Data Structures and Algorithms

Week 11 - Problems on Shortest Paths and SCCs

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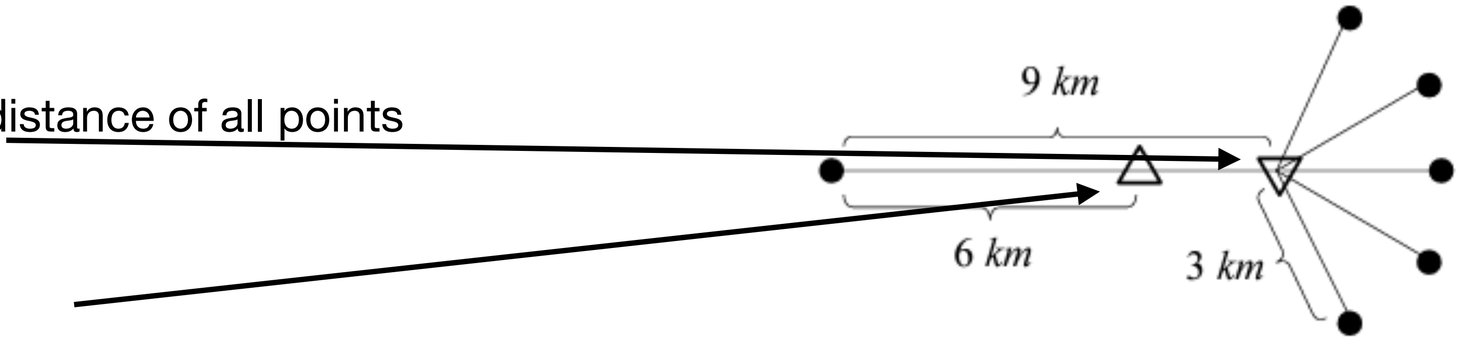
Optimal Meeting Problem

- Given a set of points, $Q = \{q_1, q_1, \ldots, q_n\}$, on a road network $G := (V, E)$ the *optimal meeting point* is the point in G with the smallest sum of distances to all points in Q
  - A more general problem is the facility location problem!
  - Quirky uses: In strategy games like **WorldofWarcraft**, a computer player may need this query to decide the routes of its warriors.
- Two commonly used functions to model the problem:
  - $\bar{x} = \arg\min_x \Sigma_i d(q_i, x)$
    - Minimizes the total travel distance of all points
  - $\bar{x} = \arg\min_x \max_i d(q_i, x)$
    - Minimizes total travel time



(a) Optimal Meeting Points

Optimal Meeting Problem: Solution

- Use Floyd-Warshall to identify shortest path for each (i, j)
- For each vertex i:
  - Identify the node which is the farthest
- Maintain a global smallest maximum distance found and the corresponding vertex k
- Return k as the answer!
- Time Complexity = Time complexity for Floyd-Warshall Algorithm
- **Approximate algorithms exist with better time complexity!**

Backup path problem

- Give G= (V,E) with a source, **s**, and destination, **t**, find two shortest paths from **s to t** that the number of shared edges between the two paths is minimised
  - Use Dijkstra's alg to find the shortest path from **s** to **t.** Let us call this path **p**
  - Increase the weights (to ∞)in **p so as to penalise them in the next run of the algorithm**
  - Rerun Dijkstra's alg to discover backup path **p'** (here the number of shared paths is 0)