Markov-Chain Based Strategic Betting

Table of Contents:

Section 1: Specification table

Base Model	Snakes and Ladders Modelled as A Betting game
Extension Assumptions	Instead of starting in at the bottom of the board and working your way up, a player can start at any balance square. The game is adjusted such that the squares represent amounts of money a player has. Rolling a dice can take them up a ladder to a higher balance or a lower balance (representing winning or losing a bet). And rolling a dice involves randomly picking a bet to play. There are two absorbing states now, winning, obtaining the highest balance possible or going bankrupt, $0. The balance a player has influences their betting options. This game can also be adjusted such that a player can select a bet on their own.
Techniques Showcased	Markov Chains: Simulate a betting session with Markov chains, with states representing the balances Monte Carlo: Simulate a games, results will be aggregated over many simulations of a game Heurestics: Finds the best possible betting strategy, a collection of 'best bets' to be played at each balance state
Modelling Question 1	On average do players win or lose and what is the spread outcomes? How quickly does it a player typically win or lose and how does starting balance influence this?
Modelling Question 2	Can we use a betting strategy to increase our odds of winning? If so what is a good (or locally optimum) betting strategy where a betting strategy is a optimal bet for each balance.

Section 2: Introduction

This project explores the dynamics of betting under uncertainty by adapting the mechanics of the board game *Snakes and Ladders* into a stochastic financial model. The aim is to evaluate how player outcomes are shaped by randomness and strategy, with a focus on both win likelihood and game duration.

We focus on two key questions:

- **Q1:** On average, do players win or lose in this system? What is the spread of outcomes, and how quickly do players typically reach an absorbing state (Win or Lose)?
- **Q2:** Can a player improve their odds of winning by using a strategy that selects the optimal bet for each balance? If so, what does an effective or locally optimal betting strategy look like?

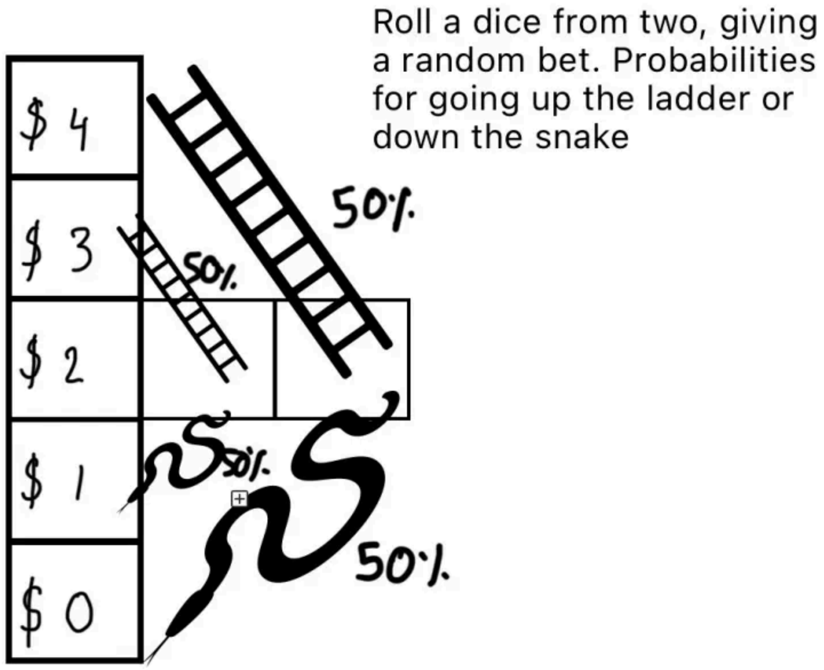
To investigate these, we extend the *Snakes and Ladders* framework. Each square is reinterpreted as a balance state (e.g. $1 to $7), and transitions between states are triggered by betting outcomes instead of dice rolls. A move up represents a winning bet (increased balance), and a move down represents a loss. The game ends when the player reaches an absorbing state: either $0 (bankrupt) or $8 (win).

This creates a **stochastic, discrete-state system** where:

- Balance evolves over time based on probabilistic bets
- Absorbing states reflect game outcomes (win or loss)
- Players can use strategies to influence the transitions

A small example is shown in figure 1, of how this model works. Each square (balance state) is connected to two possible transitions: one that increases balance (ladder/up) and one that decreases balance (snake/down), with associated probabilities (e.g., 50/50). The player starts at a midpoint (e.g., $2), and transitions continue until they reach an **absorbing state,** either **$0 balance (bankrupt) or Maximum balance (win).** Although the board has spaces on the right, it technically never ends up there and is always in one of the balance squares.

Figure 1



We use three main techniques to explore the problem:

- **Markov Chains** model the system structure. Transition matrices define the probability of moving between balances, and we compute absorption probabilities and expected steps using canonical forms.
- **Monte Carlo Simulations** are used to empirically replicate thousands of betting sessions, allowing us to estimate spread, volatility, and real-world outcome frequencies.
- **Heuristics and Optimisation** (e.g. simulated annealing) are applied to find betting strategies that maximise win rates by adapting decisions based on current balance.

By combining simulation, stochastic modelling, and optimisation, we can both **build** and **interpret** a realistic model of betting outcomes. This allows us to reflect on whether players are statistically "destined" to lose or whether behaviour (strategy) can significantly alter the odds - a finding that has relevance in real-world gambling and decision-making systems involving probabilistic risk.

Section 3: Model Description

3.1 Original Model: Snakes and Ladders

The base model is inspired by the classic board game *Snakes and Ladders*, where players roll a die to progress through a board filled with snakes and ladders. Snakes cause players to fall back while ladders accelerate their progress toward the goal. This original version is a **discrete, deterministic model**, with fixed transitions determined entirely by the player's die roll and landing position. The only source of randomness is the die itself, and all transitions thereafter are pre-defined.

3.1.1 Original Model Assumptions

Original Model Assumptions:

- **Discrete**: Board has discrete positions (squares 1 to 100).

- **Deterministic transitions**: Snake/ladder transitions are fixed.
- **Stochastic element**: Only the die roll is random.
- **Memoryless**: Markov assumption holds (next state depends only on current state and die roll).
- **Absorbing**: Final square is terminal.

> Note: Since the **extension of this model fundamentally changes** the **purpose of the mode**l, we can't compare to the "base model" of Snakes and Ladders for analysis in this report. Our 'base model' is effectively the random betting selection and the 'extended model' is where individual strategy choice is possible.

3.2 Extended Model: Stochastic Betting Framework

We extend this model into a **stochastic betting framework** by reinterpreting each square as a monetary balance ranging from $0 to $8. The player begins with a balance (say $4), and each move corresponds to the outcome of a chosen bet. Each available bet has an associated probability of increasing or decreasing the balance by specific amounts. The game ends once the player either reaches $0 (bankruptcy) or $8 (maximum balance), both of which are **absorbing states**. At intermediate balances, the player selects a bet to play, and the result of the bet determines whether the balance increases, decreases, or remains unchanged. We also create a version of this model where a player can select their own bets for each balance.

This formulation creates a **discrete, stochastic, and numerical model**. It is **linear** in structure, as each transition depends only on the current balance and chosen action, and not on the player's past trajectory. The state space consists of integer-valued balances from $0 to $8, and transitions between states are defined probabilistically. Each state (balance) has a set of available bets, with each bet specifying: A win/loss probability AND a resulting balance change if the bet succeeds or fails

3.2.1 Markov Chains Explanation

To formalise the model, we construct **Markov Chains** where each state represents a possible balance. A transition matrix is generated for each strategy or scenario, where `P[i][j]` represents the probability of moving from balance `$i` to balance `$j`. The absorbing states ($0 and $8) are modelled with self-loops (`P[0][0] = 1`, `P[8][8] = 1`), indicating that once reached, the process terminates. For a randomly selected bet at each balance, we construct a single transition matrix where transition probabilities incorporate both the randomness of bet selection and the stochastic outcomes of the bets themselves.

3.2.2 Monte Carlo Explanation

To evaluate performance, we apply **Monte Carlo simulation**, replicating thousands of game sessions for each strategy. Each simulation starts at a balance of $4 and proceeds step-by-step based on the defined bet probabilities until an absorbing state is reached. Monte Carlo allows us to empirically estimate the likelihood of success (reaching $8), the average number of steps until game termination, and the distribution of outcomes across all sessions. This approach is particularly useful in highlighting variance in strategy performance information that analytical methods alone might not easily reveal.

3.2.3 Heuristics Explanation

Since part of our extension involves allowing the selection of bet type for each balance, we can use Heuristics find an optimal betting strategy based on some outcome we choose. To include bet decision making in our Markov Chain, rather than defining a full Markov Decision Process, we construct **strategy-specific transition matrices**. That is, for each type of bet, we define a matrix based solely on that bet's transition probabilities. A strategy is then defined as a specification of which bet to play at each balance. During simulation, as the balance evolves, we dynamically switch between these matrices based on the current balance and the bet selected by the strategy. For example, if the player starts at $2 and the strategy specifies using Bet B at $2, we use the Bet B matrix. If the result transitions the player to $3, and the strategy maps $3 to Bet C, we then use the Bet C matrix for future transitions. This structure allows us to simulate and compare complex deterministic strategies without implementing the full complexity of a multi-action MDP.

Now after configuring the model this way we have a heuristic problem we can solve. We can pick a random betting strategy (a betting strategy entails what bet type a player should play each balance) and evaluate it based on some performance metric that we choose (e.g. maybe a good betting strategy means it wins a lot in simulation) and run simulated annealing.

3.2.4 Model Assumptions

- The player's balance is restricted to integer values between $1 and $7.
- A player can start from any balance but we may focus on a particular starting balance for testing.
- Each betting action has a fixed probability of success or failure, with outcomes that adjust the balance by a specific amount.
- The game terminates upon reaching either Bankruptcy or Win, which are absorbing states.
- There is no degree of 'Win', so winning a very low odds bet at $7 does not pay off more than winning a high odds bet at $7 as the outcome are both wins
- The process is memoryless: the next balance depends only on the current state and selected bet.
- Strategies are defined as mappings from balance states to individual bets.
- Transition probabilities remain constant over time and are not affected by player history or external conditions.
- Monte Carlo simulations assume each game is independent and identically distributed.

Section 4: Results

4.1 Creating the Model

To explore the first modelling question, I constructed a Markov chain to simulate the behaviour of a player placing **random bets**. The model assumes that players choose from a set of available bets uniformly at random capturing the unpredictability of real-world betting behaviour.

Each state in the transition matrix represents a balance from $1 to $7, with absorbing states at $0 (Loss) and $8 (Win). The transition probabilities reflect real-world constraints: a win increases the balance, a loss decreases it, and available bets vary by current balance.

Figure 2 shows all the possible bets that can be played from each balance and the chances of winning a losing the bet.

Figure 2

		Balance After		Percentage %	
Balance	Bet Type	Win	Lose	Win	Lose
1	A	2	0	60	40
1	B	3	0	40	60
1	C	4	0	25	75
1	D	8	0	5	95
2	A	3	1	55	45
2	B	4	0	40	60
2	C	5	1	25	75
2	D	8	0	10	90
3	A	4	2	75	25
3	B	5	1	45	55
3	C	6	1	20	80
3	D	8	0	10	90
4	A	6	2	40	60
4	B	8	0	25	75
4	C	5	3	75	25
4	D	7	3	30	70
5	A	6	4	70	30
5	B	7	3	40	60
5	C	8	2	20	80
5	D	8	0	35	65
6	A	7	5	60	40
6	B	8	3	25	75
6	C	8	4	35	65
6	D	7	5	50	50
7	A	8	6	35	65
7	B	8	5	40	60
7	C	8	4	45	55
7	D	8	3	50	50

Based on these bets, we can convert this into a transition matrix, assuming that each bet is equally likely to be played (25% chance to play each one). Figure 3 shows the transition matrix.

Figure 3

	Win	7	6	5	4	3	2	1	Lose
Win	1	0	0	0	0	0	0	0	0
7	0.425	0	0.1625	0.15	0.1375	0.125	0	0	0
6	0.15	0.275	0	0.225	0.1625	0.1875	0	0	0
5	0.1375	0.1	0.175	0	0.075	0.15	0.2	0	0.1625
4	0.0625	0.075	0.1	0.1875	0	0.2375	0.15	0	0.1875
3	0.025	0	0.05	0.1125	0.1875	0	0.0625	0.3375	0.225
2	0.025	0	0	0.0625	0.1	0.1375	0	0.3	0.375
1	0.0125	0	0	0	0.0625	0.1	0.15	0	0.675
Lose	0	0	0	0	0	0	0	0	1

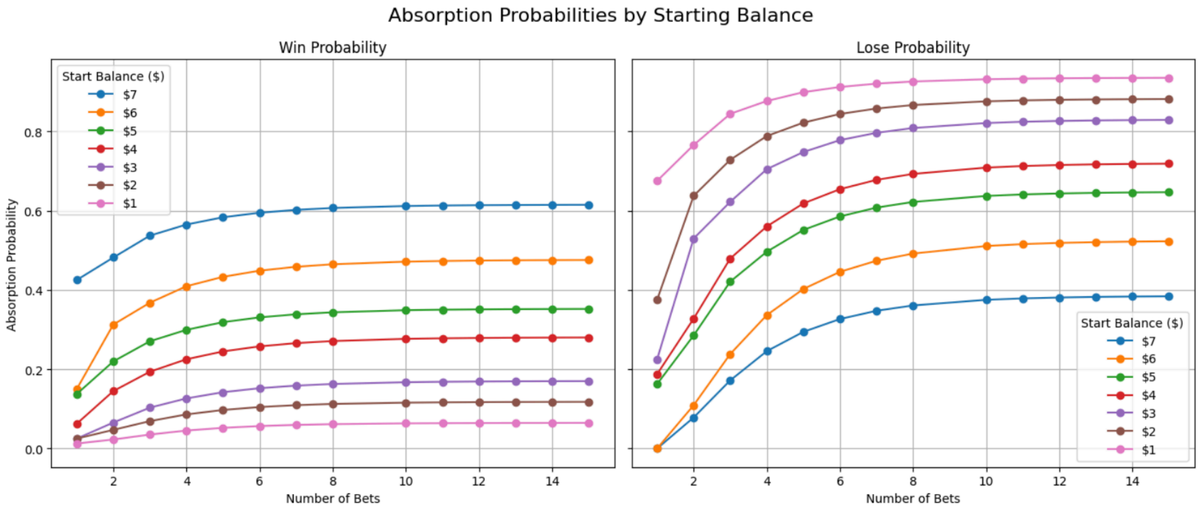
So given our premise where we pick a random bet let's try to answer this question. To answer the modelling question we can use Markov Chains properties and our transition matrix.

4.2 Absorption Probabilities: Matrix Exponentiation

To explore how players transition through the betting system, we first use the **Markov Chain property** of matrix exponentiation. By raising the transition matrix to the power of *n*, we estimate the **probability of being in a particular state (Win or Lose)** after *n* betting rounds, assuming the player starts with a specific balance ($1 to $7).

This allows us to investigate **how quickly players are absorbed**, whether they tend to win or lose, and how these outcomes change depending on starting balance.

Figure 4



4.2.1 Description of Chart

The figure shows the probability of reaching either the **Win** or **Lose** absorbing states over 15 bets, starting from balances $1 to $7. The left chart tracks **Win probabilities**, while the right shows **Lose probabilities**. Each curve corresponds to a different starting balance.

4.2.2 Behaviour of Individual Balances

Players starting at **lower balances** (such as $1 or $2) experience **sharp increases in their loss probability** very early on, with almost negligible improvement in win chances even after multiple steps. For example, the win probability for a $1 start remains below 10% even after 15 bets. In contrast, a player **starting at $7** has a noticeably **better trajectory**, reaching around 61% win probability by step 15. Nonetheless, even this highest balance sees **loss probabilities plateau** at nearly 40%, **reflecting significant risk despite initial advantage**. Mid-range balances like $4 and $5 show more gradual movement in both win and loss curves, sitting somewhere between the extremes.

4.2.3 Patterns

The curves for all balances tend to stabilise between steps 10 and 15, indicating that by this point nearly all players have been absorbed into one of the two final states. The win probability curves rise more slowly and plateau lower compared to the sharper and more dominant rise of the loss curves. This asymmetry points to the system's structural bias, more possible betting paths leading to the Losing rather than Winning, especially from lower balances. As the starting balance increases, win probability improves, but at a diminishing rate suggesting non-linear returns on higher balances.

4.2.4 Insights and Links to the Modelling Question

These results directly address the modelling question: *on average, do players win or lose, and how quickly?* Under random betting, players are significantly more likely to lose, even at the most favourable starting balance ($7), the win rate only reaches 61%. This shows that initial advantage is not enough to guarantee success.

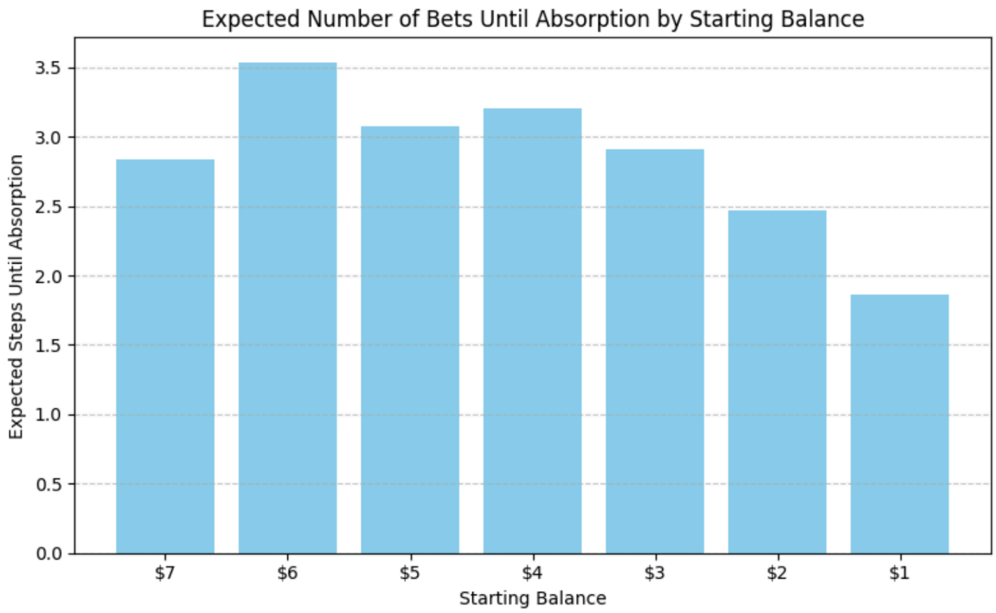
The loss-heavy outcome is largely due to the model's structure: there are more paths leading to the **Lose** state, especially from lower balances. As the by around step 8, we can see the probabilities plateauing, which gives us an idea of how long typical betting games go for until winning or losing.

4.3 Expected Time till Absorption

To get a better understanding of how quickly players win or lose, we can use: $N = (I - Q)^{-1}$

This matrix gives the expected number of times the process visits transient states before absorption, i.e., before a player wins or loses. By multiplying N by a column vector of 1s, we obtain the **expected number of steps until absorption** from each starting balance.

Figure 5



4.3.1 Description of Chart

The bar chart in figure 5 displays the average number of bets it takes for a player to be absorbed (i.e. reach the Win or Lose state), starting from each balance between $1 and $7. We observe a **non-linear relationship** between starting balance and absorption time. The values range between ~2.5 to ~3.5 steps (even shorter than the what we estimated in the previous Section 4.2.4)

4.3.2 How Do Individual Balances Behave?

Lower balances (e.g. $1, $2) result in faster absorption, typically under 3 steps. These players are **closer to the Lose state** and have **fewer transition options**, leading to shorter games. **Higher balances** (like $6 and $7) result in **longer games**. These players are further from both absorption states and have more available transitions, increasing the time spent in transient states. **Notably**, $6 has the **longest expected absorption time**, even longer than $7. This suggests that from $6, there are more "sideways" transitions (e.g. to $5 or back to $7), resulting in **extended gameplay due to back-and-forth movement**.

4.3.3 Patterns

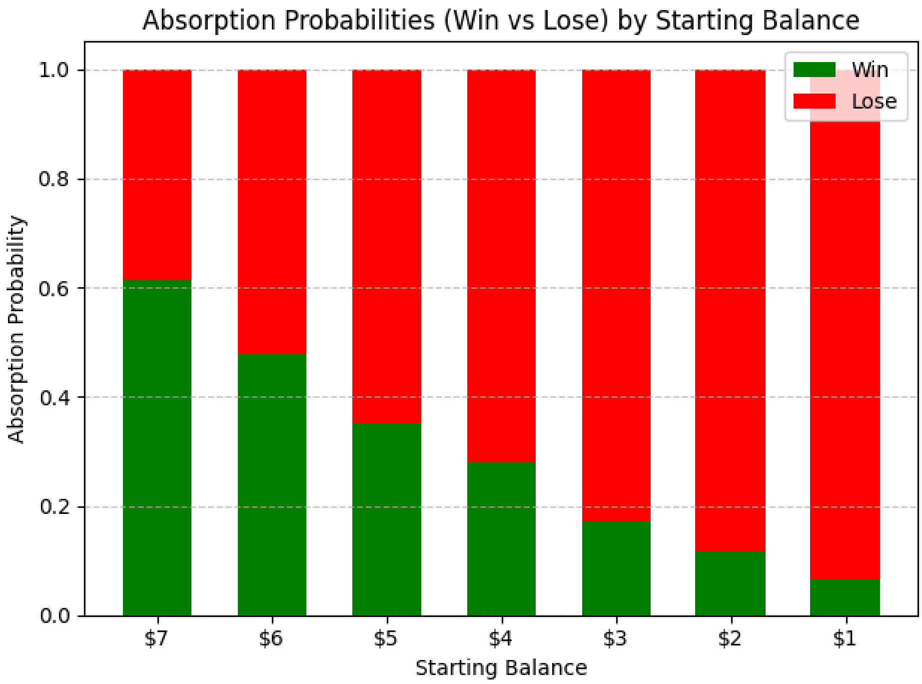
There's a **clear upward trend from low to high balances**, with **$6 showing an unexpected peak**. This suggests **players starting at high balances do not immediately win** they often enter loops or descend into other balances before eventually being absorbed.

4.3.4 Connections to question

Relating back to our question 'how quickly do people win or lose and does starting balances influence this?', it's clear **Low balances** lead to **quicker absorption (winning or losing), while** h**igher balances** take longer to win or lose, caused by more available transitions The spike at **$6** implies greater volatility which tells us about the nature of how players approach winning or losing from higher balances, where they oscillate (not always) between balances rather than progressing linearly towards winning or losing.

4.4 Likelihood of Winning Versus Losing

Figure 6



4.4.1 Description of Chart

Figure 6 shows the **probability of being absorbed** into either the **Win** (green) or **Lose** (red) state for each starting balance from $1 to $7. These probabilities are computed using the matrix equation $B = NR$, where B_{ij} gives the chance of being absorbed in state j given the chain started in state i. Each stacked bar sums to 1, indicating certainty of absorption, but the proportion of wins vs losses varies substantially by starting balance.

4.4.2 How Do Individual Balances Behave?

Only starting at $7 yields a win rate above 50% (specifically ~61%). This is expected, as it is only one transition away from the absorbing Win state and has limited exposure to the Lose state due to fewer downward transitions. **Starting from $6 or lower drastically reduces win probabilities**. For example, from $6, the chance of reaching the Win state drops to ~46%, and from $1 it falls to under 10%. These balances have a greater number of downward transitions, increasing the likelihood of hitting the Lose state before a win can be reached. The **probability of losing exceeds 50% for all balances except $7**, indicating that loss is the more probable outcome across the board in this random betting scenario.

4.4.3 Pattern Across All Balances

There is a **clear increase in win probability** as starting balance increases. However, the relationship is **non-linear**: the biggest drop-off occurs between $7 and $6, while changes between $3, $2, and $1 are less steep. This reflects how small increases in balance **do not uniformly improve win chances**, due to structural features of the transition matrix (e.g. frequent paths toward loss).

4.4.4 Connection to Modelling Question

Relating back to the question *On average do players win or lose, and how does starting balance influence this?* Under random betting, **players are far more likely to lose**, especially from lower balances. Even starting at $7 just one step from the Win state only gives a 61% chance of winning, with probabilities dropping steeply as the starting balance decreases. This shows that **starting balance strongly influences win likelihood**, but even high balances cannot guarantee success.

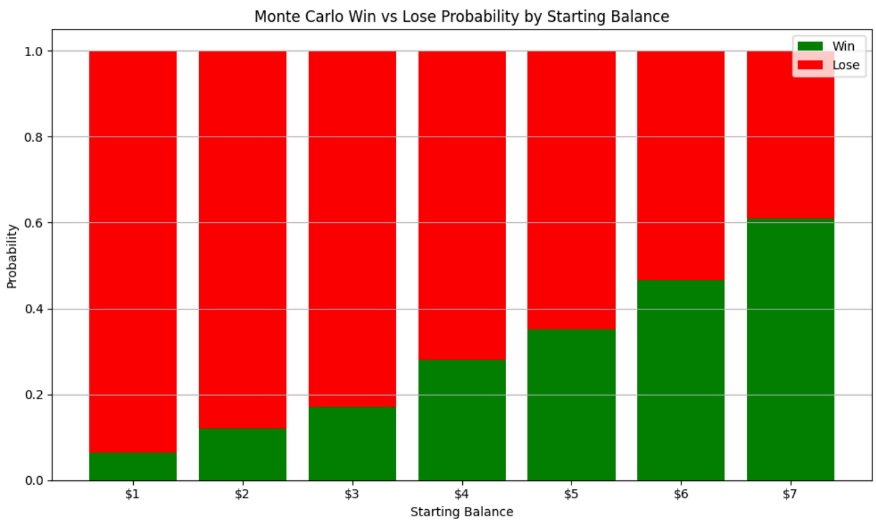
The model's structure amplifies this effect, while win paths require consistent upward transitions, loss can occur through many downward routes. This structural unfairness makes the model **inherently loss-biased**, especially under random action. Thus, random betting rarely overcomes positional disadvantage and **players starting far from the Win state are overwhelmingly absorbed into loss**, both frequently and quickly.

4.5 Monte Carlo Simulation Results

Monte Carlo simulation provides a complementary approach to Markov Chains by modelling many random games and tracking observed outcomes. We can use this to confirm some of our results from the theory application Markov Properties and also answer the spread part of our modelling question: *On average do players win or lose and **what is the spread of outcomes.*** We'll analyse overall absorption probabilities (win vs loss) and distribution of steps until absorption (game duration)

4.5.1 Simulated Absorption Probabilities (Win vs Lose)

Figure 7



4.5.1.1 Description of Chart

Figure 7 shows the probability of winning (green) versus losing (red) based on **1000 Monte Carlo simulations** per starting balance from $1 to $7. Players randomly choose bets from the available options at each state.

4.5.1.2 How Do Individual Balances Behave?

Low balances (e.g. $1, $2) show an overwhelming skew towards loss, with win rates under 10%. **High balances** (especially $7) give players the best shot at winning, with a win rate above 60%. Intermediate balances show a roughly linear trend between these two extremes.

4.5.1.3 Patterns Across All Balances

There is a **consistent upward trend** in win probability from $1 to $7, mirroring the behaviour predicted by the Markov model. Lower balances lose more often not just because of proximity to the lose state, but also due to the lack of upward betting options.

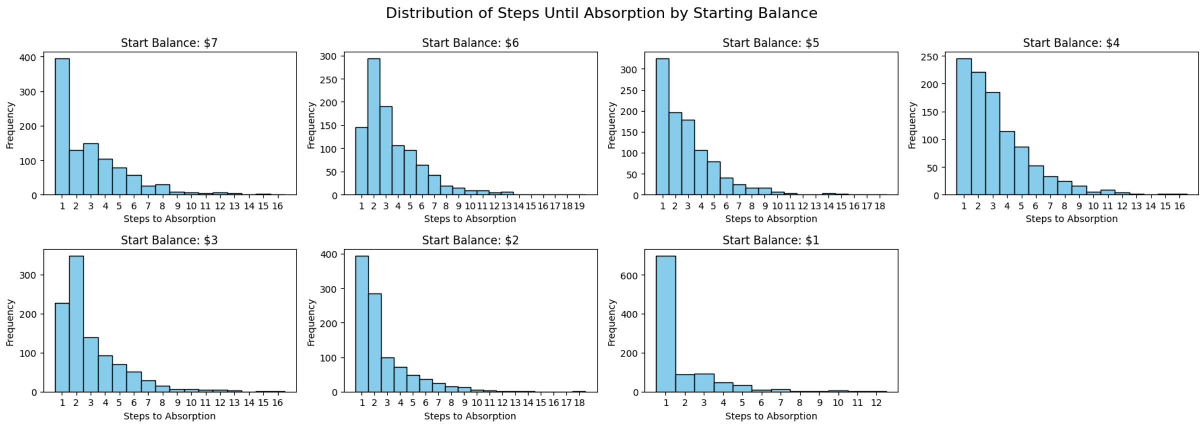
4.5.1.4 Connection to the Modelling Question

This simulation directly reinforces the modelling question's focus: *on average, do players win or lose, and how does starting balance influence this?* The answer remains clear, **players are more likely to lose**, especially when starting from lower balances. Even under repeated simulation, **losses dominate**, replicating the structural biases we saw in the transition matrix.

The chart also helps address *what is the spread*: we observe a **consistent loss-heavy distribution across all starting balances**, with win probabilities only rising meaningfully when starting near $7. Thus, **starting balance remains a key driver**, and **random betting fails to overcome the model's inherent bias** towards loss, regardless of run volume.

4.5.2 Simulated Time to Absorption (Steps Until Win/Loss)

Figure 8



4.5.2.1 Description of Chart

Figure 8 shows the distribution of the **number of steps taken before being absorbed** (either in Win or Lose) for each starting balance. Each histogram is generated from 1000 simulations.

4.5.2.2 How Do Individual Balances Behave?

$1–$3 balances show strong left-skew: most players are absorbed within 1–4 steps. **$6–$7 balances** have wider distributions: games often take 5–10+ steps before absorption.

4.5.2.3 Patterns Across All Balances

Lower balances are absorbed **quickly and consistently**, often via loss. In contrast, higher balances lead to **longer and more variable** gameplay.

There is also a subtle difference between $6 and $7: **$6 leads to longer games** on average, consistent with earlier findings from the expected steps chart.

4.5.2.4 **Connection to the Modelling Question**

This simulation directly answers the spread component of the modelling question: *what is the spread of outcomes?* The histograms show that **outcomes from low starting balances ($1–$3) are tightly concentrated**, typically ending within just **1 to 4 steps**, reflecting rapid and predictable exits, usually via loss.

In contrast, **higher starting balances ($6–$7) show a much wider spread**, with outcomes ranging from **3 to 15+ steps**. This greater variability suggests more complex game paths and higher uncertainty in how long a game might take before resolving.

Thus, **the spread of outcomes is strongly dependent on starting balance** players starting lower tend to exit quickly, while those starting higher experience longer and more varied trajectories. This pattern highlights the role of branching structure in the transition matrices and reinforces the idea that **starting position not only affects outcomes, but also the duration and volatility of gameplay**.

4.6 Strategy Selection Introduced

So far, our results have been limited by the assumption of random betting, each player selected from all available bets at random, regardless of their balance. However, this doesn't reflect how real players might behave. What if players **followed a specific betting strategy** instead?

To explore this, we now allow players to **select one fixed betting option per balance.** For example, from balance $4, a player could always choose Bet C. This lets us model more realistic, strategic behaviours and evaluate how they affect outcomes.

Figure 9

Bet A

Bet A	Win	$7	$6	$5	$4	$3	$2	$1	Lose
Win	1	0	0	0	0	0	0	0	0
$7	0.35	0	0.65	0	0	0	0	0	0
$6	0	0.6	0	0.4	0	0	0	0	0
$5	0	0	0.7	0	0.3	0	0	0	0
$4	0	0	0.4	0	0	0	0.6	0	0
$3	0	0	0	0	0.75	0	0.25	0	0
$2	0	0	0	0	0	0.55	0	0.45	0
$1	0	0	0	0	0	0	0.6	0	0.4
Lose	0	0	0	0	0	0	0	0	1

Bet B

Bet B	Win	$7	$6	$5	$4	$3	$2	$1	Lose
Win	1	0	0	0	0	0	0	0	0
$7	0.4	0	0	0.6	0	0	0	0	0
$6	0.25	0	0	0	0	0.75	0	0	0
$5	0	0.4	0	0	0	0.6	0	0	0
$4	0.25	0	0	0	0	0	0	0	0.75
$3	0	0	0	0.45	0	0	0	0.55	0
$2	0	0	0	0	0.4	0	0	0	0.6
$1	0	0	0	0	0	0.4	0	0	0.6
Lose	0	0	0	0	0	0	0	0	1

Bet C	Win	$7	$6	$5	$4	$3	$2	$1	Lose
Win	1	0	0	0	0	0	0	0	0
$7	0.45	0	0	0	0.55	0	0	0	0
$6	0.35	0	0	0	0.65	0	0	0	0
$5	0.2	0	0	0	0	0	0.8	0	0
$4	0	0	0	0.75	0	0.25	0	0	0
$3	0	0	0.2	0	0	0	0	0.8	0
$2	0	0	0	0.25	0	0	0	0.75	0
$1	0	0	0	0	0.25	0	0	0	0.75
Lose	0	0	0	0	0	0	0	0	1

Bet C

Bet D	Win	$7	$6	$5	$4	$3	$2	$1	Lose
Win	1	0	0	0	0	0	0	0	0
$7	0.5	0	0	0	0	0.5	0	0	0
$6	0	0.5	0	0.5	0	0	0	0	0
$5	0.35	0	0	0	0	0	0	0	0.65
$4	0	0.3	0	0	0	0.7	0	0	0
$3	0.1	0	0	0	0	0	0	0	0.9
$2	0.1	0	0	0	0	0	0	0	0.9
$1	0.05	0	0	0	0	0	0	0	0.95
Lose	0	0	0	0	0	0	0	0	1

Bet D

4.6.1 Description of Strategy Framework

Figure 9 displays the **transition matrices for four individual bets (A, B, C, D)**. These matrices represent the probability of moving between balances after placing a specific bet — for example, winning Bet A from $5 moves you to $6 with 60% chance, while losing moves you to $4.

To simulate strategy-based play, we assign **a single bet to each balance level**, forming a **full strategy** across balances $1 to $7. In this example, we simulate **a randomly selected strategy** — each balance picks one of the four matrices at random and follows it consistently during the simulation.

4.6.2 How Do Random Strategies Perform?

Figure 10 shows the result of simulating a randomly selected strategy (Figure 11) **1000 times from each starting balance**. The chart tracks the **probability of ending up in the Win (green) or Lose (red) absorbing states**.

Figure 10

Figure 11

Balance	$7	$6	$5	$4	$3	$2	$1
Strategy	B	C	B	D	D	A	A

Figure 11 tell us what bet will be played at each balance.

From Figure 11 we see a pattern similar to the random betting case, l**ower balances** like $1 and $2 continue to show high loss rates. **Higher balances**, particularly $7, still achieve over 60% win probability. Intermediate balances vary depending on which bet they were randomly assigned.

4.7 Strategy Optimisation Using Simulated Annealing

Now that we've introduced betting strategies, we ask: Can we use a betting strategy to increase our odds of winning? If so what is a good (or locally optimum) betting strategy where a betting strategy is a optimal bet for each balance. To answer this, we apply **simulated annealing**, where we searches the strategy space while allowing some random exploration to avoid local optima.

4.7.1 Description of Method

We attempt to maximise a player's final balance after **10 betting rounds**, assuming they start at **balance $4**. The win state is encoded as balance 8, and the lose state as balance 0. The strategy is scored based on the **average final balance** across 100 simulated games, encouraging short-term success.

Key simulation settings:

- 100 perturbations per annealing step
- Initial temperature of 50 (for broader early exploration)
- Cooling factor of 0.9

Figure 12 shows the best strategy discovered via this process:

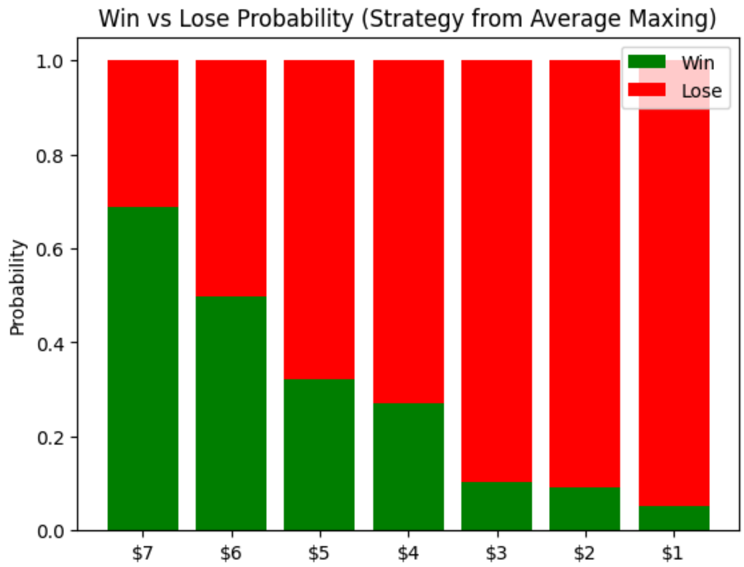
Figure 12

Balance	$7	$6	$5	$4	$3	$2	$1
Strategy	A	D	B	A	D	B	D

Which yielded a maximised average final state of 6.56

The strategy was optimised **only for starting at $4**, but we test it across **all balances** to see if the optimisation generalises.

Figure 13



4.7.2 Behaviour of the Optimised Strategy

Figure 13 shows the win/lose probabilities from simulating 1000 games per starting balance using this strategy. **Higher balances** ($6 and $7) achieve noticeably improved win rates compared to the random strategy. **Balance $4**, the optimised target, shows **slight improvement**, though not dramatic. Lower balances still yield mostly losses this was expected, since the optimisation was not focused on them.

4.7.3 Patterns and Trends

We observe that **Improvements are localised** to the region near the optimisation target. $6 and $7 benefit from the same strategic selections, suggesting those strategies provide beneficial paths even when starting further away. **No substantial boost at lower balances**, which reinforces the idea that the loss-heavy nature of the system is difficult to overcome when starting far from the win state. The strategy's win probability at $6 is among the highest so far indicating potential **indirect gains** from a locally optimised strategy.

4.7.4 Insights and Links to Modelling Question

Relating to our question, can strategy selection improve win probability? Yes but based on these results, only to an extent. This experiment shows that even when optimising from a specific balance, **nearby balances can also benefit**, especially those that share similar paths to the win state.

4.8 Win Optimisation using Simulated Annealing

4.8.1 Description of Method

Instead of maximising average balance like before, we now **directly maximise the number of wins** from a fixed starting balance. Specifically, we:

- Start from **balance $4**
- Run full simulations until **absorption** (i.e. win or lose)
- Track the **total number of wins** out of 100 games
- Use **simulated annealing** to find the strategy that produces the most wins

To reduce runtime, we:

- Lowered the **initial temperature** to speed up convergence
- Focused on a **single target balance ($4)** again, then tested results across all balances

Key simulation settings:

- 100 perturbations per annealing step
- Initial temperature of 10
- Cooling factor of 0.9

The best strategy found is shown in Figure 14:

Figure 14

Balance	$7	$6	$5	$4	$3	$2	$1
Strategy	B	A	A	C	A	A	A

It achieved a **max win count of 96 out of 100** from balance $4 which seems like a significant improvement compared to the previous attempt already.›

Figure 15

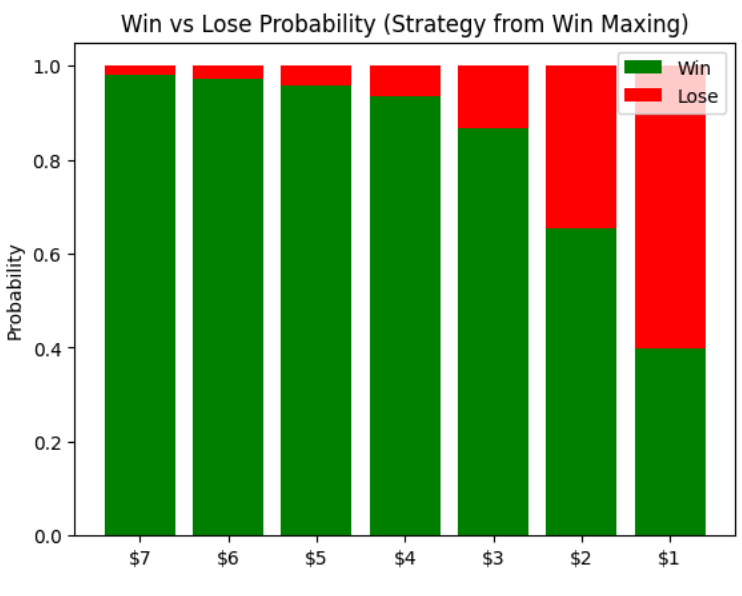
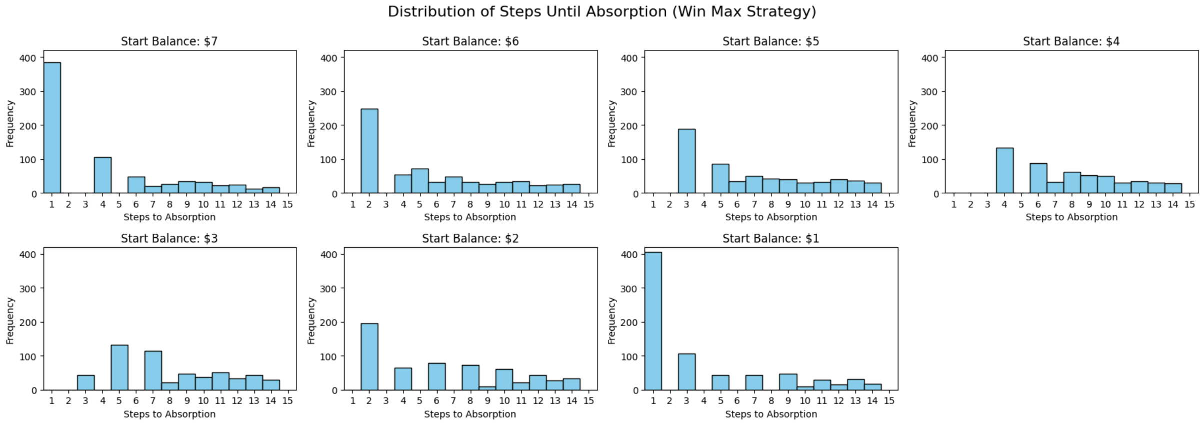


Figure 16



4.8.2 Description of Charts

Figure 15 shows the **final win vs lose absorption probabilities** from each starting balance ($1–$7) under the **optimised strategy** trained using simulated annealing from the $4 balance. The goal was to maximise wins starting from $4, but we test its performance across all balances.

Figure 16 complements this by plotting the **distribution of steps until absorption** (either Win or Lose) for each starting balance under the same strategy. These histograms give insight into **how quickly** outcomes are reached, not just **how likely** they are.

4.8.3 How Do Individual Balances Behave?

- **Higher balances ($6–$7)**:
  - In Figure 15, they now show **over 90% win probabilities**, approaching certainty.
  - Figure 16 shows these states also absorb relatively quickly, with a **majority of wins in under 5–6 steps**, indicating **fast and consistent outcomes** under the strategy.
- **Mid balances ($4–$5)**:
  - Figure 15: The win rate at $4 (the training target) **jumps to ~96%**, up from ~45% in earlier randomised strategies.
  - Figure 16: These balances show a **broader spread** in absorption time (steps range up to 15+), suggesting **slightly more volatility**, but still largely successful.
- **Lower balances ($1–$2)**:
  - In Figure 15, win probabilities are much lower (~50% for $2, ~40% for $1), despite the strategy.
  - Figure 16 shows very **fast absorption, t**ypically in **1–4 steps** indicating outcomes are determined quickly, usually as a **loss due to proximity to the absorbing Lose state.**

4.8.4 Patterns Across Charts

The win probability curve in Figure 15 **follows a non-linear pattern** rapid increase between $3 and $6, with near-plateaus at the top. The histogram patterns in Figure 16 match this, higher balances not only win more often but tend to **reach absorption in fewer steps**, particularly for $6 and $7. Lower balances cluster around shorter games, with a **sharp left-skew** toward early losses. Importantly, this strategy, though trained for $4, **generalises** well to all other balances This implies some strategies are **robust across nearby states**, benefiting from the shared transition structure of the Markov model.

4.8.5 Insights and Links to Modelling Question

This analysis directly addresses the core modelling question: Can we use a betting strategy to increase our odds of winning? The answer is clearly yes. **Strategic betting significantly improves win outcomes**, especially from mid and high balances. This contrasts with random or fixed strategies, which capped win rates at ~60%. **The improvement at**

$4 demonstrates that strategies can exploit favourable paths not visible to random betting. **Limitations persist for low balances**, where the influence of proximity to the Lose state dominates even strategic play.

In real-world analogies, this supports the idea that success isn't just about how much balance a player starts with, **it's how they bet that counts**. Figure 14 is an example of good betting strategy that the modelling question asks for but is not necessarily the best available strategy.

4.9 Overall Reflections and Real-World Relevance

4.9.1 Limitations of the Approach

It's important to note that **simulated annealing does not guarantee a globally optimal solution**. While this strategy performs very well, other strategies may yield even better results particularly if we adjusted our objective function or annealing parameters.

4.9.2 Theoretical Relevance: EV Betting

In the real world, a common technique is **Positive Expected Value (EV) betting**, where bets are chosen based on their long-term payoff:

$$\text{EV} = (\text{Amount Won} \times \text{Chance of Winning}) - (\text{Amount Lost} \times \text{Chance of Losing})$$

Placing only bets with positive EV is expected to yield profit over time. Though our model doesn't explicitly calculate EV, it's possible that **simulated annealing implicitly favours strategies with high EV** since such strategies naturally lead to more wins. This could explain the observed effectiveness. A more advanced system could integrate EV directly into the optimisation logic.

4.9.4 Broader Implications

This work demonstrates how even **simple heuristic search** (like simulated annealing) can discover **effective real-world betting strategies** from scratch. If this were extended with more sophisticated models such as reinforcement learning or dynamic programming we could unlock even **more powerful strategies** with better performance and adaptability.

This suggests that even in seemingly luck-based systems like gambling, **strategy matters**. If players can estimate their current position and apply adaptive bets based on that state, they can dramatically improve their outcomes a lesson that could generalise beyond gambling to decision-making under uncertainty more broadly.

4.9.5 Final Summary of Answers to Modelling Questions Implications

1. **Do players win or lose on average?:** Players **lose more often**, especially at **lower balances**. Only **$7** gives >50% chance of winning without strategy.
2. **How quickly do players reach an outcome?:** Most outcomes occur within 1**–8 bets**. **Lower balances absorb faster** (usually as losses), while higher balances take longer due to more paths. The average bets for each balance is approximately 2 to 3
3. **How does starting balance influence results?:** Higher balances give **more options** and **higher win rates**. Lower balances are close to the Lose state and have limited recovery paths.
4. **Can strategy improve win rates?: Yes.** Strategic betting (via simulated annealing) can **double win rates** (e.g., from ~45% to **96% at $4**) and push **$6–$7 to >90%**.

Section 5: List of Algorithms and Concepts

Markov Chains & Stochastic Modelling

- **Discrete-Time Markov Chain (DTMC):** Modelled game states as a Markov process with 9 states (Win, Lose, $1–$7), where transitions depend only on the current balance and chosen bet.
- **Transition Matrices:** Custom matrices (A–D) represent the probability of moving between balances under different betting strategies.
- **Absorbing States:** Win and Lose are absorbing; once reached, players cannot leave. This structure allows calculation of absorption probabilities.
- **Fundamental Matrix** $N = (I - Q)^{-1}$: Used to compute expected number of steps in transient states before absorption (Section 4.2.2, 4.5.2).
- **Absorption Probability via Fundamental Matrix (B = NR): C**alculates the **probability of being absorbed** in a particular absorbing state, given a transient starting state. Used in Section 4.4 to compute the probability of ending in Win vs Lose given a starting balance.

Monte Carlo Simulation

- **Random Walk Simulations** Repeated trials of games using random and strategic betting paths to empirically estimate win/loss probabilities and number of steps to absorption.
- **Step Distribution Analysis** Histogram-based visualisations captured the spread of steps until absorption (Section 4.5.2).

Strategy Optimisation (Heuristics)

- **Simulated Annealing:** Heuristic search method used to find a locally optimal strategy by iteratively tweaking bet selections for each balance (Section 4.7–4.8). Parameters included, Temperature-based control for exploration, Perturbation of strategies per iteration, Score function based on win count or final balance

Decision Strategy Design

- **Betting Matrices (A, B, C, D):** Represented four different bet behaviours with varied risk profiles.
- **Strategy Representation:** Strategies were encoded as mappings from balance ($1–$7) to specific bet matrices.

Evaluation Metrics

- **Win vs Lose Absorption Probability:** Calculated via both analytical methods (Markov chain) and simulations.
- **Expected Steps Until Absorption:** Measured to assess outcome spread and game length.
- **Spread and Skew Analysis:** Examined distribution skewness to reveal volatility at different starting balances.

Key Modelling Concepts

- **Structural Bias:** Showed how even with fair-looking transitions, the system favours loss due to multiple paths toward the Lose state.
- **Initial Conditions Sensitivity:** Demonstrated how early game position (starting balance) strongly dictates long-term outcomes.
- **Heuristics vs Global Optimisation:** Explored the limits of local optimisation approaches like simulated annealing and their impact across adjacent states.