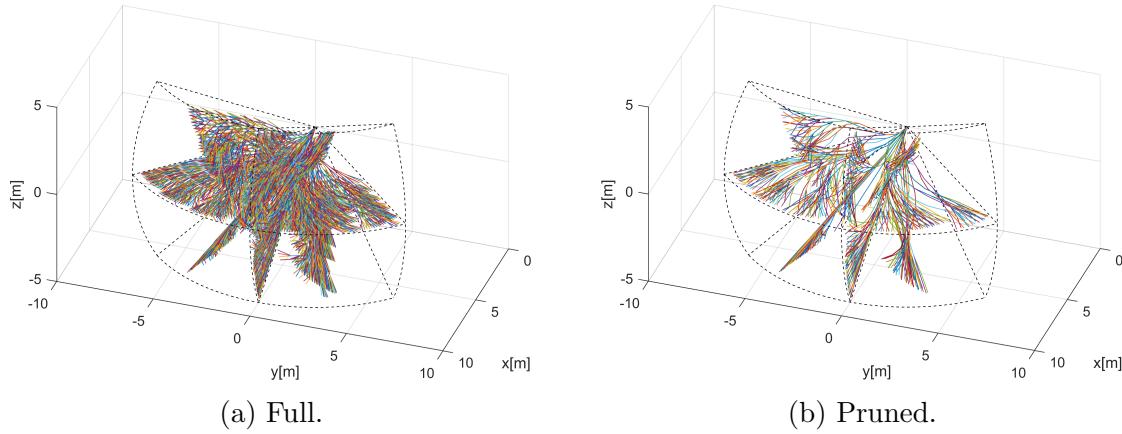


Figure 7.37: ACAS-like reach set computation example.

**Computation Methods Performance Comparison** (tab. 7.52) gives overview of memory consumption and *Coverage Ratio*.

**Node count:** *Full Node Count* shows how much memory it takes to compute *Reach set*. *Pruned Node Count* shows how much *memory* is needed for storage.

*Note.* Total size of *full/pruned Reach Set* depends on Node implementation. The Object oriented prototype implementation in Matlab for example avoidance grid taken up to 1 megabyte of system memory. The effective implementation would take up to 100 kilobytes.

*Constrained expansion* (alg. 6.1) have different selection rate, depending on method. The survival rate directly reflects strictness of selection criteria. The rate of *node pruned* is summarized in (eq.7.4)

Nodes pruned	
<i>chaotic</i>	: 78.93%
<i>harmonic</i>	: 18.50%
<i>ACAS – like</i>	: 79.05%

(7.4)

The *interpretation or results* for each reach set estimation method is like follow:

1. *Chaotic* - the main exploration drive is *Coverage Rate*, the *Trajectory segments* are not usually smooth. For our *Movement Automaton* there is only one *Smooth Movement* : Straight. Other 8 are considered *Chaotic Movements*. Impact of this fact is significant, because 4/5 of nodes were pruned.
2. *Harmonic* - the main exploration drive is *Smoothness* of contained *Trajectories*. The *Trajectory segments* which are getting further away from *cell center* are not feasible. If *Smooth Movements* set size is considered, the Smooth/Chaotic ratio is 1/8 for our *Movement Automaton* implementation. The low node count was expected in this approach. Other Contributing factor is *Trajectory Footprint Length* for uniqueness selection, which is not a tuning parameter in this method, and it is set to most strict selection.
3. *ACAS-like* - the main drive is to create set consisting from *multiple 2D separation planes*. The expansion method applies full movement set on *candidate node*. The *Separation plane movement subset* is determining, which node will be selected for further expansion. The size of separation plane subset to size of movement set rate is 1 : 3. There are four separation planes: horizontal, vertical, slash and backslash each containing full 2D plane reach set approximation which caused high node prune rate. Nodes used rate should get lower with increasing grid size.

**Trajectories count:** *Full trajectories count* shows how many *leaf nodes* were existing during calculation process without pruning. The difference between *full node count* and *full trajectories count* is count of inner tree nodes. *Pruned trajectories count* shows how many *leaf nodes* are used in run-time of *avoidance algorithm*. The difference between *pruned node count* and *pruned trajectories count* shows the count of inner nodes in active reach set.

The most of *waste leaf nodes* are removed during *layer pruning*: function *reachSet.purgeSameFootprint()* (alg. 6.1). The *Waste trajectories* or *unused leaf nodes count* have significant impact. Because *leaf nodes* are side product of *Node Expansion procedure* the amount of *pruned trajectories* is around 90 % regardless of the used method. The results are summarized in (eq. 7.5)

Trajectories pruned	
<i>chaotic</i>	: 91.24%
<i>harmonic</i>	: 88.21%
<i>ACAS – like</i>	: 89.43%

(7.5)

Calculation method	Node count		Trajectories		Coverage ratio	Parameters
	full	pruned	full	pruned		
chaotic	6727	1417	4557	399	90%	spread:15
harmonic	1724	1405	1528	180	30%	spread:9
combined	-	2405	-	435	95%	CH spread:15 H spread:9 tree comb.
ACAS-like	11294	2366	7437	786	74.95%	Separations: H/V/S/BS Coverage pruning: disabled

Table 7.52: *Reduced reach set* computation methods performance

**Coverage ratio:** (def. 19) is showing how much maneuvering versatility of *Reach Set*. *Full Reach Set Approximation* have coverage ratio of 100 %. It is possible to construct *Reference Reach Set* without constrained expansion method which contains all possible *trajectory footprints*. Following observations for *coverage ratio* can be made:

1. *Chaotic* reach set estimation method by design select *Nodes* which have probability of *trajectory footprint* diversification. The high coverage ratio was achieved at values around 90 %.
2. *Harmonic* reach set estimation method by design selects most smooth trajectories which cause low *trajectory footprint* diversity. Fairly high coverage ratio of 30 % has been achieved.
3. *Combined* reach set estimation method takes two reach set and combine their trajectory trees into single trajectory tree. It is given that *Coverage ratio* will achieve at least maximal coverage ratio of original reach sets. Harmonic reach set supplemented narrow smooth trajectories which were throw away previously. This increased overall *coverage ratio* to 95 %.
4. *ACAS-like* reach set estimation method contained 4 separation planes, which caused that it was similar to *Chaotic Reach Set Approximation* for given *Avoidance Grid*, in terms of performance. The coverage ratio For 2D plane was 100 %.

# Chapter 8

## (W) Conclusion and Future Work

*To be done here:*

- Conclusion of the work (This chapter can not be predicted for now)

### 8.1 (W) Simulation Highlights

- (W) Simulation Observations Summary- Use summary in Conclusion and future work on specific.
- (W) Static Obstacles Avoidance - TODO: main points of building, slalom, maze scenarios - link artifacts and performance criteria.
- (W) Constraints Avoidance TODO constraints main point, main loop processing, breach chance ? etc...
- (W) Unsupervised Intruder Avoidance -TODO emergency intruder avoidance emphasis navigation, Emergency avoidance contribution main points.
- (W) Supervised (UTM) Intruder Avoidance - TODO UTM contribution main points.

### 8.2 (W) Other Methods Comparison

1. Vector field avoidance [149]
2. Potential field [150]

### 8.3 (W) Lessons learned

What can be done differently

- The discretization - euclidian grid vs planar grid
- Intruder modeling ideas, the linear intersection without body volume
- The probabilistic/vs rating approach

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