# Data Structures & Algorithms

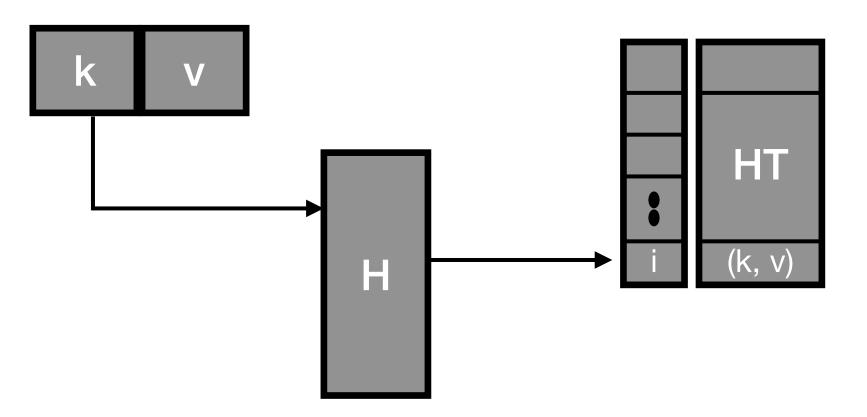
Week 3 - Hashing (HashTables, HashMaps, Dictionaries)

Subodh Sharma, Rahul Garg {svs,rahulgarg}@iitd.ac.in

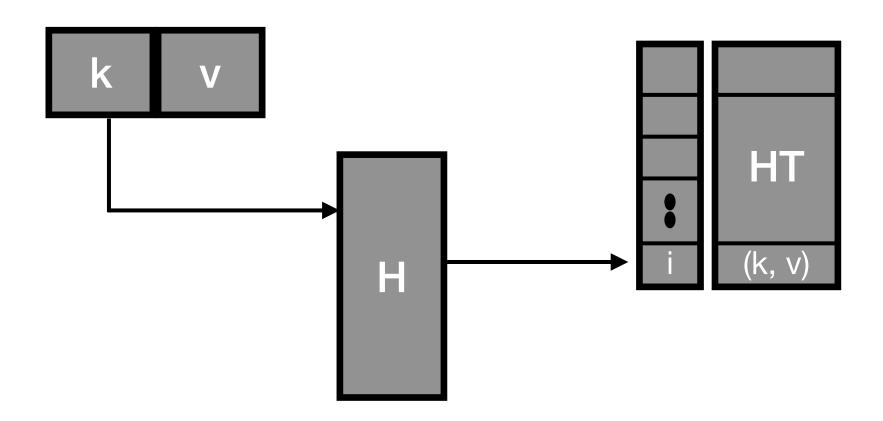
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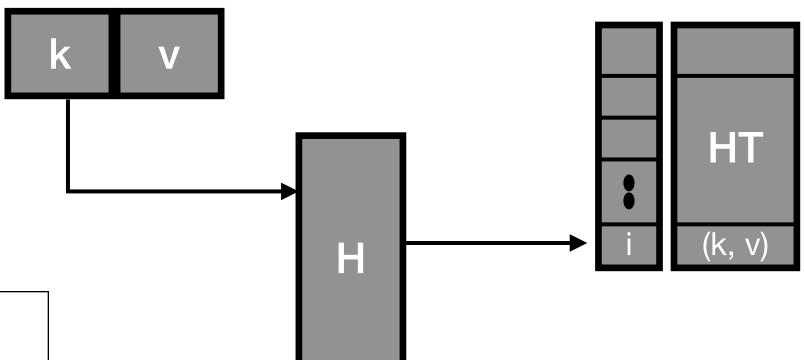


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	LL, no dup	Sorted Array wit h Binary Search	Hash Table
Insert	O(n)	O(log n)	O(1)
Delete	O(n)	O(log n)	O(1)
Contains	O(n)	O(log n)	O(1)



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  - Closed Hashing: Linear probing, Quadratic probing, Double Hashing

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- Dynamic: Dynamic resizing of HT should be possibles

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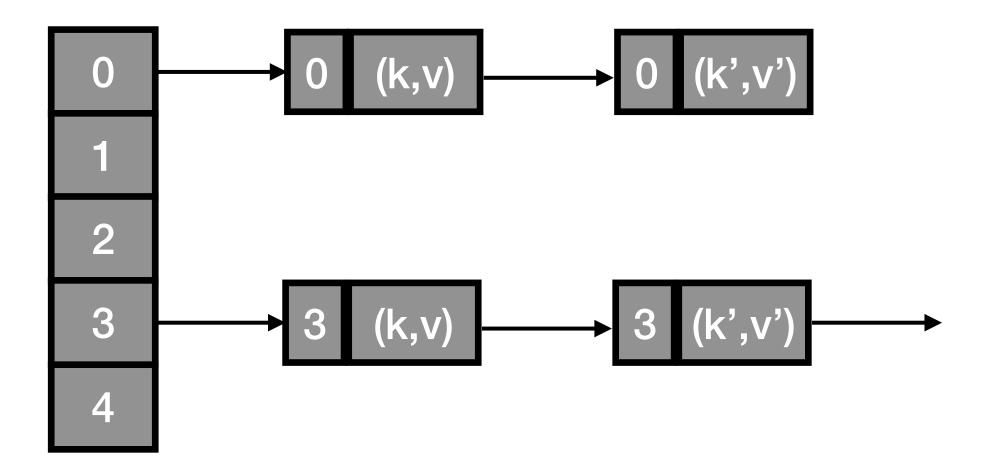
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  - Avalanche effect: Small change in the input produces significant change in the output
    - $\forall x, y : d_H(x, y) = 1 \Rightarrow P(H(x)_j \neq H(y)_j) \ge 0.5$

• For find/insert/delete an element **e** 

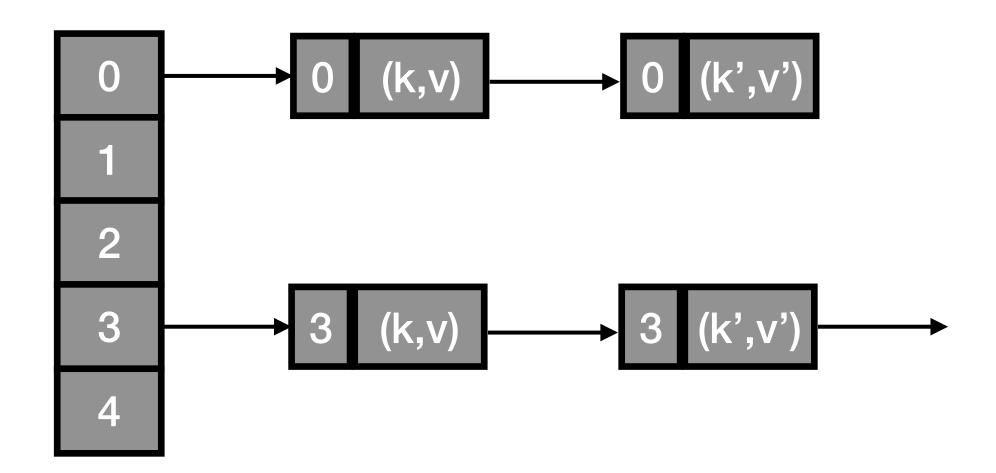
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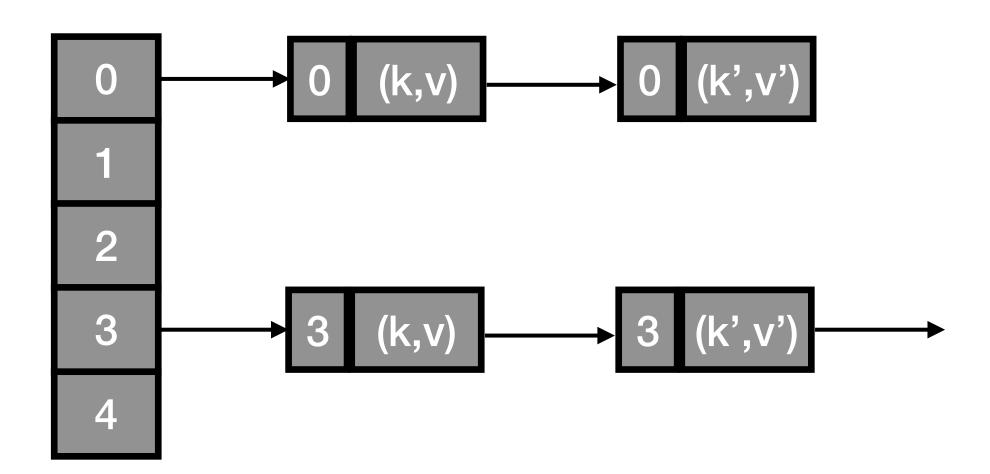
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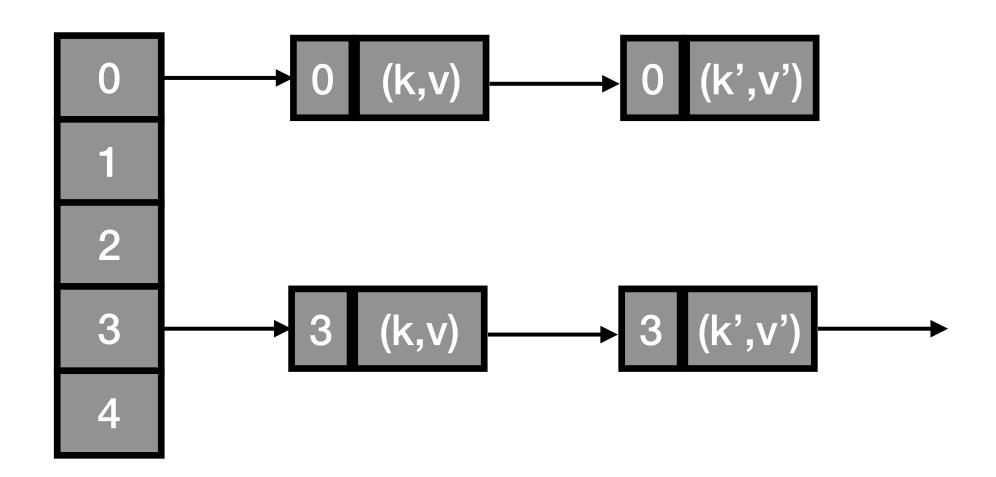
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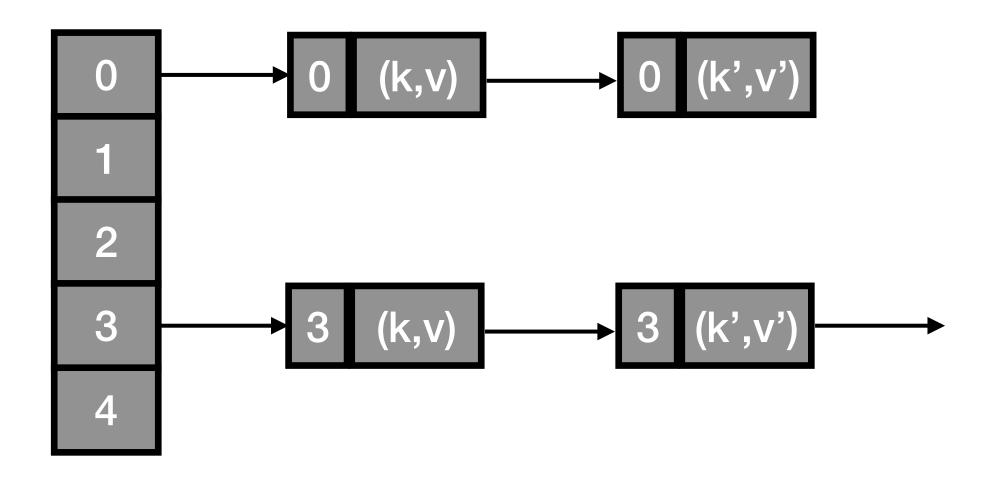
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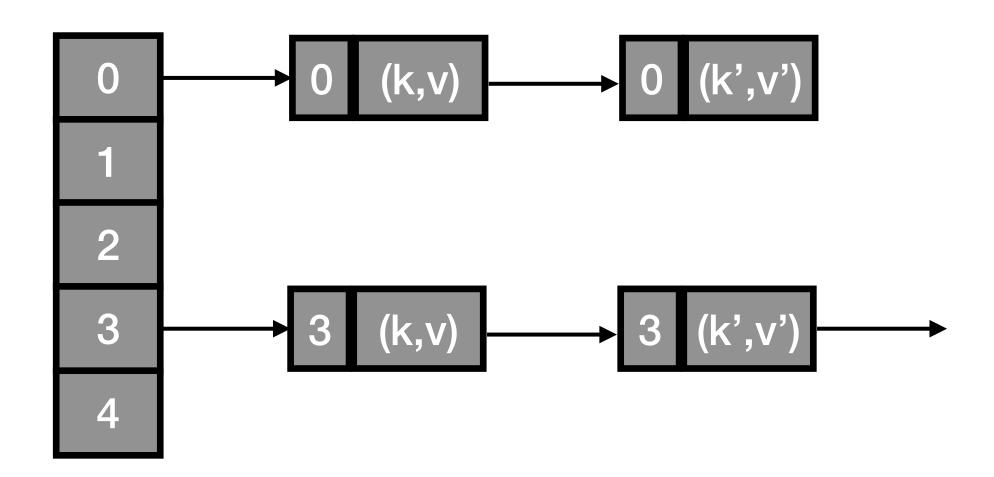


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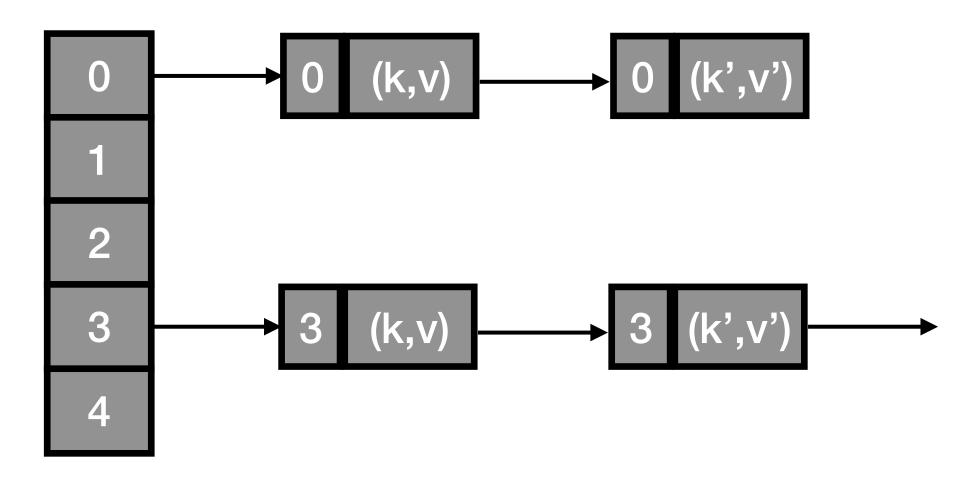


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  - Threshold for resizing: When  $\alpha > 0.7$ , **HT** is doubled in size

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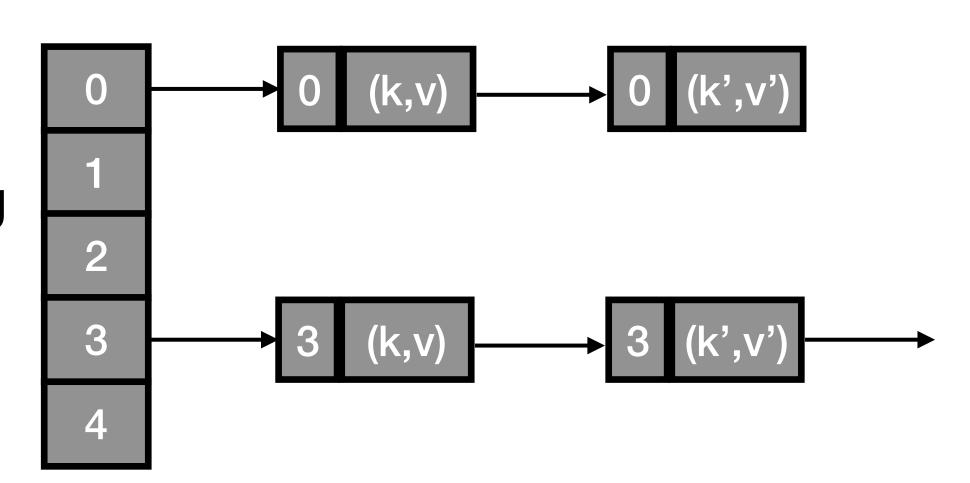
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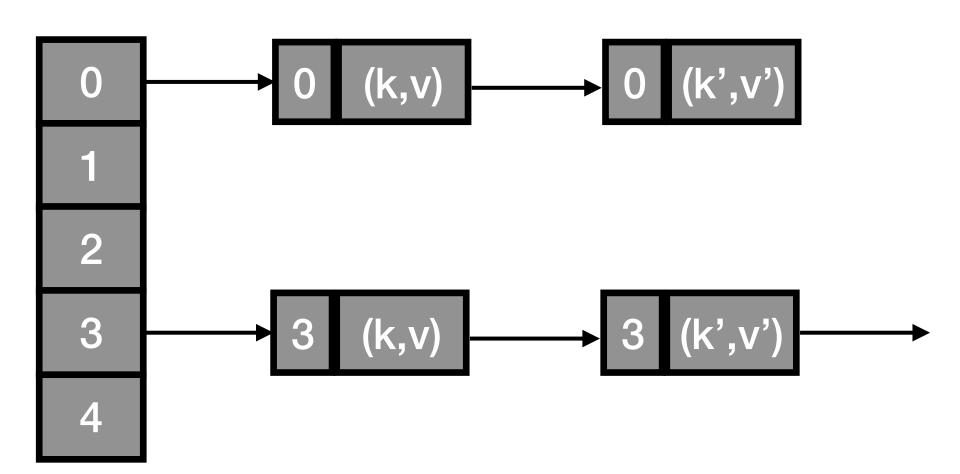


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Assumption: We use a simple uniform hash function

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**H(x)** 

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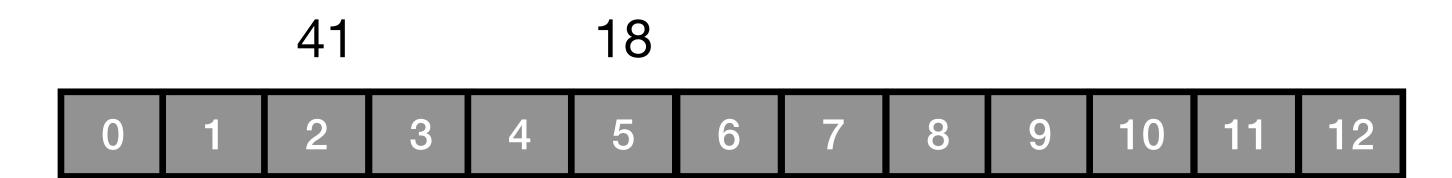
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 2
 3
 4
 5
 6
 7
 8
 9
 10
 11
 12

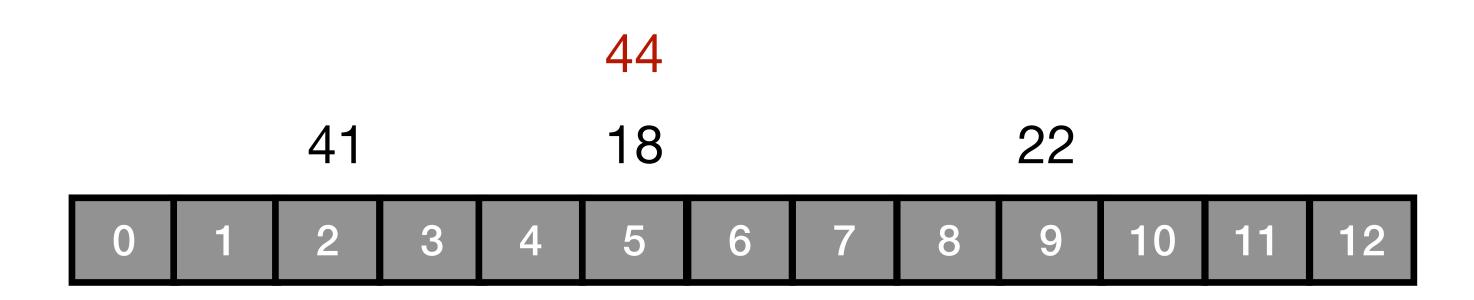
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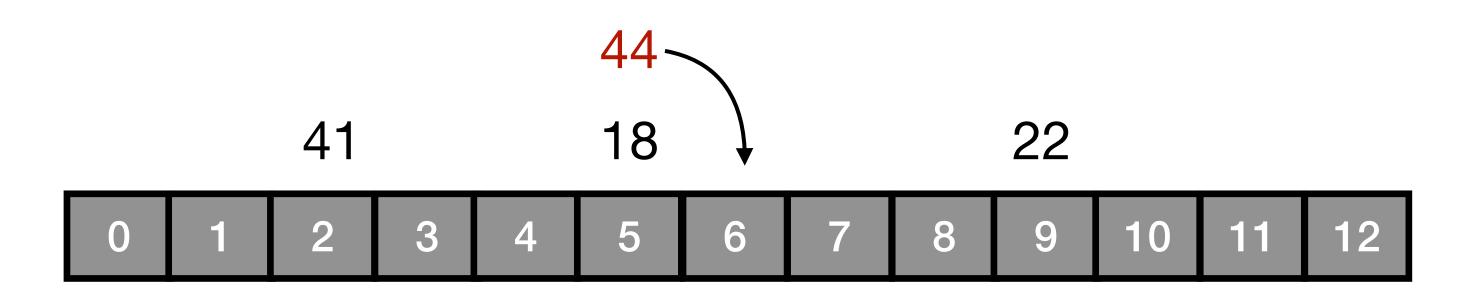
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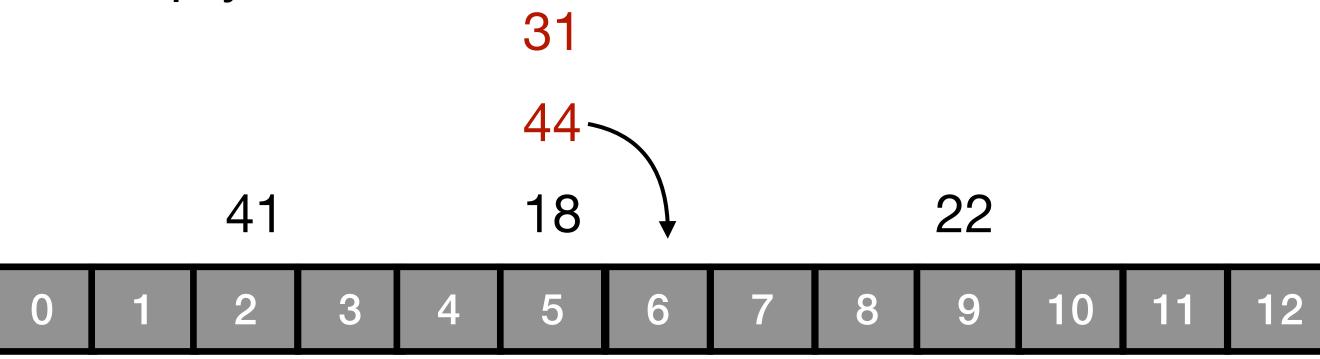
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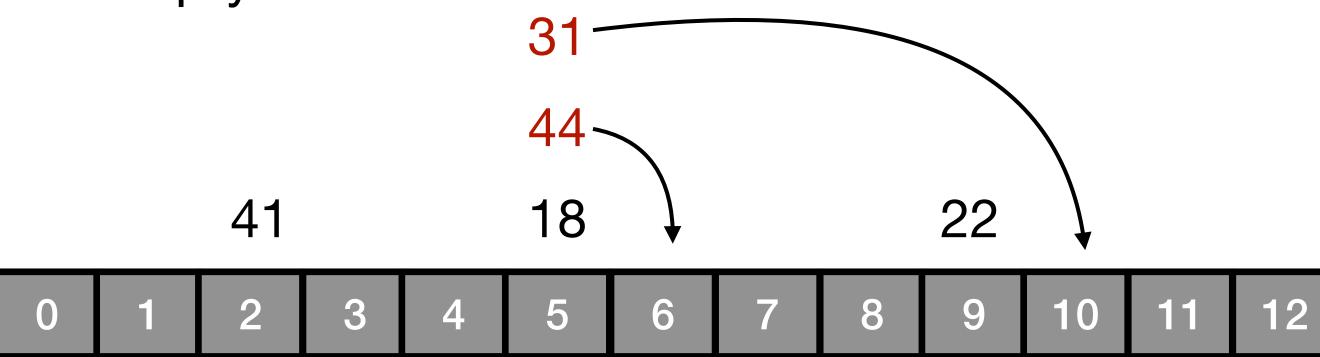
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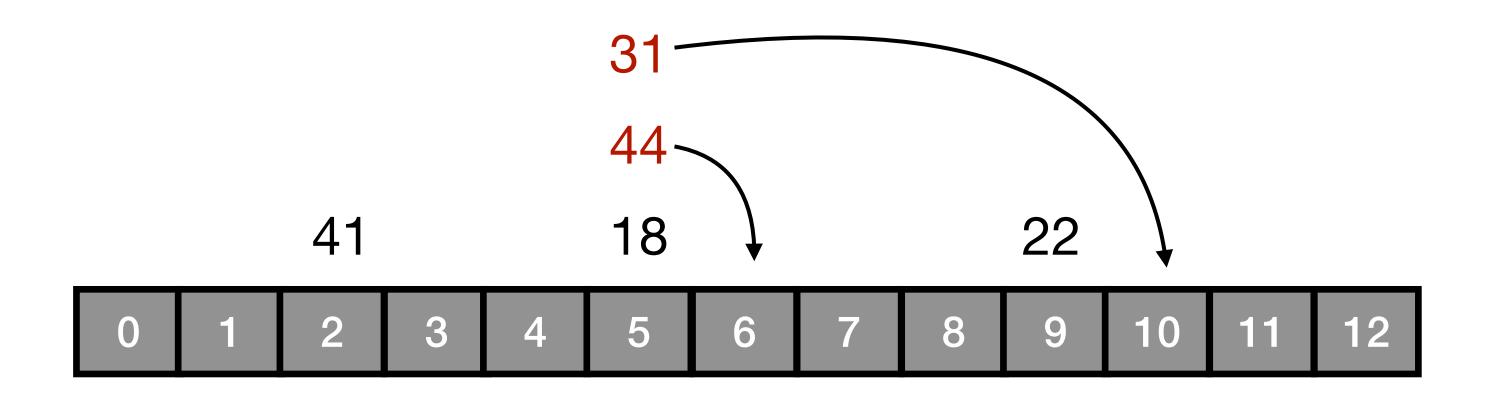
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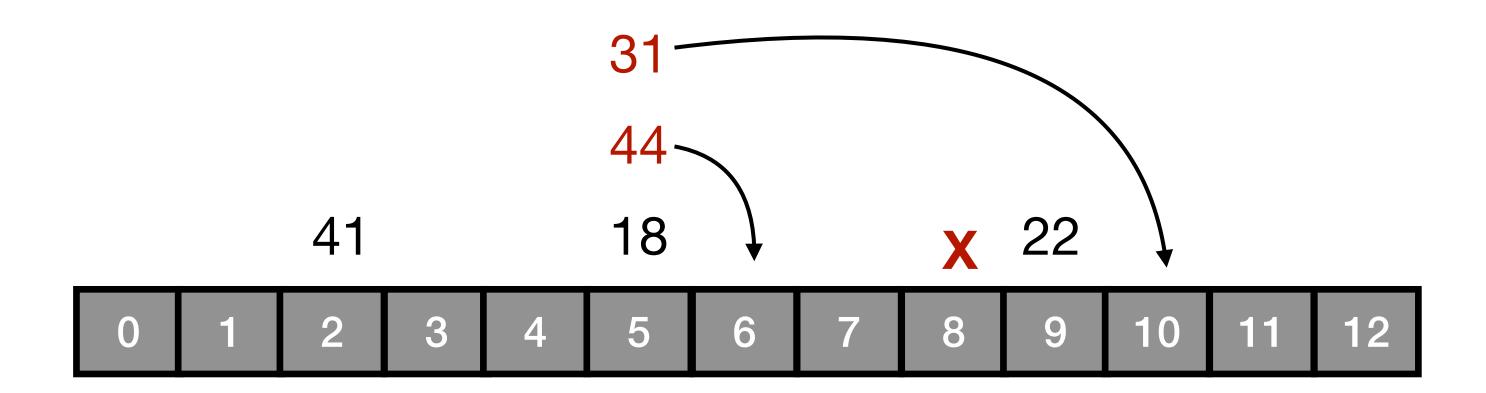
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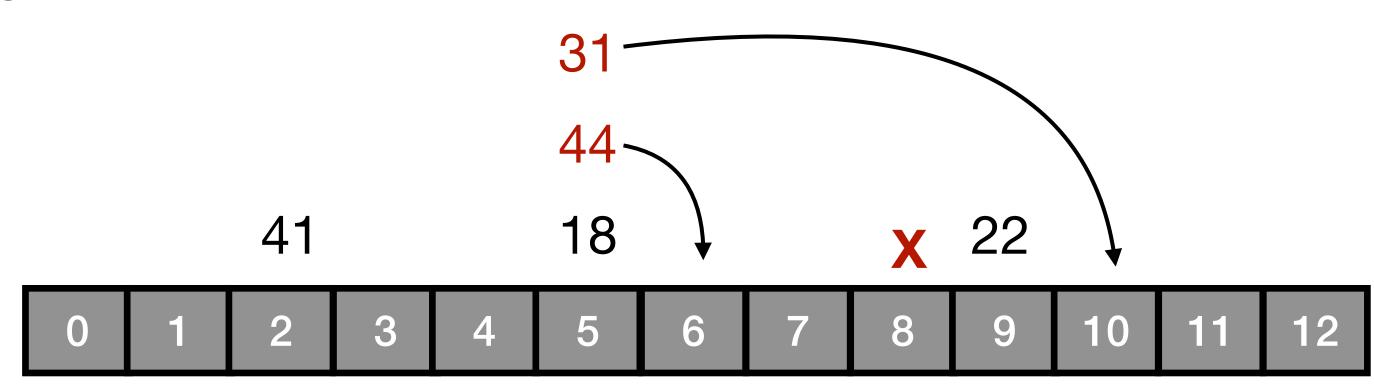
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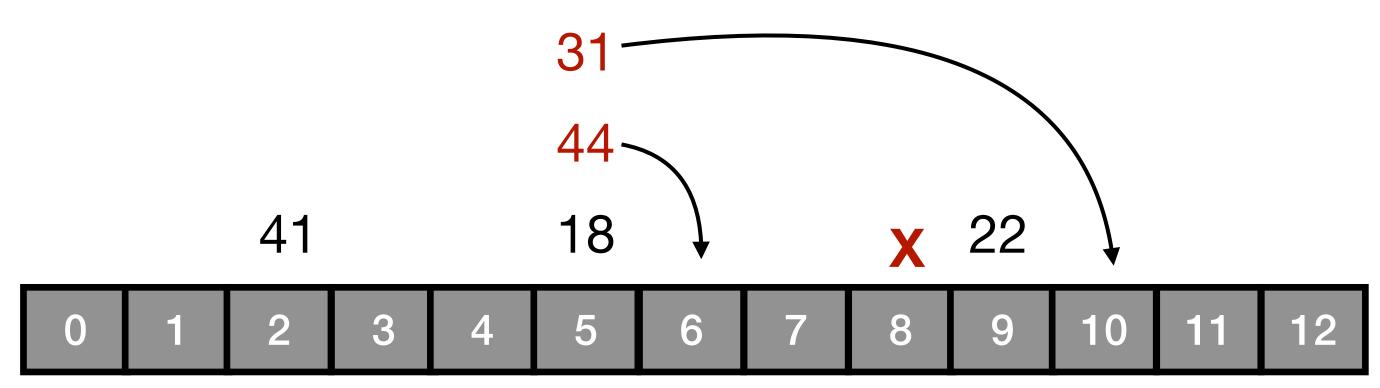
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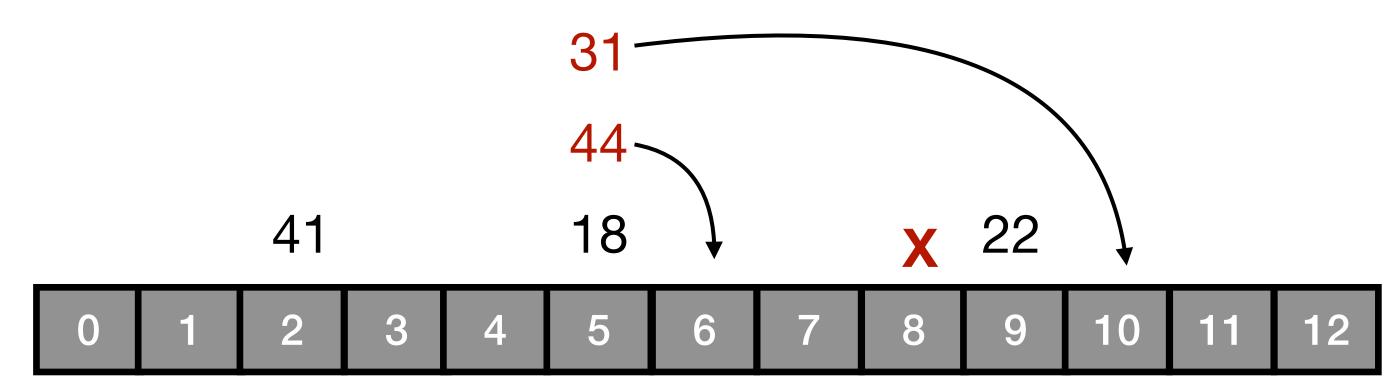


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- Q: What is the solution to the above problem?

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- EXERCISE: Try with different table sizes, different levels of loading (the number of keys inserted), and different input distributions.

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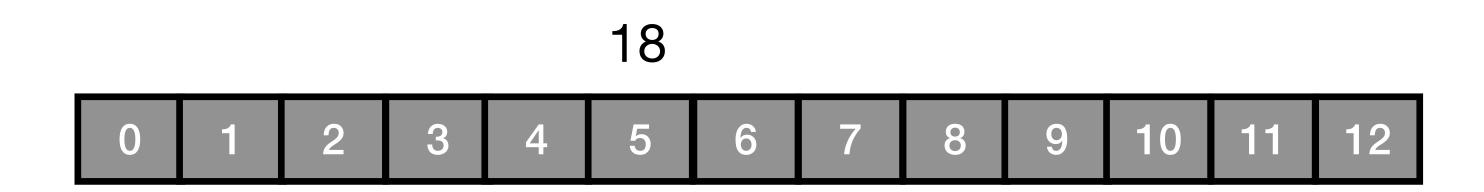
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void DoubleHashingInsert(int k){
  if (tableIsFull(HT))
    throw std::runtime_error("Error");
  uint probe = hashFunc1(k);
  uint offset = hashFunc2(k);
  while(TableOccupied(probe)){
    probe = (probe + offset) % m;
  }
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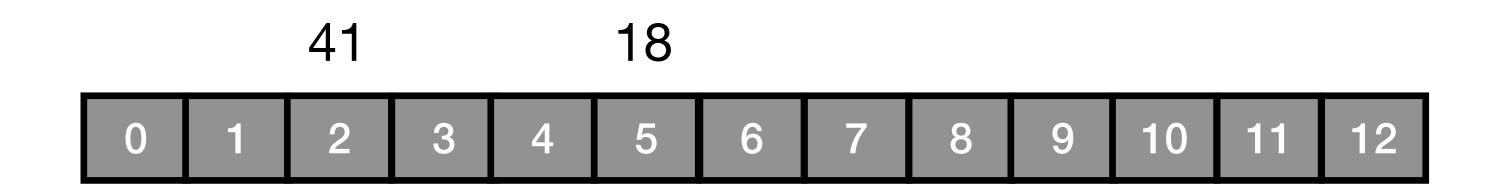
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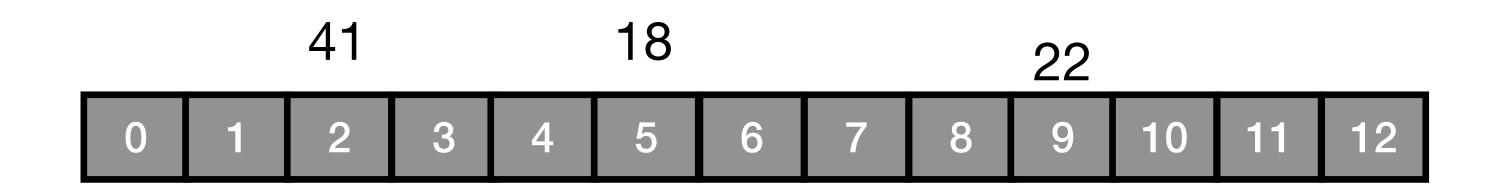
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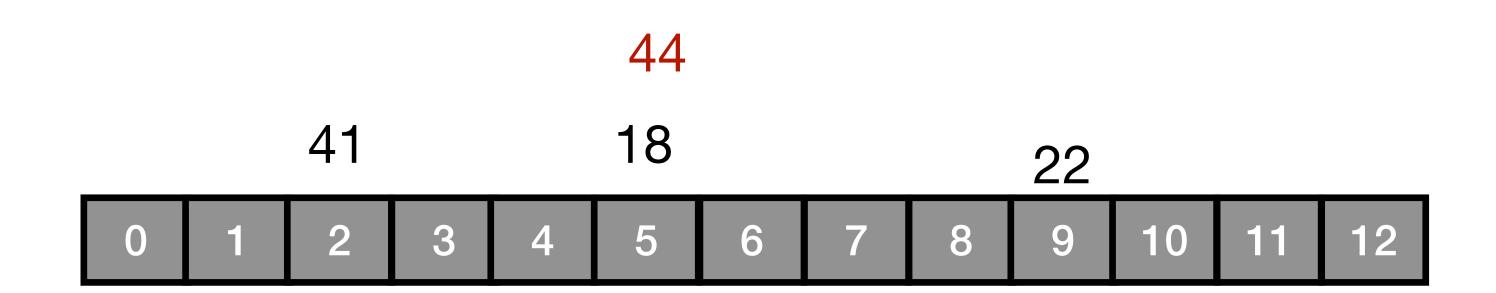
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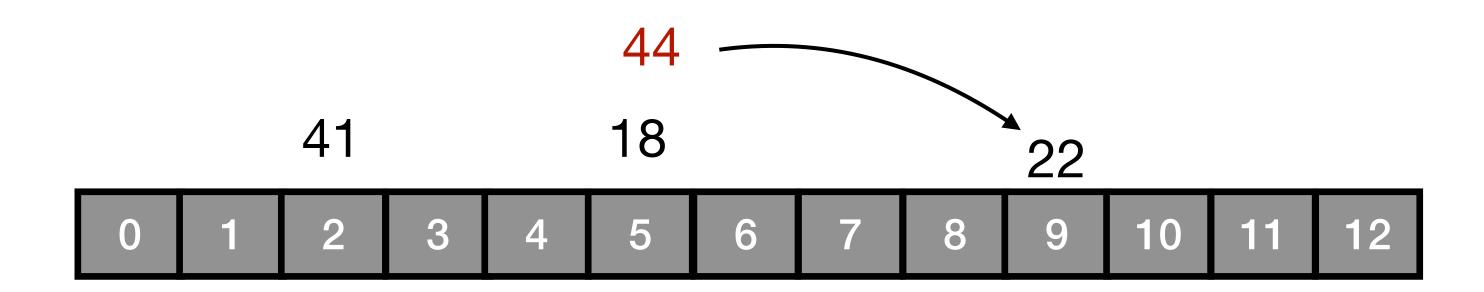
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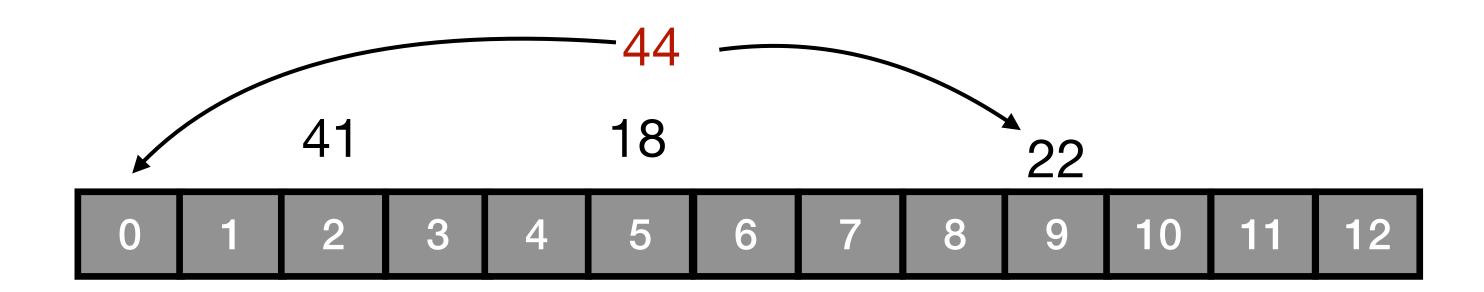
- $H_1(k)$ : k mod 13
- $H_2(k)$ : 8 (k mod 8)
- Insert keys: 18, 41, 22, 44, 59, 32, 31



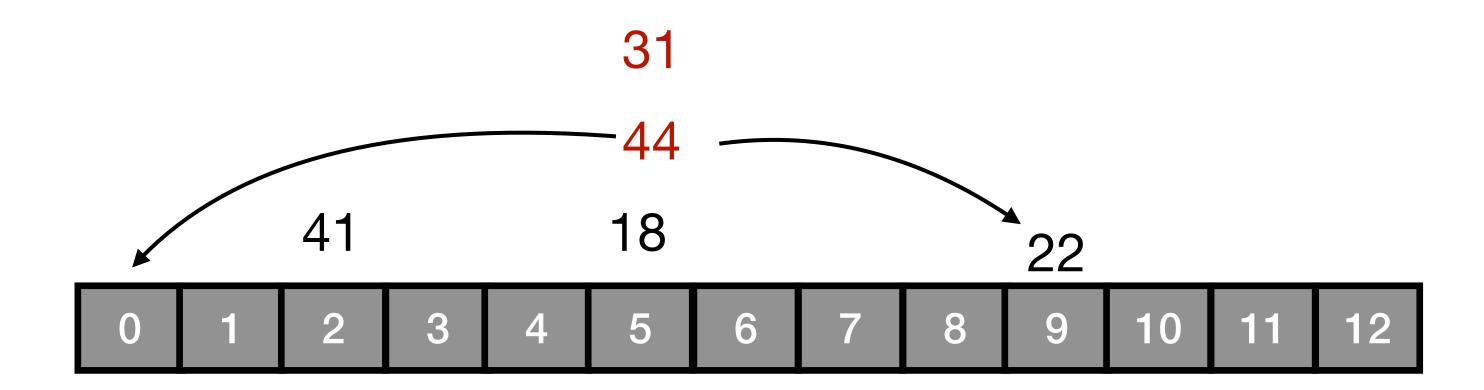
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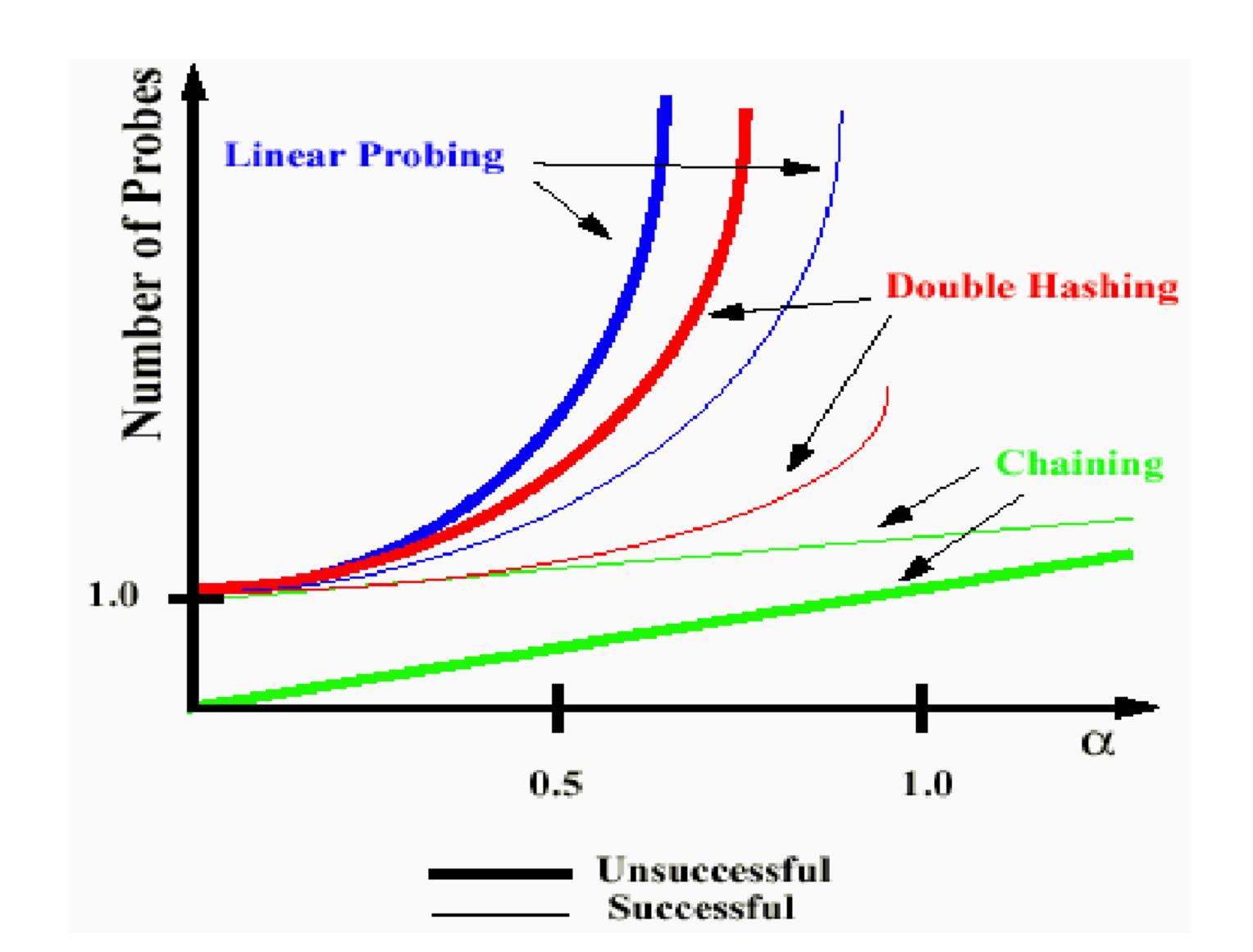
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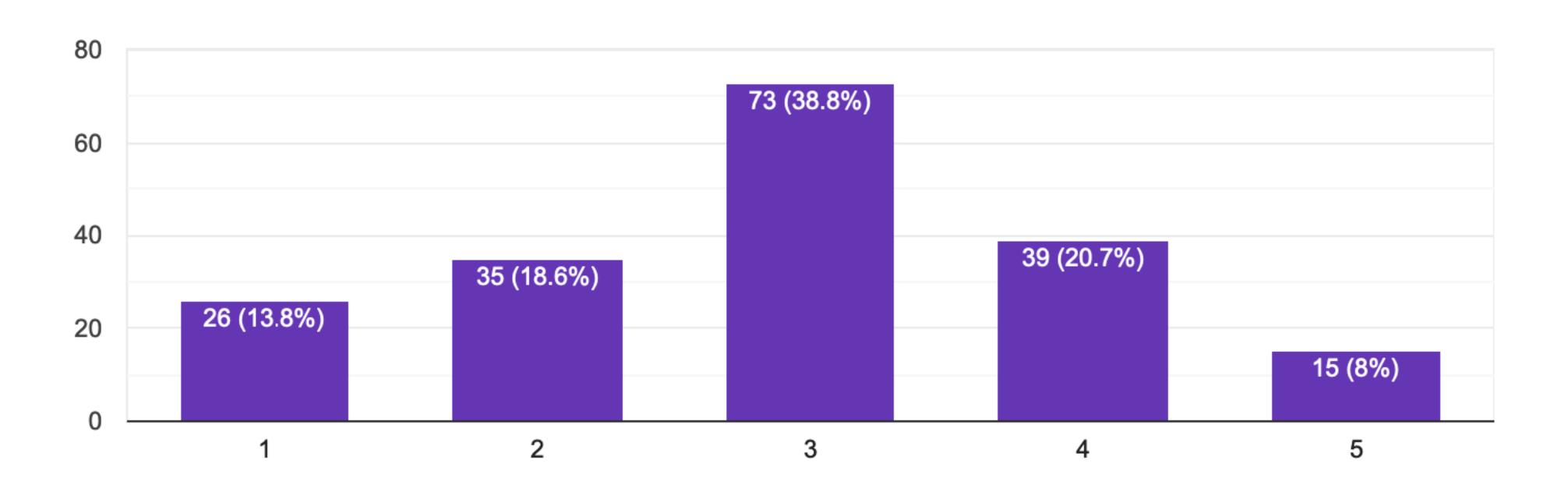
### **Expected Number of Probes**



How comfortable were you with C++ Classes prior to the start of Assignment 1?

188 responses

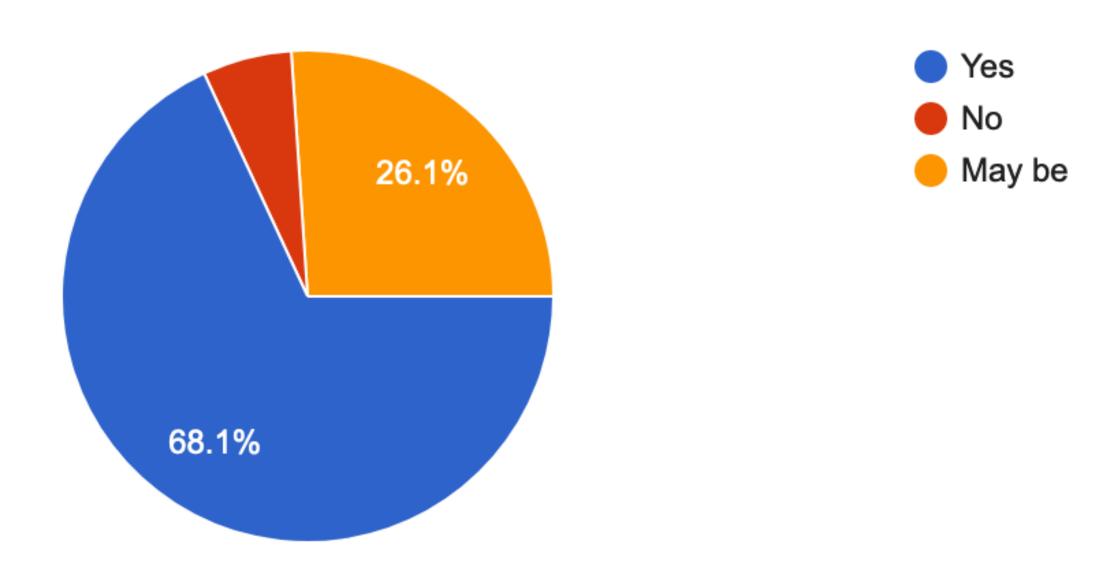




After Assignment 1 submission do you feel more confident with C++ programming?

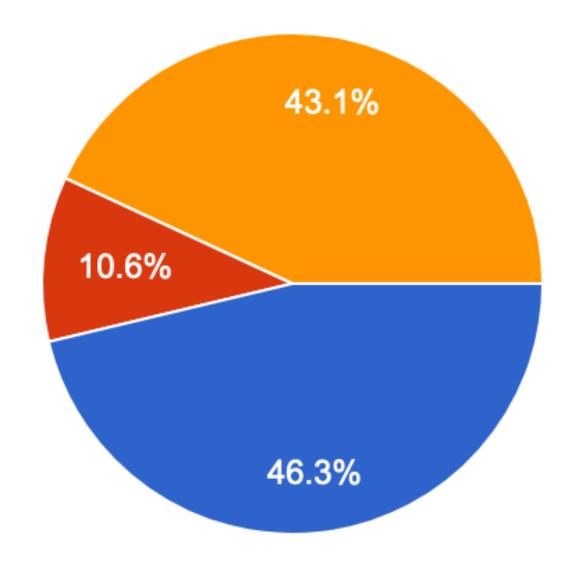


188 responses



Do you believe that the classes are moving too swiftly?

188 responses



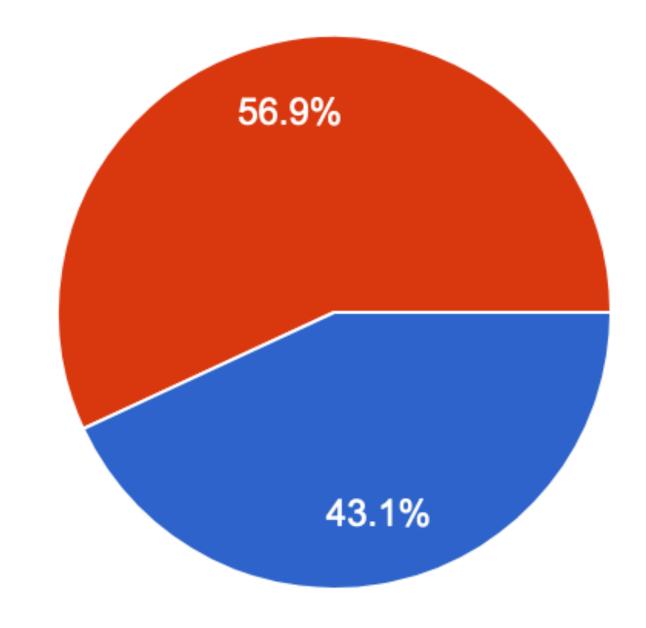


- Yes -- I am unable to keep up
- No -- I am fine with the pace
- Mixed -- On some topics the pace is fine but on others I felt it was too quick

Would you want tutorials on programming in your lab sessions?

Сору

188 responses

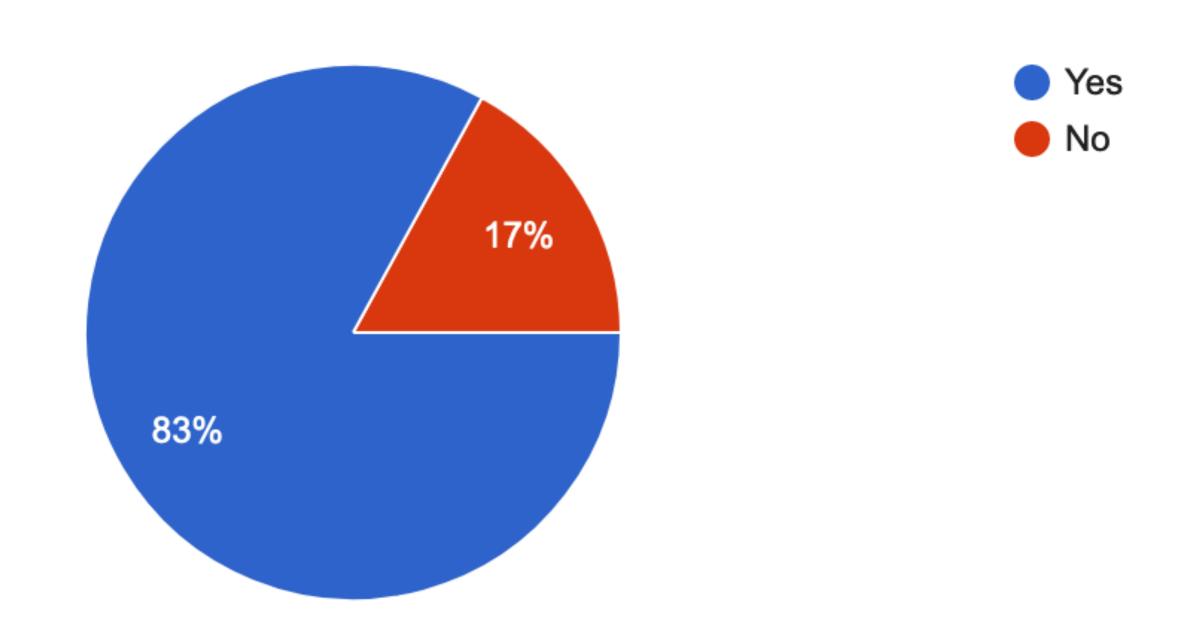


Yes

 No - I would like to use the lab time for CP and practice lab questions Does the advance disclosure of weekly class topics aid in your lecture preparation?

188 responses





Please upload slide simultaneously l

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Class feels a bit rushed, professors are just amazing but it would be better if class were slower.....

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Give more reading assignments and practice questions.

more programmes should be provided in the class

Maybe have a doubt session for clearing the doubts of the students

Like I feel that in the lab the students are not that serious like what can be done is to organize a sort of contest in lab obviously ungraded but with a leaderboard sort of thing with questions not disclosed at the

Class feels a bit rushed, professors are just amazing but it would be better if class were slower.....

Give more reading assignments and practice questions.

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Maybe have a doubt session for clearing the doubts of the students

Certainly, it seems that grasping data structures and algorithms, or indeed any subject, hinges largely on one's personal commitment. Assuming that attending a mere three-hour lecture per week would confer a profound understanding of their intricacies is rather unrealistic. In my view, a more systematic approach to teaching data structures and algorithms would require around nine hours of weekly instruction, delving into sufficient detail to comprehensively elucidate all underlying concepts while effectively fostering motivation.

However, given the constraints imposed by institutional regulations, realizing such an intensive teaching schedule is unfeasible. Thus, in my estimation, a more pragmatic approach involves proactively acquainting oneself with the subject matter through self-guided study from recommended resources prior to class sessions. These interactive sessions could then be optimized for addressing queries and igniting enthusiasm through the exploration of novel topics.

Having shifted my perspective away from harboring overly ambitious expectations from class hours and embracing the aforementioned strategy, I have found myself considerably content with the pace and depth of my learning journey.