

6.2 (R) Space Discretization - Avoidance Grid

Operation Space: The *Operation Space* is a space where UAS can effectively surveillance its surroundings.

The *Discrete Situation Evaluation* is bounded to UAS specific position and orientation in fixed time t_i . To enable deterministic evaluation the operation space needs to be segmented into finite set of space portions. The finite operation space segmentation is usually done by Grid segmentation, which distributes space into portions with solid boundary.

The *Main Sensor* is LiDAR (problems ??-??). The effective occupancy computation [1] is given by clustering LiDAR field of vision into polar coordinates grid.

The point scanned by LiDAR, where UAS position is center of local coordinate frame and UAS heading is defining the main axes is given as:

$$point = [distance, horizontal^\circ, vertical^\circ].$$

Note. For polar/euclidean transformations and local/global coordinate frames refer to background theory (sec. ??).

The right side of UAS $horizontal^\circ \in] - \pi, 0[$, the left side of UAS $horizontal^\circ \in [0, \pi]$, the down side of UAS $vertical^\circ \in] - \pi, 0[$, the top side of UAS $vertical^\circ \in [0, \pi]$

LiDAR Reading Space Segmentation: The polar space can be separated into cells, which bounds the portion the space, similar to euclidean space grid. The reason for this segmentation is LiDAR reading density¹. The polar space portions state can be assessed directly, the polar \rightarrow euclidean coordinate frame transformation is not time-effective. The polar space assessment of Lidar Data has minimal complexity and it is cost effective. [2].

Definition 1. *Space partition - cell* The cell is a portion of space in UAS local polar coordinate frame, given by:

1. Distance Range - bounded by $distance_{start} < distance_{end}$ in \mathbb{R}^+ .
2. Horizontal Range - bounded by $horizontal_{start}^\circ < horizontal_{end}^\circ \in] - \pi, \pi]$.
3. Vertical Range - bounded by $vertical_{start}^\circ < vertical_{end}^\circ \in] - \pi, \pi]$.

The bounded space for cell is defined as:

$$BoundedSpace(cell) = \dots$$

$$\left\{ point \in \mathbb{R}^3 \text{ where : } \begin{pmatrix} cell.distance_{start} < point.distance \leq cell.distance_{end}, \\ cell.horizontal_{start}^\circ < point.horizontal^\circ \leq cell.horizontal_{end}^\circ, \\ cell.vertical_{start}^\circ < point.vertical^\circ \leq cell.vertical_{end}^\circ \end{pmatrix} \right\} \quad (6.1)$$

For one LiDAR Scan the hits set is given as set of all points which lands in bounded cell space:

$$LidarHits(cell) = \{point \in LidarScan : point \in BoundedSpace(cell)\} \quad (6.2)$$

¹Example rotary LiDAR Velodyne VL-16 specs: https://www.cadden.fr/wp-content/uploads/2017/02/Velodyne_VLP-16-Puck.pdf

The passing hits for cell are hits which are going through the cell (passing), but it lands in distance greater than $cell.distance_{end}$, defined as:

$$PassingHits(cell) = \dots \left\{ \begin{array}{l} point \in LidarScan \text{ where :} \\ \left(\begin{array}{l} cell.distance_{end} < point.distance \\ cell.horizontal_{start}^{\circ} < point.horizontal^{\circ} \leq cell.horizontal_{end}^{\circ}, \\ cell.vertical_{start}^{\circ} < point.vertical^{\circ} \leq cell.vertical_{end}^{\circ} \end{array} \right) \end{array} \right\} \quad (6.3)$$

Note. The cells with same distance range form layers. The greater the distance from coordinate frame origin the greater volume of the cell.

Effective Operation Space - Avoidance Grid: Let start with example, the UAS (fig. 6.1).

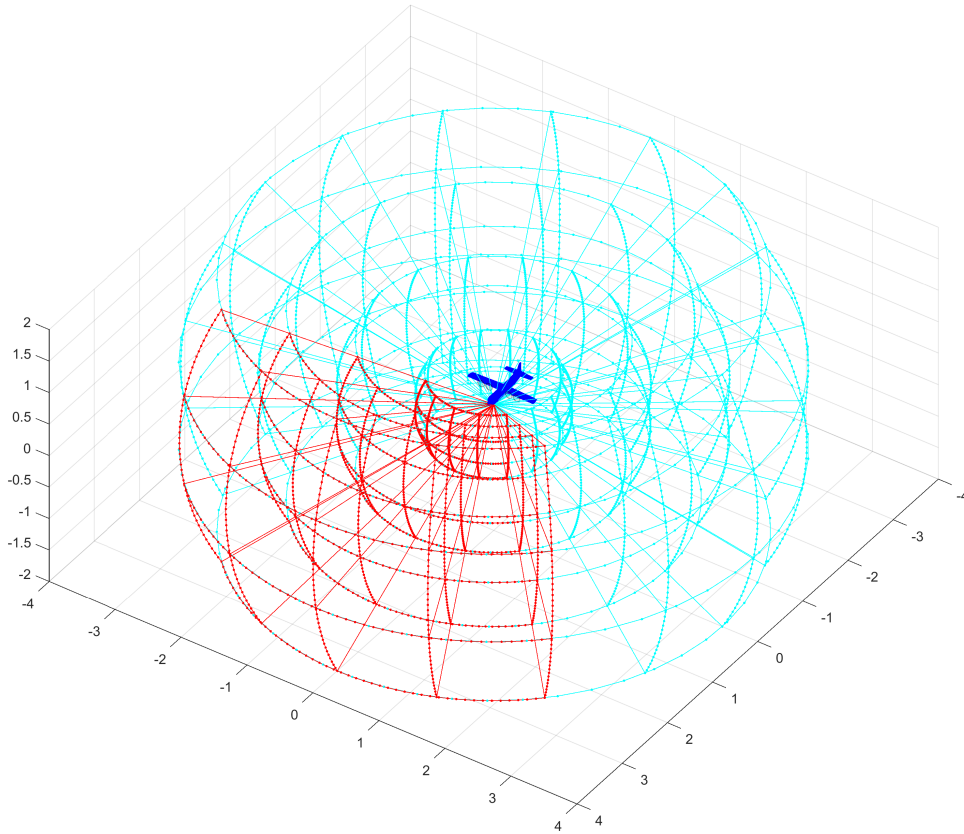


Figure 6.1: Example: The *LiDAR* reading segmentation - cells.

The *full LiDAR Swipe* (cyan and red lines) of UAS (blue plane) has *shape* of conical cylinder. Under *ideal circumstances* the *LiDAR swipe* would have *ball shape*, but in real cases the *craft body portion* where *LiDAR* is mounted is unused.

The *frontal portion* (red line) is a set of cells where *UAS* can make maneuver. The *red portion* size is determined by [3]:

1. *Sensors ranges* - the union of *effective sensor ranges* defines the maximal *effective space boundary*, because there is no reason to asses situation over *effective sensor range*.
2. *Information sources impact* - there is no real impact on *effective space boundary*.
3. *UAS maneuverability* - the *Reach Set* (sec. ??) gives optimal *effective space boundary*, because there is no need to assess the situation out of *reachable space* or its vicinity.
4. *Computation power* - the *Reach Set Evaluation* and *Intersection* algorithms are *scaling* with *effective space boundary*.
5. *Airworthiness requirements* - the *regulations* can impose some minimal requirements on *effective space boundary*.

Definition 2. *Avoidance Grid* The effective space boundary (fig. 6.1 red lines) given by a portion of space in *UAS* local polar coordinate frame, bounded by:

1. Distance Range - bounded by $distance_{start} < distance_{end}$ in \mathbb{R}^+ .
2. Horizontal Range - bounded by $horizontal_{start}^o < horizontal_{start}^o \in] - \pi, \pi]$.
3. Vertical Range - bounded by $vertical_{start}^o < vertical_{start}^o \in] - \pi, \pi]$.

Separated into layers depending on the distance and layer count:

$$\begin{aligned} layer_{start}^i &= (i - 1) \times \frac{distance_{end} - distance_{start}}{layerCount} \\ layer_{end}^i &= i \times \frac{distance_{end} - distance_{start}}{layerCount} \end{aligned} ; \quad i \in 1 \dots I \quad (6.4)$$

Layer horizontal/vertical separations defined by horizontal/vertically cell count:

$$\begin{aligned} horizontal_{start}^j &= (j - 1) \times \frac{horizontal_{end}^o - horizontal_{start}^o}{horizontalCount} \\ horizontal_{end}^j &= j \times \frac{horizontal_{end}^o - horizontal_{start}^o}{horizontalCount} \end{aligned} ; \quad j \in 1 \dots J \quad (6.5)$$

$$\begin{aligned} vertical_{start}^k &= (k - 1) \times \frac{vertical_{end}^o - vertical_{start}^o}{verticalCount} \\ vertical_{end}^k &= k \times \frac{vertical_{end}^o - vertical_{start}^o}{verticalCount} \end{aligned} ; \quad k \in 1 \dots K \quad (6.6)$$

Then cell_{i,j,k} given by (def. 1) is member cell of Avoidance Grid for boundaries:

1. Cell Distance Range (eq. 6.4) depending on layer index *i*.
2. Cell Horizontal Range (eq. 6.5) depending on horizontal index *j*.

3. Cell Vertical Range (eq. 6.6) depending on horizontal index k .

The example of Avoidance Grid Cells is given in (fig. 6.1 red boundary).

The Avoidance Grid is then given as set of cells:

$$AvoidanceGrid = \bigcup_{cell_{i,j,k}} \forall i \in 1 \dots I, j \in 1 \dots J, k \in 1 \dots K \quad (6.7)$$

Trajectory Intersection: The *trajectory* intersection with *Avoidance Grid* is solved in context of *Reach Set Approximation* (def. ??).

Note. The *trajectory intersection* function does not have an impact on *Reach Set Approximation*, because its done prior the flight.

Grid Scaling: For *Sensor Field* there is *effective sensor boundary* given as set:

$$Boundary(Sensor \in SensorField) = \{points \in \mathbb{R}^3 : \text{where reliable}\} \quad (6.8)$$

The *Boundary* for sensor fields is then given as *union of all singe sensor boundaries*:

$$Boundary(SensorField) = \bigcap_{\forall Sensors} Boundary(Sensor \in SensorField) \quad (6.9)$$

Depending on boundary properties it can be projected into maximal avoidance grid boundary values:

$$Boundary(SensorField) \rightarrow AvoidanceGrid : \begin{matrix} \max(distanceRange) \\ \max(horizontalRange) \\ \max(verticalRange) \end{matrix} \quad (6.10)$$

Our approach taken worst LiDAR performance into account [4] and following parameters for avoidance grid were calculated:

1. distance range $[0m, 10m]$,
2. horizontal range $] - 180^\circ, 180^\circ]$,
3. vertical range $[-30^\circ, 30^\circ]$.

The *count of layers* is derived from *average distance traveled by one movement application*:

$$layerCount = \frac{|distanceRange|}{\text{avg. length}(movement \in MovementSet)} \quad (6.11)$$

The *layer length* is based on *our movement set* (tab. ??, ??) the average movement length is 1 m, therefore the *layer count* is 10.

The *efficient boundary* is given by *Reach Set*. Estimate reach set coverage space using *ellipsoidal toolbox* [5] up to given *sensor field* maximal distance:

$$Boundary(ReachSet) = Ellipsoid(UASSystem, distance) \quad (6.12)$$

The values for *Reach Set Boundary* with distance 10 m was following:

1. distance range $[0m, 10m]$,
2. horizontal range $[-45^\circ, 45^\circ]$,
3. vertical range $[-45^\circ, 45^\circ]$,

The *Avoidance Grid* boundary is given as *intersection* of all boundaries:

$$Boundary(AvoidanceGrid) = Boundary(ReachSet) \cap Boundary(SensorField) \quad (6.13)$$

The values for *Avoidance Grid Boundary* for our UAS system (sec. ??) following:

1. distance range $[0m, 10m]$,
2. horizontal range $[-45^\circ, 45^\circ]$,
3. vertical range $[-45^\circ, 45^\circ]$,
4. layer count 10, layer distance 1m.

The *horizontal cell count* and *vertical cell count* was estimated by *rule of thumb* to have value 7 and 5.

Cell in Avoidance Grid Properties: For each cell $\vec{p} \in \mathbb{R}^3$ in the there are properties to be checked:

1. *Is there visibility to the cell ?* - how good is an observation of the cell by Sensor Field.
2. *Is there threat present ?* - how sure the data fusion is that there is eminent threat in the cell.
3. *Is the cell reachable ?* - if there is any trajectory which can get UAS to that cell without too much threat along the way.

The answers to these questions will be given later (tab. ??).

Bibliography

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