Contents

1	Discrete and Engineering Mathematics	[illegible]
2.	Theory of Computation	105
3	Digital Logic	1[illegible]
4.	Computer Organization & Architecture	2[illegible]
5.	Programming & Data Structures	[illegible]
6.	Algorithms	240
7.	Compiler Design	1[illegible]
8.	Operating System	4[illegible]
9.	Databases	478
10.	Computer Networks	520
11.	General Aptitude	572
12.	Conventional Questions & Solutions	6[illegible]
		[illegible]

Unit - I

Discrete and Engineering Mathematics

■ **Contents**

Sl.	Topic	Page No.
1.	Mathematical Logic	3
2.	Set Theory and Algebra	17
3.	Combinatorics	42
4.	Graph Theory	54
5.	Probability	68
6.	Linear Algebra	82
7.	Calculus	97

UNIT I Discrete and Engineering Mathematics

Syllabus :

Mathematical Logic: Propositional and first order logic.

Set Theory & Algebra: Sets, relations, functions, partial orders and lattices, Groups.

Combinatorics: Counting, recurrence relations, generating functions.

Graph Theory: Connectivity, matching, coloring.

Probability: Random variables, uniform, normal, exponential, poisson and binomial distributions, mean, median, mode and standard deviation, conditional probability and Bayes' theorem.

Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability, maxima and minima, Mean value theorems, Definite Integrals.

Analysis of Previous GATE Papers

Exam Year	1 Mark Questions	2 Marks Questions	3 Marks Questions	Total Marks
1990	1	2	–	5
1991	–	–	–	2
1992	3	–	3	25
1993	0	–	–	6
1994	4	5	–	14
1995	4	5	4	19
1996	7	7	–	21
1997	4	7	–	10
1998	7	7	1	28
1999	4	5	2	24
2000	2	5	2	22
2001	4	2	–	12
2002	3	4	2	20
2003	3	16	–	35
2004	5	11	–	27
2005	0	7	–	25
2006	3	0	–	23

Exam Year	1 Mark Questions	2 Marks Questions	Total Marks
2007	4	5	22
2008	4	10	24
2009	4	8	16
2010	0	0	24
2011	–	5	19
2012	6	5	16
2013	6	3	12
2014 (Set-1)	8	9	24
2014 (Set-2)	5	8	21
2015 (Set-1)	7	8	23
2015 (Set-2)	5	8	21
2015 (Set-3)	7	8	19
2016 (Set-1)	0	6	13
2016 (Set-2)	6	3	12

1 Mathematical Logic

1.1 Consider which of the following well-formed formulas are valid:

(a) $(P \to Q) \wedge (Q \to R) \to (P \to R)$

(b) $(P \to Q) \to (\neg P \to \neg Q)$

(c) $(P \wedge (P \vee (Q \wedge R))) \to R$

(d) $(P \to (Q \to R)) \to (P \to Q) \to (P \to R)$.

[1998 : 2 Marks]

1.2 Which of the following predicate calculus statements is/are valid.

(a) $(\forall x)[P(x) \to Q(x)] \to [(\forall x)P(x) \to (\forall x)Q(x)]$

(b) $(\exists x)[P(x) \to Q(x)] \to [(\exists x)P(x) \to (\exists x)Q(x)]$

(c) $(\forall x)[P(x) \to Q(x)] \to [(\forall x)P(x) \to (\forall x)Q(x)]$

(d) $(\exists x)[P(x) \to Q(x)] \to [(\forall x)P(x) \to (\exists x)Q(x)]$

[1998 : 1 Mark]

1.3 Which of the following pairs are logically equivalent?

(a) $p \to q$ and $\neg p \to \neg q$

(b) $p \to q$ and $\neg q \to \neg p$

(c) $p \to (q \to r)$ and $(p \to q) \to r$

(d) $p \to (q \to r)$ and $(p \to q) \to (p \to r)$

[1997 : 1 Mark]

1.4 The proposition $(p \wedge q) \to (p \to q)$ is

(a) a contradiction (b) $(p \to q) \to (p \wedge q)$

(c) $(p \to q) \to (p \vee q)$ (d) a tautology

[1991 : 1 Mark]

1.5 Let p and q be propositions. Using only one truth table, show that $p \to q$ is not equivalent to $p \to \neg q$ if p is true or false.

[1994 : 3 Marks]

1.6 If the propositions $p \to q$ is false, then the truth value of the proposition $\neg p \vee q$ is

(a) true (b) multiple values

(c) false (d) cannot be determined

[1996 : 2 Marks]

1.7 Which one of the following is false? Read as AND, $+$ as OR, $\bar{x}$ as NOT x and $\to$ as one way implication

(a) $(x + y) \to \bar{x} \to y$

(b) $(x \to y) \to (x + y) \to y$

(c) $x \to (x + y)$

(d) $(x + y) \to (x \to y) \to y$

[1994 : 2 Marks]

1.8 Let a, b, c, d be propositions. Assume that the equivalence $a \to (b \to c) \equiv (a \wedge b) \to c$ holds. Then the truth value of the formula $(a \to (b \to c)) \to ((a \to b) \to (a \to c))$ is always

(a) True

(b) False

(c) Same as the truth value of a

(d) Same as the truth value of c

[2000 : 2 Marks]

1.9 Which is the converse of the following assertion?

I walk if it is raining.

(a) Unless it is raining I walk.

(b) If I walk it is raining.

(c) If it is not raining then I walk.

(d) I walk only if it is raining.

[2001 : 1 Mark]

1.10 Consider two well-formed formulas in propositional logic

$F_1 : P \to \neg P$

$F_2 : (P \to \neg P) \to (\neg P \to P)$

Which of the following statement is correct?

(a) F_1 is satisfiable, F_2 is valid

(b) F_1 is unsatisfiable, F_2 is satisfiable

(c) F_1 is unsatisfiable, F_2 is valid

(d) F_1 and F_2 are both satisfiable

[2001 : 1 Mark]

1.11 If X and Y denote 2-bit inputs to a logic gate and the following is the truth table for the gate (X is the most significant bit)

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

(a) $Z = X \cdot Y$ (b) $Z = X + Y$

(c) $Z = X \cdot \bar{Y}$ (d) $Z = \bar{X} \cdot Y$

[2003 : 1 Mark]

1.12 Which of the following is a valid first order formula? (Here x and y are first order variables and P is a unary predicate)

(a) $(\forall x)(\exists y)[P(x) \to (\forall y)P(y)] \to (\forall x)[P(x) \to P(x)]$

(b) $(\forall x)(\exists y)[P(x) \to P(y)] \to (\exists x)[P(x) \to P(x)]$

(c) $(\forall x)(\exists y)[P(x) \vee P(y)] \to (\exists x)(\forall y)[P(x) \to P(y)]$

(d) $(\forall x)(\exists y)[P(x) \to P(y)] \to (\exists x)(\forall y)[P(x) \to P(y)]$

[2003 : 2 Marks]

1.13 Consider the following [illegible] and the two interpretations I_1 and I_2.

[illegible]

I_1: [illegible]

P_1: [illegible]

Q_1: [illegible]

I_2: [illegible]

Which of the following statements is true?

(a) [illegible] satisfies [illegible]

(b) [illegible] satisfies [illegible]

(c) Neither [illegible] satisfies [illegible]

(d) Both [illegible] satisfy [illegible]

[2006 : 2 Marks]

1.14 The following resolution rule is used in logic programming. Derive clause [illegible] from clauses [illegible].

Which of the following statements is valid regarding rule R1 [illegible]?

(a) [illegible]

(b) [illegible]

(c) [illegible] is satisfiable and only [illegible]

(d) [illegible] is satisfiable [illegible] P and Q are mutually exclusive

[2006 : 2 Marks]

1.15 Identify the correct translation in logic from the following statement [illegible]

Note: [illegible] is a [illegible] variable [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2006 : 1 Mark]

1.16 Let [illegible] and [illegible] be predicates. Let [illegible] be a constant. Let [illegible] be the following statement.

[illegible]

Which of the following is equivalent to [illegible]?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[IT-2005 : 1 Mark]

1.17 Let [illegible] be propositional statements. Consider the following statements:

P: [illegible] → (¬r → q)

Q: [illegible] → r

R: [illegible] → r

S: [illegible] → q

Which of the above statements are valid?

(a) P and Q only (b) P and R only

(c) P and S only (d) P, Q, R and S

[2001 : 2 Marks]

1.18 Consider the following propositional statement:

$(P \to Q) \vee (Q \to R) \to (P \to Q) \to (P \to R)$

(a) not satisfiable (b) satisfiable

(c) valid (d) [illegible]

(e) Contradiction

[2004 : 2 Marks]

1.19 Let P, Q and R be three atomic propositional statements. Let X denote $(P \to Q) \to R$ and Y denote $(P \to R) \to (Q \to R)$. Which one of the following is a tautology?

(a) X → Y (b) Y → X

(c) X ↔ Y (d) X ∨ Y → Y

[2005 : 2 Marks]

1.20 What is the first order predicate calculus statement equivalent to the following? Every student who is [illegible] for some course.

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2003 : 2 Marks]

1.21 Let P(x) and Q(x) be arbitrary predicates. Which of the following statements is always TRUE?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[IT-2005 : 2 Marks]

1.22 Consider the following first order logic formula in which R is a binary relation symbol.

[illegible]

The [illegible] is

satisfiable and valid.

[illegible]

[IES 2006 : 2 Marks]

1.23 [illegible]

[2005 : 2 Marks]

1.24 [illegible]

[2005 : 2 Marks]

1.25 [illegible]

[2006 : 2 Marks]

1.26 [illegible]

A	B	A ∨ B
True	True	True
True	False	True
False	True	True
False	False	False

[illegible] (DCT) [illegible] with higher precedence. [illegible] Which one of the following [illegible]

[2006 : 2 Marks]

1.27 Let Graph(x) [illegible] is a graph. Let Connected(x) be a predicate which denotes that x is connected. Which of the following first order logic sentences [illegible] represents the statement "Not every graph is connected"?

[illegible]

[2009 : 2 Marks]

1.27 Which of the following [illegible] TRUE [illegible] in all [illegible] Universe [illegible]?

[illegible]

[2007 : 2 Marks]

1.28 Which [illegible] first order [illegible] valid?

[illegible]

[IT-2007 : 2 Marks]

1.29 [illegible]

[illegible]

[2008 : 1 Mark]

1.30 Which of the following [illegible] is [illegible] valid? [illegible] variable and [illegible] formula with [illegible] variable

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[IT-2006 : 5 Marks]

1.31 Which of the following is the negation of

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[IT-2006 : 2 Marks]

1.32 [illegible] the following expressions are equivalent?

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

(a) Only 1 and 2 (b) Only [illegible]
(c) Only 1, 2 and 3 (d) All of the above

[2006 : 2 Marks]

1.33 Which one of the following is the most appropriate logical formula to represent the statement:

"Hailstones and icicles are made of water"?

The following predicates are used:

[illegible]

S(x): x is a silver ornament

F(x): x is made of water

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2008 : 2 Marks]

1.34 The binary operator [illegible] is defined as:

P	Q	P [illegible] Q
T	T	T
T	F	T
F	T	T
F	F	T

Which one of the following is logically equivalent to P [illegible] Q?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

1.35 Consider the following well-formed formulae:

I. [illegible] II. [illegible]
III. [illegible] IV. [illegible]

Which of the above are equivalent?

(a) I and II (b) I and IV
(c) II and III (d) I and IV

[2009 : 2 Marks]

1.36 Suppose the predicate F(x, y, t) is used to represent the statement that person x can fool person y at time t. Which one of the following represents the statement "Everyone can fool someone at some time"?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2010 : 2 Marks]

1.37 The truth table of the following propositional formula is [illegible]

[illegible]

(a) F(x) is a prime number
(b) F(x) is a prime number for at least one value of x greater than 1
(c) F(x) is always true for prime values of x
(d) F(x) is false for some prime values of x greater than 1 and x

[2011 : 2 Marks]

1.38 Consider the following logical inferences.

I_1: It is not the case that Ganesh and Kumar will both be elected.
Therefore, Ganesh will not be elected.
Inference: Kumar was not elected.

I_2: It is not the case that both Ganesh and Kumar will be elected.
Therefore, Ganesh will not be elected.
Inference: Kumar was not elected.

Which of the following is TRUE?

(a) Both I_1 and I_2 are correct inferences
(b) I_1 is correct but I_2 is not a correct inference
(c) I_2 is correct but I_1 is not a correct inference
(d) Neither I_1 nor I_2 is a correct inference

[2012 : 1 Mark]

1.36 What is the most precise relation of the following statements?

"Some real numbers are rational"

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2018 : 1 Mark]

1.40 What is the logical contradiction of the following statement?

"Some [illegible]"

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2018 : 2 Marks]

1.41 Which one of the following is NOT logically equivalent to $\neg(p \to q)$?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2018 : 2 Marks]

1.42 Consider the statement

"Not all dogs have tails"

The logically equivalent statement is "[illegible] dogs have tails". Which one of the following options represents the above statement?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2024 (Sec 1) : 1 Mark]

1.43 Which one of the following, propositional logic formulas is TRUE when exactly two of p, q and r are TRUE?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2014 (Set-1) : 2 Marks]

1.44 Which one of the following Boolean expressions is a Boolean tautology?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2014 (Set-3) : 2 Marks]

1.45 Consider the following statements:

P: Smartphones are not cheap

Q: Smartphones are not good

L: Laptop

M: Q implies P

N: P implies L

Which one of the following statements is M and N is CORRECT?

(a) [illegible] (b) Only M is TRUE

(c) Only N is TRUE (d) M and N is TRUE

[2014 (Set-3) : 1 Mark]

1.46 The CORRECT formula for the statement "not all rainy days are windy" is

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2014 (Set-3) : 2 Marks]

1.47 Which one of the following is NOT equivalent to $p \to q$?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2013 (Sec 1) : 1 Mark]

1.48 The binary operation $\oplus$ is defined by the following truth table

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Which one of the following [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2013 (Set 1) : 2 Marks]

1.48 Consider the following statements:

S1: If [illegible] then [illegible] will not be [illegible].

S2: If [illegible] then [illegible].

Which one of the following [illegible] is true?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

2013 (Set-2) : 1 Mark]

1.49 [illegible] well formed formula [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2013 (Set-2) : 2 Marks]

1.50 [illegible] people [illegible] Type 1 people always lie [illegible] Type 2 people always lie [illegible] Type 2 [illegible] 1 [illegible] Type 2 [illegible] 1 [illegible] Type 2 [illegible] 1

Explanations: Mathematical Logic

1.1 (a)

[illegible]

To apply rules of inference, we [illegible]

For example,

Statement (b): $(P \to Q) \to (\neg P \to \neg Q)$

$\equiv (\neg P \vee Q) \to (\neg P \vee \neg Q)$

$\equiv \neg(\neg P \vee Q) \vee (\neg P \vee \neg Q)$

$\equiv (P \wedge \neg Q) \vee (\neg P \vee \neg Q)$

$\equiv (P \vee \neg P) \wedge (\neg Q \vee \neg P)$

$\equiv T \wedge (\neg Q \vee \neg P)$

$\equiv \neg Q \vee \neg P$

[illegible]

1.2 (a)

According to [illegible]

$\neg\forall x P(x) \vee \forall x Q(x) \equiv \exists x \neg P(x) \vee \forall x Q(x)$

$\neg(\forall x P(x) \vee \forall x Q(x)) \equiv \exists x \neg P(x) \wedge \exists x \neg Q(x)$

$\exists x(P(x) \vee \forall x Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$

$\exists x(P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists x Q(x)$

Simplifying [illegible]

1.3 (a):

[illegible]

Therefore, [illegible] is [illegible] and [illegible]

(b) [illegible]

[illegible]

So [illegible] is not a tautology.

(c) [illegible]

[illegible]

So [illegible] is a tautology.

(d) [illegible]

[illegible]

So [illegible] is not a tautology.

1.4 (b):

The expression [illegible]

[illegible]

1.5 Hint: [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

From the truth table [illegible] is not a tautology. [illegible] $p \to q$ [illegible] $\neg p \to \neg q$

1.6 (d):

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Now since [illegible] is given true, we can use the truth table as follows:

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible] truth table we have to find the truth value of [illegible] $p \to (p \to q)$

[illegible]

The truth value of $p \to q$ [illegible] truth table is given below.

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Since in the second [illegible] given expression is semi-decidable, it is undecidable. Therefore, the truth value of proposition [illegible] p [illegible] cannot be determined.

1.7 (d)

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

1.8 (a)

[illegible]

1.9 (a)

Let p : [illegible]

q : [illegible]

[illegible] only if [illegible]

$p \rightarrow q$

Converse of p → q is $q \rightarrow p$

Now we can write [illegible]

[illegible] Only [illegible] $q \rightarrow p$

1.10 (a)

[illegible]

So I_1 is [illegible] f_1 is satisfiable but not valid.

[illegible]

[illegible]

[illegible]

[illegible]

So f_1 is satisfiable but not valid.

1.11 (a)

[illegible]

[illegible]

[illegible]

Now we can write [illegible]

[illegible]

[illegible]

[illegible]

1.12 (b)

[illegible] Since [illegible] is a first order formula.

1.13 (d)

[illegible]

[illegible]

Now [illegible]

[illegible]

Now [illegible] is a prime or not, [illegible]

[illegible] is a prime number if and only if [illegible] divides [illegible] is not prime.

which means that [illegible] number if and only if no number divides [illegible] implies that [illegible] relevant prime.

[illegible] divides x. For all values of x this is true.

Hence [illegible] is a composite number.

Now, consider

[illegible] is a composite number if and only if [illegible] does not divide [illegible]. This is not a contradiction.

Which means that x is a composite number if and only if no number [illegible] divides x. Hence x is a composite.

Since x always divides x, the statement is true.

∴ Both T_1 and T_2 are false.

1.14 (d):

There are [illegible] from classes P, Q, R. [illegible] such that $(P \vee R) \wedge (Q \vee \neg R) \rightarrow P \vee Q$

∴ (d) is true.

[illegible]

$\neg P \wedge Q \rightarrow (P \vee R) \wedge (Q \vee \neg R)$

∴ only [illegible] may be true.

∴ [illegible] is false.

1.15 (d):

The statements [illegible] in the classroom [illegible] for all [illegible]

[illegible]

1.16 (c):

Given [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

which is [illegible] the given expression.

[illegible]

1.17 (b):

[illegible]

[illegible]

which is a [illegible] of statements called [illegible]

[illegible]

[illegible]

[illegible]

[

1.21 (b)

[illegible]

[illegible]

[illegible]

Alternate solution

We can state the given statement NOT (every [illegible] connected) as there exists a graph [illegible]

- [illegible]
- [illegible]
- [illegible]

[illegible] The negation of the statement is false.

Therefore option (d) [illegible] graph are connected.

1.27 (a)

[illegible] assignment problem. In order all [illegible] with [illegible] to find out the proportion of correct which evaluate to true, to the total number of clauses is equal to: $\frac{2}{3}$

[illegible] All of these proportions [illegible] choice [illegible] is the correct answer.

1.28 (a)

[illegible] discussion [illegible] propositional predicate

1.29 (a)

There which is [illegible] whole [illegible] equivalent [illegible] is the logical representation.

1.30 (b)

Quantifier [illegible]

[illegible]

[illegible] predicate [illegible]

[illegible]

Let [illegible]

We will check the validity of the [illegible] The [illegible] is [illegible] implication [illegible]

[illegible]

This implication [illegible] that [illegible] true.

[illegible]

So the [illegible]

The value of the [illegible] is true [illegible]

1.31 (b)

[illegible]

[illegible]

The negation of the [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Which [illegible]

1.32 (b)

[illegible]

[illegible]

[illegible]

$$pq + pq = 2pq$$
$$= pq(1+q)(1+p)$$
$$p + p^2$$
$$= (p+p^2)(p+1)$$
$$p + p^2$$

[illegible]

$$pq + pq = 2pq$$
$$= pq(1+q)(1+p)$$
$$p + p^2$$
$$= (p+1)(p+p)$$
$$p + p$$

[illegible] and [illegible] [illegible] [illegible] [illegible]

1.33 (d)

The [illegible] of [illegible] minimum [illegible]

[illegible]

which is [illegible] [illegible] [illegible]

[illegible]

[illegible]

1.34 (b)

The [illegible] [illegible] [illegible] [illegible] [illegible]

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

From the truth table we can see that the above truth table represents the truth table of $P \to Q$ and consequently it represents the material implication.

Thus $P \to Q \equiv \neg P \vee Q \equiv \neg Q \to \neg P$ that means the contrapositive of the statement $P \to Q$ is $\neg Q \to \neg P$.

Now we are ready to translate. We choose the best logical equivalence as follows:

Choose (a): $\neg Q \to \neg P \equiv Q \vee \neg P \equiv \neg Q \to \neg P$

Choose (b): $\neg P \to \neg Q \equiv P \vee \neg Q \equiv P \to Q$

Choose (c): $P \to Q \equiv \neg P \to \neg Q \equiv P \to Q$

Choose (d): $P \to Q \equiv \neg P \to \neg Q \equiv \neg P \to Q$

Accordingly, the statement equivalent to $P \to Q$ is equivalent to $\neg P \to Q$.

1.38 (b) :

$$\text{I} \quad \neg\exists x(P(x) \wedge \neg T(x))$$

and $\text{IV} \quad \forall x(P(x) \to T(x))$

Clearly statement I and IV are equivalent.

$$\text{II} \quad \neg\exists x(T(x) \wedge \neg P(x)) \equiv \forall x(T(x) \to P(x))$$

and $\text{III} \quad \forall x(T(x) \to P(x))$

Clearly II and III are not equivalent to each other and to I and IV.

1.39 (b) :

$\forall x \exists y(x + y = 0)$ is true

$\equiv \neg(\exists x \forall y \neg(x + y = 0))$

$\equiv$ we have that there does not exists an all people not all $\neg$:

$\equiv$ there is no person who is not all people

1.40 (a) :

$P(x)$ is true, then

$x \neq 1$ and $x \neq$

x is 2 takes into account the case $x = 1$, one of the conditions is false and hence the statement which is correct by the truth table is false or the number

So The statement is false if x is a prime number.

1.41 (b) :

Let p represent

$\neg$ [Sachin scores well now he is injured]

$\therefore \neg p \to q$

$\therefore q$

$\therefore \neg p$

Clearly $\neg p$ is true so we will be interested in the truth table of $\neg p$.

Hence, we have the following truth table:

p	q	$\neg p$	$\neg p \to q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

which corresponds to $(\neg p \to q) \to q$

$\equiv [(\neg p \to q) \wedge \neg q] \to q$

$\equiv [\neg(\neg p \to q) \vee q] \to q$

$\neg p \to q$

$\neg(\neg p \to q) \vee q \equiv \neg p \wedge \neg q \vee q$

which is a tautology

So I_2 is the correct interpretation.

1.42 (a) :

Symbolic representation is found

$\forall x(\text{Person}(x) \to \text{Friend}(x))$

1.43 (d) :

Given that the statements are true

$\forall x(\text{Person}(x) \to \text{Friend}(x))$ is correct

$\forall x(\text{Friend}(x) \to \text{Person}(x))$

$\exists x(\text{Friend}(x) \wedge \text{Person}(x))$

Alternatively

$\exists x(\text{Friend}(x) \wedge \text{Person}(x))$ gives

There is a person who is my friend and who is not a friend

So $\exists x(\text{Friend}(x) \wedge \neg\text{Person}(x))$

Ravi does not exists my friend who is not a person

i.e. none of my friends are not a person

So it is false statement.

1.44 (d) :

$\forall x(P(x) \to Q(x)) \equiv \forall x(\neg P(x) \vee Q(x))$

$\text{Then } \exists x(P(x) \wedge \neg Q(x)) \equiv \neg\forall x(P(x) \to Q(x))$

Given $\exists x(P(x) \wedge \neg Q(x))$

$$\equiv \neg\forall x(P(x) \to Q(x))$$

$$\equiv \neg\forall x(\neg P(x) \vee Q(x))$$

$$\equiv \neg\forall x(P(x) \to Q(x))$$

so the statement $\exists x(P(x) \wedge \neg Q(x))$ is equivalent to $\neg\forall x(P(x) \to Q(x))$

$\forall x(\neg y = x \to P(x)) \equiv \forall x(\neg y = x \to P(x))$

$\equiv \forall x(P(x) \to Q(x))$

[illegible]

(d) [illegible]

$\therefore$ Only (c) is not logically equivalent to $\forall x(P(x) \to Q(x))$.

1.43 (d)

(a) [illegible] = gold(x)
All stones are not gold.

(b) [illegible] = gold(stone(x))
$\neg$[illegible] gold are stones

(c) [illegible]
There exists gold which is not a stone or no gold exists.

(d) [illegible] = gold(x)
Not all [illegible] is gold i.e. there exists some metal which is not gold.

1.44 (b)

[illegible]

■ [illegible]
[illegible]
■ [illegible]
■ [illegible]
[illegible]
which is not a tautology.

1.45 (a)

[illegible] = mobile is cheap
[illegible] = Small mobile phones are not cheap

[illegible]

Q: Cheap mobile phones are not good

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Since P and Q are equivalent, all three of L, M, N are true.

Hence option (d) is right.

1.46 (d)

[illegible]

[illegible]

[illegible]

Alternate Method

Not Thinking degree can be assumed as [illegible] which is a [illegible]

[illegible]

1.47 (b)

[illegible]

Option (a) is $p \to q$ ■ [illegible]

Option (b) is $p \to q$ ■ $p \to q$

Option (c) is $p \to q$ ■ $p \to q$ which is not equivalent to $p \to q$.

1.48 (d)

The given truth table corresponds to $p \to q$ + $p \to q$ = [illegible]

It is [illegible] both commutative and associative.

1.49 (a)

C : Person is [illegible].
K : Person is [illegible].
E : Person is elected.

S_1 : C → E
S_2 : K → E
S_3 : E → K.

So [illegible] S_1 and S_2 : C → E = [illegible] → K = [illegible] → E

We can write [illegible] which is same as [illegible] → C, which is same as [illegible].

1.50 (d)

Since P = R ■ [illegible] and the connectives in both the sides are same, [illegible].

Option (d) is the right answer.

1.51 (c)

[illegible]

Case 1: [illegible]

The statement is true as it is [illegible] I am telling the truth.

There are two cases possible

Case 1: [illegible]

Case 2: [illegible]

In both case we [illegible]

Case 3: [illegible] truth or false.

[illegible]

So the statement [illegible] truth or false.

[illegible] truth or false.

[illegible] truth or false.

[illegible] truth or false.

[illegible] truth or false.

[illegible] truth or false.

[illegible] truth or false.

[illegible] truth or false.

So option (c) is correct.

1.52 Sol.

We will try to make

$$[illegible]$$

$$[illegible]$$

$$[illegible] \quad (1)$$

$$[illegible] \quad (2)$$

Now we can [illegible]

[illegible]

[illegible]

[illegible] cannot be satisfied by any [illegible]

In addition (1) must be satisfied which [illegible] and [illegible] is not [illegible]

Now the only [illegible]

[illegible]

1.53 Sol.

[illegible]

Take [illegible]

$$[illegible]$$

Take [illegible]

$$[illegible]$$

Take [illegible]

$$[illegible]$$

Take [illegible]

$$[illegible]$$

So the number of [illegible]

1.54 (d)

[illegible]

[illegible]

Let [illegible] be true, so we have

$$[illegible]$$

Now [illegible] false.

[illegible]

Now [illegible] true, [illegible] false

[illegible] is false.

So, [illegible] = 2078

■■■■

2 Set Theory and Algebra

2.1 State whether the following statement is true or false:

[illegible]

[1987 : 1 Mark]

2.2 (a) How many binary relations are there on a set A with n elements?

(b) How many [illegible] functions are there from A to A with n elements?

[1987 : 1 Mark]

2.3 The complement of the element a in the lattice L is given in figure [illegible]

[1988 : 2 Marks]

2.4 [illegible]

[1988 : 2 Marks]

2.5 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1991 : 2 Marks]

2.6 Let A be a finite set of size n. The number of [illegible] is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[1993 : 1 Mark]

2.7 [illegible] Then

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1994 : 2 Marks]

2.8 Let R be symmetric and transitive binary relation on a set A. Then

(a) R is reflexive and hence an equivalence relation

(b) R is reflexive and hence an equivalence relation

(c) R is reflexive and hence an equivalence relation

(d) None of the above

[1995 : 1 Mark]

2.9 The number of [illegible]

(a) 2 (b) [illegible]

(c) [illegible] (d) None of the above

[1995 : 1 Mark]

2.10 [illegible] Then

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1995 : 1 Marks]

2.11 [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[1996 : 1 Mark]

2.12 Let $X = \{a, b, c, d, e\}$. [illegible] The number of edges in G. [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[1996 : 1 Mark]

2.13 Suppose X and Y are sets and $|X|$ and $|Y|$ are their cardinalities. [illegible]

[illegible] 97 functions from X to Y. From this one can conclude that

(a) $|X| = 1$, $|Y| = 97$

(b) $|X| = 97$, $|Y| = 1$

(c) $|X| = 97$, $|Y| = 97$

(d) None of the above

[1996 : 1 Mark]

2.14 Which of the following statements is false?

(a) The set of rational numbers is countable

(b) The set of irrational numbers is countable

(c) The set of real numbers is closed under addition

(d) The set of real numbers is closed under multiplication

(e) The set of real numbers is closed under division

[1996 : 1 Mark]

2.15 Let A denote the set of real numbers. Let $f : A \times A \to A \times A$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of $f(x, y)$ is

(a) $f^{-1}(x, y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

(b) $f^{-1}(x, y) = (x - y, x + y)$

(c) $f^{-1}(x, y) = \left(\frac{x}{2}, \frac{y}{2}, \frac{x}{2}, \frac{y}{2}\right)$

(d) $f^{-1}(x, y) = (2x - y, 2x - y)$

[1996 : 2 Marks]

2.16 Let R be a non-empty relation on a set A defined by $A \times A$. Which of the following statements is false?

(a) R is symmetric

(b) R is reflexive and transitive

(c) R is antisymmetric and transitive

(d) R is negative and transitive

(e) R is not reflexive and not symmetric

[1996 : 2 Marks]

2.17 Which one of the following statements is false?

(a) The set of rational numbers forms a finite set because it can be put in one-to-one correspondence with the set of integers

(b) The set $\{0, 1\}$ is closed under multiplication modulo 2 but not under multiplication modulo 3

(c) The set of all strings over the alphabet Σ forms a group with respect to concatenation

(d) Let G be a set of G is a subgroup of the group $(G, \cdot)$ then $a \cdot a = e$ for any pair of elements $a, b \in G$

[1998 : 2 Marks]

2.18 The number of equivalence relations on the set $\{1, 2, 3, 4\}$ is

(a) 7 (b) 15

(c) 24 (d) 4

[1997 : 1 Mark]

2.19 Suppose $A = \{1, 2, 3, 4\}$. The number of elements in the largest equivalence relation on A is

(a) 4 (b) 7

(c) 9 (d) 16

[1998 : 1 Mark]

2.20 Let R_1 and R_2 be equivalence relations on a set A. Consider the following assertions:

(i) $R_1 \cup R_2$ is an equivalence relation

(ii) $R_1 \cap R_2$ is an equivalence relation

Which of the following is correct?

(a) Both assertions are true

(b) Assertion (i) is true but assertion (ii) is false

(c) Assertion (ii) is true but assertion (i) is false

(d) Both assertions are false

[1996 : 1 Mark]

2.21 The number of functions from an m element set to an n element set is

(a) $m + n$ (b) m

(c) n^m (d) $m \cdot n$

[1996 : 1 Mark]

2.22 The binary relation $S = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on the set $A = \{1, 2, 3\}$ is

(a) Reflexive, symmetric and transitive

(b) Neither reflexive nor irreflexive but transitive

(c) Irreflexive, symmetric and transitive

(d) Irreflexive and anti-symmetric

[1998 : 2 Marks]

2.23 Suppose $A = \{a, b, c, d\}$ and Π_1 is the following partition of A:

$$\Pi_1 = \{\{a, b\}, \{c\}, \{d\}\}$$

(a) List the ordered pairs of the equivalence relation induced by Π_1

(b) Draw the matrix of the above equivalence relation

[1998 : 2 Marks]

2.24 Let $(A, *)$ be a semigroup. Which one of the following statements is true?

(a) Show that for every a and b in A, $a * b = b * a$

(b) Show that for every a and b in A, $a * b = a$

(c) Show that for every a, b, c in A, $a * b = a * c$

[1998 : 2 Marks]

2.25 The number of binary relations on a set with n elements is

(a) n^2 (b) $2n$

(c) 2^{n^2} (d) None of these

[1999 : 1 Mark]

2.26 (a) Consider the following:

Let R be a relation on S which is symmetric and transitive. Then R is a reflexive. But $x \in S$ $x \in S$ $x \in S$ be arbitrary.

Consider any y such that $x R y$. Now because R is symmetric, $y R x$. Now $x R y$ and $y R x$ implies $x R x$ by transitivity.

Hence proved. Given by Mr. Negligent

(b) Give an example of a relation on S which is symmetric and transitive but not reflexive.

[1998 : 2 Marks]

2.27 A relation R is defined on the set of real numbers as $x R y$ if $x - y$ is even. Which of the following statements is true?

(a) R is not a partial order relation

(b) R is an equivalence relation having 1 equivalence class

(c) R is an equivalence relation having 2 equivalence classes

(d) R is an equivalence relation having 3 equivalence classes

[2000 : 2 Marks]

2.28 Let R be a relation on the set of real numbers. Which of the following statements always hold?

(a) $R(R(R)) = R(R)$ (b) $R(R \cup R(R)) = R$

(c) $R(R \cap R(R)) = R(R)$ (d) $R \subset R(R)$

[2000 : 2 Marks]

2.29 Consider the following statements:

S_1: R is a relation on a set A. If R is symmetric and transitive, then R is reflexive. But $x \in A$. $x \in A$ $x \in A$ be arbitrary.

Consider any y such that $x R y$. Now because R is symmetric, $y R x$. Now $x R y$ and $y R x$ implies $x R x$ by transitivity.

Hence proved. Given by Mr. Negligent

S_2: There exists a relation R on a set A such that R is symmetric and transitive but not reflexive.

Which of the statements S_1 and S_2 is/are correct?

(a) Only S_1 is correct

(b) Only S_2 is correct

(c) Both S_1 and S_2 are correct

(d) None of S_1 and S_2 is correct

[2001 : 2 Marks]

2.30 Let S be a finite set, and let E and F be subsets of S. Consider the following statements about images.

$S1$: $f(E \cap F) = f(E) \cap f(F)$

$S2$: $f(E \cup F) = f(E) \cup f(F)$

(a) Only $S1$ is correct

(b) Only $S2$ is correct

(c) Both $S1$ and $S2$ are correct

(d) None of $S1$ and $S2$ is correct

[2001 : 2 Marks]

2.31 The binary relation $S = \{(x, y) \mid x \text{ is the father of } y\}$ on $A = \{1, 2, 3\}$ is

(a) Neither reflexive nor symmetric

(b) Symmetric and transitive

(c) Transitive and reflexive

(d) Reflexive and symmetric

[2002 : 2 Marks]

2.32 Consider the set S of all strings over the alphabet $\Sigma = \{0, 1\}$. S with the concatenation operation for strings

(a) does not form a group

(b) forms an abelian group

(c) does not have an identity element

(d) forms a group if the empty string is removed from S

[2002 : 1 Mark]

2.84 [illegible]

[illegible]

[2006 : 2 Marks]

2.85 [illegible]

[illegible]

[2006 : 2 Marks]

2.86 [illegible]

[illegible]

[2004 : 1 Mark]

2.87 [illegible]

[illegible]

[2004 : 1 Mark]

2.88 [illegible]

[illegible]

[IT-2004 : 1 Mark]

2.89 [illegible]

[illegible]

[IT-2004 : 1 Mark]

2.90 [illegible]

[illegible]

[2004 : 1 Mark]

2.91 [illegible]

[illegible]

[2004 : 2 Marks]

2.92 [illegible]

[illegible]

[2004 : 2 Marks]

2.42 [illegible]

Which one of the following is [illegible]?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[2005 : 1 Mark]

2.43 [illegible] Hasse Diagram of the [illegible]

[illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2005 : 1 Mark]

2.44 [illegible] multiplicative modulo 15. The inverse of 11 and [illegible] are

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2005 : 1 Mark]

2.45 [illegible] relations [illegible]

[illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2005 : 2 Marks]

2.46 [illegible]

Which one of the following [illegible]?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2005 : 2 Marks]

2.47 Consider [illegible] matrices [illegible]

$$\begin{bmatrix} a & b & c \\ c & b & a \\ 0 & 0 & e \end{bmatrix}$$

where a, b, c, d, e and [illegible] are real numbers and $a \neq c$. Under the usual multiplication operation, the matrices

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2005 : 2 Marks]

2.48 Let [illegible] function from set A to set B, [illegible] function from B to C, and [illegible] function from A to C. [illegible] of A. Which of the following statements always [illegible] such functions?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[IT-2005 : 2 Marks]

2.49 Let S be a set with n elements. Let [illegible] be a collection of distinct subsets of S such that [illegible] $R_2 = S$. What is the minimum possible size of C?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) n!

[IT-2005 : 2 Marks]

2.50 [illegible] set N of natural numbers and [illegible] operation [illegible] $N \times N \to N$ satisfies [illegible] for [illegible] $(x, y) = (y, x)$, [illegible] N. Which of the following properties have [illegible]?

I. [illegible]

II. [illegible]

III. [illegible]

(a) [illegible] only (b) II and III only

(c) I and III only (d) None of these

[IT 2006 : 2 Marks]

2.51 Let X, Y, Z be sets of [illegible] respectively. Let $W = X \cap Y$ and B be the power set of all subsets of W. The number of functions from B to C is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 1 Mark]

2.52 [illegible] modulo 10 [illegible] group [illegible] possible [illegible]

(a) [illegible]
(b) 2 [illegible]
(c) It does not have an inverse
(d) It does [illegible] inverse

[2006 : 1 Mark]

2.53 A relation R is defined as ordered pairs [illegible]

Then R is

(a) Neither [illegible] equivalence Relation
(b) A Partial Order [illegible]
(c) A Total Order
(d) An Equivalence Relation

[2006 : 1 Mark]

2.54 Let R be a relation defined as

$X = \{(p, q) : p \text{ divides } q\}$

$Y = \{(p, q) : p \text{ is prime and } q \text{ is prime}\}$

Which one of the following is true?

(a) $X \subset Y$
(b) $X \supset Y$
(c) $X = Y$
(d) $X \cap Y = \phi$ and $X \cup Y = \mathbb{N} \times \mathbb{N}$

[2006 : 2 Marks]

2.55 Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of n distinct prime numbers and let $S_1, S_2, \ldots, S_n$ be subsets of P such that $S_i \neq S_j$ for $i \neq j$. Let the function $f(x)$ be defined as the number of elements in P that divide x. Let $f(S_i) = |\{p \in P : p \text{ divides } S_i\}|$.

Then $\sum_{i=1}^{n} f(S_i) =$

(a) $2n$ (b) 2^n
(c) $2^n - 1$ (d) $2n + 1$

[2006 : 2 Marks]

2.56 Let P, Q, R be [illegible] denote the symmetric difference operator defined as $P \Delta Q = (P \cup Q) - (P \cap Q)$. Using Venn diagrams [illegible] which of the following is true? [illegible]

I. $(P \Delta Q) \Delta R = (P \cup Q \cup R) - (P \cap Q \cap R)$
II. $(P \Delta Q) \Delta R = (P \cup Q \cup R) - (P \cap Q \cap R)$

(a) I only (b) II only
(c) Neither I nor II (d) Both I and II

[IT-2008 : 2 Marks]

2.57 What [illegible] of the [illegible] X defined as $X = \{x \mid x \text{ divisible by } 2 \text{ or } 3\}$

(a) 26 (b) 28
(c) 27 (d) 16

[IT-2006 : 2 Marks]

2.58 Let S be a set of 3 elements. The number of ordered pairs in the equivalence relation on S is

(a) [illegible] (b) 27 and 3
(c) 27 and 9 (d) [illegible] and 1

[2006 : 1 Mark]

2.59 Consider the set $A = \{a, b, c, d\}$. Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$ on A:

$\pi_1 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$, $\pi_2 = \{\{a, b\}, \{c, d\}\}$,

$\pi_3 = \{\{a\}, \{b, c, d\}\}$

Let $\pi = \{\pi_1, \pi_2, \pi_3, \pi_4\}$ be the set of all partitions of A. For $\pi_i, \pi_j \in \pi$, $\pi_i \leq \pi_j$ if and only if π_i refines π_j. The poset $(\pi, \leq)$ is shown in the figure below.

[2007 : 2 Marks]

2.60 A partial order P is defined on the set of natural numbers as follows. Here x | y denotes x divides y.

(i) $x \mid y \Rightarrow x \mathrm{P} y$
(ii) $x \mid y$ if and only if $x \mathrm{P} y$ and $y \mathrm{P} x$
(iii) $x \mathrm{P} y$ and $y \mathrm{P} z \Rightarrow x \mathrm{P} z$

Consider the following pairs of natural numbers:

(i) (101, 202) (ii) (11, 101)
(iii) (145, 290) (iv) (0, 101)

Which of these ordered pairs of natural numbers are contained in P?

(a) (i) and (ii) (b) (i) and (iv)
(c) (i) and (iii) (d) (ii) and (iv)

[IT-2007 : 2 Marks]

2.61 [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 1 Mark]

2.62 Consider the following Hasse diagrams

Which of the following statements is true?

(a) [illegible] and (iii) only (b) [illegible] and (iii) only

(c) [illegible] (d) [illegible] and (iii) only

[IT-2006 : 2 Marks]

2.63 Which one of the following is NOT isomorphic to a group of order 4 (Group)?

(a) Rational numbers

(b) Integer numbers

(c) Positive real numbers under multiplication

(d) Positive real numbers under addition

[2009 : 2 Mark]

2.64 Consider the binary relation $R = \{(x, y), (x, z), (y, z)\}$ defined on a set $\{x, y, z\}$. Which one of the following is TRUE?

(a) R is symmetric but NOT antisymmetric

(b) R is NOT symmetric but antisymmetric

(c) R is both symmetric and antisymmetric

(d) R is neither symmetric nor antisymmetric

[2009 : 1 Mark]

2.65 Let the composition table of a group be given below

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

Which one of the following is true?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2009 : 1 Marks]

2.66 What is the greatest number of distinct relations that can be defined on a set of n elements?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2010 : 1 Mark]

2.67 Consider a set S = {a, b, c}, where a and b are cube roots of unity. The operation '.' is multiplication operation. The structure (S, .) forms

(a) a group (b) a ring

(c) an integral domain (d) a field

[2010 : 1 Mark]

2.68 How many [illegible] are there on a set of n elements?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2013 : 2 Marks]

2.69 [illegible] defined as $x * y = x - y$. Which one of the following statements is true?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2013 : 1 Mark]

2.70 [illegible] The order of the group is

[2014 (Set 1) : 2 Marks]

2.71 Consider the following two subsets of the set S of integers between [illegible] and [illegible]. Let the difference of the two elements be the absolute difference of the two elements in Q.

Consider the following two statements:

S1: There is a subset of S such that [illegible] the [illegible] other subsets.

S2: There is a subset of S such that [illegible] smaller than [illegible] other subsets.

Which one of the following is CORRECT?

(a) S1 is true and S2 is true

(b) S1 is true and S2 is false

(c) S1 is false and S2 is true

(d) Neither S1 nor S2 is true

[2014 (Set-2) : 2 Marks]

2.72 Let X and Y be finite sets and $f: X \to Y$ be a function. Which one of the following statements is TRUE?

(a) For any subsets A and B of Y,

$f(A \cap B) = f(A) \cap f(B)$

(b) For any subsets A and B of X,

$f(A \cap B) = f(A) \cap f(B)$

(c) For any subsets A and B of X,

$f(A \cap B) = \min(f(A), f(B))$

(d) For any subsets A and B of X,

$f(A \cap B) \subseteq f(A) \cap f(B)$

[2014 (Set-4)] : 1 Mark

2.73 Let G be a group with 15 elements. Let H be a subgroup of G. It is known that $H \neq G$ and H is not the trivial subgroup. The sum of all elements of H is

[2014 (Set-4)] : 1 Mark

2.74 Consider the set of all functions $f: \{0, 1, 2, \ldots, 2014\} \to \{0, 1, 2, \ldots, 2014\}$ such that $f(f(i)) = i$ for all $0 \le i \le 2014$. Consider the following statements:

P: For each such f, there exists i such that $f(i) = i$.

Q: For each such f, there exists i such that $f(i) = 1007$.

R: Each such f is onto.

Which one of the following is TRUE?

(a) P is true and R is false

(b) Only Q is true

(c) Only P and Q are true

(d) Only P is true

[2014 (Set-3)] : 2 Marks

2.75 There are two elements x and y in a group $(G, *)$ such that x is the only element in the group satisfying $x * x = x$ and y is the only element satisfying $y * y = y$. Let z be the identity element. The maximum number of elements in such a group is ____

[2014 (Set-3)] : 2 Marks

2.76 If $f(x) = 1 - x$ and $g(x) = \frac{x}{x-1}$, then $\frac{f(g(x))}{g(f(x))}$ is

(a) $\frac{f(x)}{g(x)}$ (b) $\frac{x}{x}$

(c) $\frac{g(x)}{f(x)}$ (d) $\frac{x}{1-x}$

[2015 (Set-1)] : 1 Mark

2.77 Consider the set $A = \{1, 2, 3, 4\}$ ordered by $\le$. Let $B \subseteq A$ and $B = \{x \in A \mid x^2 \le 9\}$. Which of the following statements are true?

1. B is a lattice
2. B is a chain
3. B is a lattice
4. B is a chain

(a) 1 and 2 only (b) 2 and 3 only

(c) 1, 2 and 3 only (d) 1, 2 and 4 only

[2015 (Set-1)] : 1 Mark

2.78 Suppose $(P, \le)$ and $(Q, \le)$ are partial orders represented by the following Hasse diagram:

Define a relation $\le$ on $P \times Q$ by $(p_1, q_1) \le (p_2, q_2)$ if $p_1 \le p_2$ and $q_1 \le q_2$. Let $R = \{(p, q) \in P \times Q \mid p = q\}$. Let S be the set of all pairwise incomparable elements of $P \times Q$. Let p_1 be the maximum number of elements in a chain in $P \times Q$ and p_2 be the maximum number of elements in S. Then

(a) $p_1 = 6$, $p_2 = 3$ (b) $p_1 = 4$, $p_2 = 3$

(c) $p_1 = 6$, $p_2 = 2$ (d) $p_1 = 4$, $p_2 = 2$

[2015 (Set-1)] : 2 Marks

2.79 Let R be the relation on the set of positive integers such that $(a, b) \in R$ if and only if a and b have a common divisor other than 1. Which one of the following statements about R is true?

(a) R is symmetric and reflexive but not transitive

(b) R is reflexive but not symmetric and not transitive

(c) R is transitive but not reflexive and not symmetric

(d) R is symmetric but not reflexive and not transitive

[2015 (Set-3)] : 1 Mark

2.80 The cardinality of the power set $\{0, 1, 2, \ldots, 9\}$ is ____

[2015 (Set-4)] : 1 Mark

2.81 The number of onto functions (surjective functions) from $X = \{1, 2, 3\}$ to $Y = \{x, y, z\}$ is _____.

[2015 (Set-B): 3 Marks]

2.82 Let X and Y denote the sets of 10 students and 5 courses, respectively. Let R be the set of all possible functions from X to Y. The number of elements in R is _____.

2.83 Let P and Q denote sets and let $P \# Q$ denote $P \cap Q$. Let R and S denote sets and let $R \# S$ denote $R \cup S$. Consider the following propositions.

S1: $(P \# Q) \# R = P \# (Q \# R)$

S2: $(P \# R) = (P \# Q) \# R$

Which of the following is true for the Boolean variables P, Q and R?

(a) Only S1 is true

(b) Only S2 is true

(c) Both S1 and S2 are true

(d) Neither S1 nor S2 are true

[2015 (Set-D): 1 Mark]

2.84 Suppose G is the power set of the set $S = \{1, 2, 3, 4, 5\}$. Let $F \subseteq G$ denote the set of elements in F which are the complements of F. For any $F, H \in G$, let $F \# H$ be the set of all elements in F which are not in H. Which one of the following is true?

(a) $\forall F, H \in G, F \# H = H \# F$

(b) $\exists F, H \in G$ such that $F \# H = \{1, 2, 3\}$

(c) $\forall F \in G$, $\exists H \in G$ such that $F \# H = \{1, 2, 3\}$

(d) $\forall F \in G$, $\exists H \in G$ such that $F \# H = F$

[2015 (Set-A): 1 Mark]

2.85 The R-S-T relation on the set of ordered pairs of positive integers such that $(p, q) \in R$ if $p + q = 5$ is _____.

(a) Both reflexive and symmetric

(b) Reflexive but not symmetric

(c) Symmetric but not reflexive

(d) Neither reflexive nor symmetric

[2015 (Set-D): 2 Marks]

2.86 A function $f: N \to N$, defined on the set of positive integers N, satisfies the following properties:

$f(n) = f(n+2)$ if $n > 2$

$f(n) = f(n) + n$ if $n > 2$ and

Let $S = \{y \mid y = f(n) \text{ for some } n \in N\}$. Then the least distance between any two distinct elements in S is _____.

[2016 (Set-D): 1 Mark]

2.87 A binary relation R on $N \times N$ is defined as follows: $(m, n) R (p, q)$ if $m \le p$ and $n \le q$. Consider the following propositions:

P: R is reflexive

Q: R is transitive

Which one of the following statements is true?

(a) Both P and Q are true

(b) P is false and Q is true

(c) P is true and Q is false

(d) Both P and Q are false

[2016 (Set-B): 2 Marks]

2.88 Consider a set S of 250 elements. Let P_1, P_2 and P_3 be properties satisfied by elements in S. Let N_1, N_2 and N_3 be the number of elements in S satisfying properties P_1, P_2 and P_3 respectively. Let N_4 be the number of elements in S satisfying none of the properties. Which one of the following statements is true?

(a) $N_1 + N_2 + N_3 + N_4 = 250$

(b) $N_1 + N_2 + N_3 - N_4 = 250$

(c) $N_1 - N_2 + N_3 + N_4 = 250$

(d) $N_1 - N_2 + N_3 - N_4 = 250$

[2016 (Set-B): 2 Marks]

2.89 Consider a set S of 250 elements. Let P_1, P_2 and P_3 be properties satisfied by elements in S. Let N_1, N_2 and N_3 be the number of elements in S satisfying properties P_1, P_2 and P_3 respectively. Let N_4 be the number of elements in S satisfying none of the properties. Which one of the following statements is true?

(a) $N_1 + N_2 + N_3 + N_4 = 250$

(b) $N_1 + N_2 + N_3 - N_4 = 250$

(c) $N_1 - N_2 + N_3 + N_4 = 250$

(d) $N_1 - N_2 + N_3 - N_4 = 250$

[2016 (Set-B): 2 Marks]

2.90 Consider a set S of 250 elements. Let P_1, P_2 and P_3 be properties satisfied by elements in S. Let N_1, N_2 and N_3 be the number of elements in S satisfying properties P_1, P_2 and P_3 respectively. Let N_4 be the number of elements in S satisfying none of the properties. Which one of the following statements is true?

(a) $N_1 + N_2 + N_3 + N_4 = 250$

(b) $N_1 + N_2 + N_3 - N_4 = 250$

(c) $N_1 - N_2 + N_3 + N_4 = 250$

(d) $N_1 - N_2 + N_3 - N_4 = 250$

[2016 (Set-B): 2 Marks]

2.91 Consider a set S of 250 elements. Let P_1, P_2 and P_3 be properties satisfied by elements in S. Let N_1, N_2 and N_3 be the number of elements in S satisfying properties P_1, P_2 and P_3 respectively. Let N_4 be the number of elements in S satisfying none of the properties. Which one of the following statements is true?

(a) $N_1 + N_2 + N_3 + N_4 = 250$

(b) $N_1 + N_2 + N_3 - N_4 = 250$

(c) $N_1 - N_2 + N_3 + N_4 = 250$

(d) $N_1 - N_2 + N_3 - N_4 = 250$

[2016 (Set-B): 2 Marks]

2.92 Consider a set S of 250 elements. Let P_1, P_2 and P_3 be properties satisfied by elements in S. Let N_1, N_2 and N_3 be the number of elements in S satisfying properties P_1, P_2 and P_3 respectively. Let N_4 be the number of elements in S satisfying none of the properties. Which one of the following statements is true?

(a) $N_1 + N_2 + N_3 + N_4 = 250$

(b) $N_1 + N_2 + N_3 - N_4 = 250$

(c) $N_1 - N_2 + N_3 + N_4 = 250$

(d) $N_1 - N_2 + N_3 - N_4 = 250$

[2016 (Set-B): 2 Marks]

■■■

Answers Antisymmetric Set Theory and Algebra

2.1	(a)	2.2	(a)	2.3	(c)	2.4	(d)	2.5	(c)	2.6	(d)	2.7	(a)	2.8	(b)	2.9	(a)
2.10	(c)	2.11	(b)	2.12	(c)	2.13	(c)	2.14	(b)	2.15	(c)	2.16	(b)	2.17	(c)	2.18	(a)
2.19	(b)	2.20	(c)	2.21	(c)	2.22	(b)	2.23	(a)	2.24	(b)	2.25	(c)	2.26	(a)	2.27	(c)
2.28	(b)	2.29	(c)	2.30	(c)	2.31	(a)	2.32	(d)	2.33	(a)	2.34	(d)	2.35	(c)	2.36	(b)
2.37	(c)	2.38	(c)	2.39	(c)	2.40	(d)	2.41	(a)	2.42	(a)	2.43	(b)	2.44	(c)	2.45	(a)
2.46	(a)	2.47	(b)	2.48	(a)	2.49	(c)	2.50	(a)	2.51	(d)	2.52	(c)	2.53	(a)	2.54	(a)
2.55	(a)	2.56	(b)	2.57	(b)	2.58	(b)	2.59	(c)	2.60	(d)	2.61	(d)	2.62	(a)	2.63	(c)
2.64	(c)	2.65	(a)	2.66	(c)	2.67	(c)	2.68	(c)	2.69	(c)	2.70	(c)	2.71	(c)	2.72	(d)
2.73	(b)	2.74	(a)	2.75	(c)	2.76	(d)	2.77	(d)	2.78	(b)	2.79	(c)	2.80	(c)	2.81	(b)
2.82	(b)																

Explanations Set Theory and Algebra

2.1 Sol.

A relation is said to be equivalence relation if

(i) Reflexive

(ii) Symmetric, and

(iii) Transitive.

Union of two reflexive relations and two symmetric relations are reflexive and symmetric respectively. However, union of two transitive relations [illegible] transitive. Union of two equivalence relations [illegible] an equivalence relation.

Example:

Let S_1 and S_2 be set $A = \{1, 2, 3\}$

$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ is an equivalence relation.

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$ is an equivalence relation.

$R_1 \cup R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1)\}$ is not an equivalence relation.

Because $(1, 2)$ & $(2, 3)$ needs $(1, 3)$ element for transitive relation.

2.2 Sol.

(a) [illegible]

Number of binary relations on a set A with n elements is 2^{n^2}

(b) Number of one to one functions from a set A with n elements to set B with n elements is $n!$

So the number of one to one functions from a set A with n elements to itself is $n! = n!$.

2.3 Sol.

The complement of an element is defined if the element is in the universal set and it will be unique.

∴ The complement of the element is in the universal set.

2.4 Sol.

The relation over $\{1, 2, 3\}$ that satisfies the reflexive property is $\{(1, 1), (2, 2), (3, 3)\}$ and it is symmetric.

So for symmetric relations

$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)\}$

Now, for transitive closure of the relation we

$= \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (2, 1), (1, 2)\}$

So for transitive closure of the relation we

$= \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (2, 1), (1, 2)\}$

2.5 Sol.

$S = S_1 \cup S_2 \cup S_3 \cup \ldots \cup S_n$

For finite number of sets, union of sets is always finite. If all S_i are finite, then S will be finite.

2.6 (b).

Number of relations in $A \times A = n^2$

∴ Number of subsets of the power set $A \times A$

$= 2^{n^2}$.

2.7 (b)

[illegible]

2.8 (b)

A relation which is symmetric and transitive [illegible] reflexive relation.

(i) [illegible] is symmetric and transitive but not reflexive.

(ii) [illegible] is symmetric, transitive and reflexive.

(iii) A relation [illegible] symmetric and transitive [illegible] reflexive.

2.9 (d)

[illegible]

Given set [illegible]

So [illegible]

[illegible] number of elements in power set [illegible]

2.10 (d)

(i) Closure Property: Multiplication [illegible] matrices [illegible] Matrix multiplication [illegible] closure property.

(ii) Associative Property: Matrix multiplication [illegible]

$(AB)C = A(BC)$

Where A, B, C are [illegible] matrices.

(iii) Identity Element: Identity matrix [illegible] identity element for matrix multiplication [illegible]

(iv) Inverse Element: [illegible] singular matrix [illegible]

(v) Commutative: Matrix multiplication is not commutative.

$AB \neq BA$

Where A, B are non singular matrices.

Matrices [illegible] commutative.

For [illegible] but [illegible]

2.11 (d)

[illegible]

Representing [illegible] using Venn diagram [illegible] follows

[illegible]

2.12 (b)

$x = \{8, 4, 6, 12, 24\}$

The Hasse diagram [illegible]

[illegible]

2.13 (b)

[illegible]

The number of elements in $X \cup Y = 100$

Given the number of elements in $X \cap Y = 67$

[illegible]

So [illegible]

2.14 (b)

[illegible]

Therefore the set of rational numbers doesn't form a [illegible] under multiplication.

2.15 (a)

[illegible]

Solve [illegible]

[illegible]

[illegible]

So [illegible]

[illegible]

[illegible]

So [illegible]

[illegible]

$$y = \frac{1 - 2x}{3}$$

So the joint pdf of X and Y is given by

$$f(x, y) = f_X(x) \cdot f_Y(y) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

So x_1 and x_2 are independent variables and y_1 and y_2 are independent variables.

$$f(x, y) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

Alternate Method

Given $f(x, y) = (x - y)^2$ in $-1 \le x \le 1$

Take some random values of pairs (x, y)

$f(x, y) = (1 - 1)^2 = 0$, $f(x, y) = (0 - 1)^2 = 1$

Now the correct answer must be (B). Only option (B) satisfies.

Alternate option (A) may be selected as the best option, because (C) and (D) are not 2, 3.

2.16 (b)

(i) **Reflexive**

$A \times A = A \times A$

So (A, A) elements will be present in R.

So R is not reflexive.

(ii) **Symmetric**

If (A, B) belongs to R, then (B, A) also must be present in R. So relation R is symmetric relation.

(iii) **Transitive**

If (A, B) belongs to R, (B, C) belongs to R, then (A, C) must be present in R.

For example,

$A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{5, 6\}$

$A \times B$ = {(1, 3), (1, 4), (2, 3), (2, 4)}

$B \times C$ = {(3, 5), (3, 6), (4, 5), (4, 6)}

But $A \times C$ = {(1, 5), (1, 6), (2, 5), (2, 6)}

So relation R is not a transitive relation.

2.17 (b)

The given set of 30 strings can be divided into 2 groups, one containing those with even number of 1's and the other containing those with odd number of 1's.

2.18 (d)

Corresponding to each part of the set {1, 2, 3, 4}, there exists a unique equivalence relation. Since there are four elements in the set, we have the set of 4 elements and so 4 block, one block, three elements and two block, two elements and one block.

$$S(4, 1) = 1$$

$$S(4, 2) = \binom{4}{1} S(1, 1) + \binom{4}{2} S(2, 1) = 4 + 3 = 7$$

$$S(4, 3) = \binom{4}{2} S(2, 1) = 6$$

$$S(4, 4) = \binom{4}{3} S(3, 3) = 1$$

Total = 1 + 7 + 6 + 1 = 15

So the number of equivalence relations on the set {1, 2, 3, 4} is 15.

2.19 (b)

A is infinite set of 3 elements. The largest equivalence relation on A is the universal set A. So, the number of elements in the new partition is 1.

2.20 (d)

A relation is said to be an equivalence relation if it satisfies

(i) Reflexive

(ii) Symmetric

(iii) Transitive

Reflexive and symmetric properties are satisfied by the given relation.

Transitive property is not satisfied. So it is not an equivalence relation.

Therefore, R is not an equivalence relation but only a partial order relation.

2.21 (b)

The number of functions from A to B = n^m

2.22 (b)

The relation R is not transitive in A, as (a, b) ∈ R and (b, a) ∈ R but (a, a) ∉ R.

So, the relation R is not an equivalence relation.

The relation R is reflexive and symmetric but not transitive.

Therefore, R is a partial order relation.

So option (b) is correct.

2.23 Sol.

Given [illegible]

$$A_1 = \{a, b, c, d\}$$

(a) The ordered pairs of the relation [illegible] relation is defined by, i.e.

$$R = \{(a, b), (a, c), (a, d), (b, c), (b, d), (c, d)\}$$

(b) The diagraph of the above equivalence relation is [illegible]

2.24 Sol.

Given [illegible]

The contrapositive statement [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

2.25 Sol.

The minimum number of elements in a binary relation [illegible] elements = [illegible]

[illegible] binary relations, either to appear or a binary relation [illegible] appear or a binary relation.

$\therefore$ Number of binary relations $= 2^{n^2}$.

2.26 Sol.

(iii) According to Mr. X, it is only can select the [illegible] Mr. X is [illegible] Mr. X [illegible] implies Mr. X.

[illegible] Mr. X [illegible] Mr. X.

However, [illegible] according to symmetricity, it is also not present. So [illegible] in relation, it need not be [illegible].

For Example: Empty relation is both symmetric and antisymmetric. However, [illegible] relation is [illegible].

(iv) Examples of relations which are neither symmetric and antisymmetric relations are:

i. Empty relation is symmetric and antisymmetric but not reflexive.

ii. Relation $R = \{(x, y) : x, y \in \mathbb{N}, x + y = 5\}$ is neither symmetric and antisymmetric relation.

2.27 Sol.

A relation R on a set A is said to be reflexive if $(x, x) \in R$ for every element $x \in A$.

(a) Given relation

(i) x is even and y is even

(ii) x is even and y is odd

Therefore, relation R is equivalence relation because it is

i. Reflexive

$x + x = 2x$ = even

$\therefore (x, x) \in R$ for all $x \in A$ [illegible]

ii. Symmetric

$x + y$ = even [illegible] $y + x$ is also even

$\therefore$ [illegible]

iii. Transitive

If $x + y$ = even and $y + z$ = even

Then $x + y + y + z$ = even

$\Rightarrow x + 2y + z$ = even

$\Rightarrow x + z$ = even

$\therefore$ R is [illegible] Transitive.

$\therefore$ relation R is an equivalence relation which [illegible] relation [illegible] equivalence [illegible] of all even integers.

[illegible]

2.28 (d):

[illegible] only common element between [illegible] and [illegible]

[illegible]

2.29 (b):

(i) Relation R_1 is not reflexive because [illegible] the set of integers

[illegible]

$\therefore$ [illegible] reflexive relation

(ii) If $(x, y) \in R_1$ then [illegible]

[illegible]

$\therefore$ R_1 is symmetric relation

(iii) [illegible] R_1 and [illegible]

[illegible]

[illegible]

$\therefore$ R_1 is transitive relation

Since R_1 is not reflexive, symmetric and transitive so R_1 is an equivalence relation.

(iv) $(x, y) \in R_2$ then [illegible] set of integers

[illegible]

[illegible]

$\therefore$ [illegible] relation

Since R_2 is not reflexive, it is not an equivalence relation.

(v) $(x, y) \in R_3$ [illegible] set of [illegible]

[illegible]

$\therefore$ R_3 is reflexive relation.

[illegible]

$\therefore$ R_3 is symmetric relation.

[illegible]

[illegible]

[illegible]

[illegible]

(vi) [illegible] set of [illegible] numbers

[illegible]

[illegible]

$\therefore$ R_4 is reflexive relation.

[illegible]

[illegible]

$\therefore$ R_4 is not symmetric relation.

[illegible]

[illegible]

$\therefore$ R_4 is not transitive relation.

Since R_4 is reflexive and [illegible] symmetric, so R_4 is not an equivalence relation.

2.30 (b):

R_1: [illegible] set of integers

[illegible] set of odd integers

[illegible] set of even integers

[illegible]

Thus [illegible]

R_2: Let two irrational numbers [illegible] are respectively $(1+\sqrt{2})$ and $(1-\sqrt{2})$.

So [illegible]

[illegible] rational number

Thus [illegible] since both R_1 and R_2 are not equivalence relation.

2.31 (a)

Given a function [illegible]

[illegible]

[illegible]

Therefore R is reflexive and R_2 is false.

2.32 (d):

[illegible] transitive and symmetric but not reflexive.

2.33 (a)

[illegible]

[illegible]

[illegible]

[illegible] is an equivalence relation only if the following conditions are satisfied:

1. [illegible]
2. [illegible] associative operation
3. [illegible] identity
4. [illegible] element of [illegible]

Condition 1: [illegible] for any $x \in \Sigma^*$ [illegible]

$x \in \Sigma^* \Rightarrow x \in \Sigma^*$

Condition 2: For any strings $x, y, z \in \Sigma^*$,

$x \in [illegible] \Rightarrow x \in [illegible]$

Suppose we have for example in:

$x = [illegible] = [illegible] = 001$ then

$[illegible] = x[illegible]$

$= 01[illegible] = 001[illegible]$

$= 01[illegible]$

$[illegible] = x[illegible]$

$= 001[illegible] = 001[illegible]$

$= 01[illegible]$

Condition 4: [illegible] string [illegible]

$xy = [illegible]$

[illegible] Σ^* [illegible]

Condition 4: There is no [illegible] for Σ^*

[illegible]

So Σ^* with the concatenation [illegible] does form a monoid.

2.34 (d)

[illegible] $p(x) \rightarrow p(y)$ [illegible]

$\Rightarrow p(x) \rightarrow p(y)$

Now in option (d) True, $p(y)$ [illegible]

$p(x)$ [illegible]

2.35 (c)

[illegible] pairs [illegible] $(a, b), (b, a), (a, a)$ [illegible] Substitute them [illegible] one [illegible] and see which [illegible] satisfies the equation.

The given relations are:

$(a \vee b) \wedge (a \vee b) = a$ ....(i)

$(a \vee b) \wedge (a \vee b) =$ ....(ii)

[illegible]

[illegible] equation (i) [illegible]

$a \wedge b = a$

Now RHS of equation (ii) =

Therefore [illegible] RHS. This [illegible] (a, b) is not a solution [illegible]

Similarly by [illegible] the remaining [illegible] possible solution pairs.

[illegible] be found for only two pairs (b, a) and (a, a). [illegible] will satisfy both [illegible] equations.

Therefore choice (c) is correct.

2.36 (b)

$S = \{(x, y) \mid y = x - 1, \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$

$= \{(0, 1), (1, 2), (2, 3), (3, 4), \ldots\}$

Now in R, there is the equivalence of S.

$R_1 = \{(0, 0), (1, 1), (2, 2), (3, 3), \ldots\}$

$\{(0, 1), (1, 2), (2, 3), (3, 4), \ldots\}$

$\{(0, 2), (1, 3), (2, 4), \ldots\}$

$\{(0, 3), (1, 4), (2, 5), \ldots\}$

Let R_1 be [illegible] equivalence class of R

$(0, 1), (1, 2) \in S \Rightarrow (0, 2) \in R_2$

$(0, 2), (2, 3) \in S \Rightarrow (0, 3) \in R_3$

$(0, 3), (3, 4) \in S \Rightarrow (0, 4) \in R_4$

and so on ...

Also $(1, 2), (2, 3) \in S \Rightarrow (1, 3) \in R_2$

$(1, 3), (3, 4) \in S \Rightarrow (1, 4) \in R_3$

$(1, 4), (4, 5) \in S \Rightarrow (1, 5) \in R_4$

and so on ...

$\Rightarrow R = \{(0, 0), (0, 1), (0, 2), (0, 3), \ldots, (1, 2), (1, 3), (1, 4), \ldots\}$

Now the equivalence class of S is $R_1 = R_2 = R_3 = \{(0, 0), (0, 1), (0, 2), \ldots\}$, $R_2 = \{(1, 2), (1, 3), (1, 4), \ldots\}$.

Option (b) is correct.

2.37 (d)

[illegible] triangle must be the same number of upper triangle elements mirror. Diagonal elements are [illegible]. Therefore when we add [illegible] are the same. Then, per triangle and diagonal elements. Since the total [illegible] number, number $(n - 1)$ elements should be in [illegible] and [illegible] element.

Total number of elements in diagonal upper triangle

$= n + (n - 1) + (n - 2) + \ldots + 1$

$= n(n - 1)/2$

Now in the [illegible] these elements can be arranged [illegible] ways. All these elements

$= [illegible] = [illegible]$

Since there is no [illegible] in triangle elements the answer is power $(2, n(n + 1)/2)$ which [illegible].

2.35 (d)

[illegible]

2.36 (c)

[illegible]

2.37 (d)

Step 1:

[illegible]

Step 3

[illegible]

Step 5:

[illegible]

2.41 (a)

The Hasse diagram of the given poset is

Since [illegible] is the [illegible] non empty subset of [illegible] and [illegible] has a GLB.

Now the [illegible] poset [illegible] GLB of {1, 2} and {1, 2, 3} is {1}.

The Hasse diagram may be drawn as

Now the given Hasse diagram represents a complete lattice as every non empty subset has both a LUB and GLB. In addition, [illegible] is a bounded lattice, which is also a sub lattice of the given poset, hence a complete lattice.

Therefore, [illegible] is true.

2.42 (a)

$$X = (A \cup B) \cap C$$
$$= (A \cap B) \cup (B \cap C)$$
$$= (A \cap B) \cup (B \cap C)$$
$$= A \cap B \cap C$$
$$Y = (A \cup C) \cap (B \cup C)$$
$$= (A \cap C) \cup (B \cap C)$$
$$= (A \cap C) \cup (B \cap C)$$
$$= (A \cap C) \cup (B \cap C)$$
$$= (A \cap B) \cup (B \cap C)$$
$$= A \cap B \cap C \quad \text{(since } A \cap C = \phi\text{)}$$
$$= A \cap B \cap C \quad \text{(by commutative property)}$$
$$\therefore X = Y$$

2.43 (b)

The poset [illegible] is a chain hence every pair of elements has LUB and GLB. The poset is a distributive lattice. The distributive law [illegible] holds for [illegible] elements.

$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$

$x + (y \cdot z) = (x + y) \cdot (x + z)$

Where + and · are [illegible] of [illegible] in the given poset [illegible]

$0 + x = x + 0 = x$, $1 \cdot x = x \cdot 1 = x$

$0 \cdot x = x \cdot 0 = 0$, $1 + x = x + 1 = 1$

So it is a distributive lattice. Given element 1 has 2 complements and element 0 has 2 complements in a distributive lattice, complement is unique, so they are unique.

2.44 (c)

The set $S = \{1, 2, \ldots, 9, 10, 11, 12\}$ is a group under multiplication modulo 13.

The identity element for this group is 1 since,

$1 \times x = x \times 1 = x$

Now, [illegible] = 4, [illegible] = 4.

Since [illegible] and [illegible] = 1

$\text{order}(4) = 4$ and $[illegible] = 1$

[illegible] = 4 [illegible] is not a generator, unique

$\text{order}(7) = [illegible]$ = inverse of 7 in 7.

$(7 \cdot 7) \bmod 13 = 6$

[illegible] element of set S 7 is not a generator

$(7 \cdot 6) \bmod 13 = 9$ and $(7 \cdot 9) \bmod 13 = 1$

[illegible] = 3

So 4 and 7 are respectively 2 and 3 generators of the group.

2.45 (b)

R = {(x, y) : x is the relative of y} is an equivalence relation on the set of [illegible].

Let [illegible] = {(x, y) : x and y are [illegible] of [illegible]}

[illegible] = {(x, y) : x is the father of y}

[illegible] is reflexive

Now $(x, y) \in S$

$\Rightarrow (x, y) \in R$ and $(x, y) \in R$

$\Rightarrow (y, x) \in R$ and $(y, x) \in R$

(Since R and R are symmetric)

$\Rightarrow (y, x) \in R$

$\therefore (y, x) \in R \cap S$

$\Rightarrow (y, x) \in R \cap S$

Thus $R \cap S$ is symmetric

Now consider

$(x, y) \in (R \cap S) \cap S$

$\Rightarrow (x, y) \in (R \cap S)$

and (x, y) and $(y, z) \in S$

$\Rightarrow (x, y) \in R$ and (x, y) and $(y, z) \in S$

(Since $R \cap S \subseteq R$ and $R \cap S \subseteq S$)

$\Rightarrow (x, z) \in R \cap S$

$\therefore R \cap S$ is transitive and since $R \cap S$ is reflexive, symmetric and transitive

$\therefore R \cap S$ is equivalence relation

Statement (S) is true and statement (R) is false

Option (C) is correct.

3.46 (ABD)

Consider the given diagram shown below

Indeg $(v_1) = 0$

Indeg $(v_2) = 0$ (v1 → v2)

Hence the given graph is not a tree

As we can see from the diagram, it is not a connected graph

$\therefore$ It should be a forest, not be a tree

$\therefore$ Answer is (ABD)

3.47 (bd)

(a) The set U is closed under addition of upper triangular matrices, all result is an upper triangular matrix.

(b) Matrix multiplication is associative $(A \cdot B) \cdot C = (A \cdot B) \cdot C$.

(c) Take two elements

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and this belongs to U. But A^{-1} is not in U as A^{-1} is lower triangular matrix.

(d) If $A \in U$ then A^{-1} exists. It is given that A is upper triangular matrix. A^{-1} exists, every row of the upper triangular matrix is non-singular and A^{-1} is unique inverse.

$\therefore$ the set U has a unique inverse, inverse exists for every

3.48 (cd)

If $\text{sum} = g(x) = y^2$

Then we can the following parameters as given

Dominant vertex of the graph is not

given case → x^2 is a dominant vertex

$\therefore$ domain of $g(x)$ is the set of all of x.

Thus non-negative integer is one of the above mentioned cases

$h(x) = x^2 + y^2$

which also are to be true since sum is non-negative

So option (d) is true

3.49 (bc)

The set of all real valued functions in which is positive for all $x \in A$

i.e. $f(x) > 0$ is the set of all functions f where f is the set of all functions f such that $f(x) > 0$. We have a function which says that the f is a function from A to B and $f(x) > 0$

Hence $f(x)$ is a function from A to B and $f(x) > 0$

Hence $f(x)$ is a function from A to B and $f(x) > 0$

So the correct options are (bc)

3.50 (a)

I. $f(x) = x - 1 = 0$

$x + 1 = 0 \Rightarrow x = -1 + x = 0$

$x = 0$

Here 0 is integer and $x \in A$

So it is a function

II. $f(x) = \text{min}(x, y)$

$\text{min}(x, y) = x = \text{min}(x, y)$

Take 0 the only value of x which will satisfy the equation $f(x) =$

So it is a function, and $f(x) = 0$

So II has a function.

III: [illegible]

[illegible]

Now [illegible] has a solution

[illegible] in the power [illegible]

[illegible] III has a solution [illegible]

Hence, I and II are identical [illegible].

2.33 (d)

Given [illegible] $|X| = n$, [illegible] $|Y| = y$ and $|X| = x$

$$|X \times Y| = xy$$

So, [illegible]

$$\overline{X} = \overline{x}$$

$$\overline{Y} = \overline{y}$$

So the number of [illegible] relations from X to Y = $|X|^{|Y|}$

$$= 2^{xy} = 2^{mn}$$

2.34 (b):

Let [illegible] $A = \{1, 2, 3, 4, 5, 6, 7\}$

Here [illegible] the [illegible] any [illegible] we [illegible] that

$$x^2 + x = 1 \pmod 7$$

x	1	2	3	4	5	6	0
	1	2	3	4	5	6	0
x^2	2	4	6	0	3	5	1
x^2+x	3	6	0	4	1	5	2

[illegible] there [illegible] no [illegible] integer, [illegible] true

[illegible] $\gcd = 1$

[illegible] $\gcd = 1$

From the table we see that [illegible] not [illegible]

So, $\gcd(3, 7) \neq 1$

[illegible] 3 is [illegible] and [illegible]

[illegible]

Hence [illegible] has no [illegible].

2.35 (d):

[illegible] and [illegible]

[illegible]

So [illegible]

[illegible] This [illegible] relation

2.36 (d):

Consider the following Venn diagram [illegible]

$$X = \{E, F\}, \quad \{E, F, G\}$$

$$Y = \{E, F, G\}, \quad \{E, F\}$$

So, $X = Y$

[illegible] from [illegible] which we

$$X = (E \cap F) \cup (F \cap G)$$

$$= \overline{E \cap F} \cap \overline{F \cap G}$$

$$= (\overline{E} \cup \overline{F}) \cap (\overline{F} \cup \overline{G})$$

$$= (\overline{E} \cap \overline{F}) \cup (\overline{E} \cap \overline{G}) \cup (\overline{F} \cap \overline{F}) \cup (\overline{F} \cap \overline{G})$$

$$= (\overline{E} \cap \overline{F}) \cup (\overline{E} \cap \overline{G}) \cup \overline{F} \cup (\overline{F} \cap \overline{G})$$

$$= \overline{F} \cup (\overline{E} \cap \overline{G})$$

$$= (\overline{E} \cap \overline{G}) \cup \overline{F}$$

Similarly, $Y = (\overline{E} \cap \overline{F}) \cup (\overline{F} \cap \overline{G}) \cup (\overline{E} \cap \overline{G}) \cup \overline{F}$

$$= (\overline{E} \cap \overline{F}) \cup (\overline{F} \cap \overline{G}) \cup (\overline{E} \cap \overline{G}) \cup \overline{F}$$

$$= \overline{F} \cup (\overline{F} \cap \overline{G}) \cup (\overline{E} \cap \overline{G})$$

$$= \overline{F} \cup (\overline{E} \cap \overline{G}) \cup (\overline{F} \cap \overline{G})$$

$$= (\overline{E} \cap \overline{G}) \cup \overline{F}

[illegible]

[illegible] = 5

[illegible] and [illegible] = 8

$\therefore \sum_{i=1}^{5} f(i) = 2 + 3 + 3 + 3 = 12$

[illegible]

[illegible]

Now [illegible] = 5

S = {1, 3, 8, [illegible]}

[illegible] = number of 3 elements subsets = $^5C_3$ = 10

$\therefore$ n = 10

The 10 [illegible] {1, 3, 8}, {1, 2, 8}, {1, [illegible], 3}, {1, 3, 4}, {1, 3, 5}, {1, 4, 8}, {2, 5, 8}, {3, 8, 9}, {2, 4, [illegible], [illegible]}

[illegible]

$\sum_{i=1}^{5} f(i) = 8 \times 3 + 2 \times 2 = 28$

Since [illegible] m = 5 × 5 + 6 [illegible]

but [illegible] = 2 × 10 = 20 [illegible] 2(6) = 30

[illegible] correctly corresponds.

(Correct choice is (d))

[illegible]

$\sum f(i) = f(1) + f(2) + \ldots + f(n)$

Since [illegible] $= {}^nC_1$

$\sum f(i) = n \times {}^nC_1 = n^2$

2.16 (b)

$P(A \cap B) = P(A) \times P(B)$

where A is [illegible] between p and q

I. $P(PS) = P(A \cap B) = 6 = p(1-p)(1-p)(1-p)(1-p) + p(1-p)(1-p)(1-p)(1-p) = 6 \times p(1-p)$

[illegible]

[illegible]

So statement is true.

II. [illegible] $= p(1-p)(1-p)(1-p)(1-p) + p(1-p)(1-p)(1-p)(1-p) = 2p(1-p)(1-p)(1-p)(1-p)$

[illegible]

[illegible]

So statement II is true.

2.17 (b)

[illegible] = 1

[illegible]

[illegible] = 1

[illegible] = 2

[illegible] = 3

[illegible] = 4

[illegible] = 5

[illegible] = 6

[illegible] = 7

[illegible] = 8

[illegible] = 9

[illegible] = 10

[illegible] = 11

[illegible] = 12

[illegible] = 13

[illegible] = 14

[illegible] = 15

[illegible] = 16

[illegible] = 17

[illegible] = 18

[illegible] = 19

[illegible] = 20

[illegible] = 21

[illegible] = 22

[illegible] = 23

[illegible] = 24

[illegible] = 25

[illegible] = 26

[illegible] = 27

[illegible] = 28

[illegible] = 29

[illegible] = 30

[illegible] = 31

[illegible] = 32

[illegible] = 33

[illegible] = 34

[illegible] = 35

[illegible] = 36

[illegible] = 37

[illegible] = 38

[illegible] = 39

[illegible] = 40

[illegible] = 41

[illegible] = 42

[illegible] = 43

[illegible] = 44

[illegible] = 45

[illegible] = 46

[illegible] = 47

[illegible] = 48

[illegible] = 49

[illegible] = 50

[illegible] = 51

[illegible] = 52

[illegible] = 53

[illegible] = 54

[illegible] = 55

[illegible] = 56

[illegible] = 57

[illegible] = 58

[illegible] = 59

[illegible] = 60

[illegible] = 61

[illegible] = 62

[illegible] = 63

[illegible] = 64

[illegible] = 65

[illegible] = 66

[illegible] = 67

[illegible] = 68

[illegible] = 69

[illegible] = 70

[illegible] = 71

[illegible] = 72

[illegible] = 73

[illegible] = 74

[illegible] = 75

[illegible] = 76

[illegible] = 77

[illegible] = 78

[illegible] = 79

[illegible] = 80

[illegible] = 81

[illegible] = 82

[illegible] = 83

[illegible] = 84

[illegible] = 85

[illegible] = 86

[illegible] = 87

[illegible] = 88

[illegible]

2.18 (b)

[illegible]

[illegible]

[illegible]

We can be shown (c).

2.80 [illegible]

In this problem

[illegible]

[illegible]

[illegible]

[illegible]

Since all the above elements are powers of [illegible]

Hence [illegible] is cyclic.

Similarly

[illegible]

Since all the above elements are powers of [illegible]

2.15 (a)

[illegible]

Elements = [illegible]

Hence [illegible] = [illegible].

2.16 (a)

The [illegible] is always known as group. When [illegible] = 7 [illegible] [illegible] [illegible] [illegible] [illegible] group.

or

This [illegible] that [illegible] [illegible] [illegible] [illegible] [illegible]

2.71 (ii)

Given Facts & Lowing details:

[illegible]

[illegible] symmetrical [illegible] relation.

S1: [illegible] subset [illegible]

S2: There [illegible] subset of [illegible] has [illegible] subset.

S3: [illegible] which is subset of [illegible] than [illegible] subset.

[illegible] minimum [illegible]

[illegible]

(Note: [illegible]

[illegible] smaller than every other subset

[illegible]

(Note: [illegible]

[illegible] S1 and S3 are correct.

2.72 (b)

Example:

[illegible]

(a) [illegible]

[illegible]

(b) [illegible]

[illegible] is false

(c) [illegible]

[illegible] is false

(d) [illegible]

[illegible]

[illegible] is true

2.73 Sol.

Order of subgroup divides order of group (Lagrange's theorem). Given order of group, 6, 9 or 15. So it is given that the subgroup has at least 1 element and it is unique and trivial group, therefore the order of subgroup can't be 1, 3 or 15.

Hence [illegible]

2.74 (b)

Since it gives that of [illegible]

[illegible]

Verification of the first [illegible]

Statement P: [illegible] symmetric function or identity function which is not true. [illegible] there are more [illegible] than [illegible] function.

Example: [illegible]

Statement Q: [illegible] symmetric [illegible] identity function.

This is true, [illegible]

Statement R: [illegible] function is [illegible] which is true, since [illegible]

2.75 Sol.

[illegible]

[illegible]

[illegible]

[illegible]

Now [illegible]

[illegible]

[illegible]

[illegible] and [illegible]

There are only [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

All other combinations are equal to one of these [illegible]

[illegible] already proved)

[illegible]

[illegible]

[illegible]

[illegible]

and so on...

So the group [illegible]

[illegible]

2.76 (b)

$$a(t) = 1 \quad , \quad b(t) = \frac{1}{t-1}$$

[illegible]

$$= \frac{1}{1-x} - \frac{x-1}{1-x}$$

[illegible]

$$\frac{f(x)}{g(x)} = \frac{x}{1-x} = \frac{x}{(1-x)(1-x)} = \frac{-x}{(x-1)^2}$$

$$\therefore \frac{g(x)}{f(x)} = \frac{f(x)}{g(x)}$$

2.79 Let:

$A = \{x, y, z\}$

1. $\{x, y, z\}$ is trivially power set of A and is [illegible]
2. $\{x, y\}$ is true. Empty set is [illegible]
3. $\{\{x, y\}\}$ is true as power set of A must contain $\{x, y\}$ as a element subset of A.
4. $\{x, y\}$ is false. Power set of A must contain $\{x, y\}$ as element not as [illegible]
5. $\{x, y\}$ is [illegible] solution of 2^A.

2.78 (d):

A and B contents

[illegible]

$\therefore$ [illegible] $= 2! = 2$ elements

If a is given, then only b will be [illegible] the [illegible]

Number of [illegible] $= 2! = 2 \times 2 \times 1 = 6$.

$\therefore$ possessing distributive property $= 1$

[illegible] distributive property $=$

$= 1 - \frac{6}{9} = 0.667$ which is between 0.5 and 1.

2.79 (d):

[illegible]

(a) [illegible]

(b) [illegible]

(c) R is symmetric

[illegible] common [illegible] that 1, [illegible] and also [illegible] distance and [illegible] common divisor [illegible] 1

(d) R is [illegible]

If [illegible] and [illegible] then [illegible] R [illegible]

Example: $(2, 8) \in R$ and $(8, 2) \in R$ [illegible] $(2, 2) \notin R$

$\therefore$ R is [illegible] but relative not transitive.

2.80 (a):

$\therefore$ $S = \{0, 1, 2, \ldots, 10\}$

$|S| = 11$

$T(x) =$ [illegible]

2.81 (b):

The number of onto functions from $A \to B$ where $|A| = m$ and $|B| = n$ is given by the formula

$= {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - \ldots - {}^nC_{n-1} 1^m$

Here $m = 4$ and $n = 3$

Substituting in above formula we get

$3^4 - {}^3C_1 2^4 + {}^3C_2 1^4$

$= 81 - 48 + 3 = 36$

2.82 (b):

Total possible relations $= 2^{20} = 1048$

Negative of onto functions

$= 20P_2 = 20 \times 19$

$\therefore$ Required probability $= \frac{20 \times 19}{2^{20}} = 0.018$

8.28 (b):

$$\forall x \in X,\ x \cdot x = x$$

This is the NAND operation. NAND is commutative but not associative.

So only (b) is true.

8.29 (d):

$X = \{1, 2, 3, 4\}$

Let be the power set of X $= P(X) = P(X)$

$P = \{\{\}, \{1\}, \{2\}, \ldots, \{1, 2\}, \{1, 3\}, \ldots, \{1, 2, 3\}, \ldots, \{1, 2, 3, 4\}\}$

$|P| = 2^4 = 16$ elements

$\emptyset \in P \Rightarrow \emptyset \subset X$

$\emptyset \cap X = \emptyset$, $\emptyset \cup X = X$

(a) $\forall X \in P, \exists Y \in P$ means for every subset of X there exists a subset Y such that $X \cap Y = \emptyset$.

This is false.

For example, $X = \{1\}$ and there is no subset Y such that $X \cap Y = \emptyset$.

(b) $\forall X \in P, \exists Y \in P$ means for every subset X there exists a subset Y such that X and Y have no common element. This is clearly false.

Simply any non-empty set X and Y have at least one element in common.

(c) $\forall X \in P, \exists Y \in P$ and $X \cap Y = \emptyset$ means that for every subset X and every subset Y we have $X \cap Y = \emptyset$.

This is true only if the set can be seen from the example $X = \{1\}$, $Y = \{2, 3, 4\}$ where $X \subseteq Y$.

(d) $\forall X \in P, \forall Y \in P, (X \cap Y = \emptyset \Rightarrow X \subseteq Y)$ means that for any two subsets X and Y, $X \cap Y = \emptyset \Rightarrow X \subseteq Y$ i.e. $X \cap Y = \emptyset \Rightarrow X$

This is true only if $X = \emptyset$ as per the boolean algebra.

LHS $= X \cap Y = \emptyset$

RHS $= X = \emptyset$

$\Rightarrow$ RHS $= X = \emptyset$ $\Rightarrow$ $X \cap Y = \emptyset$ $\Rightarrow$ LHS = RHS.

8.30 (a):

$R = \{(x, y) \mid x - y = 2\}$

(i) Check for reflexive property.

$\forall x \in X$, $(x, x) \in R \Rightarrow x - x = 2$

$\Rightarrow 0 = 2$ which is false.

So relation R is not reflexive.

(ii) Check for symmetric property.

If $(x, y) \in R$ then $(y, x) \in R$

$(x, y) \in R \Rightarrow x - y = 2$

$(y, x) \in R \Rightarrow y - x = -2$

If $x - y = 2$ is true then $y - x = -2$ is false.

So relation R is not symmetric.

8.31 (b):

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ \frac{x+1}{2} & \text{if } x \text{ is odd} \end{cases}$$

$f: X \to X$

Now $f(2) = f\left(\frac{2}{2}\right) = f(1)$

$f(3) = f(3+1) = f(4) = f\left(\frac{4}{2}\right) = f(2)$

$f\left(\frac{4}{2}\right) = f(2) = 1$

So $f(1) = f(2) = f(3) = f(4) = f(5)$

Now let us find $f(f(x)) = f(f(x)) = f(f(1)) = f\left(\frac{1}{2}\right)$

$= f(1) = f(2) = f(f(2))$

Now let us find $f(3)$:

$f(3) = f(f(3)) = f(4) = f\left(\frac{4}{2}\right) = f(2)$

$f(4) = f(f(4)) = f\left(\frac{4}{2}\right) = f(2)$

So $f(3) = f(4) = f(2)$

Thus $f(x)$ will be equal to one of $f(1)$, $f(2)$ or $f(3)$.

[illegible] maximum number of distinct [illegible] is only [illegible]

Thus [illegible]

Since [illegible]

Point is [illegible]

All other x values will give only one of these three values

2.57 (b)

[illegible]

P : R is reflexive

Q : R is transitive

[illegible]

While [illegible] R is reflexive

[illegible]

[illegible]

[illegible]

Now let us take a counter example

[illegible]

[illegible]

[illegible]

So [illegible] R is not transitive.

So P is true and Q is false. Choice (b) is correct.

2.58 (b)

I. If there is a tree of 8 components out of 24 components, each of which is connected with exactly 3 components, then the number of edges in the tree is 16

II. Every component of U contains exactly odd number of components in [illegible]

III. Every component of U contains exactly even number of components in U

Only II is true.

3 Combinatorics

3.1 (a) Solve the following recurrence relation:

$T(n) = T(n-1) + n$

$T(1) = 1$

(b) What is the generating function for the sequence 1, 1, 1, 1, 1, 1, 1, 1, ...?

[1997 : 2 Marks]

3.2 Solve the following recurrence relation:

$T(n) = T\left(\frac{n}{2}\right) + 1$

$T(1) = 1$

[1998 : 8 Marks]

3.3 How many substrings of length two are there in a character string of length n?

[1999 : 2 Marks]

3.4 The number of binary strings of length n in which no two consecutive 0's appear is

(a) $n!$ (b) 2^n

(c) ^nC_n (d) None of these

[1999 : 1 Mark]

3.5 The number of binary strings of length n in which no two consecutive 0's appear is

(a) $n!$ (b) n^2

(c) $\binom{n+1}{2}$ (d) $\frac{n(n+1)}{2}$

[1994 : 2 Marks]

3.6 In a small community of 89 people, there are 18 persons who speak English, 19 people who speak Hindi and 20 people who speak Kannada. 8 persons speak both English and Hindi, 7 persons speak both Hindi and Kannada, whereas 6 persons speak both Kannada and English. How many people speak all three languages?

(a) 9 (b) 8

(c) 7 (d) 6

[1998 : 2 Marks]

3.7 Solve the following recurrence relation:

$x_n = 2x_{n-1} - x_{n-2}$

$x_1 = 2$

[1998 : 2 Marks]

3.8 The sum of the positive integers x and y is 100. What is the maximum value for the product xy?

(a) 2500 (b) 2600

(c) 2700 (d) None of these

[1999 : 2 Marks]

3.9 The minimum number of elements that must be added to make it a group is

(a) 3 (b) 6

(c) 9 (d) 12

[2000 : 1 Mark]

3.10 How many 4-digit numbers have at least one digit divisible by 11?

(a) 2401 (b) 3276

(c) 3333 (d) 4545

[2001 : 2 Marks]

3.11 The solution to the recurrence equation $T(2^k) = 3T(2^{k-1}) + 2^k$, $T(1) = 1$ is

(a) 2^k (b) $\frac{(3^k - 2^k)}{2}$

(c) $3^{\log_2 k}$ (d) $2^k \log_2 k$

[2002 : 1 Mark]

3.12 The minimum number of colours needed to colour a simple connected graph in such a way that no two adjacent vertices have the same colour is

(a) 2 (b) 3

(c) 4 (d) $n - \left\lceil \frac{n}{2} \right\rceil$

[2002 : 1 Mark]

3.13 A set S consists of 3 distinct integers arranged in ascending order. How many distinct non-empty subsets of S can be there such that (i) each subset has odd number of elements, (ii) the sum of the elements of each subset is even, and (iii) the product of the elements of each subset is divisible by 3?

(a) 2 (b) 10

(c) 63 (d) 255

[2005 : 1 Mark]

3.14 [illegible] property with the condition that every [illegible] should be accompanied by [illegible] will [illegible] husband. The number of [illegible] pairs possible with the property is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2003 : 1 Mark]

3.15 Mali has a [illegible] in which every English letter is drawn [illegible] times. The [illegible] such that [illegible] and in any [illegible] consecutive letters are different. Let [illegible] be the [illegible] with [illegible] letters. What is the maximum value of [illegible] for which [illegible] requirement?

(a) 4 (b) 8

(c) 7 (d) 6

[2004 : 2 Marks]

3.16 In how many ways can we divide [illegible] balls [illegible] into [illegible] distinct [illegible] such that [illegible] and each bin contains exactly one ball?

(a) 4! (b) [illegible]

(c) [illegible] (d) [illegible]

[IT 2005 : 2 Marks]

3.17 [illegible] where p and q are distinct prime numbers. How many [illegible] are there in the group [illegible]?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[IT 2005 : 2 Marks]

3.18 What is the [illegible] of the [illegible] such that [illegible] and [illegible] modulo 5

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

3.19 [illegible] $= \sum_{i=0}^{n} c_i x^i$ where [illegible] $x = 1$. What is [illegible]?

(a) [illegible] (b) $i + 1$

(c) [illegible] (d) $\emptyset$

[2006 : 2 Marks]

3.20 The set of elements in [illegible] is [illegible] [illegible] [illegible]

Common Data for Q.3.24 & Q.3.25

Let x_n denote the number of binary strings of length n that contain no two consecutive 0's.

3.24 Which of the following recurrence relations is satisfied by x_n?

(a) $x_n = 2x_{n-1}$ (b) $x_n = x_{n-1} + 1$

(c) $x_n = x_{n-1} + x_{n-2}$ (d) $x_n = x_{n-1} + x_{n-2} + 1$

[2005 : 1 Marks]

3.25 The value of x_5 is

(a) 5 (b) 7

(c) 8 (d) 13

[2005 : 2 Marks]

3.26 The sum of the first 100 positive integers divisible by 6 is

(a) 267 (b) [illegible]

(c) 289 (d) 300

[IT-2005 : 2 Marks]

3.27 In how many ways can a committee of 5 people be chosen from 10 people if Mr. X refuses to serve on the committee if Mr. Y is a member of the committee?

(a) $\frac{10!}{5!\,5!} - \frac{8!}{5!\,3!}$

(b) $\frac{10!}{5!\,5!} - \frac{9!}{4!\,5!}$

(c) $\frac{9!}{5!\,4!}$

(d) $\frac{9!}{5!\,4!} + \frac{8!}{5!\,3!}$

[2005 : 2 Marks]

3.28 A permutation is a sequence of numbers in which each number is followed by its image under the permutation. For example, (1, 2, 3) and (2, 3, 1) are two distinct permutations of the set {1, 2, 3}. The set of all possible permutations of {1, 2, 3} is {(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)}. Consider the set {1, 2, 3, 4, 5, 6, 7}. The number of 7! permutations of {1, 2, 3, 4, 5, 6, 7} that do not contain the pattern (2, 3, 4) is _____.

[2014 (Set-1) : 2 Marks]

3.29 The number of divisors of the positive integer 2014 is _____.

[2014 (Set-2) : 2 Marks]

3.30 $\sum_{i=1}^{\infty} \frac{i}{2^i} =$ _____

[2016 (Set-1) : 2 Marks]

3.31 [illegible] represent the number of ways of [illegible] consisting of [illegible] coins. What is the recurrence relation for a_n?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2013 (Set-1) : 2 Marks]

3.32 The number of divisors of 2016 is _____

[2013 (Set-2) : 1 Mark]

3.33 The number of 4 digit numbers [illegible] that can be formed using 1, 2, 3, 4 is

[2013 (Set-2) : 1 Mark]

3.34 Let a_n be the number of bit-strings of length n that do NOT contain two consecutive 0's. Which one of the following is the correct recurrence relation for a_n?

(a) $a_n = a_{n-1} + a_{n-2}$ (b) $a_n = 2a_{n-1} + a_{n-2}$

(c) $a_n = 2a_{n-1} + a_{n-2} + 2a_{n-3}$ (d) $a_n = 2a_{n-1} + 2a_{n-2}$

[2016 (Set-1) : 1 Mark]

3.35 The coefficient of x^5 in $(x+1)(x+2)\cdots(x+10)$ is

[2016 (Set-1) : 2 Marks]

3.36 Consider the recurrence relation $a_n = 9a_{n-1} - 20a_{n-2}$ for $n \geq 2$. If $a_0 = 1$, $a_1 = 9$, the value of a_2 is

[2016 (Set-1) : 2 Marks]

■■■■

Answers Combinatorics

8.1	(a)	8.5	(d)	8.6	(d)	8.7	(c)	8.8	(c)	8.9	(a)	8.11	(b)	8.12	(d)	8.13	(c)
8.14	(b)	8.15	(d)	8.16	(a)	8.17	(b)	8.18	(c)	8.19	(a)	8.20	(a)	8.21	(a)	8.22	(d)
8.23	(c)	8.24	(d)	8.25	(d)	8.26	(b)	8.27	(a)	8.28	(a)	8.29	(b)				

Explanations Combinatorics

8.1 (a)

(a) [illegible]

$T(n) = T(n-1) + n$

$T(n) - T(n-1) = n$

For Homogeneous solution

$[E - E_{-1}] = 0$

$1 - [illegible] = 0$

$= 1$

Homogeneous solution

$T_h = C(1)^n = C$

For Particular solution

Let particular solution include $d_1 n$

$\Rightarrow T_p = d_1 n$ [illegible] $d_1 = d_1 (n-1) + n$

$\Rightarrow d_1 n = d_1 n - d_1 + n$

$\Rightarrow d_1 n = d_1 n - d_1 + n$

$\Rightarrow d_1 + 2d_1 = 1$

$d_1 = 1/2$ and $d_1 = 1$

$\Rightarrow d_1 = d_1$ and $d_1 = [illegible]$

$\Rightarrow d_1 = [illegible]$

So particular solution

$\left[\frac{1}{2} \quad \frac{1}{n}\right] = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$

So complete solution is

$T(n) = C + \frac{1}{2}(n^2 + n)$

Given $T(1) = 1$

$1 = C + \frac{1}{2}(1^2 + 1)$

$1 = C + 1$

$C = 1$

Therefore complete solution [illegible]

solution is $T(n) = \frac{n(n+1)}{2}$

(d) The Fibonacci numbers are defined as

$x_0 = 1, x_1 =$

$x_n = x_{n-1} + x_{n-2}, n \geq 2$

$\Rightarrow \sum_{n=2}^{\infty} x_n z^n = \sum_{n=2}^{\infty} x_{n-1} z^n + \sum_{n=2}^{\infty} x_{n-2} z^n$

$\Rightarrow \sum_{n=2}^{\infty} x_n z^n = z \sum_{n=2}^{\infty} x_{n-1} z^{n-1} + z^2 \sum_{n=2}^{\infty} x_{n-2} z^{n-2}$

$\Rightarrow$ Let $X(z) = \sum_{n=0}^{\infty} x_n z^n$ [illegible]

So $x_0 = 1, x_1 = 1$ [illegible]

$\Rightarrow$ Let $X(z) = \sum_{n=0}^{\infty} x_n z^n$ [illegible]

$\Rightarrow X(z) = \frac{1}{1 - z - z^2}$

8.5 Sol.

$x(n) = \frac{-1}{2} \frac{n}{2} + 1$

$\Rightarrow$ [illegible] $\frac{n}{n}$

Let $n = 2^k$.

$\Rightarrow$ [illegible]

$\Rightarrow T(n) = T(n-1) + [illegible]$

Let $T(n) = x_k$

$x_k - x_{k-1} = 1$

For Homogeneous solution

$x_k - x_{k-1} = 1$

$1 - 1 = 0$

$1 = 1$

So homogeneous solution is $x_k = C(1)^k = C$

For Particular solution

Let [illegible] $d_1 k$

$d_1 k - d_1 (k-1) = 1$

$\Rightarrow d_1 = 1$

Therefore solution is

[illegible]

$x_k = C + k$

$T(2^k) = C + k$

$T(n) = C + \log_2 n$

Given $T(1) = 1$

$\Rightarrow C = 1$

So complete solution is $T(n) = \log_2 n + 1$

33 Sol.

[illegible]

Number of strings of length 0 = 1 (only 0)

Number of strings of length 1 = 1

a, b, c, d

Number of substrings of length 1 = [illegible]

aa, bb, cc, dd

Number of substrings of length 2 = [illegible]

aaa, bbb, ccc, ddd

Number of substrings of length 3 = [illegible]

aaaa, bbbb, cccc, dddd

[illegible] number of substrings

$= 1 + (1 + 2 + 3 + 4)$

(Sum = [illegible] natural numbers)

$= 1 + \frac{4 \cdot (4+1)}{2} = 11$

Therefore, total number of substrings [illegible] characters can be constructed from a character alphabet of length 4 is $\frac{n(n+1)}{2} + 1$.

34 Sol.

[illegible]

Number of ways for arranging [illegible] = 11 ways

[illegible]

Required number of [illegible]

[illegible]

35 Sol.

The number of strings of length 2 = [illegible]

The number of substrings of length 1 = [illegible]

The number of substrings of length 2 = [illegible]

The number of substrings of length 3 = [illegible]

[illegible]

The number of substrings of length n is 1

Required number of substrings

$= 2 + 3 + 4 + \ldots + (n+1)$

= sum of first n natural numbers

$= \frac{n(n+1)}{2}$

36 Sol.

[illegible] English

[illegible] Hindi

[illegible] Kannada

[illegible]

[illegible]

[illegible]

3.6 (d):

Number of ways for distribution of [illegible] among [illegible] = [illegible]

The number of ways for distribution of [illegible] among [illegible] = [illegible]

Number of ways for distribution of [illegible] among [illegible] = [illegible]

$= {}^{15}C_{10} = {}^{15}C_{5} = 3003$

Number of ways for distribution of [illegible] among [illegible] = [illegible]

$= {}^{15}C_{10} = {}^{15}C_{5} = 3003$

$\therefore$ Total number of ways = [illegible] × 3003 × 3003 = 2[illegible]

3.9 (b):

Let the number of [illegible] = [illegible]

Number of ways = [illegible]

Required number of words from the same [illegible] = 8.

Using pigeonhole principle

$$\left\lceil \frac{n-1}{8} \right\rceil + 1 = 9$$

$$\left\lceil \frac{n-1}{8} \right\rceil = 8$$

$$\lceil n-1 \rceil = 8$$

$$n = 9$$

So the minimum number of [illegible] from [illegible] is 9. Thus a block of 32 words is sufficient to guarantee that three words are from same [illegible] and is 9.

3.10 (b):

The [illegible] to be [illegible] digit even numbers starting with 0 and must end with [illegible]

[illegible] condition for 0 [illegible] as well as condition given will [illegible] with 0 separately.

So the total number of 4 digit even numbers ending with zero 4 digits [illegible] in 3, 4, 6, 8

4 digit even numbers ending with [illegible]

[illegible]

4 digit even numbers ending with [illegible]

[illegible]

So the total number of 4 digit even numbers

[illegible]

3.11 (b):

[illegible]

[illegible]

[illegible]

[illegible]

So from [illegible] homogeneous solutions

$$x_1 = {}^{8}D_{3} = 3$$

$$x_2 = {}^{8}D_{1} = 1$$

$$= 8$$

Homogeneous solutions

$$x_3 = {}^{15}D_{8}$$

$$[illegible] = {}^{15}D_{8}$$

The particular solution

[illegible]

$$d = -\frac{1}{2}$$

Therefore the complete solution is

$$D_{15} = {}^{15}D_{8} - \frac{1}{2}$$

Given, [illegible]

$$1 = {}^{15}D_{8} - \frac{1}{2}$$

$$1 = {}^{15}D_{8} - \frac{1}{2}$$

$$[illegible] = \frac{3}{2}$$

So the required solution is

$$D_{15} = \frac{[illegible]}{2} - \frac{1}{2}$$

$$D_{15} = \frac{[illegible]}{2}$$

3.12 (d):

[illegible] minimum number of [illegible] will be seven the [illegible] will be

= 2, when [illegible]

= 8, when [illegible]

Therefore [illegible] = 2 [illegible] and [illegible] odd.

3.13 (b):

This corresponds to the [illegible] problem. [illegible] with 3 [illegible] with 4 elements. The number of [illegible]

[illegible]

8.14 (i)

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]

8.15 (ii)

[illegible]

Since [illegible] pairs of [illegible] are possible [illegible] ordered

[illegible]

[illegible]

[illegible]

[illegible]

8.16 (iii)

[illegible]

$$D_n = \sum_{i=0}^{n} (-1)^i \frac{n!}{i!}$$

$$D_5 = \sum_{i=0}^{5} (-1)^i \frac{5!}{i!} = \frac{5!}{0!} - \frac{5!}{1!} + \frac{5!}{2!} - \frac{5!}{3!}$$

$$= 120 - 120 + 60 - 20 = 40 \text{ derangements.}$$

8.17 (iv)

[illegible]

[illegible]

$$\phi(n) = n(1-\frac{1}{p_1})(1-\frac{1}{p_2})\ldots$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

8.18 (iv)

[illegible]

[illegible]

[illegible]

[illegible]

8.19 (iii)

$$\frac{1}{(1-x)^n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r x^r$$

Put $n = 3$,

$$\frac{1}{(1-x)^3} = \sum_{r=0}^{\infty} {}^{3+r-1}C_r x^r$$

$$= \sum_{r=0}^{\infty} {}^{r+2}C_r x^r$$

$$= \sum_{r=0}^{\infty} \frac{(r+2)(r+1)}{2!} x^r$$

$$= \sum_{r=0}^{\infty} \frac{(r+2)(r+1)}{2} x^r$$

$$= \sum_{r=0}^{\infty} \frac{(r+2)(r+1)}{2} x^r$$

Since [illegible]

$$= \sum_{r=0}^{\infty} a_r x^r$$

[illegible]

8.20 (iv)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

1.21 (b)

Consider the following diagram:

Pole 1 has one more only right to move the top disk on it to pole 3.

Let us denote right move by 'R' and left move by 'L'. Now, starting from (0, 0, 0) to (0, 0, 1), there is only one move, which is 'R'. Now, starting from (0, 0, 1) to (0, 1, 1), there is only one move, which is 'L'.

Similarly to move from (0, 1, 1) to (1, 1, 1), there is only one move, which is 'R'. Now, starting from (1, 1, 1) to (1, 1, 0), there is only one move, which is 'L'.

The number of moves required to move from (0, 0, 0) to (1, 1, 0) is equal to the number of permutations of 'R' and 'L' consisting of 10 'R's and 10 'L's.

Applying formula of permutation with repetition, we get the answer as $\frac{20!}{10!\,10!} = {}^{20}C_{10}$.

1.22 (d)

The solution to each of the sub-problems S_{ij} may be stored in a two dimensional array. Now, after solving S_{11}, the solution should be given to S_{12} and S_{21}. Now, after solving S_{12} and S_{21}, the solution should be given to S_{13}, S_{22} and S_{31} and so on. So, we can subtract the sum of the number of moves to get the answer.

Now, the number of S_{ij} moves should be equal to each other. So, in S_{11}, C should then move to 1 as the 1st move. So, (4, 4) to (5, 1). Now, from (5, 1) to (10, 10) choose 1 box to make 9 moves and 9 moves to move over which can be done in $\frac{10!}{9!}$ ways $= {}^{10}C_1$ ways.

∴ The number of ways which can move from (5, 1) to (10, 10) with 1 move is $= {}^{10}C_1 \times {}^{9}C_1 = \frac{10!}{9!} \times 1 = 10$.

∴ Number of ways which can move from (0, 0) to (10, 10) with 1 move, 10 moves and 9 moves is

$$\frac{(20)!}{(10)!} \times {}^{10}C_1 \times {}^{9}C_1 = \frac{(20)!}{(10)!} \times 10 \times 9 \text{ ways}$$

which is option (d).

1.23 (b)

$P = 2 + 5 + 8 + 11 + 14 + \ldots + (3k - 1)$

$= \frac{k}{2}[2 \times 2 + (k - 1) \times 3]$

$Q = 3 + 7 + 11 + 15 + \ldots + (4k - 1)$ [illegible]

P is an AP with $a = 2$, $d = 3$ and $n = k$.

Q is an AP with $a = 3$, $d = 4$ and $n = k$.

$$P = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{k}{2}[2 \times 2 + (k - 1) \times 3]$$

$$= \frac{k}{2}[4 + 3k - 3] = k^2$$

$$Q = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{k}{2}[2 \times 3 + (k - 1) \times 4]$$

$$= \frac{k}{2}[6 + 4k - 4]$$

$$= \frac{k}{2}[2k + 2]$$

$$= k^2 + k$$

Clearly, $P < Q < 2P$.

1.24 (c)

[illegible] consecutive [illegible]

First [illegible] $y = 1$

$x_2 = 2$

$x_1 = 2$

$x_3 = 2$

$x_2 = 2$

$x_4 = 2$

Now, [illegible] $= 2$ [illegible] choose [illegible] $x_1 = x_2 + x_3$ satisfies the numbers obtained from the recursion.

3.95 Sol:

$x_1 = 3,\ x_2 = 4$

$\Rightarrow x_3 = 5 - x_1 - x_2 = 5 - 3 - 4 = -2$

$\Rightarrow x_4 = 7 - x_1 - x_3 = 7 - 3 + 2 = 6$

$\Rightarrow x_5 = x_1 + x_4 = 3 + 6 = 9$

So, given statement is correct.

3.96 (b)

FOC: $x_1 \times 2 \times 3 \times \ldots \times 99$

$\left|\begin{matrix} 3 & 1 & 1 & 2 \\ 1 & & & \end{matrix}\right.$

So, there are 27 multiples of 11 from 1 to 300, so 300 will not be a multiple of 11 and so, the number of zeros is 27.

Only 31 will be a prime number. Thus, the number of zeros in 11 and 31 is 27 only.

So total number of 11's = 27 + 1 = 28

So exponent of 11 is 28.

3.97 (a)

The 5 balls can be distributed among 6 distinct boxes in $C(5+6-1, 5) = C(10, 5) = 252$ ways.

Value of the distribution $C(n+k-1, k) = \frac{(n+k-1)!}{k!(n-1)!} = \frac{(n+k-1)!}{(n-1)!}$

Since they are indistinguishable by product rule total ways = 252 × 5! = 252 × 120 = 30240.

3.98 Sol:

In a 20 person committee, one person is to be selected to find out the solution.

$x_1 + x_2 = 20$

$\Rightarrow x_1 = 0 \Rightarrow x_2 = 1 \Rightarrow 0 + 1 = 1$

For 1 solution, in 20 member committee, have 0 males and 1 female. The number of ways of permutation of 0 male and 1 female is

$= 0! \times 1! = 1$

Now, for 2 solutions in the case $x_1 = 1$ $x_2 = 0$ (female member)

$\Rightarrow x_1 + x_2 = 1 \Rightarrow x_2 = 1 - 1 = 0$

For 2 solutions, in 20 member could have 1 male and 1 female. The number of permutation of 1 male and 1 female is

$= \frac{2!}{1!1!} = 2$

So, for 1 or 2, 50, 60, 70, 80, 90 and 100 are the other solutions. Total number of solutions are 8.

$\frac{7!}{4!3!} = 35,\ \frac{8}{6!2!} = 28,\ \frac{9!}{5!4!} = 126,\ \frac{10!}{6!4!} = 210$

So, total number of 20 members

$= 1 + 2 + 35 + 28 + 126 + 210 = 402$

3.99 Sol:

[illegible]

Now, in the [illegible] number to pick.

Now, we make a probability distribution table for r

r	0	1	2
p(r)	1/3	1/2	1/6

From the table, we can easily find the expected value of r.

$E(r) = \sum r_i p(r_i) = 0 \times \frac{1}{3} + 1 \times \frac{1}{2} + 2 \times \frac{1}{6}$

$= \frac{6}{6} = 1$ [illegible] multiplication table [illegible] accuracy.

3.100 Sol:

[illegible]

Factorising 20 4 [illegible] by successively dividing by primes we get

$20 = 2 \times 10 = 2 \times 2 \times 5$

Now, we use the formula

[illegible] is the prime factorization of [illegible]

then the number of divisors [illegible] is given by $(n_1+1)(n_2+1)\ldots(n_k+1)$.

Now since $20 = 2^2 \times 5^1$, the number of divisors [illegible]

$= 2 \times 2 \times 2 = 8$

3.101 Sol:

$\sum_{r=0}^{n} \frac{1}{r!(n-r)!} = \frac{1}{n!}\sum_{r=0}^{n} \frac{n!}{r!(n-r)!}$

$= \frac{1}{n!}\left[\frac{n!}{0!n!} + \frac{n!}{1!(n-1)!} + \frac{n!}{2!(n-2)!} + \ldots + \frac{n!}{n!0!}\right]$

$= \frac{1}{n!}$ [illegible] the number in above series [illegible]

$= \frac{99}{100} = 0.99$

4.82 (d)

a_n = [illegible] strings of length n with no 1's in consecutive 1's

$a_1 = 2$ [illegible strings = 1, 0]

$a_2 = 3$ [illegible strings = 00, 01, 10]

$a_3 = 5$ [illegible strings = 000, 001, 010, 100, 101]

Option (a):

$a_1 = a_2 = a_3 = 2$ [illegible]

$\Rightarrow a_1 = a_2 = a_3 = 2^1$ [illegible]

a_n [illegible] $\forall n \geq 1$ [illegible] $a_n = 2$ which is False

Option (b):

$a_1 = a_2 = 2a_3$ [illegible]

$\Rightarrow a_1 = a_2 = 2a_3 = 2^1$

[illegible] which is False.

Option (c):

$a_1 = 2a_2 = a_3 = 2^1$ [illegible]

$\Rightarrow a_1 = 2a_2 = a_3 = 2^1$

[illegible] which is False.

Option (d):

$a_1 = 2a_2 + a_3$ [illegible]

$\Rightarrow a_1 = 2a_2 + 2a_3 = 2$

[illegible] $2 \times 2 + 1 = 5$ which is False.

$\therefore$ Option (d) [illegible] a_{n-1} [illegible] is correct.

4.84 (50)

According to DCC outcomes only by [illegible] we get that the prime factors of 2100 are $2 \times 2 \times 3 \times 5 \times 5 \times 7$

$= 2^2 \times 3 \times 5^2 \times 7$

If $N = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ [illegible] then number of factors of N are $(\alpha_1+1)(\alpha_2+1) \cdots (\alpha_k+1)$

$\therefore$ For 2100 we have $(2+1)(1+1)(2+1)(1+1) = 36$ factors.

4.85 (50)

We consider the case for generating prime numbers where $i = 2, 3, 5, 7$ for 1 digit only 2, 3 digits only 2 and 3 digits only 5 and 7. The following tree diagram gives all the 4 digit numbers satisfying the given condition.

All the 4 digit numbers satisfying the given condition

$\Rightarrow$ Number of 4 digit numbers = Number of 4 digit numbers = 15

4.86 (50)

Let a_n be the number of ternary strings that do not contain two consecutive 1's. We need to develop a recurrence relation for a_n.

Consider 1 bit strings 0, 1.

So $a_1 = 2$

Consider 2 bit strings 00, 01, 10, 11.

Out of this, one string 11 is not acceptable as it contains consecutive 1's.

So $a_2 = 3$

Consider 3 bit strings

Consider some [illegible] strings of length 1 to 100 [illegible] Possible [illegible] consecutive 1's.

So $a_1 = 2$. Then [illegible] a_1, a_2, a_3 [illegible] $a_n = a_{n-1} + a_{n-2}$ is correct.

4.87 (50)

We need to find coefficient of x^5 in $(x^2 + x^3 + x^4 + x^5)^5$

$$= (x^2(1 + x + x^2 + x^3))^5$$

$$= x^{10}(1 - x)^{-5}(1 - x^4)^5$$

$$= \frac{x^{10}}{(1-x)^5} \cdot x^{4} \sum_{i=0}^{5} (-1)^i C(5, i) x^{4i}$$

$$= x^{10} \sum_{i=0}^{5} (-1)^i C(5, i) x^{4i}$$

So we need to find x^{5-10} or x^{-5} to put $i = 0$.

So coefficient of x^5 is $(-1)^0 C(5, 0) = C(5, 0) = 1$

[illegible]

Given [illegible]

Generating function [illegible]

Now [illegible]

Since [illegible]

[illegible]

■■■

4 Graph Theory

4.1 Which of the following graphs are planar? Give reasons.

[1998 : 8 Marks]

(a) G1 only (b) G1 and G2
(c) G2 only (d) G2 and G3

4.2 A graph is planar if and only if

(a) it does not contain subgraphs homeomorphic to K_5 and $K_{3,3}$
(b) it does not contain subgraphs isomorphic to K_5 and $K_{3,3}$
(c) it does not contain subgraphs isomorphic to K_5 and $K_{4,4}$
(d) it does not contain subgraphs homeomorphic to K_5 or $K_{3,3}$

[1999 : 2 Marks]

4.3 The maximum number of possible edges in an undirected graph with n vertices and k components is

[2001 : 2 Marks]

4.4 A connected planar graph with minimum number of vertices has

(a) 2 edges, 3 vertices
(b) 3 edges, 3 vertices
(c) 3 edges, 4 vertices
(d) 4 edges, 4 vertices

[1992 : 1 Mark]

4.5 Maximum number of edges in a planar graph with n vertices is

[1998 : 2 Marks]

4.6 The number of distinct simple graphs with upto three nodes is

(a) 5 (b) 7
(c) 8 (d) 9

[1994 : 1 Mark]

4.7 Prove that in a finite graph, the number of vertices of odd degree is even.

[1996 : 3 Marks]

4.8 Let G be a connected undirected graph with distinct edge weights. Let e_{max} be the edge with maximum weight and e_{min} be the edge with minimum weight. Which of the following statements is false?

(a) Every minimum spanning tree of G must contain e_{min}
(b) If e_{max} is in a minimum spanning tree, then it is a minimum weight edge in G
(c) No minimum spanning tree contains e_{max}
(d) G has a unique minimum spanning tree

[2001 : 2 Marks]

4.9 How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{v_1, v_2, \ldots, v_n\}$ of n vertices?

(a) $n(n-1)/2$ (b) 2^n
(c) $n!$ (d) $2^{\frac{n(n-1)}{2}}$

[2001 : 2 Marks]

4.10 Minimum number of edges in a connected graph with n nodes is

(a) n (b) n – 1
(c) n + 1 (d) max (n, e)

[2002 : 1 Mark]

4.11 Let G be a connected graph on n vertices and m edges. Let G' be a connected graph obtained from G by removing an edge from G. The number of components in the resultant graph must necessarily lie between

(a) 1 and m (b) 1 and n – 1
(c) 1 and m – n + 1 (d) 2 and n – 1

[2003 : 1 Mark]

4.12 How many perfect matchings are there in a complete graph on 6 vertices?

(a) 5 (b) 24
(c) 120 (d) 15

[2003 : 2 Marks]

4.13 A graph $G = (V, E)$ satisfies $|E| \geq |V| - 1$. The minimum number of edges in G and the maximum number of edges in G respectively, if G is connected are

(a) 3 (b) 2
(c) 6 (d) 5

[2003 : 2 Marks]

4.14 The minimum number of colours required to colour the following graph such that no two adjacent vertices are assigned the same colour is

(a) 2 (b) 4
(c) 3 (d) 5

[2004 : 2 Marks]

4.15 How many graphs on 6 labelled vertices exist with 3 edges? [illegible]

(a) [illegible] (b) $\sum_{i=1}^{3} [illegible]$

(c) [illegible] (d) $\sum_{i=1}^{3} \frac{[illegible]}{[illegible]} C_i$

[2005 : 2 Marks]

4.16 What is the maximum number of edges in a simple disconnected graph with 6 vertices?

(a) 6 (b) 9
(c) 10 (d) 15

[IT-2004 : 1 Mark]

4.17 What is the number of vertices in a connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?

(a) 10 (b) 11
(c) 9 (d) 12

[IT-2004 : 2 Marks]

4.18 Let $G(V, E)$ be a simple connected planar graph with 13 vertices and 19 edges. Then, the number of faces in the [illegible] is

(a) 7 (b) 8
(c) 9 (d) 12

[2005 : 1 Mark]

4.19 Which one of the following graphs is NOT a tree?

(a)
(b)
(c)
(d)

[2005 : 2 Marks]

4.20 Edge weights of a connected simple graph are positive. Then any subset of edges that connects all the vertices and has minimum total weight is a

(a) Hamiltonian cycle (b) path
(c) hamiltonian path (d) tree

[IT-2006 : 1 Mark]

4.21 Consider an undirected unweighted graph G. Let u be a vertex in G. Let T be a breadth-first spanning tree of G rooted at u. Let T be a depth-first spanning tree of G rooted at u. Let T' be a minimum spanning tree of G. Let T' be a shortest path spanning tree of G rooted at u. Let T' be a single tree. The number of the elements in the set $\{u, v\}$ for which the distance between u and v in T is the same as the distance between u and v in G is

(a) 1,2,3 (b) 1,3
(c) 1,2 (d) 1,3

[IT-2006 : 2 Marks]

Common Data for Q.4.22 & Q.4.23

The [illegible] graph G is a connected simple graph with [illegible] vertices. The vertices of G are numbered [illegible]. The [illegible] edges in G are [illegible].

4.22 The number of [illegible] in G is

(a) 1 (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

4.84 The maximum degree of a vertex in a [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

4.85 The number of connected components in [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

4.86 Let G be a connected planar graph with 10 vertices. If the number of faces is 2, then G has

(a) 9 edges and 2 vertices

(b) 9 edges and 3 vertices

(c) 10 edges and 2 vertices

(d) 10 edges and 3 vertices

[2007 : 1 Mark]

4.87 Consider G = (V, E) where V = {1, 2, 3, 4, 5} and E [illegible]

Which of the following is a topological sort of G?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 1 Mark]

4.88 Which of the following graphs is isomorphic to [illegible]

(a) [illegible]

(b) [illegible]

(c) The complete graph on [illegible] vertices

(d) None of these

[2007 : 2 Marks]

4.89 What is the size of the smallest Vertex Cover, Maximal Independent Set and Minimum Dominating Set in [illegible]

(a) [illegible] (b) 4

(c) 3 (d) 2

[IT-2008 : 1 Mark]

(a) 3 (b) 2

(c) 4 (d) [illegible]

[IT-2008 : 1 Mark]

4.90 G is a simple connected planar graph with 13 vertices and 19 edges. Each face has at least three edges. The number of faces in a plane embedding of G is

(a) 2 (b) 3

(c) 4 (d) 5

[IT-2008 : 2 Marks]

4.91 What is the chromatic number of the complement of a simple connected graph which has exactly one vertex of degree n-1? (n is the number of vertices in the graph)

(a) 2 (b) 3

(c) [illegible] (d) n

[2008 : 1 Mark]

4.92 Which one of the following statements is TRUE for any simple connected undirected graph with more than 2 vertices?

(a) No two vertices have the same degree.

(b) At least two vertices have the same degree.

(c) All vertices have the same degree.

(d) All vertices have different degree.

[2008 : 1 Mark]

4.93 Let G = (V, E) be a graph. Define [illegible] where k is the number of connected components in G. If S and T are two subsets of V such that S ⊆ T, then [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2010 : 1 Mark]

4.94 Let G be a connected undirected graph with more than two vertices. If exactly two vertices of G have degree 1, then the number of vertices in the graph with degree 2 is [illegible]

(a) 0 (b) [illegible]

(c) 1 (d) [illegible]

[2010 : 1 Mark]

4.85 The degree sequence of a simple graph is the sequence of the degrees of the vertices of the graph in non-increasing order. Which of the following sequences can be the degree sequence of any graph?

I. 7, 6, 5, 4, 3, 2, 1
II. 6, 6, 5, 4, 3, 2, 1
III. 7, 6, 5, 4, 4, 3, 2
IV. 6, 5, 5, 4, 3, 2, 1

(a) I and II (b) III and IV
(c) I and III (d) II and IV

[2010 : 2 Marks]

4.86 P and Q are graphs with the following structures.

Which one of the following statements is TRUE in relation to these graphs?

(a) P is planar but Q is non-planar
(b) Both P and Q are planar
(c) P is non-planar but Q is planar
(d) Neither P nor Q is planar

[2010 : 2 Marks]

4.87 Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is connected, the number of bounded faces in any embedding of G in the plane is equal to

(a) 7 (b) 6
(c) 5 (d) 4

[2012 : 1 Mark]

4.88 Which of the following graphs is isomorphic to

(a)

(b)

(c)

(d)

[2013 : 2 Marks]

4.89 Let G be a connected undirected graph on 6 vertices. Two vertices of G are labelled u and v. Exactly one simple path between u and v has length 4. The degree sequence of G is

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2012 : 2 Marks]

4.90 Consider an undirected unweighted graph G with vertex set {a, b, c, d, e}. The adjacency matrix of G is [illegible]. Let G' be the graph obtained by adding an edge between a and b to G. How many spanning trees does G' have?

(a) 5 (b) 1
(c) 7 (d) 8

[2015 : 1 Mark]

4.91 Which of the following statements is/are TRUE for undirected graphs?

P: Number of odd degree vertices is even
Q: Sum of degrees of all vertices is even

(a) P only (b) Q only
(c) Both P and Q (d) Neither P nor Q

[2013 : 1 Mark]

4.92 Let G = (V, E) be an undirected graph with a set V of vertices and E the set of edges. Then it is known that any spanning tree of G has exactly |V| − 1 edges.

Consider the following graph algorithm that takes as input G and two disjoint subsets V_1 and V_2 of V.

(i) Let T = (V, E) be a spanning tree of G. Let E_1 = {(u, v) ∈ E | u ∈ V_1 or v ∈ V_1}
(ii) Let T = (V, E) be a spanning tree of G. Let E_2 = {(u, v) ∈ E | u ∈ V_2 or v ∈ V_2}
(iii) Let E_3 = E_1 ∩ E_2. Let E_3 = {(u, v) ∈ E_3 | there is a path in T between u and v}
(iv) Let T = (V, E) be a spanning tree of G. Let E_4 = {(u, v) ∈ E | u ∈ V_1 and v ∈ V_2}

[2014 (Set-1) : 1 Mark]

4.93 Consider an undirected unweighted graph G. Let u be a vertex in G and let T be a breadth-first spanning tree of G rooted at u. Let T' be a depth-first spanning tree of G rooted at u. Let |E| denote the number of edges in G. Let |T| denote the number of edges in T. Let |T'| denote the number of edges in T'.

The number of edges in G is ____

[2011 (Set-1) : 2 Marks]

4.94 An undirected complete graph K_n with $V = \{v_1, v_2, \ldots, v_n\}$ is called a complete graph. A cycle is a simple connected graph with exactly two vertices of degree 1. Let P_n and C_n denote the complete graph and the cycle with n vertices respectively. Which of the following statements is/are TRUE?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2014 (Set-1) : 2 Marks]

4.46 [illegible]

[2014 (Set-1&3) : 1 Mark]

4.47 [illegible]

[2014 (Set-2) : 2 Marks]

4.47 [illegible] for the weighted graph [illegible]

[2014 (Set-2) : 2 Marks]

4.48 [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2014 (Set-3&4) : 2 Marks]

4.49 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2014 (Set-4) : 2 Marks]

4.50 [illegible]

[2013 (Set-1) : 2 Marks]

4.51 A graph is self complementary if it is isomorphic to its complement. Let all self complementary graphs on [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2013 (Set-2) : 2 Marks]

4.52 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2013 (Set-2) : 2 Marks]

4.53 [illegible]

[2013 (Set-4) : 1 Mark]

4.54 [illegible]

[2013 (Set-3) : 2 Marks]

4.55 [illegible]

[2013 (Set-2) : [illegible] Mark]

■■■■

Answers Graph Theory

4.1	(c)	4.2	(b)	4.3	(c)	4.6	(c)	4.5	(c)	4.6	(d)	4.7	(b)	4.8	(c)	4.9	(a)
4.10	(d)	4.11	(c)	4.12	(d)	4.13	(c)	4.14	(c)	4.15	(b)	4.16	(c)	4.17	(c)	4.18	(c)
4.19	(c)	4.20	(c)	4.21	(c)	4.22	(b)	4.23	(b)	4.24	(c)	4.25	(b)	4.26	(c)	4.27	(d)
4.28	(c)	4.29	(c)	4.30	(d)	4.31	(c)	4.32	(b)	4.33	(c)	4.34	(c)	4.35	(c)	4.36	(d)
4.37	(c)	4.38	(b)	4.39	(c)	4.40	(c)	4.41	(c)	4.42	(c)	4.43	(c)	4.44	(c)	4.45	(b)
4.46	(b)	4.47	(d)	4.48	(b)	4.49	(b)	4.50	(c)	4.51	(d)	4.52	(b)				

Explanations Graph Theory

4.1 (c)

G_1 is $K_{3,3}$ which is a well-known non planar graph.

Given G_2 is homeomorphic to K_5 so it is non planar graph.

The above graph shows that it is planar.

G_4 is contracted to K_5 which is well known non planar graph. Therefore, G_4 is non planar graph.

4.2 (b)

Kuratowski's theorem: A graph is planar if and only if it does not contain subgraph homeomorphic to K_5 or $K_{3,3}$.

4.3 (c)

The maximum number of edges in a simple and disconnected graph with 'n' vertices and 'k' components is $\frac{(n-k)(n-k+1)}{2}$.

4.4 (c)

K_5 is smallest non planar graph. It has 5 number of minimum vertices.

The minimum number of vertices in a connected graph with 11 edges in K_5 is $\frac{5(5-1)}{2} = 10$.

4.5 Sol.

Maximum number of edges in a connected, simple, undirected graph with 8 vertices is 8.

4.6 (c)

Assume that the vertices are labelled. Number of distinct simple graphs with 3 nodes = 8

(i) [illegible]

Number of distinct simple graphs with 4 nodes = 2

(ii) [illegible]

(iii) [illegible]

Number of distinct simple graphs with 5 nodes = 4

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(iv) [illegible]

Therefore, the number of distinct simple graphs with 3 nodes = [illegible]

4.7 (b)

Adjacency matrix is symmetrical for a graph $G = (V, E)$. Because in vertices degree is

$$\sum_{v \in V} \text{degree}(v) = 2|E|$$

In G_1 and G_2 degree of only the set of vertices of odd degree and hence there are even odd degree in a undirected graph $G = (V, E)$ [illegible]

4.8 (c)

K_5 is smallest non planar graph. Number of minimum vertices.

The minimum number of vertices in a connected graph with 11 edges in K_5 is $\frac{5(5-1)}{2} = 10$.

4.9 Sol.

Maximum number of edges in a connected, simple, undirected graph with 8 vertices is 8.

4.10 (d)

Assume that the vertices are labelled. Number of distinct simple graphs with 3 nodes = 8

(i) [illegible]

Number of distinct simple graphs with 4 nodes = 2

(ii) [illegible]

(iii) [illegible]

Number of distinct simple graphs with 5 nodes = 4

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(iv) [illegible]

Therefore, the number of distinct simple graphs with 3 nodes = [illegible]

4.11 (c)

Adjacency matrix is symmetrical for a graph $G = (V, E)$. Because in vertices degree is

$$\sum_{v \in V} \text{degree}(v) = 2|E|$$

In G_1 and G_2 degree of only the set of vertices of odd degree and hence there are even odd degree in a undirected graph $G = (V, E)$ [illegible]

$$\sum_{v \in V} \text{degree}(v) = \sum_{v \in V_1} \text{degree}(v) + \sum_{v \in V_2} \text{degree}(v) = 2e$$

Since sum is even but $\sum_{v \in V_1} \text{degree}(v)$ is odd, the sum on the other hand side of the equation must be odd. The total number of vertices in V_1 must be even. Since V_1 is even, V_2 must be even.

109 (a)

(a) A connected graph is defined to be a tree if it contains no cycle. Minimum spanning tree of G must contain v_{11} the vertex of degree four will pick v_{11} first.

(b) The v_{22} is a vertex in the spanning tree of G. Since it is removed from the graph, the edges $v_{11}v_{22}$ must be removed.

(c) A minimum spanning tree of G is obtained by removal of edges $v_{11}v_{22}$ and $v_{11}v_{33}$. Since edges are of unit weight, any spanning tree will be a minimum spanning tree.

(d) Since the given graph is a tree, it is the only minimum spanning tree.

110 (b)

In a graph G with n vertices, maximum number of edges = $\frac{n(n-1)}{2}$.

There are two ways for a edge to be in G or may appear in graph or may be absent in graph. So there are two options for each edge.

Total number of graphs with n vertices = $2^{\frac{n(n-1)}{2}}$.

111 (b)

The number of edges in complete graph with n vertices is $\frac{n(n-1)}{2}$.

112 (d)

Given $|E| \le 3|V| - 6$ ... (1)

Let δ be the minimum degree of the graph.

Since G cannot have self loops or parallel edges, we have

$$\delta \le \frac{2|E|}{|V|} \quad \text{... (2)}$$

Substituting eq (1) in eq (2) we get

113 (c)

Given G is connected, the minimum number of edges in G is equal to the number of vertices in G. Hence G must be a cycle graph as shown below.

Complete graph on vertices as shown below.

Between two vertices, if edges is removed, the number of components will be n − 1 irrespective of the total number of edges in the graph.

The maximum number of components possible is reached when each component is a tree in which case the number of components will be n − 1.

Thus, the number of connected components may vary between 1 and n − 1.

114 (b)

The number of edges in a complete graph with n vertices, where n is even, is equal to the number of edges in a complete bipartite graph with two vertex sets of size n/2 each.

$$= \frac{n(n-1)}{2}$$

The number of edges in a complete bipartite graph with two vertex sets of size n/2 each is

$$\text{Number of perfect matchings} = \frac{(n/2)!}{(n/2)/2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 2 \times 1} = 15$$

115 (d)

Given $|E| \le 3|V| - 6$ ... (1)

Let δ be the minimum degree of the graph.

Since G cannot have self loops or parallel edges, we have

$$\delta \le \frac{2|E|}{|V|} \quad \text{... (2)}$$

Substituting eq (1) in eq (2) we get

$$\Rightarrow \frac{2}{|V|}\{|V| - 4\}$$

$$= \frac{2}{|V|}\{8 - 4\}$$

$$= \frac{8}{V}$$

Given that, connected degree 4 vertices in the graph are equal to 8

4.14 (a)

An assignment of the vertices 1, 2, 3, 4 and 5 to the vertices of the graph is as shown below. Clearly the graph is a complete graph.

So, clearly it is a complete graph.

(Note: This graph is a planar graph. It is a complete graph, and the chromatic number of a planar complete graph is 4.)

4.15 (b)

Total number of edges are 1+2+3+4+5 = 15

$$\Rightarrow \frac{(n-1)(n-2)}{2} = \frac{(5-1)(5-2)}{2} = 6 \text{ edges}$$

Now we have 15 edges and we need $\frac{(n-1)(n-2)}{2} = 6$ edges.

So we need to remove $\binom{n-1}{2} - 6$ edges. Each such edge deleted represents a spanning graph on the given vertices.

Total number of such graphs

$$\frac{(n-1)(n-2)}{2}\binom{n-1}{2} - 6 = \frac{(n-1)(n-2)}{2}\binom{n-1}{2} - \frac{(n-1)(n-2)}{2}C_2$$

Now, since $\binom{n-1}{2} = {}^{n-1}C_2$

$$= \frac{(n-1)(n-2)}{2}\,{}^{n-1}C_2 - \frac{(n-1)(n-2)}{2}\,{}^{n-2}C_2$$

$$= \frac{(n-1)(n-2)}{2}\,{}^{n-1}C_2$$

Since $n = 5$,

$$\frac{(5-1)(5-2)}{2}\,{}^{5-1}C_2 = \frac{(5-1)(5-2)}{2}\,{}^{4}C_2$$

and so, the result.

$$= \frac{(n-1)(n-2)}{2}\,{}^{n-1}C_2 - \frac{(n-1)(n-2)}{2}\,{}^{n-2}C_2$$

Since, required number of reduced edges

$$= \frac{(n-1)(n-2)}{2}\,{}^{n-1}C_2 + \frac{(n-1)(n-2)}{2}\,{}^{n-2}C_2$$

$$= \frac{(n-1)(n-2)}{2}\left[{}^{n-1}C_2 + {}^{n-2}C_2\right]$$

$$= \sum_{k=1}^{n-1} k\,{}^{n-k}C_k$$

which is the result.

4.16 (a)

Max number of edges in a connected graph is $n - 1$, which occurs from one edge.

4.17 (b)

Sum of degrees of all the vertices = 2 × Number of edges

$2 \times 27 = 1 \times 3 + 2 \times 3 + 3 \times 3 + 4 \times 3 = 27 \times 2$

$v = 13$

Number of vertices = 3 + 3 + 3 + 4 = 13

4.18 (b)

Given, $v = 10$, $E = 15$

Let the total number of regions.

$$R = E - V + 2$$

$$= 15 - 10 + 2 = 7$$

4.19 (b)

Given graph is K_5, which is known to be non planar. K_3, K_4 and K_5 are the only complete graphs which are planar.

4.20 (d)

Number of edges in a connected tree = n − 1. Tree is a connected graph which does not form a cycle. So it is called as tree.

4.21 (b)

Vertex connectivity of a complete graph is K_n is $n - 1$ and minimum edge connectivity is

[illegible]

4.22 (b)

[illegible]

4.23 (b)

[illegible]

4.24 (b)

[illegible]

4.25 (b)

[illegible]

4.26 (b)

[illegible]

4.27 (c)

Whenever in a graph a vertex has even degree, then it will always have an odd number of

(a) Simple graph: A simple graph is said to be a cycle if it is even, every vertex in the graph has even degree. A regular graph: A regular graph is said to be regular if every vertex has the same degree.

(b) [illegible] because every vertex degree is odd. (38)

(c) G_2 has [illegible] vertices and [illegible] edges. [illegible] G_1 and G_2 are [illegible] G_1 and G_2 are 21, 22 denote which [illegible] number.

Since every vertex of the [illegible] has even degree. Also [illegible] G_1 and G_2 are connected. So it is an Euler graph.

4.28 (b)

Maximal independent sets:

[illegible]

Maximal minimal independent set

{b, d, f, g}

Size = 4

4.29 (d)

Since [illegible] edges, this graph. This graph is not bipartite and [illegible] the chromatic number is at least 3.

A proper colouring with 3 colors is shown below:

Hence the minimum number of colors needed to color given graph is equal to 3.

4.30 (d)

In a cycle of a multigraph, there [illegible] vertices [illegible] must be even.

Now 5 odd vertices [illegible] of vertices which have odd degree.

Subgraph is [illegible]

of all vertices which are connected to it. [illegible] degree and [illegible] degree.

So [illegible] have even degree, also since the graph is connected, that graph [illegible] Euler graph and hence an Euler graph.

4.31 (a)

It is not a simple connected graph [illegible] cycle of odd length then [illegible] since the vertices can be alternately colored with two colors. [illegible] and for the second [illegible] and [illegible].

Alternatively, a simple connected graph with no cycle of odd length must be bipartite. [illegible] the chromatic number [illegible] all [illegible] sequence always [illegible] a bipartite graph [illegible] there [illegible] a partition of a bipartite graph and [illegible] only two part [illegible].

Note: Since it is given that the graph is connected, it cannot be a null graph.

4.32 (b)

In a simple connected [illegible] the graph [illegible] two vertices [illegible] vertices must have same degree [illegible]. This is not true, the [illegible] vertices [illegible] graph with [illegible] vertices having different degrees is not possible. [illegible] Handshaking Theorem [illegible]

4.33 (b)

Given $\sum_{i=1}^{n} d(v_i) = 8$ and degree

We know [illegible] $\sum_{i=1}^{n} d(v_i) = 2R_1$

where R_1 is the number of edges in G.

Edges [illegible] $\sum d(v_i) = 2|E|$

$\Rightarrow 8 = 2|E|$

$\Rightarrow |E| = R_1$

$\Rightarrow |E| = R_1 = |S| - 1$ [illegible] $R_1 = |V| - 1$

Where $|S|$ = number of vertices of tree is one.

$|V|$ is number of vertices of tree T

$\Rightarrow |S| = |V| - 1$

$\Rightarrow |S| = |V|$

4.84 (d)

[illegible]

Now consider a tree with 3 nodes [illegible]

[illegible]

Now [illegible] number of leaves and degree of 3. But the [illegible] leaves but the [illegible] number of leaves.

Now consider a tree with 4 nodes [illegible] the root and one child. [illegible]

[illegible]

[illegible]

[illegible]

Similarly [illegible]

4.85 (c)

[illegible] Hamiltonian path in [illegible] graph or [illegible]

The algorithm is:

1. [illegible]
2. [illegible] "1" from [illegible] graph [illegible] that vertex number: [illegible]
3. [illegible] this sequence [illegible] order
4. [illegible] graph
5. [illegible] 1

   The [illegible] sequence is not a graph sequence [illegible] in step 1, [illegible] of input [illegible] in the sequence. [illegible] the algorithm will [illegible] number of 1s and zero number of 0s.

   [illegible]

[illegible]

[illegible]	(Step 1)
[illegible]	(Step 2)
[illegible]	(Step 3)
Sequence is a graph	(Step 4)
[illegible]	(Step 1)
[illegible]	(Step 2)
[illegible]	(Step 3)
Sequence is a graph	(Step 4)
[illegible]	(Step 1)
[illegible]	(Step 2)
[illegible]	(Step 3)
Sequence is a graph	(Step 4)

[illegible] sequence is [illegible] graph.

The initial sequence corresponds to following graph

[illegible]

Similarly for sequence 2

[illegible]

[illegible]	(Step 1)
[illegible]	(Step 2)
[illegible]	(Step 3)
Sequence is a graph	(Step 4)
[illegible]	(Step 1)
[illegible]	(Step 2)
[illegible]	(Step 3)
Sequence is a graph	(Step 4)
[illegible]	(Step 1)
[illegible]	(Step 2)
[illegible]	(Step 3)
Sequence is a graph	(Step 4)

The sequence is not a graph (first step), since no pair [illegible] graph. So algorithm ends.

[illegible] graph.

Similarly [illegible] graph and [illegible] sequence of graph.

[illegible]

4.36 (a)

The planar embeddings of K_5 and Q_3 are shown below

Hence, given graphs are planar.

4.37 (a)

[illegible]

[illegible]

[illegible]

[illegible]

4.38 (b)

Given invariants are verified.

Step 1: [illegible]

Step 2: Since [illegible] given graph

[illegible]

Degree sequence of graph in option (b) is (1, 1, 2, 2, 2, 3, 3).

Degree sequence of graph in option (c) is (1, 1, 2, 2, 2, 2, 2).

Degree sequence of graph in option (d) is (1, 1, 2, 2, 2, 2, 2).

Degree sequence of graph in option (e) is (1, 1, 2, 2, 2, 2, 2).

[illegible]

Step 3: [illegible] given graph, we check the number of cycles.

In given graph there is one cycle of length 3 but in Graph (a) there is no cycle of length 3.

Graph (b) has one cycle of length 3.

So only Graph (b) can be isomorphic to given Graph.

4.39 (a)

The graph represents K_5.

[illegible]

4.40 (c)

[illegible]

The [illegible] between [illegible] possible choosing any edge is 21.

[illegible]

Hence [illegible]

[illegible]

4.41 (c)

[illegible]

4.42 (d)

$G = (V, E)$ [illegible]

$G_S = (V, E_S)$ where E_S [illegible]

[illegible]

4.43 (d)

[illegible]

Draw above diagram

(i) The four corner vertices have degree each which gives a total of 4 × 3 = 12 degrees.

(ii) The 16 side vertices have degree each which gives a total of 16 × 5 = 80 degrees.

(iii) The 16 interior vertices have each degree 8 giving a total of 16 × 8 = 128 degrees.

So total degrees of the graph

12 + 80 + 128 = 220 degrees.

So the number of edges in the given connected graph

$\frac{\text{Total degrees}}{2}$

Therefore the number of edges in the graph = 220/2 = 110

1.44 Sol.:

Use the Dijkstra's algorithm.

The sequence of steps in the algorithm for each graph is shown below.

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

1.45 Sol.:

Bipartite graph is a graph whose vertices can be partitioned into two disjoint sets such that every edge connects a vertex in one set to a vertex in the other set.

(a) [illegible] edges maximum

(b) [illegible] edges maximum

(c) [illegible] edges maximum

(d) [illegible] edges maximum

(e) [illegible] edges maximum

(f) [illegible] edges maximum

∴ Maximum number of edges in a bipartite graph with 6 vertices = 9 edges.

1.46 (b)

[illegible] then it is a self complement. [illegible] complementary graph.

[illegible]

[illegible]

[illegible]

[illegible]

The answer of the above equation is [illegible].

So, [illegible] graph which is self complementary.

1.47 Sol.:

[illegible] edges with [illegible] weights. Take all of them. The weights of edges can be chosen in [illegible] ways as shown below.

Thus [illegible] minimum spanning tree [illegible] × 2 = 8]

1.48 (b)

The number of edges in a spanning tree of a graph G with n vertices and k components is [illegible] = n − k.

1.49 (a)

[illegible]

[illegible]

— [illegible]

Total edges = [illegible]

[illegible]

1.50 Sol.:

Number of vertices [illegible] = 10

[illegible]

Minimum degree [illegible] = 3

[illegible] = 2 [illegible] for connected planar graphs

[illegible]

Since every region is bounded by at least 3 edges and since every edge is exactly shared by 2 regions, we have [illegible]

[illegible]

Substituting this in [illegible] we get.

[illegible]

[illegible]

4.51 (d)

[illegible] self-complementary graph has exactly [illegible] number of edges. [illegible] graph is [illegible] edges.

[illegible] must be divisible by [illegible] so only [illegible] is [illegible] and [illegible] = 4.

4.52 (d)

An edge is called bridge if it is removed [illegible] disconnect the graph into [illegible] components.

In a cycle there is no edge between any pair of vertices [illegible] edge. Thus [illegible] bridge cannot be part of a simple cycle.

4.53 (d)

x_0 : Number of leaf nodes degree 0;

x_1 : Number of nodes with degree 1

x_2 : Number of nodes with degree 2

We have following equations for binary tree

$n = x_0 + x_1 + x_2$ (where n is the total number of nodes in the tree) ....(1)

Summation for total number of edges which are children and the one which is not a child of any node.

We get $2 = n \times 1 + x_2 \times 2 - 1$

$= n + 2x_2 - 1$ ....(2)

Combining equations (1) and (2) we get

$x_0 + x_1 + x_2 = n$; $x_1 + 2x_2 = 1$

$\Rightarrow x_0 = x_2 + 1$

Therefore the number of nodes of degree two is one more than number of leaf nodes.

$x_0 = 200 + 1 = 199$

4.54 (b)

Given $|V| = 100$ vertices and

$|E| = 500$ edges

Key weight of MST of G = 900

In KRT:

Number of vertices = 100

Number of edges = 99

If each edge of G is assigned by a unique MST weight [illegible] MST increased by [illegible]

Total 199 edges [illegible] = 446 is increased.

Total weight in new MST

= 900 + 446 = 1346

4.55 (d)

Euler's theorem says that every planar graph satisfies the inequality [illegible] is a necessary condition to prove any given planar graph.

■■■■

5 Probability

5.1 Let A and B be any two arbitrary events. Which of the following is/are true?

(a) $P(A \cup B) = P(A) + P(B)$

(b) $P(A \cap B) = P(A) \cdot P(B)$

(c) $P(A \mid B) = P(A) + P(B)$

(d) $P(A \cup B) = P(A) + P(B)$

[2011 : 1 Mark]

5.2 A bag contains 10 red balls and 10 black balls. Balls are drawn one after another without replacement. The probability that the 4th ball drawn is black and the 5th ball is white is

(a) $\frac{2}{19}$ (b) $\frac{10}{19}$

(c) $\frac{1}{19}$ (d) $\frac{9}{19}$

[1998 : 2 Marks]

5.3 Two dice are thrown simultaneously. The probability that the sum of the two numbers is 7 is

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$

(c) $\frac{7}{36}$ (d) $\frac{1}{12}$

[1996 : 1 Mark]

5.4 The probability that two independent events A and B both occur is 0.16 and the probability that neither A nor B occurs is 0.36. The probability that A occurs is

(a) $\frac{2}{32}$ or $\frac{4}{32}$ (b) $\frac{1}{32}$ or $\frac{3}{32}$

(c) $\frac{1}{32}$ or $\frac{7}{32}$ (d) $\frac{1}{32}$ or $\frac{3}{32}$

[1996 : 2 Marks]

5.5 The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and rain tomorrow?

(a) 0.3 (b) 0.35

(c) 0.45 (d) 0.6

[1995 : 1 Mark]

5.6 A die is rolled. Let E be the event "the number obtained is even" and let F be the event "the number obtained is a prime number". If a fair die is rolled and the number obtained is even, then the probability of the event that the number obtained is prime is

(a) 1/3 (b) 2/3

(c) 1/2 (d) 1/6

[1995 : 1 Mark]

5.7 Suppose X is a random variable with probability density function $f(x) = 2x$, $0 \le x \le 1$. Which of the following statement(s) is/are true?

(a) There is a sample space on which X has the uniform distribution.

(b) There is a sample space on which X has the value zero with probability 1/2.

(c) There is a sample space on which X has a normal distribution.

(d) None of the above

[1999 : 1 Mark]

5.8 Consider two events E_1 and E_2 such that probability of E_1, $P(E_1) = \frac{1}{3}$, conditional probability $P(E_2 | E_1) = \frac{1}{2}$, and probability $P(E_1 \cup E_2) = \frac{1}{4}$. Which of the following statement(s) is/are true?

(a) $P(E_2 | E_1) = \frac{2}{3}$

(b) Events E_1 and E_2 are mutually exclusive.

(c) Events E_1 and E_2 are independent.

(d) $P\left(\frac{E_1}{E_2}\right) = \frac{1}{3}$

[1998 : 2 Marks]

5.9 E_1 and E_2 are events in a probability space satisfying the following conditions:

- $P(E_1) = P(E_2)$
- $P(E_1 \cap E_2) = 1$
- E_1 and E_2 are independent

The value of $P(E_1)$, the probability of the event E_1, is

(a) 0 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

[2008 : 2 Marks]

5.10 A coin is tossed three times and a sixth is rolled. What is the probability that they are observed on the same day?

(a) $\frac{1}{36}$ (b) $\frac{1}{18}$

(c) $\frac{1}{9}$ (d) $\frac{2}{3}$

[2001 : 2 Marks]

5.11 Four fair coins are tossed simultaneously. The probability that at least one head and one tail appear is

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{15}{16}$

[2002 : 2 Marks]

5.12 Let $P(B)$ denote the probability of the event B. Given $P(A) = 1$, $P(B) = 1/2$, and A and B are independent, the values of $P(A|B)$ and $P(B|A)$ respectively are

(a) 1/4, 1/2 (b) 1/2, 1/4

(c) 1/2, 1 (d) 1, 1/2

[2003 : 1 Mark]

5.13 A program consists of two modules executed sequentially. Let $f_1(t)$ and $f_2(t)$ respectively be the probability density functions of time taken by executing the two modules. The probability density function of the overall time taken to execute the program is given by

(a) $f_1(t) + f_2(t)$ (b) $\int_0^t f_1(t-x) f_2(x)\,dx$

(c) $\int_0^t f_1(x) f_2(t-x)\,dx$ (d) $\max(f_1(t), f_2(t))$

[2005 : 2 Marks]

5.14 If a fair coin is tossed four times. What is the probability that two heads and two tails will result?

(a) 0.50 (b) 0.25

(c) 0.38 (d) 0.75

[2004 : 1 Mark]

5.15 In a population of N people, n people are infected with a disease. In a random sample of size m, k people are found to be infected. The population size N is estimated to be $\frac{mn}{k}$. This estimate is the maximum likelihood estimate under the assumption that the sampling distribution is

would. What is the probability that a randomly selected child belongs to a family with two children?

(a) 0.36 (b) 0.50

(c) 0.40 (d) 0.25

[2001 : 1 Mark]

5.18 The X and Y have exponential distributions and independent random variables with mean 1. If $Z = \min(X, Y)$, then the mean of Z is given by

(a) $(X + Y)/2$ (b) $\min(x, y)$

(c) $\min(x, y) + 1/2$ (d) $x = y$

[2004 : 2 Marks]

5.17 A multiple choice test has 100 questions. Each question has four alternatives of one mark each, with no negative marking. Each incorrect answer reduces 0.25 marks. Suppose 100 students select all their answers randomly with uniform probability. The standard deviation of the marks obtained by all these students is

(a) 0 (b) 25.00

(c) 50.00 (d) 8.66

[2004 : 2 Marks]

5.18 Two random binary strings S_1 and S_2 are chosen independently with uniform probability. The probability that the Hamming distance between the two strings is the number of bit positions where the two strings differ is

(a) $C_1 p^2$ (b) $C_2 p^2$

(c) $C_1 p^2$ (d) $C_2 p^2$

[2004 : 2 Marks]

5.19 A point is randomly selected with uniform probability in the X-Y plane within the rectangle with corners $(0, 0)$, $(1, 0)$, $(1, 2)$ and $(0, 2)$. If p is the expected value of the distance of the point from the origin, the expected value of p is

(a) 0.8 (b) 1

(c) 1.0 (d) 0.5

[2004 : 2 Marks]

5.20 Let $f(x)$ be the probability density function of a random variable X. The probability that $a < X < b$ is

(a) $f(b) - f(a)$ (b) $f(a) - f(b)$

(c) $\int_a^b f(x)\,dx$ (d) $\int_a^b x f(x)\,dx$

[2005 : 1 Mark]

5.21 A bag contains 10 blue marbles, 20 green marbles and 30 red marbles. A marble is drawn from the bag, its colour recorded and it is put back in the bag. This process is repeated 3 times. The probability that exactly two marbles drawn have the same colour is

(a) 0.27 (b) 0.30
(c) 0.54 (d) 0.62

[IT-2005 : 2 Marks]

5.22 A fair coin is tossed repeatedly. Let X be the outcome of two successive tosses in the same. Assuming that the tosses are independent, the expected value of X is

(a) 0 (b) 4
(c) 5 (d) 8

[IT-2005 : 2 Marks]

5.23 The certain town, the probability that it will rain on any day is known to be 0.6. Moreover, the temperature is known to be 0.6. Moreover, the temperature is known to be equal to 0°C with probability 0.4. The temperature at noon is known to be equal to 0°C with probability 0.4. The temperature at noon is known to be equal to 0°C with probability 0.4. The temperature at noon

5.80 [illegible] either computer science or mathematics everyday. This studies [illegible] [illegible], Let the probability that the [illegible] mathematics on any day is 0.6. Let the probability that [illegible] computer science the next day is 0.4. Given that [illegible] computer science on Monday, what is the probability that [illegible] computer science on Wednesday?

(a) [illegible] (b) 0.56
(c) 0.4 (d) 0.6

[2005 : 2 Marks]

5.81 Let X be a random variable following normal distribution with mean +1 and variance 1. Let Y be another random variable with mean −1 and variance 1. If $E(X^2) = 2$, then the standard deviation of Y is

(a) 2 (b) 3
(c) $\sqrt{3}$ (d) 1

[2008 : 2 Marks]

5.82 An unbiased die with 6 faces, numbered 1 to 6, is thrown. The probability that the face value is odd is 0.5 of the probability that the face value is even. The probability of getting any even numbered face is the same.
If the die is thrown twice, the probability that the sum of the values is even given that the first value is 5 is 0.75, which is [illegible]. The following statements about the probability of the sum of the values are [illegible]:

(a) [illegible] (b) [illegible]
(c) 0.485 (d) [illegible]

[2009 : 2 Marks]

5.83 Consider a communication system. It consists of the probability of a single error in a binary [illegible] is p. It uses each [illegible] to a [illegible] repetition. This scheme [illegible] a computer with a probability of q. What is the probability of a computer error after decoding?

(a) $pq = (1-p)(1-q) + pq(1-p) + (1-q)p$
(b) $1 - pq$ (c) pq

[2010 : 2 Marks]

5.84 What is the probability that a divisor of 1067 is a multiple of 10?

(a) [illegible] (b) 0.625

(c) [illegible] (d) 1.9823

[2010 : 2 Marks]

5.85 If two fair coins are tossed and at least one of the coins is known to be a head, what is the probability that both the coins are heads?

(a) 0.8 (b) 0.1
(c) 0.5 (d) 0.25

[2011 : 2 Marks]

5.86 If the difference between the expectation of the square of a random variable $E[X^2]$ and the square of its expectation $[E(X)]^2$ is denoted by D, then

(a) D = 0 (b) D ≥ 0
(c) D ≤ 0 (d) D > 0

[2010 : 2 Marks]

5.87 Consider the finite sequence of random variables $X = [x_1, x_2, \ldots, x_n]$. Let μ_X be the mean and σ_X be the standard deviation of X. Let another random variable Y be defined as $y_i = ax_i + b$, where a and b are positive constants. Let μ_Y be the mean and σ_Y be the standard deviation of Y. Which one of the following statements is INCORRECT?

(a) Index position of median of X in X is the same as the index position of median of Y in Y
(b) Index position of median of X in X is the same as the index position of median of Y in Y
(c) $\mu_Y = a\mu_X + b$
(d) $\sigma_Y = a\sigma_X + b$

[2011 : 2 Marks]

5.88 A deck of cards has cards numbered from 1 to 6. The deck is shuffled thoroughly. Two cards are then removed at random from the deck. What is the probability that the two cards selected will have numbers in consecutive order, or the sum of the numbers on the cards is 7?

(a) 1/3 (b) 4/15
(c) 1/9 (d) 2/9

[2011 : 2 Marks]

5.89 Consider a random variable X that takes values −1 and 1 with probability 0.5 each. The value of the cumulative distribution function F(x) at x = −1 and x = 1 is

(a) 0 and 0.5 (b) 0.5 and 1
(c) 1 and 0.5 (d) 0.25 and 0.75

[2012 : 1 Mark]

3.39 Suppose a fair six sided die is rolled twice. If the value on the die is x, $x \geq 3$ then x is called a success. What is the probability that the sum of the values is less than or equal to 7?

(a) 1/21 (b) 5/12

(c) 2/3 (d) 1/6

[2012 : 2 Marks]

3.40 Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this period?

(a) 0.5768 (b) 0.0498

(c) 0.0892 (d) 0.0620

[2013 : 1 Mark]

3.41 Suppose you have a line segment of length 1. A point chosen uniformly at random. What is the expected length of the shorter segment is _____.

[2014 (Set-1) : 1 Mark]

3.42 Four tainted coins are mixed. The probability that the coin of Uncertainty being 2/3 is 2/3. The value of X is _____.

[2014 (Set-1) : 2 Marks]

3.43 The security system at an IT firm is a group of 10 cameras, out of which two are working. To check whether the system is functional, the officials choose four of the cameras randomly and switch them on (without replacement). The system is considered functional if at least three of the four cameras are found to be working. Let the probability that the system is deemed functional be denoted by p.

Then $100p =$ _____.

[2016 (Set-3) : 1 Mark]

3.44 The probability that a given positive integer lies between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is

[2014 (Set-2) : 2 Marks]

3.45 Let S be a sample space and two mutually exclusive events A and B are such that $P(A) = 0.4$. If B denotes the complement of event B, then the maximum value of $P(A \cap B)$ is _____.

[2014 (Set-1) : 2 Marks]

3.46 Suppose X for $i = 1, 2, 3$ are independent and identically distributed random variables with probability mass function $P(X_i = 1) = P(X_i = 0) = 1/2$ for $i = 1, 2, 3$. If the random variable $Y = X_1X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $P(Y = 1 | X_1 = 1) =$ _____

[2015 (Set-4) : 2 Marks]

3.47 A probability density function on the interval [0, 1] is given by $2x^2$ and outside this interval the value of the function is zero. The cumulative distribution function

[2010 (Set-1) : 1 Mark]

3.48 Consider the following experiment.

Step 1. Fair coin is tossed.

Step 2. If the outcomes are HEADS, HEADS then stop.

Step 3. If the outcomes are either (TAILS, HEADS) or (HEADS, TAILS), then toss the coin once more.

Step 4. If the outcome is (TAILS, TAILS), then go to Step 1.

The probability that the experiment stops at Step 2 is _____ (up to two decimal places)

[2016 (Set-1) : 2 Marks]

3.49 Suppose there is a shop that sells equal number of light bulbs of two different types. The probability of a light bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that a light bulb chosen uniformly at random lasts more than 100 hours is

[2016 (Set-2) : 1 Mark]

■■■■

Answers Probability

5.1	(c)	5.2	(c)	5.3	(d)	5.4	(c)	5.5	(d)	5.6	(c)	5.7	(c)	5.8	(c)	5.9	(c)
5.10	(b)	5.11	(c)	5.12	(d)	5.13	(c)	5.14	(a)	5.15	(b)	5.16	(c)	5.17	(d)	5.18	(a)
5.19	(d)	5.20	(c)	5.21	(b)	5.22	(a)	5.23	(c)	5.24	(a)	5.25	(c)	5.26	(d)	5.27	(b)
5.28	(b)	5.29	(c)	5.30	(c)	5.31	(a)	5.32	(b)	5.33	(a)	5.34	(a)	5.35	(a)	5.36	(a)
5.37	(d)	5.38	(a)	5.39	(c)	5.40	(b)	5.41	(c)								

Explanations Probability

5.1 (c)

(a) $P(A \cap B) = P(A)P(B)$ [illegible] false since [illegible] if and only if A and B are independent events.

(b) $P(A \cup B) = P(A) + P(B)$ is true only if [illegible] if A and B are mutually exclusive.

(c) $P(A \cap B) = P(A) \times P(B)$ [illegible]

(d) $P(A \cap B) \le P(A) = P(B)$ is false.

Since $P(A \cap B) \le P(A) \le P(B)$

5.2 (c)

Required one 10 watts bulb [illegible] is an [illegible] probability

$$= \frac{{}^{10}C_1 \times {}^{8}C_1}{{}^{18}C_2} = \frac{10 \times 16}{\frac{18 \times 17}{2}} = \frac{2}{9}$$

5.3 (d)

Each head one of dice will have 6 ways

$= 1 - P(\text{none of the dice shows 6 facing up})$

$$= 1 - \left(\frac{5}{6} \times \frac{5}{6}\right) = \frac{11}{36}$$

5.4 (c)

The probability [illegible] the bottom card of a randomly shuffled deck is $= \frac{4}{52}$ [illegible] there are 51 remaining cards.

From the remaining 51 cards [illegible] probability that the [illegible] is $\frac{3}{51}$

So, required probability $= \frac{4}{52} \cdot \frac{3}{51}$.

5.5 (d)

$P(\text{Rain today}) = 0.3$

$P(\text{Rain tomorrow}) = 0.8$

$P(\text{Rain today} \mid \text{Rain tomorrow}) = 0.7$

$P(\text{Rain today} \mid \text{Rain tomorrow}) =$

$P(\text{Rain today} \cap \text{Rain tomorrow})$

$= P(\text{Rain today}) \times P(\text{Rain tomorrow}) = P(\text{Rain today} \mid \text{Rain tomorrow})$

$0.3 \times 0.8 = 0.7 \times P(\text{Rain today} \cap \text{Rain tomorrow})$

$P(\text{Rain today} \cap \text{Rain tomorrow})$

$= 0.3 \times 0.8 \div 0.7 = 0.3$

So, the probability that it will rain today and tomorrow is 0.3

5.6 (c)

Probability of getting an odd number on rolling of dice $= \frac{3}{6} = \frac{1}{2}$.

Now using binomial distribution

$P(\text{Exactly one odd number appears in three throws})$

$$= {}^3C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2$$

$$= 3 \times \left(\frac{1}{2}\right)^3 = \frac{3}{8}$$

5.7 (c)

[illegible] X is [illegible] X is [illegible] than 3. So [illegible] at which $X > 3$

5.8 (c)

$$P(A_1) = \frac{1}{2}, \quad P(T_1) = \frac{1}{3}$$

and $P(F_1 \cap F_2) = \frac{1}{5}$

$\therefore$ $P(F_1 \cup F_2)$

$= P(F_1) + P(F_2) - P(F_1 \cap F_2)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{5} = \frac{19}{30}$

Therefore probability of failure = 63%.

Therefore (c) is correct.

(d) For independent events

$P(E_1 \cap E_2) = P(E_1)P(E_2)$

Here $P(E_1 \cap E_2) = \frac{1}{5}$

$P(E_1)P(E_2) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$\therefore P(E_1 \cap E_2) \neq P(E_1)P(E_2)$

$\therefore$ Events E_1 and E_2 are not independent.

Option (d) is correct.

(a) Since E_1 and E_2 are mutually exclusive

$\therefore$ Conditional probability

$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{0}{3}$

$\therefore$ Option (a) $P(E_1|E_2) = 0$ is correct.

5.9 (d):

Given that

(i) $P(E_1) = P(E_2) = \alpha$

(ii) $P(E_1 \cap E_2) = \frac{1}{9}$

(iii) E_1 and E_2 are independent

$P(E_1 \cap E_2) = P(E_1)P(E_2)$

$= \alpha \times \alpha = \alpha^2$

Now,

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$1 = \alpha + \alpha - \alpha^2$

$1 = 2\alpha - \alpha^2$

$\alpha^2 - 2\alpha + 1 = 0$

$(\alpha - 1)^2 = 0$

$\alpha = 1$

So, $P(E_1) = P(E_2) = \alpha = 1$

5.10 (b):

Sum of the digits = 9

$\therefore$ Number = 108, 207, 306, 405, 504, 603, 702, 801, 900

Number of numbers = 9

Required probability $= \frac{7}{9} = \frac{1}{9}$

5.11 (d):

Given that the bag contains 3 red balls and 2 blue balls.

Probability of getting red ball

$= \frac{3}{5}$

Probability of getting blue ball $= \frac{2}{5}$

$\therefore$ Probability of getting red ball $= \frac{3}{5}$

$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{1}{5}$

Or

Alternate Method:

Total number of balls = 5

Total number of ways of selecting 3 balls = $^5C_3 = 10$

Number of ways of selecting 3 red balls = $^3C_3 = 1$

Probability of selecting 3 red balls $= \frac{1}{10}$

Number of ways of selecting 2 blue balls and 1 red ball $= {}^2C_2 \times {}^3C_1 = 3$

Probability of selecting 2 blue balls and 1 red ball $= \frac{3}{10}$

Probability of selecting 3 red balls or 2 blue balls and 1 red ball $= \frac{1}{10} + \frac{3}{10} = \frac{4}{10} = \frac{2}{5}$

5.12 (d):

Given that $P(A) = \frac{1}{2}$

$P(B) = \frac{1}{3}$

Since two events are independent

$P(A \cap B) = \frac{1}{6}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{3}$

5.13 (c):

Let the random variable X = first and second moments be represented by X_1 and X_2 respectively.

$\therefore$ $X = X_1 + X_2$ is a random variable.

Now, the joint density function

$$g(x) = \int_{-\infty}^{\infty} f(x_1, x_2)\,dx_2$$

$$= \int_{-\infty}^{\infty} f(x_1)\,f(x_2 - x_1)\,dx_1$$

$$= \int_{-\infty}^{\infty} f_1(x_1)\,f_2(x_2 - x_1)\,dx_1$$

which is the convolution of f_1 and f_2 where $f = f_1 * f_2$

Given statement is therefore false.

3.14 (a)

Given $E(X) = 12$

$V(X) = 18$

Apply Taylor's formula for binomial distribution

$$P(X = 0) = {}^{n}C_0 (1-p)^n \left[1 - \frac{1}{2}\right]^{1-n}$$

$$= {}^{n}C_0 \left(\frac{1}{2}\right)^n (1-p)^n$$

$$= \frac{1}{2^n} = \frac{8}{27} = \frac{2}{9}$$

3.15 (b)

Total number of children

$= 50 \times 0 + 30 \times 2 + 20 \times 1 = 80$

Favourable cases = Total number of children who come from 2 children families

$= 30 \times 2 = 60$

Required probability that a randomly picked child belongs to a 2 children family

$= \frac{60}{80} = \frac{3}{4}$

3.16 (d)

$f(x) = \lambda e^{-\lambda x}, x \geq 0$

X and Y are two independent exponentially distributed random variables with λ_1 and λ_2 parameters of X and Y respectively.

$f_X(x) = \lambda_1 e^{-\lambda_1 x}, x \geq 0$

$f_Y(y) = \lambda_2 e^{-\lambda_2 y}, y \geq 0$

Given $Z = \min(X, Y)$

$P(Z > z) = P(X \geq z, Y \geq z)$

$= P(X \geq z) P(Y \geq z)$

$= e^{-\lambda_1 z} e^{-\lambda_2 z}$

$= e^{-(\lambda_1 + \lambda_2) z}$

Hence mean of exponential distribution = (Parameter)$^{-1}$

So $\mu_1 = \frac{1}{\lambda_1} = \mu_2 = 10$

$\mu = \frac{1}{\lambda_1 + \lambda_2} = \frac{1}{20}$

Z is random variable with parameter

$$\text{Mean of } Z = \frac{1}{\lambda_1 + \lambda_2} = \frac{1}{\frac{1}{\mu_1} + \frac{1}{\mu_2}} = \frac{\mu_1 \mu_2}{\mu_1 + \mu_2}$$

3.17 (b)

Let the number of correct answers be random variable X.

The probability distribution is given as follows

X	1	2
$P(X)$	0.75	0.25

Expected number of correct answers

$= E(X) = \Sigma X P(X)$

$= 1 \times 0.75 + 2 \times 0.25 = 1.25$

Total marks awarded for 15 questions

$= 1.25 \times 100 = \frac{75}{4}$ marks for one student

Total expected marks of 100 students

$\frac{75}{4} \times 100 = 1875$ marks

Hence correct answer is (b).

3.18 (a)

Thermometer is accurate within $\pm 1°C$, we are asking probability that it will be accurate for two successive days. Let the events A = Event that thermometer is accurate with error within $\pm 1°C$ and B = Event that thermometer is accurate with error within $\pm 1°C$

$p = 0.4 + 0.2$

Given that A + probabilities are (0, 0), (0, 1), (1, 0), (1, 1), (1, 2) only two of them (0, 0) and (1, 0) are accurate

So $P(X = 0) = {}^{2}C_0 (0.6)^2 (0.2)^0$

$$= \frac{9}{25}$$

Correct option is (a).

3.19 (d)

Length of projection vector is given

$= p = \sqrt{x^2 + y^2}$

$$\mu = x^2 + c^2$$

$$E(y) = E(x^2) + c^2$$

$$E(y) = E(x^2) + c^2$$

Now x and c are uniformly distributed between 0 and 1 and $c = 2$

Probability density function is given

$$x = \frac{1}{1-0} =$$

Probability density function of x^2

$$y = \frac{1}{2\sqrt{y}} = \frac{1}{2\sqrt{y}}$$

$$E(x^2) = \int_0^1 x^2 p(x)\,dx$$

$$= \int_0^1 x^2 \cdot 1\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$E(c^2) = \int_0^1 c^2 p(c)\,dc$$

$$= \int_0^1 c^2 \cdot 1\,dc$$

$$= \left[\frac{c^3}{3}\right]_0^1 = \frac{1}{3}$$

$$\therefore E(y) = E(x^2) + E(c^2) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

3.20 (b)

If $f(x)$ is the multivariate probability density function of a random variable X then,

$$p(a < x \le b) = p(a \le x \le b) = \int_a^b f(x)\,dx$$

3.21 (b)

The given condition corresponds to sampling with replacement and without order.

Number of balls in the same color means, the final outcome of the three draws must be a permutation of Blue, Green, Red.

Thus, $n = 3! = 6$ such permutations possible.

Now, probability of picking a Blue disc, Green second and Red third

$$= \frac{10}{20} \times \frac{5}{19} \times \frac{10}{18}$$

Required probability

$$= 6 \times \frac{10}{20} \times \frac{5}{19} \times \frac{10}{18} = \frac{1}{6}$$

3.22 (b)

$$P(X) = S(X, x, p)$$

Where S_n = number of times HH you get successive HHHD or TTTT.

p = Probability that you get an X_i heads.

Consider all the 2 tosses in get 2 heads. In Case of HH, HT, TH, TT, countable cases are HH, TT.

$$p(X = 2) = p^2 + q^2$$

So, there for $X = 3$ only THH and HTT are the successful out of total 8 outcomes.

So, $p(X = 3) = \frac{2}{8}$

In general, you can see that in every case you will have only two favourable cases and 2^n possible cases. So, for n we get probability is $\frac{2}{2^n}$

So, the probability distribution table for X the number of tosses is given below:

X	2	3	4	5	...
$P(X)$	$\frac{2}{4}$	$\frac{2}{8}$	$\frac{2}{16}$	$\frac{2}{32}$	...

$$E(X) = 2 \cdot \frac{2}{4} + 3 \cdot \frac{2}{8} + 4 \cdot \frac{2}{16} + 5 \cdot \frac{2}{32} + \ldots$$

It is multiplication of 2 and 2 and 2 and so on. So, multiply by $E(X)$ by 2 and subtracting from $E(X)$

$$E(X) = 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{8} + 5 \cdot \frac{1}{16} + \ldots$$

$$2E(X) = 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{8} + 5 \cdot \frac{1}{16} + \ldots$$

$$\therefore E(X) - 2E(X) = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

$$\text{G.P. } E(X) = 1 + \frac{1/4}{1 - 1/2}$$

$$= 1 + \frac{1}{2} = 3$$

3.23 (d)

Let X = Amit is the student

$P(X > 25)$ = Temperature more than 25°C

Total the desired probability =

$P(X > 25) = p$

The tree diagram for the problem is given below.

Given $P(B|A) = 0.6$

by rule of total probability $P(B) =$

$$\frac{1}{2} \times 0.4 + \frac{1}{2} \times x = 0.8$$

$$\Rightarrow \quad x = 0.8$$

5.24 (a):

The number of attempts of the man follows geometric distribution. It is well known that the number of trials follows geometric distribution $E(X) = 1/p$,

where p is the probability that the man succeeds in any one attempt.

Alternate Method

Let X be the number of attempts to succeed.

Let p be the probability that man succeeds in any one attempt.

Now the probability distribution table of X is given below.

x	1	2	...	x	...
$P(X = x)$	p	$(1-p)p$	...	$(1-p)^{x-1}p$	...

$$E(X) = \sum xp(x)$$

$$= 1 \times p + 2 \times (1-p)p + 3 \times (1-p)^2 p + \ldots$$

This is such that arithmetic-geometric series which can be solved as

$$E(X) = 1/p$$

5.25 (a):

The medians = 7 as the problem is given below

$$P(H) = \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{4} = \frac{7}{16}$$

5.26 (a):

Number of permutations with '2' in 1st position = 10!

Number of permutations with '2' in 2nd position = 10 × 9!

If the first digit is any of the 10 odd numbers and the 2 spaces after that are filled with any 2 integers in 9 ways!

Number of permutations with '2' in 3rd position = 10 × 9 × 9!

If the first 2 places with 2 of the 10 odd numbers and the remaining 7 places with remaining 17 numbers is arranged in 17! ways in 12th place. Also it is not possible to satisfy the given condition, since there are only 10 odd numbers available to fill in the first 2.

So the desired number of permutations which satisfy the given condition is

$$9! + 10 \times 18! + 10 \times 9 \times 17! + 10 \times 9 \times 8 \times 16! + \ldots + 10 \times 9!$$

Now the probability of this happening is given by

$$\frac{18! + 10 \times 18! + 10 \times 9 \times 17! + \ldots + 10 \times 9!}{20!}$$

Which is clearly not choices (b), (c) or (d).

$\therefore$ Answer is (a) none of these.

5.27 (a):

The sequence follows Poisson distribution with $\lambda = 20$ sequence.

The observation second = 30 − 45 = interval

$= 45/60 = 3/4$ min

So the parameter for the poisson distribution $\lambda = 20 \times 3/4 = 15$

The required probability =

$$P(X = 1) + P(X = 2) + P(X = 3)$$

$$= \frac{e^{-15}15^1}{1!} + \frac{e^{-15}15^2}{2!} + \frac{e^{-15}15^3}{3!}$$

$$= 1.1 \times 10^{-6} + 6.6 \times 10^{-6}$$

5.28 (b):

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 1 - P(A') - P(B') + P(A' \cap B')$$

$$= 1 - 0.42 - 0.28 + 0.12$$

$$= 0.42 = 42\%.$$

5.29 (b):

Probability two people have the same birthday

$= 1 - P$(all have different birthday) P(exactly two people have same birthday)

Now, P(all have different birthday)

$$= \frac{365 \times 364 \times \ldots \times 365 - 364 + 1}{365}$$

P(exactly two people have same birthday)

$= {}^{365}C_{2} \times \frac{364 \cdot 363 \cdots (365-2+1)}{365^{2}}$

$^{365}C_{2}$ ways to choose who those two people will be, and 364 times 363 day only 365 ways are left, what the birthday is.

Now

P(at least three people have the same birthday)

$= 1 - \left[\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdots \frac{365-n+1}{365} \right]$

Which is option (b).

5.50 (0.65)

Let C denotes computer science day and M denotes mechanical. The tree diagram for the problem can be represented as shown below:

Now, to calculate total probability we take into the consideration (C) and get the answer as shown below

p(C on monday and C on wednesday)

= p(C on monday, C on tuesday and C on wednesday) + p(C on monday, M on tuesday and C on wednesday)

$= 0.4 \times 0.6 \times 0.4 + 0.4 \times 0.6 \times 0.4$

$= 0.16 + 0.16 = 0.32$

5.51 (0.5)

Given $\mu_x = 2$, $\sigma_x^2 = 4$, $\mu_y = 3$ and $\sigma_y = 5$ and $\rho_{xy} = -0.5$ is unknown

given, $p(X \le 1) = p(Y \ge 8)$

Converting into standard normal variate,

$$p\left(Z \le \frac{1-2}{\sigma_x}\right) = p\left(Z \ge \frac{8-3}{\sigma_y}\right)$$

$$p\left(Z \le \frac{-1}{2}\right) = p\left(Z \ge \frac{5}{5}\right)$$

$$p(Z \le -0.5) = p(Z \ge 1) \quad \text{...(i)}$$

Now we have to find out in standard normal distribution.

$$p(Z \le -0) = p(Z \ge 0) \quad \text{...(ii)}$$

Comparing (i) and (ii) we conclude that

$$\frac{5}{\sigma_y} = 1 \Rightarrow \sigma_y = 5$$

5.52 (0.9)

It is given that

$p(x=0) = 0.9\,p(x=1)$

Now since $\sum p(x) = 1$

$\therefore p(x=0) + p(x=1) = 1$

$\Rightarrow 0.9\,p(x=1) + p(x=1) = 1$

$\Rightarrow p(x=1) = \frac{1}{1.9} = 0.5263$

Now it is given that binary error $b(x) = x + 1$

$\therefore p(2) = p(x) = p(0)$

Now error,

$p(error) = p(2) \cdot p(4) + p(1)$

$= p(2) + p(4) + p(1)$

$\therefore p(2) = p(1) = p(0)$

$= \frac{1}{9} \cdot p(error) = \frac{1}{9} \cdot 0.5263$

$= 0.1754$

This given that

$p(error \mid x=0) = 0.75$

$$= \frac{p(error \cap x=0)}{p(x=0)} = 0.75$$

$$= \frac{p(x=0 \mid error)}{p(x=0)} = 0.75$$

$$\Rightarrow p(error \mid x=0) = \frac{0.75 \times 0.5}{0.75}$$

$$= \frac{p(x=1) \cdot p(x=0)}{0.75} = \frac{0.1754 \times 0.1754}{0.75}$$

$= 0.4677 \cdots$ Calculate

5.53 (a)

The tree diagram for the problem is shown above. Through the tree we can calculate probability.

p(defective faulty) = pq + (1 − p)(1 − q)

5.34 (pq):

$p(\text{multiple of } 10^{20} \mid \text{divisible by } 10^{10})$

$= \dfrac{p(\text{multiple of } 10^{20} \text{ \& divisible by } 10^{10})}{p(\text{divisible by } 10^{10})}$

$= \dfrac{p(\text{multiple of } 10^{20} \text{ \& divisible by } 10^{10})}{p(\text{multiple of } 10^{10})}$

Since $10^{20} = 10^{10} \times 10^{10}$

$p(\text{multiple of } 10^{20}) = p(\text{multiple of } 10^{10})$

Any divisor of 10^{10} is of the form $2^a \times 5^b$ where $0 \le a \le 10$, $0 \le b \le 10$.

The number of such divisors is given by $(10+1) \times (10+1) = 121 \times 121$.

So no. of divisors of $10^{10} = 121 \times 121$.

Any number divisible by multiple of 10^{10} must be divisible by 10^{10}. The first 2 digits are 00 and 25 respectively. The number of such numbers can be found by subtracting the number of numbers divisible by 10^{10} from the total number of numbers, which will be equal to $p(\text{multiple of } 10^{10})$ as well as divisible by 10^{10}.

$\therefore p(\text{multiple of } 10^{10} \mid \text{divisible by } 10^{10})$

$$= \frac{121 - 1}{121 \times 121} = \frac{1}{121}$$

5.35 (a):

Let A be the event of rainfall in a town.

B be the event of flood in a town.

The required probability is

$$p(A \mid B) = \frac{p(A \cap B) \text{ or } p(A \cap B)}{p(A \cup B)}$$

$$= \frac{p(A \cap B)}{p(A \cup B)}$$

$p(A \cap B) = p(\text{flood in town and rain})$

$= p(HH, HT) = \frac{2}{4}$

$p(A \cup B) = p(\text{flood or rain in town})$

$p(HH, HT, TH) = \frac{3}{4}$

So required probability $= \dfrac{2/4}{3/4} = \dfrac{1}{3}$

5.36 (b):

$Var(x) = E(x^2) - [E(x)]^2 = 0$

where $Var(x)$ is the variance of x,

So its variance is σ_x^2 and hence never negative.

$k = 0$.

5.37 (d):

Standard deviation is affected by scale but not by shift of origin.

$\sigma_x = \sigma_y$

$\sigma_x = \sigma_z$

So it could be either σ_y or σ_z. As σ_x is more, so standard deviation cannot be negative.

So $\sigma_x = \sigma_z$ is the answer.

5.38 (a):

The five entrances are {1, 2, 3, 4, 5}

So sample space = 5 ordered pairs.

Number of ways in which 2 persons have to choose each pair

$p(1^{st} \text{ and } 2^{nd} \text{ same}) = p(1^{st} \text{ and } 2^{nd} \text{ same}) = 1$

$= \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$

$= \dfrac{5}{25} = \dfrac{1}{5}$

5.39 (d):

The pmf of the random variable is

x	1	11
$P(x)$	0.5	0.5

The cumulative distribution function $F(x)$ is the probability that x is less than or equal to x as given below:

x	1	11
$F(x)$	0.5	1.0

So correct option is (d).

5.40 (bc):

If first throw is 1, 2 or 3 then sum is even. So 18 possible ordered pairs. Set of pairs are {(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)} and {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)}

If first throw is 4, 5 or 6 then second throw is odd and therefore the only way $S_n = 8$ is if the second was 6. Which leads to 3 possibilities.

So the required probability becomes $\frac{18}{36}$ or $\frac{1}{2}$.

From above diagram,

$$\text{prob} = P(1) = \frac{1}{2}\times\frac{3}{10}+\frac{1}{2}\times\frac{1}{6} = \frac{14}{30} = \frac{7}{15}$$

5.11 (a)

Poisson formula for $(X = x)$ given as

$$\frac{e^{-\lambda}\lambda^{x}}{x!}$$

x = 3 customers

λ = 1 customer

So $\lambda = nP = 8 \times 1 = 8$

Probability of observing fewer than 3 customers

$= P(X < 3) = P(X \le 2)$

$= P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{e^{-8}8^{0}}{0!} + \frac{e^{-8}8^{1}}{1!} + \frac{e^{-8}8^{2}}{2!} = \frac{17}{e^{8}}$$

Option (a) is correct.

5.12 (b)

Suppose we break a stick of unit length at a point chosen uniformly at random. Then the length of the longer piece is a random variable X uniformly distributed on the interval $[1/2, 1]$

i.e. $\alpha = 0$ and $\beta = 1/2$

In uniform distribution $E(X) = (\beta + \alpha)/2$

So the expected length of the longer piece $= 1/2(1/2 + 1) = 3/4 = 0.75$

5.13 (c)

Sample space $= 6^4 = 1296$

F, S, S, F $\rightarrow$ 4 ways $\left(\frac{4!}{2!} = 4\right)$

F, S, S, S $\rightarrow$ 6 ways $\left(\frac{4}{3!2!} = 6\right)$

Probability of event to be 2S $= \frac{6+4}{1296} = \frac{5}{648}$

$\Rightarrow$ $x = 10$

5.14 (c)

The tree diagram for the problem is shown below:

Required probability

$$= \frac{{}^{70}C_{5}\times{}^{30}C_{1}}{{}^{100}C_{6}} + \frac{{}^{70}C_{6}}{{}^{100}C_{6}}$$

$$= \frac{2.1}{3.0} + \frac{1}{3.7} = \frac{35}{3.6}$$

$$p = 0.1176$$

$\Rightarrow$ $100p = 11.76$

5.15 (b)

$1 \le x \le 100$

Fix is not divisible by 2, 3 or 5 $= 100 - P(2) - P(3) - P(5)$ is divisible by 2, 3 or 5

$$= 1 - \left[\left\lfloor\frac{100}{2}\right\rfloor + \left\lfloor\frac{100}{3}\right\rfloor + \left\lfloor\frac{100}{5}\right\rfloor\right]$$

$$= 100 - \left[\frac{100}{2} + \frac{100}{3} + \frac{100}{5}\right]$$

$$= \frac{26}{100} = 0.26$$

5.16 (c)

It is given that A and B are mutually exclusive also it is given that $A \cup B = S$

So in this case events A and B are collectively exhaustive

So if two events A and B are both mutually exclusive and collectively exhaustive, then $P(A \cap B) = P(\phi) = 1 \Rightarrow P(B) = 1 - P(A)$

Now we have to maximize $P(A)P(B)$

$= P(A)[1 - P(A)]$

Let $P(A) = x$

Now $P(A)[1 - P(A)] = x(1 - x) = x - x^2$

Say $y = x - x^2$

$$\frac{dy}{dx} = 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = -2 < 0 \left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} = -2 < 0$$

y has maximum at $y = 1/2$

$$v_{max} = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = 0.25$$

5.47 **Sol:**

The p.d.f. for X_1, X_2 and X_3 are given in the problem is as shown below

$$X_1 : \begin{vmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} \quad X_2 : \begin{vmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} \quad X_3 : \begin{vmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix}$$

Given: $Y = X_1X_2 + X_3$

The required probability $= P(Y = 0 \mid X_3 = 0)$

$$= \frac{P(Y = 0, X_3 = 0)}{P(X_3 = 0)} \quad \text{(1)}$$

Now

$P(X_3 = 0) = 1/2$ (from p.d.f. of X_3) (2)

$P(Y = 0, X_3 = 0)$

$= P(X_1X_2 + X_3 = 0, X_3 = 0)$

$= P(X_1X_2 + 0 = 0, X_3 = 0)$

$= P(X_1X_2 = 0, X_3 = 0)$

$= P(X_1 = 0, X_2 = 0, X_3 = 0) + P(X_1 = 0, X_2 = 1, X_3 = 0) + P(X_1 = 1, X_2 = 0, X_3 = 0)$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8} \quad \text{(3)}$$

$$\text{also } P(Y = 0, X_3 = 0) = \frac{3}{8} \quad \text{(4)}$$

Now substituting (2) and (3) in (1), we get

The required probability =

$P(Y = 0 \mid X_3 = 0)$

$$= \frac{3/8}{1/2} = \frac{3}{4} = 0.75$$

5.48 **Sol:**

Given: $f(x) = \frac{1}{x^2}$, $1 \le x \le 2$

$= 0$, elsewhere

So $\int_1^2 f(x)\,dx = 1$

$$\Rightarrow \int_1^2 \frac{1}{x^2}\,dx = 1$$

$$\Rightarrow \left[-\frac{1}{x}\right]_1^2 = 1$$

$$\left[-\frac{1}{2} + \frac{1}{1}\right] = 1$$

$$\Rightarrow \frac{1}{2} = 1$$

$$\Rightarrow c = \frac{1}{2} = 0.5$$

5.49 **Sol:**

The tree diagram for the given situation.

The desired output is 1

Now by total probability

probability $= P(1) = 0.5 \times 0.5 + 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5$

If p is the probability of 1

$p = 0.5 \times 0.5$

and $q = 0.5 \times 0.5$

so required $= P(1) = \frac{0.5 \times 0.5}{1 - 0.5 \times 0.5} = \frac{0.25}{0.75}$

$= \frac{1}{3} = 0.33$ (up to 2 decimal places)

5.50 **Sol:**

Defective $= 100$ bulbs $= 0.3 \times 0.7 + 0.2 \times 0.4$

$= 0.35 + 0.2 = 0.55$

■■■■

6 Linear Algebra

6.1 [illegible] which [illegible] the null space [illegible] equal [illegible]?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[1987 : 2 Marks]

6.2 A square matrix is singular when [illegible]

(a) The columns are linearly independent

(b) The rows are linearly independent

(c) The columns are linearly dependent

(d) None of the above

[1987 : 2 Marks]

6.3 If A and B are real symmetric matrices of size $n \times n$, then, which one of the following is true?

(a) $AA^T = I$ (b) $A = A^T$

(c) $AB = BA$ (d) $(AB)^T = BA$

[1984 : 2 Marks]

6.4 The rank of the following $(n+1) \times (n+1)$ matrix where a is a real number is

$$\begin{bmatrix} 1 & a & a^2 & \cdots & a^n \\ 1 & a & a^2 & \cdots & a^n \\ \vdots & & & & \vdots \\ 1 & a & a^2 & \cdots & a^n \end{bmatrix}$$

(a) 1

(b) 2

(c) n

(d) Depends on the value of a

[1991 : 1 Mark]

6.5 Let $AX = B$ be a system of linear equations where A is an $m \times n$ matrix and B is $m \times 1$ column vector. X is an $n \times 1$ [illegible]

Which of the following is false?

(a) The system has a solution if and only if both $[A|B]$ and A have the same rank

(b) If $m = n$ and B is the zero vector, then the system has infinitely many solutions

(c) If $m = n$ and B is non zero vector, then the system has a unique solution

(d) The system will have only trivial solution if $m = n$, B is the zero vector and rank $(A) = n$

[1995 : 1 Mark]

6.6 [illegible] and [illegible]

[illegible]

(a) if $a = b$ or $b = c$, n is an integer

(b) always

(c) never

(d) if $a = b = c$, $n = 0$

[1986 : 2 Marks]

6.7 The determinant of the matrix

$$\begin{bmatrix} 3 & -5 & 1 & 1 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ is}$$

(a) 1 (b) [illegible]

(c) 11 (d) [illegible]

[1997 : 1 Mark]

6.8 [illegible] (a_{ij}) be a [illegible] square matrix and I_n be the [illegible] identity matrix of order n. [illegible] the matrix A. Then AI_n is [illegible] that matrix

(a) row [illegible] same as the second row

(b) row [illegible] same as the second row of A

(c) column [illegible] same as the second column of A

(d) zero matrix

[1997 : 2 Marks]

6.9 Consider the following system of equations

$x + 2y = 6$

$3x + 6y = 12$

$2x + 4y = 12$

This system

(a) [illegible]
(b) [illegible]
(c) [illegible] number of solutions
(d) [illegible] number of solutions

[1996 : 1 Mark]

6.10 The rank of the matrix [illegible] is

$$\begin{vmatrix} [illegible] & 4 & 5 & 6 \\ 0 & [illegible] & 7 & 8 \\ 5 & 2 & [illegible] & 3 \\ [illegible] & [illegible] & 14 & 21 \end{vmatrix}$$

(a) 3 (b) [illegible]
(c) 2 (d) 4

[1999 : 2 Marks]

6.11 Consider the following determinant:

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Which of the following statements is true for Δ?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[1998 : 2 Marks]

6.12 [illegible]

[illegible]

The sum of the elements of the array is

(a) 0 (b) [illegible]
(c) [illegible] (d) $n^{(n+1)}$

[2000 : 1 Mark]

6.13 The determinant of the matrix $\begin{vmatrix} [illegible] & 3 & 5 & 7 \\ 8 & [illegible] & 7 & 9 \\ 2 & [illegible] & 2 & 3 \\ [illegible] & [illegible] & [illegible] & 1 \end{vmatrix}$ is

(a) 4 (b) 0
(c) [illegible] (d) 1

[2001 : 2 Marks]

6.14 Consider the following statements:

S_1 : The sum of two singular $n \times n$ matrices may be non-singular.

S_2 : The product of two non-singular matrices may be singular.

Which of the following statements is correct?

(a) S_1 and S_2 are both true
(b) S_1 is true, S_2 is false
(c) S_1 is false, S_2 is true
(d) S_1 and S_2 are both false

[2002 : 1 Mark]

6.15 The rank of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is

(a) 4 (b) 2
(c) [illegible] (d) 0

[2002 : 1 Mark]

6.16 Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 36 & 12 \\ 0 & 2 & 45 & 34 \\ 0 & 0 & -1 & 1[illegible] \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[2003 : 2 Marks]

6.17 Consider the following system of linear equations:

$$\begin{bmatrix} 3 & 1 & -1 \\ 4 & 2 & 12 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Notice that the second and the third columns of the coefficient matrix are linearly dependent. How many non-zero solutions does the system have?

(a) 0 (b) 1
(c) 2 (d) infinitely many

[2003 : 2 Marks]

6.18 Let $A = [a_{ij}]$ be a 3×3 matrix with real entries. If $|A| = 1$, then $|B^{-1}|$ is

(a) $|A^{-1}|$
(b) $|A|$
(c) $|A^2|$
(d) [illegible]

[2001 : 1 Mark]

6.19 What values of x, y, z satisfy the following system of linear equations?

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 3 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 6 \\ 8 \\ 12 \end{vmatrix}$$

(a) $x = 6, y = 8, z = 3$
(b) $x = 12, y = [illegible], z = [illegible]$
(c) $x = 6, y = 0, z = -1$
(d) $x = [illegible], y = [illegible], z = 3$

[IT-2004 : 1 Mark]

Q.20 Let A be an $n \times n$ matrix of the following form

$$A = \begin{vmatrix} 8 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 & & 0 & 0 & 0 \\ \dots & & & & & & & & \\ \dots & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 8 \end{vmatrix}$$

What is the rank of the matrix A^2?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[IIT-2004 : 2 Marks]

Q.21 If matrix $X = \begin{bmatrix} 0 & 1 \\ -a^2 - a - 1 & -a \end{bmatrix}$ and $X^2 - X - I = 0$ (I is the identity matrix and 0 is the zero matrix), then the inverse of X is

(a) $\begin{bmatrix} 1 & a \\ a^2 & a \end{bmatrix}$ (b) [illegible]

(c) $\begin{bmatrix} a & 1 \\ -a^2 - a - 1 & -a \end{bmatrix}$ (d) [illegible]

[IIT-2004 : 2 Marks]

Q.22 How many solutions does the following system of linear equations have?

$$x + 6y = -1$$
$$x + y = 2$$
$$x + 6y = 4$$

(a) infinitely many

(b) two distinct solutions

(c) unique

(d) none of these

[2004 : 2 Marks]

Q.23 The determinant of the matrix given below is

$$\begin{matrix} 3 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 3 & 3 & 0 & 1 \\ 1 & 2 & 0 & 1 \end{matrix}$$

(a) -1 (b) 0

(c) 1 (d) 2

[IIT-2005 : 1 Mark]

Q.24 Consider the following system of equations in three real variables x_1, x_2 and x_3:

$$2x_1 - x_2 + 3x_3 = 1$$
$$x_1 + 2x_2 - 5x_3 = 2$$
$$x_1 + 7x_2 + x_3 = 3$$

This system of equations has:

(a) no solution

(b) unique solution

(c) more than one but a finite number of solutions

(d) infinitely many solutions

[2005 : 2 Marks]

Q.25 What are the eigenvalues of the following 2×2 matrix?

$$\begin{matrix} 2 & 1 \\ -1 & 5 \end{matrix}$$

(a) 1 and 1 (b) 1 and 6

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

Q.26 E is an $m \times n$ real matrix, b is an $m \times 1$ real vector. Suppose there are two $n \times 1$ vectors, x and y such that $Ex = b$, and $Ey = b$, $x \neq y$.

Which of the following statements is/are false?

(a) Determinant of E is zero

(b) There are infinitely many solutions to $Ex = b$

(c) There exists a $z \neq 0$ such that $Ez = 0$

(d) E must have two identical rows

[2008 : 2 Marks]

Q.27 What are the eigenvalues of the matrix P given below?

$$P = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

(a) $0, -\sqrt{2}, +\sqrt{2}$ (b) [illegible]

(c) 0, 0, 0 (d) 0, 0, 1

[IIT-2004 : 2 Marks]

Q.28 Let A be the 2×2 matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. What is the maximum value of $x^T Ax$ where the maximum is taken over all x that are unit vectors?

[illegible]

(a) 2 (b) [illegible]
(c) [illegible] (d) [illegible]

[ST-2005 : 1 Mark]

6.31 Consider the set of polynomials [illegible] in X [illegible] $= x_1$ [illegible] $= 0$, where F [illegible] $x_2]^T$. Which of the following [illegible]?

(a) [illegible] is a basis for [illegible] subspace X.
(b) [illegible] is a linearly independent set [illegible] X [illegible] therefore is a basis for X.
(c) [illegible]
(d) None of the above

[2007 : 2 Marks]

6.32 Consider the following equations

$$x_1 + x_2 = 2x_3 \quad (1)$$
$$x_1 = 2x_3 \quad (2)$$
$$x_1 + 2x_2 = 4 \quad (3)$$

Incorporating into these equations, the possible solution is

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2008 : 1 Mark]

6.33 [illegible] the following matrices have [illegible]

$$[illegible] \begin{bmatrix} 1 & 2 \\ [illegible] \end{bmatrix} \quad \begin{bmatrix} -1 & 0 \\ [illegible] & -1 \end{bmatrix}$$

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

6.34 [illegible] $\mathbf{x}$ [illegible] $\mathbf{x}$ [illegible] $\mathbf{x}$ [illegible] $\mathbf{x}$ [illegible] correct?

S1: [illegible] are represented [illegible] of the other two.
S2: The [illegible] of $\mathbf{x}$ [illegible] can be represented as a linear combination of the other two vectors.
S3: $\mathbf{x}\mathbf{x}^T = 0$ [illegible]
S4: [illegible] inverse.

(a) S2 and S3 (b) [illegible] S4
(c) S1 and S4 (d) [illegible]

[IT-2009 : 3 Marks]

6.35 Consider the matrix

$$A = \begin{bmatrix} [illegible] \end{bmatrix}$$

The eigenvalues corresponding to A are

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2010 : 2 Marks]

6.36 Consider the matrix of [illegible]

$$[illegible] = 8$$
$$\begin{vmatrix} 6 & [illegible] \\ [illegible] & 8 \end{vmatrix}$$

Which one of the following [illegible] for $C(M)$ [illegible] of the [illegible] of the matrix?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2011 : 2 Marks]

6.37 [illegible] $= a_{11} = -1$ and $a_{22} = 1$.

Then the zero-free [illegible] of the matrix A [illegible]

(a) [illegible] and [illegible]
(b) [illegible] and [illegible]
(c) [illegible] and [illegible]
(d) [illegible] and [illegible]

[2013 : 1 Mark]

6.38 [illegible] the following does NOT equal

$$[illegible]$$

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2013 : 1 Mark]

6.39 Consider the following system of linear equations

$$3x - 2y = 1$$
$$4x - 3y = 1$$
$$-x + y + z = 0$$
$$x - y + 2z = 0$$

The number of solutions for this system is

[2015 (Set-1) : 1 Mark]

6.38 [illegible] is

[2014 (Set-I) : 1 Marks]

6.39 [illegible]

$$A = \begin{bmatrix} 2 & \\ & 7 \end{bmatrix}$$

[illegible]

[2014 (Set-III) : 1 Marks]

6.40 The product of the non-zero eigenvalues of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is _____.

[2014 (Set-2) : 2 Marks]

6.41 Which one of the following statements is TRUE about a square matrix with real entries?

(a) If the sum of the eigenvalues of the matrix is positive, the determinant of the matrix is positive.

(b) If the trace of the matrix is positive, all its eigenvalues are positive.

(c) If the determinant of the matrix is positive, all its eigenvalues are positive.

(d) If the product of the eigenvalues is positive, all the eigenvalues are positive.

[2011 (Set-2) : 1 Mark]

6.42 If V_1 and V_2 are 3-dimensional subspaces of a 6-dimensional vector space V, then the smallest possible dimension of $V_1 \cap V_2$ is _____.

[2014 (Set-3) : 1 Mark]

6.43 The minimum number of accurate measurements required to determine the eigenvalues of the matrix $P(V) = V - V(V^T V)^{-1} V^T$ where V is an $m \times n$ matrix, $m > n$, and V^T is its transpose, is _____.

[2014 (Set-2) : 1 Mark]

6.44 Consider the following matrix [illegible] and [illegible]. The eigenvalues of the matrix [illegible] and [illegible]. What are the values of [illegible] and [illegible]?

$$A = \begin{bmatrix} [illegible] \\ [illegible] \end{bmatrix}$$

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2013 (Set-1) : 2 Marks]

6.45 The trace of the inverse of the matrix

$$\begin{bmatrix} [illegible] \end{bmatrix} = _____.$$

[2013 (Set-2) : 1 Mark]

6.46 Consider the following system of linear equations represented in matrix form

$$\begin{bmatrix} 5 & 1 & 15 \\ [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] \end{bmatrix}$$

1. Add the third row to the second row.
2. [illegible] from the first row [illegible].

The resulting [illegible] is _____

[2013 (Set-2) : 2 Marks]

6.47 In the given matrix $\begin{bmatrix} [illegible] & 1 & 2 \\ [illegible] & 1 & [illegible] \\ [illegible] & 2 & 1 \end{bmatrix}$ one of the eigenvalues is [illegible]. The eigenvalues are arranged in ascending order of their magnitudes.

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2013 (Set-3) : 1 Mark]

6.48 [illegible] $\begin{bmatrix} 2 & 2 \\ [illegible] & 9 \end{bmatrix}$

[illegible]

[2013 (Set-1) : 1 Mark]

6.29 [illegible]

[illegible] [2015 (Set-3) : 2 Marks]

6.30 [illegible]

[illegible] [2016 (Set-1) : 1 Marks]

6.31 [illegible]

I. [illegible]

II. [illegible]

III. [illegible]

Which [illegible] the following is CORRECT?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2016 (Set-2) : 1 Marks]

6.32 [illegible]

[illegible] [2016 (Set-3) : 1 Marks]

■■■■

Answers : Linear Algebra

6.1	(c)	6.2	(c)	6.3	(d)	6.4	(a)	6.5	(a)	6.6	(a)	6.7	(b)	6.8	(c)	6.9	(b)
6.10	(a)	6.11	(b)	6.12	(a)	6.13	(a)	6.14	(a)	6.15	(c)	6.16	(c)	6.17	(b)	6.18	(c)
6.19	(d)	6.20	(b)	6.21	(a)	6.22	(c)	6.23	(b)	6.24	(d)	6.25	(d)	6.26	(a)	6.27	(b)
6.28	(a)	6.29	(d)	6.30	(c)	6.31	(d)	6.32	(d)	6.33	(a)	6.34	(a)	6.35	(d)	6.36	(a)
6.37	(d)	6.38	(b)	6.39	(c)	6.40	(a)										

Explanations : Linear Algebra

6.1 (c)

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

6.2 (c)

[illegible]

Therefore [illegible] linearly dependent, [illegible] consider.

6.3 (d)

The matrix M is said to be symmetric if $M^T = M$

$M^2 = M^T M = MM^T = M$

Since A and B are symmetric $A^T = A$ & $B^T = B$.

6.4 (a)

If the rows of the given matrix are [illegible] the matrix has only one independent row.

[illegible] matrix = No. of independent rows of the matrix.

∴ Rank of given matrix = 1

6.5 (a)

Following are the possible solutions for a system of linear equations:

(i) If matrix A and augmented matrix [A:B] have same rank then the system has unique solution if the rank is equal to the number of variables.

(ii) If rank of A and augmented matrix [A:B] have same rank but it is less than the number of variables, then the system has infinite solutions.

(iii) If matrix A and augmented matrix [A:B] have different rank, which is less than the number of variables, then system has no solution.

Therefore, option (a) is the only correct answer.

6.6 (b)

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and $$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$AB = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} a\cos\theta - c\sin\theta & b\cos\theta - d\sin\theta \\ a\sin\theta + c\cos\theta & b\sin\theta + d\cos\theta \end{bmatrix}$$

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} a\cos\theta + b\sin\theta & -a\sin\theta + b\cos\theta \\ c\cos\theta + d\sin\theta & -c\sin\theta + d\cos\theta \end{bmatrix}$$

$AB = BA$ if $b\sin\theta = c\sin\theta$

and $a\sin\theta = d\sin\theta$

$\sin\theta = 0$ or $a = d$

$\sin\theta = 0$ or $b = c$

$\sin\theta = 0$ or $a = d$ or $b = c$

6.7 (b)

Inverse of matrix is

$$\begin{bmatrix} 6 & -5 & 1 \\ 11 & 2 & -4 & 3 \\ 11 & 1 & -4 & 3 \\ 8 & 0 & 2 & 1 \end{bmatrix}$$

Above matrix is a square matrix and its upper triangular matrix for elementary row transformation (upper triangular matrix)

Determinant of matrix

$= 6 \times 2 \times 1 \times 1 = 12$

6.8 (a)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Trace of matrix is defined by sum of diagonal elements of matrix. Therefore, trace of matrix A is

$$\text{Tr}(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\text{Tr}(A^2) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}^2 = \begin{vmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{vmatrix}$$

A^2 is the matrix obtained by multiplication of matrix A with itself. Therefore, A^2 is a square matrix.

6.9 (b)

Given equations are

$$x - 2y = 5$$
$$4x + 3y = 2$$
$$3x + 8y = 6$$

Above equations can be written as

$$\begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 6 \end{bmatrix}$$

Augmented matrix [A:B] is given as

$$\begin{bmatrix} 1 & -2 & 5 \\ 4 & 3 & 2 \\ 3 & 8 & 6 \end{bmatrix}$$

Performing gauss-Elimination on the above matrix

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & 5 \\ 4 & 3 & 2 & 22 \\ 3 & 8 & 6 & 6 \end{array}\right] \xrightarrow[\text{and } R_3 - 3R_1]{R_2 - 4R_1}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & 5 \\ 0 & 11 & -18 & 2 \\ 0 & 14 & -9 & -9 \end{array}\right] \xrightarrow{R_3 - \frac{14}{11}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & 5 \\ 0 & 11 & -18 & 2 \\ 0 & 0 & 135 & -45 \end{array}\right]$$

The rank of matrix is 3 and order of matrix [A:B] is 3.

Therefore, the system is consistent and has unique solution.

6.10 (c)

The given matrix is

$$\begin{bmatrix} 4 & 8 & 7 \\ 0 & 0 & 0 \\ 4 & 8 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

Rank = 2

Now we find the rank of A

$$\begin{vmatrix} 4 & 8 & 0 \\ 0 & 0 & 8 \\ 4 & 8 & 0 \end{vmatrix} = 4\begin{vmatrix} 0 & 8 \\ 8 & 0 \end{vmatrix}$$

$$= 4 \times (-64) = -256 \neq 0$$

∴ Rank of the matrix = 3

6.11 (b)

The determinant of the matrix is the product of the entry on the main diagonal

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & b & 4 \\ 0 & 0 & ab \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & (b-0) & (4-0) \\ 0 & (0-0) & (ab-0) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 0 & (b-0) & -(0-4) \\ 0 & 0 & (ab-0) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 0 & b & 4 \\ 0 & 0 & ab \end{vmatrix}$$

$$= 1(b-0)(ab-0) - 2(0-0)(ab-0) + 3(0-0)(0-0)$$

$$= (b)(ab) - 2(0)(ab-0) + 3(0)(0-0)$$

$$= (b)(ab) - 2(0)(ab-0) + 3(0)(0-0)$$

$$= (b)(ab) + 0 + 0 = ab$$

So ab is a factor of A.

6.12 (a)

The matrix $X(t)$ is by definition

$$X(t) = \begin{bmatrix} 0 & 1 & 2 & \cdots & 1 & 0 \\ 0 & 0 & 1 & \cdots & 1 & 1 \\ 0 & 0 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ (n-1) & (n-2) & (n-3) & \cdots & 0 & 0 \end{bmatrix}$$

so of course antisymmetric matrix and hence sum of the elements of any antisymmetric matrix is 0.

6.13 (d)

Alternate Method:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} = n\sum_{i=1}^{n} a_{i(n-i)} \quad \text{[illegible]}$$

$$= \frac{n(n-1)}{2} \cdot n = \frac{n^2(n-1)}{2} \quad \text{[illegible]}$$

6.14 (a)

The matrix is $\begin{vmatrix} 2 & 0 & 0 & 0 \\ 5 & 7 & 0 & 0 \\ 2 & 3 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$

Rank of the matrix is determined by expanding the first row of the matrix

$$\text{Determinant} = \begin{vmatrix} 2 & 0 & 0 \\ 5 & 7 & 0 \\ 2 & 3 & 2 \end{vmatrix} = 2 \times \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 2 \times 1 \times (2) = 4$$

Therefore, the determinant of given matrix is 4.

6.15 (b)

R_1 is true

Consider the two elementary matrices

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Since A and B is idempotent

Consider $(A + B)$ is an elementary matrix

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, so R_2 is false.

Now consider two matrices, 1 matrices

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$C = D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

However $(C - D)$ is a singular matrix, so [illegible] matrix.

Therefore, both B and A are true.

6.16 (a) :

The given matrix is $\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$

Above matrix is invertible and diagonalizable because the given matrix is symmetric.

6.16 Sol:

We have to find [illegible]

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The given matrix is a unique triangular matrix and [illegible] matrix are diagonal. Therefore upper triangular matrix.

So the given [illegible] and the matrix have [illegible]

$\lambda_1 =$

6.17 (b) :

The augmented matrix for the given system is

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & -1 & 3 \\ 2 & 1 & 1 & 7 \end{array}\right]$$

Performing Gauss-Jordan method on the above matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -1 & -5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -3 & -8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & \frac{-8}{3} \end{array}\right]$$

Now the augmented matrix is [illegible] the system has no solution.

6.18 (b)

$\lambda_1, \lambda_2, \lambda_3$ [illegible]

Given $ABC = I$

$\Rightarrow (ABC)^{-1} = I^{-1}$

$\Rightarrow \quad A^{-1} = B^{-1}C$

$\Rightarrow \quad A^{-1}(A^{-1})^{-1} = A^{-1}B^{-1}C$

$\Rightarrow \quad I = A^{-1}(A^{-1})^{-1}$

$\quad\quad I = A^{-1}B^{-1}C$

$\quad\quad (I)^{-1} = (A^{-1}B^{-1}C)^{-1}$

$\quad\quad \Rightarrow (C)^{-1}(B^{-1})^{-1}(A^{-1})^{-1}$

$\quad\quad = CBA$

6.19 (a)

[illegible]

$1 \times y = 3^{-1} \times 4 + 1^{-1} \times 3 = 6$

$1 \times y = 3^{-1} \times 1 + 1^{-1} \times 4 = 5$

$y = y = 3^{-1} \times 1 + 1^{-1} \times 3 = 12$

Out of $x = 3$, $y = 6$, $z = 12$, we can conclude that [illegible] calculated.

6.20 (a)

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & & & & & & & & \\ \cdots & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 2 \end{bmatrix}$$

A can be expanded using first row as $|A_n| = 2|A_{n-1}| + |A_{n-2}|$.

So find [illegible] as the given matrix with a TRIDIAGONAL [illegible] $|A_n|$ is calculated as P_n.

Then recurrence relation is

$$P_n = 2P_{n-1} - P_{n-2}$$

$$\Rightarrow P_n - 2P_{n-1} + P_{n-2} = 0$$

This is a homogeneous system [illegible] is

[illegible]

Now [illegible] solution

$$T = [illegible] = 8 \text{ and } C_2 = [illegible]$$

We get

$$[illegible]$$

Alternatively, [illegible] for $n = 3$ and $n = 2$

$$d_1 = [illegible] \text{ and } d_2 = [illegible] = 8$$

[illegible] and [illegible] in [illegible] solution. [illegible] respectively.

6.31 (b)

Given X [illegible]

and $X^2 - X - I = 0$ [illegible]

We need to find X^n.

[illegible] for [illegible] we get

$$I = X - X^2$$

Multiply [illegible] by X [illegible] both sides

$$X^{-1} = [illegible] - X$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} [illegible] \end{bmatrix}$$

$$= \begin{bmatrix} [illegible] \end{bmatrix}$$

Which is option (b).

6.32 (c):

$$x - 3y = [illegible]$$

$$x - y = 2$$

$$x + 3y = 3$$

The augmented matrix is

$$\left[\begin{array}{cc|c} 1 & -3 & [illegible] \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right]$$

Using elementary row transformations we get,

$$\left[\begin{array}{cc|c} 1 & -3 & [illegible] \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{cc|c} 1 & -3 & [illegible] \\ 0 & 4 & [illegible] \\ 0 & 6 & [illegible] \end{array}\right]$$

[illegible]

Rank $[A] = 2$

Rank $[A|B] = 2$

Since Rank $[A|B]$ = Rank $[A]$ = number of unknowns, the system has unique solution.

[illegible]

$$\left[\begin{array}{cc|c} -1 & 6 & [illegible] \\ 0 & 4 & [illegible] \\ 0 & 0 & 0 \end{array}\right]$$

Rank $[A|B] = 2$ = number of non-zero rows in $[A|B]$.

Rank $[A] = 2$ = number of non-zero rows in $[A]$.

Rank $[A|B]$ = Rank $[A] = 2$ = number of unknowns.

$\therefore$ The [illegible] solution exists. Consistent and unique.

6.33 (c)

$$A = \begin{bmatrix} 0 & 1 & 0 & 4 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 0 & 0 & 1 \end{bmatrix}$$

By expanding the determinant with minor of element

$$\Delta = -1 \begin{vmatrix} 0 & 1 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

$$= -1 \times 0 \times \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= -1 \times 0 \times 1 = 0$$

6.34 (b)

The augmented matrix for the given system is

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Using partial pivot method, we have multiplying we get,

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[R_3 \to R_3]{R_2 \to R_2}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 0 & -1 \times 2 & 1 \times 2 & 1 \times 2 \\ 0 & 0 \times 2 & 0 \times 2 & 0 \times 2 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3]{R_2 \to R_2} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 0 & -1 \times 2 & 1 \times 2 & 1 \times 2 \\ 0 & 0 & 2 & 1 \end{array}\right]$$

Rank $[A] = 3$

Rank $[A|B] = 3$

Since Rank $[A|B]$ = Rank $[A]$ = number of unknowns, the system has unique solution.

6.95 (b)

[illegible]

The characteristic equation of the matrix is given by

[illegible]

[illegible]

$\lambda^3 - 7\lambda - 6 = 0$

$\lambda = [illegible]$

$\therefore$ The eigenvalues are [illegible] and 3

6.96 (b)

[illegible]

[illegible]

[illegible]

(a) [illegible]

(b) The number of linearly independent eigenvectors corresponding to the eigenvalue 3 is [illegible]

(c) There are [illegible] linearly independent eigenvectors corresponding to the eigenvalue 3. Therefore the matrix is diagonalizable.

(d) Found two linearly independent eigenvectors corresponding to the eigenvalue 3 [illegible] identical [illegible] It is not necessary that 2 identical eigenvalues [illegible] the matrix [illegible] be diagonalizable.

6.97 (b)

[illegible]

[illegible]

Since, the [illegible] matrix

So the eigenvalues [illegible]

[illegible]

Product of eigenvalues

[illegible]

Only choice with sum of eigenvalues [illegible] and product of eigenvalues [illegible]

So [illegible] the correct option.

6.98 (b)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The eigenvalues are [illegible]

The required answer is the maximum eigenvalue of $A = \frac{\sqrt{11}+\sqrt{3}}{2}$

6.99 (b)

[illegible]

[illegible]

[illegible] Since all [illegible] x_1, x_2, x_3 [illegible] $x_1 = x_2 = x_3$ [illegible] be expressed as [illegible] $[0, 1]$ [illegible]

6.100 (b)

The augmented matrix for the system is

[illegible]

[illegible]

[illegible]

$= 8$

$\therefore$ [illegible]

Choose option (b) [illegible]

6.81 (a)

Eigenvalues of $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & 0 \\ 0 & 0-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-\lambda) = 0$$

$$\lambda = 0 \text{ or } \lambda = 1$$

Eigenvalues of $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\begin{vmatrix} -\lambda & 1 \\ 0 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 = 0$$

$$\lambda = 0, 0$$

Eigenvalues of $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 - 1 = 0$$

$$1 - 2\lambda + \lambda^2 - 1 = 0$$

$$1-\lambda = 1, -1$$

$$\lambda = 1 \text{ or } 1+i$$

Eigenvalues of $\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$

$$\begin{vmatrix} -1-\lambda & 0 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(-1-\lambda)(1-\lambda) = 0$$

$$(\lambda+1)(\lambda-1) = 0$$

$$\lambda = -1, -1$$

Only one matrix has one eigenvalue 1 which is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence choose (a)

6.82 (d)

S1 and S2:

Since M has zero determinant, its rank is less than 3. But M is of size 3 × 3, then its rank is one. So, the three column vectors of M are linearly dependent. Which means there exists a linear combination of these vectors which evaluates to 0 i.e. $k_1C_1 + k_2C_2 + k_3C_3 = 0$

Now any one of C_1 can be written as a linear combination of the other two as

$$C_1 = \frac{k_2}{k_1}C_2 - \frac{k_3}{k_1}C_3$$

$$= -\frac{k_2}{k_1}C_2 + \frac{k_3}{k_1}C_3 + 0C_4 + \dots + \frac{k_n}{k_1}C_n$$

So, rank is less than the number of columns.

So, all minors are zero.

S3: $MX = 0$ is a homogeneous system and such an equation where $|M| = 0$ has non trivial solutions.

S4: Since M is singular, M has no inverse. So S4 is false.

So S1, S2 are true and S3 only, so choose (a).

6.83 (a)

Sum of eigenvalues = trace $(A) = 2 + y$

Product of eigenvalues = $|A| = 2y - 3$

$\therefore$ $4 + 8 = 2 + y$ ... (i)

$4 \times 8 = 2y - 3$ ... (ii)

$\therefore$ $8 - y = 12$ (i)

$2y - 3 = 32$ ... (ii)

$\therefore$ Solving (i) and (ii) we get

$$y = 4 \text{ and } x = 10.$$

6.84 (b)

Since the given matrix is upper triangular, its eigenvalues are the diagonal elements i.e. the eigenvalues are 1, 4 and 3.

6.85 (b)

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Eigen (A) are the roots of the characteristic equation given below:

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1-\lambda) - 1 = 0$$

$$-(1-\lambda)(1+\lambda) - 1 = 0$$

$$\lambda^2 - 2 = 0$$

$$\lambda = \pm\sqrt{2}$$

Eigenvalues of A are $\sqrt{2}$ and $-\sqrt{2}$ respectively.

So eigenvalues of $A^2 = (\sqrt{2})^2$ and $(-\sqrt{2})^2$

$$= 2 \text{ and } 2$$

$$= 2^2 \text{ and } 2^2$$

$$= 2^2 \sqrt{2} \text{ and } 2^2 \sqrt{2}.$$

6.36 (a):

The given matrix can be transformed into the matrix given in (a) only by the elementary operations of the type $R_i \to kR_i$ or $R_i \to R_i + kR_j$ only as shown below.

Option (a):

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & x & x^2 \\ 1 & x & x^2 \end{vmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{vmatrix} 1 & x-1 & x^2-x \\ 1 & x-1 & x^2-x \\ 1 & x+1 & x^2-1 \end{vmatrix}$$

Option (b):

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & x & x^2 \end{vmatrix}$$

Option (d):

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{vmatrix} 0 & x+y & x^2-y^2 \\ 0 & y+z & y^2-z^2 \\ 1 & x & x^2 \end{vmatrix}$$

Option (c): We can never obtain the given matrix as it can not be converted into a row by multiplying a column with -1 which will change the sign of the determinant as we can see below.

$$\begin{vmatrix} x & x^2 & 0 & x+1 & x(x+1) \\ y & y^2 & -1 & y+1 & y(y+1) \\ z & z^2 & 1 & z+1 & z(z+1) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x(x+1) & 0 & x & 1 \\ 0 & y(y+1) & -1 & y+1 & 1 \\ 0 & z(z+1) & 1 & z+1 & 1 \end{vmatrix}$$

6.37 Sol:

Performing gauss elimination on the augmented matrix we obtain as,

$$\left[\begin{array}{cccc|c} 2 & 2 & 6 & 1 & \\ 4 & 3 & 7 & 1 & \\ & 1 & 1 & 2 & \\ 2 & 2 & 7 & 0 & \end{array}\right]$$

We can write it as

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & \frac{1}{2} & \\ 0 & 1 & 8 & 8 & \\ 0 & 0 & 26 & 21 & \\ 0 & 0 & 25 & 21 & \end{array}\right] \xrightarrow{R_4 \to R_4 - R_3} \left[\begin{array}{cccc|c} 1 & 1 & 3 & \frac{1}{2} & \\ 0 & 1 & 8 & 8 & \\ 0 & 0 & 26 & 21 & \\ 0 & 0 & 0 & 0 & \end{array}\right]$$

Rank (A) = 3

Rank (A/B) = 3

Rank (A) = Rank (A/B) = No. of variables = 3.

So unique soln.

So option (a) is correct.

6.38 Sol:

The value of the determinant of a matrix is equal to the product of its eigenvalues. So any pair of matrices with same eigenvalues will have same determinant.

6.39 Sol:

$$A = \begin{bmatrix} 2 & & \\ -1 & 1 & 1 \\ 7 & & 5 \end{bmatrix}$$

$A = [X_1, X_2, X_3]$

So rank of $A = 3$ if

$$A = \begin{bmatrix} 2 & -5 & 13 \\ -1 & 36 & 29 \\ 7 & 63 & 36 \end{bmatrix}$$

So as X_1, X_2, X_3 now are linearly dependent $|A| = 0$.

6.40 Sol:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & & & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & & & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ let } X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$AX = \lambda X$

$\Rightarrow x_1 + x_5 = \lambda x_1$

$\Rightarrow x_2 + x_4 = \lambda x_2 \Rightarrow \lambda x_2 = \lambda x_1$

(i) $\lambda \neq 0$

$\Rightarrow x_2 = x_1 = 0$

$x_3 = \lambda x_3 = x_3 = 0$

$\Rightarrow x_1 + x_5 = \lambda x_1 \Rightarrow 2x_1 = \lambda x_1$

$\Rightarrow \lambda = 2$

$\Rightarrow x_3 = x_3 + x_4 = \lambda x_3 \Rightarrow 2x_3 = \lambda x_3$

$\Rightarrow \lambda = 2$

(ii) $\lambda = 0$

$\therefore$ Eigenvalue $\lambda = 0$

$\therefore$ There are 2 distinct eigen values 0 & 2

Product of distinct eigen values $= 2 \times 0 = 0$

6.41 (a)

Sum of eigenvalues = Trace (A)

Product of eigenvalues = |A|

(a) If the trace of the matrix is positive and the determinant of the matrix is negative, then at least one of the eigenvalues is negative.

This statement is false. It could not be that all eigenvalues would be positive and the product would be negative.

(b) If the trace of the matrix is positive and the eigenvalues are positive

This is false since the sum of positive and negative eigenvalues is positive.

(c) If the determinant of the matrix is positive, all the eigenvalues are positive

This is false since product of two negative numbers is positive

(d) If the product of the trace and determinant of the matrix is positive, all the eigenvalues are positive

This statement is false since negative eigenvalues and determinant is negative. Then all eigenvalues could not be positive.

6.42 Sol.

$$V = \{p, b, c, d, e, f\}$$

$\Rightarrow V_1 = \{p, b, c, d\}$ and $V_2 = \{p, e, f\}$

Smallest possible dimension of $V_1 \cap V_2 = 2$

6.43 Sol.

$P(X) = X^3 - 4X^2 - 6X + 3$

$= X(X^2 - 4X - 6) + 3 = X(X(X^2 + 4X + 3) + 3$

$= X(X(X^2 + 4X + 3) + 3)$

Only one temporary variable is required

temp = X*X

temp = temp + 4*X

temp = X*temp

temp = X*temp

temp = temp + 3

temp = X*temp

temp = temp + 3

$\therefore$ 7 arithmetic operations are done.

6.44 (d):

Trace = Sum of eigenvalues

$1 + x = 6$

$\Rightarrow x = 5$

Determinant = Product of eigenvalues

$3x = 4k, \ -7$

$3 = 4k, \ -7$

$\Rightarrow k = -\frac{3}{4}$

$\Rightarrow k = \frac{3}{4}$

$\Rightarrow k = 3$

6.45 Sol.

$|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 6 \\ 2 & 4-\lambda \end{vmatrix} = (4-\lambda)(4-\lambda) - 12 = 0$$

$\lambda^2 - 8\lambda + 4 = 0$

$\lambda = \frac{8 \pm \sqrt{64 - 16}}{2} = 4 \pm 2$

$\Rightarrow \lambda = 6, \ 2$

$\therefore$ The smaller eigenvalue is 2

6.46 Sol.

Given vectors 1 and 2 are orthogonal to each other, the type of R 1 R2 and C 1 C2 respectively. Hence, it will be unchanged from the original matrix.

So the required determinant

$$= \begin{vmatrix} 5 & 4 & 40 \\ 7 & 3 & 105 \\ 8 & 2 & 100 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 4 & 40 \\ 7 & 3 & 105 \\ 8 & 2 & 100 \end{vmatrix} \quad C_3 \to 100C_3 \quad \begin{vmatrix} 5 & 4 & 0 \\ 7 & 3 & 0 \\ 8 & 2 & 0 \end{vmatrix} = 0$$

So the required determinant = 0.

6.47 (b):

$$\text{Trace} = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

Given eigenvalue $\lambda = 1$

Let λ be the eigenvector. Then $(A - \lambda I)X = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 2 \\ 0 & 1-\lambda & 1 \\ 1 & 2 & 1-\lambda \end{vmatrix} X = 0$$

put $\lambda = 1$

$$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} -x_2 - 2x_3 \\ x_3 \\ x_1 - 2x_2 \end{vmatrix} = 0$$

put $x_3 = k$

$\Rightarrow$ put $x_2 = -2k$ and $x_1 = 4k$

So the eigen vector $= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

The ratio are $x_1/x_2 = \frac{1}{1/2} = 2$

and $x_2/x_3 = \frac{-1/2}{-1/2} = 1$

Only option (b) $(1, 2, 1)$ has the same ratio and the value is an eigen vector

or

Alternate Method:

$AX = \lambda X$

Check each option with eigen vector to find the respective eigen value. Hence option

Option (a): $\begin{vmatrix} 1 & 1 & 2 \\ 0 & & 3 \\ 1 & 2 & 1 \end{vmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 1 \end{bmatrix}$ (false)

Option (b): $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ (true)

and so on

only choice (b) satisfies and so it is the correct answer

Q.18 **Sol:**

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} \begin{vmatrix} 1 & C_{12} \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 8 \\ -4 & 0 \end{vmatrix}$$

This is an upper triangular matrix. Hence diagonal elements are 1. Hence we will use only the cofactors of the elements of the second column, as follows

$C_{12} = 2 \quad C_{22} = 4$

$C_{12} \times C_{22} = 2$

$\Rightarrow \quad C_{12} = 1$

$C_{21} \times C_{12} + C_{22} = 8$

$\Rightarrow \quad 4 + C_{22} = 8$

$\Rightarrow \quad C_{22} = 4$

Q.19 **Sol:**

$$px + qy + rz = 0$$
$$qx + ry + pz = 0$$
$$rx + py + qz = 0$$

Let $A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix}$. The system is $AX = 0$

This is a homogeneous system. Such a system has a trivial solution $X = 0$.

So $\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

$p(r^2 - p^2) - q(q^2 - pr) + r(pq - r^2) = 0$

$p^3 + q^3 + r^3 - 3pqr = 0$

$p = q = r$ satisfies the above equation

Also if $p + q + r = 0$ then it can be proved that the determinant vanishes. Hence we can simply take the above form.

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} \xrightarrow{R_1 + R_2 + R_3} \begin{vmatrix} p+q+r & p+q+r & p+q+r \\ q & r & p \\ r & p & q \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

Therefore the correct option is (d) which is $p = q = r = 0$ or $p = q = r$.

Q.20 **Sol:**

Two eigen values are 8 and 2 of a 3×3 matrix. The third eigen value $= \lambda$ (say) $= 1$

Now $|A| = \lambda_1 \lambda_2 \lambda_3$

$\Rightarrow \quad |A| = (8 \times 2 \times 1) = 16$

$= 3 \times 5 = 15$

Q.21 **Sol:**

I. A linear system may still be inconsistent or inconsistent

II. A linear system may still be equal to one of the other, may also be inconsistent.

III. A homogeneous system may be equal to zero. A homogeneous may have solution or consistency

So only III is correct.

Q.22 **Sol:**

Eigenvalues $= 1, 2, 4 \Rightarrow |A| = 1 \times 2 \times 4 = 8$

So $\left|A^{-1}\right| = \frac{1}{|A|} = \frac{1}{8} = \frac{2}{16} = 0.125$

7D Calculus

7.1 If at every point of a curve $y = f(x)$ the slope is given by $\frac{-2x}{y}$, then the curve is

(a) A straight line (b) A parabola
(c) A circle (d) An ellipse

[1995 : 1 Mark]

7.2 The formula used to compute an approximation to the second derivative of a function f at a point x_0 is

(a) $\frac{f(x_0+h)-f(x_0-h)}{2}$

(b) $\frac{f(x_0+h)-f(x_0-h)}{2h}$

(c) $\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}$

(d) $\frac{f(x_0+2h)-2f(x_0+h)+f(x_0-h)}{h^2}$

[1999 : 1 Mark]

7.3 What is the maximum value of the function $f(x) = 2x^3 - 3x^2 - 12x + 5$ in the interval $[0, 3]$?

(a) 0 (b) 10
(c) 19 (d) 5

[1997 : 2 Marks]

7.4 Consider the function $y = |x|$ in the interval $[-1, 1]$. In this interval, the function is

(a) continuous and differentiable
(b) continuous but not differentiable
(c) differentiable but not continuous
(d) neither continuous nor differentiable

[1998 : 1 Mark]

7.5 What is the value of $\int_0^{2\pi} |\sin x|\,dx$?

(a) 1 (b) 0
(c) 1 (d) π

[IT-2006 : 2 Marks]

7.6 The following contour is in a region of analyticity of $f(z)$. The value of $\oint_C f(z)\,dz$ is

(a) π (b) $\sqrt{\pi}$
(c) $\sqrt{\pi/2}$ (d) 0

[IT-2006 : 2 Marks]

7.7 If $f(x)$ is the function defined as follows, what is the minimum value of $f(x)$ for $x \in [0, 2]$?

$$f(x) = \begin{cases} 26 & \text{when } x \le 3 \\ 8x & \text{when } x > 3 \end{cases}$$

(a) 2 (b) 8
(c) 26 (d) 24

[IT-2006 : 2 Marks]

7.8 Consider the following two statements on the function $f(x) = |x|$:

P: $f(x)$ is continuous for all real values of x.
Q: $f(x)$ is differentiable for all real values of x.

Which of the following is CORRECT?

(a) P is true and Q is false
(b) P is false and Q is true
(c) Both P and Q are true
(d) Both P and Q are false

[2005 : 1 Mark]

7.9 $\lim_{x \to \infty} \frac{x - \sin x}{x + \cos x}$ equals

(a) 1 (b) 1
(c) ∞ (d) $-\infty$

[2006 : 1 Mark]

7.10 A point on a curve is said to be an extremum if it is a local minimum or a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is

(a) 0 (b) 1
(c) 3 (d) 2

[2004 : 2 Marks]

7.11 $\int_1^e (1 + \ln x)(1 + x \ln x)\,dx$ evaluates to

(a) 0 (b) 1
(c) $\ln e$ (d) $e \ln e$

[2009 : 2 Marks]

7.12 What is the value of $\lim_{x \to 0} \left(1 + \frac{x}{n}\right)^n$?

(a) 0 (b) e^x
(c) e^{-x} (d) 1

[2010 : 1 Mark]

7.13 Given $y = x^2$, what will be the value of the definite integral

$$\int_{x=0}^{x=1} \frac{dy}{dx}\, dx$$

(a) 0 (b) 2
(c) 1 (d) 3

[2012 : 2 Marks]

7.14 Consider the function $f(x) = \sin(x)$ in the interval $x \in [\pi/4, 7\pi/4]$. The number and nature of the local maxima of this function are

(a) One at $x = \pi/2$
(b) One at $x = 3\pi/2$
(c) One at $x = \pi/2$ and one at $x = 3\pi/2$
(d) One at $x = \pi/4$ and one at $x = 3\pi/2$

[2013 : 1 Mark]

7.15 Which one of the following functions is continuous at $x = 3$?

(a) $f(x) = \begin{cases} 2, & \text{if } x = 3 \\ x - 1, & \text{if } x > 3 \\ \frac{x-3}{x+3}, & \text{if } x < 3 \end{cases}$

(b) $f(x) = \begin{cases} 4, & \text{if } x = 3 \\ 6 - x, & \text{if } x \le 3 \end{cases}$

(c) $f(x) = \begin{cases} x - 3, & \text{if } x \ge 3 \\ x + 3, & \text{if } x < 3 \end{cases}$

(d) $f(x) = \frac{1}{x^2 - 9}$, if $x \ne 3$

[2013 : 1 Mark]

7.16 Let the function

$$f(x) = \begin{vmatrix} \sin\theta & \cos\theta & \sin\theta \\ \sin(x+\theta) & \cos(x+\theta) & \sin(x+\theta) \\ \sin(2x+\theta) & \cos(2x+\theta) & \sin(2x+\theta) \end{vmatrix}$$

where $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. $f(x)$ denotes the derivative of $f(x)$ with respect to x. Which one of the following statements is true?

(a) There is a θ in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ such that $f(x) = 0$
(b) There is a θ in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ such that $f(x) = 1$
(c) There is a θ in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ such that $f(x) = -1$
(d) Both (a) and (b) above

[2014 (Set-1) : 1 Mark]

7.17 The matrix $[x]$ satisfies the following equation, $[x] + [x]^2 + [x]^3 = 0$. The value of $[x]$ is

[2014 (Set-1) : 2 Marks]

7.18 A function $f(x)$ is continuous in the interval $[0, 1]$. It is known that $f(0) = f(1) = 0$ and $f(1) = 1$. Which one of the following statements must be true?

(a) There exists a x in $[0, 1]$ such that $f(x) = f(1 - x)$
(b) There exists a x in $[0, 1]$ such that $f(x) = -f(1 - x)$
(c) The maximum value of the function in the interval $[0, 1]$ is 1
(d) There exists a x in $[0, 1]$ such that $f(x) = -f(-x)$

[2014 (Set-1) : 2 Marks]

7.19 A monic polynomial in $f(x)$ of degree 3 has a root $x = -1$, $x = 2$ and $x = 3$. Which one of the following statements is true?

(a) $f(x) + f(-x) = 0$ (b) $f(x) + f(4 - x) = 0$
(c) $f(x) - f(-x) = 0$ (d) $f(x) - f(4 - x) = 0$

[2014 (Set-2) : 1 Mark]

7.20 If $\int_0^{\pi} x \sin x\, dx = \pi$, then $\int_0^{\pi} x^2 \sin x\, dx$ is equal to _____.

[2014 (Set-3) : 1 Mark]

7.21 The value of the integral given below is

$$\int_{-1}^{1} \sin^3 x\, dx$$

(a) 2 (b) 1
(c) –1 (d) 0

[2014 (Set-3) : 2 Marks]

7.22 $\lim_{x \to 0} x^x$ is

(a) e^{-1} (b) 0
(c) 1 (d) Not defined

[2015 (Set-1) : 1 Mark]

8. $\lim_{x \to 0} \left(\frac{\sin x}{x} + \frac{\cos x}{x}\right)$ is equal to 0.

(a) True (b) False

[2014 (Set-1) : 1 Mark]

9. $\lim_{x \to 0} \left(\frac{\sin x}{x} + \frac{\cos x}{x}\right)$ is equal to 0.

(a) True (b) False

[2014 (Set-1) : 1 Mark]

(c) Both 1 and 2 (d) Neither 1 nor 2

[2014 (Set-1) : 1 Mark]

7.23 $\int_{x}^{2x} \frac{\sin(t+x)}{x^2}\,dx =$ _____

[2016 (Set-1) : 2 Marks]

7.24 Let $y(x) = x^{-1}$ and A denote the area of the region bounded by $y(x)$ and the X-axis, when x varies from $x = 1$ to $x = 10$. Which of the following statements is/are true?

1. A is a finite number
2. A is infinite
3. A is a monotonic function

(a) 2 only (b) 3 only
(c) 2 and 3 only (d) 1, 2 and 3

[2015 (Set-3) : 2 Marks]

7.25 The value of $\lim_{x \to 0} (1 + x^2)^{\frac{1}{x}}$ is

(a) 0 (b) 1
(c) e (d) e^2

[2016 (Set-3) : 1 Mark]

7.26 The convergence of $\int_{x=0}^{\infty} \frac{1}{x^{\alpha}}\,dx = \frac{1}{\alpha}$ $\forall \alpha$ where $\alpha > 0$ then $\int_{1}^{\infty} \frac{x \ln x}{(1+x^2)^2}\,dx$ is

(a) $\frac{1}{2}\left[\ln 2 - \frac{1}{2}\right]$
(b) $\frac{1}{2}\left[\ln 2 - \frac{1}{2}\right]$
(c) $\frac{1}{2}\left[\ln 2 - \frac{1}{2}\right]$
(d) $\frac{1}{2}\left[\ln 2 - \frac{1}{2}\right]$

[2015 (Set-4) : 2 Marks]

7.27 $\lim_{x \to 4} \frac{\sin(x-4)}{x-4} =$ _____

[2016 (Set-1) : 1 Mark]

7.28 Let $g(x)$ be a polynomial and $f(x) = \int_{0}^{x} g(t)\,dt$. If the degree of $f(x)$ is 6, then the degree of $f(x^2)$ is

[2016 (Set-2) : 1 Mark]

■■■

Answers Calculus

7.1	(d)	7.2	(d)	7.3	(b)	7.4	(b)	7.5	(b)	7.6	(c)	7.7	(b)	7.8	(a)	7.9	(a)
7.10	(b)	7.11	(d)	7.12	(b)	7.13	(d)	7.14	(b)	7.15	(a)	7.16	(a)	7.17	(a)	7.18	(a)
7.21	(a)	7.22	(d)	7.23	(d)	7.25	(c)	7.26	(a)								

Explanations Calculus

7.1 (d)

Given that slope of the curve at any point is equal to the slope of tangent i.e.

$$\frac{dy}{dx} = -\frac{2x}{x}$$

i.e.,

$$\frac{dy}{dx} = -\frac{2x}{x}$$

$$dy = \frac{-2x\,dx}{x}$$

Integrating both sides

$$\int dy = -2 \times \int \frac{x}{x}\,dx$$

$$\frac{y^2}{2} = -x^2 + c$$

This equation represents $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

So the curve is an ellipse with $a = 1$ and $b = \sqrt{2}$

7.2 (d)

The Newton method to compute an approximation of the derivative of a function $f(x)$ at a point x_0 is

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h}$$

So an approximation for the second derivative of a function $f(x)$ at a point x_0 is

$$f'(x_1) = \frac{f(x_1+h) - f(x_1-h)}{h}$$

$$= \frac{\left[\frac{f(x_1+h) - f(x_1)}{h} - \frac{f(x_1) - f(x_1-h)}{h}\right]}{h}$$

$$= \frac{f(x_1+h) - 2f(x_1) + f(x_1-h)}{h^2}$$

7.3 (b)

We need to evaluate maximum of

$$f(x) = x^2 - 3x + 6$$

in the interval [0, 2]

First find local maximum if any by putting $f'(x) = 0$

$$f'(x) = 3x^2 - 2x - 1$$

$$4x = 2$$

$$x = \frac{1}{2}$$

$$f''(x) = 6x - 2$$

So: $x = \frac{1}{2}$

is a point of local minimum. So there is no point of local maximum.

Now calculate the values of f at end points of the interval and at local minimum if any in this case to find out local maximum.

x	$f(x)$
0	6
2	6

Clearly the absolute maximum is at $x = 0$ and 2 and maximum value is 10

7.4 (b)

The function $y = |x|$ in the interval [−1, 1] is

x is continuous and differentiable everywhere except at $x = 0$, where it is continuous but not differentiable

Since $y = |x|$ contains 0, which is in the interval it is continuous but not differentiable

7.5 (b)

For the integral $\int_0^{\pi} (x - \pi)^2 \cos x\, dx$

$f(x) = (x - \pi)^2 \cos x$

$f(2\pi - x) = (2\pi - x - \pi)^2 \cos(2\pi - x)$

$= (\pi - x)^2 \cos x = (x - \pi)^2 \cos x = f(x)$

So $\int_0^{\pi} (x - \pi)^2 \cos x\, dx = 0$

Since

$\int_0^{2a} f(x)\, dx = 0$ whenever $f(2a - x) = -f(x)$

7.6 (b)

$$I = \int_0^{\infty} e^{-x^2} dx$$

On comparing with area under the standard normal curve we get $\mu = 0$, $\sigma = 1$.

So we get

$$\frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-\frac{x^2}{2}} dx = 0.5$$

So, the integral will be equal

$$\int_0^{\infty} e^{-x^2} dx = 0.5\sqrt{2\pi}$$

$$= \sqrt{\frac{\pi}{2}}$$

which is option (b).

7.7 (b)

The function attains its minimum value will be when it is maximum since it is subtracted by function.

The maximum value of $f(x)$ is 8.2.

At $x = 2.5$ the function has the value

$$f(x) = 25(2.5)^2 - 2(2.5)$$

$$= 2.0833 - 2 = 0.0833$$

So, $x = 2.5$ is 2.0833 for $x \le 2.5$

$x + 2x \le 2$

But it must be the function $x \ge 2.5$, putting 2.5 in the function we get the minimum value of the function which is

$$0.0833 \times 25 = 1.25 = 2.166$$

Now comparing the minimum value 2.0833 of the first function with minimum value 2.166 of the second function we get the overall minimum of this function is $-2.0833 = -25/12$, which is option (b).

7.8 (a)

$|x|$ is continuous and differentiable everywhere except at $x = 0$. At $x = 0$ it is continuous but not differentiable.

Since $f(x)$ is real in $(-\infty, \infty)$, it will also be continuous but not differentiable.

7.9 (a)

$$\lim_{x \to \infty} \frac{x - \sin x}{x + \cos x} = \lim_{x \to \infty} \frac{1 - \sin x/x}{1 + \cos x/x}$$

$$= \frac{\lim_{x \to \infty} (1 - \sin x/x)}{\lim_{x \to \infty} (1 + \cos x/x)}$$

$$= \frac{1 - \lim_{x \to \infty} \frac{\sin x}{x}}{1 + \lim_{x \to \infty} \frac{\cos x}{x}}$$

$$= \frac{1 - 0}{1 + 0} = 1$$

7.10 (b)

$$y = x^4 - 4x^3 + 2x^2 + 97$$

$$\frac{dy}{dx} = 12x^2 - 12x^2 + 4x = 0$$

$$x(12x^2 - 12x + 4) = 0$$

$$x = 0$$

or $12x^2 - 12x + 4 = 0$

$$x^2 - x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$\frac{d^2y}{dx^2} = 36x^2 - 24x + 4$$

Now at $x = 0$

$$\frac{d^2y}{dx^2} = 4 > 0$$

$\therefore f(x)$ has a minimum at $x = 0$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 0 \text{ and } \left.\frac{d^3y}{dx^3}\right|_{x=1} = 24 \neq 0$$

Since $x = 1$ satisfies the second derivative test, $f(x)$ has no extremum at $x = 1$. So $f(x)$ has only one point of extremum (at $x = 0$).

7.11 (b)

Since

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{0} \frac{\ln(1+x)}{1+x^2}\,dx$$

$$= \int_{-\infty}^{0} \frac{1 - \sin\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{4} - x\right)}\,dx$$

Since $\sin(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \int_{-\infty}^{0} \frac{\sin\frac{\pi}{4} - \sin x}{1 + \sin\frac{\pi}{4}\sin x}\,dx$$

$$= \int_{-\infty}^{0} \frac{\frac{1}{\sqrt{2}} - \sin x}{1 + \frac{1}{\sqrt{2}}\sin x}\,dx$$

$$= \int_{-\infty}^{0} \frac{1 - \frac{\sqrt{2}\sin x}{1 + \sin x}}{1 + \frac{\sqrt{2}\sin x}{1 + \sin x}}\,dx$$

$$= \int_{-\infty}^{0} \frac{1 + \sin x - \sqrt{2}\sin x}{1 + \sin x + \sqrt{2}\sin x}\,dx$$

$$= \int_{-\infty}^{0} \frac{\cos x}{2}\,dx$$

$$= \int_{-\infty}^{0} \cos x\,dx$$

$$= \left[\sin x\right]_{-\infty}^{0}$$

$$= \sin\left[\sin\frac{\pi}{4}\right] - \lim_{x \to -\infty} \sin(x)$$

$$= \sin\left[\sin 45^\circ\right] - \sin(-\infty)$$

$$= \sin\left[\frac{1}{\sqrt{2}}\right] - 0 = \frac{1}{\sqrt{2}}\sin 2$$

7.12 (b)

$$\lim_{n \to \infty} \left(1 - \frac{1}{n^2}\right)^{n^2} = \lim_{n \to \infty} \left(1 - \frac{1}{n^2}\right)^{n^2}$$

$$= \left[\lim_{n \to \infty} \left(1 - \frac{1}{n^2}\right)^{n^2}\right]^{\lim_{n \to \infty} \frac{n^2}{n^2}}$$

Now we use the standard limit

$$\lim_{n \to \infty} \left(1 + \frac{1}{n^2}\right)^{n^2} = e^1$$

So equation becomes $= (e^{-1})^1 = e^{-1}$

7.13 (d):

According to Dirichlet's theorem

$$\text{So, } \int_0^{\pi} \frac{\cos x + \sin x}{\cos x - \sin x}\,dx = \int_0^{\pi} \frac{e^{ix}}{e^{ix}}\,dx$$

$$= \int_0^{\pi} e^{ix}\,dx$$

$$= \frac{e^{i\pi} - e^{0}}{2i} = \frac{1}{2i}\left(e^{i\pi} - 1\right)$$

$$= \frac{1}{2i}\left(-1 - 1\right) = 1$$

$$= \frac{2}{2} = 1$$

7.14 (b):

From the plot it is clear that, we can easily accommodate the range $[0, 2\pi]$. Hence only one real root is possible.

7.15 (a):

$$f(x) = \begin{cases} x^2 & \text{if } x \le 3 \\ x + 1 & \text{if } x > 3 \end{cases}$$

$$g(x) = \begin{cases} \dfrac{x+3}{x-3} & \text{if } x \le 3 \\ x - 3 & \text{if } x > 3 \end{cases}$$

$$\lim_{x \to 3^-} g(x) = \lim_{x \to 3^-} \frac{x+3}{x-3} = \frac{3+3}{3-3} = \frac{6}{0} = \infty$$

$$\lim_{x \to 3^+} g(x) = \lim_{x \to 3^+} x - 3 = 3 - 3 = 0$$

So, $\lim_{x \to 3} g(x)$ does not exist.

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} f(x) = f(3) = 9$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} f(x) = 3 + 1 = 4$$

So, $\lim_{x \to 3} f(x)$ does not exist.

Option (a) is correct.

7.16 (c):

$$f(x) = \begin{cases} \sin(\pi/x) & \text{if } x \le 0 \\ \cos(\pi/2) & \text{if } 0 < x < 1 \\ \tan(\pi/2) & \text{if } x \ge 1 \end{cases}$$

$$f(0+) = 0$$

So, as $x \to 0$ we get $f = \infty$ is always discontinuity at $x = 0$. It will reduce to zero, since I and II now will become zero.

$$f(0-) = 0$$

So, as $x \to 0$ we get $f = \infty$ is always discontinuity at $x = 0$. It will reduce to zero, since I and III now will become zero.

So $f(x)$ is continuous at $x = 0$. Now in the interval $[0, 1]$, the function $f(x)$ is continuous and differentiable. Now check the given interval $[0, 2]$ contain any multiple of $\pi/2$ where $\cos(\pi/2)$ is neither continuous nor differentiable.

Since all the three conditions of Rolle's theorem are satisfied, the conclusion of Rolle's theorem holds.

I: $\int_0^2 f(x)\,dx = \int_0^1 \cos(\pi/2)\,dx + \int_1^2 \tan(\pi/2)\,dx = 0$

Statement is true.

II: $\int_0^2 f(x)\,dx = \int_0^1 \cos(\pi/2)\,dx + \int_1^2 \tan(\pi/2)\,dx$

$\int_0^2 f(x)\,dx = 0$ is true, since the only way it can be false is $\int_0^2 f(x)\,dx = 0$ for all values of x, which is possible only if $f(x)$ is a constant which is untrue.

Therefore only II, and III are true.

7.17 (b):

$$f(x) = x\cos x$$

$$f'(x) = x\cos x - \sin x$$

$$f''(x) = -x\sin x - \cos x - \cos x$$

$$f''(x) + y(x) = -x\sin x - 2\cos x + x\cos x = 0$$

$$\Rightarrow -x\sin x - 2\cos x + x\cos x = 0$$

$$\Rightarrow -x\sin x - 2\cos x = 0$$

$$\Rightarrow x\sin x + 2\cos x = 0$$

$$\Rightarrow x\sin x = -2\cos x$$

7.18 (b):

Consider $y(x)$ as

(i) There exists $c \in (0, 1)$ such that $f(c) = f(0) = 0$

If $x = 0$ is a root of $f(x)$

The given function satisfies the conditions of Rolle's theorem.

$$g(x) = f(x) - f(x) + 1 = 0$$

Now, from the mean value theorem, we know that the above statement is true, at some point c.

Now we check for $g(x)$.

$$g(0) = f(0) - f(0) = 1$$

$$g(1) = f(1) - f(1) = 1 - 1 = 0$$

$$g(0) \times g(1) = 0 \times 0 = 0 \le 0$$

So, by intermediate value theorem, there exists a point $c \in (0, 1)$ such that $g(c) = 0$.

7.19 (a)

$x = 1, x = 2$ and $x = 3$ are roots.

$f(x) = (x-1)(x-2)(x-3)$

$\Rightarrow f(0) = (-1)(-2)(-3) = -6$

$\Rightarrow f(4) = (3)(2)(1) = 6$ and $f(5) = 24$

$f(0) \times f(4) = -36 < 0$

$\therefore f(x) = 0$ has a root in $(0, 4)$

Alternate method

Since the function is continuous and differentiable, there exists a root in the interval $(0, 4)$. By intermediate value theorem, there must be a root c such that $f(c) = 0$.

7.20 Sol:

$$\Rightarrow \int_{-\pi}^{\pi} x \sin x \, dx = \pi$$

$$\int_{-\pi}^{\pi} x \sin x \, dx = \int_{-\pi}^{\pi} |x \sin x| \, dx = \pi$$

$$\Rightarrow \int_{-\pi}^{\pi} |x \sin x| \, dx = \int_{-\pi}^{\pi} -x \sin x \, dx = 2\pi$$

$$\Rightarrow -\cos x \sin x \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} (-\cos x \cos x) \, dx = 2\pi$$

$$\Rightarrow \int_{-\pi}^{\pi} \cos^2 x \, dx = 2\pi$$

$$\Rightarrow k = 1$$

7.21 (a)

The given integral is positive and we get

$$\int_{-\pi}^{\pi} x^2 \sin x \, dx = \int_{-\pi}^{0} x^2 \sin x \, dx + \int_{0}^{\pi} x^2 \sin x \, dx$$

$$= 0 - \int_{0}^{\pi} x^2 \sin x \, dx + \int_{0}^{\pi} x^2 \sin x \, dx = 2\int_{0}^{\pi} x^2 \sin x \, dx$$

Since, the integrand is even, we get

$$\int_{0}^{\pi} x^2 \sin x \, dx = (-x^2 \cos x - 2x \sin x - 2\cos x)\Big|_{0}^{\pi} = \pi^2 - 4$$

7.22 (c)

$$x = \lim_{n \to \infty} x^{1/n}$$

$$\ln(x) = \lim_{n \to \infty} \ln(x^{1/n})$$

$$\ln(x) = \lim_{n \to \infty} \frac{\ln x}{n}$$

Now, using L' Hospital's rule

$$\ln(x) = \lim_{n \to \infty} \frac{1/x}{1}$$

$$\ln(x) = 0 \Rightarrow x = e^0 = 1$$

7.23 Sol:

$$\int_{0}^{\pi/2} \sin(\ln \tan x) \, dx = \int_{0}^{\pi/2} \frac{-1}{\tan x} \sec^2 x \, dx$$

Let $\frac{1}{\tan x} = t \Rightarrow -\sec^2 x \, dx = dt$

If $x = 0$, $t = \infty$ and $x = \frac{\pi}{2}$, $t = 0$

$$\int_{0}^{\pi/2} \sin(\ln \tan x) \, dx = \int_{\infty}^{0} \sin t \, dt$$

$$= |\cos t|_{\infty}^{0} = \cos 0 - \cos \infty = 1$$

7.24 (b)

$$f(x) = \frac{1}{\sin x}$$

Statement 1: $f(x)$ is continuous in $[-1, 1]$. Let us check the statement.

We need to check $\lim_{x \to 0} f(x) = 0$

$$\text{Left limit} = \lim_{x \to 0^-} \frac{1}{\sin x} = \lim_{x \to 0^-} \frac{1}{x}$$

$$= \lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\text{Right limit} = \lim_{x \to 0^+} \frac{1}{\sin x} = \lim_{x \to 0^+} \frac{1}{x}$$

$$= \lim_{x \to 0^+} \frac{1}{x} = +\infty$$

[illegible]

Statement 2: [illegible] is [illegible] Since [illegible] the function is not [illegible]

[illegible]

Statement 3: A is [illegible]

$$A = \left|\int [illegible]\right| \quad \left|\int [illegible]\right|$$

$$= \left|\begin{matrix} [illegible] \\ [illegible] \end{matrix}\right| \quad \begin{matrix} [illegible] \\ [illegible] \end{matrix}$$

$$= \left|\begin{matrix} [illegible] \\ [illegible] \end{matrix}\right| \quad \begin{matrix} [illegible] \\ [illegible] \end{matrix}$$

So A is [illegible]

[illegible]

7.85 (c)

Let $y = \ln_{[illegible]}(1+x^2)$

$$\ln(y) = \ln(\ln(1+x^2))$$

$$= \lim_{[illegible]} \frac{\ln(1+x^2)}{[illegible]}$$

[illegible]

$$= \lim_{[illegible]} \frac{\frac{1}{[illegible]}[illegible]}{[illegible]}$$

$$= \lim_{[illegible]} \frac{[illegible]}{[illegible]}$$

Again [illegible] applying [illegible] rule

$$\ln(y) = \lim_{[illegible]} \frac{[illegible]}{[illegible]}$$

$$\ln(y) = [illegible]$$

7.86 (a)

$$[illegible] \quad \text{... (1)}$$

$$[illegible] = \frac{1}{[illegible]} \ln [illegible]$$

$$[illegible] \quad \text{... (2)}$$

So [illegible] from (1) and (2), since [illegible] we get [illegible] as follows

Equation (2) [illegible]

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

7.87 Sol.

$$\lim_{[illegible]} \frac{[illegible]}{[illegible]}$$

[illegible]

So the [illegible] is $[illegible] = \frac{[illegible]}{5}$

7.88 Sol.

[illegible]

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

Now $[illegible]$

$$[illegible]$$

$$[illegible]$$

Clearly [illegible] is 3.

Unit - II

Theory of Computation

Contents

Sl.	Topic	Page No
1.	Finite Automata : Regular Languages	107
2.	Push Down Automata CFL & DCFL	125
3.	Turing Machines RE, REC and Undecidability	19

UNIT II Theory of Computation

Syllabus: [illegible]

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1988	[illegible]	2	[illegible]	[illegible]	6
1989	[illegible]	2	[illegible]	[illegible]	4
1990	[illegible]	[illegible]	[illegible]	[illegible]	10
1991	1	4	[illegible]	1	[illegible]
1992	1	2	[illegible]	[illegible]	5
1993	2	2	[illegible]	[illegible]	7
1994	[illegible]	3	[illegible]	[illegible]	7
1995	5	3	[illegible]	[illegible]	11
1996	[illegible]	[illegible]	[illegible]	[illegible]	5
2000	3	1	[illegible]	[illegible]	6
2001	3	2	[illegible]	[illegible]	7
2002	1	6	[illegible]	1	18
2003	[illegible]	6	[illegible]	[illegible]	15
2004	1	[illegible]	[illegible]	[illegible]	8
2006	[illegible]	6	[illegible]	[illegible]	14
2008	2	5	[illegible]	[illegible]	12

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	2	[illegible]	22
2010	[illegible]	5	17
2011	4	3	11
2012	[illegible]	[illegible]	7
2013	[illegible]	3	8
2014	4	[illegible]	[illegible]
2015	2	3	8
2014 Set-1	2	4	8
2014 Set-2	[illegible]	2	6
2014 Set-3	[illegible]	3	8
2015 Set-1	[illegible]	2	5
2015 Set-2	[illegible]	[illegible]	7
2015 Set-3	[illegible]	[illegible]	3
2016 Set-1	3	3	9
2016 Set-2	3	[illegible]	11

1 Finite Automata : Regular Languages

1.1 How many substrings of length 3 can be extracted or can be formed from a character string of length [illegible] [illegible]

[1999 : 2 Marks]

1.2 Let R_1 and R_2 be regular sets defined over [illegible]

(a) $R_1 \cup R_2$ is non-regular

(b) $R_1 \cap R_2$ is regular

(c) $\Sigma^* - R_1$ is regular

(d) R_1^* is not regular

[1990 : 2 Marks]

1.3 Let $r = 1+1 \cdot 0^*$, $s = 1^*0$ and $t = 1^*0^*$ be three regular expressions. Which [illegible] are true?

(a) $L(r) = L(s)$ and $L(s) = L(t)$

(b) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$

(c) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(r)$

(d) $L(r) = L(s)$ and $L(s) = L(t)$

[1991 : 2 Marks]

1.4 Which of the following regular expressions [illegible]

(a) $(a^* + b^*)^* = (a + b)^*$

(b) $(a^* + b^*)^* = (a^* + b^*)^*$

(c) $(a + b)^* = a^* + b^*$

(d) $(a + b)^* = a^* + b^*$

[1992 : 2 Marks]

1.5 State Transition Diagram without labels or annotations. A DFSM (Finite State Machine) can be designed to accept any sequence of any [illegible] defined over the Decimal Digits.

[1994 : 1 Mark]

1.6 The number of substrings of length 3 that can be formed from a character string of length $n \geq 3$

(a) n (b) $n-2$

(c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n-2)}{2}$

[1994 : 2 Marks]

1.7 The regular expression for the language recognized by the finite state automata

[1994 : 2 Marks]

1.8 Which of the following pairs of DFA's are equivalent? [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[1995 : 2 Marks]

1.9 A finite state machine with the following state table has a single input X and a single output Z.

present state	next state, z=0	next state, z=1
A	B,0	A,1
B	C,1	D,1
C	A,0	B,1
D	D,1	C,0

If the initial state is unknown, then the shortest input sequence to reach the final state, D is

(a) 01 (b) 10

(c) 101 (d) 110

[1995 : 2 Marks]

1.10 Which two of the following four regular expressions are equivalent?

(i) $(00)^* (\varepsilon + 0)$ (ii) $0(00)^*$

(iii) 0^* (iv) $0(00)^*$

(a) (i) and (ii) (b) (ii) and (iii)

(c) (i) and (iv) (d) (iii) and (iv)

[1996 : 1 Mark]

1.11 Let L_1 be a regular language over $\Sigma = \{a, b\}$. Which of the following is true?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[1996 : 1 Mark]

1.12 Given $\Sigma = \{0, 1\}$, which one of the following is a regular language over Σ?
(a) Set of all strings over Σ
(b) Set of all languages over Σ
(c) Set of all regular languages over Σ
(d) Set of all languages over Σ accepted by a Turing Machine

[1997 : 1 Mark]

1.13 Which one of the following regular expressions represents the set of all strings not containing 00 as a substring?
(a) $0^*(1 + 01)^*$
(b) $0^*1(1 + 01)^*$
(c) $(0 + 1)^*$
(d) $0^*(1 + 01)^*$

[1997 : 2 Marks]

1.14 If the regular set A is represented by $A = (0 + 1)^*$ and the regular set B is represented by $B = (00)^*(1 + 0)$, which of the following is true?
(a) $A \subset B$
(b) $B \subset A$
(c) A and B are incomparable
(d) $A = B$

[1998 : 1 Mark]

1.15 Which of the following statements is/are true? Type-0 grammars are Finite state automata.
(a) The minimum number of states required in a finite state automata to accept all binary strings of length 25 or more is 26.
(b) [illegible] binary strings in which the number of 0's is the same as the number of 1's.
(c) The set of binary strings in which the number of 0's is the same as the number of 1's.
(d) The set $\{1, 101, 11011, 1111, \ldots\}$

[1998 : 1 Mark]

1.16 The string 1101 does not belong to the regular language represented by
(a) $(0^*1^*)^*$
(b) $(10 + 11)^*$
(c) $(0 + 1)^*(00 + 11)^*$
(d) $(00 + 1)^*(0 + 1)^*$

[1998 : 1 Mark]

1.17 How many substrings of length 2 can be formed from a character string of length n?
(a) n
(b) n − 1
(c) 2n
(d) $\frac{n(n-1)}{2}$

[1998 : 1 Mark]

1.18 Let L be the set of all binary strings such that every string begins and ends with the same symbol. The number of states in the minimum state deterministic finite automaton accepting L is
(a) 2
(b) 4
(c) 6
(d) 8

[1998 : 2 Marks]

1.19 Consider the regular expression $(0 + 1)(0 + 1)\ldots$ n times. The minimum state finite automaton that recognizes the language represented by this regular expression contains
(a) n states
(b) n + 1 states
(c) n + 2 states
(d) None of these

[1999 : 1 Mark]

1.20 Let $\Sigma = \{a, b\}$. The language $L = \{a^n b^n \mid n \geq 0\}$ is a regular language. Consider the two regular languages $L_1 = a^*b^*$ and $L_2 = (a + b)^*$. Which of the following is true?
(a) $L = L_1$
(b) $L = L_2$
(c) $L = L_1 \cap L_2$
(d) $L = L_1 \cup L_2$

[2000 : 1 Mark]

1.21 Let L be the set of all binary strings whose last two symbols are the same. The number of states in the minimum state deterministic finite automaton accepting L is
(a) 2
(b) 3
(c) 4
(d) 5

[2000 : 1 Mark]

1.22 Which of the following statements is/are true?
(a) Every subset of a regular set is regular
(b) Every finite subset of a non-regular set is regular
(c) Every finite language is regular
(d) Intersection of a regular language and a context free language is always context free

[2000 : 2 Marks]

1.23 Consider the following statements:
S1: $\{a^n \mid n \geq 0\}$ is a regular language
S2: $\{a^n \mid n \geq 0\}$ is a context free language

Which of the following statements S_1 and S_2 is/are correct?

(a) Both S_1 and S_2 are correct
(b) Only S_1 is correct
(c) Both S_1 and S_2 are incorrect
(d) None of S_1 and S_2 is correct

[2001 : 1 Mark]

1.24 Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at most

(a) N^2 (b) 2^N
(c) $2N$ (d) $N!$

[2001 : 1 Mark]

1.25 Consider a DFA over $\Sigma = \{0, 1\}$ accepting all strings which, when treated as a binary number, are multiples of 3. What is the minimum number of states in the DFA?

(a) 4 (b) 5
(c) 3 (d) 6

[2002 : 2 Marks]

1.26 Consider the following languages

$L_1 = \{ww \mid w \in \{a, b\}^*\}$

$L_2 = \{ww^R \mid w \in \{0, 1\}^*, w^R$ is the reversal of $w\}$

$L_3 = \{a^p \mid p$ is a prime number$\}$

$L_4 = \{a^n \mid n$ is an integer$\}$

Which of the languages are context-free?

(a) Only L_1 and L_2 (b) Only L_2, L_3 and L_4
(c) Only L_1, L_2 and L_3 (d) Only L_4

[2001 : 2 Marks]

1.27 The finite state machine described by the following state diagram with a single input and two outputs, where each transition is labelled by the input, and the corresponding output

(a) Outputs the sum of the present and the previous bits of the input
(b) Outputs 1 whenever the input sequence contains 11
(c) Outputs 0 whenever the input sequence contains 0
(d) None of the above

[2002 : 2 Marks]

1.28 An arithmetic expression consists of the letters a, b, c and operators +, −, *, /, ↑ (exponentiation). Each letter and each operator is a single character. Assuming ↑ has the highest precedence and is right associative, and + has the lowest precedence, which of the following is a valid postfix expression?

(a) a b c ↑ * − (b) a b c ↑ − *
(c) a b − c ↑ * (d) a b − c * ↑

[2002 : 2 Marks]

1.29 Consider the following deterministic finite automaton M.

Let S denote the set of states in M, and Σ denote the set of symbols. Let δ be the transition function of M. The number of strings in Σ^* that are accepted by M is

(a) 1 (b) 2
(c) 3 (d) 4

[2003 : 2 Marks]

1.30 Consider the NFA M shown below

Let the language accepted by M be L_1. Let L_2 be the language accepted by the NFA M_1 obtained by changing the acceptance status of q_0 in M. Let L_3 be the language accepted by the NFA M_2 obtained by changing the acceptance status of q_1 in M. Which of the following statements is true?

(a) $L_1 = \{0, 1\}^*$ (b) $L_1 = \{0, 1\}^+$
(c) $L_2 \subseteq L_1$ (d) $L_3 = L_1$

[2003 : 2 Marks]

1.31 Which one of the following regular expressions represents the set of all binary numbers that are divisible by 3?

(a) $(0^* + 1(01^*1)^*1)^*$ (b) $(0 + 1(1 + 01)^*)^*$
(c) $(0 + 1(1 + 01)^*1)^*$ (d) $(0^*1^*)^*$

[IIT-2004 : 1 Mark]

1.32 Let $M = (K, \Sigma, \delta, s, F)$ be the finite state automaton, where

$K = \{s, t, u, v\}$, $\Sigma = \{a, b\}$, $F = \{v\}$

$\delta(s, a) = \{t\}$, $\delta(t, b) = \{u\}$, $\delta(u, a) = \{v\}$, $\delta(v, b) = \{v\}$

A grammar to generate the language over [illegible] is [illegible] where $V = \{S, A, B\}$, $\Sigma = \{a\}$

Which one of the following set of rules makes $L(G) = L(a^*)$?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[IT-2004 : 2 Marks]

1.83 The following three state automaton [illegible] accepts binary strings in which the number of 0's and 1's are respectively

(a) divisible by 3 & 2 (b) [illegible]

(c) [illegible] (d) divisible by 3 & 2

[2004 : 2 Marks]

1.84 Which of the following [illegible] PCFG [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[IT 2005 : 1 Mark]

1.85 The language $\{0^i 1^j 2^k \mid i = j + k\}$ is

(a) [illegible]

(b) Context free but not regular

(c) Recursive but its complement is not recursively enumerable

(d) Not context free.

[IT 2006 : 1 Mark]

1.86 Consider the non-deterministic finite automaton (NFA) shown in the figure.

Let X is the state in the equivalent Deterministic finite automaton (DFA) obtained by the NFA with $\{a\}$ as the input alphabet. Let $L(X)$ be the language accepted by the DFA with X as the only accepting state. Which of the following statements is true? [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[IT 2006 : 2 Marks]

1.87 Consider the grammar with non-terminals $\{S, X, Y, Z, W\}$ and terminals $\{a, b\}$ with the following production rules:

$S \to XY$

$X \to Za$

$Z \to Xa \mid a$

$Y \to Wb$

$W \to Sb$

where S is the start symbol. [illegible] the set of minimal states in the equivalent DFA. We wish to construct a deterministic finite automaton (DFA) to recognize the same language. What is the minimum number of states required for the DFA?

(a) 3 (b) 4

(c) 4 (d) 5

[IT-2005 : 2 Marks]

1.88 A language L satisfies the Pumping Lemma for regular languages. It satisfies the Pumping Lemma for context free languages. Which of the following statements is/are true? [illegible]

(a) [illegible]

(b) [illegible] but not necessarily a regular language.

(c) [illegible]

(d) None of these

[IT-2005 : 2 Marks]

1.89 Consider the machine M

The language recognized by M is

(a) [illegible]

(b) [illegible]

(c) [illegible]

[illegible] according

[2006 : 2 Marks]

1.40 The finite state machine represents a finite state machine which takes as input a binary number. [illegible] Transition

What is the following is TRUE?

(a) It computes 1's complement of the input number

(b) It computes 2's complement of the input number

(c) It accepts odd number

(d) It accepts the input number

[2006 : 2 Marks]

1.41 In the minimum state DFA [illegible] the only final state

Consider the strings [illegible] and [illegible]. Which of the following statements is true?

(a) The automaton accepts [illegible] and rejects [illegible]

(b) The automaton accepts each of [illegible], [illegible] and [illegible]

(c) The automaton rejects each of [illegible], [illegible] and [illegible]

(d) The automaton accepts [illegible] and rejects [illegible] and [illegible]

[IT 2006 : 1 Mark]

1.42 Which regular expression best describes the language accepted by the minimum state finite automaton below?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[IT-2006 : 1 Mark]

1.43 Consider the regular grammar below

[illegible]

The DFA [illegible] accepts the language generated by the grammar above

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[IT-2006 : 2 Marks]

1.44 Which of the following statements are true?

(a) Every language has a regular grammar

(b) Every language has a regular expression

(c) Every regular language has a context free grammar

(d) Every regular language has a Turing machine

[IT-2006 : 2 Marks]

Directions for Q.1.45 to Q.1.46:

In the regular language over {a, b} consider the following strings:

I. [illegible]

II. prefix [illegible]

III. [illegible]

IV. [illegible]

1.45 Which of the above strings belong to the regular language?

(a) Both I and IV (b) Only I

(c) Only IV (d) Both I and III

[IT-2006 : 2 Marks]

1.46 Which of the above strings belong to the language whose minimal DFA has 3 states?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[IT-2006 : 2 Marks]

1.47 Let [illegible] be the set of all binary strings of length [illegible] and [illegible] be the number of 1's in [illegible]. Which one of the following languages is not regular?

(a) [illegible]

(b) [illegible]

(c) [illegible]

[illegible]

[illegible]

[IT-2006 : 2 Marks]

1.48 Consider the regular language [illegible]

[illegible]

[IT-2006 : 2 Marks]

1.49 [illegible]

[IT-2007 : 1 Mark]

1.50 [illegible]

[2007 : 2 Marks]

1.51 [illegible]

[2007 : 2 Marks]

Common Data for Q.1.52 & Q.1.53

Consider the following state transition diagram

1.52 The language accepted by the automaton is given by the regular expression

[illegible]

[2007 : 2 Marks]

1.53 The minimum number of states required to be added to make the DFA [illegible]

[illegible]

[2007 : 2 Marks]

1.54 [illegible]

[2007 : 2 Marks]

1.55 [illegible]

[IT-2007 : 2 Marks]

1.56 Consider the following DFA [illegible]

What language does the DFA accept?

(a) All strings [illegible]

(b) All strings of 0's and 1's which have [illegible]

(c) All strings of 0's and 1's which have [illegible]

(d) All strings of 0's and 1's which have [illegible]

[IT-2007 : 2 Marks]

1.57 Consider the grammar [illegible]

$S \to aB \mid bA$

$A \to a \mid aS \mid bAA$

$B \to b \mid bS \mid aBB$

Consider the following strings.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[IT 2006 : 2 Marks]

1.67 [illegible] The [illegible] indicated by [illegible] indicated by [illegible] respectively.

The [illegible] which [illegible] the [illegible] is:

(a) None of the above

[IT-2005 : 2 Marks]

Common Data for Q.1.68 to Q.1.69:

Consider the regular expression

[illegible]

1.68 Which of the following [illegible] is/are equivalent to the regular expression [illegible]

[IT-2007 : 2 Marks]

1.69 Which of the [illegible] language is represented by the regular expression [illegible]?

[IT-2007 : 2 Marks]

1.60 Which one of the regular expressions given below defines the same set of strings as defined by the regular expression 0^*?

(a) $(0+1)^*$
(b) $(0+1)^*1(0+1)^*$
(c) $(0^*1)^*(0+1)^*$
(d) $(0+1)^*1(0+1)^*1(0+1)^*$

[IT-2007 : 2 Marks]

1.61 Which of the following regular expressions over {0, 1} represents all strings that do not contain the substring 100?

(a) $0^*1^*0^*$
(b) $0^*1(0+1)^*$
(c) $0^*1^*0^*1^*$
(d) $(0+1)^*1(0+1)^*1(0+1)^*$

[IT-2008 : 1 Mark]

1.62 Let M be an NFA with n states and let M′ be the minimal DFA that accepts the same language. Which of the following is ABSOLUTELY true?

(a) $m \le 2^n$
(b) $m \le n$
(c) M has more than n states
(d) $m = 2^n$

[IT-2008 : 1 Mark]

1.63 In the finite automata shown in the DFA accepts the strings which begin with the following language over the alphabet {0, 1}

(a) Set of all strings that do not end with 01
(b) Set of all strings that do not end with 00
(c) Set of all strings that do not contain the substring 00
(d) The set described by the regular expression $(0+1)^*(0+1)$

[IT-2008 : 2 Marks]

1.64 Which of the following languages is/are non-regular?

L_1 = {0^n | n is an integer ≥ 100}

L_2 = {w | w reads the same forward and backward}

L_3 = {w | w ∈ {0, 1}* has an equal number of 0's and an even number of 1's}

(a) L_2 and L_3 only
(b) L_1 and L_2 only
(c) L_2 only
(d) L_3 only

[IT-2008 : 2 Marks]

1.65 Consider the following pair of finite automata M_1 accepts L_1 and M_2 accepts L_2. Which one of the following is TRUE?

(a) $L_1 = L_2$
(b) $L_1 \cap L_2 = \phi$
(c) $L_1 \cap L_2 = \phi$
(d) $L_1 \subset L_2$ or $L_2 \subset L_1$

[IT-2008 : 2 Marks]

1.66 Given below are two finite state automata (→ indicates the start and ⊥ indicates a final state)

Which of the following pairs of strings is accepted by both the automata?

(a) aaa, bbb
(b) aaa, aa
(c) aa, bb
(d) bb, bbb

(c)

(d)

(e)

(f)

[2005 : 2 Marks]

1.67 Match List-I with List-II and select the correct answer using the given codes below the lists.

List-I

A.

D.

C.

D.

List-II

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

Codes:

	A	B	C	D
(a)	3	1	3	4
(b)	1	3	2	4
(c)	[illegible]	2	[illegible]	4
(d)	[illegible]	2		[illegible]

[2005 : 2 Marks]

1.68 [illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

1.69 Which one of the following languages over the alphabet {0, 1} is described by the regular expression (0 + 1)*0(0 + 1)*0(0 + 1)*?

(a) The set of all strings containing two 0's

(b) The set of all strings containing at most two 0's

(c) The set of all strings containing at least two 0's

(d) The set of all strings beginning and ending with either 0 or 1

[2008 : 1 Mark]

1.70 The following DFA accepts the set of all strings over {0, 1} that

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2009 : 2 Marks]

1.75 Let $L = \{a, b\}^*$. [illegible] has exactly number of [illegible] number of [illegible] which [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2010 : 2 Marks]

1.76 [illegible] Let [illegible] minimum [illegible] of states in a non-deterministic finite automaton that accepts [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2010 : 2 Marks]

1.77 The [illegible] analysis for a modern computer language [illegible] the power of [illegible] and pushdown automata

(a) Finite state automata
(b) Deterministic pushdown automata
(c) Non-deterministic pushdown automata
(d) Turing machines

[2011 : 1 Mark]

1.78 [illegible] the language [illegible] For example, [illegible] the language represented by the regular expression p^*q^* and $\{p, q\}^* \mid n = N\}$. Then which [illegible] is/are regular?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2011 : 1 Mark]

1.79 Definition of a language L with alphabet {a} is given as follows:

$L = \{a^{n!} \mid n \geq 0\}$, and n is a positive integer constant.

What is the minimum number of states in a deterministic finite automaton that accepts L?

(a) $n + 1$
(b) $n + 2$
(c) $n!$
(d) $n! + 1$

[2011 : 2 Marks]

1.80 A deterministic finite automaton (DFA) D with alphabet $\Sigma = \{a, b\}$ is shown below:

What is the number of states in a minimal DFA which accepts the same language as D?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2011 : 2 Marks]

1.81 What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ε is the empty string.

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2012 : 1 Mark]

1.82 Given the language $L = \{ab, aa, baa\}$, which of the following strings are in L^*?

1. [illegible] 2. [illegible]
3. [illegible] 4. [illegible]

(a) 1, 2 and 4 (b) 2, 3 and 4
(c) 1, 2 and 3 (d) 1, 3 and 4

[2015 : 1 Mark]

1.79 Consider the set of strings on {0, 1} in which every substring of length two has at most two zeros. For example, 001110 and 011001 are in the language, but 1100 is not. Let L be the language of length two. Let S be the language. A possible minimal DFA that accepts this language is shown below.

The missing arcs in the DFA are

(a)

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]		
[illegible]	[illegible]			
[illegible]		[illegible]		

(b)

	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]		[illegible]
[illegible]				
[illegible]			[illegible]	
		[illegible]		

(c)

	[illegible]	[illegible]	[illegible]	
[illegible]				[illegible]
[illegible]		[illegible]		
[illegible]		[illegible]		

(d)

	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]		[illegible]
[illegible]				
[illegible]	[illegible]			
[illegible]		[illegible]		

[2015 : 2 Marks]

1.80 Consider the languages $L_1 = \phi$ and $L_2 = \{\phi\}$. Which one of the following represents $L_1^* \cap L_2^*$?

(a) ϕ (b) $\{\phi\}$
(c) $\{\phi, \phi\}$ (d) $\{\phi, \phi, \phi\}$

[2014 : 1 Mark]

1.81 Consider the DFA as given below.

Which of the following are TRUE?

1. Complement of L(M) is context-free
2. L(M) = {(01)^n | n ≥ 1}
3. Every language accepted by M is in the minimal DFA
4. Complement L(M) over {0, 1} is "infinite"

(a) 1 and 3 only (b) 1 and 4 only
(c) 2 and 3 only (d) 2 and 4 only

[2013 : 2 Marks]

1.82 Which of the following statements is/are TRUE?

(a) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.
(b) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.
(c) The language $L = \{ww \mid w \in \{a, b\}^*\}$ is regular.
(d) The language $L = \{ww^R \mid w \in \{a, b\}^*\}$ is regular.

[2014 (Set-1) : 1 Mark]

1.83 Consider the finite automaton as in the following figure.

What is the set of reachable states from the state q_0?

(a) $\{q_0, q_1, q_2\}$ (b) $\{q_0, q_1\}$
(c) $\{q_0, q_1, q_2, q_3\}$ (d) $\{q_1\}$

[2014 (Set-1) : 1 Mark]

1.84 Which of the regular expressions given below represent the following DFA?

I. [illegible]
II. [illegible]
III. [illegible]
(a) I and II only (b) I, II and III only
(c) I and III only (d) II and III only
[2011 (Sec-1) : 2 Marks]

1.85 If L_1 [illegible] and $L_2 = \{a^n b^n \mid n \geq 0\}$ [illegible]
I. [illegible] is context-free language
II. $L_1 \cap L_2$ [illegible]
Which one of the following is CORRECT?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
[2014 (Set-3) : 1 Mark]

1.86 [illegible] $L_1 = \{w \in (a+b)^* \mid w$ has at least as many occurrences of (ab) as (ba)$\}$ [illegible] $L_2 = \{w \in (a+b)^* \mid w$ has at least as many occurrences of (aa) as (bb)$\}$. Which one of the following is TRUE?
(a) L_1 is regular but not L_2
(b) L_2 is regular but not L_1
(c) Both L_1 and L_2 are regular
(d) Neither L_1 nor L_2 are regular
[2011 (Sec-3) : 2 Marks]

1.87 The length of the shortest string NOT in the language defined by the regular expression is _____.
[2014 (Set-3) : 1 Mark]

1.88

Consider the DFAs shown above. The minimized state in the minimized DFA that accepts the language $(a|b)^*a(a|b)^*$ is _____
[2015 (Set-1) : 2 Marks]

1.89 Consider alphabet $\Sigma = \{0, 1\}$. Let the strings S_1 and S_2 be the second strings X_1, X_2 and X_3 respectively in the corresponding lexicographic enumeration. X_1, X_2 and X_3 satisfy the following:
$X_1 = 1X_1$
$X_2 = 0X_1 + 1X_2$
$X_3 = 0X_2 + 1X_3$
Which one of the following is the value of S_2?
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
[2015 (Set-2) : 2 Marks]

1.90 Which of the following languages is/are regular?
$L_1 = \{a^m b^n \mid m = n$ and $m, n \geq 0\}$
$L_2 = \{a^m b^n \mid m = n$ and $m, n \geq 0\}$
(a) L_1 only (b) L_2 only
(c) L_1 and L_2 both (d) Neither L_1 nor L_2
[2016 (Set-2) : 2 Marks]

1.91 The number of states in the minimal deterministic finite automaton corresponding to the regular expression $(0+1)^*(10)$ is _____.
[2015 (Set-3) : 2 Marks]

1.92 Let L be the language represented by the regular expression $2^*(0|1)1^*2^*$ where $\Sigma = \{0, 1, 2\}$. What is the minimum number of states in a DFA that recognizes L (complement of L)?
(a) 2 (b) 3
(c) 4 (d) 5
[2015 (Set-3) : 1 Mark]

1.93 Which of the statements below is/are true for the given grammar?
[illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
[2016 (Set-1) : 1 Mark]

1.94 Which of the following decision problems are decidable?
I. Given NFAs N_1 & N_2 is $L(N_1) \cap L(N_2) = \phi$?
II. Given a CFG G = (N, T, P, S) and a string $w \in T^*$, is $w \in L(G)$?
III. Given CFG G_1 and G_2, is $L(G_1) = L(G_2)$?
IV. Given TM M is $L(M) = \phi$?
(a) I and IV only (b) I and III only
(c) III and IV only (d) II and IV only
[2018 (Set-1) : 1 Mark]

1.95 Which one of the following regular expressions represents the language whose set of all binary strings contains two consecutive 0's and two consecutive 1's?

(a) $(0+1)^*00(11)^*$

(b) $(0+1)^*(00+11)(0+1)^*$

(c) $(0+1)^*00(0+1)^*11(0+1)^*$

(d) $(0+1)^*(00+11)(0+1)^*$

[2016 (Set – I) 1 Mark]

1.96 The number of distinct [illegible] that can be produced by the regular expression $(0+1)^*$ is ____

[2016 (Set – II) 1 Mark]

1.97 Consider the following two statements:

I. If a language is accepted by a DFA then its complement is also accepted by the DFA.

II. [illegible] regular language [illegible] regular.

Which one of the following is correct?

(a) Only I is true

(b) Only II is true

(c) Both I and II are true

(d) Both I and II are false

[2016 (Set – II) 1 Mark]

Answers: Finite Automata & Regular Languages

1.8	(d)	1.9	(a, c)	1.10	(b)	1.11	(d)	1.12	(a)	1.13	(b)	1.14	(c)	1.15	(a)	1.16	(b)
1.17	(d)	1.18	(d)	1.19	(a)	1.20	(a, d)	1.21	(b)	1.22	(b)	1.23	(b)	1.24	(c)	1.25	(b)
1.26	(d)	1.27	(a)	1.28	(b)	1.29	(d)	1.30	(b)	1.31	(a)	1.32	(b)	1.33	(d)	1.34	(b)
1.35	(a)	1.36	(b)	1.37	(a)	1.38	(b)	1.39	(a)	1.40	(b)	1.41	(d)	1.42	(a)	1.43	(a)
1.44	(c)	1.45	(a)	1.46	(a)	1.47	(c)	1.48	(d)	1.49	(b)	1.50	(a)	1.51	(c)	1.52	(a)
1.53	(b)	1.54	(a)	1.55	(a)	1.56	(c)	1.57	(b)	1.58	(a)	1.59	(a)	1.60	(c)	1.61	(a)
1.62	(a)	1.63	(d)	1.64	(b)	1.65	(a)	1.66	(d)	1.67	(c)	1.68	(a)	1.69	(c)	1.70	(c)
1.71	(b)	1.72	(a)	1.73	(b)	1.74	(b)	1.75	(a)	1.76	(a)	1.77	(b)	1.78	(a)	1.79	(d)
1.80	(a)	1.81	(d)	1.82	(a)	1.83	(a)	1.84	(b)	1.85	(a)	1.86	(a)	1.87	(c)	1.88	(b)
1.89	(a)	1.90	(d)	1.91	(c)	1.92	(b)	1.93	(b)	1.94	(d)	1.95	(b)	1.96	(b)	1.97	(b)

Explanations: Finite Automata & Regular Languages

1.1 (a)

[illegible] substrings of all [illegible] strings of length [illegible] from characters [illegible] of length $n = 2^{(n-1)} + 1$

Example: Let the length of string be 4.

$|x| = 4$ and $x = ABCD$

Set of substrings = {[illegible]}

Set of pairs of [illegible] = {[illegible]}

Set of substrings of length 3 is {ABC, BCD}

Set of substrings of length 4 is {ABCD}

Set of substrings of length 5 is {[illegible]}

Set of substrings of length 6 is {[illegible]}

So, total substrings of all lengths = [illegible] of length 1 is

$$1 + 4 + 2 + 1 + 1 = 1 + \frac{4 \times (4-1)}{2} + 1$$

1.2 (d)

Since regular languages are closed under union, intersection, complement, concatenation, kleene closure as well as kleene closure.

So, statements (a), (b), (c) and (d) are true.

1.3 (b, c)

$$L = 1^* + 0^*$$

So, the language corresponds to [illegible] starting with 1 followed by any number of [illegible]

So, $L(r^*) \subseteq L(r)$

$$L^* = 1^* 0^*$$

The language consists of all strings starting with C. Since it has to start with a lower case letter, it must start with some L. So it has more restrictions imposed on it.

So $L(G) \subset L(G_1)$

Note: G_1 is also (a) and (c) are actually same.

1.4 (Bonus)

According to definition of regular expression

$(a+b)^* = (a+b)^*$

1.5 (Bonus)

[illegible] Sol: Finite State machine can be designed to add even numbers of zero [illegible] length sequence. It will have finite memory and [illegible] given input [illegible]

1.6 (Bonus)

Number of substrings of all lengths including the empty string that can be formed from a character string of length n is:

$$\text{length is: } \frac{n(n+1)}{2} + 1.$$

Example: If the length of string be 4 $|w| = 4$ and $w = abcd$

No. of substrings of length 1 is 4

No. of substrings of length 2 is 3 (a, b, c, d)

No. of substrings of length 3 is 2 (ab, bc, cd)

No. of substrings of length 4 is 1 (abcd)

No. of substrings of length 0 is 1 (empty string)

So, total substrings of all lengths including empty string of length 0 is

$$4 + 3 + 2 + 1 + 1 = \frac{4 \times (4+1)}{2} + 1 = 11$$

1.7 (Bonus)

since G_1 could state is just some more than

Regular expression = RE of A + RE of B

$= 0^* + 0^*1(0+1)^*$

$= 0^* + 0^*1(0+1)^*$

1.8 (Bonus)

$L = \{x^n y^n \mid n \geq 1\}$

The language L produces all strings having equal number of x and y and follows the rules.

(i) $R \to xRy \mid xy$

R is language produced by above grammar contains all strings having equal number of x and y and y follows x.

(ii) $xy \mid xxyy$

This is a context free grammar, starting xxyy which can contain equal no. of x and y.

(iii) xy^+

This is a regular grammar, starting with x and ends with y followed by one or more x and y generated from xy. Above RR can generate strings which contain equal no. of x and y.

So only (i) and (ii) are the correct languages.

1.9 (Bonus)

The state diagram which represents the given state table is

From state A to reach C the shortest sequence is

A [illegible] B [illegible] C

A [illegible] D [illegible] C

From B to reach C

B [illegible] C

B [illegible] D [illegible] C

From C to reach C

C [illegible] D [illegible] C

C [illegible] B [illegible] C

From D to reach C

D [illegible] C

D [illegible] B [illegible] C

So if with the shortest input sequence to reach the final state C, it can never be the initial state.

1.10 (b)

(i) [illegible]

[illegible] = {001^n | n ≥ 0}

Because number of 0's and 1's are equal, any number of 0's

(ii) (00) [illegible] number of 0's

(iii) 0* [illegible]

(iv) [illegible] number of 0's

So (i) and (iii) are same.

1.11 (d)

(a) L = {a^n | n is a prime number} is context free language. This language finds [illegible] comparison between number of a's and number of b's.

(b) L = {a^n b^n | n ≥ 1} is context free language. There is comparison between number of a's and number of b's.

(c) L = {a^n | n is more than 100} is context free language. There is comparison between number of a's and number of b's.

(d) L = {a^n b^n c^n | n ≥ 1} is context sensitive language. Because number of a's, number of b's and number of c's are independent. There is no comparison between number of a's and number of b's.

1.12 (b)

(a) The set of all strings over Σ* which is countably infinite.

(b) Set of all language over Σ is 2^Σ*. According to Cantor's theorem if S is countably infinite set, then its power set 2^S is uncountable.

So 2^Σ* is uncountable because Σ* is countably infinite.

(c) Set of all regular languages over Σ is countably infinite.

(d) Set of all languages over Σ is countably infinite set of all Type 3 languages which is countably infinite.

1.13 (d)

(a) L.L. (regular expression) = (0+1)* (0+1) generates strings which do not contain 100 as substring. However, it also contains the string which contain 100 as substring but ends with 100 is consistent.

(b) L.L. (regular expression) = (0+1)* 100 generates strings which do not contain 100 as substring which contain 100 as substring.

1.14 (d)

$$S \to bS^2 \mid ba \mid a$$

So $T = \{a(ba)^n \mid n \geq 0\}$

$a(ba)^n \cap T^* = \Lambda$

So $A = B$

1.15 (b)

The numbers 1, 2, 3, 4, 5, ..., 2^n are represented by 1, 10, 11, 100, ...

The problem is the only representation is in need by 0's.

The only 1's representation is shown in 2^n.

The DFA for above language is

So the numbers 1, 2, 3, 4, 5, ..., 2^n are [illegible] with leading zero [illegible] is a state automaton.

1.16 (a)

(i) The string 101 occurs as long as [illegible] represented by f(00*10, 0*(00 + 1)) because [illegible] appears in a string that 1 and 0 only appear in pairs.

(ii) [illegible] generate strings with odd number of 0's [illegible] 1101.

1.17 (d)

Number of substrings of 111 [illegible] from a [illegible] strings of length n is $\frac{(n-1)(n-2)}{2}$.

Since [illegible] the number of substrings [illegible]

1.18 (b):

The number of states in the minimum state [illegible] Kleene's theorem [illegible] Kleene's theorem.

1.19 (b):

The minimum state finite automata [illegible] recognizes the language represented by regular expression $(0+1)^*(0+1)^*$.

This language contains all strings [illegible] exactly length n.

$n+1$ states are required to count the length upto n.

Next [illegible] minimal DFA.

For example for $n=3$ the diagram is shown below

1.20 (d):

According to the rules of regular expression

$$(a_1+a_2)^* = (a_1+a_2)^*$$

Now [illegible] $(a+b)^*$

So [illegible]

1.21 (a):

The language can be described by the grammar

$S \to$ [illegible]

$\to$ [illegible]

$\to$ [illegible]

So above language is regular [illegible]

1.22 (d):

[illegible]

$L = \{$ [illegible] $\}$

$L = \{$ [illegible] $\}$

[illegible]

1.23 (a):

$L_1 = \{$ [illegible] $\}$

[illegible] produced by [illegible] which is regular language.

Thus [illegible] for L_1 is [illegible]

$L_2 =$ [illegible] and a [illegible] language not regular [illegible]

So [illegible] but L_2 is not [illegible].

1.24 (b):

The [illegible] accepting the maximum number of [illegible] minimized DFA is 2[illegible].

1.25 (d):

A DFA over $\Sigma = \{$ [illegible] $\}$ accepting all strings which [illegible] divisible by 8 [illegible] number of states [illegible] $n \geq 8$ states.

1.26 (d):

$L_1 =$ [illegible]

is not a regular language (Which [illegible] number [illegible] initial [illegible] is [illegible])

$L_2 =$ [illegible]

is the [illegible]

[illegible] language [illegible] is [illegible] strong matching [illegible]

$L_3 = \{$ [illegible] $\}$

is regular language, which contains [illegible] having even number of [illegible]

$L_4 = \{$ [illegible] $\}$

[illegible] is a context sensitive language (CSL) [illegible] the [illegible] is infinite [illegible])

1.77 (c)

The state diagram represents the DFSM which corresponds to accepting the strings and generating the output.

Also it represents generous table. Output is not for corresponding inputs.

1.78 (b)

The minimal finite automaton with 3 states which accepts the language $L = \{w \mid w \text{ is divisible by } 3\}$ is as follows:

1.79 (c)

The given bit pattern can be represented as

1 1

The total strings can be filled in $2^n = 16$ ways. The above bits are 16 such strings in the machine. Now all of these are accepted by the machine. Accept state of the complement is given below.

								accepted
1	0	0	1	0	0			✓
1	0	0	1	0		1		✓
1	0	0	1	1	0	1		✓
1	0	1			1	1		✓
1	0	1		0	0	1		✓
1	1	0	1	0	0	1		✓
1	1	1	1	0	0	1	1	✓

Only these seven strings given above are accepted. They other strings of these 16 strings are rejected since they could not reach the final state.

∴ Correct answer is (c).

1.80 (b)

The given machine M is

Now the complement of machine M is

In the case of DFA, $L(\overline{M}) = L(M)$. So, in the case of NFA, this is not true. In fact $L(\overline{M})$ and $L(M)$ are not complement.

$L(\overline{M}) \cap L(M) = L(\overline{M})$ = whatever is left of $\overline{M}$ accordingly, finally, we get

$$L_{\overline{M}} = L(\overline{M}) = (0+1)^* - (0+1)^* 1 (0+1)^*$$
$$= (0+1)^* 0 (0+1)^*$$

1.81 (b)

In choice (a), (c) and (d), there is the power choice which represents the $a^n b^n$ and a^n respectively and hence all these are context free languages.

In choice (b), the language is not a context free language since "aab" is always generated.

So, choice (b) is not a context free language.

1.82 (b)

$$S(A, a) = A \qquad A \to aA$$
$$S(A, b) = B \quad \blacksquare \quad A \to bB$$
$$S(B, a) = B \quad \blacksquare \quad B \to aB$$
$$S(B, b) = A \quad \blacksquare \quad B \to bA$$

Since A is final state, we need to put $A \to \epsilon$.

So the new grammar is obtained by adding the productions $A \to aA$, $A \to bB$, $B \to aB$, $B \to bA$, $A \to \epsilon$.

1.83 (a)

The given finite state automata is a composite string $w \in \{a, b\}^*$ where the number of b's is multiple of 3 and the number of a's is odd. Hence (a).

1.84 (a)

Given regular expression is infinite set (because a^* and b^* are infinite strings). Accepted strings are always finite in length by definition.

1.85 (a)

Given language is finite. Hence it is regular language.

1.38 (a)

Writing Y and Z in terms of memory cells using Moore's method, we get

$$Y = X0 + Y0 + X1$$
$$Z = X0 + Z1 + Y0$$

Clearly, $Y = Z$

1.39 (b)

The given grammar generates strings in which X and Y have same

$$S \to Z0X \mid W0Y$$
$$Z \to 0Z1 \mid \varepsilon$$
$$W \to 1W \mid \varepsilon$$

When only strings in Z and W is equivalent to $S \to 0Z1 \mid 1W$.

When only strings in S $\to$ 0Z1 | 1W

So, $L(S) = \{0^n1^n\}$

So, only strings generated by the given grammar is the set of strings of the form 0^n1^n. So minimum number of states required will be 3 as minimal DFA the count of number of states is 3.

1.40 (d)

(i) This is regular as satisfies the pumping lemma for regular language.

(ii) This is CFL as satisfies the pumping lemma for CFL.

By satisfying pumping lemma, we can never say that a language is regular or CFL. It can only be used to prove that a certain language is not regular or not CFL. In case, the language violates the corresponding pumping lemma.

So, for regular or non-regular language we satisfy pumping lemma for regular language.

Similarly for CFL and non-CFL we satisfy pumping lemma for CFL.

So satisfying pumping lemma doesn't never confirming about the type of language.

1.41 (b)

(a) is false since Σ^* is accepted by finite FA.

(b) is true.

(c) is false since non-regular contains $\{0^n1^n\}$ as a subset, but is being rejected by the machine.

(d) is false, since there are many infinite sets of strings which are not accepted by M.

1.42 (b)

$L = \{ba, ba + bb\}$

1.43 (b)

The given right linear grammar can be converted into the following DFA.

The minimum number of strings over the alphabet {a, b} which are not in the set of strings accepted by the DFA.

So Myhill Nerode equivalence classes for this language is containing the set of strings ending with a and b respectively.

$S \to bC \mid aB + \varepsilon$ | L_1 (a*) is even

$A \to aa \mid ba \mid \varepsilon$ | L_2 (a*) is odd

1.44 (d)

(a) Every language has a regular superset.
True, Σ^* is such a superset.

(b) Every language has a regular subset.
True, ϕ is such a subset.

(c) Every subset of a finite language is regular.
False, if $W \subseteq \Sigma^*$, $L = \{0^n1^n\}$ is not regular.

(d) Every subset of a finite language is regular.
True. Every subset of a finite set is finite by definition. Every finite set is regular. Hence, every subset of a finite language is regular.

1.46 (pt):

Prefix L_1 with L_2 and L_1L_2 is a regular language.

Since all this is a regular language, option a CFL.

1.48 (a):

Option (a): $L(L_1) = (a+b)^*$. Then regular language $L_2 = \{a^n b^n : n \geq 0\}$ is a context free language. Given that L_2 is not regular.

Option (b): $L(L_1) = (a+b)^*$, $L(L_2) = \{a^n b^n : n \geq 0\}$. Then regular language $L_1 - L_2$ is context free language. So option (b) is not correct to show that regular language L_2 need not be regular.

Option (c): $L(L_1) = \{ab\}^*$. Then regular language $L_1 \cap L_2 = \{ab\}^*$ is not regular.

So option (c) is not correct to show that regular language L_2 need not be regular.

Option (d): $L = \{a^n b^n : n \geq 0\}$ is not a regular language. So not regular.

1.49 (d):

Choice (a) is regular since it is finite.

Choice (b) is regular since although computation would be very large, it is finite and hence it produces a DFA and hence accepted by DFA.

Note: $n_1(x) = n_0(x) = 2 \Rightarrow n_1(x) - n_0(x) = 0$ and $n_1(x) + n_0(x) = 4$.

Choice (c) is also regular since the number of 0's and 1's are equal in the string, as a whole, and this cannot be done by a finite automata. It has finite memory and has no stack for counting up to infinite. Therefore, choice (c) is not regular.

Choice (d) is regular since the number of 1's is exactly $3 \times$ the number of 0's. Since 3 and number of 1's are divisible by 3 and this can be recognized by a DFA with 3 states.

Alternate: This language belongs to the language of choice (c) as shown below.

1.50 (a):

$L = \{w \mid w \in (0+1)^*$ the number of 0's and 1's in w are equal$\}$

Since L is context free language.

The minimum number of deterministic finite automata accepting the language L has at most 6 states.

1.51 (a)

[illegible] $\{w \mid w \in \{0,1\}^*\}$ [illegible]. For [illegible] and [illegible] [illegible] and get minimum expression

$$r = [0(0+1)^*] + [(0+1)^*1]$$

Now put input as regular string, without [illegible] any new [illegible] generated here.

Example if we put [illegible] $= 0(0+1)^*$ [illegible] $+ (0+1)^*1$ [illegible] which [illegible] by the first part.

Similarly if we put [illegible] $(0+1)^*1$ then [illegible] $(0+1)^*1$ which is already [illegible] by the second part.

So [illegible]

So the regular expression is [illegible]

$$r = [0(0+1)^*] + [(0+1)^*1]$$

That is, L includes all binary strings which start and end with the same symbol. So minimum DFA [illegible] [illegible] therefore L is regular.

1.52 (b)

q_0 is [illegible] starting state and [illegible] [illegible] Dividing [illegible].

q_0 is the [illegible] state. From q_0 [illegible] regular expression [illegible] combination of a and b [illegible] by q_0 or q_1. So q_0 and q_1 [illegible] [illegible] state and [illegible] the [illegible].

[illegible] $= b^*(a+b)^*$.

1.53 (b)

The minimum state DFA is given below for the regular expression [illegible] by:

d. DFA contains minimum [illegible] q_0 and q_1.

1.54 (c)

Given $S \to xS \mid xSS \mid \varepsilon$

$B \to x \mid yB \mid zb$

$B \to y \mid z = x$

Substitute in S production

$S \to yz \mid xS \mid xSS \mid \varepsilon$

Now solution of S is

$S \to xS \mid yzS$

$S \to (x + yz)S$

So $L(G) = L = (x+yz)^* + (x+yz)^*(yz)(x+yz)^*$

G1 generates every string in which [illegible] before [illegible].

G2: $S \to yz \mid xS \mid xB$

$B \to z \mid xB$

Substitute B in S production

$S \to yz \mid xS \mid xxB$ or $S \to yz \mid xS$

Now solution of S is

$S \to (x+yz)S$

So $L(G2) = L = (x+yz)^*$

G2 generates every string in which [illegible] is followed by [illegible].

1.55 (d)

Given DFA [illegible] as q0, q1 as q1, q2 as q2, q3 as q3.

Accepted [illegible] $= 0 \bmod 3$, $L = 1 \bmod 3$, [illegible] $= 0 \bmod 3$ [illegible] $= 0 \bmod 3$. So number is even [illegible].

q0 is final state means where number of a is even and number of b is odd.

q10 is final state means where number of a is odd and number of b is even.

1.56 (b)

The grammar cannot generate any string starting with a and ending with b, neither aa nor bb.

[illegible] generated by the grammar. The derivation for [illegible] and [illegible] is shown below

[illegible]

[illegible]

So only [illegible] can be generated from this grammar.

1.57 (b):

$L(P) \cap L(Q) = \{a^n b^n c^n\}$ [illegible] all strings in common to L and Q. Note that aa is common to both P and Q and hence must be accepted by any PDA accepting $L(P) \cap L(Q)$. However, all of the given machines reject it. Therefore option (b) is not correct. Hence, the right answer.

1.58 (d):

The language of the given regular expression r is [illegible] the strings ending with ab. Option (d) is the correct machine for this language.

1.59 (a):

The option (a) [illegible] permutation automata [illegible] in [illegible] permutation [illegible] as shown below.

The [illegible] clearly accepts all strings containing the substring abb, which is same as regular expression r.

1.60 (d):

$R = (a + b + c)^*(ab + bc)^*$ has following equivalent DFA

Which is minimized. Transition from [illegible] to [illegible] on [illegible] transition from [illegible] to [illegible] does not affect the language.

[illegible]

[illegible]

which is equivalent to

[illegible]

which is equivalent to

[illegible]

Equivalent regular expression [illegible] $= (a+b)^*(ab+bc)(a+b+c)^*$

1.61 (c):

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) Any [illegible]

1.62 (c):

A state in a DFA will be [illegible] of the set of [illegible] equivalent NFA.

So, [illegible] maximum number of states in the equivalent DFA will be 2^n, while the [illegible] NFA is set with [illegible] number of states.

So, minimum number of states in equivalent DFA is [illegible] of states in equivalent DFA is 2^n.

1.63 (d):

Interchanging final and non final states of DFA is used for complementation.

Given DFA generates all strings ending with [illegible]

Complement of DFA will [illegible] all strings ending with [illegible] and with [illegible]

Complement of given DFA is

[illegible]

Ans [illegible]

[illegible] is regular. Since 10000 is finite

[illegible] machine, [illegible]

[illegible] where [illegible] is finite

language which is known to be [illegible]

Ans [illegible]

[illegible] DFA accepting all strings containing

[illegible] minimum number of accepting [illegible]

[illegible] the same [illegible]

Containing a consecutive 1's.

[illegible]

Ans [illegible]

The transition table for Y and Z is as follows

Y:

	a	b
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Z:

	a	b
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Now since Z starts and Y accepting states so the product automaton Z × Y and since [illegible] states

The table for Z × Y is as follows

	a	b
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Since [illegible] Y is [illegible] and Z is also accepting state in above [illegible]

[illegible] the transition table and Kleene's [illegible] given

Now there are only [illegible] 1, 2 & 3. It can be [illegible] Q & R or S & T [illegible] 2 possible tables.

[illegible]

	a	b
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

	a	b
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Rewriting [illegible] in same order [illegible]

B. So as given in choices we get

Table 1.

	a	b
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Table 2.

	a	b
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Now [illegible] table [illegible] [illegible] answer.

Ans (C)

[illegible] expression

[illegible]

[illegible]

[illegible] 4.

[illegible]

Note: [illegible] NFA for initial state of [illegible] also has final state of [illegible] in such single expression.

Ans [illegible]

[illegible] is regular [illegible] expression [illegible]

[illegible] TC, but not regular since here, we need to [illegible] same with [illegible]

[illegible]

4. [illegible]

1.69 [illegible]

[illegible]

1.70 [illegible]

[illegible]

1.71 [illegible]

[illegible]

1.72 [illegible]

[illegible]

[illegible]

1.73 [illegible]

[illegible]

1.74 [illegible]

[illegible]

1.75 [illegible]

[illegible]

[illegible]

1.76 [illegible]

[illegible]

1.77 [illegible]

[illegible]

1.78 [illegible]

[illegible]

1.79 [illegible]

[illegible]

[illegible]

3. [illegible]

4. Strings (0) [illegible] in [illegible] (5) does not belong to [illegible]. Only 1, 2 and 4 belong to L.

1.79 (d)

Notice that [illegible] are named accepting. [illegible] ending [illegible] which [illegible] accepting.

[illegible] should go to [illegible] accepting.

[illegible] 1 [illegible] 01. [illegible] should go to [illegible] time. [illegible] string to this point [illegible] 1 [illegible] 01 [illegible] 1 [illegible] 01 [illegible] 1 [illegible] 01

[illegible] = [illegible] 1 is present, then the language accepted by the given DFA is [illegible] ending with 1.

[illegible] and [illegible] are equivalent to given DFA.

1.85 (a)

$L_1 = \{a^n \mid n \geq 0\}$

$L_2 = \{a^n \mid n \geq 0\}$

I. $L_1 \cdot L_2 =$ [illegible] $a^n b^n$ is regular.

II. [illegible] a^n is [illegible]

→ [illegible]

[illegible]

1.86 (d)

A machine which accepts L_1 must have finite memory since there is no need to keep [illegible] number of 1's in memory. The problem is because whenever [illegible] DFA [illegible] as shown below.

[illegible]

So the difference between number of 1's and number of 0's can't be $L_1 = 1$.

Since finite memory machine is not possible.

L_2 requires infinite memory since any number of 0's can come in front of 1 but can't [illegible] between. So the difference between the number of 0's and 1's can be [illegible]. So this language requires infinite memory. But FA has only finite memory.

So L_2 is not regular.

1.87 (a)

Given the [illegible] regular expression, from a, b, [illegible] that is not generated by the given regular expression is bbbbbb.

In this case minimum string 100 generated by the given regular expression is bbb, where length is 3.

1.88 (b)

$L(X) = \{a^n \mid n = 5\}$ [illegible] with 5

$L(Y) = \{a^n \mid n \text{ is even}\}$ [illegible] with 5

$L(X) \cap L(Y) = \{a^n \mid n = 5\}$ [illegible] is common

∴ Number of states =

minimal DFA which accepts single language.

1.89 (d)

Converting the given regular expression into a state diagram we get.

From the [illegible] we can write

$L_1 = \{0(1+0)^*\}$

1.90 (a)

[illegible] $|w| \geq 0$]

[illegible] $|w| \geq 0$, [illegible] we get [illegible] and [illegible] and get the regular expression:

[illegible] $= 1(1^*)^* = 1(1^*)^* = 1(1^*)^* = 1^*$

Now particular [illegible] which [illegible] and any [illegible] string to the [illegible] [illegible] can be generated with the [illegible] connected or [illegible] or both.

So the given language = $\{a^n b^n \mid n \geq 0\}$ which is clearly regular.

∴ L_1 is regular.

$L_2 = \{a^n b^n \mid n \geq 0\}$. This language [illegible] [illegible] between number of a's and b's. So L_2 is not regular language.

$L_3 = \{a^n b^n c^n \mid n \geq 0\}$ = [illegible] is not regular language.

1.91 (b)

The given regular expression $(0+1)^*(00)$ corresponds to binary string ending with 00. To accept the minimal string '00', we need 3 states. So any state is required since this is a machine which accepts 'ending with' type language.

So we need only 3 states. The design of the minimal DFA is shown below.

∴ Minimum number of states in the DFA = 3.

1.92 [illegible]

- [illegible]

  Here strings of the language L containing the substring 0011

  To accept the minimal string "0011" we need 5 states, since subtracting machine without required transitions we need only 5 states.

  Since the complementary DFA will accept complementary language, and [illegible] complementary DFA is minimum number states as the minimal DFA, therefore L will have only 5 states as shown below.

  Minimum DFA is:

  Minimum DFA is:

1.93 [illegible]

[illegible]

[illegible]

1.94 [illegible]

I. Disjointness is decidable – [illegible] – Decidable

II. Membership of CFL's – Decidable

III. Equivalence of CFL's – Undecidable

IV. Ambiguity of CFG's – Undecidable

(a) II and IV only is the correct answer

1.95 [illegible]

[illegible]

Given [illegible]

Choice (b) is correct.

Choice (c): 00 which does not belong to given language

Choice (d): always ends with 11 [illegible] minimal generator 00, 11

1.96 [illegible]

L = [illegible]

Min DFA: [illegible]

[illegible]

1.97 [illegible]

- [illegible] are accepting, [illegible] must be [illegible] and cannot be S^+
- [illegible] is [illegible] $\Delta = \phi$ which is equivalent to [illegible]

■■■■

2 Push Down Automata: CFL & DCFL

2.1 [illegible]

(a) [illegible]

(b) It [illegible] the [illegible] [illegible]

(c) [illegible]

(d) [illegible]

[1995 : 2 Marks]

2.2 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1997 : 2 Marks]

2.3 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1999 : 2 Marks]

2.4 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2002 : 2 Marks]

2.5 [illegible] grammar [illegible] string [illegible] length [illegible] in G is Chomsky normal form?

(a) [illegible]

(b) [illegible]

[1992 : 2 Marks]

2.6 [illegible] feature [illegible] not be [illegible] by context free [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1991 : 2 Marks]

2.7 Consider the grammar with the following productions

$S \rightarrow aS|bS$

$S \rightarrow aS|b$

$S \rightarrow bS|ab$

$S \rightarrow b|ab$

[illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1993 : 2 Marks]

2.8 If T_1 and T_2 are context free languages and R is regular [illegible] the languages [illegible] Which of the following [illegible]

(a) [illegible]

(b) [illegible]

(c) $T_1 \cap R$

(d) $T_1 \cap T_2$

[1996 : 2 Marks]

2.9 [illegible] grammar with production rules

$S \rightarrow [illegible]$

$A \rightarrow [illegible]$

$B \rightarrow [illegible]$

$C \rightarrow [illegible]$

[illegible]

[1998 : 2 Marks]

2.10 Which of the following languages over $\{a, b\}$ is accepted [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1997 : 2 Marks]

2.11 Which of the following statements is false?

(a) Every finite subset of [illegible] is regular

(b) Every subset of a regular set is regular

(c) [illegible]

(d) [illegible]

[1995 : 2 Marks]

2.13 Consider the following statements about context free grammars

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1999 : 1 Mark]

2.14 Let L_1 be a regular language and L_2 be a context free language. Let L_3 be the set of all languages accepted by a pushdown automata. Which of the following statements are true?

(a) $L_2 \cap L_3$ (b) $L_1 \cap L_2$

(c) $L_2 \cup L_3$ (d) None of the above

[1999 : 1 Mark]

2.15 If L_1 is a context free language and L_2 is a regular language, which of the following statements are false?

(a) $L_1 \cap L_2$ is a context free

(b) $L_1 - L_2$ is a context free

(c) $L_2 - L_1$ is a context free

(d) $L_1 \cup L_2$ is regular [1999 : 2 Marks]

2.16 Which of the following statements is true?

(a) The language accepted by a non deterministic pushdown automata but not accepted by a deterministic pushdown automata

(b) The union of two context free languages is a context free language

(c) The intersection of two context free languages is a context free language

(d) The complement of a context free language is a context free language

[2001 : 1 Mark]

2.18 The [illegible] accepted by a finite automata is known as Regular Language. Which of the following statements is true?

(a) [illegible]

(b) [illegible]

(c) deterministic finite state [illegible]

(d) [illegible]

[2002 : 1 Mark]

2.17 Let $G = (\{S\}, \{a, b\}, R, S)$ be a context free grammar where the set R is

$S \to aSb \mid bSa \mid SS \mid \varepsilon$

Which of the following statements is true?

(a) G is not ambiguous

(b) There exists a type 0 language that cannot be generated by G

(c) There exists a deterministic pushdown automata that accepts L(G)

(d) Which an Type 0 deterministic finite automata can recognize L(G)

[2008 : 2 Marks]

2.18 Which one of the following statements is false?

(a) [illegible] such that all the context free grammars generating L are ambiguous

(b) An unambiguous context free grammar always has a unique parse tree for each string of the language generated by it

(c) Turing machines and recursive functions are equivalent ways of describing computations

(d) A finite state automata can remember an arbitrary number of states

[IT-2006 : 1 Mark]

2.19 Let $\Sigma = \{a, b\}$. Let L be the language of all strings such that

$R = \{a, b\}$, $P = \{\}$, $S = \{a, b\}$, $T = \{a\}$ and

$\Delta = \{(a, a), (a, b), (b, a), (b, b), (a, a), (a, b), (b, a), (b, b)\}$

Consider the set of all strings in Σ^* which are accepted by the automaton (M)?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[IT-2004 : 2 Marks]

2.20 The language $\{a^m b^n c^{m+n} \mid m, n \geq 1\}$ is

(a) regular

(b) context free but not regular

(c) [illegible]

(d) [illegible]

[2004 : 2 Marks]

2.21 Consider the following grammar G

$S \to bS \mid aA \mid b$

$A \to bA \mid aB$

$B \to bB \mid aS \mid a$

Let $N_a(w)$ and $N_b(w)$ denote the number of a's and b's in a string w respectively. The language $L(G) \subseteq \{a, b\}^*$ generated by G is

(a) $\{w \mid N_a(w) = 2N_b(w)\}$

(b) $\{w \mid N_a(w) = 3N_b(w)\}$

(c) $\{w \mid N_a(w) = [illegible]\}$

(d) $\{w \mid N_a(w) = [illegible]\}$

[2004 : 2 Marks]

2.82 Let L be a regular language and M be a context free language. Let L and M be defined over the alphabet Σ. Let $\overline{L}$ and $\overline{M}$ denote the complements of L and M respectively.

Which of the following statements is false? The complement of L and M respectively

(a) The complement of L is regular and M is context free

(b) The complement of L is context free

(c) L is a regular language

(d) None of the above

[IT-2005 : 2 Marks]

2.83 Let T be a finite state automaton with a single accepting state. The FSA will accept a string if it reaches the accepting state. The state is initialized with the start state. The start state is not an accepting state. The FSA has the following transitions: $\delta(q_0, a) = q_1$, $\delta(q_1, a) = q_2$, $\delta(q_2, a) = q_0$, $\delta(q_0, b) = q_0$, $\delta(q_1, b) = q_1$, $\delta(q_2, b) = q_2$. The start state is q_0

(c) always deterministic context-free language
(d) [illegible]

[IT 2006 : 2 Marks]

2.33 Consider the pushdown automaton (PDA) given below [illegible] The PDA accepts the language [illegible]

The transitions of the PDA are defined below as follows [illegible]

The language accepted by the PDA is

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[IT 2006 : 2 Marks]

2.34 [illegible] common to both, S is start symbol, [illegible] are terminals, and [illegible] are empty strings.

S → aSbb | c

A → bbA | ε

[illegible]

(a) [illegible]
(b) [illegible]

[IT 2006 : 2 Marks]

2.35 Consider the following statements about the context-free grammar

G = {S → aS, S → bS, S → b}

1. G is ambiguous
2. G produces all strings with equal number of a's and b's
3. G can be accepted by a deterministic PDA

Which of the above statements are true?

(a) 1 only (b) [illegible]
(c) 2 and 3 only (d) [illegible]

[2006 : 2 Marks]

2.36 Consider a context-free grammar G with the following production rules [illegible] recognized by the [illegible] which [illegible] the following [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2007 : 1 Mark]

2.37 Consider the following grammar [illegible] representing [illegible] specified in capital letters.

G1: [illegible] → WHILE [illegible] DO [illegible]
[illegible] → [illegible]
[illegible] → [illegible]

G2: [illegible] → WHILE [illegible] DO [illegible]
[illegible] → [illegible]
[illegible] → [illegible]
[illegible] → [illegible]
[illegible] → [illegible]

Which one of the following statements is true?

(a) G1 generates a context-free language and G2 is regular
(b) G1 generates a regular language and G2 is context-free
(c) Both G1 and G2 are regular
(d) Both G1 and G2 are context-free but neither of them is regular

[IT-2007 : 2 Marks]

2.38 The language L = {a^i b^j | i ≠ j} over the alphabet {a, b} is

(a) regular
(b) context-free but not deterministic CFL
(c) is a regular language
(d) is not a deterministic CFL but is a CFL

[IT-2007 : 2 Marks]

2.39 Which of the following statements is/are true?

(a) Every NFA can be converted to an equivalent DFA
(b) Every non-deterministic finite automaton has an equivalent deterministic finite automaton

(c) Every regular language is a context-free language

(d) Context-free languages are closed under intersection

[2006 : 2 Marks]

2.35 Which of the following statements are true?

1. Every left recursive grammar can be converted to an equivalent right recursive grammar.
2. All regular languages can be represented by context-free grammars.
3. The language represented by a deterministic pushdown automaton are closed under complementation. [illegible]
4. The context-free languages are closed under intersection.
5. The context-free languages are closed under intersection. [illegible]

(a) [illegible]

(b) [illegible]

[2005 : 2 Marks]

2.36 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I

A. Checking for identifier in a lexical analyzer

B. Syntax of formal arithmetic in the accepted form of a function argument in the number of parameters passed to a function

C. Arithmetic expressions and nested loop statements

D. Definition

List-II

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

Codes:

	A	B	C	D
(a)	1	[illegible]	3	1
(b)	4	1	3	2
(c)	4	1	2	1
(d)	[illegible]	3	4	2

[2008 : 2 Marks]

2.37 Consider the following languages:

$L_1 = \{a^i b^j c^k : j = k + i\}$

$L_2 = \{a^i b^j : j = 2i, i \geq 0\}$

Which of the following is true?

(a) L_1 is not a CFL but L_2 is

(b) $L_1 \cap L_2 = \phi$ and L_2 is not regular

(c) $L_1 \cup L_2$ is not a CFL and L_2 is

(d) There exists a DPDA for $L_1 \cup L_2$ and there exists a DFA that accepts L_2

[IES 2009 : 2 Marks]

2.38 Consider a CFG with the following productions.

$S \rightarrow AA \mid B$

$A \rightarrow [illegible]$

$B \rightarrow [illegible]$

[illegible] terminals. The language generated by this grammar is

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible] containing [illegible]

[IES-2009 : 2 Marks]

Directions for Question 2.42 to 2.44:

A CFG G is given with the following production rules where S is the start symbol. A is non-terminal and a and b are terminals.

$S \rightarrow aSbA$

$A \rightarrow aAb \mid bAa \mid \epsilon$

2.42 Which of the following strings are generated by the grammar rules?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[IES-2008 : 2 Marks]

2.43 Two leftmost derivations for above Question have many steps and exhibit left-recursive parsing and how many steps are in the derivations?

(a) 6 and 1 (b) 6 and 2

(c) 7 and 2 (d) [illegible]

[IES 2008 : 3 Marks]

2.44 $S \rightarrow aSb \mid ab \mid \epsilon$

The [illegible] generated by the above grammar over the alphabet $\{a, b\}$ is [illegible]

(c) all odd lengths
(d) all odd lengths, beginning with a
(e) strings that begin and end with the same symbol
(f) strings of length greater than 2

[2004 : 1 Mark]

2.45 Which one of the following is FALSE?
(a) There is a unique minimal DFA for every regular language
(b) Every NFA can be converted to an equivalent DFA
(c) Complement of a context free language is recursive
(d) Every regular language is accepted by a Turing machine

[2005 : 1 Mark]

2.46 Match all items in Group 1 with correct options from those given in Group 2

Group 1
P. Regular expression
Q. Parikh's theorem
R. Pumping lemma
S. Regular grammar

Group 2
1. Syntax analysis
2. Text generation
3. Lexical analysis
4. Finite Automata

(a) P-4, Q-1, R-2, S-3
(b) P-3, Q-1, R-4, S-2
(c) P-3, Q-4, R-1, S-2
(d) P-2, Q-1, R-4, S-3

[2006 : 1 Mark]

2.47 Consider the following state table for a FSM with two inputs A and B, one input and one output.

Present State	Next State A=0, B=0	A=0, B=1	A=1, B=0	A=1, B=1	Output
0	0	2	0	2	1
1	1	3	1	3	2
2	0	0	0	0	0
3	1	0	1	2	1
4	4	1	4	1	3
5	5	1	5	3	2
6	6	1	6	3	0

If the initial state is 0, the minimum number of flip-flops required to implement the machine is _____ with Output = 1?

(a) 2 (b) 3
(c) 5 (d) 8

[2009 : 2 Marks]

2.48 Let $L = L_1 \cap L_2$ where L_1 and L_2 are languages as defined below.

$L_1 = \{a^n b^n c^n \mid n \geq 0\}$
$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$

Then L is

(a) not recursive
(b) recursive
(c) context free but not regular
(d) recursively enumerable but not context free

[2008 : 2 Marks]

2.49 Consider the languages $L_1 = \{0^i 1^j \mid i \neq j, i = 10j\}$, $L_2 = \{0^i 1^j \mid i = j\}$, $L_3 = \{0^i 1^j \mid i = 2j\}$. Which one of the following statements is true?

(a) Only L_2 is recursive
(b) Only L_2 and L_3 are recursive
(c) Only L_1 and L_3 are recursive
(d) All are recursive

[2010 : 2 Marks]

2.50 Consider the languages L_1, L_2 and L_3 as given below.

$L_1 = \{0^p \mid p \text{ is prime}\}$
$L_2 = \{1^n \mid n \text{ is even}\}$ and
$L_3 = \{0^p 1^q \mid p \text{ is prime and } p = q\}$

Which of the following statements is NOT TRUE?

(a) Push Down Automata (PDA) can be used to recognize $L_1 \cup L_2$
(b) L_1 is a regular language
(c) All the three languages are context free
(d) Turing machines can be used to recognize all the languages

[2011 : 2 Marks]

2.51 Consider the Turing machines

$L_1 = \{0^n \mid n \geq 0\}$
$L_2 = \{0^n 1^n \mid n \geq 0\}$

Which one of the following statements is TRUE?

(a) L_2 is context-free
(b) L_1 is recursive
(c) Complement of L_2 is recursive
(d) Complement of L_1 is context-free but not regular

[2013 : 2 Marks]

2.52 Consider the following languages over the alphabet $\Sigma = \{0, 1\}$

$L_1 = \{0^n 1^n \mid n \geq 0\}$

$L_2 = \{ww^R \mid w \in \{0, 1\}^*\}$

$L_3 = \{ww \mid w \in \{0, 1\}^*\}$

Which one of the following statements is true? (Note: w^R is the reverse of the string w. Which of these languages are deterministic context-free languages?)

(a) None of the languages
(b) Only L_1
(c) Only L_1 and L_2
(d) All the three languages

[2014 (Set-3) : 2 Marks]

2.53 Consider the NPDA $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ where $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, Z_0\}$, $F = \{q_3\}$, and δ is defined as follows. Q is the set of states, Γ is the input alphabet, Γ is the stack alphabet, q_0 is the initial state, Z_0 is the initial stack symbol, and F is the set of accepting states. The transition function δ is as follows:

Which one of the following sequences of states is possible for the given input to be the same number of 0's and 1's?

(a) 0, 1, 2, 3
(b) 0, 1, 2, 3
(c) 0, 1, 2, 3
(d) 0, 1, 2, 3

[2014 (Set-3) : 2 Marks]

2.54 Which of the following languages is/are context-free?

$L_1 = \{a^n b^m c^k \mid n, m, k \geq 1\}$

$L_2 = \{a^n b^m c^k \mid m = n + k\}$

$L_3 = \{a^n b^m c^k \mid n = m + k\}$

(a) L_1 only
(b) L_2 only
(c) L_1 and L_2 only
(d) L_1 and L_3 only
(e) L_2 and L_3 only
(f) L_3 only

[2015 (Set-3) : 1 Mark]

2.55 Consider the context-free grammars over the alphabet $\{a, b\}$ defined by the two rules, G_1 and G_2, as follows:

G_1: $S \to aS \mid bS \mid a$

G_2: $S \to aS \mid bS \mid aA \mid bA$, $A \to aA \mid bA \mid a$

Which one of the following pairs of languages is generated by G_1 and G_2 respectively?

(a) $\{w \mid w \in (a+b)^* \text{ and } w \text{ has at least one } a\}$ and $\{a^n b^n \mid n \geq 1\}$
(b) $\{a^n b^n \mid n \geq 1\}$ and $\{w \mid w \in (a+b)^* \text{ and } w \text{ has at least one } a\}$
(c) $\{w \mid w \in (a+b)^* \text{ and } w \text{ has at least one } a\}$ and $\{w \mid w \in (a+b)^* \text{ and } w \text{ has at least one } a\}$
(d) $\{a^n b^n \mid n \geq 1\}$ and $\{a^n b^n \mid n \geq 1\}$

[2016 (Set-1) : 2 Marks]

2.56 Consider the transition diagram of a PDA given below. The input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.

Which one of the following is TRUE?

(a) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automaton
(b) $L = \{a^n b^n \mid n \geq 0\}$ and is accepted by any deterministic PDA
(c) $L = \{a^n b^n \mid n \geq 0\}$ and is accepted by any Turing machine that halts on every input
(d) $L = \{a^n b^n \mid n \geq 0\}$ and is accepted by a deterministic context-free

[2016 (Set-1) : 2 Marks]

2.57 Consider the following languages:

$L_1 = \{a^n b^m c^k \mid n, m, k \geq 0\}$

$L_2 = \{a^n b^n c^k \mid n, k \geq 0\}$

Which one of the following is TRUE?

(a) Both L_1 and L_2 are context-free.
(b) L_1 is inherently ambiguous but L_2 is not ambiguous.
(c) L_2 is inherently ambiguous but L_1 is not ambiguous.
(d) Neither L_1 nor L_2 is context-free.

[2016 (Set-2) : 2 Marks]

2.39 [illegible]

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) Both [illegible]

[2016 (Set 2) : 1 Mark]

2.40 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2016 (Set 3) : 2 Marks]

Answers Push Down Automata: CFL & DCFL

2.2	(c)	2.3	(a, c)	2.4	(a,d)	2.5	(c)	2.6	(a)	2.7	(c)	2.8	(c)	2.10	(a)	2.11	(b)
2.12	(a)	2.13	(c)	2.14	(a,c)	2.15	(c)	2.16	(b)	2.17	(a)	2.18	(c)	2.19	(c)	2.20	(b)
2.21	(a)	2.22	(c)	2.23	(b)	2.24	(d)	2.25	(a)	2.26	(b)	2.27	(c)	2.28	(b)	2.29	(c)
2.30	(d)	2.31	(a)	2.32	(b)	2.33	(b)	2.34	(c)	2.35	(d)	2.36	(b)	2.37	(b)	2.38	(c)
2.39	(c)	2.40	(b)	2.41	(b)	2.42	(d)	2.43	(a)	2.44	(b)	2.45	(b)	2.46	(b)	2.47	(a)
2.48	(c)	2.49	(b)	2.50	(a)	2.51	(a)	2.52	(c)	2.53	(a)	2.54	(c)	2.55	(b)	2.56	(c)
2.57	(a)	2.58	(c)	2.59	(b)												

Explanations Push Down Automata: CFL & DCFL

2.1 [illegible]

[illegible]

1. [illegible]

or

2. [illegible]

or

3. [illegible]

2.2 (c)

[illegible] languages. [illegible] following two features which [illegible]

1. [illegible]
2. [illegible]

2.3 (a, c)

[illegible] all the [illegible] in [illegible] Kleene [illegible]

2.4 (a, d)

[illegible] languages are closed under union, [illegible] closed. However, [illegible] languages are not closed under [illegible]

8.5 (b)

A context-free grammar is in Chomsky Normal Form if all productions are in one of two simple forms either:

1. $A \to BC$ where A, B and C are variables or
2. $A \to a$ where a is a terminal and is alternative

So for any string of length n, since production of type $A \to BC$ is used $n-1$ times to produce sentential form of length n consisting only of variables and terminals, the length of derivation is $2n-1$.

Since length of derivation of any possible string is $2n-1$, the grammar is in Chomsky Normal Form.

Example:

$$S \to XY$$
$$Y \to XY$$
$$X \to a$$
$$Y \to b$$

Derivation for string aaabbb in above CNF is:

$$S \to XY$$
$$\to aY$$
$$\to aXY$$
$$\to aaY$$
$$\to aab$$

Length $= 2 \times 3 - 1 = 6 - 1 = 5$

$$= 2n - 1$$

8.6 (a)

(a) The language corresponding to regular expression $(a+b)^*$ is context free language. Since context free language is closed under union, the language corresponding to regular expression $(a+b)^* + (a+b)^*$ can be represented by context free grammar.

(b) The language corresponding to regular expression of $(a+b)^*$ is recursively enumerable language. Since recursively enumerable language is closed under union, the language corresponding to regular expression $(a+b)^* + (a+b)^*$ can be represented by context free grammar.

(c) The language corresponding to regular expression has been defined below as follows:

$L = \{ww \mid w \in (a+b)^*\}$

This language is context free language and this language is recursively enumerable language. Since recursively enumerable language is closed under union, this language can be represented by context free grammar.

(d) Because the recursively enumerable languages of arbitrary length are not recursive, the language can be represented by context free grammar.

8.7 (a)

S → aSb | ab

S → aSb | ab

S → aSb | ab

S → aSb | ab

Since the LHS of the production $S \to aSb$ contains more than one variable, it is not context free grammar. For example in $aSbS \to aaSbbS$, the production has length of LHS, 4 greater than length of RHS. Hence it is not the production for context free grammar. It is all the productions for context free grammar. Hence it is type 1 (CSG).

8.8 (b)

(a) L_1 is context free language and type 1 language. Then $L_1 \cap L_2$ is context free language because context free languages are closed under intersection.

(b) Since context free languages are not closed under complement, $L_1 - L_2$ is not context free language and not type 1 language.

(c) L_1L_2 is context free language and L_1 is regular language. Since context free languages are closed under regular concatenation.

(d) $L_1 \cup L_2$ is context free language. The language $L_1 \cup L_2$ is surely context free language because context free languages are closed under union.

8.9 Sol:

No. of variables are

$V = \{A, B\}$

Turing S and T are all in every possible combination, we get

(a) $S \to ATA \mid ATC \mid BAC \mid BAC \mid AC \mid C \mid BC$

$A \to aA \mid a$

$T \to bT \mid b$

$C \to c$

The above equivalent grammar of C contains unit production.

$S \to C$

Move S → C with C → c and put S → c.

Since unit production is not allowed

$S \to c$

So eliminate unit production without production rules and unit rules

G: $S \to ATA \mid ATC \mid BAC \mid BAC \mid AC \mid c \mid BC$

$A \to aA \mid a$

$T \to bT \mid b$

$C \to c$

2.10 (b)

The language

$L = \{wcw^R \mid w \in \{a, b\}^*\}$

is accepted by DPDA (finite state machine with a stack) by choosing the middle symbol in the machine (using the stack) and so it is DCFL. For other languages, PDA appears to be nondeterministic and hence the stack is used to keep track of the symbols which appear before the middle symbol and then these symbols are matched with top of stack and popped. At the end of the string, the stack is empty and accepted. Thus, the initial stack symbol Z_0 is popped.

The DPDA for above language is

2.11 (d)

(a) Regular languages are closed under union. So $L_1 \cup L_2$ is regular.

(b) Let $\Sigma = \{a, b\}$. Σ^* is regular so

$L_1 = \{w \mid w \in \Sigma^*\}$

$L_2 = \{w \mid w \in \Sigma^*\}$

Here, L_1 & L_2 are regular over Σ^*

$L_1 \cap L_2 = \{a^n b^n \mid n \geq 0\}$

However, L_1 is CFL and L_2 is regular so by closed under intersection, intersection of regular and CFL need not be regular.

(c) Regular languages are closed under intersection. So the intersection of two regular sets is regular.

2.12 (d)

Statement (a) is regular and closed under union and kleene closure and not closed under intersection and complement.

2.13 (a)

The machine can operate as a mere PDA accepting strings:

(i) Acceptance by empty stack

(ii) Acceptance by final state

Since there exists a procedure to convert PDA accepting by final state to PDA accepting by empty stack and vice versa. So acceptance or recognition power of PDA of both approaches are same. Therefore $L_P = L_R$

2.14 (a, d)

(a) Since CFL is closed under regular difference so $L_1 - L_2 = CFL - \text{regular} = CFL$. So option (a) is false.

(b) Since context free language is closed under reversal so $L = L_1 L_2 L_1^R L_2^R$ is CFL.

So option (b) is true.

(c) Context free language is not closed under complementation. So $\overline{L_1}$ is not context free language. So option (c) is false.

(d) Regular languages are closed under complementation. So $\overline{L_2}$ is regular.

So option (d) is true.

2.15 (b)

(a) False. CFL can be divided into two categories.

While language in other category cannot be accepted by a deterministic machine with one counter.

(b) True, since CFL is not closed under intersection, intersection of CFL is not CFL. Thus

(c) False. Since CFL is not closed under intersection and complement.

2.16 (d)

A PDA with a stack limited to containing 10 symbols is finite memory and hence equivalent to DFA. So the language accepted by a PDA which has stack limited to 10 symbols is described as regular.

2.17 (d)

The grammar $S \to aSbS \mid bSaS \mid \varepsilon$

(a) Given string is 3 letter string which belongs to L(G). But it has a number of derivation trees, which makes it ambiguous. Some derivation trees are given below.

(b) There exists a type 3 GG such that L(G) = L(G') where G' is a type 2 grammar. L(G) only if L(G').

(c) [illegible] $L(G) = \{w \mid n_a(w) = n_b(w)\}$ [illegible] which is equal to $\{a^n b^n\}$.

This language is [illegible] context free language.

∴ There exists a DPDA that accepts it.

(d) [illegible] given language is regular [illegible] difference between number of 'a's and number of 'b's is even.

∴ No DFA exists to accept it.

2.15 (a):

(a) This statement [illegible] context free.

(b) Always [illegible] is to solve [illegible] transformations.

(c) The set of all languages accepted by all NPDAs is [illegible] CFLs, which is a proper subset of the set of all languages accepted by all DPDAs, which is the set of DCFLs. Hence option (c) is false.

(d) Finite languages [illegible] regular. So true.

2.16 (b):

This machine accepts the language

$L = \{w \mid n_a(w) = n_b(w)\}$

This language allows only strings with equal number of a's and b's. Thus, the strings of the form $a^n b^n$ which is context free language.

Hence, the given machine accepts a CFL.

∴ This machine is equivalent to a PDA.

2.17 (a):

∴ [illegible]

(a) DPDA can accept this language. When a's and b's are in input, [illegible] stack and when a's appear in input [illegible] stack and popped out. After all the [illegible] stack is empty, [illegible] $n_a(w) = n_b(w)$ and the string is accepted. Also, it is [illegible]. Now since a DPDA [illegible] the language is DCFL and hence context free. Thus, the language is not regular, [illegible] count and compare [illegible] and is [illegible] not be done by any FA.

2.18 (b):

Draw the DFA of the given grammar as shown below:

Clearly, the machine accepts $\{a, ba, bba, \ldots\}$ where $a \in \{a, b\}$.

2.19 (d):

(a) $L_1 \cap L_2 = \{a^n b^n c^n\}$ = Regular (CSL) = Regular (CSL)^c = Regular (CSL) = CSL.

∴ CSL may or may not be regular. So [illegible] (a) and (b) are false.

(c) CSL is a subset of CFL. So option (c) is false. So, answer is (d) none of these.

2.20 (d):

In NFA, we may have a dead configuration. That means we may not [illegible] to any alphabet from this state.

We may have strings accepted in NFA ending in final state although [illegible] symbol of the string [illegible] symbol driving [illegible] of the string and [illegible] final state.

[illegible] Neither DFA nor NFA are [illegible] PDA.

Neither DFA nor NFA are [illegible] TM.

2.24 (a)

[illegible]

2.25 (a)

[illegible]

2.26 (b)

[illegible]

2.27 (d)

[illegible]

2.28 (b)

[illegible]

2.29 (b)

[illegible]

2.30 (c)

[illegible]

2.31 (a)

[illegible]

2.32 (b)

[illegible]

2.33 (b):

[illegible]

1. [illegible]

2. [illegible]
3. [illegible]

2.34 (c):

[illegible]

2.35 (d):

[illegible]

$S \to a, CB$

$B \to b$

$F \to i$

Since LHS of every production is a single nonterminal, it is clearly CNF.

[illegible]

$S \to a, CB$

$B \to b$

$L \to L, E$

$E \to E+F$

$F \to i$

[illegible] LHS of some of the productions in [illegible] not a single nonterminal.

2.36 (b):

[illegible] is a DCFL since a DFA can recognize the language. [illegible] and number of [illegible] but [illegible] is impossible [illegible] stack symbol only. [illegible] accepted. [illegible] is rejected. [illegible] DCFL [illegible] DCFL.

2.37 (d)

1. [illegible]
2. [illegible]
3. [illegible]
4. For every subset [illegible] number of [illegible] and state [illegible] NFA.

2.38 (c)

1. True
2. [illegible] productions can be removed from [illegible] CNF.
   [illegible] productions can be [illegible] CNF.
3. True
4. [illegible] in Chomsky normal form, [illegible] product [illegible]

A [illegible] parse tree generated by CNF according is shown below

2.39 (d)

[illegible]

[illegible] equal to [illegible] respectively.

The grammar $X \to X.X \mid X.X \mid X^*$ generates the arithmetic expressions with matched pairs of parentheses and the postfix operator.

The grammar $S \to bSb \mid b$ generates the palindrome strings.

2.10 (b) :

It is CFL (pushdown automata) and regular with [illegible]

It is CFL (pushdown automata) and prefix with each [illegible]

It is CFL every string [illegible]

It is CFL every string has [illegible]

So, $L_1 \cap L_2 = \phi$, the option (b) is true

Union of two CFL is a CFL and so option (c) is false

[illegible] CFL and hence decidable [illegible]

So option (d) is false

2.11 (b) :

$S \to [illegible]$

$D \to [illegible]$

So, S generates $\{0^n 1^n \mid n \geq 1\}$

$S \to aA$

$A \to bA \mid bC$

$A \to [illegible]$

So, S generates $\{0^p 1^q 0^r \mid min\ p, q > 1\}$

$= \{0^p 1^q 0^r \mid m, n, q \geq 0\}$

$= \{0^m 1^n 0^k \mid i, j, k \geq 0\}$

$L(S) = \{0^i 1^j 0^k \mid i, j, k \geq 0\} \cap \{0^i 1^j 0^k \mid i \geq 1\}$

Which is same as option (b)

2.12 (d) :

$S \to aS$

$S \to bA$

$S \to [illegible]$

$S \to a|bAb$

$S \to a|bA|aA$

$S \to [illegible]$

2.13 (a) :

$S \to aS$	1
$S \to bA$	2
$S \to aabab$	3
$S \to aabbabab$	4
$S \to aabbabab$	5
$S \to aabbaaa$	6

Thus, a parse tree can be constructed as shown below.

2.14 (b) :

[illegible]

So the grammar [illegible] length palindromes.

Notice that [illegible] strings [illegible] ends with the same [illegible] cannot be generated by above grammar.

[illegible] generated by this grammar are [illegible] odd length palindromes and [illegible]

2.15 (c) :

It is well known that [illegible] DCFL is a proper subclass of the set of CFLs. NPDA's which accept [illegible] must be class of CFL's [illegible] which [illegible]

2.16 (b) :

P=1, Q=1, R=1

2.17 (a) :

The transition diagram [illegible] TM is [illegible]

[illegible] length [illegible] machine [illegible] output 1, [illegible]

In fact [illegible] length [illegible] string [illegible] 101. When [illegible] while reading [illegible] 01.

2.32 (a)

$L_1 = \{a^i b^j c^k \mid i, j, k \geq 0\}$

$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$

$T_1 = \{a, b, c\}$

L_1 is CFL. L_2 is regular. First use closure property to get a CFL.

$L = L_1 \cap L_2 = CFL \cap Reg = CFL$

However, since a context free language is more complex than CFL cannot distinguish by closure property, we need to find based on concept here.

Any string belonging to L data must have equal number of a's, b's and c's. This can be checked by a single PDA by matching the only one allowed by each L_1 and L_2.

i.e. $L = L_1 \cap L_2 = \{a^n b^n c^n\}$

Now this is clearly a context sensitive, but not a CFL.

2.33 (a)

$L_1 = \{a^i b^j c^k \mid i = j\}$

$L_2 = \{a^i b^j c^k \mid j = k\}$

$L_3 = \{a^i b^j c^k \mid i = k\}$

$L = \{a^n b^n c^n \mid n \geq 0\}$

All of L_1, L_2, L_3 and L are context free, since L_1, L_2 and L_3 are simple enough to produce a PDA. However, L requires two comparisons between i and j and j and k and cannot be performed in a PDA.

2.34 (b)

Since L_1 involves the comparison of p and q, it cannot be context free.

Statement (b) is more [illegible]

2.35 (a)

$L_1 = \{a^p b^q \mid p, q \geq 0\}$ regular language

$L_2 = \{a^p b^q c^r \mid p, q, r \geq 0, p = r\}$ CFL

(a) $L_3 = CFL \cap CFL$

(a) $L_3 = L_1$ [illegible] from because CFL [illegible] regular intersection.

(b) Complement of the intersection

$L_3 = \overline{CFL_1 \cap CFL_2} = CFL_1 \cup CFL_2$

Now every CFL is RECURSIVE so that complement [illegible]

(c) Complement [illegible] to convert [illegible] regular language [illegible] and under complement [illegible] complement [illegible] regular.

2.36 (b)

L_1 = [illegible] DCFL

L_2 = [illegible] L_1 [illegible]

L_3 = [illegible] DCFL

2.37 (b)

The language accepted is

$L(M) = \{a^n b^n \mid n \geq 0\}$ which is the set of complement of L.

Only accepting DFA [illegible] the language and hence will be accepted.

The following transitions have [illegible] will be [illegible] in the DFA

[illegible]	[illegible]
[illegible]	[illegible]

[illegible] (b)

[illegible]

2.51 (d) :

[illegible]

2.55 (d) :

G : [illegible]

G_p : [illegible]

G_1 : [illegible]

Now substitute in $S \to [illegible]$

We get $S \to [illegible]$

So, $L(G) = [illegible]$

$G_1 = [illegible]$

Substitute in $A \to [illegible]$

$\to A \to [illegible]$

Now substitute A and B in $S \to [illegible]$

$S \to [illegible]$

So $L(G_1) = [illegible]$

So correct answer is choice (d).

2.56 (d) :

The Turing machine accepts $[illegible]$ in all strings. On [illegible] it will be rejected.

The language accepted is $[illegible]$

So, $L = \{[illegible] \mid [illegible] \geq 1\}$

$= \{[illegible] \mid [illegible] \geq 0\}$

Clearly this is a context regular CFL and hence it [illegible] in CFL.

2.57 (b) :

L_1 has [illegible] comparisons and hence is CFL but L_2 has 3 comparisons and hence not CFL. So $L_1 \cap L_2$ is CFL.

So [illegible] is correct.

2.58 (d) :

G_1 : [illegible]

G_2 : [illegible]

$L_1 = L(G_1) = \{[illegible]\}$

$L_2 = L(G_2) = \{[illegible]\} = \{[illegible]\}$

Clearly L_1 is regular as L_2 is regular. So Turing machine [illegible] is decidable.

2.59 (b) :

[illegible] variable dependency graph.

So only [illegible] variable.

■■■■

3 Turing Machine: RE, REC and Undecidability

3.1 Which of the following problems are undecidable?

(a) Membership problem for context free language

(b) Whether a given context free language is regular

(c) Whether a Turing machine halts on all inputs

(d) Membership problem for context free languages

[1999 : 2 Marks]

3.2 Turing machines are

(a) more powerful than finite automata but less powerful than PDA

(b) Turing machines are equivalent to PDA

(c) Turing machines are equivalent to finite automata

(d) None of the above

[1999 : 2 Marks]

3.3 Decision problem for context free languages are

(a) equivalence of context free languages

(b) always recursively decidable

(c) also called type 0 languages

(d) recognizable by Turing machines

[1998 : 2 Marks]

3.4 Which one of the following statements is true?

(a) Turing machine recognizable languages are closed under union and complement

(b) Turing machine recognizable languages are closed under intersection

(c) Turing machine recognizable languages are closed under complement

(d) Turing machine recognizable languages are closed under union

[1997 : 2 Marks]

3.5 Which of the following statements is false?

(a) Regular grammar is context free grammar

(b) Non deterministic PDA is deterministic PDA

(c) Non deterministic PDA is deterministic PDA

(d) Multi tape Turing machine is equivalent to single tape Turing machine

[1994 : 2 Marks]

3.6 Which of the following statements are possible for a recursive language?

(a) Regular grammar is context free grammar

(b) Non deterministic PDA is deterministic PDA

(c) Non deterministic PDA is deterministic PDA

(d) Multi tape Turing machine is equivalent to single tape Turing machine

[1994 : 2 Marks]

3.7 Which of the following statements are true?

(a) The halting problem for Turing machine is undecidable.

(b) Determining whether context free grammar is ambiguous is undecidable.

(c) Given two context free grammars G_1 and G_2, it is undecidable whether $L(G_1) = L(G_2)$

(d) Given two regular grammars G_1 and G_2, it is undecidable whether $L(G_1) = L(G_2)$

[1996 : 1 Mark]

3.8 Which one of the following is not decidable?

(a) Given a Turing machine M, a string w and an integer k, M accepts w in k steps

(b) R_1 is a subset of R_2 for regular expressions

(c) Language accepted by a given finite state machine is empty

(d) Language accepted by a given context free grammar is empty

[1997 : 2 Marks]

3.9 Regarding the language accepted by finite automata, which of the following statements is false?

(a) Non deterministic finite automata are equivalent to deterministic finite automata

(b) Non deterministic pushdown automata are equivalent to deterministic pushdown automata

(c) Non deterministic Turing machines are equivalent to deterministic Turing machines

(d) Multi tape Turing machines are equivalent to single tape Turing machines

[1996 : 1 Mark]

8.20 Which of the following is true?

(a) The complement of a recursive language is recursive

(b) The complement of a recursively enumerable language is recursive

(c) The complement of a recursive language is recursively enumerable

(d) The complement of a context-free language is recursive

[2008 : 2 Marks]

8.21 The Chomsky hierarchy

(a) Includes Turing machines

(b) Includes context-free languages

(c) Includes regular languages

(d) Includes finite state automata

[2002 : 2 Marks]

8.18 Consider the following problems

P_1: Does a given finite state machine accept a given string?

P_2: Does a given context-free grammar generate a given string?

Which of the following statement is true?

(a) P_1 and P_2 are decidable

(b) Neither P_1 nor P_2 are decidable

(c) Only P_1 is undecidable

(d) Only P_2 is undecidable

[2002 : 2 Marks]

8.19 Consider the following Problem X

Given a Turing machine M over {0, 1} and a word w over {0, 1}, does the Turing machine M halt on the input w?

Which of the following statements about X is correct?

(a) X is decidable

(b) X is undecidable but partially decidable

(c) X is undecidable and not even partially decidable

(d) X is a halting problem

[2003 : 2 Marks]

8.14 It is known that P ≠ NP. Consider the language L defined as follows

$$L = \begin{cases} \{0, 1\}^* & \text{if } P = NP \\ \phi & \text{otherwise} \end{cases}$$

Which of the following statement is true?

(a) L is recursive

(b) L is recursively enumerable but not recursive

(c) L is context-free but not recursive

(d) Whether L is recursive or not is not known at present even if P = NP

[2003 : 1 Mark]

8.15 The strings of language L are recursively enumerable. A recursive language is a language whose strings are accepted by a Turing machine. Which of the following statements is true?

(a) L is necessarily finite

(b) L is regular but not necessarily finite

(c) L is context-free but not necessarily finite

(d) L is recursive but not necessarily finite

[2003 : 1 Mark]

8.16 A single-tape Turing Machine M has two states q_0 and q_1, of which q_0 is the initial state. The tape alphabet of M is {0, 1, B}. The input alphabet is {0, 1}. The symbol B is the blank symbol used to indicate end of input string. The transition function of M is defined in the following table.

	0	1	B
q_0	$(q_1, 1, R)$	$(q_1, 1, R)$	$(H, 1, L)$
q_1	$(q_1, 0, R)$	$(q_0, 1, L)$	(q_1, B, L)

The table entry $(q_1, 1, R)$ in row q_0 and column 1 means that if M is in state q_0 and reads 1 on the current tape cell, then it will write 1 on the same tape cell, move one cell to the right and go to state q_1. Which of the following statements is true about M?

(a) M does not halt on any string in $(0 + 1)^*$

(b) M halts on all strings in $(0 + 1)^*$

(c) M halts on all strings ending in 0

(d) M halts on all strings ending in 1

[2003 : 2 Marks]

8.17 Consider the decision problems P_1, P_2 and P_3. P_1 is known to be decidable and P_2 is undecidable. Which one of the following is TRUE?

(a) P_3 is decidable if P_1 is reducible to P_3

(b) P_3 is undecidable if P_3 is reducible to P_2

(c) P_3 is undecidable if P_2 is reducible to P_3

(d) P_3 is decidable if P_3 is reducible to P_2's complement

[2003 : 2 Marks]

3.18 Let L_1 be a recursive language and let L_2 be a recursively enumerable but not recursive language. Which one of the following is TRUE?

(a) $\overline{L_1}$ is recursive and $\overline{L_2}$ is recursively enumerable

(b) $\overline{L_1}$ is recursive and $\overline{L_2}$ is not recursively enumerable

(c) $\overline{L_1}$ and $\overline{L_2}$ are recursively enumerable

(d) $\overline{L_1}$ is recursively enumerable and $\overline{L_2}$ is recursive

[2002 : 2 Marks]

3.19 Assume that $\Sigma = \{a, b\}$ and the alphabet of the language is defined as follows:

$L = \{a^n b^n \mid n \geq 0\}$ | distinct $n = 2$ and distinct $m \geq 4\}$

Which one of the following statements is true?

(a) L is recursively enumerable but not recursive

(b) L is recursive, but not context free

(c) L is context free but not regular

(d) L is regular

[2006 : 2 Marks]

3.20 Let L_1 be a regular language, L_2 be a deterministic context free language and L_3 be a context free language, but not recursive, language. Which one of the following statements is false?

(a) $\overline{L_1}$ is a deterministic CFL

(b) $L_1 \cap L_2$ is recursive

(c) $L_2 \cap L_3$ is a context free

(d) $L_1 \cup L_2 \cup L_3$ is recursively enumerable

[2006 : 2 Marks]

3.21 Which of the following problems is undecidable?

(a) Membership problem for CFL

(b) Ambiguity problem for CFG

(c) Finiteness problem for FSA

(d) Equivalence problem for FSA

[2007 : 1 Mark]

3.22 Which of the following statements is false?

(a) It is decidable whether a context free language is empty or not

(b) It is decidable whether a context free language is finite or not

(c) It is decidable whether a context free language is regular or not

(d) It is decidable whether a context free language is context free or not

[2008 : 1 Mark]

3.23 Which of the following are undecidable?

1. Whether the intersection of two regular languages is infinite or not
2. Whether a given context free language is regular
3. Whether two pushdown automata accept the same language
4. Whether a given grammar is context free

(a) 2 and 3 (b) 1 and 4

(c) 3 and 4 (d) 2 and 4

[2008 : 1 Mark]

3.24 Let L_1 be a recursive language. Let L_2 and L_3 be languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

(a) $L_2 \cap L_3$ is recursively enumerable

(b) $L_1 - L_2$ is recursively enumerable

(c) $L_2 - L_1$ is recursively enumerable

(d) $L_2 \cap \overline{L_3}$ is recursively enumerable

[2010 : 1 Mark]

3.25 Which of the following pairs of DFA/PDA/REGEXP expressions proved equivalent?

(a) Deterministic finite automata (DFA) and Non-deterministic finite automata (NFA)

(b) Deterministic pushdown automata (DPDA) and Non-deterministic pushdown automata (NPDA)

(c) Deterministic finite state Turing machine and Non-deterministic single state Turing machine

(d) Single tape Turing machine and Two stack Turing machine

[2011 : 1 Mark]

3.26 Which of the following statements is/are correct?

1. If L is a context free language, then is L always recursive?
2. If L is a recursive language, then is $\overline{L}$ always recursive?
3. If L is a regular language, then is $\overline{L}$ also regular?
4. If L is a recursive language, then is $\overline{L}$ also recursive?

(a) 1, 2, 3, 4 (b) 1, 2

(c) 2, 3, 4 (d) 3, 4

[2013 : 1 Mark]

3.32 [illegible]

1. [illegible]
2. [illegible]
3. Turing decidable languages are closed under union and complementation
4. Turing recognizable languages are closed under union and intersection

(a) 1 and 4 only (b) [illegible]
(c) [illegible] (d) [illegible]

[2013 : 1 Mark]

3.29 Which one of the following statements is true?

1. G is CFG. Is $L(G) = \Sigma^*$?
2. G is CFG. Is $L(G) = \Sigma^*$?
3. M is Turing machine. Is $L(M)$ regular?
4. [illegible]

(a) 3 only (b) 3 and 4 only
(c) [illegible] (d) 2 and 3 only

[2013 : 2 Marks]

3.30 Let L be a language and $\overline{L}$ be its complement. Which one of the following is NOT a valid possibility?

(a) Neither L nor $\overline{L}$ is recursively enumerable (re).
(b) One of L and $\overline{L}$ is re but not recursive; the other is not re.
(c) Both L and $\overline{L}$ are re but not recursive.
(d) Both L and $\overline{L}$ are recursive.

[2014 (Set 1) : 1 Marks]

3.31 Let M be a Turing machine accepting the language L on the alphabet $\Sigma = \{0, 1\}$. Let it be given that L is infinite and M accepts every string in $\{0, 1\}^*$. Then, L is

(a) decidable and recursively enumerable
(b) undecidable but recursively enumerable
(c) undecidable and not recursively enumerable
(d) decidable but not recursively enumerable

[2014 (Set 2) : 1 Marks]

3.32 Let L_1, L_2, L_3 be three languages such that L_1 is recursively enumerable, L_2 is recursive, and L_3 is neither recursive nor recursively enumerable. Which one of the following is NOT a valid statement?

(a) $L_1 \cap L_2$ is recursively enumerable
(b) $L_1 \cap L_3$ is recursively enumerable
(c) $L_2 \cap L_3$ is recursively enumerable
(d) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

[2014 (Set 1-Set 2) : 1 Mark]

3.33 Let Σ be a finite set of symbols and let Σ^* be the power set of Σ. Which one of the following is TRUE?

(a) Both Σ^* and Σ are countable
(b) Σ^* is countable and Σ is uncountable
(c) Σ is countable and Σ^* is uncountable
(d) Σ and Σ^* are uncountable.

[2014 (Set-3) : 1 Mark]

3.34 Which one of the following problems is undecidable?

(a) Deciding if a given context-free grammar is ambiguous.
(b) Deciding whether a context-free language generated by a given context-free grammar is empty.
(c) Deciding if the language generated by a given context-free grammar is empty.
(d) Deciding if a given context-free language is regular.

[2014 (Set-3) : 2 Marks]

3.35 For any two languages L_1 and L_2 such that L_1 is a subset of L_2, consider the following statements and recursively enumerable (re) and recursive (r). Which one of the following statements is true?

1. $\overline{L_1}$ (complement of L_1) is recursive
2. $\overline{L_2}$ (complement of L_2) is recursive
3. $\overline{L_1}$ is recursive
4. $L_2 - L_1$ is recursively enumerable

(a) 1 only (b) 4 only
(c) 3 and 4 only (d) 2 and 4 only

[2015 (Set-1) : 1 Mark]

8.86 Consider the following statements

1. [illegible] Turing decidable [illegible] decidable.
2. There exists some language which is in NP but is not Turing decidable.
3. If L is a language in P, then L is Turing decidable.

Which of the above statements are true?

(a) Only 1 (b) [illegible]
(c) Only [illegible] (d) [illegible]

[ME-II (Sec-2): 1 Mark]

8.87 Let X be a recursive language and Y be a recursively enumerable language. Let L and Z be two languages such that X reduces to L and Z reduces to Y. Which of the following statements is/are TRUE?

(a) If L is recursively enumerable and Z is recursive.
(b) If L is recursive and Z is recursively enumerable.
(c) If L is recursively enumerable and Z is recursive.
(d) If L is recursively enumerable and Z is recursively enumerable.

[2016 (Sec-2): 2 Marks]

8.88 Consider the following types of languages L_1: Regular, L_2: Context-free, L_3: Recursive, L_4: Recursively enumerable.

Which of the following are TRUE?

I. $L_1 \cap L_2$ is recursively enumerable.
II. $L_2 \cap L_3$ is recursive.
III. $\overline{L_3} \cap L_4$ is recursive.
IV. $L_1 \cap L_2 \cap L_3$ is context-free.

(a) I only (b) [illegible] III only
(c) I and II only (d) [illegible]

[2016 (Sec-2): 1 Mark]

8.89 Consider the following languages

- $L_1 = \{M \mid M$ halts on at least one input$\}$
- $L_2 = \{M \mid M$ halts on input 2016 given all inputs$\}$
- $L_3 = \{M \mid M$ accepts $\}$

where M is a Turing machine. $\langle M \rangle$ denotes a string representing of M. Which one of the following is/are TRUE?

(a) L_1 is recursive and L_2, L_3 are not recursive.
(b) L_2 is recursive and L_1, L_3 are not recursive.
(c) L_1, L_2 are recursive and L_3 is not recursive.
(d) L_1, L_2, L_3 are recursive.

[2016 (Sec-2): 2 Marks]

■■■■

Answers Turing Machine, RE, REC and Undecidability

8.1	(b, d)	8.2	(b, c)	8.3	(a, d)	8.4	(c)	8.5	(a, c)	8.6	(a)	8.7	(c)						
8.8	(b)	8.9	(b)	8.10	(c)	8.11	(b)	8.12	(a)	8.13	(b)	8.14	(a)	8.15	(b)	8.16	(a)		
8.17	(a)	8.18	(b)	8.19	(d)	8.20	(b)	8.21	(b)	8.22	(d)	8.23	(d)	8.24	(b)	8.25	(b)		
8.26	(b)	8.27	(d)	8.28	(c)	8.29	(d)	8.30	(c)	8.31	(b)	8.32	(d)	8.33	(c)	8.34	(a)		
8.35	(d)	8.36	(d)	8.37	(c)	8.38	(d)	8.39	(c)										

Explanations Turing Machines, RE, REC and Undecidability

8.1 (Bonus)

(a) Decidable Membership [illegible] CFG. [illegible]

(b) Undecidable Regularity of context free language [illegible] CFG is regular.

(c) Decidable Finite state automata [illegible] the answer is always yes.

(d) Undecidable Type 0 language [illegible] membership [illegible] problem.

8.2 (Bonus)

(a) [illegible] within [illegible] steps.
This can be decided by simulating the TM [illegible] steps.

(b) [illegible] specific [illegible] input. State entry [illegible] This problem [illegible] problem also become undecidable.

(c) It is undecidable whether a given language [illegible] the product of two numbers. Indeed, this [illegible] of all decide whether the given TM accepts [illegible]. But only halting [illegible] is recursively enumerable.

(d) It is undecidable.

8.3 (a, d)

(a) True. Every CFL is a context free language whereas there exists context free languages which are not CFL for example context free language [illegible] a proper superset of context free language.
$L_1 \subseteq L_2$

(b) False. Since there exist some undecidable languages which are not context free. So [illegible] recursively enumerable languages [illegible].

(c) False. Because context free language is called type 2 language.

(d) True. Type 0 language are recursively enumerable and recognizable by a turing machine.

8.4 (b)

Every finite language over some finite alphabet is regular because it only finite language. DFA or NFA can be created by brute force.

8.5 (a, d)

(a) There exists no algorithm to convert every NFA to DFA. Because in general NFA accepts infinite [illegible].

(b) Since there exists no equivalence between NFA and DFA and recognition power of both are same.

(c) [illegible] power of NFM [illegible] determinism [illegible] NFM [illegible] deterministic turing machine [illegible].

(d) Since [illegible] machines and finite automata [illegible] statement is true [illegible] not true.

8.6 (a)

(a) Every regular language [illegible] problem. So it is trivially decidable.

(b) [illegible] CFG [illegible] is PDA since the recognition power is same.

(c) Since the recognition power of non-deterministic PDA is [illegible] PDA, some [illegible] ambiguous [illegible] by non-deterministic PDA but not by deterministic PDA. So there exists no algorithm for converting non-deterministic PDA to deterministic PDA.

(d) [illegible]

8.7 (d)

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

8.8 (b)

(a) Given a Turing machine [illegible] machine (UTM).

(b) [illegible] problem.

(c) [illegible] problem. [illegible] (PCP)

(d) Emptiness of CFL is decidable problem.

8.9 (b)

(a) [illegible] NFA [illegible] DFA. [illegible] NFA and DFA is same.

(b) [illegible] NFA to DFA and [illegible]

by NFA but not by DFA [illegible] NFA and DFA are same.

Example:

[illegible] by NFA [illegible] by DFA.

(c) [illegible] power of NFA [illegible] DFA.

(d) [illegible] machine [illegible] machine [illegible] power of [illegible]

8.10 (a)

The [illegible] implementation. [illegible] implementation.

So [illegible] decidable.

8.11 (a) :

The [illegible] machine [illegible] machine (UTM).

(a) [illegible] decidable [illegible]

(b) The [illegible] decidable.

8.12 (a)

[illegible] of DFA [illegible] language [illegible] and [illegible] are decidable [illegible].

R_1 and R_2 are not decidable.

8.13 (b)

Problem [illegible] problem. [illegible] machine [illegible] problem. [illegible] problem of [illegible] machine [illegible] decidable. [illegible] machine [illegible] decidable. [illegible] machine [illegible] decidable but partially [illegible]

8.14 (b)

[illegible]

8.15 (d)

The [illegible] is undecidable [illegible]

[illegible]

8.16 (d)

[illegible]

[illegible]

8.17 (d)

[illegible]

8.18 (b)

[illegible]

8.19 (b)

[illegible]

8.20 (b)

[illegible]

1. [illegible]
2. [illegible] $L_2 = L_1$ is RE.

[illegible]

[illegible]

3.81 (b):

[illegible]

3.82 (a):

[illegible]

3.83 (d):

[illegible]

3.84 (d):

[illegible]

3.85 (d):

Given L_1 is RE and L_2 and L_3 are RE but not REC.

Consider

$L_1 \cap L_2 = L_3 \cap L$

$= (RE \cap RE) \cap (RE \cap RE)$

$= (RE \cap RE) \cap (RE \cap RE)$

$= RE \cap RE$

$RE \cap RE$

$= RE$

Choose option.

Choice (a): $L_1 \cap L_2 = L_3 \cap L_4$

$= RE \cap RE = RE \cap RE$

Now the complement of a RE language is CO-RE language, so it is not RE.

$\therefore L_1 = RE \cap RE$ but not RE or CO-RE

$\therefore$ Choice (a) is false.

Choice (b):

$L_1 \cap L_2 = (RE \cap RE)$ but not RE or CO-RE

$= RE \cap RE = RE$

So choice (b) is true.

Choice (d):

$L_1 \cap L_2 = (RE \cap RE)$ but not RE or CO-RE

$= RE \cap RE = RE$

$\therefore$ Choice (d) is true.

3.86 (b)

$TM = NTM$

$DPDA \subset NPDA$

$DFA = NFA$

Since $DFA = NFA$ and $DPDA \subset NPDA$.

3.87 (a):

[illegible]

Complement of accepted language may or may not be undecidable. [illegible]

[illegible] is regular, $\bar{L}$ is also regular.

Because [illegible] number of [illegible] decidable.

[illegible] $\bar{L}$ is also integer [illegible]

[illegible] decidable.

So correct option is (a) [illegible] decidable.

8.28 (d):

1. Non-deterministic finite automata can be given finite state machine equivalent to deterministic finite automata. [illegible] initial state [illegible] transition function [illegible] finite
2. Turing machine to [illegible] type-0 (RE) language [illegible] recognizable [illegible] complement is not decidable
3. [illegible] decidable [illegible] CFG [illegible] and complement [illegible]
4. [illegible] recognizable [illegible] CFG language [illegible] intersection [illegible]

So option (d) is correct.

8.29 (d):

1. [illegible]
   This is the [illegible] problem of CFG which is decidable.
2. [illegible] is Σ*.
   This is the [illegible] problem of CFG which is undecidable.
3. [illegible] machine is [illegible] regular.
   This is a regularity problem of Rice's theorem which is undecidable.
4. [illegible] = L(M1) = L(M2)
   This is the equivalence problem of regular language which is decidable.

So only 2 is undecidable.

8.30 (d):

Option (a): [illegible] is not [illegible] possible [illegible] and [illegible] would become [illegible] the L.E. Fermat.

Option (b): [illegible] by the [illegible] theorem.

8.31 (d):

When a TM accepts a finite language it is type-0 [illegible] and hence [illegible] is the [illegible] recognizable.

But the problem [illegible] is accurately enumerable [illegible] given input [illegible] condition.

8.32 (d):

If A is a CFL then DT → A is RE is a one way function, which is true.

Option (d): [illegible] RE [illegible] not RE [illegible] inverse of [illegible] function [illegible] false.

8.33 (c):

Cantor's Diagonal Theorem: The set of all [illegible] is countably infinite [illegible] countably infinite.

Now, 2^N is countably infinite. So by Cantor's Diagonal Theorem, 2^N is uncountably infinite.

8.34 (a):

Ambiguity problem of context free grammar is undecidable.

8.35 (d):

L1 is CFL → L1 = $\overline{CFL}$ = RE L = DF.

L2 is RE → $\overline{RE}$ = L2 is not DF

Statement 1: L1 is recursive [illegible] complement of CFL is always CSL and hence recursive.

Statement 2: L2 is recursive [illegible] false because L2 is not DF.

Statement 3: L2 is CFL is false [illegible] not CFL.

Statement 4: L2 is [illegible] false because [illegible] = RE [illegible] = CFL [illegible] = RE = RE [illegible] L2 = DF.

So only statements 1 and 4 are true.

8.36 (d):

Statement 1: [illegible] decidable languages are closed under complementation.

Statement 2: False, because L ⊆ NP ⊆ PSPACE.

Statement 3: [illegible] language is [illegible] PSPACE.

Statement 4: True, [illegible] NP is decidable.

3.37 (a):

$X \to RE$

$Y \to RE$ but not REC.

$Y \leq X$ and $X \leq Y$

Since we know that REC, RE, CFL are closed under reduction.

Now, if Y is RE but not REC, then $\overline{Y}$ is not RE.

Now $\overline{Y} \leq Y$

If X is RE $\Rightarrow \overline{Y}$ is RE

Contrapositive: If Y is not RE $\Rightarrow \overline{Y}$ is not RE

Now $X \leq \overline{Y}$

If X is REC, $\overline{Y}$ is also REC

So $\overline{Y}$ is REC $\Rightarrow X$ is REC

So Y is not REC and X is RE

Choice (a) is correct.

3.38 (a):

$L_1 \to$ Decidable

$L_2 \to$ CFL

$L_3 \to$ RE

$L_4 \to$ RE

$L_5 \to L_1 \to$ RE

$$\overline{L_5} \cup L_4 = \overline{RE} \cup RE = RE \cup RE$$
$$= RE \cap RE = RE$$

So I is true.

II. $\overline{L_5} \cup L_2$ is recursive

$$\overline{L_5} \cup L_2 = \overline{CFL} \cup REC = \overline{REC} \cup REC$$
$$= \overline{REC} \cup REC$$
$$= REC \cup \overline{REC} = REC$$

So II is true.

III: $\overline{L_1} \cup L_3$ is CFL.

$$\overline{L_1} \cup L_3 = \overline{Decidable} \cup RE$$
$$= REC \cap \overline{CFL} = CFL$$

So III is true.

IV: $L_1 \cup L_2$ is CFL

$$L_1 \cup L_2 = REC \cup \overline{CFL} = REC \cup \overline{REC}$$
$$= REC \cup CFL = CFL$$

Since $L_1 \cup L_2$ will not be a CFL, so IV is false.

So only I, II and III are true.

3.39 (a):

$$L_5 = \{a^n \mid n \in GCD(p, q)\}$$

Since this language is a non-trivial property of RE language, it is undecidable by Rice's theorem.

So L_5 is not recursive.

L_1, L_2 are recursively enumerable since $TM(L)$ is RE and RE is closed under concatenation and a^n is a CFL. Hence, L_1, L_2 are RE.

■■■■

Unit - III

Digital Logic

■ **Contents**

Sl. No.	Topic	Page No.
1.	Logic Functions and Minimization	62
2.	Combinational Circuits	138
3.	Sequential Circuits	184
4.	Number Systems	105

UNIT III Digital Logic

Syllabus : Logic functions, Minimization, Design and synthesis of combinational and sequential circuits, Number representation and computer arithmetic (fixed and floating point).

Analysis of Previous GATE Papers

Exam Year	1 Mark Quiz	2 Marks Quiz	3 Marks Quiz	5 Marks Quiz	Total Marks
1990		8		3	31
1991				2	12
1992					
1993					
1994					
1995					
1996	3	4		2	21
1997	4	3			7
1998		2			
1999	2	2			8
2000	1	4			9
2001	1	3			7
2002	4	4		1	17
2003	1	4			9
2004	4	6			14
2005	4	4			12
2006	1	5			11

Exam Year	1 Mark Quiz	2 Marks Quiz	Total Marks
2007	3	3	10
2008	4	2	9
2009	2	1	4
2010	3	2	7
2011	2	3	8
2012	2	1	4
2013	3	1	5
2014 Set-1	2	1	4
2014 Set-2	3	1	5
2014 Set-3	2	2	6
2015 Set-1	1	2	5
2015 Set-2	1	3	7
2016 Set-1	2	3	8
2016 Set-2	2	2	6
2017 Set-1	2	2	7
2017 Set-2	3	2	7

1 Logic Functions and Minimization

1.1 Let x and y be two Boolean variables and $x \oplus y = xy + \bar{x}\bar{y}$. Then $x \oplus y$ is equivalent to

(a) $x + y$ (b) $\bar{x} + \bar{y}$

(c) x (d) 1

[1997 : 1 Mark]

1.2 Let $f(x, y, z) = \sum(1, 3, 4, 6)$ be a Boolean function. Which one of the following is a prime implicant of f?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1997 : 2 Marks]

1.3 [illegible] the logic circuit shown in the figure below. The function [illegible] of minimum delay in the circuit is equivalent to

(a) [illegible]

(b) $f(w, x, y, z) = \sum(0, 8, 12, 14, 15)$

(c) [illegible]

(d) [illegible]

The minimum delay is

(a) 20, 10 (b) 30

(c) 30, 20, 10 (d) 20, 10, 10

[1997 : 2 Marks]

1.4 The simplified expression for the Boolean function $f(x, y, z)$ is

$$x + y + z + 1$$
$$xy + 0$$
$$xz + xz + 1$$
$$xz + xz + 0$$

Now the following statements are made in respect of f:

(a) 0000 (b) 1001

(c) 1011 (d) 1101

[2000 : 2 Marks]

1.5 Given the following Karnaugh map, which one of the following represents the minimal sum of products of the map?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2001 : 1 Mark]

1.6 Minimum SOP for $f(w, x, y, z)$ corresponding to the sample map below is

(a) $xz + xy$ (b) $xz + xy$

(c) $xy + xz$ (d) None of these

[2002 : 1 Mark]

1.7 Consider the following circuit with inputs x and y and output f. Given that

$f_1(x, y, z) = \sum(0, 1, 3, 4)$

$f_2(x, y, z) = \sum(0, 6, 7)$ and

$f(x, y, z) = \sum(0, 1, 3)$

(a) [illegible] (b) [illegible]

(c) $\sum(0, 1, 4, 6)$ (d) None of these

[2003 : 2 Marks]

1.8 $f(x, y) = xy + 1$. Simplified expression for $f(x, y, z)$ is

(a) $x + y$ (b) xyz

(c) $xy + z$ (d) None of these

[2003 : 2 Marks]

1.9 The Boolean expression for the output of the multiplexer in the figure is

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2000 : 2 Marks]

1.10 Consider the combinational circuit composed of CMOS gates and [illegible]

The [illegible] buffer has delay $t_d = 2$ ns, $X = 4$ ns as shown in the figure. Both NOR gates and all wires have zero delay. Assume that all gate outputs and wires are initially at logic level 0 at time 0. The following waveform [illegible] is the [illegible] interval from 0 to 5 ns

(a) 1 (b) 2
(c) [illegible] (d) 4

[2000 : 2 Marks]

1.11 The Boolean function $f(x, y) = x'y + xy'$ is equivalent to

(a) $x + y'$ (b) $x + y$
(c) $x + y'$ (d) $x' + y$

[2000 : 1 Mark]

1.12 What is the minimum number of NAND gates required to implement a 2-input exclusive-OR function without using any other logic gates?

(a) 3 (b) 4
(c) 5 (d) 6

[2004 : 1 Mark]

1.13 Evaluate the [illegible] $AB + AB' + ABC + ABC' + AB'C'$ in sum of products form.

(a) $AB + AC$ (b) $AB + AC + AB$
(c) $AB + AC$ (d) $AB + AC + AB$

[2004 : 2 Marks]

1.14 What are the essential prime implicants of the following Boolean function?

$$f(w, x, y, z) = w'z + w'x' + wx$$

(a) [illegible] and [illegible] (b) [illegible] and [illegible]
(c) [illegible] only (d) [illegible] and [illegible]

[2001 : 1 Mark]

1.15 [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these.

[2003 : 1 Mark]

1.16 A [illegible] is [illegible] and [illegible] in the CMOS position [illegible] is connected to [illegible] and [illegible] position [illegible] is connected to [illegible]. Two of these [illegible] are [illegible] and [illegible] are connected to [illegible] as shown below.

What function does this circuit perform?

(a) $S1 \cdot S2$ (b) $S1 + S2$
(c) $\overline{S1 + S2}$ (d) $S1 \oplus S2$

[2001 : 1 Mark]

1.17 Consider the following circuit.

What [illegible] the following is TRUE?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) Same as X, Y, Z is [illegible]

[2005 : 1 Mark]

1.18 The following [illegible] corresponding to [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 1 Mark]

1.19 Given two [illegible] and [illegible] and [illegible] in [illegible]. [illegible] represents the [illegible] and the [illegible] is

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

1.20 Consider [illegible] Suppose [illegible] the input is [illegible] [illegible] The function [illegible] [illegible] we would like the [illegible] to change from [illegible] to [illegible] [illegible] [illegible] Which of the following [illegible] will ensure [illegible] the [illegible] property [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

1.21 We represent the [illegible] [illegible] and [illegible] [illegible] for [illegible] [illegible]

[illegible] is obtained by adding [illegible] [illegible] is [illegible] by [illegible] and [illegible] by [illegible]

Which one of the following [illegible] is [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

1.22 Consider [illegible] [illegible] [illegible]

Tanker Answer Q.1.26 & Q.1.27:
Consider the following expression

$$ad = ac + bcd$$

1.26 Which of the following Karnaugh Maps correctly represents the expression?

(a)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	×	×		
$\bar{a}b$	✓			
ab	×	✓		×
$a\bar{b}$	×			×

(b)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	×	×		
$\bar{a}b$	×			
ab		✓		✓
$a\bar{b}$	×	×		✓

(c)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$		×		
$\bar{a}b$	×	×		✓
ab		✓		✓
$a\bar{b}$	×			✓

(d)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$		×		
$\bar{a}b$	×	×		
ab	×	✓		✓
$a\bar{b}$	×		✓	✓

[2005 : 2 Marks]

1.27 Which of the following expressions does not correspond to the Karnaugh Map provided in the above question?

(a) $ac + ad + abc = ad$

(b) $ac + cd = ad = cd + ad$

(c) $ac + ad + abc = ad$

(d) $\bar{a}\bar{c}\bar{d} + a\bar{c}\bar{d} + ac\bar{d} + abc$

[2005 : 2 Marks]

1.28 The function $f(x, y, z) = \sum(0, 1, 2, 4, 5, 6)$. Which of the following expressions is a NOR equivalent of f?

(a) $(x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(\bar{x}+y+z)$

(b) $(x+y+z)(x+y+\bar{z})(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})$

(c) $(x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(x+\bar{y}+\bar{z})$

(d) $(x+y+z)(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})$

[2007 : 2 Marks]

1.29 Define the exclusive-OR (XOR) function on Boolean variables X and Y as $X \oplus Y = X\bar{Y} + \bar{X}Y$.
Let $Z = X \oplus Z$. Consider the following assertions P, Q and R.
P: $X = Y \oplus Z$
Q: $Y = X \oplus Z$
R: $X \oplus Y \oplus Z = 1$
Which of the following is TRUE?
(a) Only P and Q are valid
(b) Only Q and R are valid
(c) Only P and R are valid
(d) All P, Q, R are valid

[2007 : 2 Marks]

1.30 In a look ahead carry generator, the carry generate function G_i and the carry propagate function P_i for inputs A_i and B_i are defined as:
$P_i = A_i \oplus B_i$ and $G_i = A_i B_i$
The expressions for the sum bit S_i and the carry bit C_{i+1} of the look ahead carry adder are defined as: $S_i = P_i \oplus C_i$ and $C_{i+1} = G_i + P_i C_i$ where C_i is the carry bit of the previous stage.
Consider a two-level logic implementation of the look ahead carry generator. Assume that P_i and G_i are available for the carry generator and that P-AND and OR gates are available. The number of AND gates and OR gates required to implement the look ahead carry generator for the bits C_2, C_3, C_4 and C_5 respectively are:
(a) 6, 3 (b) 6, 5
(c) 10, 5 (d) 10, 3

[2007 : 2 Marks]

1.31 The line T in the following figure is stuck at 0. The correct test for T is

When all the following inputs (X1, X2, X3, X4) will detect the fault T:
(a) 0, 0, 0, 1 (b) 0, 0, 1, 1
(c) 0, 1, 1, 1 (d) None of these

[2007 : 2 Marks]

1.32 In the Karnaugh map shown below, X denotes don't care term. What is the minimum number of 2-input NAND gates required to implement the minimum sum of products expression?

(a) $\overline{bc} + \overline{ad}$ (b) $ab + cd + bc$

(c) $\overline{bc} + \overline{a}b\,d$ (d) $ab + cd + \overline{c}\,\overline{d}$

[2008 : 1 Mark]

1.33 Given f_1, f_2 and f_3 are in sum of products form as in decimal. Find the circuit

$f_1 = \Sigma m(1, 6, 7, 8)$

$f_2 = \Sigma m(1, 6, 8, 9)$

$f_3 = \Sigma m(1, 6, 8, 9)$

The f_3 is

(a) $\Sigma m(1, 6, 9)$ (b) $\Sigma m(1, 8)$

(c) $\Sigma m(1, 8, 9)$ (d) $\Sigma m(1, 6, 8)$

[2008 : 1 Mark]

1.34 A set of Boolean functions is said to be functionally complete if all Boolean functions can be realized using the connections of NOT functionally complete?

(a) EX-NOR (b) implication operation

(c) OR operation (d) NAND

[2009 : 1 Mark]

1.35 If P, Q, R are Boolean variables, then $(P + \overline{Q})(P\overline{Q} + PR)(P + R) = Q$ simplifies to

(a) PQ (b) $P + R$

(c) $P\overline{Q} + R$ (d) $P(Q + R)$

[2009 : 2 Marks]

1.36 What is the minimum number of 2-input gates required to implement the Boolean function $(AB + C)$ assuming that we have only 2-input NOR gates?

(a) 2 (b) 3

(c) 4 (d) 5

[2009 : 1 Mark]

1.37 If m_i denotes the minterm corresponding to PQR, $P\overline{Q}R + PQR + Q\overline{R} + \overline{P}\,\overline{Q}\,\overline{R}$ is

(a) $\Sigma m(1, 3, 5, 7)$

(b) $\Sigma m(1, 3, 4, 6)$

(c) $\Sigma m(1, 3, 6, 7)$

(d) $\Sigma m(1, 4, 6, 7)$

[2010 : 1 Mark]

1.38 The simplified SOP (sum of products) form of the Boolean expression $(P + Q + R)(P + Q + \overline{R})(P + \overline{Q} + R)$ is

(a) $P + QR$ (b) $P + \overline{Q}R$

(c) $P + Q$ (d) $P + \overline{Q}\,\overline{R}$

[2010 : 2 Marks]

1.39 What is the Boolean expression for the output of the combination logic circuit of NAND gates given below?

(a) $Q + \overline{R}$ (b) $P + Q$

(c) $\overline{P} + \overline{Q}$ (d) $P + Q + R$

[2010 : 2 Marks]

1.40 Which one of the following circuits is a NOT equivalent to a 2 input NAND (exclusive NOR) gate?

(a)

(b)

(c)

(d)

[2011 : 1 Mark]

1.41 The decimal value 0.3 in IEEE-754 single precision floating point format is represented as

(a) [illegible] and exponent value [illegible]

(b) [illegible] and exponent value [illegible]

(c) [illegible] and exponent value [illegible]

(d) no exact representation

[2015 : 1 Mark]

1.42 What is the minimal form of the Karnaugh map shown below? Assume that X denotes a don't care term.

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2015 : 2 Marks]

1.43 Which one of the following expressions does NOT represent exclusive-NOR of P and Q?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2013 : 1 Mark]

1.44 Consider the following Boolean expression for F:

$$F(P, Q, R, S) = PQ + PQR + PQRS$$

The minimum number of literals in an equivalent sum of products form for F is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2014 (Set-1) : 1 Mark]

1.45 The dual of a Boolean function $F(x_1, x_2, \ldots, x_n, +, \cdot)$ is a function F^D defined as $F^D(x_1, x_2, \ldots, x_n, +, \cdot) = \overline{F(\overline{x_1}, \overline{x_2}, \ldots, \overline{x_n}, \cdot, +)}$. A function is said to be self-dual if $F = F^D$. The number of self-dual functions with n Boolean variables is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2014 (Set-3) : 1 Mark]

1.46 Consider the following minimum sum of products expression for F:

$$F(P, Q, R, S) = \sum (\text{illegible})$$

The minterms 2, 7, 8 and 13 are the don't care terms. The minimum number of product terms for F is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2014 (Set-3) : 1 Mark]

1.47 The Boolean function F is defined on two variables P and Q as follows:

Consider the Boolean variable X defined as $X = (P \oplus Q)$.

$F(P, Q) = (P \cdot Q) + (P \cdot X) + (Q \cdot X)$

The equivalent expression for F is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2011 (Set-3) : 2 Marks]

1.48 Consider the two functions

$$f(X, Y, Z) = X \cdot Y \cdot Z + X \cdot Y \cdot \overline{Z} + \text{illegible}$$

$$g(X, Y, Z) = X \cdot Y \cdot Z + X \cdot Y \cdot \overline{Z} + X \cdot Y$$

Which one of the following statements

(a) Both f and g are functionally complete

(b) Only f is functionally complete

(c) Only g is functionally complete

(d) Neither f nor g is functionally complete

[2015 (Set-1) : 2 Marks]

1.49 The number of prime implicants and essential prime implicants of the following Boolean expression is ____.

$$F(P, Q, R, S) = \sum m(0, 2, 5, 7, 8, 10, 13, 15)$$

[2016 (Set-2) : 2 Marks]

1.50 The number of prime implicants of the function $f(w, x, y, z) = \sum(0, 2, 4, 5, 6, 7, 8, 10, 12, 14)$ is ____.

[2016 (Set-3) : 2 Marks]

1.51 Given the Boolean function $F = PQ + QR$, where P, Q, R are Boolean variables, and $\overline{F}$ is the complement of F. The following expressions are

S1: $F = \Sigma(0, 5, 6)$

S2: $F = \Sigma(0, 1, 2, 3, 4)$

S3: $F = \Pi(1, 3, 4)$

S4: $F = \Pi(0, 1, 2, 3, 4)$

Which of the following is true?

(a) S1 false, S2 true, S3 true, S4 false

(b) S1 true, S2 false, S3 false, S4 true

(c) S1 false, S2 false, S3 false, S4 true

(d) S1 true, S2 true, S3 true, S4 false

[2013 (Set 4) : 2 Marks]

1.32 Consider the Boolean expression F with four inputs w, x, y, z given by $F = \Sigma(1, 4, 5, 6, 9, 11, 12, 13, 14)$ and $d = \Sigma(2, 3, 7, 8, 10, 15)$. Then a minimal sum of products expression for F is

(a) $xz + \bar{x}\bar{z}$ (b) $xz + \bar{z}\bar{y}$

(c) $\bar{z}\bar{y} + xy$ (d) $xy + \bar{x}\bar{z}$

[2016 (Set 1) : 1 Mark]

1.33 Two variables x_1 and x_2 are to be selected from a Boolean function $f(x_1, x_2, x_3, x_4)$ given by $f = \Sigma(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$. Which one of the following must always be 0?

(a) $x_1 \oplus x_2 \oplus x_3 = 0$

(b) $x_1 + x_2 + x_3 = 0$

(c) $x_1 \oplus x_2 = x_3 \oplus x_4$

(d) $x_1 + x_2 = x_3 + x_4 = 0$

[2016 (Set 2) : 1 Mark]

■■■■

Answers: Logic Functions and Minimization

1.1	(b)	1.2	(a)	1.3	(b)	1.4	(c)	1.5	(a)	1.6	(b)	1.7	(c)	1.8	(c)	1.9	(c)
1.10	(d)	1.11	(d)	1.12	(b)	1.13	(b)	1.14	(a)	1.15	(c)	1.16	(c)	1.17	(d)	1.18	(a)
1.19	(a)	1.20	(a)	1.21	(a)	1.22	(c)	1.23	(c)	1.24	(b)	1.25	(c)	1.26	(a)	1.27	(a)
1.28	(d)	1.29	(d)	1.30	(b)	1.31	(c)	1.32	(a)	1.33	(c)	1.34	(a)	1.35	(a)	1.36	(b)
1.37	(a)	1.38	(b)	1.39	(a)	1.40	(d)	1.41	(c)	1.42	(b)	1.43	(d)	1.44	(a)	1.45	(d)
1.46	(a)	1.47	(d)	1.48	(b)	1.51	(a)	1.52	(a)	1.53	(a)						

Explanations: Logic Functions and Minimization

1.1 (b)

$$S = x + y$$

The value of $x \oplus y$

$$= x \oplus (x + y)$$

$$= (x + y) + x$$

$$= (x + y) + x$$

$$= x + y + x$$

By consequence $x + x = x$

$$= x + y = S$$

Therefore

$$x \oplus y = S$$

1.2 (a)

Here there are two prime implicants, one of which is essential prime implicant.

1.3 (b)

By [illegible]

$f = f_1 f_2 + f_3$

$f_1 = \Sigma(0, 8, [illegible])$, $f_2 = \Sigma(0, 1, 8, 9, [illegible])$, $f_3 = \Sigma(0, [illegible])$

The common minterms of f_1 and f_2 are 5

Therefore,

$f = f_3 = \Sigma(5)$

Let $f = \Sigma(0, 8, 5)$ can be represented as follows:

(i) $f = \Sigma(0, 5)$

(ii) $f_2 = \Sigma 0$

Therefore option (d) is correct choice.

1.4 (a)

Truth table is as follows

(i) Given $x = 0$, $y = 1$, $z = 0$ and $w = 0$

(i) $x + y + z = 0 + 1 + 0 = 1$

(ii) $xy = 0 \cdot 1 = 0$

(iii) $xz = x = 0 \cdot 0 = 0$

However given $xz + w = 1$

Therefore option (a) is not the solution.

(ii) Given $x = 1$, $y = 0$, $z = 0$ and $w = 1$

(i) $x + y + z = 1 + 0 + 0 = 1$

(ii) $xy = 1 \cdot 0 = 0$

However given $xy = 1$

Therefore option (b) is not the solution.

(iii) Given $x = 1$, $y = 0$, $z = 0$ and $w = 1$

(i) $x + y + z = 1 + 0 + 0 = 1$

(ii) $xy = 1 \cdot 0 = 0$

(iii) $xz + w = 1 \cdot 0 + 1 = 1$

(iv) $x + y + xz' = 1 + 0 + 0 \cdot 1 = 0$

(iv) Given $x = 1$, $y = 0$, $z = 0$ and $w = 0$

(i) $x + y + z = 1 + 0 + 0 = 1$

(ii) $xy = 1 \cdot 0 = 0$

(iii) $xz + w = 0 \cdot 0 + 0 = 0$

However given $xz + w = 1$

Therefore option (c) is not the solution.

1.5 (a)

Minimal SOP of the given K-map is

$= xy + yz$

1.6 (b)

Minimum SOP for $f(w, x, y, z)$

$= xz + wz'$

1.7 (a)

$y = \overline{f \cdot f_3}$, $f_1 = f(x, y) + f_3$ ...(1)

Given $f_2(x, y, z) = \Sigma(0, 1, 2, 3, 5, 6)$ and

$g(x, y, z) = \Sigma m(0, 1, 6)$

$= f_2 = \Sigma m(0, 1)$ ...(2)

Common minterms of functions f_1 and f_2 are minterms in $f \cdot f_3$.

Therefore $f_3 = f_2(x, y, z) = \Sigma m(0, 1, 6)$

$\Rightarrow f_3 = f(x, y, z) = \Sigma m(0, 1, 4)$

Therefore given circuit answer is option (a).

1.8 (b)

$= \Sigma(0, 1, 2, 3)$

$= f(x', y', z', w')$

$= f(x, y, z)$

$= f(x', y', z')$

$= (x' + y') + z = x'y' + z$

1.9 (d)

Final Boolean expression for [illegible]

$= wy + wy' + xz + xz'$

Total literal count = 10 literals

Minimum [illegible] as follows

So $w, x, y, z = 0, 1, 0, 1$ or $w, x, y, z = 1, 0, 1, 0$

Total number of rounds = 9 seconds

1.10 (d) :

Consider the given TTL OR gate circuit. G_1 and G_2 are identical OR gates. G_3 is a NOT gate.

1. The two inputs of the NOR gate G_1 are tied together and connected to the output of gate G_3.
2. The identity gate G_2 input is connected to the output of gate G_1 and output waveform.
3. To find gate G_1 output waveform, it is required to identify G_2 output waveform.

The output of G_1 is constant at 0 during interval from 0 to 100 sec.

Hence option (d).

1.11 (b) :

$$x(t) = x_1 - x_2 = x_1(t) + x_1(t-1) - x_2$$

$$= x(t) + x(t-1) \qquad x(t) = u(t) - u(t-1)$$

$$= x(t) + x(t-1) \qquad x(t-1) = u(t-1) - u(t-2)$$

$$= x(t) + y(t)$$

1.12 (b) :

A 2-input Exclusive OR function can be implemented using NAND gates.

1st NAND gate

Inputs: A, B

Output: $(AB)'$

2nd NAND gate

Inputs: $(AB)', A$

Output: $A' + AB$

3rd NAND gate

Inputs: $(AB)', B$

Output: $B' + AB$

4th NAND gate

Inputs: $A' + AB, B' + AB$

Output: $AB + AB'$ (Exclusive OR function)

1.13 (b) :

K map

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$	0	0	1	1
C	1	1	0	1

Equivalent expression of above K map is $A\bar{C} + \bar{A}C + AC$

1.14 (d) :

$$(A \oplus B) \oplus C$$

$$(A\bar{B} + \bar{A}B)\bar{C} + \overline{(A\bar{B} + \bar{A}B)}C$$

$$A\bar{B}\bar{C} + \bar{A}B\bar{C} + (\bar{A} + B)(A + \bar{B})C$$

$$A\bar{B}\bar{C} + \bar{A}B\bar{C} + ABC + \bar{A}\bar{B}C$$

$$= A(\bar{B}\bar{C} + BC) + \bar{A}(B\bar{C} + \bar{B}C)$$

1.15 (d) :

In Db is ON when both switch S1 and S2 are in same state, either open or closed.

S1	S2	Bulb
0	0	0
0	1	0
1	0	0
1	1	0

Above truth table derives the NOR expression

1.13 (a)

The given circuit is

$$Y = xy + yz$$
$$= xy(z + \bar{z}) + yz(x + \bar{x})\quad [x + \bar{x} = 1]$$
$$= xyz + x\bar{y}z + xyz + xyz$$

Hence,

The value of Y and the corresponding truth table

1.18 (a)

$f(x, y, z) = \Sigma(1, 4, 6, 9, 11, 12)$

The K-map for f is

$\bar{B}\bar{C}\bar{D} + A\bar{C}D + AB\bar{C}$

1.19 (a)

$$c_1 = a_1b_1 + b_1c_1 + a_1c_1 = a_1b_1 \quad \ldots(1)$$

[illegible] input carry $c_0 = 0$

$$s_1 = a_1b_1 + a_1c_1 + b_1c_1$$
$$= a_1b_1 + a_1(b_1c_1) + a_1b_1c_1 \quad \ldots(2)$$
$$s_2 = a_2b_2 + a_2c_2 + b_2c_2$$
$$= a_2b_2 + b_2(a_2 + c_2) + a_2c_2$$
$$= a_2b_2 + a_2b_2 + a_2b_2c_2 + a_2b_2c_2 + a_2c_2$$
$$= a_2b_2 + a_2b_2c_2 + a_2b_2c_2 + a_2c_2$$

1.20 (a)

Given: $x, y, z = 2, 5, 6, 10, 11, 13, 14$

Draw the K-map

$wxyz \rightarrow 0101 \quad (5, 13)$
$wxyz \rightarrow 1010 \quad (10)$
$wxyz \rightarrow 0110 \quad (6)$
$wxyz \rightarrow 1011 \quad (11)$
$wxyz \rightarrow 1101 \quad (13)$

1.21 (a)

[illegible]

The [illegible] condition is

[illegible]

So the [illegible] condition is

[illegible]

1.22 (b)

Four binary number b, b, b, b, the corresponding gray code h, h, h, h, is given in Table-1

Table-1

For a [illegible] number [illegible] b_3 b_2 b_1 b_0 [illegible] to R_3 R_2 R_1 R_0 the corresponding gray code of binary number [illegible] is given in Table-[illegible]

From the above we get the equations for x_1, x_2, x_3 and x_4.

Two-level x_3 is such that

$x_3 = (x_1, x_2, b_1, b_2)$

$= \text{and}(x_1, \overline{x_2}, b_1, b_2)$

1.24 (a)

Let the number of state x_i be the value of the function $f(x_1, x_2, \ldots, x_n)$ is the number of variable whose decimal equivalent. Then every switching function can be expressed in the form $f(x_1, x_2, \ldots, x_n) = \sum m(x_1, x_2, \ldots, x_n)$. A function x_i is called 0 if it has corresponding minterm. Consequently, in order to find the function, it is necessary to calculate the number of which can have value either 0 or 1. Since there are 2^n possibilities, we have

OR

Let a function $f = \overline{A}\,\overline{B} + A\,\overline{B} + B$ is a function variables. The possible combinations are shown as

1. Taking each combination minterms then number of functions is $2^{2^n} = 16$
2. Taking any two combination minterms the number of functions becomes $\binom{16}{2}$
3. Taking any three combination minterms $\binom{16}{3}$
4. Taking four combination minterms, the number of functions are $\binom{16}{4}$
5. Taking no combination minterms, the number of functions are $\binom{16}{0}$

Total number of functions is 2 for four variables are

$\binom{16}{0} + \binom{16}{1} + \binom{16}{2} + \binom{16}{3} + \binom{16}{4} = 2^{16}$

Similarly for 3 variables, total number of functions are $\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3} + \binom{8}{4} = 2^8$

1.25 (b)

Given function $y(x) = \sum(0, 1, 6, 7, 12, 13)$

Corresponding K-map is

$f(w, x, y, z) = w'x'y'z' + w'x'y'z + w'x'yz$
$+ w'xyz' + w'xyz + wx'y'z'$
$+ wx'y'z + wx'yz + wxyz'$
$= w'x'(y'z' + y'z + yz) + w'x(yz' + yz)$
$+ wx'(y'z' + y'z + yz) + wx(yz' + yz)$
$= w'x' + w'x + wx' + wx$
$= w(x' + x) + w(x' + x)$
$= w + w$

Hence $y(x)$ is independent of variable x.

1.26 (a)

Given function $f(a, b, c, d) = \sum(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

Simplified SOP for K-MAP is

$= a\,\overline{b}\,\overline{c}\,\overline{d} + a\,\overline{b}\,\overline{c}\,d + a\,\overline{b}\,c\,\overline{d} + a\,\overline{b}\,c\,d + \overline{a}\,\overline{b}\,\overline{c}\,\overline{d} + \overline{a}\,\overline{b}\,\overline{c}\,d + \overline{a}\,\overline{b}\,c\,\overline{d} + \overline{a}\,\overline{b}\,c\,d + \overline{a}\,b\,\overline{c}\,\overline{d} + \overline{a}\,b\,\overline{c}\,d + \overline{a}\,b\,c\,\overline{d} + \overline{a}\,b\,c\,d + a\,b\,\overline{c}\,\overline{d} + a\,b\,\overline{c}\,d + a\,b\,c\,\overline{d} + a\,b\,c\,d$

Note: We use the following rules

$a + \overline{a} = 1$

$\overline{a} + \overline{a} = \overline{a} = a$

$a + a = a = \overline{a}$

1.27 (d)

$f = ad + ac + bd + bc$

1.87 (a) (i)

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Assumption (c):

[illegible] expression [illegible] to make [illegible] 1, which is not equivalent to given [illegible]

1.88 (a) (i)

[illegible]

The K-map for [illegible]

[illegible]

[illegible]

[illegible]

1.34 (b)

[illegible]

1.35 (b)

$$= (PQ' + PR + PQ' + QR)(PR' + Q')$$
$$= (PQ' + PR + PQ' + QR)(PR' + Q')$$
$$= (PQ' + PR + PQ' + QR)(PR' + Q')$$
$$= (PQ' + PR + PQ' + QR)(PR' + Q')$$
$$= (PQ' + PR + PQ' + QR)(PR' + Q')$$
$$= PQ' + QR$$
$$[\because (1 + P) = P]$$

OR

(1) (2) (3)

$$= \bar{Q} \quad PQ \quad PR \quad \bar{P}R + \bar{Q}$$

Using AND-OR-Invert (AOI) gates

$$(P + Q)(P + R) + \bar{Q} = (P + \bar{P}R) + PQ + PR + \bar{Q} + \bar{Q}$$
$$= Q + P + \bar{P}R + Q$$

The simplified expression is

$$= PQ + PR + Q = PQ + PR + Q$$

1.36 (b)

$$AB + C = (A + C)(B + C)$$

using [illegible] theorem, the simplified expression is in POS form. Hence 2 level NOR-NOR circuit is required to implement POS form.

1.37 (d)

$$= PQ + QR + PR$$

Sum of product form

$$= PQ(R + \bar{R}) + \bar{R}(P + \bar{P}Q) + PR(Q + \bar{Q})$$
$$= PQR + PQ\bar{R} + \bar{R}P + \bar{R}\bar{P}Q + PQR + P\bar{Q}R$$
$$= PQR + PQ\bar{R} + PR + \bar{P}\bar{R}Q \quad [\because A + A = A]$$
$$= m_6 + m_5 + m_7 + m_3$$

1.38 (b)

$$= \bar{Q} + \bar{R}(P + \bar{Q}) + R(P + Q)$$

Interchanging the brackets and simplifying, we get the boolean expression, $= P + QR$.

1.39 (b)

Boolean Expression $A = \overline{P \cdot \overline{Q + Q} + R}$

$$= \overline{P \cdot \overline{Q + Q} \cdot \overline{R}} = \overline{P \cdot Q + \bar{Q} \cdot \bar{R}}$$
$$= \overline{(P + \bar{Q})(Q + \bar{R})} = (\bar{P} + Q)(\bar{Q} + R)$$

$$B = \overline{\overline{P + P + Q} \cdot \overline{R}}$$
$$= \overline{\bar{P} \cdot \overline{R + Q} \cdot \bar{R}} = \overline{\bar{P} \cdot (\bar{R} \cdot \bar{Q}) \cdot \bar{R}}$$
$$= \overline{(P + \overline{R + Q})(Q + \overline{R})} = (P + R + Q)(Q + R)$$

So $C = A \cdot B$

$$= (\overline{P + \overline{R + Q}})(Q + \overline{R})(P + R + Q)(Q + R)$$
$$= (\overline{P + R + Q})(Q + \overline{R})(P + R + Q)(Q + R)$$
$$= (\bar{P}\bar{R}\bar{Q} + \bar{Q}\bar{R})(PQ + PR + QR + RR)$$
$$= \bar{P}\bar{R}\bar{Q}PQ + \bar{P}\bar{R}\bar{Q}PR + \bar{Q}\bar{R}PQ + \bar{Q}\bar{R}PR + \bar{Q}\bar{R}QR$$

$$\because AA = A$$
$$= A + A = A$$

$$= \bar{P}\bar{R} + \bar{P}\bar{R}\bar{Q}P + \bar{Q}\bar{R}P$$
$$= \bar{P}\bar{R} + \bar{P}\bar{R}\bar{Q}P + \bar{Q}\bar{R}P$$
$$= \bar{P}\bar{R} + PQ$$
$$= \bar{P}\bar{R} + PQ$$
$$= Q + R$$

1.40 (b)

$$\overline{xy + z} = \overline{xy + z\bar{x}} = \overline{x(y + \bar{x})} = \overline{x(y + z)}$$
$$= \overline{xy + xz} = \overline{x(y + z)}$$

[illegible]

$= [illegible]$

[illegible]

[illegible]

[illegible]

$= \overline{(x+y)(x+y)}$

$= (x+y)\overline{(x+y)}$

$= x\bar{y} + \bar{x}y = x \oplus y$

$\therefore$ Option (b) is not XNOR.

1.41 (d):

$[illegible] = 0.1010 \times 2^{[illegible]}$

$= 0.0101 \times 2^{[illegible]}$

$\therefore$ exponent $= 1$ and mantissa $= 0.0101$

$\therefore$ (d) is correct.

1.42 (b):

Two quad-2 multiplexers can be used to implement the logic shown.

1.43 (b):

Truth table of NAND is as under

$\overline{(x \cdot 1)y} = 1(x' + x\bar{y})$

$= x'(1 \cdot y) + x(\bar{y} \cdot 1)$

$x'y + x\bar{y}$ ...(i)

Option (a) is not a NAND.

Option (b) $x'(x+y) = xy + x'y' \neq x'y + x\bar{y}$

Option (c) $x'(x+y) = xy + x'y' \neq x'y + x\bar{y}$

Option (d) $x(x+y) = xy + x\bar{y} \neq x'y + x\bar{y}$

Option (b) is correct.

1.44 (d):

$F = PQ + PQR + \bar{P}QR$

$F = PQ + QR(P + \bar{P}) = QR$

$F = PQ + QR + QS$

1.45 (d):

$n = 4$ (4 boolean variables)

[illegible] (C) [illegible]

[illegible] (B) [illegible]

[illegible] (A) [illegible]

[illegible] (D) [illegible]

Total cell function

Number of minterms = Number of [illegible]

Since, the function [illegible] (B, C, D), (A, C, D), (A, B, D), (A, B, C) are all normal sum of minimal pairs, only one combination of minimal pairs covers all the function is also self dual.

[illegible]

Total cost $= 2 \times 2 \times 2 \times 2 = 16 = 2^4$

[illegible]

[illegible] number of terms

Hence [illegible]

Answer [illegible]

1.46 [illegible]

[illegible]

1.47 [illegible]

[illegible]

$= [illegible]$

$= [illegible]$

$= [illegible]$

$= PQ : PQ = P \oplus Q$

1.48 (b)

$(X \oplus Y) \oplus 1 = X' \oplus Y \oplus 1 = X \oplus Y' = Y \oplus X'$

[illegible]

Hence it is functionally complete.

[illegible]

1.49 [illegible]

$[D' + AB' + AC' + AC'D + AC'D']$

$= [D' + AC'D + AB' + AC' + AC'D]$

$= [D' + AC' + AB' + AC' + AC'D]$

$= [D + AC' + AB' + AC + AD]$

$= [D + AC' + AB' + AC + AD]$ [illegible] D'

$AD = D' + A)$

$= [D' + A' + AC' + AB' + AC]$

[illegible] $A' + AC' = A'$

[illegible] $A' + AC' + AB' = A' + A(C' + B') = A' + C' + B'$

$= [D' + A' + C' + B']$

$= ABCD$

1.50 [illegible]

[illegible]

[illegible]

1.51 [illegible]

$F = F' + QF = \Sigma(0, 1, 2, 3, 7) = \Pi(4, 5, 6)$

S1: $F = \Sigma(1, 5, 7)$ false

S2: $F = \Sigma(0, 1, 2, 3, 7)$ true

S3: $F = \Sigma(1, 5, 7)$ true

S4: $F = \Sigma(0, 1, 2, 3, 7)$ false

$\Rightarrow F = \Sigma(0, 1, 2, 3, 7) = \Pi(4, 5, 6)$

1.52 (a)

0	0	0
0	1	1
1	0	1
1	1	1

[illegible] ...(1)

[illegible] ...(2)

[illegible] ...(3)

[illegible]

1.18 [illegible]

It is given [illegible] $x_1 \oplus x_2 \oplus x_3 = 0$

	x_1	[illegible]	[illegible]	[illegible]	[illegible]
0	0	0	0	0	[illegible]
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	[illegible]
4	0	1	0	0	
5	0	1	0	1	[illegible]
6	0	1	1	0	[illegible]
7	0	1	1	1	[illegible]
8	1	0	0	0	
9	1	0	0	1	[illegible]
10	1	0	1	0	[illegible]
11	1	0	1	1	[illegible]
12	1	1	0	0	[illegible]
13	1	1	0	1	[illegible]
14	1	1	1	0	[illegible]
15	1	1	1	1	[illegible]

Let the minterms

$m_0, m_2, m_3, m_5, m_6, m_8, m_{10}, m_{12}$ and m_{15}

$x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$

[illegible]

Option (a) fails whenever

$x_1 = x_2 = x_3 = x_4 = 1$

Option (b) fails whenever

x_1, x_2, x_3 and x_4 are [illegible] and for [illegible] more combinations

Option (c) fails whenever

x_1, x_2, x_3 and x_4 are [illegible] and for few more combinations

Option (d) satisfies for every combination.

Hence option (d) is correct.

■■■■

2 Combinational Circuits

2.1 Consider the circuit in figure below. It implements

(a) $A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C}$ (b) $A + B + C$

(c) $A + B + \bar{C}$ (d) $AB + B\bar{C} + \bar{B}C$

[1996 : 2 Marks]

2.2 What is the equivalent Boolean expression in product-of-sum form for the Karnaugh map given in figure

(a) $\bar{D}\bar{C} + \bar{D}\bar{B}$

(b) $(\bar{C} + \bar{D})(\bar{B} + \bar{C})(\bar{B} + \bar{D})$

(c) $(\bar{B} + \bar{D})(\bar{B} + \bar{C})$

(d) $(\bar{C} + \bar{D})(\bar{B} + \bar{D})$

[1996 : 1 Marks]

2.3 What is the following function implemented by the Karnaugh map shown below?

AB \ CD	00	01	11	10
00	0	0	1	0
01	x	x	1	x
11	0	0	1	0
10	0	0	1	0

(a) $\bar{B}\bar{D} + C\bar{D}$

(b) $\bar{D}(\bar{B} + \bar{C})$

(c) $\bar{B}\bar{D} + A\bar{D}$

(d) $(\bar{C} + \bar{D})(\bar{B} + \bar{D}) + \bar{D}(\bar{A} + \bar{D})$

[1998 : 1 Mark]

2.4 Which of the following sets of components are sufficient to implement any arbitrary Boolean function.

(a) XOR gates, NOT gates

(b) 2-to-1 multiplexers

(c) AND gates, XOR gates

(d) Three-input gates that output 1 if the majority of the inputs are 1, 0 otherwise

[1999 : 2 Marks]

2.5 Which function does NOT implement the same function as the one given below?

$xy \backslash zw$	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

(a) $x \oplus y$

(b) $x \oplus z$

(c) $(w \oplus x) \oplus (y \oplus z)$

(d) None of these above

[2000 : 2 Marks]

2.6 Consider the logic shown below. The output of 2:1 Mux is given by the expression $(x_1\bar{x}_2 + x_2)$

What is the function implemented?

(a) $F = x_1 + x_2$ (b) $F = x_1x_2 + \bar{x}_1\bar{x}_2$

(c) $F = x_1x_2 + x_1\bar{x}_2$ (d) $F = x_1 + \bar{x}_2$

[2001 : 2 Marks]

2.7 Consider the following multiplexer with I_0, I_1, I_2, I_3 as the four inputs, selected by two address line combinations $(A_1A_0) = (0, 0), (0, 1), (1, 0), (1, 1)$ respectively to the output Y of the multiplexer. EN is the Enable input.

The truth table of the multiplexer is represented by the above expression.

(a) xxxx (b) 0 1 x
(c) x 1 1 x (d) None of these

[2002 : 2 Marks]

2.8 Transform the following logic circuit (without expressing the switching function) into an equivalent logic circuit that employs only 8 NAND gates each with 2 inputs.

[2002 : 2 Marks]

2.9 Consider the ALU shown below.

If the operation is done in 2's complement representation, which of the following operations can be performed using the control lines S_1 and S_0 (assuming addition and subtraction are performed)?

(a) $A = B$ and $A = B$ for $S_1 = 1$
(b) $A = B$ and $A = 1$ for $S_1 = B$
(c) $A = B$ for $S_1 = B$ or A
(d) $A = B$ and $A = B$ and $A = 1$

[2001 : 2 Marks]

2.10 A decimal number as digit in hexadecimal is represented by 1000. The 1000... 9 by 1001. A combinational circuit is to be designed which takes 2-bit hexadecimal input and outputs the 2's complement of the input. Only AND, OR and NOT gates are to be used. What is the minimum number of gates required?

(a) 3 (b) 5
(c) 4 (d) 7

[2004 : 2 Marks]

2.11 The circuit shown below implements a 2 input NOR gate using two 2:1 MUX. Inverted signal is selected as the upper input. What are the values of signals X, Y, Z?

(a) 1, 0, B (b) 1, 0, A
(c) 0, 1, B (d) 0, 1, A

[2005 : 2 Marks]

2.12 The minimum number of combinational gates with two inputs required to implement the function represented by the following Karnaugh map.

Which of the product terms given below is an essential prime implicant of this function?

(a) QRS (b) PQS
(c) PRS (d) QRS

[2006 : 2 Marks]

2.13 The majority function is a Boolean function for which the value is the value of the majority of the variables. Consider the following three-input majority function. Draw a gate level diagram for the circuit using only NAND gates.

(a) NOR, AND (b) NOR, NOR
(c) OR, OR (d) OR, AND

[2008 : 2 Marks]

2.14 Consider the circuit below. Which one of the following statements is true about the circuit?

(a) $x = 1$, $y = 1$, $z = 1$ (b) $xz = xy + yz$
(c) $x = y = z = yz$ (d) $x = 1$, $y = z = yz$

[2006 : 2 Marks]

2.15 The [illegible] in the [illegible] AND gate used to [illegible] & [illegible] [illegible]

(a) $X1 = 1$, $X2 = 0$, $X3 = 0$
(b) $X1 = 1$, $X2 = 1$, $X3 = 1$
(c) $X1 = 0$, $X2 = 0$, $X3 = 0$
(d) $X1 = 0$, $X2 = 0$, $X3 = 1$

[2001 : 1 Mark]

2.16 How many [illegible] 2 line to 4 line decoders are needed to construct a 6 to 64 line decoder without using any other logic gates?
(a) 7 (b) 8
(c) 9 (d) 10

[2005 : 1 Mark]

2.17 Suppose a sequential circuit and the circuit is allowed to have a single memory. Boolean functions of 4 variables. What is the minimum size of the memory required?
(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2001 : 2 Marks]

2.18 [illegible] Boolean [illegible] [illegible]

(a) [illegible]
(b) [illegible]
(c) independent of the variable
(d) dependent on all the variables

[2002 : 1 Mark]

2.19 What Boolean function does the circuit below realize?

(a) $xz + yz$ (b) $xz + xz$
(c) $xy + yz$ (d) $xy + yz$

[2006 : 1 Mark]

2.20 [illegible] multiplexer [illegible]

(a) $P + Q = 2$ (b) $P + Q = 1$
(c) $P - Q = R$ (d) $\overline{P - Q} = 1$

[2010 : 1 Mark]

2.21 [illegible] if and only if the inputs are equal.

Inputs			Outputs			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	0	0	0	1	1	0
1	0	0	0	0	0	1
0	1	0	0	0	1	1
0	0	1	0	1	0	1
1	1	1	1	1	1	1

What functions does the circuit represent?
(a) Priority encoder (b) Decoder
(c) Multiplexer (d) Demultiplexer

[2013 : 1 Mark]

2.22 [illegible] multiplexer with [illegible] lines S_1 and S_0 given below.

The minimum sum of products form of the Boolean expression for the output F of the multiplexer is

(a) $\bar{P}Q + Q\bar{R} + PQR$

(b) $PQ + PQ\bar{R} + PQ\bar{P} + PQP$

(c) $PQP + PQR + QR + P\bar{Q}R$

(d) None of

[2014 (Set-1) : 2 Marks]

2.33 Consider the following combinational function block which involves four Boolean variables P, Q, R and S. Let X be a Boolean variable corresponding to the output of the block.

```
F(P, Q, R, S) {
  if (P == 1) {
    return Q;
  }
  else {
    return R;
  }
}
```

Which one of the following Boolean expressions is the most simplified representation of X?

(a) 2:1 Multiplexer

(b) Half adder

(c) Multiplexer

(d) Full Adder

[2014 (Set-1) : 1 Mark]

2.34 A half adder is implemented with XOR and AND gates. A full adder is implemented with two half adders and one OR gate. The propagation delay of an XOR gate is twice that of an AND/OR gate. The propagation delay of an AND/OR gate is 1.2 nanoseconds.

A 4-bit ripple carry binary adder is implemented by using four full adders. The total propagation time in this 4-bit binary adder in nanoseconds is _____.

[2015 (Set-3) : 2 Marks]

2.35 Consider the following 2:1 multiplexer circuit in SOP form.

The minimal sum of product form of the output X is

(a) $\bar{P}Q + PQ\bar{S}$ (b) $PQ + \bar{S}$

(c) $PQ + PQ\bar{S}$ (d) $\bar{P}\bar{Q}R + P\bar{Q}S$

[2016 (Set-1) : 2 Marks]

2.36 Consider a 4-bit carry look ahead adder for adding two 4-bit integers. The time to perform addition in this adder is

(a) 4 ns (b) 8 ns

(c) 16 ns (d) 32 ns

[2016 (Set-1) : 2 Marks]

2.37 Consider an eight bit ripple carry adder for two unsigned integers. Each gate in the adder has a propagation delay of 2 ns. If the inputs to the adder are 10101010 and 01010101 (decimal values 170 and 85), the time (in ns) after which the correct sum is available at the output of the adder is _____.

[2016 (Set-2) : 1 Mark]

■■■■

Answers: Combinational Circuits

2.1	(a)	2.2	(d)	2.3	(d)	2.4	(b) or (c)	2.5	(c)	2.6	(c)	2.7	(a)	2.8	(a)	2.9	(b)
2.11	(d)	2.12	(d)	2.13	(c)	2.14	(a)	2.15	(a)	2.16	(c)	2.17	(c)	2.18	(c)	2.19	(b)
2.20	(b)	2.21	(a)	2.22	(a)	2.23	(c)	2.24	(b)	2.25	(b)						

Explanations: Combinational Circuits

2.1 (b)

The given function $F(A, B, C, D)$ can be plotted as follows:

$F(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

$= \sum m(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

$= 1$ (Logic 1)

2.2 (d)

The Karnaugh map for SOP form is given as

The Karnaugh map for POS form is given as

POS form $= (A + B)(A + C)(A + D)(B + C + D)$

2.3 (b)

Karnaugh map for the function is given as follows:

Corresponding Karnaugh map is

$F(A, B, C, D) = \sum m(0, 2, 4, 6, 8, 10, 12, 14)$

$= \sum m(0, 2, 4, 6, 8, 10, 12, 14)$

2.4 (b, c)

Universal gates are those using which any type of logic function can be realized. Examples of universal gates are NAND and NOR.

$XOR_1 = (OR, AND, NOT)$

$XOR_2 = (NAND)$

$XOR_3 = (NOR)$

$XOR_4 = (OR, AND)$

Only XOR_1 is not a universal gate.

2.5 (b)

Karnaugh map is

From SOP form we get the following

$f(w, x, y, z) = \sum m(1, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15)$

$= (x + z)(w + y + z)(w + x + y)$

Thus, optimal cost of implementation is given as 8 units.

For 2:1 MUX

2:1 MUX [illegible] is given as

$f(x, y, z) = y(x + z)$

OR

$f(x, y, z) = [illegible] + (x + z)$

Option (c) [illegible] is the [illegible] given $y = [illegible]$

2.4 (a)

Function [illegible] is

$y = (x_1' + 1)x_1 = x_1$

The output is

$z = x_1' + x_1 x_2$

$z = x_1(x_1' + x_2) = x_1 x_2$

2.5 (b)

Given [illegible] function [illegible] is [illegible] over [illegible]

$f(x, y, z)$

$= \sum(0, 1, 3, 4, 5, 7) = \bar{A}_1\bar{A}_0 + \bar{A}_1 A_0 + A_1\bar{A}_0 + A_1 A_0$

$= x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$

$= x\bar{y}(\bar{z} + z) + xy(\bar{z} + z)$

$= x\bar{y} + xy$

[illegible] expression [illegible]

2.6 (d)

Given [illegible] is

Step 1: Converting AND [illegible]

Step-2: Converting [illegible] inhibited OR

Since inhibited [illegible] NAND

[illegible]

Step 3: Converting NOT [illegible] NAND

[illegible]

2.7 (a)

[illegible] $C_1 = [illegible]$

Now $S_0 = A_0 B_0' + B_0 A_0'$

and $C_0 = A_0 + A_0 B_0$

and $S_1 = A_1 \oplus B_1 \oplus C_0$

if $C_1 = [illegible]$

Now $S_1 = A_1 A_0' + A_1 B_1'$

and $C_1 = A_0 + A_1 B_1$

[illegible] over $A = 0, A = 1$ and [illegible]

2.10 (b)

[illegible] Lines [illegible]

$f(w, x, y, z) = \Sigma(5, 6, 7, 8, 9)$

The [illegible] are [illegible]

[illegible]

$f(w, x, y, z) = \Sigma(0, 1, 2, 3, 4, 10, 11, 12, 13, 14, 15)$

So output

Solution is correct

8.11 (d) :

[illegible] A will be selected when [illegible] is high.

So total function will be [illegible]

[illegible]

8.12 (b) :

[illegible]

8.13 (c) :

The [illegible] should be [illegible]

[illegible] The output [illegible] The output [illegible] be low only if [illegible] are high.

[illegible] variables are low. [illegible]

8.14 (b) :

Let the [illegible] be [illegible]

[illegible]

8.15 (a) :

[illegible]

8.16 (b) :

[illegible] decoder

[illegible]

8.17 (b) :

[illegible] variable [illegible] line to 3 line.

8.18 (d) :

[illegible] Hence dependent on all variables.

2.16 (b)

[illegible]

2.17 (b)

[illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

2.18 (b)

[illegible]

2.19 (d)

[illegible]

2.20 (b)

[illegible]

2.21 (b)

[illegible]

2.22 (d)

[illegible]

2.23 (c)

[illegible]

2.24 Sol.

[illegible]

[illegible]

2.25 (d)

[illegible]

2.26 (b)

[illegible]

2.27 Sol.

[illegible]

[illegible]

3 Sequential Circuits

3.1 [illegible] is a combinational circuit and PLA is a sequential circuit.

[1996 : 2 Marks]

3.2 [illegible] the maximum clock frequency at which the circuit in figure can be operated. Assume that propagation delay of the edge triggered flip-flop and AND gate is 10 ns. Assume that the setup time for the D input of the flip-flop is negligible.

[1991 : 2 Marks]

3.3 The following arrangement of a counter and D flip-flop.

Has Q as the initial state of D flip-flop. [illegible] After the clock cycle the output state P, Q is represented by

(a) [illegible] 0 (b) 1, 1

(c) [illegible] 0 (d) [illegible]

[2000 : 2 Marks]

3.4 Consider the following circuit with initial state $Q_2 = Q_1 = 1$. The D flip-flops are positive edge triggered and have a propagation delay of 10 ns and hold time of 0 ns.

Assume the following timing diagram of X and the clock period of 100 ns. Which one is the correct clock?

[2001 : 2 Marks]

3.5 Consider the sequential circuit given below. Let $Q_1 = 1$, $Q_2 = Q_0 = 0$. The state of the circuit is given by the value of $4Q_2 + 2Q_1 + Q_0$.

Which one of the following is the correct state sequence of the circuit?

(a) 1, 3, 4, 6, 0, 0, 2, [illegible] (b) [illegible]

(c) 1, 2, 3, 5, 3, 3, 1, [illegible] (d) [illegible]

[2001 : 2 Marks]

3.6 A [illegible] synchronous sequential circuit behaves as follows:

[illegible] the number of 0's and 1's respectively in initial state of the circuit $Q_1 = Q_0 = 1$. The circuit outputs 00 until one of the following conditions holds.

1. $x_2 = x_1$. In this case, the output of the circuit is 10 and all subsequent outputs are 10.

[illegible] Initially, the output of the [illegible] sequence detector is [illegible].

What is the minimum number of [illegible] required in the state transition diagram of the above circuit?

(a) 5 (b) 6

(c) 7 (d) 8

[2008 : 2 Marks]

8.7 In an SR latch, [illegible] by cross-coupling two NAND gates. Assuming the inputs are S and R, then it will [illegible] in

(a) S = 0, R = 1 (b) S = 1, R = 0

(c) S = 0, R = 0 (d) [illegible]

[2003 : 1 Mark]

8.8 Consider the [illegible] implementation of a 2-bit counter using T flip-flops following the equation $Q_1^+ = Q_1 \oplus Q_0$ as shown below:

To complete the circuit, the input T_0 should be

(a) Q_0' (b) $Q_1 + Q_0$

(c) $Q_1 \oplus Q_0'$ (d) $Q_1 \oplus Q_0$

[2004 : 2 Marks]

8.9 How many pulses are needed to change the content of a decade counter from 1001 (9) to 1000 (8) assuming it is a BCD counter?

(a) 1 (b) 10

(c) 9 (d) 8

[2005 : 1 Mark]

8.10 When all the following input sequences are always given to the circuit, which one will exhibit a race condition?

(a) A B C
0 0 0
[illegible]
1

(b) A B C
[illegible]

(c) A B C
1 0
1 1 1
1 1 1

(d) A B C
[illegible] 1
0
[illegible]

[2006 : 2 Marks]

8.11 Consider the following [illegible] and its corresponding [illegible] encoded representation.

Consider the following [illegible] diagram. Let A_1 represent the higher order bit of the input. Assume the propagation delay of each gate is negligible.

Find the sum of the equivalent of the output sequence. The output sequence is [illegible] clock pulses 1 through [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

8.13 Consider the following [illegible]

The flip-flops are initialized to [illegible]. Let the initial state be [illegible]. The transition sequence is

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2005 : 2 Marks]

8.09 [illegible]

[2006 : 1 Mark]

8.11 [illegible] diagram [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

8.13 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2007 : 1 Mark]

8.16 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2007 : 2 Marks]

8.17 [illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

Figure

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2007 : 2 Marks]

8.18 Consider the [illegible] state diagram and its [illegible] by a JK flip-flop.

The combinational circuit generates J and K as [illegible] and Q. The Boolean expressions for J and K are

(a) [illegible]
(b) [illegible]
(c) [illegible]

[2008 : 2 Marks]

8.19 In the sequential circuit shown below [illegible] of the outputs Q, Q [illegible] the next four values of Q_1Q_0?

(a) 1.1.0.0.0.0 (b) 0.0.1 0.1.1.0
(c) 1.0.0.0.0.1 (d) 1.1.0.0.0.1

[2010 : 2 Marks]

8.20 The minimum number of D flip flops needed to design a mod 209 counter is

(a) 9 (b) 8
(c) 512 (d) 256

[2011 : 1 Mark]

Common Data Q.8.21 & Q.8.22

Consider the following circuit involving two D-type flip flops and an AND gate.

8.21 If the same input pattern is applied to the clock pulse, the initial values of the flip flop outputs Q_1Q_0 are 00, what will be the value of Q_1Q_0 after the clock pulse?

(a) 00 (b) 01
(c) 10 (d) 11

[2011 : 2 Marks]

8.22 If the flip flops are reset to 0 initially, what is the total number of distinct outputs (including reset) generated by the circuit in response to the clock pulses?

(a) 3 (b) 4
(c) 5 (d) 6

[2011 : 2 Marks]

8.23 Consider the state table

Assume that the flip flops are initially reset to zero. The minimum number of NAND gates required to implement the D flip flops is

(a) X (b) X + Y
(c) X ⊕ Y (d) Y

[2012 : 1 Mark]

8.24 Consider a 2's complement representation of an integer stored in 8 bits. The result is expressed in 16 bits if the integer is sign extended to 16 bits.

(a) 2's complement representation
(b) 1's complement representation
(c) 2's complement representation
(d) 1's complement representation

[2014 (Set-2) : 1 Mark]

8.25

The above synchronous sequential circuit uses JK flip flops. Initial state of Q_1Q_0 = 00. The state sequence for the next 4 clock cycles is

(a) 01, 10, 01, 10 (b) 01, 10, 11, 00
(c) 00, 01, 10, 11 (d) 00, 01, 11, 10

[2014 (Set-3) : 2 Marks]

8.26 Consider a 4-bit Johnson counter with an initial value of 0000. The sequence of the counter is

(a) 0, 1, 3, 7, 15, 14, 12, 8, 0
(b) 0, 1, 3, 6, 7, 9, 11, 13, 15, 0
(c) 0, 2, 4, 6, 8, 10, 12, 14, 0
(d) 0, 8, 12, 14, 15, 7, 3, 1, 0

[2015 (Set-1) : 1 Mark]

8.27 An edge triggered D flip flop is connected to a positive edge triggered JK flip flop as follows. The D input of the D flip flop is connected to the J input and the complement of the J input is connected to the K input of the JK flip flop. The Q output of the D flip flop is connected to the clock input of the D flip flop, the clock input of the JK flip flop is connected to the output of the D flip flop. Initially, the output of the D flip flop is reset to zero. The JK flip flop is reset to one. The clock to the D flip flop is a periodic signal with a period of 10 ns. The output of the JK flip flop is

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
[illegible] (Set-I) 2 Marks]

4.26 [illegible] 2 Marks]

4.27 [illegible] [2018 (Set-I) : 1 Mark]

■■■■

Answers : Sequential Circuits

4.1	(a)	4.2	(a)	4.3	(b)	4.4	(a)	4.5	(c)	4.6	(d)	4.7	(d)	4.8	(d)	4.9	(c)	4.10	(b)
4.11	(d)	4.12	(c)	4.13	(c)	4.14	(a)	4.15	(a)	4.16	(d)	4.17	(c)	4.18	(d)	4.19	(a)	4.20	(a)
4.21	(d)	4.22	(b)	4.23	(a)	4.24	(c)	4.25	(c)	4.26	(d)	4.27	(c)						

Explanations : Sequential Circuits

4.1 (a)

[illegible]

4.2 (a)

[illegible]

4.3 (b)

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]		[illegible]	[illegible]
1	[illegible]	[illegible]	1	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

4.4 (b)

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	0	0	[illegible]	
[illegible]	0	0	1	1
[illegible]			1	[illegible]
[illegible]		0	1	[illegible]
[illegible]	[illegible]	[illegible]	1	[illegible]
[illegible]	[illegible]	0	0	[illegible]
0	0	[illegible]		

[illegible]

3.6 (b)

The [illegible] in [illegible] is [illegible], [illegible], [illegible], [illegible].

3.7 (a)

1. [illegible] outputs on clock [illegible] corresponding [illegible] 0
2. If $D_n = x$ = [illegible] the output [illegible] the [illegible] clock [illegible].

Based on [illegible] the state diagram of the [illegible]

[illegible]

3.8 (b)

[illegible] S and R [illegible] 0

[illegible]

[illegible] SR Latch [illegible] NAND [illegible]

S	R	Q
[illegible]	[illegible]	Invalid state $Q_n = Q_{n+1} = 1$
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible] $Q_n = 1$ and $Q_{n+1} = 1$

3.9 (d)

The [illegible] is [illegible] = 2 [illegible] = 1 [illegible]

[illegible] = 2m (1, 3)

[illegible] = $\bar{x}_1 \bar{x}_0 + x_1 x_0 = x_1 \oplus x_0$

3.10 (b)

[illegible]

So [illegible] option 2 [illegible] output as 1.

3.11 (d)

Check [illegible] from X are given [illegible]

[illegible]

3.12 (d) :

[illegible]

3.13 (d) :

1. [illegible] flip-flop [illegible] Q_n and T [illegible] Q_{n+1}.
2. Flip-flop [illegible] the negative edge [illegible] Q [illegible] the [illegible] Q [illegible] the [illegible] [illegible] changing at the positive edge of the clock]
3. [illegible] T [illegible] will change at the [illegible] will [illegible] clock [illegible]
4. [illegible] clock rise [illegible] both [illegible] and [illegible].

Hence correct option (b).

[illegible]

Hence correct option (b).

Output Q_2 [illegible] with [illegible] the input [illegible] [illegible] the [illegible] [illegible] the [illegible] [illegible] the [illegible] [illegible] [illegible]

[illegible]

3.17 (a):

[illegible]

3.18 (d):

[illegible]

3.19 (a):

[illegible]

[illegible]

3.20 (a):

[illegible]

3.21 (d):

[illegible]

3.22 (b):

[illegible]

3.23 (d):

[illegible]

3.24 (c):

[illegible]

326 (d):

Clock	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

327 (d):

Clock	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

328 (b):

[illegible]

Where $T_A = T_C$

Clock	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

The sequence generated by this counter is [illegible]

[illegible] flip-flops are required.

329 (d):

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Number Systems

4.1 Fill in the blanks:

In the two's complement representation of a binary number, when the sign of the number is positive, the leading ____ and the ____

[1990 : 8 Marks]

4.2 For the digits in Decimal representation the range of the ...

[1991 : 2 Marks]

4.3 Consider the number given in decimal expression.

$$10^3 \times 4 + 10^2 \times 7 + 10 \times 3 + 8$$

The number 10 is the conjugate representation of the number ....

4.4 Using the 8087 automatic expression with the 80186 CPU requires that the 8087 CPU is operated in ____.

4.5 When two 16-bit binary numbers $A = a_{15}a_{14}a_{13}a_{12}$ $B = b_{15}b_{14}b_{13}b_{12}$ are multiplied, the product of the resulting is given by

4.6 A floating point number representation uses 32 bits. Digital representation uses a 24-bit fractional mantissa. The value of the exponent is 6.

(a) Find the range of the exponent is ____

(b) The range of the exponent is ____ if base of higher representation is used 64 format.

[1990 : 3 Marks]

4.7 A 8031 uses the look up table for multiplication of two 8-bit unsigned numbers. The size of 8031 required is

(a) 256 × 8 (b) 65k × 8

(c) 16k × 16 (d) 65k × 16

[1996 : 1 Mark]

4.8 Booth's algorithm for integer multiplication gives wrong result when the multiplier number is

(a) 101010 (b) ...

(c) 111111 (d) 000000....001

(e) 111111....111

[1996 : 1 Mark]

4.9 Consider the following floating point number representation.

The exponent is in the excess-127 representation and mantissa is in the sign magnitude representation. The range of the magnitude of the normalized numbers in this representation is

(a) 0 to 1 (b) 0.5 to 1

(c) 1/256 to 0.5 (d) 1/256 to 1

[1996 : 1 Mark]

4.10 Consider the circuit in binary which takes two bit binary inputs b_1b_0 and b_3b_2 and outputs the binary product of b_1b_0 and b_3b_2. The circuit implements

(c) Binary to octal conversion
(d) Binary to hexadecimal conversion

[1996 : 2 Marks]

4.11 Given $(173)_8$ [illegible] The [illegible] is

(a) 16 (b) 9
(c) [illegible] (d) 8

[1997 : 2 Marks]

4.12 Excess-3 coding is a binary coding [illegible]

(a) 0 = 1000, 1 = 1001 (b) 0 = 0011, 1 = 0100
(c) 0 = 0011 + 1000 = 1011 (d) 0 = 0011 + 1100 = 1111

[1998 : 1 Mark]

4.13 Two's complement representation [illegible]

(a) Sign magnitude (b) 1's complement
(c) 2's complement (d) None of these

[1999 : 2 Marks]

4.14 The number 2001 in excess-3 representation is

(a) 0010 0001 (b) 0110 0101
(c) 0010 0101 (d) 1000 0101

[2000 : 1 Mark]

4.15 Consider the values A = 2C5, B = 7F, C = 1F4, D = 1F4. C = 2D. [illegible] Which of the following [illegible] correct [illegible]

X = A + B
Y = A + C
X = X + C
Y = Y − B

(a) X = 1A0, Y = [illegible] (b) X = 6C0, Y = 1F0
(c) X = 1A0, Y = 6C0 (d) X = 6C0, Y = 1F0

[2000 : 2 Marks]

4.16 The 2's complement representation of $(-239)_{10}$ is equivalent to

(a) AD1₁₆ (b) 1DA₁₆
(c) [illegible] (d) 0.47

[2001 : 2 Marks]

4.17 The decimal value 0.25

(a) is equivalent to binary 0.1
(b) is equivalent to binary 0.01
(c) is equivalent to hexadecimal 0.01
(d) can not be represented precisely in binary

[2002 : 1 Mark]

4.18 The 2's complement representation of the decimal value −15 is

(a) 1111 (b) 1111
(c) [illegible] (d) 0001

[2003 : 1 Mark]

4.19 Sign magnitude representation [illegible]

(a) [illegible]
(b) [illegible] additive
(c) [illegible]
(d) [illegible]

[2002 : 1 Mark]

4.20 Assume all numbers are in 2's complement representation. Which of the following numbers is divisible by 5? (11 011)

(a) 11 001 (b) 11 010
(c) 11 011 (d) 11 101

[2004 : 1 Mark]

4.21 Which of the following is a valid [illegible] floating point number in IEEE 754 [illegible]

Bit position: 31 [illegible] 0
[illegible] | [illegible] | [illegible]
Sign bit | Exponent | Mantissa

[illegible] represents [illegible] binary in the [illegible] and mantissa [illegible] respectively. Then the floating point number represented is

(a) [illegible]
(b) [illegible]

What is the minimum difference between two successive real numbers representable in this system?

(a) 2^{-11} (b) 2^{-8}
(c) 2^{-7} (d) 2^{-4}

[2008 : 2 Marks]

4.22 $(1234)_5$ (in base 5 number system) is equal to $(x)_7$ (in base 7 number system). The possible value of x is

(a) 5, 13 (b) 10, 17
(c) 5, 26 (d) 5, 11

[2004 : 1 Mark]

4.24 What is the result of [illegible] in sign magnitude, three-digit fixed point notation, with rounding?

(a) [illegible] = −111 (b) 0.51
(c) [illegible] 111 = 7 (d) [illegible]

(c) [illegible] respectively
(d) [illegible] respectively

[2004 : 1 Mark]

4.24 An 8-bit binary lookahead adder, which is formed by [illegible] using 4-bit CLA blocks, [illegible] NOR gates only. Assuming that all the inputs are available, find the minimum time [illegible] uncomplemented forms and the delay of each gate is one nanosecond, what is the overall propagation delay of the adder? [illegible] Assume that [illegible] have [illegible] NOR [illegible] gates.

(a) [illegible] ns (b) [illegible] ns
(c) [illegible] ns (d) [illegible] ns

[2004 : 2 Marks]

4.25 [illegible] = 1111 1000 [illegible] 0000 0010 h. [illegible] the [illegible] numbers. [illegible] processor is [illegible] memory?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

4.26 The number $(2.34)_8$ is equivalent to

(a) $(2.7)_{16}$ and $(2.3)_{16}$
(b) $(2.736)_8$ and $(2.3112)_8$
(c) $(2.7)_{16}$ and $(2.3)_{16}$
(d) $(2.36)_{16}$ and $(2.3)_{16}$

[2005 : 2 Marks]

4.27 The range of integers that can be represented by an [illegible] is

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2005 : 1 Mark]

4.28 The hexadecimal representation of $(657)_8$ is

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2005 : 1 Mark]

4.29 [illegible] 8-bit [illegible] 37 will be coded as

(a) [illegible]
(b) [illegible]

[2005 : 1 Mark]

4.30 $(2.4)_8$ [illegible] evaluates to

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

Common Data for Q.4.31 & Q.4.32

Consider the following floating point format

[illegible]

[illegible] magnitude form.

4.31 The decimal number [illegible] is represented in the floating point format [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

4.32 The normalized representation for the above format is specified as follows. The mantissa has an implicit [illegible] bit to the left of the binary point. [illegible] is specified in excess [illegible] field. The normalized representation of the above number is $(0.25) \times 2^{[illegible]}$

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

4.33 The [illegible] of the [illegible] numbers [illegible] and [illegible] are

(a) [illegible] and an overflow
(b) [illegible] and an overflow
(c) [illegible] and no overflow
(d) [illegible] and no overflow

[2006 : 1 Mark]

4.34 The multiplicand and the multiplier are multiplied by multiplier $X = x_3x_2x_1x_0$ using bit-pair recoding. Booth's algorithm, partial products are generated as [illegible] in the table.

[illegible]	[illegible]	[illegible]	[illegible]	Partial Product
1	0	1	0	[illegible]
2	0		1	[illegible]
1	0	1	0	[illegible]
4	0		1	[illegible]
5	1	2	0	[illegible]
6	1		1	[illegible]
7	1	1	0	[illegible]
8			1	[illegible]

[illegible] Partial Product

(b) [illegible] and Y (d) [illegible] and Y
(c) [illegible] and Y (d) [illegible] Y

[2006 : 2 Marks]

4.45 [illegible]
(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

4.46 [illegible] floating point number in IEEE 754 single precision format
[illegible]
The value of the mantissa in decimal form is
(a) −16 (b) [illegible]
(c) −28 (d) None of these

[2008 : 1 Mark]

4.47 [illegible] floating point representation [illegible]
(a) the normalized value [illegible]
(b) the normalized value [illegible]
(c) the normalized value [illegible]
(d) the [illegible]

[2008 : 1 Mark]

4.48 [illegible] = 11, is [illegible]
(a) decimal 10 (b) decimal [illegible]
(c) decimal 10 and 11 (d) the value is 2

[2008 : 1 Mark]

4.49 The two numbers given above are multiplied using the Booth's algorithm.
Multiplicand [illegible]
Multiplier [illegible]
How many additions/subtractions are required in the [illegible] method? [illegible]
(a) 6 (b) 8
(c) [illegible] (d) 12

[2008 : 2 Marks]

4.50 [illegible] is equivalent to
(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2009 : 1 Mark]

4.51 [illegible] 16 bit signed integer [illegible] 2's complement [illegible] 2's complement representation of [illegible]
(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2010 : 1 Mark]

4.52 [illegible] integer [illegible] 2's complement [illegible] number is [illegible]
(a) −256 (b) 128
(c) 127 (d) 0

[2018 : 1 Mark]

4.53 The base for octal of [illegible] is _____.

$$\frac{81^{5}}{9^{3}} = 3^{x}$$

[2011 (Set 2) : 1 Mark]

4.54 Consider the equation $(123)_x = (321)_y$ [illegible] unknown. The number of possible solutions is _____.

[2014 (Set 2) : 1 Mark]

4.55 [illegible] variable is represented using [illegible] precision IEEE floating point format. [illegible] that uses [illegible] bits for exponent and [illegible] for mantissa. A certain variable [illegible] assigned the decimal value of 14.25. Its representation in hexadecimal notation is
(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2014 (Set-2) : 2 Marks]

4.56 [illegible] $(32)_x = (23)_y$ [illegible] and [illegible]. The number of possible [illegible] is _____.

[2015 (Set-1) : 2 Marks]

4.57 [illegible] 2's complement representation [illegible] 111 111 [illegible] 011 in decimal representation is _____.

[2016 (Set-1) : 1 Mark]

4.58 Let X be the number represented by 16-bit integer in 2's complement representation. The [illegible] number of digits [illegible] in the integer value representation. Then X [illegible] is _____.

[2016 (Set-2) : 1 Mark]

■■■■

Answers Number Systems

4.7	(d)	4.8	(a)	4.9	(d)	4.10	(d)	4.11	(c)	4.12	(b)	4.13	(a,b)	4.14	(c)	4.15	(a)
4.16	(c)	4.17	(b)	4.18	(b)	4.19	(d)	4.20	(a)	4.21	(c)	4.22	(d)	4.23	(c)	4.24	(a)
4.25	(a)	4.26	(a)	4.27	(c)	4.28	(b)	4.29	(a)	4.30	(b)	4.31	(d)	4.32	(d)	4.33	(a)
4.34	(c)	4.35	(a)	4.36	(c)	4.37	(d)	4.38	(d)	4.39	(b)	4.40	(b)	4.41	(a)	4.42	(b)
4.43	(a)																

Explanations Number Systems

4.1 (c)

When two's complement & sign magnitude numbers are added, the result is

This result is in sign magnitude form.

Therefore, when two's complement is in sign magnitude form

Result in sign magnitude form is 127

Sign magnitude form of 127 is

4.2 (c)

The equivalent of the number $(1.1)^2$ is

4.3 (c)

The number in hexadecimal system is given as

$= 2C \times 8 + 9 \times 9 + 1 \times 5 + 6 \times 1$

$= 2C \times 8 = 2C \times 7 + 2C \times 1$

The value in decimal = [illegible]

The value in decimal system = 8.3[illegible]

The hexadecimal equivalent of the given decimal number is given as:

$(8776)_{10} = (1000\ 0111\ 0001\ 0001)_2$

The number of '1's in the binary representation of the number is 6

4.4 (c)

Octal to binary conversion

4.5 (c)

$C = A \times B$

$C_{15}C_{14}C_{13}C_{12}C_{11}C_{10}C_9C_8C_7C_6C_5C_4C_3C_2C_1C_0 = (a_7a_6a_5a_4a_3a_2a_1a_0) \times (b_7b_6b_5b_4b_3b_2b_1b_0)$

[illegible]

The sum of the product of the given numbers

$A_7 = a_7b_7 + a_6b_6$

4.6 (c)

The given string can be represented as follows

[illegible]

Required = [illegible] + 64

Above 7 bits are sufficient to represent the following count values.

The number of bits required can be inferred by the formula:

$n = \lceil \log_2(\text{Max count} + 1) \rceil$ [illegible]

(a) The range of the exponent is the range of the [illegible] decimal numbers, which is [illegible]

(b) The range of the exponent if the value is represented in excess-63 format is [illegible]

4.7 (d)

Decoder can be used for multiplication of two 8-bit numbers. Size of ROM required is

$= 2^{16} \times 16$

$= 2^{6} \times 16$

$= 64 \times 16$

$= 64\ K \times 16$

4.8 (a)

Put the multiplicand multiplier as

[illegible]

[illegible]

[illegible]

[illegible]

(b) [illegible]

[illegible]

The [illegible] gives [illegible] because [illegible] maximum number of 1's.

4.9 (d):

[illegible]

Mantissa

The range of the mantissa of the [illegible] is [illegible].

4.10 (d):

When [illegible] and b_2 is 1, then 100 is added to the number.

$b_1b_2b_3b_4 = (1100)_1 = (12)_{10}$

[illegible] 12, 100 is added.

Therefore the result in binary for radix 12 is [illegible].

4.11 (d):

(a) When the radix is 10

Then $(1024)_{10} = 1000 + 24 + 1 = (1025)_{10}$

Option (a) is correct.

(b) When radix is 8

$\therefore (1024)_8 = (1 \times 8^3) + (0 \times 8^2) + (2 \times 8^1) + (4 \times 8^0) = (1044)_{10}$

and $(1024)_8 = (1000)_8$

$(1000)_8 \neq (1025)_{10}$

Therefore option (b) is incorrect.

(c) When the radix is 6

$(1024)_6 = (1000)_6 = (216)_{10}$

and $(1024)_6 = (4000)_6$

Therefore $(1024)_6 \neq (1025)_{10}$

Option (c) is incorrect.

(d) When the radix is 4

$(1024)_4 = (1000)_4 = (256)_{10}$

and $(1024)_4 = (4000)_4$

Therefore $(1024)_4 \neq (1025)_{10}$

Option (d) is incorrect.

Alternative method:

$(1024)_x = (1025)_{10}$

$\Rightarrow 2x^3 + 2x + 4 = x^3 + x + 5$

$\Rightarrow x^3 - 4x - 1 = 0$

$\Rightarrow x^3 - 2x^2 + 2x^2 - 4x - x - 1 = 0$

$\Rightarrow (x^2 - 2)(x - 2) + (x - 1)(x - 2) = 0$

$\Rightarrow (x - 2)(x^2 + x - 1) = 0$

$\Rightarrow x = 2, 1.3$

$\therefore x = 2$ satisfies the given equation.

4.12 (b):

[illegible] can be expressed in 2's complement form in 8 bits as follows:

[illegible]

Decoding to decimal number [illegible]

[illegible]

Alternative Method:

$(-100)_{10}$ [illegible] 2's complement [illegible] as follows

$(100 \times 2^0) = (1 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$

$= 64 + 8 + 4 = 76$

$\therefore -100 + 76 = -67$

4.13 (b):

[illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

4.14 (d):

The 2's complement number represents the [illegible] sample binary.

So, [illegible] binary representation [illegible]

[illegible]

4.15 (b):

$A = 2.0 \times 10^4$

$B = -2.0 \times 10^4$

$C = 1.0$

(a) $A + B + C$

$= (2.0 \times 10^4) + (-2.0 \times 10^4) + 1.0$

(ii) $Y = A + 1$

$= 2.0 \times 10^{2} + 1.0$

$= 2.0 \times 10^{2} + 0.0100\ldots1 \times 10^{2}$

$= 2.0100\ldots1 \times 10^{2} = 2.0 \times 10^{2}$

(iii) $X = X - C = 0.0 + 1.0 = 1.0$

(iv) $Y = Y + 1$

$= 1.0 \times 10^{2} + 0.0 = 2.0 \times 10^{2} = 0.0$

Therefore after the expression values of X and Y are 1.0 and 0.0 respectively.

4.16 (b)

2's complement representation of a negative number is determined as follows:

(i) Determine a binary representation of the positive value of the number.

(ii) Take the 2's complement of binary representation in

$-89_{10} = 2's\ complement\ of\ 0000\ 1011$

$= 1110\ 0101 = (1101)_{2}$

Therefore option (b) is correct.

4.17 (b)

$0.75 \times 2 = 1.50;\quad 0.50 \times 2 = 1.00$

Therefore the decimal value 0.75 is equivalent to binary 0.11

4.18 (b)

The 2's complement of negative number is represented by the binary number that when added to a positive number of the same magnitude results in 2000

$+12 = 0111$

The 2's complement representation of the decimal value −12 is

-12 in 2's complement $= 1100$

4.19 (d)

Sign-magnitude representation is commonly used in floating point number representation. The number's sign (positive or negative) is stored in a separate bit, while the number's magnitude is represented by the remaining bits.

4.20 (a)

	2's Complement	Decimal Value
(a)	0111	55
(b)	1110	−8
(c)	0101	41
(d)	1110	87
(e)	0000	7

Explanation:

(a) 25 is divisible by 5 so the binary value 110011 is divisible by 101.

4.21 (d)

Short fig. Floating point number representation

$\ldots = (-1)^{s} \times 2^{(e-127)} \times 1.m$, if the exponent

$e = 10000111 = 135$

Mantissa:

Mantissa $= 1.m = 1.0110000000000000000000 = 1.0$

Maximum value is

$= (-1)^{s} \times 2^{(e-127)} \times 1.m = 1 \times 2^{(135-127)} \times 1.0 = 256$

Now minimum value is

$= (-1)^{s} \times 2^{(e-127)} \times 1.m = 1 \times 2^{(1-127)} \times 1.0$

$= 1.0 \times 2^{-126} = 2^{-126}$

$= 1.0 \times 2^{-126} = 2^{-126}$

4.22 (d)

(i) $(75)_{10}$ if $x = 9$

$(75)_{10} = 7 \times 8 + 5 \times 8^{0}$

$= 56 + 5 = 61$

(ii) $(75)_{10}$ if $x = 11$

$(75)_{10} = 6 \times 11^{1} + 5 \times 11^{0}$

$= 66 + 5 = 71$

Alternative Method

$(75)_{10} = (65)_{x}$

$\Rightarrow 7x + 5 = 6y + 5$

$\Rightarrow 7x = 6y \Rightarrow \ldots$ (i)

The options (b), (c), (d) satisfy (i). Now check

$7 \times 5 = 5 \times 7 = 35$

$\therefore$ Option (d) is correct.

4.23 (a)

$(110)_{2} = 1 \times 2^{2} + 1 \times 2^{1} + 0 \times 2^{0}$

$= 4 + 2 + 0 = (6)_{10}$

The expression evaluates to 6.0.

The value evaluated by the expression is 6.0 and 10.0.

4.24 (a)

As per the given description for a ripple carry adder, the delay is 8 time units, and 2 time units delay is added for generating the output carry.

Hence that is clearly is prime state.

[illegible]

[illegible]

4.25 (b)

$x = (1011.10)$

$= 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 1 + 2^{-1} \times 0$

$= 2^3 + 2^2 + 2^1 + 2^0 + 2^{-1} \times 0$

$= 8 + 4 + 2 + 1 + 0 = 15$

$y = (0.0101)$

$= 2^{-1} \times 1 + 2^{-2} \times 1$

$= 0 + 0.25 = 0.25$

So the value of AB in 2's complement representation

$AB = 1000.1000$

4.26 (d)

$(1.6875)_{10} = (0.1011)_2$

$= (0.1010\ 0111\ 0010\ \ldots)_2$

$= (1.0172)_2$

$= (0.1\ 1\ 1\ 0\ 0.1\ 1\ 1\ 0\ 0.1\ 0\ 1\ 1\ \ldots)_2$

$= (1.8210282)_2$

4.27 (a)

The component of a number represent the exponent number. If a number is stored by the most significant bit as sign bit and the base = 2, biased exponent plus one represents the magnitude. So the exponent = [illegible] + 1 = [illegible]

4.28 (b)

$(45)_{10} = (0101101)_2$

$= (0.0101101)_2$

$= (0.A\ 2)_{16}$

4.29 (a)

2's complement of $(-27) = (11001101)$

Approach of [illegible] and start from right end

Letting pairs of two zeroes which are encircled as

$00 \rightarrow 0$

$01 \rightarrow 11$

$10 \rightarrow$ [illegible]

$11 \rightarrow 0$

4.30 (a)

$x = (0.1100)$

$= 0 + 0 + 0.5 + 0.25 = 0.75$

$28.4_{10} = (0.0111\ \ldots)$

$= 16 + 2 + 1 + 0.5 + \ldots$

$29.5 \times 0.75 = 22.125 = 1000.1_2$

$(0.0111 \times 0.5 = 0.01$ and if we [illegible] the [illegible] of dividend [illegible] get 1000.

4.31 (d)

The decimal number is 0.375 × 2

We have to find binary floating point representation [illegible] normalization.

Decimal [illegible] = 19 − 8 = 11

Representing 11 in binary

$11_{10} = (0011011)_2$

Representation of mantissa in binary

$mantissa_2 = (0.0011\ 0000\ 0000\ 0000)$

Floating point representation in 32 bits

Sign	Exponent	Mantissa
0	00001011	00110000000000000000

[illegible]

$= (41B0000)_H$

4.32 (b)

The decimal number is 0.0156 × 2

We have to find the mantissa [illegible] in IEEE [illegible] notation

The [illegible] representation of 0.0156 is 0.0011 1100 0010 1 1

[illegible]

After normalization we have

1.1110 0010 0101 × 2

Mantissa exponent = 10 − 6 = 4

Representation biased exponent 64 [illegible]

$exp_2 = (0000100)_2$

Floating point representation of the number is as follows

Sign	Exponent	Mantissa
0	00000100	[illegible]

[illegible]

$= (1A4E0)_H$

4.33 (a):

[illegible]

Option (a) is correct

4.34 (c):

[illegible]	Multiplicand bits multiplier			Partial product
	[illegible]	[illegible]	[illegible]	
1	0	0	0	[illegible]
2	0	0	1	[illegible]
3	0	1	0	[illegible]
4	0	1	1	[illegible]
5	1	0	0	[illegible]
6	1	0	1	[illegible]
7	1	1	0	[illegible]
8	1	1	1	[illegible]

[illegible]

[illegible]

4.35 (a):

$(C18D)_H = (1100\ 0001\ 1000\ 1101)_B$

$= (1100\ 0001\ 1000\ 1101.100)_{2\times 4}$

$= (301\ 301\ 301\ 301.100)_{2\times 4}$

$= (1011\ 1011\ 0111\ 0111.100)_{2\times 4}$

$= (BBB7.8)_H$

Binary addition and subtraction are decimal addition and subtraction.

$0 - 0 = 0$, $0 - 1 = 1$, $1 - 0 = 1$, $1 - 1 = 0$

$0 + 1 = 1$ with 1 carry.

4.36 (a):

Sign bit = 1 [illegible] number is negative

Exponent = 10000011

Exponent is coded with 127 bias in IEEE single precision format

E_{actual} = Actual exponent = 10000011 − 127 = 131 − 127 = 4

Mantissa bits = 10100000000000000000000

[illegible] implied [illegible]

Hence the Number = -1.101×2^4

$= -110.1_2 = -6.5$

4.37 (d):

[illegible] 0 together with [illegible] function [illegible] exclusive sign expression [illegible] specific value.

4.38 (d):

[illegible]

[illegible] exactly [illegible] bits [illegible] exactly [illegible] bits [illegible] be [illegible] error.

4.39 (b):

Applying boolean algebra:

[illegible]

[illegible] 4 subtractions [illegible] operations.

4.40 (b):

$(0.2\ 7)_8 = (0.010\ 111)_B$

[illegible]

4.41 (a):

8's complement representation of 8 is 8's $(D)_H$

$(-27)_{10} = (11110011)_B$

2's complement of 11110011 = P =

$\bar{P} + 1 = 1\ 1\ 1\ 1\ 0000\ 1101$

$\bar{P} = 1111\ 1000\ 0011\ 1010$

$= 0000\ 0111\ 1000\ 0101$

$= 1 \times 2^{11} + 1 \times 2^{10} + 1 \times 2^{9} + 1 \times 2^{8} + 1 \times 2^{7} + 1 \times 2^{6} + 1 \times 2^{5} + 1 \times 2^{4} + 1 \times 2^{3} + 1 \times 2^{2} + 1 \times 2^{1} + 1 \times 2^{0}$

$= 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1024 - 1 = 1023$

$8 \times 8 = 64 \times 1023 = (63936)_{10}$

Convert to binary

2	[illegible]	[illegible]
2	[illegible]	[illegible]
2	[illegible]	[illegible]
2	[illegible]	[illegible]
2	[illegible]	[illegible]
2	[illegible]	1
2	[illegible]	0
2	150	[illegible]
2	80	[illegible]
2	30	[illegible]
2	3	
2	1	1
2	5	1
	2	

[illegible]

$8 \times 7 = 000.1100000000.00$

Normalization of:

$8 \times 7 = 00.0011 \; 1101.0111$

Normalization of 8×7:

$\frac{[illegible]}{[illegible]} = [illegible]_{16}$

Alternative Method.

2's complement of Dividend no. $[illegible]_{16}$

2's complement of $8 \times 7 = [illegible]_{16}$

Left shift [illegible] $= [illegible]_{16}$

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

1.12 Sol.:

Range of 2's complement numbers

$(-2^{n-1}) \le x \le (2^{n-1} - 1)$

So minimum value expressible in 8 bits is

$-2^7 = -128$

maximum unsigned integer stored in 8 bits is

$2^8 - 1 = 127$

Exception (b) is correct.

1.13 Sol.:

[illegible]

$\Rightarrow$ [illegible]

$\Rightarrow$ [illegible]

$\Rightarrow$ [illegible]

$\Rightarrow$ [illegible]

1.14 Sol.:

[illegible]

$25 + 10 + 8 = xy + 8$

$33 = 8 + xy$

$\Rightarrow xy = 25$

Possible combinations

The radix y should be greater than [illegible]

$x = 1, y = 25$

$x = 5, y = 5$

$x = 25, y = 1$

$x = 2, y = 8$

Available y values satisfy the given equation

1. $x = 1, y = 25$
2. $x = 5, y = 5$
3. $x = 2, y = 10$

3 possibilities

1.15 Sol.:

[illegible]

$x = 1100$

$\Sigma = 187 = 8 = 1011 \ldots 1000000$

[illegible]

1.16 Sol.:

[illegible]

[illegible]

$\Rightarrow$ [illegible]

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(iv) [illegible]

(v) [illegible]

(vi) [illegible]

Total 3 possible solutions

4.47 Sol:

[illegible]

[illegible] — [illegible]

[illegible] Decimal Equivalent [illegible]

4.48 Sol:

Number of bits = 8

Range of unsigned integer that can be represented

$= [illegible]$

$= [illegible]$

$\therefore$ [illegible]

Range of magnitude represented in 2's complement

$= [illegible]$

$= [illegible]$

$\therefore$ [illegible]

Hence

$\therefore$ [illegible]

Unit . IV

Computer Organization & Architecture

■ Contents

Sl.	Topic	Page No.
6.	Cache and Main Memory	208
7.	Instructions, Pipelining and Addressing Modes	222
8.	CPU Control Design and Interfaces	241
9.	Secondary Memory & DMA	267

UNIT IV Computer Organization & Architecture

Syllabus : [illegible]

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques	2 Marks Ques	3 Marks Ques	5 Marks Ques	Total Marks
1990	[illegible]	2	[illegible]	[illegible]	9
1991	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1992	–	–	[illegible]	[illegible]	–
1993	–	–	[illegible]	–	–
1994	–	–	[illegible]	–	–
1995	–	–	[illegible]	–	–
1996	2	1	[illegible]	2	10
1997	[illegible]	1	[illegible]	[illegible]	9
1998	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1999	1	5	[illegible]	[illegible]	21
2000	0	[illegible]	[illegible]	2	15
2001	0	2	[illegible]	2	12
2002	0	3	[illegible]	[illegible]	3
2003	2	[illegible]	[illegible]	[illegible]	2
2004	[illegible]	[illegible]	[illegible]	–	18
2005	4	9	[illegible]	[illegible]	21
2006	[illegible]	7	[illegible]	–	14

Exam Year	1 Mark Ques	2 Marks Ques	Total Marks
2007	2	6	14
2008	–	[illegible]	24
2009	2	4	10
2010	[illegible]	4	9
20[illegible]	[illegible]	4	9
2012	2	2	9
2013	1	4	9
2014 Set 1	1	[illegible]	7
2014 Set 2	1	2	7
2014 Set 3	1	2	5
2015 Set 1	1	[illegible]	5
2015 Set 2	1	2	5
2015 Set 3	[illegible]	2	5
2016 Set 1	1	2	5
2016 Set 2	[illegible]	5	11

1 Cache and Main Memory

1.1 A three-way set associative cache memory has a total of [illegible] blocks divided into [illegible] sets. The main memory has a total of [illegible] blocks and each block has a size of 256 [illegible]

(i) How many bits are required for addressing the main memory?

(ii) How many bits are needed for [illegible] in the TAG, SET and WORD fields?

[1999 : 2 Marks]

1.2 [illegible] level virtual memory. The memory access time for main memory [illegible] and the memory access time [illegible] to the secondary memory [illegible] for the hit ratio. Assume that for 90% of the time the word is in 50 percent of the main memory.

[1999 : 2 Marks]

1.3 The main memory of a computer has 2048 blocks and the cache has 64 blocks. Each block has 32 words. Assume the set-associative mapping scheme with 8 blocks per set. The block [illegible]

(a) [illegible] of the cache

(b) [illegible] of the cache

(c) [illegible] of the tag

(d) [illegible]

[1998 : 1 Mark]

1.4 Which of the following statements is true?

(a) Paging [illegible]

(b) Paging [illegible] protected address space

(c) Paging [illegible] of program [illegible] where the program will be loaded in physical memory

(d) Paging [illegible] the physical memory size [illegible]

[1999 : 2 Marks]

1.5 A computer [illegible] 2 MB RAM. The [illegible] memory [illegible]

(a) [illegible] 256 [illegible] each memory

(b) [illegible] 16 million [illegible] each memory

(c) [illegible] with 16 million [illegible] each memory

(d) [illegible] each memory

[2000 : 2 Marks]

1.6 Which [illegible] direct?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[1991 : 1 Mark]

1.7 [illegible] put in one cache block [illegible]

(a) [illegible] program

(b) [illegible] program

(c) [illegible]

(d) None of the above

[2001 : 1 Mark]

1.8 Which of the following statements is false?

(a) Virtual memory implements the translation of a program's address space into physical memory addresses

(b) Virtual memory allows other benefits to exceed the size of the physical memory

(c) Virtual memory increases the degree of multiprogramming

(d) Virtual memory reduces the context switching overhead

[2001 : 1 Mark]

1.9 Consider a system with 2 level cache. Access [illegible] cache [illegible] and [illegible] respectively. The hit ratio of level 1 and level 2 cache is 0.8 and 0.9, respectively. What is the average access time [illegible] the cache [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 1 Mark]

1.10 Consider a small two-way set-associative cache memory, consisting of four blocks. For choosing the block to be replaced, use the least recently used (LRU) scheme. The number of cache misses for the following sequence of block addresses is:
8, 12, 1, 12, 8

(a) 2 (b) [illegible]
(c) [illegible] (d) [illegible]

[2001 : 2 Marks]

1.11 A cache uses RAM for memory and has a total 64 lines. It has to be mapped [illegible] memory gets mapped to cache?

(a) 10 (b) [illegible]
(c) 1 (d) [illegible]

[2005 : 2 Marks]

1.12 In general, the TLB of a computer system is used to improve performance because

(a) Virtual memory [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2005 : 1 Mark]

1.13 Consider a direct mapped cache of size 32 KB with block size 32 bytes. The CPU generates 32 bit addresses. The number of bits needed for tag, index and the offset are respectively

(a) 10, 17 (b) 10, 22
(c) 15, 17 (d) [illegible]

[2005 : 2 Marks]

1.14 A cache line stores 16 words. The main memory has latency 60 ns and bandwidth 1 GB/s. The time required to fetch the entire cache line from the main memory is

(a) 82 ns (b) 64 ns
(c) 48 ns (d) [illegible] ns

[2006 : 2 Marks]

1.15 A computer system has a level-1 cache (access time 2 cycles), a level-2 cache (access time 8 cycles) and a level-3 cache (access time 20 cycles). The cache hit ratio for different levels is as follows:

	Cache L1	Cache L2	Cache L3
Hit ratio	[illegible]	[illegible]	[illegible]

Assuming that the main memory access time is 50 cycles, the average memory access time (AMAT) in cycles is:

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

1.16 Consider a direct mapped cache with 16 blocks, each of size 32 bytes. The memory block with address 0 is mapped to cache block with address 0. Which of the following memory addresses will be mapped to cache block with address 0?

(a) 512 (b) [illegible]
(c) 1024 (d) [illegible]

[2006 : 2 Marks]

1.17 A CPU has a cache with block size 64 bytes. The main memory has 16 banks, each bank being 16 bytes wide. Consecutive words are mapped on consecutive banks (with wrap around). A block read from the cache takes 1 cycle to send the address to the main memory and multiple cycles of parallel bank accesses for the contents. Assuming that no time is lost between successive transfers on the bus, the minimum number of cycles needed to fill a cache block is

(a) 10 (b) [illegible]
(c) 17 (d) [illegible]

[2006 : 2 Marks]

Common Data for Q.1.17 & Q.1.18

Consider two cache organizations: The first one is 32 KB 2-way set associative with 32-byte block size. The second is of the same size but direct mapped. The size of an address is 32 bits. In the 2-way set associative cache, the tag field is 20 bits, the index field is 9 bits, and the offset field is 3 bits. The tag field is 21 bits, the index field is 8 bits, and the offset field is 3 bits.

1.17 The number of bits in

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

1.18 The number of bits in

(a) 24 bits (b) 25 bits
(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

Common Data for Q. 1.19 & Q. 1.20

A CPU [illegible] 2D array mapped [illegible] byte block size. Suppose [illegible] two dimensional array of size [illegible] with elements [illegible] byte each. Consider the following two C code segments, P1 and P2.

```
P1 : for (i = 0; i < 512; i++) {
        for (j = 0; j < 512; j++) {
            x = x + A[i][j];
        }
    }

P2 : for (i = 0; i < 512; i++) {
        for (j = 0; j < 512; j++) {
            x = x + A[j][i];
        }
    }
```

P1 and P2 are executed independently with the same initial state, namely, [illegible] in the cache [illegible]. Let the number of cache misses [illegible] by P1 be M_1 and that by P2 be M_2.

1.19 The value of M_1 is

(a) 0 (b) 2048
(c) 2048 (d) 32214

[2006 : 2 Marks]

1.20 The value of the ratio M_1/M_2 is

(a) 0 (b) [illegible]
(c) 1 (d) 16

[2006 : 2 Marks]

1.21 Consider a 4 [illegible] cache memory consisting of 128 lines with a block size of 512 words. The CPU generates a 32-bit address. It is known that the cache memory stores the most recent 64 blocks of the main memory. The number of bits in the TAG field of an address is

(a) 9, 8 (b) 7, 9
(c) 7, 8 (d) 8, 9

[2007 : 1 Mark]

Common Data for Q. 1.22 & Q. 1.23

Consider a machine with a byte addressable main memory of 16 Mbytes and a direct mapped cache memory of 32 Kbytes associated with the system. A block of memory consists of 16 bytes in the cache memory. Assume that the cache is initially empty. The processor fetches the bytes of data from consecutive memory locations starting from location 1000H

Common Data for Q.1.28 to 1.30

[illegible]

```
double ARR[ ][ ];
int i;
// Initialize array ARR to 0.0
for (i = 0; i < 2048; i++)
    for (j = 0; j < 2048; j++)
        ARR[i][j] = 0.0;
```

[illegible]

1.28 The total size of the page table in the system is

(a) [illegible] (b) [illegible]

(c) 8 Mbits (d) 16 Mbits

[2008 : 2 Marks]

1.29 Which of the following statements about the [illegible] is correct?

(a) ARR[0][0] (b) ARR[1][0]

(c) ARR[0][1] (d) ARR[1][1]

[2008 : 2 Marks]

1.30 The cache hit ratio for this [illegible] logic is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

1.31 How many 32 K × 1 RAM chips are needed to provide a memory capacity of 256 K-bytes?

(a) 8 (b) 16

(c) 32 (d) 128

[2009 : 1 Mark]

1.32 Consider a 4 way set associative cache memory consisting of total 16 cache blocks. The main memory consists of 256 blocks and the [illegible] memory access is in the following order:

0, 255, 1, 4, 8, 5, 188, 179, 210, 129, [illegible], 98, 132, 232

Which one of the following memory blocks will NOT be in cache if LRU replacement policy is used?

(a) 5 (b) 8

(c) 129 (d) 232

[2009 : 2 Marks]

1.33 A [illegible] memory unit has the capacity of 8 megawords (word size [illegible]). The [illegible] chip has 16 rows of cells with 16 cells in each row. The time taken for a single refresh operation is 100 nanoseconds. The time required to perform one refresh operation on all the memory cells in the memory unit is

(a) [illegible] (b) [illegible] nanoseconds

(c) [illegible] nanoseconds

(d) [illegible] nanoseconds

[2010 : 1 Mark]

Common Data for Q. 1.34 & Q. 1.35

A computer system has an L1 and L2 cache, an [illegible] cache and a main memory. [illegible] The hit ratio of L1 cache is 0.8 and of L2 cache is 0.9. The time taken to access a block in L1 cache is 2 nanoseconds, L2 cache is 10 nanoseconds, [illegible] cache is 50 nanoseconds and main memory is 500 nanoseconds. The cache and main memory unit are organized as [illegible]

1.34 What fraction of misses in L1 cache and hits in L2 cache will result in a block transfer from L2 cache to L1 cache? What is the time taken to access a block in main memory?

(a) 2 nanoseconds (b) 20 nanoseconds

(c) [illegible] nanoseconds (d) 90 nanoseconds

[2010 : 2 Marks]

1.35 When there is a miss in both L1 cache and L2 cache, first a block is transferred from main memory to L2 cache, and then a block is transferred from L2 cache to L1 cache. What is the total time taken for these transfers?

(a) [illegible] nanoseconds (b) [illegible] nanoseconds

(c) [illegible] nanoseconds (d) 586 nanoseconds

[2010 : 2 Marks]

1.36 An [illegible] random-access read/write block cache is organized as [illegible] blocks, each of size [illegible] bytes. The [illegible] memory address is 16 bits. The [illegible] tag in the address to each cache block is stored. Which of the following is the size of the tag field?

[illegible]

(c) Value or
(d) Modified value

Assume that is the minimum number of bits needed to identify the memory block mapped in the cache.
What is the total size of memory needed at the cache controller to store meta data (excluding the cache)?

(a) 4864 bits (b) 8448 bits
(c) 6656 bits (d) 5376 bits

[2011 : 2 Marks]

Linked Answer for Q. 1.37 & Q. 1.38

A cache has 64 kBytes, 4 way set associative, write back data cache with block size of 32 Bytes. The processor generates 32-bit addresses. Each cache tag directory entry contains, in addition to address tag, 2 valid bits, 1 modified bit and 2 replacement bits.

1.37 The number of bits in the tag field of an address is

(a) 11 (b) 14
(c) 16 (d) 27

[2012 : 2 Marks]

1.38 The size of the cache tag directory is

(a) 160 bits (b) 768 bits
(c) 40 Bytes (d) 56 Bytes

[2012 : 2 Marks]

1.39 In a 4 way set associative cache, the cache is divided into 4 sets, each of which consists of 4 lines. The cache uses LRU replacement scheme. The memory blocks are accessed in the sequence 0, 8, 16, 24, 32, 40, 48, 56, 0, 8, 16, 24, 32, 40, 48, 56. The main memory block numbered 0 can be mapped to any one of the cache lines from

(a) (block address) mod (4)
(b) (block address) mod (8)
(c) (block address) mod (16)
(d) (block address) mod (32)

[2013 : 1 Mark]

1.40 A RISC instruction set has 32 bit words. Its data bus is 16 bits. The number of 2-word instructions that can be stored in a 16 K × 16 ROM is

(a) 4 (b) 5
(c) 6 (d) 7

[2013 : 1 Marks]

1.41 An array sequence of cache block addresses is being accessed in a 4 way set associative cache. The number of bits in a cache block address is 32. Two consecutive accesses to the same block will never be evicted due to the LRU replacement policy. What is the minimum number of bits required to uniquely identify the cache block?

(a) 27 (b) 29
(c) 30 (d) 32

[2014 (Set 1) : 2 Marks]

1.42 A cache memory is organized as 16 blocks of 8 words. The word length is 32 bits. The size of the physical address space is 4 GB. The number of bits in the TAG field is

[2014 (Set 3) : 1 Mark]

1.43 In designing a set associative cache memory, the tag field for each block is an important parameter. Which one of the following is correct in this context?

(a) A smaller tag field implies better spatial locality
(b) A smaller tag field implies a smaller cache lookup time, but lower cache hit ratio
(c) A smaller tag field implies a larger cache hit ratio, but longer cache lookup time
(d) A smaller block size implies lower tag field miss penalty

[2014 (Set 2) : 2 Marks]

1.44 Which one of the following is a power-of-two, doubling width, doubling the capacity and block size multiplying, which one of the following is/are not affected?

(a) Width of a memory
(b) Width of an address decoder
(c) Width of a row select decoder
(d) Width of a column multiplexer

[2014 (Set 3) : 2 Marks]

1.45 The access time is 1 nanosecond for a read operation with a hit in cache, 5 nanoseconds for a read operation with a miss in cache, 2 nanoseconds for a write operation with a hit in cache and 10 nanoseconds for a write operation with a miss in cache. Execution of a sequence of instructions takes 100 instruction fetch

[illegible] machine, [illegible] type of [illegible] and [illegible] [illegible] The [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[2011 (Set-3) : 2 Marks]

1.46 [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

Explanations Cache and Main Memory

1.1 (Sol.)

(i) Main memory size

= [illegible]

= [illegible] × 256 words

= [illegible] words

= [illegible] words

Now [illegible] in main memory [illegible]

(ii) Set associative cache

Block size = 256 words = 2^8 words

∴ No. of [illegible] = [illegible]

Number of [illegible]

∴ No. of [illegible] = [illegible] = 2

Number of [illegible] = SET OFFSET is 3

Number of [illegible] in TAG = 19 − 18 = 1 bit

1.2 (Sol.)

Cache access time

$t_c = 10^{-9}$ sec

Memory access time

$t_m = 10^{-8}$ sec

Access efficiency,

$\eta = t_c/t_a = 0.9$

Average access time,

$t_a = t_c/\eta = 1.11 \times 10^{-9}$ sec

Hit ratio H

$t_a = [H \times t_c + (1 - H)t_m]$

$1.11 \times 10^{-9} = [H \times 10^{-9} + (1 - H) \times 10^{-8}]$

$0.9 = 10^{-9}/[H \times 10^{-9} + (1 - H) \times 10^{-8}]$

$H(10^{-9} - 10^{-8}) = 0.9 - 1$

$H = \frac{0.1}{1 - 0.9} = 1$

For hit ratio H = [illegible] the 20%

1.3 (D)

Cache [illegible] cache

$= \frac{\text{[illegible]}}{\text{[illegible]}}$

$= \frac{1}{8} = 8$

[illegible]

Thus [illegible] the [illegible] cache

1.4 (a)

Virtual memory is a concept which allows [illegible] the physical memory size [illegible]

1.5 (b)

(a) [illegible] memory [illegible] by graphics card

(b) [illegible] MB

(c) [illegible] MB

(d) [illegible]

(e) [illegible] by card

1.6 (b)

[illegible] the processor. Therefore, [illegible] driver

1.7 (b)

The [illegible]

1. Temporal [illegible] is [illegible] memory [illegible]
2. Spatial [illegible] memory [illegible]

[illegible]

1.8 (d)

[illegible] more program [illegible] blocks in the physical memory [illegible] normal memory [illegible] virtual memory [illegible] independent [illegible] memory space [illegible] multiprogramming.

However virtual memory [illegible]

1.9 (c).

Access time

$= [illegible]$

$= [illegible]$

$= 18.9$ ns.

1.10 (c)

Sequence: [illegible]

Assume the LRU [illegible]

Page Frames

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Page Faults	Miss	Miss	Miss	Hits

or

Total [illegible] = 4

1.11 (b) [illegible]

[illegible]

Number of [illegible] in a memory cycle

$= 100$/memory cycle time [illegible]

$= [illegible]$ time

$= [illegible]$

So time [illegible] memory cycle

$= [illegible]$

$= [illegible]$ ns

So percentage of [illegible]

$= [illegible]$

1.12 (b).

Increasing the RAM [illegible] primary memory [illegible] [illegible]

1.13 (a)

[illegible]

$= [illegible]$

$= [illegible]$

$= [illegible]$ bytes $= [illegible]$

Size of [illegible] $= [illegible] = 17$ [illegible]

[illegible]

1.14 (a)

[illegible] is 10 bytes in [illegible]

$= [illegible]$

[illegible] is 82 ns.

Total time required [illegible]

$= [illegible] + 82 = [illegible]$ ns

1.15 (a)

1. Direct: Number of [illegible]

   $= [illegible]$ blocks

   Size [illegible] $= 10$

   Number of [illegible] $= 4 = 2^2$ words

   Size of [illegible]

   Tag bits $= [illegible] = 18$

   Each [illegible]

   Total tag bits $= [illegible] \times 8$ bits.

2. Associative: Number of blocks in cache.

   $= 1024 = 2^{10}$ [illegible]

   Number of sets in cache $= 2^{10}/2 = 2^9$ sets

   Bits to represent sets $= 9$

   Size of [illegible] $= 2^2$ words

   Bits to represent words $= 2$

   Tag bits $= 30 - [illegible] = 19$

   Each block will have [illegible] tag bits. So

   Total tag bits $= 1K \times 19$ bits.

3. 2-way Number of [illegible]

   $= [illegible] = 2^9$ blocks

   Number of sets in cache $= [illegible]$

   Bits to represent [illegible] $= 9$

[illegible]

1.18 (d) :

[illegible]

1.19 (d) :

[illegible]

1.20 (b) :

[illegible]

1.21 (d) :

[illegible]

1.23 [illegible]

[illegible]

Total tag memory size = [illegible]

Total data memory size = [illegible]

So total memory size = [illegible] = [illegible] KB

Block size = [illegible] B

∴ # blocks in main memory = [illegible]

$$= \frac{[illegible]}{[illegible]} = [illegible]$$

lines in cache = [illegible]

Block size = [illegible] B

Physical address = [illegible]

[illegible]

[illegible]

[

1.25 (d):

2 way set associative cache

Cache memory size = 64 kB

Block size = 128 B

$$\text{number of sets} = \frac{64 \times 1K}{128} = 64 \times 2^{10} = 2^{16}$$

$$\text{number of sets} = \frac{2^{16}}{2} = 2^{15}$$

32 bit virtual address

Tag (17 bits)	Set (15 bits)	Byte offset (7 bits)

Tag memory size

= Number of sets × No. of tags in a set × Number of bits in a tag

$= 2^{15} \times 2 \times 17$ bits

$= 68 \times 2^{10}$ bits $= 68$ kbits

1.26 (b):

Array is APK[1024, 1024]

$$\begin{bmatrix} a[0][0] & a[0][1] & \cdots & a[0][1023] \\ a[1][0] & a[1][1] & \cdots & a[1][1023] \\ \vdots & & & \\ a[1023][0] & a[1023][1] & \cdots & a[1023][1023] \end{bmatrix}$$

1 element size = 8 B

Block size = 16 B

∴ No. of blocks in 2 blocks = 2

No. of elements = 2 elements

∴ No. of blocks required to store array = 512 blocks

Array:

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

1.30 (c):

Name = [illegible]

a[0][0] = [illegible]

a[0][1] = [illegible]

a[1][0] = [illegible]

a[1][1] = [illegible]

Loop unroll is followed by [illegible]

$$\therefore \text{Hit ratio} = \frac{1024}{1024 + 1024} = \frac{1024}{2048} = \frac{1}{2} = 50\%$$

1.31 (d):

$$\text{Block size} = 32 \times 8 = 256 \text{ b}$$

$$\text{Array size} = 256 \text{ b} \times 8 = 2048 \text{ b}$$

$$\text{Number of sets} = \frac{256 \text{ b} \times 8}{32 \times 8 \times 1} = \frac{2^8 \times 2^3}{2^5 \times 2^1} = 2^5 = 32$$

To require 8 parallel lines and 32 each parallel line

Actual [illegible] compare logic [illegible]

1.32 (d):

There are total 1 [illegible] lines [illegible]

[illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

1.83 (d)

[illegible]

Total access time = [illegible] main memory and 3 times memory bus access time [illegible]

2	Access time of L1 cache
20	Access time of L2 cache
200	Access time of MM

[illegible]

1.37 [illegible]

[illegible]

[illegible]

[illegible]

1.38 [illegible]

[illegible]

1.39 [illegible]

[illegible]

1.40 [illegible]

[illegible]

1.41 [illegible]

[illegible]

1.42 Sol.:

[illegible]

1.43 [illegible]

[illegible]

1.44 [illegible]

[illegible]

1.45 Sol.

Average seek time

= [illegible]

= [illegible]

[illegible]

Rotational time = [illegible]

[illegible]

Average disk time = [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Average inventory needs time

$$\frac{[illegible]}{[illegible]} = [illegible]$$

1.46 Sol:

$T_{[illegible]} = [illegible]$

$T_{[illegible]} = [illegible]$

$h = [illegible]$

$T_{avg} = [illegible] \times T_{[illegible]} + [illegible] \times T_{[illegible]}$

$= [illegible]$

$= [illegible]$

1.47 Sol:

Block size = [illegible] bytes = $2^{[illegible]}$ bytes

Select 7 [illegible]

Index = [illegible] bits [∵ number of lines = [illegible]]

Minimum memory size = [illegible] bytes

Physical address = [illegible] bits

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

∴ Address range = [illegible] to [illegible] (inclusive)

1.48 Sol:

Minimum memory size = [illegible] GB

Block size = [illegible]

Tag = [illegible] bits

Number of sets = [illegible] GB

∴ Total [illegible] number of lines = [illegible]

So number of sets = [illegible] lines

1.49 Sol:

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

So number of sets = $\frac{512\text{ K}}{8} = \frac{2^{[illegible]}}{2^{[illegible]}} = 2^{[illegible]}$

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

So TAG size = [illegible] bits.

1.50 Sol:

1. Miss rate = [illegible]
   T_c = [illegible] ns
   T_p = 20 ns
   T_{avg} = [illegible] ns
2. Miss rate = [illegible]
   T_p = [illegible] ns
3. Miss rate = [illegible]
   T_{avg} = [illegible] ns
4. Miss rate = [illegible]
   T_{avg} = [illegible] ns
5. Miss rate = [illegible]
   T_{avg} = [illegible] ns
6. Miss rate = [illegible]
   T_{avg} = [illegible] ns
7. Miss rate = [illegible]
   T_{avg} = [illegible] ns
8. Miss rate = [illegible]
   T_{avg} = [illegible] ns

So [illegible] is the answer.

■■■■

2 Instructions : Pipelining and Addressing Modes

2.1 The [illegible] of addressing [illegible] in a microprocessor system is minimized by

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1991 : 2 Marks]

2.2 Comparing the [illegible] in a pipelined CPU with [illegible] in a non-pipelined [illegible] but identical CPU we can say that

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible] time taken for [illegible] instruction fetch cycle

[2000 : 1 Mark]

2.3 [illegible] given below [illegible]

X. Indirect addressing Y. Stack

Z. Immediate addressing [illegible]

4. Auto decrement [illegible] commands

[illegible]

(a) [illegible]

(b) [illegible]

[2000 : 1 Mark]

2.4 [illegible] the following [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2003 : 1 Mark]

2.5 [illegible] with [illegible] beginning and [illegible] the [illegible] affected [illegible]

(a) [illegible] and the hardware [illegible]

(b) [illegible] and the [illegible] padded bits

(c) [illegible]

(d) [illegible]

[2002 : 1 Mark]

2.6 In a 2-stage [illegible]

(a) [illegible] between [illegible]

(b) [illegible]

(c) [illegible] bit [illegible] memory access

(d) [illegible]

[2002 : 1 Mark]

2.7 [illegible] pipeline [illegible]

(a) [illegible] have different [illegible]

(b) [illegible]

(c) [illegible]

(d) All of the above

[2002 : 2 Marks]

2.8 [illegible] single ALU, consider the following [illegible]

1. [illegible] instruction [illegible]
2. [illegible]
3. [illegible]

[illegible] caused:

(a) [illegible] (b) [illegible]

(c) [illegible] (d) All the [illegible]

[2003 : 1 Mark]

Common Data for Q.2.9 & Q.2.10

[illegible] the following [illegible] the [illegible] The mnemonics of [illegible] are shown as [illegible]

	MOV	[illegible]	[illegible]
	MOV	[illegible]	[illegible]
	CMP	[illegible]	Compare [illegible] with [illegible]
	[illegible]	[illegible]	Jump to [illegible] if flag [illegible]
	SUB	[illegible]	[illegible]
	ROR	[illegible]	Rotate right [illegible] by one bit

[illegible]

2.8 [illegible] the contents of register A to A, the value of register B after the program [illegible] will be

(a) [illegible] of bits in A

(b) [illegible] bits in A

(c) A

(d) A

[2004 : 2 Marks]

2.10 [illegible] instruction [illegible] Assume [illegible] the value of register A after execution of the same set of instructions?

(a) [illegible]

(b) [illegible]

(c) [illegible] by one bit

(d) [illegible]

[2005 : 2 Marks]

2.11 [illegible] of the following addressing modes [illegible] program relocation at run time?

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

(a) [illegible] and 3 (b) [illegible]

(c) [illegible] and 3 (d) [illegible]

[2004 : 1 Mark]

2.12 [illegible] of the following additions will result in an overflow?

(i) [illegible] (ii) [illegible]

(iii) [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

2.13 [illegible] instruction pipelining to speed up the performance of a CPU. [illegible] is a memory reference [illegible]

```
Tx ← M[100]
M[100] ← R2
M[100] ← R3
enable interrupts
Ty ← R1 + Tx
R2 ← M[100]
M[100] ← R2
R1 ← R3
R1 ← Ty
Tz ← M[100]
R2 ← R3
R1 ← R2
R2 ← R1
Tz ← M[100]
```

[2004 : 2 Marks]

2.14 [illegible] pipelined [illegible] Loop: [illegible] = 2; [illegible] = 2000; [illegible] (1, 28, 30, [illegible]) where the [illegible] instruction [illegible] stages [illegible]

	S1	S2	S3	S4
I1	1	2	1	2
I2	2	1	2	1
I3	1	1	2	1
I4	2	1	2	1

The [illegible] (a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

2.15 In an [illegible] of a floating point CPU, the speed ratio for floating point [illegible] is 20% and the speed ratio for fixed point add has been [illegible] to 10%. What is the overall speedup achieved if the ratio of the number of floating point operations to the number of fixed point operations is 1 : 3 and the floating point instruction [illegible] the execution time of the fixed point instruction in the original design?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

Common Data for Q.2.16 to Q.2.18

Consider the following program segment for a hypothetical CPU having three user registers R1, R2 and R3.

Instructions	Operation	Instruction Size (in words)
[illegible]	[illegible]	2
[illegible]	[illegible]	1
[illegible]	[illegible]	1
[illegible]	[illegible]	2
[illegible]	[illegible]	1

2.36 Consider that the memory is word addressable with 32 bits and the instruction is stored starting from memory location 184. Shortest Path is a program which [illegible] the [illegible] instruction [illegible] in the stack with the

(a) 1040 (b) 1041
(c) 1042 (d) 1043

[2004 : 2 Marks]

2.37 Consider that the memory is word addressable with 16 bits. A [illegible] instruction [illegible] in 1000 (decimal) [illegible] machine. What will be the actual address produced in [illegible]

(a) 1007 (b) [illegible]
(c) 1005 (d) 1006

[2004 : 2 Marks]

2.38 Total [illegible] cycles required for [illegible] instruction is

Assume:
R0 contains [illegible]
Stack contains
A[illegible] operand in memory
[illegible] cycle
Instruction [illegible]
2 bytes per word
The total number of clock cycles required to execute the program is

(a) 25 (b) 31
(c) 35 (d) 37

[2004 : 2 Marks]

2.39 A [illegible] has the stage delays as 150, 120, 150 and 110 nanoseconds respectively. Registers have [illegible] of 5 nanoseconds [illegible]. Assuming a constant clock rate, find total time taken to process 1000 data items on this pipeline will be

(a) [illegible] microseconds
(b) [illegible] microseconds
(c) [illegible] microseconds
(d) [illegible] microseconds

[2004 : 2 Marks]

2.40 Consider the [illegible] machine [illegible]

[illegible]

[illegible] addressing mode with [illegible] under [illegible]. The second operand [illegible] is [illegible] addressing mode. [illegible] addressing mode [illegible] and the [illegible] respectively. The total [illegible] of the instruction [illegible] for [illegible] designed [illegible] and [illegible] addressing modes. During execution of [illegible] instruction, the [illegible] address [illegible] is determined [illegible].

The [illegible] of memory [illegible] is [illegible]

(a) 3 (b) 4
(c) 5 (d) 6

[2005 : 2 Marks]

2.41 Match List-I (with Instructions) with the correct [illegible] and choose the [illegible] given below the lists.

List-I	List-II
A. [illegible]	1. [illegible]
B. while [illegible]	2. [illegible]
C. int [illegible]	3. [illegible]
	4. [illegible]

Codes:

	A	B	C
(a)	[illegible]	2	1
(b)	[illegible]	3	[illegible]
(c)	3	2	1
(d)	[illegible]	[illegible]	[illegible]

[2005 : 2 Marks]

2.42 A 5 stage pipeline [illegible] has the following sequence of stages

IF — Instruction fetch from memory
ID — Instruction decode and operand fetch,
EX — Execute ALU operation for data and address computation,
MA — Data memory access — for write access the pipeline stalls [illegible] cycle,
WB — Register write back.

(a) [illegible] microseconds
(b) [illegible] microseconds
(c) [illegible] microseconds
(d) [illegible] microseconds

[2001 : 2 Marks]

Consider the following sequence of instructions:

I1: [illegible]

I2: [illegible]

I3: [illegible]

Instructions are executed in a pipelined manner.

What is the minimum number of clock cycles taken to completely execute the above sequence of instructions starting from the beginning of I1?

(a) 5 (b) 10

(c) 12 (d) 16

[2005 : 2 Marks]

Common Data for Q.2.23 & Q.2.24

Consider a [illegible] bit microprocessor with [illegible]

The MAR, the buses and all the registers in the data path are [illegible] bits wide. All operations including memory read operation [illegible] are assumed to be carried out in the ALU. Two clock cycles are needed for memory read operation — the first one for loading address in the MAR and the second one for loading data from the memory into the MDR.

2.23 The instruction fetch [illegible] takes the register transfer interpretation [illegible]. The minimum number of clock cycles needed for execution cycle of the instruction is

(a) 2 (b) 3

(c) 4 (d) 6

[2005 : 2 Marks]

2.24 The instruction "Add R0, mem" is a two word instruction. Assuming that PC is incremented during the first cycle of the fetch cycle, the register transfer interpretation is

Fetch [illegible]

[illegible] ← M[PC],

The minimum number of CPU clock cycles needed for the execution cycle of this instruction is

(a) 2 (b) 3

(c) 4 (d) 5

[2005 : 2 Marks]

2.25 [illegible] bit [illegible] for a pipelined microprocessor. [illegible] has [illegible] stages with execution times of [illegible] respectively. [illegible] pipeline [illegible] instruction time. [illegible] pipeline design, [illegible] speedup [illegible] for executing [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

2.26 A CPU has 24-bit instructions. A program starts at address 800 (hexadecimal). Its first instruction is [illegible]. The following is a list of programs [illegible] in decimal?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 1 Mark]

2.27 Which of the following statements is/are true?

(i) [illegible]

(ii) [illegible] of array elements with [illegible] array instruction

(iii) It reduces memory fragmentation of data

(iv) It enables faster address calculations than absolute addressing

[2006 : 2 Marks]

2.28 [illegible] instructions [illegible] and [illegible] [illegible] and [illegible] [illegible] the following program instructions.

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Which of the statements below is/are true about the program instructions?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2008 : 2 Marks]

2.29 A CPU has five-stage pipeline and runs at 1 GHz frequency. Instruction fetch happens in the first stage of the pipeline. A conditional branch

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

2.50 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible] [2008 : 2 Marks]

2.51 The data path shown in the figure computes the number of [illegible] the 8-bit input word corresponding to [illegible] the shift register. The initial contents of the shift register are [illegible] is the least significant bit of this register.

The microprogram for the control is shown in the table below with missing control words for [illegible]

[illegible] the number of [illegible] microinstructions [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

Common Data for Q.2.52 & 2.53

A pipelined processor consists of four instruction pipeline with the following stages: Instruction fetch (IF), instruction decode (ID), Execute (EX), and Write back (WB). The corresponding operations are [illegible] load and store instructions are carried out in the EX stage. The sequence of instructions corresponding to the statements [illegible] The values of variables P, Q, R, S and T are stored in the register R0, R1, R2, R3 and R4 respectively. [illegible] the same [illegible]

ADD	R0, R1, R2	R2 ← R1 + R0
MUL	R3, R2, R1	R1 ← R2 × R3
SUB	R4, R3, R0	R0 ← R3 − R4
DIV	R0, R4, R1	R1 ← R4 / R0
STORE	R0, X	X ← R0

2.52 [illegible] dependencies. Write-After-Read (WAR) dependencies and Write-after-Write (WAW) dependencies. The number of [illegible] respectively:

(a) 2, 3, 4 (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

2.53 [illegible] The IF, ID, EX, WB stages [illegible] for the ADD, SUB, MUL and DIV operations. Operand forwarding [illegible] required to execute the sequence of instructions is:

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

2.34 A processor takes 12 cycles on average to complete an instruction. The average clock rate of the processor is 1.5 GHz. Suppose a pipelined processor takes 5 cycles per instruction, what is the speedup assuming that every instruction is executed?

(a) 2.4 (b) 3

(c) 9 (d) 6

[2007 : 1 Mark]

2.35 Data forwarding techniques can be used to speed up the execution of a program. Consider the following pipelined machine with IF, ID, EX, MEM, WB stages.

I1: R1 ← R2 + R3, R2 ← R4 + R5, R1 ← R6 + R7

I2: R1 ← R1 + R2, R2 ← R4 + R5, R1 ← R2

I3: R1 ← R1 + R2, R2 ← R4 + R5, R1 ← R2

I4: R1 ← R1 + R2, R2 ← R4 + R5, R1 ← R2

In which of the following pairs, will the result of execution of the 4 instructions be same as executing the instructions in sequence of the instructions shown below?

(a) (I1 and I2) (b) (I2 and I3)

(c) (I3 and I4) (d) (I1 and I4)

[2007 : 2 Marks]

2.36 Delayed branch is a technique for improving the performance of pipelined instruction execution. Consider the following instructions and their execution times.

Instruction	Execution time (cycles)	Comments
Add R1, R2, R3	1	R1 ← R2 + R3
Sub R4, R5, R6	1	R4 ← R5 − R6
Jnz	2	Jump if not zero
Load	2	Load from memory

Consider the following sequence of instructions.

Loop: Load R2, 1000	Load R2 from memory location 1000
Add R1, R2, R3	Add R1 to R2 and store result in R3
Sub R4, R5, R6	Subtract R5 from R4 and store result in R6
Jnz R2, Loop	Decrement R2
Jnz R1, Loop	Decrement R1
Mul R1, R2, R3	Multiply R1 and R2 and store result in R3
Store R3, 1000	Store R3 in memory location 1000

What is the minimum number of cycles needed to execute the instructions shown above assuming each instruction takes one cycle to execute?

(a) 5 (b) 6

(c) 8 (d) 14

[2007 : 2 Marks]

2.37 Consider a pipelined processor. Instructions have four stages:

IF: Instruction Fetch

ID: Decode

WO: Write Operand

The IF, ID and WO stages take one clock cycle each for any instruction. The result of an instruction is available at the EX stage of the instruction. The MEM and WB stages take one clock cycle each for all instructions. Consider the following sequence of instructions:

Add	R0,	R1,	R2	R3 ← R1 + R2
MUL	R4,	R5,	R6	R4 ← R5 * R6
SUB	R0,	R0,	R4	R0 ← R0 − R4

(a) 7 (b) 9

(c) 10 (d) 14

[2007 : 2 Marks]

Common Data for Q.2.38 to Q.2.40

Consider the following program segment. Here R1, R2 and R3 are the general purpose registers.

Instruction	Operation	Instruction size (bytes)
MOV R1, (3000H)	R1 ← M[3000H]	3
LOOP:		
MOV R2, (R1)	R2 ← M[R1]	1
ADD R2, R1	R2 ← R1 + R2	1
MOV (R1), R2	M[R1] ← R2	1
INC R1	R1 ← R1 + 1	1
DEC R3	R3 ← R3 − 1	1
BNZ LOOP	Branch on not zero	2
HALT	Stop	

Assume that the content of memory location 3000H is 10 and the content of the register R3 is 100. The content of each memory location from 2000H to 2064H is 55H. The processor has 64 bytes of memory and the instruction begins execution from memory location 1000H. All the numbers are in hexadecimal.

2.39 [illegible] memory is word addressable. The number of memory words [illegible] the program sample size is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

2.39 Assume that the memory is word [illegible] After the execution of the program, the number of memory bits in 2001 is

(a) [illegible] (b) 101

(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

2.40 [illegible] interrupt occurs during the execution of the instruction [illegible] What [illegible] will be pushed on to the stack?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

2.41 [illegible] 2007, part one additional [illegible] following [illegible] After the execution of this additional [illegible] the contents of the [illegible] are

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 1 Mark]

2.42 [illegible] Q.42 – Q.44 is based on the following [illegible] with X [illegible] is added to the contents of location X with X decremented by one word [illegible] the effective address is calculated [illegible] Q.42 [illegible] the effective address equal to the contents of location X with X [illegible] Q.43 [illegible] the effective address is calculated [illegible] the effective address equal to the contents of location X with X decremented by one word [illegible] the effective address equal to the [illegible] Q.44 [illegible] the following [illegible] from the above [illegible] the addition operation [illegible] the stack.

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

2.43 A [illegible] pipeline single cycle processor [illegible] at 200 MHz [illegible] pipeline [illegible] with [illegible] per stage [illegible] 2.0 nsec, 1.5 nsec and 2.5 nsec [illegible] The speed up of the pipeline processor for a large number of instructions is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

2.44 Which of the following [illegible] is/are [illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

2.45 Which of the following [illegible] pipelined processor?

1. Increasing the [illegible]
2. [illegible]
3. [illegible]

(a) 1 and 2 only (b) [illegible]

(c) [illegible] only (d) 1, 2 and 3

[2006 : 2 Marks]

Common Data for Q.2.46 & Q.2.47

Delayed branch [illegible] helps in the scheduling of instructions.

2.46 For all delayed conditional branch instructions, [illegible]

(a) [illegible] following the conditional [illegible] in memory is executed

(b) The first instruction in the taken branch [illegible]

(c) The first instruction in the taken path is executed

(d) [illegible] the pipeline [illegible] of instruction execution

[2006 : 2 Marks]

2.47 The [illegible] pipelined [illegible] with one branch delay slot.

I1: [illegible] R7 + R5
I2: [illegible] R7 [illegible]
I3: [illegible] I2 + [illegible]
I4: [illegible] Memory [I3]; R1
[illegible]
Which of the [illegible] I1, I2, I3 and I4 [illegible] legitimately access the value [illegible] without program modification?

(a) [illegible] (b) I4
(c) [illegible] (d) I1

[2008 : 2 Marks]

2.48 Consider a hypothetical processor. The number of cycles needed by the four instructions I1, I2, I3, I4 in a sequence P1, P2, P3, P4 is shown below:

	P1	P2	P3	P4
I1	[illegible]	1	1	1
I2	[illegible]	3	2	[illegible]
I3	[illegible]	[illegible]	1	2
I4	[illegible]	2	2	[illegible]

What is the minimum number of cycles needed to execute the following group?

for i = [illegible] do P1; I2; I3; I4 [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

2.49 A five-stage pipelined processor has Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Instruction Execute (IE), and Write Operand (WO) stages. The IF, ID, OF and WO stages take 1 clock cycle each for any instruction. The IE stage takes 1 clock cycle for ADD and SUB instructions, 3 clock cycles for MUL instruction, and 6 clock cycles for DIV instruction respectively. Operand forwarding is used in the pipeline. What is the number of clock cycles needed to execute the following sequence of instructions?

Instruction	Meaning of Instruction
I_1: MUL R_2, R_0, R_1	$R_2 \leftarrow R_0 * R_1$
I_2: DIV R_4, R_2, R_3	$R_4 \leftarrow R_2 / R_3$
I_3: ADD R_5, R_4, R_0	$R_5 \leftarrow R_4 + R_0$
I_4: SUB R_6, R_5, R_3	$R_6 \leftarrow R_5 - R_3$

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2010 : 2 Marks]

2.50 The expression below uses six temporary variables [illegible]

[illegible]
s = 1
t = 10
u = 20
a = [illegible] b
[illegible]
[illegible]
b = [illegible]
[illegible]
d = [illegible]
return d;

Assuming that all operations take their operands from registers, what is the minimum number of registers needed to evaluate the expression without spilling?

(a) 3 (b) 4
(c) [illegible] (d) 6

[2010 : 2 Marks]

2.51 Consider a hypothetical processor with an instruction of type LW R1, 20(R2), which during execution reads a 32-bit word from memory and stores it in register R1. The effective address of the memory location is obtained by the addition of a constant 20 and the contents of register R2. Which of the following addressing modes does this instruction use to access memory?

(a) Immediate Addressing
(b) Register Addressing
(c) Register Indirect Scaled Addressing
(d) Base Indexed Addressing

[2011 : 1 Mark]

2.52 Consider evaluating the following expression tree on a machine with load-store architecture in which memory can be accessed only through load and store instructions. The variables a, b, c, d and e are initially stored in memory. The temporary operands and the result of this expression can be evaluated in the register file of the processor. There are no common subexpressions. Operands can be fetched from memory into the register file, but the result of an operation must be stored in memory. What is the minimum number of registers needed to evaluate this expression?

(a) 2 (b) 6
(c) 5 (d) 9

[2011 : 2 Marks]

2.33 Consider an instruction pipeline with four stages (S1, S2, S3 and S4) each with combinational circuit only. The pipeline registers are required between each stage and at the end of the last stage. Delays for the stages and for the pipeline registers are as follows:

What is the speed up ratio of the pipelined unit in steady state under ideal conditions when compared to the corresponding non-pipelined unit in steady state?

(a) 1.0 (b) 2.5
(c) 1.1 (d) 3.0

[2011 : 2 Marks]

2.34 Register renaming is done in pipelined processors

(a) as an alternative to register allocation at compile time
(b) for efficient access to function parameters and local variables
(c) to handle certain kinds of hazards
(d) as part of vector processing

[2012 : 1 Mark]

2.35 Consider the following sequence of instructions

MUL R5, R0, R1
DIV R6, R5, R7
ADD R8, R6, R2
Assume that MUL takes 4 cycles and DIV takes 5 cycles in execution stage and all other operations take 1 cycle. Assume that there is no forwarding. Which one of the following is a correct sequence of pipeline stages for the instruction sequence given above?

(a) Instruction fetch
(b) Operand fetch
(c) Execute
(d) Result write back

[2013 : 2 Marks]

2.36 Consider an instruction pipeline with five stages without any branch prediction. Each instruction is fetched, decoded, executed, accessed memory and finally written back. The stage delays for FI, DI, FO, EI and WO are 6 ns, 7 ns, 10 ns, 8 ns and 6 ns respectively. There are intermediate storage buffers after each stage and the delay of each buffer is 1 ns. A program consisting of 12 instructions $I_1, I_2, I_3, \ldots, I_{12}$ is executed in this pipelined processor. Instruction I_4 is the only branch instruction and its branch will be taken. If the branch is taken, the time (in ns) needed to complete the program is

(a) 87 (b) 90
(c) 132 (d) 135

[2013 : 2 Marks]

2.37 A machine has a 32 bit architecture, with 1 word long instructions. It has 64 registers, each of which is 32 bits long. It needs to support 45 instructions, which have immediate operand in addition to two register operands. Assuming that the immediate operand is an unsigned integer, the maximum value of the immediate operand is _____.

[2014 (Set-I) : 1 Mark]

2.38 Consider a 5-stage instruction pipeline, in which all stages are perfectly balanced. Assume that there is no cycle time overhead of pipelining. When an instruction is executed in this 5-stage pipeline, the speed up achieved with respect to non-pipelined execution is 2.5. If the stage delay of the pipeline is 2 ns, the non-pipelined cycle time is

[2014 (Set-1) : 2 Marks]

2.39 Consider two processors P_1 and P_2 executing the same instruction set. Assume that under identical conditions, for the same input, a program takes longer on P_2 than on P_1. Which of the following is/are possible?

(a) P_2 has a higher clock rate but fewer stages than P_1
(b) P_2 has a higher clock rate but more stages than P_1
(c) P_2 has a lower clock rate but fewer stages than P_1
(d) P_2 has a lower clock rate but more stages than P_1

[2014 (Set-1) : 2 Marks]

of P is 2.5 GHz. Then the clock frequency of P is [illegible] GHz.

[2011 (Set 1) : 2 Marks]

2.60 Consider the following pipelined processors with the following characteristics. Assume that they are implemented on identical circuit technology.

P1: Four stage pipeline with stage latencies 1 ns, 2 ns, 2 ns, 1 ns.

P2: Four stage pipeline with stage latencies 1.5 ns, 1.5 ns, 1.5 ns, 1.5 ns.

P3: Five stage pipeline with stage latencies 0.5 ns, 1 ns, 1 ns, 0.5 ns, 1 ns.

P4: Five stage pipeline with stage latencies 0.5 ns, 0.5 ns, 1 ns, 1 ns, 1 ns.

Which processor has the highest peak clock frequency

(a) P1 (b) P2

(c) P3 (d) P4

[2014 (Set 1) : 1 Mark]

2.61 An instruction pipeline has five stages, namely instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MA), and register write back (WB) with stage latencies 0.5 ns, 2 ns, 2 ns, 1 ns and 0.75 ns, respectively (including the latch delay). The pipeline is clocked frequency. Its designers have decided to split the ID/RF stage into two stages (ID, RF), and the EX stage into three stages (EX1, EX2, EX3) each of latency 1 ns. Assuming ideal conditions, the speed-up of the new design with the new pipeline is _____.

[2014 (Set-2) : 2 Marks]

2.62 Two processors based on three address instruction formats, including one address, two address and three address instructions are used to execute a program. The instructions are of the following types:

S1: Arithmetic operations

S2: Arithmetic operations with one operand in memory

S3: Arithmetic operations with two operands in memory

(a) Instructions S1 and S2 only (b) Only S2 and S3

(c) Only S1 and S3 (d) All S1, S2 and S3

[2015 (Set-1) : 1 Mark]

2.63 Consider a non-pipelined processor with a clock rate of 2.5 gigahertz and average cycles per instruction of four. The same processor is upgraded to a pipelined processor with five stages but the clock rate of the pipelined processor is 2 gigahertz. Assume that there are no stalls in the pipeline. The speed-up of the pipelined processor is _____.

[2015 (Set-1) : 2 Marks]

2.64 Consider the sequence of machine instructions given below:

MUL R5, R6, R7	R5 = R6 * R7
DIV R8, R9, R10	R8 = R9 / R10
ADD R3, R5, R8	R3 = R5 + R8
STR R3, R0, R4	R3 → R0 + R4

In the above sequence, R0 to R10 are general purpose registers. In the instructions shown, the first register stores the result of the operation performed on the second and third registers. This sequence of instructions is executed in a pipelined processor with the following 4 stages: (1) Instruction Fetch and Decode (IF), (2) Operand Fetch (OF), (3) Perform Operation (PO) and (4) Write back the Result (WR). The IF, OF and WR stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD and STR, 2 clock cycles for MUL and 3 clock cycles for DIV instruction. The pipelined processor uses operand forwarding from the PO stage to the OF stage. The number of clock cycles taken to execute the above sequence of instructions is _____.

[2015 (Set-2) : 2 Marks]

2.65 Consider a processor with byte addressable memory. Assume that it supports a stack organized in the main memory. Assume that the contents of the Program Counter (PC) and Stack Pointer (SP) are 4000 and 5000, respectively. The stack grows towards lower addresses. The stack pointer (SP) points to the topmost valid element of the stack. Assume that the current value of SP is 0100. The CALL [illegible]

instruction and assume the first memory location opcode and the second word is the operand address of the subroutine (one word = 2 bytes). The CALL instruction is implemented as follows:

- Store the contents of PC in the stack
- Store the contents of PSW register in the stack
- Load the starting address of the subroutine in PC

The content of PC just before fetch of the CALL instruction is 0F00₁₆. The address of the CALL instruction, the address of the stack pointer is

(a) 0F84₁₆ (b) 0F86₁₆
(c) 0F04₁₆ (d) 0F02₁₆

[2014 (Set 2): 2 Marks]

2.66 Consider the following pseudo-sequence of pipeline instructions I_1 to I_3. Each of these instructions is a 16-bit instruction.

I_1: R1, R2, R3

where operation OP is performed on the contents of registers R2 and R3. The result is stored in register R1.

I_1: ADD R1, R2, R3 I_2: MUL R5, R1, R3
I_3: SUB R4, R1, R5 I_4: ADD R6, R2, R4
I_5: MUL R7, R5, R4

Consider the following two statements:

S1: There is no data dependence between instructions I_1 and I_3

S2: There is no data dependence between instructions I_2 and I_4

S3: Within an instruction pipeline, any data dependence always creates one or more stalls.

Which of the above statements is/are correct?

(a) Only S1 is true
(b) Only S2 is true
(c) Only S1 and S3 are true
(d) Only S2 and S3 are true

[2013 (Set 2): 2 Marks]

2.67 Consider the following reservation table for a pipeline having three stages S_1, S_2 and S_3.

	Time → 1	2	3	4	5
S_1	X				X
S_2		X		X	
S_3			X		

The minimum average latency (MAL) is _____.

[2014 (Set 3): 2 Marks]

2.68 Three stage pipeline has stage delays 300, 500, 400 and 400 picoseconds. The first stage (with delay 300 picoseconds) is replaced with a functionally equivalent design involving two stages with respective delays 200 and 300 picoseconds. The throughput increase of the pipeline is _____ percent.

[2016 (Set-1): 2 Marks]

2.69 A processor has 40 distinct instructions and 24 general purpose registers. A 32-bit instruction word has two operands followed by an immediate operand. The number of bits available for the immediate operand field is _____.

[2016 (Set-2): 1 Mark]

2.70 Consider a processor with 64 registers and an instruction set of size twelve. Each instruction has exactly two operands, each operand being a register. Instruction is to be stored in memory in an integer number of bytes. The instruction format has an opcode field followed by two operand fields. If a program has 100 instructions, the amount of memory (in bytes) consumed by the program text is _____.

[2016 (Set-3): 2 Marks]

2.71 Consider a 5-stage pipeline processor with a clock cycle time of 2 ns. The first stage of the pipeline has a delay of 1.5 ns, the second stage has a delay of 1 ns, the third stage has a delay of 1.5 ns, the fourth stage has a delay of 1 ns, and the fifth stage has a delay of 1.5 ns. Assuming that there are no stalls in the pipeline, the latency of the pipeline is _____ ns.

[2016 (Set-3): 2 Marks]

2.72 Suppose instructions I and G can be computed in stages F and H respectively. Given two instances I_1 and I_2 and two instances of G_1, it is required to implement the computation $F(I_1, G_1)$ for each I_1 using a pipelined processor. The minimum time required to complete the computation is _____ nanoseconds.

[2016 (Set-3): 2 Marks]

■■■■

Answers Instructions, Pipelining and Addressing Modes

2.1	(b)	2.2	(b)	2.3	(c)	2.4	(c)	2.5	(a)	2.6	(b)	2.7	(c)	2.8	(a)	2.9	(b)
2.10	(a)	2.11	(c)	2.12	(b)	2.13	(d)	2.14	(c)	2.15	(d)	2.16	(c)	2.17	(b)	2.18	(b)
2.19	(c)	2.20	(d)	2.21	(c)	2.22	(a)	2.23	(b)	2.24	(b)	2.25	(b)	2.26	(a)	2.27	(b)
2.28	(d)	2.29	(a)	2.30	(a)	2.31	(d)	2.32	(a)	2.33	(a)	2.34	(b)	2.35	(b)	2.36	(b)
2.37	(b)	2.38	(d)	2.39	(a)	2.40	(c)	2.41	(c)	2.42	(b)	2.43	(c)	2.44	(d)	2.45	(d)
2.46	(b)	2.47	(c)	2.48	(d)	2.49	(b)	2.50	(b)	2.51	(d)	2.52	(d)	2.53	(b)	2.54	(c)
2.55	(d)	2.56	(b)	2.57	(c)	2.58	(a)	2.59	(d)	2.60	(b)						

Explanations Instructions, Pipelining and Addressing Modes

2.1 (b)

Vertical microprocessor is independent of size of main memory and depends only on the number of the microprocessor.

2.2 (b)

In pipelined CPU, time will be influenced. So the memory in non-pipelined CPU takes less time compared to pipelined CPU.

2.3 (c)

Indexed addressing mode is used for pointer arithmetic and array addressing mode. It is used for accessing array elements and auto increment addressing mode is used for loops.

2.4 (c)

Stack pointer is the pointer of machine language instruction that tells the operation to be performed.

2.5 (a)

Bus is the line through which the data or bytes are transmitted from one place to another. Start and stop bits are used for synchronization purpose.

2.6 (b)

When two or more address complement numbers are added, overflow is determined by

1. Both operands are positive and the result is negative.
2. Both operands are negative and the result is positive.

On multiplying two numbers of n-bits, it can produce overflow because the product of two n-bit signed numbers is a 2n-bit number.

2.7 (c)

Instructions are caused due to dependencies. Different pipelining hazards are:

1. Structural dependency: The pipeline stages have different delays.
2. Control dependency: Unconditional instructions are executed out of order.
3. Data dependency: The pipeline stages of your hardware conflicts.

2.8 (d)

(i) Only P instruction uses the result of the j instruction as an operand. Hence, write-read (WAR) hazard occurs. It is a part of data dependency.

(ii) The execution of a conditional jump instruction causes a dependency condition dependent on the result.

So (i) and (ii) are instructions required to NOP as the same instructions causes write-read (WAR) hazard.

2.9 (b)

$$A = A_0$$

The value of R after execution of the given instructions number of A_0.

2.10 (a)

Insert five instructions

DIV A, B

2.11 (a)

Based on addressing mode & relative addressing mode provide the program relocation. Relocation is not possible in following reason. In relative addressing mode the effective address field contains a relative address (or displacement D). If $R_1, R_2, R_3, \ldots R_n$ are GPR registers then A is computed as follows.

$$A = R + D$$

By changing contents of R the processor can access the immediate address referred by a block of memory. R. This address field contains displacement, the processor to move from location to location of memory to another without modifying the address in R. When used in conjunction with base register and index register addressing.

2.12 (b)

Overflow happens only when sign bit of two input numbers is 0, and the result has sign bit 1.

Sign bit of two input numbers is 1, and the result has sign bit 0.

Overflow indicates that signed arithmetic while carry is important only for unsigned arithmetic. A carry happens when there is a carry out of the most significant bit. Here quotient cannot carry but only for unsigned numbers.

2.13 (d)

Data forwarding means ALU (EX) writes to a common register and subsequently reads from that same memory location. Instead of stalling the pipeline, the value directly from the register is fed to the ALU, implied memory, so this increases the performance.

Here, the value of R_1 is 0 does not actually matter as the data will be in R_1 and R_2 in stage ID. R_0 and $R_1[100]$.

2.14 (c)

Time	1	2	3	4	5	6	7	8
I	IF	ID	EX	WB				
I		IF	ID	EX	WB			
I			IF	ID	EX	WB		
I				IF	ID	EX	WB	
I					IF	ID	EX	WB

Total time = 10 ns

2.15 (d)

In CISC system 8 floating point operation and 2 fixed point operation.

Execution time of 2 floating point operation = 1 cycle.

Total time in old system = 2 × 20 + 8 × 1 = 50 units

In Pipelined system, floating point takes = 2 × 0.8 = 1.6 units

Fixed point takes = 1 × 0.8 = 0.8 units

Total time = 1.6 × 8 + 0.8 × 2 = 13.6 units

The speed up = 50/13.6 = 3.68

2.16 (c)

Instruction	Instruction size (words)	Execution time (cycles)
MOV R_0, 2000	2	10 to 100
MOV R_1, [R0]	1	10 to 100
ADD R_1, R_1	1	10 to 100
MOV [2000], R_1	2	10 to 100
Halt	1	10 to 100

If an interrupt occurs, the CPU has to wait after completing the HALT instruction. Even address 020 executed in two cycle.

2.17 (b)

Instruction	Instruction size	Execution interval
MOV R_1, 1000	2	0.01 to 0.01
MOV R_2, [R1]	1	0.01
ADD R_2, R_2	1	0.005
[illegible]	1	0.01 – 0.05
HALT	1	0.005 – [illegible]

The interrupt address will be [illegible] 1004.

2.18 (b)

Operation	Instruction size	Reg. read Clock cycles
R_0 ← Memory [3000]	2	2 × 1 + 2 = 4
R_2 ← Memory [1000]	1	1 × 1 + 2 = 3
R_0 ← [R_0 + R_2]	1	1 = 1
Memory [3000] ← R_0	2	2 × 2 + 2 = 6
Machine Halt	1	1 + 1 = 2
		Total = 21

2.19 (d)

[illegible] times

[illegible]

When [illegible] is [illegible] maximum stage delay is max {150, 180, 100, 120}

= 180

[illegible]

[illegible]

[illegible]

[illegible]

2.20 (b)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

2.21 (b)

The correct matchings are

1. [illegible] — Immediate Addressing Mode
2. While(A[i]) — Autoincrement
3. [illegible] — Indirect Addressing Mode

2.22 (b)

This is RISC pipeline and an ARM pipeline. Data forwarding [illegible] branch penalty is 1.

Number of cycles = 8 cycles

2.23 (b)

[illegible]

The sequence of instructions takes [illegible] following cycles

I cycle: [illegible]

II cycle: [illegible]

III cycle: [illegible]

[illegible]

2.24 (b)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

2.25 (b)

Execution time of one instruction = [illegible] execution time

where k = No. of stages in pipeline

n = No. of instructions executed time

Without pipeline execution time

[illegible]

[illegible]

Time saved in ns, [illegible] = 210 = 202.

2.26 (d)

Fixed instruction 2 bits

Input/output address of the program is 8 bits. The size of instruction is 8 bits long so the address is always in multiple of 8 bits. The address is 400 [illegible] instructions of the program.

2.27 (b)

Relative addressing can not be faster than absolute addressing as absolute address must be calculated from relative address.

(a) It makes it easier to address memory use a much smaller relative address in the instructions which results in smaller instruction size.

(c) By using the base address of array we can access array elements using relative addressing.

(d) It can also be used to release the base address in case of relocation. Instructions remain the same.

2.85 (b)

Memory location 1001 has value = 50

$R_0 \leftarrow$

$R_1 \leftarrow 1$

$R_2 \leftarrow 1001$

Same instruction address 1001 = 50.

2.86 (d)

[illegible]

2.87 (a)

[illegible]

2.88 (c)

[illegible]

2.89 (b)

[illegible]

2.90 (b)

[illegible]

The same clock cycle is required to execute all instruction.

2.91 (b)

[illegible]

2.92 (b)

[illegible]

2.93 (b)

[illegible]

2.94 (b)

[illegible]

2.95 (d)

Pipelined processor has 4 stages IF, ID, EX, WB

Clock Cycles	Instruction
1	ADD
2	SUB
3	MUL

Corresponding pipeline diagram

Clock Cycle	1	2	3	4	5	6	7	8
[illegible]	[illegible]							
[illegible]		[illegible]						
[illegible]			[illegible]					

Required minimum clock cycle = 8.

2.38 (d)

Given: M[R1] = 10

Instruction	Required Memory Transfer
$R_1 \leftarrow M[R_1]$	1
$R_1 \leftarrow M[R_1]$ and R_2	10
$M[R_1] \leftarrow R_2$	10
	21

2.39 (a)

The instruction at location 200 is executed and the memory location 300 (whose address is 100) is incremented to 301 by the ISZ loop.

2.40 (b)

Location	Operation
100	$R_1 \leftarrow M[200]$
101	$R_2 \leftarrow R_2 + R_1$
102	$R_1 \leftarrow M[R_1]$
103	$R_2 \leftarrow R_2 - R_1$
104	$M[R_1] \leftarrow R_2$
105	$R_2 \leftarrow R_2 - 1$

At the time of instruction fetch, the contents of the instruction (ISZ R_2) for the current address (106) will be pushed on to the stack.

2.41 (d)

Binary equivalent of complement of 2000 is 1111 1111 1111 1111 is equivalent to decimal 65535

1111 0000 1111 0000 is equivalent to decimal 61200

After addition operation, 65535 + 61200 = 126735

1111 0000 1111 0000 = 1000 0000 0000 0000 (overflowing result is 0)

Carry flag = 0, Overflow flag = 1, and sign flag = 0.

2.42 (b)

In cycle 18, the A is 888 = 1000 + 888. Each memory fetch takes 10 cycles.

Each cache hit is 1000 in 2 cycles. Each miss is equivalent to the 1000 in 10 cycles. It increments after the 2 cycles for the hit and the 888 and that is where we start to count. In cycle 18 we stock to incrementing. In cycle 18 and 19 it increments 888 + 888 = 1776

2.43 (d)

For non-pipelined unit, time required = 5 + 6 + 7 + 8 + 9 = 35

For pipelined unit, time = (max delay) × (no. of stages) + (latch delay) = 9 × 5 + 1 = 46

Speed up = (time in non-pipelined unit)/(time in pipelined unit) = 35/46 = 0.76

2.44 (d)

A RISC (Reduced Instruction Set Computer) architecture utilizes a simple instruction set, provides only those instructions which are frequently used in a computer, and fits each instruction in a fixed length word.

2.45 (d)

All the three statements are false for pipelining processor.

2.46 (b)

Forward indirect unconditional branch instruction in the first instruction is the fetch through pipeline stages, and evaluation is not at stake.

2.47 (b)

The pipelining results in 1 instruction issued for execution per pipeline. Delay slots will be occupied by the next instruction in the fall through path. For a branching instruction I_1 = Go to label X, some dummy slots I_2 will be there.

2.48 (d)

Nothing is needed to be executed for loop when i = 1 ⇒ 2 = 1 = 2 = 3 = 4 = 5 = 6

So total cycles needed to execute the loop = 2 × 17 = 34

2.49 (b)

Number of cycles = 17 cycles

2.32 (b)

Key sequence line

R2 ← [illegible]	.....	[illegible]
R5 ← [illegible]	.....	[illegible]
R5 ← [illegible]	.....	[illegible]
R2 ← R2 + R5	.....	[illegible]
R1 ← R1 + R2	.....	[illegible]
R2 ← R5 + R1	.....	[illegible]
R6 ← R6 + R1	.....	[illegible]
R1 ← R1 + R2	.....	[illegible]
R5 ← [illegible]	.....	[illegible]
return (R2 − R6)	.....	return (d − f)

[illegible]

2.33 (d)

LDX [illegible], 20H R2) Instruction [illegible] address can be formed by adding the constant with the R2 [illegible] the MUL instruction [illegible] the base [illegible] and 20 [illegible] address of [illegible] array. Hence the address [illegible] Based Indexed Addressing mode. The address [illegible] formed [illegible] array.

2.34 (d)

[illegible]

Load R1, [illegible]

Add R1, [illegible]

Load R2, [illegible]

Sub R2, [illegible]

Load R1, [illegible]

Load R2, [illegible]

Sub R2, [illegible]

Add R3, R2, [illegible]

[illegible] is evaluated completely [illegible] 8 registers.

2.35 (d)

Speed up = [illegible] / Pipeline execution time.

Non pipeline Execution time

= [illegible] + [illegible] + [illegible] + [illegible]

= [illegible] = 20ns

Pipeline Execution time

= (Max(stage delay) + buffer delay)

= [illegible] = 12ns

Hence speedup = 20/12 = 2.5

2.36 (c)

[illegible] pipelining to hum [illegible] thousands.

[illegible] (depends [illegible] is also used to [illegible]

[illegible] Option (c) is true.

2.37 (d)

PC holds the address of next instruction to be executed [illegible] the value of PC in MAR and [illegible] PC. Now [illegible] the value of PC in memory and [illegible] value is loaded into PC. Then [illegible] instruction service routine [illegible]

2.38 (b)

[illegible]

Stage [illegible] = [illegible]

[illegible] stage of [illegible] stage of [illegible]

Total [illegible] without overlap

[illegible]

[illegible] = 58 [illegible] = 58 − [illegible] = 56

2.39 (d)

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible] + [illegible] = [illegible] = 14 bits [illegible] field

= 2^{14} − 1 = 16383 [illegible] possible [illegible]

[illegible] spread

2.40 (b)

$$\text{Speed up}(S) = \frac{\text{pipeline}}{(1 - \text{fraction of instructions affected})}$$

Number of stalls per instruction

= 0.25 × 1 + 0.20 × 2 = 0.65

$$\text{Speed up} = \frac{5}{1 + 0.65} = 1.$$

2.41 (b)

T_1 = CPI × Clock period × [illegible]

T_1 = 1.2 CPI × Clock time, ...(1)

T_2 = 0.75 × T_1 = 0.75 × CPI × 1 nsec ...(2)

Using (1) and (2)

1.2 CPI × Clock time = 0.75 × CPI × 1 nsec

$\Rightarrow$ Cycle time $= \frac{0.33}{1.5} = 0.22$ nsec

Frequency (GHz) $= \frac{1}{0.22 \text{ nsec}} = 4.5$ GHz

2.60 (a):

Clock cycle time = Maximum of propagation delay

For L1: clock cycle time = Maximum of 8

For L2: clock cycle time = Maximum of 1.5

For L3: clock cycle time = Maximum of 1

For L4: clock cycle time = Maximum of 1.5

Minimum clock cycle time = Max of latch delay

Total clock cycle time

$$= \text{Max of } \left(\frac{1}{1.5}, \frac{1}{1.5}, \frac{1}{1.5}\right) + 1$$

$\therefore$ Total clock cycle time

2.61 (a):

T_{cell} = 1 + stabilization + cell delay × $C_{L(n)}$

T_{cell} = 1 + 1.5 × 2 + 0.5 × 2ns = 5.0ns

T_{WL} = 1 + 0.2 × 16 × 1ns = 4ns

$$\frac{t}{C} = \frac{C_{L(n)}}{C_{cell}} = \frac{0.5}{0.2} = 2.5$$

2.62 (a):

Assume 1 of 5 addresses is random and the other address accessed in processor cycles. Implied accumulator accesses are not be address TAG

2.63 (a):

Non-pipelined processor:

Time instruction execution time

= 6n × 0.5 + 2.5 = 1.0n nanoseconds.

Pipelined processor:

Time instruction execution time

= n × 2 = 0.5n nanoseconds.

Speedup = 1.0n / 0.5n = 2.0

2.64 (a):

$\therefore$ It is taken 3 clock cycles.

2.65 (a):

$\therefore$ Before CALL instruction execution, SP contents = 0FFE

While CALL execution:

(i) PC contents are pushed into SP incremented by 2 $\Rightarrow$ SP = 0FFC

(ii) Fetch contents of operand address, SP incremented by 2 $\Rightarrow$ SP = 0FFA

$\therefore$ The value of stack pointer is 0FFA.

2.66 (a):

$S_1 = S_2 = S_3 = S_4 = S_5 = S_6 = S_7 = S_8 = S_9 = S_{10} = S_{11} = S_{12} = S_{13} = S_{14} = S_{15} = S_{16}$

S_9 is an interface between two semi-expanders

Interface cost = 4 (type of S_9)

Stabilization and expansion stages may be avoided when expander matching is used.

2.67 (a):

- Collision vector (C) = (C$_7$, C$_6$, C$_5$, C$_4$, C$_3$)
- Collision latency vector: collision latency is numbers available in (C$_7$, C$_6$)
- Permissible latency is 1 then will minimum peak is 5th cycle

Latency sequence with (1)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
S_1	1	2		3	4	5	6		7	8	9			
S_2			1	2			3	4	5			6		7
S_3			2					3	4				5	

Latency cycle

$S_1 - S_2 = 1$

$S_2 - S_3 = 5$

$S_3 - S_1 = 1$

$S_1 - S_3 = 2$

(1, 5), (1, 3), (1, 5), (2, 4)

$\therefore$ Average latency = 4.5 = 4

Latency sequence with (3)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
S_1				1			2	3		4	5		6	7		8
S_2			1		2			3	4		5	6			7	
S_3		1			2	3			4	5			6	7		

Latency cycle:

$(I_1 - I_2) = 2$

$(I_2 - I_3) = 2$

$(I_3 - I_1) = 3$

so, average latency = 3.

Latency sequence with (6)

Latency cycle:

$(I_1 - I_2) = 5$

$(I_2 - I_3) = 1$

$(I_3 - I_1) = 3$

$(I_3 - I_4) = 1$

Latency cycle (5, 1, 3, 1), (5, 1, ...)

so, average latency = ...

$\therefore$ MAL value = 3

2.68 Sol:

P is stage

t_p = Max (max stage delay) = 200 ps

1 instruction = 200 ps

Number of instruction = 1000

IT_p (throughput) = 1000 instruction/...

P_2 is stage

t_p = 600 ps

1 instruction = 600 ps

Number of instruction = 1000

IT_{P_2} (throughput) = 1000 instruction/...

$1000 \times 600 = 600000$ ps

$1000 \times 1800 = 1800000$ ps

$$= \frac{1800}{600} = 3.00$$

= 200 ps

2.69 Sol:

The number of instructions immediate operand field is 8 bits.

2.70 Sol:

Cache = ... = 64 bytes = 8 bytes

200 ... = 100 ns

2.71 Sol:

Consider that 5-stage pipeline with stage latencies

$$t_1 = \frac{20}{4} = 5 \text{ ns}$$

$$t_1 = \frac{8}{4}, \frac{8}{1} = 2 \text{ ns}$$

$$t_3 = \text{max}(t_1, t_2) = 8 \text{ ns}$$

Ratio of ... = 5, 2, 8, 2 respectively. So assign 4, 1, 4, 1 units.

So max clock frequency = f_0

$$\Rightarrow \frac{1}{t_0} = 250 \text{ MHz}$$

$$= \frac{1}{4} = 250 \text{ MHz} \quad \text{...(1)}$$

Now, largest stage delay divide into half parts

i.e. 4 units into 2 units

So, equivalent throughput ... = 5 ns, 2 ns

Now, largest stage delay = 2 ns

So, calculate new frequency = $\frac{1}{6T}$

$$\frac{1}{6T} = \frac{250}{6} = 41.67 \text{ MHz}$$

[... is different from (1)]

2.72 Sol:

[illegible] C_1 take 2 ns to complete P and 3 ns respectively.

We have to find ... with a throughput of C_1 and C_2 ...

$$\text{...} = \frac{10 \times 2}{3} = \frac{20}{3} = 6.67 \text{ ns}$$

[illegible]

We have to find ... for complete of C_1 and the output.

So ... = 3 ns (result of the ... complete ... [illegible])

■■■■

3 CPU Control Design and Interfaces

8.1 When an interrupt occurs, an interrupting device

(a) [illegible] the interrupt

(b) [illegible] interrupted process [illegible] other process [illegible] the interrupt

(c) always [illegible] interrupts [illegible] processing [illegible]

(d) may change [illegible] interrupted process [illegible] and resume another process

[1997 : 1 Mark]

8.2 RAID [illegible] disk [illegible] to provide [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[1999 : 2 Marks]

8.4 Consider [illegible] CPU in [illegible] programming.

(a) Hard wired control, [illegible] microprogrammed [illegible]

(b) Hard wired control, [illegible] microprogrammed [illegible]

(c) Horizontal [illegible] microprogrammed, Hard wired control

(d) Vertical [illegible] microprogrammed, Hard wired control

[1999 : 2 Marks]

8.4 A processor [illegible] interrupt is

(a) [illegible] system [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2001 : 1 Mark]

8.5 Which [illegible] match for the [illegible]

List-I

X. [illegible]

Y. [illegible]

Z. [illegible]

List-II

I. [illegible]

II. Writing [illegible]

III. [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2001 : 2 Marks]

8.6 Consider [illegible] CPU. The registers [illegible] are 16-bit wide. [illegible] and MAR are 16-bit registers. The MDR [illegible] and the DIR [illegible] 2 CPU clock cycles and use DMA [illegible] Memory Address Register and VDU Memory [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2001 : 2 Marks]

8.7 A device [illegible] CPU [illegible]

(a) [illegible] is active (b) [illegible]

(c) [illegible] is [illegible] (d) [illegible]

[2002 : 1 Mark]

8.8 The [illegible] address [illegible]

(a) the operand [illegible]

(b) [illegible] address of the operand is [illegible] memory

(c) the subroutine containing the [illegible] of the [illegible] instruction
(d) the [illegible] of the program is implicit

[2002 : 1 Mark]

8.9 Normally microprogramming
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) None of the above

[2002 : 2 Marks]

8.10 [illegible] multiplication [illegible] bit numbers. [illegible] microcycle time. The total time of the multiplier is
(a) 60 μs (b) [illegible]
(c) [illegible] (d) 60 μs

[2002 : 1 Mark]

8.11 What is the minimum size of PCI [illegible] to access [illegible] of a 64-bit memory?
(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 1 Mark]

8.12 A CPU has only [illegible] instructions [illegible] Consider the following signals in [illegible]
D0 : T1 . Ain . Bout . [illegible]
D1 : [illegible]
T2 : [illegible]
T3 : [illegible]
T4 : [illegible]
T0 : T1 . Cin . Dout . Ain
T1 : [illegible]
T2 : [illegible]
T3 : [illegible]
T4 : [illegible]
T5 : [illegible]
T6 : T1 . [illegible]
T7 : [illegible]
T8 : [illegible]
T9 : [illegible]
T10 : [illegible]
T11 : End

Which of the following logic functions will generate the [illegible] signal Ain.
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2004 : 2 Marks]

8.13 Normally most microprocessors are provided from [illegible] Ordinarily a PIO [illegible] from [illegible] PIO [illegible] in PIO [illegible] [illegible] PIO [illegible] Which one of the following [illegible] PIO with [illegible] PIO?
(a) PIO performance is [illegible]
(b) PIO [illegible]
(c) PIO [illegible]
(d) PIO [illegible]

[2006 : 1 Mark]

8.14 Which one of the following [illegible] PIO [illegible] single [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible] are not possible but multiple [illegible] are possible
(d) [illegible] are possible but multiple [illegible] are not possible

[2006 : 1 Mark]

8.15 A [illegible] CPU uses [illegible] signals S1 to S8 in various time slots T1 to T8 to implement 4 instructions I1 to I4 as shown below.

Which of the following pairs of control signals is correct for [illegible] for connecting control signals S1 and S10 respectively? [T1 [illegible] indicates that the control signal should be generated in time slot T1 of the instruction cycle.] [2007 : 2 Marks]

(a) S5 = T1 + I2.T5 and S10 = [illegible] I2.T4 + I3.T5
(b) S6 = [illegible] and S10 = [illegible] I2.T4 + I4.T5

(c) [illegible]

(d) [illegible]

[2003 : 2 Marks]

3.16 A [illegible] is used to have [illegible] of [illegible] for [illegible] currently have a logic value of [illegible]. Similarly, [illegible] is used to have a [illegible] of [illegible] for [illegible] currently have a logic value of [illegible]. A circuit is said to have multiple stuck-at faults if [illegible] multiple stuck-at faults. [illegible] multiple stuck-at faults present in a circuit with 2 inputs is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2003 : 2 Marks]

3.17 An microprocessor with a processor has 120 signals which can be divided into a group of mutually exclusive signals as follows:

Group 1 : 70 signals, Group 2 : 20 signals, Group 3 : 20 signals, Group 4 : 6 signals, Group 5 : 4 signals

How many bits of the control word are required to be used for encoding the control signals?

(a) 6 (b) 8

(c) 28 (d) 56

[2005 : 2 Marks]

3.18 The floating point unit of a microprocessor takes a design D1 takes 20 cycles compared to 5 cycles taken by the fixed point unit. There are two more design suggestions D2 and D3. D2 takes 25 more cycles for fixed point unit and 3 cycles less cycles for floating point unit compared to design D1. D3 takes 40% less cycles for fixed point unit and 10 cycles more cycles for floating point unit compared to design D1. Run a given program which has 80% fixed point operations and 20% floating point operations, which of the following relationship represents relative performance of these designs (D1 > D2 means method D1 is faster than D2)

(a) D1 > D3 > D2 (b) D2 > D3 > D1

(c) D3 > D2 > D1 (d) D2 > D1 > D3

[2007 : 2 Marks]

3.19 Consider a CPU with all instructions requiring a clock cycle to complete execution. There are 240 instructions in an assembly language program. Assume that 20% of instructions are memory access instructions. While accessing the memory, a single wait state is used for each access. What is the minimum number of clock cycles required to execute the program?

(a) 285 (b) 285 × 10

(c) 285 × 5 (d) 285 × 20

[2008 : 2 Marks]

3.20 Which of the following is/are true of the auto increment addressing mode?

I. It is useful in creating self relocating code.

II. It is used in stack implementation. In an Instruction Set Architecture, if there is an auto increment mode, an explicit increment instruction is not required for the implementation of stack.

III. The amount of increment depends on the size of the data item accessed.

(a) I only (b) II only

(c) III only (d) II and III only

[2008 : 2 Marks]

3.21 The use of multiple accumulators will mainly impact the performance of the execution of which of the following instructions?

1. Arithmetic and logical instructions
2. register moves and exchanges
3. unconditional branches

(a) 1 only (b) 2 only

(c) 3 only (d) 1 and 3

[2008 : 2 Marks]

3.22 A CPU generates interrupts for an I/O device at a rate of 500 interrupts per second. An average interrupt handling requires 2000 instruction cycles. Average I/O device data transfer requires 100 instruction cycles. The processor has a clock rate of 500 MHz. The I/O device is busy for the entire duration of the I/O data transfer. The percentage of processor time required for I/O data transfer is

(a) 10% (b) 20%

(c) 25% (d) 50%

[2009 : 2 Marks]

3.23 Consider a CPU with a clock rate of 500 MHz executing an interrupt service routine. The interrupt is raised by the device when it is ready to transfer data. The interrupt service routine takes 1000 clock cycles. Assuming that the interrupt is raised 500 times per second, what is the percentage of time the processor is busy with the interrupt?

(a) 1% (b) 2%

(c) 5% (d) 10%

[2009 : 1 Mark]

3.24 In a non pipelined sequential processor, a program segment, which is a loop of 10 instructions, is executed to transfer 500 bytes from an I/O device to memory.

[illegible]

Assume that each instruction in this program is spread across many memory locations and that cycle stealing is used for DMA transfer. The instruction fetch and execution takes two clock cycles each.

The time required to complete the transfer is approximately (Assuming the DMA controller requires 20 clock cycles for initialization and other overheads. Each DMA transfer cycle takes two clock cycles to transfer one byte of data. Consider a computer running at 100 MHz clock rate)

(a) 8.4 (b) 4.1

(c) 5.1 (d) 9.1

[2011 : 2 Marks]

8.24 The minimum [illegible] implementation of [illegible] is

(a) 64 bits (b) [illegible] bits

(c) 1 kbits (d) 8 Kbits

[2012 : 1 Mark]

8.25 Consider a [illegible] memory system that consists of 8 memory modules attached to the system bus which is one word wide. When a write request is made, the bus is occupied for 100 nanoseconds (ns) by the data, address and control signals. During the same 100 ns, and for 500 ns thereafter, the addressed memory module busies itself in storing a single word. The internal operations in two memory modules may overlap in time, but only one request can be in progress at any time.

The maximum number of words that can be written in one second is [illegible] if only sequential writes are made to this memory system.

[2014 (Set-2) : 2 Marks]

■■■■

Answers : CPU Control Design and Interfaces

8.1	(a)	8.2	(a, b)	8.3	(b)	8.4	(c)	8.5	(a)	8.6	(c)	8.7	(a)	8.8	(b)	8.9	(d)
8.10	(c)	8.11	(b)	8.12	(d)	8.13	(c)	8.14	(b)	8.15	(d)	8.16	(b)	8.17	(b)	8.18	(c)
8.19	(b)	8.20	(c)	8.21	(c)	8.22	(d)	8.23	(a)	8.24	(d)						

Explanations : CPU Control Design and Interfaces

8.1 (a)

An interrupt is a signal from a device connected to a computer or from a program within the computer that causes the main program that is executed by the computer to stop for a short time while the interrupt is serviced. After the interrupt is serviced, the main program may resume. Some interrupts are generated by hardware devices when they need to be serviced.

8.2 (a, b)

RAID (Redundant array of inexpensive disks) is a technique that makes multiple disk drives [illegible] RAID configurations are used to provide fault tolerance, high speed.

8.3 (b)

Configurations for CPU is determining value of memory provide

- Hard-wired control → Microprogrammable control → Vertical micro programming

8.4 (c)

A processor consists of one input port, single system bus, which need execution of pipelined instructions.

8.5 (a)

- Indirect addressing → Passing address of operand in memory

- Indirect addressing [illegible]
- Type [illegible]

8.6 (a)

CPU is [illegible] actually.

[illegible]

8.7 (a)

[illegible]

8.8 (b)

[illegible]

8.9 (d)

[illegible]

8.10 (c)

[illegible]

8.11 (b)

Multiplying [illegible] bit numbers [illegible] in maximum 15 bits.

Total number of multiplications possible

$= 2^{15} \times 2^{15} = 64$ G.

[illegible] = 64 K × 6 [illegible]

8.12 (a)

We must [illegible] when all operands are 1 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

8.13 (a)

Memory [illegible] DMA assures that memory [illegible] the value set of addresses. Address bit [illegible] is assigned to [illegible] DMA [illegible] this approach [illegible] operating systems [illegible] and [illegible]

8.14 (b)

Consider the [illegible]

[illegible]

[illegible]

[illegible]

8.15 (a)

[illegible]

8.16 (b)

[illegible]

3.17 (b)

[illegible]

3.18 (d) :

[illegible]

3.19 (b) :

[illegible]

3.20 (c) :

[illegible]

3.21 (b) :

[illegible]

3.22 (c)

[illegible]

3.23 (a)

[illegible]

3.24 (d).

[illegible]

3.25 (b).

[illegible]

4D Secondary Memory & DMA

4.1 The data transfer between memory and I/O devices using programmed I/O is faster than interrupt driven I/O.

[1999 : 2 Marks]

4.2 The three [illegible] are [illegible].

[1998 : 2 Marks]

4.3 Consider a hard disk [illegible] the maximum speed.

[1997 : 1 Mark]

4.4 A [illegible] hard disk [illegible]
[illegible]
[illegible]
[illegible]
The processor [illegible] Direct Memory Access [illegible]
[illegible]

[1991 : 2 Marks]

4.5 A [illegible] hard disk [illegible]
[illegible]
Disk rotation speed = [illegible] rpm
[illegible]
[illegible]

[1994 : 2 Marks]

4.6 If the clock [illegible] 2600 rpm determine the effective data [illegible] which is defined as the number of bytes transferred per second [illegible] and memory [illegible]

[1996 : 2 Marks]

4.7 For the [illegible] of four chip I/O devices, which of the following sequences is correct?
(a) [illegible] priority to [illegible]
(b) [illegible]
(c) [illegible] single device [illegible]
(d) [illegible] separate interrupt pin on the processor for each device

[1998 : 1 Mark]

4.8 A [illegible] is required to transfer a total of [illegible] bytes. [illegible]

[illegible] [illegible]
[illegible] [illegible]

[1999 : 2 Marks]

4.9 The [illegible] given below.

A. [illegible]	1. Disk [illegible]
B. Cache	2. Tape
C. Interrupt [illegible]	3. Printer
D. Double [illegible]	4. [illegible]

Codes:
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[1997 : 1 Mark]

4.10 I/O [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) None of the [illegible]

[1997 : 1 Mark]

4.11 Formatting of floppy diskettes is [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[1996 : 1 Mark]

4.12 The main difference between CISC and RISC processor is that a RISC processor typically

(a) has a smaller instruction set

(b) has fewer addressing modes

(c) has more registers

(d) is easier to implement using hard wired logic

[1999 : 2 Marks]

4.13 The address connections between the cache memory and the processor have a width of 32 bits. Each cache block contains 16 bytes. Each byte has a unique address. The cache is 2-way set associative. The cache has a total capacity of 16 MB. The main memory has a capacity of 4 GB. The cache bits in the address of the MAB.

How many address bits are there in the X and Y fields and what is the size of the internal memory in the cache controller?

(a) 10, 6, 1024 (b) 8, 6, 512

(c) 6, 8, 2048 (d) 10, 8, 512

[2004 : 2 Marks]

4.14 A hard disk with 20 tracks per surface spins at 300 rpm. Successive tracks are separated by 1 mm. The average seek time to move the read/write head over 10 tracks is 100 ms. The average rotational latency is 100 ms. The transfer rate is 1 MB/s. The average access time of the disk system is 200 ms. If a file of 10000 bytes is to be read from the disk, what is the time required?

(a) 0.5 s (b) 1.0 s

(c) 1.5 s (d) 0.1 s

[2004 : 2 Marks]

4.15 The average access time of a disk with an outer diameter of 30 cm, an inner diameter of 10 cm and 500 rpm. The maximum storage capacity of the disk is 60 Gbytes. The disk contains a specified 1000 RPM. The main memory of a computer has 16 bit wide data path and 100 ns cycle time. If 32 bits of data is to be transferred from the disk, the percentage of memory cycles used for transfer is approximately

(a) 0.01% (b) 1%

(c) 2% (d) 10%

[2004 : 2 Marks]

Linked Answer Questions 4.16 and 4.17

A disk has 8 equally spaced tracks numbered 0 to 7. The seek time between two successive tracks is 5 ms. The disk is currently at track 3 and a request is made to read a block on track 1. The disk uses the shortest seek time first (SSTF) scheduling.

4.16 What is the total seek time for the request? Assume that the head is initially moving in the direction of increasing track numbers.

(a) 10 ms (b) 15 ms

(c) 20 ms (d) 25 ms

[2005 : 2 Marks]

4.17 If the disk has 20 sectors per track and rotates at 600 rpm, the average rotational latency for the block on track 1 is

(a) 15 ms (b) 10 ms

(c) 30 ms (d) 50 ms

[2005 : 2 Marks]

4.18 What are the two components in disk access time?

(a) Seek time, rotational latency

(b) Seek time, transfer rate

(c) Seek time, access time

(d) Rotational latency, access time

[2006 : 1 Mark]

4.19 A cache with 1 word block size is connected to a CPU. Cache access takes 1 cycle. A miss requires 4 cycles. The cache is initially empty. The CPU generates the following sequence of memory addresses (in decimal): 1, 2, 3, 4, 1, 2, 3, 4. What is the maximum performance gain of using the cache? Assume instruction mode and assume a single program and infinite cache.

(a) 16 (b) 27
(c) 32 (d) [illegible]

[2005 : 4 Marks]

4.21 [illegible] specifications.
[illegible] Rotational speed 3600 rpm. The disk controller [illegible] whenever the head is ready to [illegible] the disk controller [illegible] by [illegible] from the [illegible] disk [illegible] time [illegible] the disk controller [illegible] 20 kB per second [illegible]
(a) 10 (b) 20
(c) 40 (d) 80

[2005 : 2 Marks]

4.22 Which of the following RAID [illegible] and [illegible] achieve [illegible] and [illegible] highest [illegible] performance?
(a) [illegible] RAID 0 + RAID 1 [illegible] mirrored pairs
(b) [illegible] RAID 1 [illegible] mirrored pairs
(c) [illegible] RAID 0 [illegible] striping in [illegible]

[2006 : 1 Mark]

4.23 Which of the following [illegible] are [illegible] examples of [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) Signal processing

[2007 : 1 Mark]

4.24 Consider a disk pack with 16 surfaces, 128 tracks per surface, 256 sectors per track and 512 bytes in a sector. The capacity of the disk pack and the number of bits required to specify a particular sector in the disk are respectively
(a) 256 Megabytes, 19 bits
(b) 256 Megabytes, 28 bits
(c) 512 Megabytes, 20 bits
(d) [illegible] Gigabytes, 24 bits

[2007 : 1 Mark]

4.25 Consider a Direct Mapped Cache with 8 cache blocks (numbered 0-7). If the memory block numbers are accessed in the following order: 0, 8, 0, 6, 8, 10, 8, 20, 20, 16, 20, 18, 20, 24, 4, 12, 0, 8, 24, 16, 24, 8, 0, 12, 0, 8, 24, 16, 24, 8, 0, 12, 0, 8, 24, 16, 24, 8, 0

4.26 The address of [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

4.27 An application [illegible] each [illegible] The [illegible] of the disk [illegible] is given as [illegible] rpm. [illegible] seek time [illegible] ms [illegible] seek time [illegible] ms [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2[illegible] : 2 Marks]

4.28 [illegible] 512 [illegible] 512 [illegible] 1024 [illegible] 1024 [illegible] 512 [illegible] 1024 [illegible] 512 [illegible] 1024 [illegible] 512

Explanations Secondary Memory & DMA

4.1 Sol.

[illegible]

4.2 Sol:

[illegible]

4.3 Sol.

[illegible]

4.4 Sol.

[illegible]

4.5 Sol:

[illegible]

4.6 Sol.

[illegible]

4.7 Sol:

[illegible]

4.8 (c):

[illegible]

[illegible] including [illegible]
So correct choice option = (c)

4.9 (b):

DMA [illegible] — Disk
Source — High speed RAM
Destination — Primary
Condition code register — [illegible]

4.10 (a):

[illegible] implies [illegible] memory [illegible]

4.11 (d):

Formatting of floppy disk refers to [illegible] information on [illegible]

4.12 (b):

(a), (b), (c), (d).
[illegible] processor are:
1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible] control

4.13 (c):

Total [illegible] = 80 bits
Address [illegible] = 10 bits
Max input = 8 bit
[illegible] = [illegible] bits
Max address [illegible]
= 10 bits
Size of [illegible]
= [illegible]

4.14 (c):

[illegible] = 16 MB [illegible]
= [illegible]
Size of transfer = [illegible]
[illegible]
[illegible]

$$x = \frac{[illegible]}{[illegible]}$$

$$x = \frac{200}{[illegible]}$$

$$x = [illegible]\%$$

or $x = [illegible]\%$

4.15 (b):

[illegible] = [illegible]
Rotational frequency = [illegible]
[illegible] = [illegible]
[illegible]
Now the [illegible] is [illegible] for the DMA [illegible] memory [illegible] it has [illegible] bit word [illegible] bits
[illegible] memory cycle.
[illegible]
[illegible]
So total memory cycle = [illegible]
[illegible]

4.16 (b):

With Constant Linear Velocity, CLV, the density of bits is uniform from cylinder to cylinder. Because there are more sectors in outer cylinders, the disk must rotate slower when reading under the read-write head is moved towards outer tracks. This [illegible] used by modern CD and DVDs.
With Constant Angular Velocity, CAV, the disk rotates at a constant angular speed [illegible] the bit density decreases in outer cylinders. (These disks would have a smaller number of sectors per track on all cylinders).
[illegible] = 90 [illegible] = 240
CAV [illegible] = 90 [illegible] should be [illegible]

4.17 (a)

[illegible]

4.18 (b)

[illegible]

4.19 (c)

[illegible]

4.20 (b)

[illegible]

4.21 (c)

[illegible]

4.22 (d)

[illegible]

4.23 (b)

[illegible]

4.24 (d)

[illegible]

1.93 (d):

- [illegible] = 8.33

So, rotational delay = 30000.0 = [illegible] ms

[illegible] transfer [illegible] single block in a track

Track capacity = Track^ bytes per track = [illegible] × 500 = 50,000 bytes

In [illegible] sec, we can transfer 50,000 bytes

Transfer time to transfer 250 bytes = 0.1 × 250/50,000 sec = 0.5 ms

Avg rotational delay = 0.5 × rotational delay = 0.5 × 0.1 = 50 ms

Average seek time = 10 = 1 = 2 + ... + 400/500 = [illegible] time to move. Let a seek operation consists of 1 ms and we have 500 track crossing = 400 × 500 = [illegible]

Average time to transfer = Average seek time + Average rotational delay + Data transfer time = [illegible]

Average time for transferring 250 bytes = 24000 + 50 + 0.5 = [illegible] ms

1.94 (a):

In magnetic disk unit head is in position, the data [illegible] high density [illegible] to reach the data. [illegible] begin. The average time for head movement to take place is seek latency which is not [illegible] because a further [illegible] on the periphery of the platter.

1.95 (b):

The address [illegible] corresponds to sector no.

[illegible] × 8 × 10 × 6 + 7 × 8 × 6 = 29 = 707 × 97

1.96 (b):

The address 40, 10, [illegible]

[illegible] × 88 − 31 = 0.39

1.97 (b):

Rotational speed = 6000 rpm

$$\text{Average latency} = \frac{\left(\frac{60 \times 10^3}{6000}\right)}{2} = 5\text{ms}$$

Avg access time

= Avg latency + Avg seek time

= 5 ms + 10 ms = 15 ms

Time to read 1 library is 15 ms

Time to read 100 libraries

= 100 × 15 msec

= 1.5 sec

1.98 (d):

We can read [illegible] data from all 16 surfaces simultaneously.

Currently read is on 5th surface, sector no. 46.

How much data it read from 5th surface

= 24 × 512 = 12288 bytes.

How much [illegible] from surface 10th to 24th

= 5 × 512 × 6 = 16,640 bytes.

How much data to read from cylinder 1801 to 1999

32 × 13 × 8 × 512 = 1664 × 16 bytes.

Total data read from 20th to 1999th cylinder

12288 + 16640 + 4250 = 33178 bytes

= 33178 × 8 bytes = 265424 bits

But we need to read 40,000 bits. So we need to go to 2nd cylinder.

1.99 (b):

disk time = 4 ms

$$\text{Rotational latency} = \frac{60}{15000} \times 0.5 = 2\text{ ms}$$

Average rotational latency = 2 × 2 = 4 ms

For each sector, we require Seek time + Rotational Latency + Transfer time.

Seek time = 9 ms = 0.09 seconds per seek

Rotation speed of 512 B per sector = $\frac{0.09}{600}$

= 0.01 ms (rotation time for 1 sector)

Total time required for 1 sector

= 4 ms + 0.01 ms = 0.01 ms

For 2000 sectors = 2000 × 0.01 = 14020 ms

138 [illegible]

1500 rotations → 1 minute

1 rotation → [illegible]

Rotational latency = [illegible] minute

= $\frac{[illegible]}{1500}$ = [illegible] minute

(i) Average rotational latency = $\frac{1}{2}$ × 4 msec = 2 msec

(ii) Average seek time = 2 × average rotational delay

= 2 × 2 = 4 msec

Transfer rate = 500 × 100 bytes/sec

Sector size = 512 bytes

= 5 bytes = 2

Sec Transfer time = $\frac{512}{60 \times 10^3}$ × 1 sec

= [illegible] msec

(iii) 5 × seek time + 5 × Transfer time

= 10 × 0.01024 = 0.1024 msec

(iv) Average time = Average rotational delay + Average seek time = Controller time + Transfer time

= 2 + 4 + 0.01024 + 0.1024

= 6.11424

= 6.1 msec

139 [illegible]

Dirty count register = 10 bits

Maximum value = 2^10 = 1024 blocks

If [illegible] control variable = 64 bytes

Number of [illegible] = 1024 × 64 bytes

Scrambler Time has worst case equal

= $\left\lceil \frac{[illegible]}{64} \right\rceil$ = [illegible]

Unit - V

Programming & Data Structures

■ **Contents**

Sl.	Topic	Page No.
1.	Programming	258
2.	Arrays	297
3.	Stacks & Queues	302
4.	Linked List	310
5.	Trees	321
6.	Graphs	331
7.	Hashing	335

UNIT V Programming & Data Structures

Syllabus: Programming in C. Functions, Recursion, Parameter passing, Scope, Binding, Arrays, Stacks, Queues, Linked Lists, Trees, Binary search trees, Binary heaps.

Analysis of Previous GATE Papers

Exam Year	1 Mark Qns.	2 Marks Qns.	3 Marks Qns.	5 Marks Qns.	Total Marks
1990	–	1			2
1991	1	2		2	20
1992	–	–			
1993	2	5		3	25
1994	4	4		7	49
1995	2	1		2	18
1996	4	3		1	15
1997	2	3		2	16
1998	2	7		3	30
1999	–	4		2	18
2000	3	4		2	22
2001	–	3		1	11
2002	3	2			7
2003	3	7		–	19
2004	7	10		3	27
2005	5	5		2	17
2006	2	7		–	16

Exam Year	1 Mark Qns.	2 Marks Qns.	Total Marks
2007	2	7	16
2008	3	5	13
2009	1	4	9
2010	1	6	13
2011	1	3	7
2012	1	4	9
2013		3	8
2014 Set-1	1	3	9
2014 Set-2	3	3	9
2014 Set-3	3	3	9
2014 Set-4	3	3	9
2015 Set-1	3	2	8
2015 Set-2	4	3	10
2016 Set-1	3	5	13
2016 Set-2	3	5	13

1 Programming

1.1 Study the following program written in block structured language:

```
program [illegible];
procedure P(integer);
begin
    x := [illegible] + 10;
end;
procedure Q
var x, y : integer;
begin
    x := 3;
    y := 4;
    P(y);
    Write(x)                    ...(1)
end;
begin
    x := 7;
    y := 8;
    Q
    Write(x)                    ...(2)
end.
```

What will be printed by write statements marked (1) and (2) in the program if parameters are passed by value?

(a) 5, 8 (b) 8, 3

(c) 5, 7 (d) None of these

[1997 : 2 Marks]

1.2 In which of the following cases is it not possible to obtain? Two methods for call by reference and call by value parameter passing?

(a) Passing an expression as a parameter

(b) Passing an array as a parameter

(c) Passing a pointer as a parameter

(d) Passing an array element as a parameter

[1989 : 2 Marks]

1.3 Match the pairs in the following

List-I

A. Dangling reference

B. Activation record

C. Separate stack

D. Coercion

List-II

P. Type conversion

Q. Dynamic data structure

R. Recursive

S. Non-deterministic loop

[1990 : 2 Marks]

1.4 Indicate all the true statements from the following

(a) A programming language not supporting either recursion or pointers or arrays cannot support dynamic memory allocation

(b) Although C does not support call by name parameter passing, the effect can be correctly simulated in C

[1991 : 1 Mark]

1.5 Match the pairs in the following

List-I

A. Smalltalk

B. LISP

C. Prolog

D. VAL

List-II

P. Logic programming

Q. Object oriented programming

R. Functional programming

S. Object-oriented programming

[1994 : 2 Marks]

Common Data Questions 1.6 & 1.7:

Consider the following program shown in below

```
program PARAMETER (input, output);
var m, n : integer;
procedure P(var x, y : integer);
var m : integer;
begin
    m := [illegible]
    x := y + 1;
end;
procedure Q(x, y : integer, var z : integer);
begin
    x := y + 1;
end;
```

```
begin
    m := 0; [illegible] m, m); writeln (m);
    [illegible]
end
```

1.6 The value of m output by the program PARAM is

(a) [illegible] a local variable [illegible]

(b) 0, because m is the actual parameter that corresponds to the formal parameter [illegible]

(c) 0, because both [illegible] and [illegible] are [illegible] and y has the value 0

(d) 1, because both [illegible] are pass by reference [illegible] and [illegible] in procedure [illegible]

[1998 : 2 Marks]

1.7 The value of m output by the program PARAM is

(a) 0, because m is the actual parameter corresponding to the formal parameter Q

(b) 0, because m is the actual parameter to be passed to Q

(c) 1, because m is the actual parameter corresponding to the formal parameter Q

(d) 1, because m is the actual parameter corresponding to [illegible] Q

(e) none of the above

[1998 : 2 Marks]

1.8 What is the scope of m declared in the main program?

(a) PARAM, P, Q (b) PARAM, P

(c) PARAM, Q (d) P, Q

[1998 : 2 Marks]

1.9 What does the following code do?

```
var a, b : integer;
begin
    a := 5;
    b := 10;
    a := a - b
end;
```

(a) exchange (a) and (b)

(b) [illegible] (a) and change in (b)

(c) [illegible] (b) and change in (a)

(d) [illegible] [illegible]

[1998 : 2 Marks]

1.10 FORTRAN implementations do not permit recursive [illegible]

(a) [illegible] for variables

(b) [illegible] for variables

(c) [illegible] on all machines

(d) it is not possible to implement recursion on all machines

[1998 : 1 Mark]

1.11 [illegible] the Algol 60 [illegible] because

(a) it makes it more difficult to verify programs

(b) [illegible] the running time of the programs

(c) [illegible] the memory required for the programs

(d) it results in the compiler generating less efficient code

[1998 : 1 Mark]

1.12 In which of the following cases is it possible to obtain different results if the call-by-reference method and call-by-name parameter passing methods are used?

(a) Passing a constant value as a parameter

(b) Passing the address of an array as a parameter

(c) Passing an array element as a parameter

(d) Passing an array

[1998 : 2 Marks]

1.13 What is the value of X printed by the following program?

```
program OUTPUT (input, output);
    var X : integer;
    procedure IND (var x);
        var X : integer;
        begin
            X := 2
        End(X);
        Write ln(X);
    end
```

(a) 1 (b) 2

(c) Run time error (d) None of these

[1998 : 2 Marks]

1.14 The correct matching for the following pairs is

List - I	List - II
A. Auto increment	1. Linking loader
B. Text editor	2. Garbage collection
C. Reference counts	3. Macro instructions
D. Address relocation	4. Assembler

Codes:

(a) A - 2, B - 1, C - 4, D - 3

(b) A - 4, B - 3, C - 1, D - 2

(c) A - 4, B - 3, C - 2, D - 1

(d) A - 2, B - 1, C - 3, D - 4

[1996 : 2 Marks]

1.15 Thing of [illegible] languages

(a) that support recursion

(b) that support [illegible]

(c) that use dynamic scoping

(d) None of the above

[1997 : 1 Mark]

1.16 Consider a block of procedures [illegible]

```
Procedure A
    x : integer;
    Procedure B;
        x, y : real;
        S1
    end B
    Procedure C;
        Integer
        S2
    end C
end A;
```

The [illegible] Procedures B, C and S2 are

(a) x of A is used in B and x in B and x of A is used in S2

(b) x of B, y and x in S1 and x of B is used in S2

(c) x of B is used in B and x in B and x of A is used in S2

(d) None of the above

[1997 : 2 Marks]

1.17 Either [illegible] [illegible] exam, [illegible] of [illegible] [illegible] [illegible]

1.22 The most appropriate matching for the following pairs

List-I	List-II
X. [illegible]	1. Loops
Y. [illegible]	2. Iterators
Z. [illegible]	3. Constants

[illegible]

(a) X – 3, Y – 2, Z – [illegible]
(b) X – 1, Y – 3, Z – 2
(c) X – 2, Y – 1, Z – 3
(d) X – 3, Y – 1, Z – 2 [2000 : 1 Mark]

1.23 The following C code is [illegible]

```
struct node {
    int [illegible]
    float [illegible];
};
struct node *[illegible];
dynamic node
```

(a) An array [illegible]
(b) A structure of 2 fields, each field being a pointer to an array of [illegible]
(c) A [illegible] 2 fields [illegible]
(d) A [illegible] each element of which is a structure of type node [illegible] [2000 : 1 Mark]

1.24 The correct sequence of machine instructions following policy

List-I

X. [illegible]
Y. [illegible]
Z. [illegible]

List-II

1. [illegible]
2. [illegible]
3. Lost memory

Codes

(a) X – 1, Y – 3, Z – 2
(b) X – 2, Y – 1, Z – 3
(c) X – 3, Y – 2, Z – 1
(d) X – 3, Y – 1, Z – 2 [2000 : 1 Mark]

1.25 [illegible]

(a) multiple variables having the same memory location
(b) multiple variables having the same value
(c) multiple variables having the same identifier
(d) multiple [illegible] [2000 : 1 Mark]

1.26 Consider the following C declaration

```
struct {
    short s[5];
    union {
    float [illegible];
    long [illegible];
    } u;
} *p;
```

Assume that [illegible] bytes, short [illegible] 2 bytes, [illegible] 4 bytes and 8 bytes respectively. The memory requirement for variable p, ignoring alignment considerations, is

(a) 22 bytes (b) [illegible] bytes
(c) 18 bytes (d) [illegible] bytes [2000 : 1 Mark]

1.27 The value of x at the end of the execution of the following C program

```
int fun(int n) {
    static int count = 0;
    count = count + n;
    return count;
}

main() {
    int x = 0, i = 0;
    for (i = 0; i < 4; i++)
        x = fun(i);
}
```

(a) 10 (b) 1
(c) 0 (d) [illegible] [2000 : 2 Marks]

1.28 What is printed by the print statement in the following [illegible] call by reference parameter passing?

```
void swap(int &x, int &y) {
    int temp;
    temp = x;
    x = y;
    y = temp;
}
void main() {
    int a = 10, b = 5;
    swap(a, b);
    printf("%d", a);
}
```

```
        printf(y);
    }
    Function(y){
    {
        y = y - 4;
        z = x + y + z;
    }
```

(a) 10.8 (b) 8 [illegible]
(c) 27.7 (d) None of these

[2001 : 2 Marks]

1.29 Consider the following three C functions:

```
F1      int [illegible]
        {
            [illegible] = 10;
            return (&x);
        }
F2      int [illegible]
        {
            int *px;
            *px = 10;
            return *px;
        }
F3      int *[illegible]
        {
            int *py;
            py = (int *) malloc(sizeof(int));
            *py = 10;
            return *py;
        }
```

Which of the three functions are likely to cause problems with pointers?

(a) Only F3 (b) Only F1 and F3
(c) Only F1 and F2 (d) [illegible] F3

[2001 : 2 Marks]

1.30 Consider the following program

```
Program P2
    [illegible]
    procedure S(n: integer);
    begin
        [illegible] := 1;
        print(n);
    end;
    procedure R(n: integer);
    begin
        var n: integer;
        n := 2;
        S(n);
        print(n);
    end;
    begin {main program P2}
        n := 10;
        R(n);
        print(n);
    end.
```

If the language has dynamic scoping and parameters are passed by reference, what will be printed by the program?

(a) 10 (b) 1
(c) 2 (d) None of these

[2001 : 2 Marks]

1.31 The results returned by function calls "call by result" and "call by reference" parameter passing conventions are

(a) Always identical
(b) Identical unless aliasing occurs
(c) Identical unless exceptions occur
(d) Always different in the presence of exception

[2002 : 1 Mark]

1.32 In the Hamming [illegible]

(a) [illegible] the minimum number of bits required to uniquely identify the correct bit in error, and the error-correcting period for the code
(b) [illegible] the number of consecutive records affected by a single error, and the error-correcting period for the code
(c) The [illegible] of global variables depends on the order of function calls in a program
(d) Recursion requires the [illegible] for the recursive function to be stored on a stack because the activation [illegible] are called

[2002 : 1 Mark]

1.33 Consider the following declaration of a two-dimensional array in C:

```
char a[100][100];
```

Assuming that the main memory is byte-addressable and that the array is stored starting from memory address 0, the address of a[i][j] is

(a) 4040 (b) 4050
(c) 5040 (d) 5050

[2002 : 2 Marks]

1.84 Assume that the following variable declaration

int *a[10], b[10], c[10];

For the following expressions

1. a[3]
2. a[2][8]
3. b[1]
4. b[8][3]

Which of the following statements are true?

(a) 1, 2 and 4 only (b) 1, 2 and 3 only

(c) 2 and 3 only (d) 4 only

[2008 : 1 Mark]

1.85 In the following C language fragment, [illegible] are integer variables, and [illegible] are array of integers. The variable [illegible] is initialized to [illegible] and the array [illegible] is initialized to

the value of 2[illegible] for [illegible]

```
for (i = 0; i < n; i++)
    A[i] = 0;
for (i = 2; i < Two[illegible]; i++)
    Two[i] = Two[illegible] + Two[illegible];
    A[illegible] = A[illegible] | 1 << i;
for (i = 0; i < n; i++)
    printf("%d", A[i]);
```

The set of numbers printed by this program fragment is

(a) {n | n ∈ {0, 1, ..., n−1}}

(b) {n | n ∈ {0, 1, ..., n−1} and n is a power of 2}

(c) {n | n ∈ {0, 1, ..., n−1} and n is a prime}

(d) {}

[2008 : 2 Marks]

1.86 Consider the following program written in C language.

```
#include <stdio.h>
#define print(x) printf("%d", x)
int x;
void p(int);
void main(){
        x = 5;
        p(x);
        print(x);
}
void p(int y){
        int x = y * 2;
        if (y < 5)
        return;
        else
        printf("%d", x);
}
```

The output of this program is

(a) 12 5 (b) 25 5 10

(c) 10 5 (d) 10 5 5

[2008 : 2 Marks]

1.87 The goto statement in programming is used to

(a) [illegible]

(b) be able to alter the flow of control from the current block

(c) [illegible]

(d) [illegible]

[2001 : 1 Mark]

1.88 Consider the following C function swap.

```
void swap(int x, int y)
{
    int temp;
    temp = x;
    x = y;
    y = temp;
}
```

In order to exchange the values of two variables x and y,

(a) call swap(x, y)

(b) call swap(&x, &y)

(c) [illegible] in the calling environment

(d) [illegible] as the parameters are passed by value

[2001 : 1 Mark]

1.89 [illegible] is not taken as a keyword in C. The statement

if (x == 0) x = 1;

else x = 0;

can be replaced by the following statement

(a) x = 1 + x; (b) x = [illegible]

(c) x = x [illegible] (d) x = (!x);

[2001 : 1 Mark]

1.90 Consider the following C program which [illegible] a sequence of integers [illegible] X. Note that [illegible] in the program which indicates some missing statements. Choose the correct option to replace X in the program.

```
#include <stdio.h>
#define ROW 4
#define COL 4
int M[ROW][COL] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
11, 12, 13, 14, 15, 16};
void main();
{
    int i, j, k;
    for (i = 0; i < 4; ++i)
    {
        }
    }
    for (i = 0; i < 4; ++i)
        for (j = 0; j < 4; ++j)
            printf("%d", M[i][j]);
}
(a) for(j = 0; j < 4; ++j)
    {
        t = M[0][j];
        M[0][j] = M[3][j];
        M[3][j] = t;
    }
(b) for(j = 0; j < 4; ++j)
    {
        M[0][j] = 0;
        t = M[1][j];
        M[2][j] = M[1][j];
    }
(c) for(j = 0; j < 4; ++j)
    {
        t = M[2][j];
        M[2][j] = M[3][j];
        M[3][j] = t;
    }
(d) for(j = 0; j < 4; ++j)
    {
        M[0][j] = 0;
        t = M[1][j];
        M[2][j] = M[1][j];
        M[3][j] = t;
    }
```

[2004 : 2 Marks]

1.41 What is the output of the following program?

```
#include <stdio.h>
int func(int x);
int func(int y);
void main()
{
    int x = 5, y = 10, count;
    for (count = 1; count <= 2; count++)
    {
        x = func(x) + func(y);
        printf("%d", x);
    }
}
func(int x)
{
    int y;
    y = func(y);
    return(y);
}
func(int x)
{
    static int y = 10;
    y = y - 1;
    return(y + x);
}
(a) 45 55                    (b) 42 52
(c) 44 47                    (d) 38 38
```

[2004 : 2 Marks]

1.42 Consider the following C program:

```
#include <stdio.h>
typedef struct
{
    char *a;
    char *b;
} t;
void f1(t);
void f2(t *);
main()
{
    static t x = {"A", "B"};
    printf("%s %s\n", x.a, x.b);
    f1(x);
    printf("%s %s\n", x.a, x.b);
    f2(&x);
}

void f1(t y)
{
    y.a = "C";
    y.b = "D";
    printf("%s %s\n", y.a, y.b);
    return;
}
```

void [illegible] *p;
{
[illegible]
[illegible] = [illegible]
printf("%s [illegible]", [illegible], p = [illegible]);
[illegible]
}

What is the output generated by the program?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2001 : 2 Marks]

1.44 Consider the following C function

int [illegible]
{
[illegible]
[illegible]
[illegible]
[illegible]
}

The value returned by the function

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

1.45 Consider the following C program

[illegible]
{
int [illegible];
[illegible]
/* Assume [illegible] */
[illegible] = [illegible];
while [illegible]
{
if [illegible]
[illegible] = [illegible];
else
[illegible] = [illegible];
}
[illegible]
}

The program computes

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2004 : 2 Marks]

1.46 Choose the best matching between the programming styles in Group 1 and their characteristics in Group 2.

Group-1	Group-2
P. Functional	1. Concerned based procedural
Q. Logic	2. Implicitly typed variables
R. Object oriented	3. Statements from declarative expression evaluation
S. Imperative	4. Extensive use of recursion, lazy evaluation

Codes:

(a) P-4, Q-3, R-2, S-1
(b) P-1, Q-3, R-2, S-4
(c) P-4, Q-3, R-1, S-2
(d) P-3, Q-4, R-1, S-2

[2004 : 2 Marks]

1.47 Consider the following program segment for reversing the digits in a given integer to obtain a new integer. Let $n = d_1 d_2 \ldots d_m$.

int [illegible];
rev = 0;
while (n > 0)
{
rev = rev * 10 + n % 10;
n = n / 10;
}

The loop invariant (correct value of the partial result) is

(a) $n = d_1 d_2 \ldots d_{m-i}$ and rev = $d_m d_{m-1} \ldots d_{m-i+1}$
(b) $n = d_{m-i+1} \ldots d_{m-1} d_m$ or rev = $d_{m-i} \ldots d_2 d_1$
(c) $n = d_1 d_2 \ldots d_m$
(d) $n = d_1 d_2 \ldots d_m$ or rev = $d_1 d_2 \ldots d_m$

[2004 : 2 Marks]

1.48 Consider the following C program segment

char [illegible][20];
char *s = "[illegible]";
int length = strlen(s);
for (i = 0; i < length; i++)

[illegible]

(a) pointer
(b) string
(c) [illegible]
(d) no output or error

[2004 : 2 Marks]

1.45 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2005 : 1 Mark]

1.46 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 1 Mark]

1.47 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 1 Mark]

1.48 [illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

(a) [illegible] (b) 1 and 3 only
(c) [illegible] (d) [illegible]

[2006 : 1 Mark]

1.49 [illegible]

```
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
```

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

1.50 [illegible]

```
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
```

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

Linked Answer Questions 1.51 & 1.52.

[illegible] graph [illegible] [illegible] [illegible]

```
i = 1;
do
{
    j = i - 1;
    while(j >= 0 && A[j] > ...
    if (j >= 0) ...
}while(i < n);
flag = 1;
for (j = 0; j < n; j++)
if (j != i && E[j] == ...
if (flag) printf("Minimum ...");
else printf("Minimum does not exist");
```

1.54 Choose the correct expressions for E_1 and E_2

(a) E_1 : A[i][j] and E_2 : i = j;

(b) E_1 : A[j][i] and E_2 : i = j;

(c) E_1 : A[i][i] and E_2 : i = j;

(d) E_1 : A[j][j] and E_2 : i = j + 1.

[2006 : 2 Marks]

1.55 Choose the correct expression for E_1

(a) (A[i][j] && A[j][i]) (b) (A[i][j] && A[i][i])

(c) (A[i][j] || A[j][i]) (d) (A[i][j] || A[i][i])

[2006 : 2 Marks]

1.56 The following C function takes a single-digit positive integer in A and returns a string of digits containing the same digits as A. The string is the lexicographically smallest permutation of the digits in A.

```
char *sort(int A, char *b)
{
    int count[128], j;
    for (j = 0; j < 128; j++)
        count[j] = 0;
    while (A > 0)
    {
        A = ...
        B = ...
    }
    for (j = 0; j < 128; j++)
        if (count[j]) ...
    return ...
}
```

Choose the correct alternatives for statements A, B and B

(a) A: count[A%10]++ and B: count[b[i]] = ...

(b) A: count[A%10]++ and B: count[b[i]]++

(c) A: count[A%10] = 1 and B: count[b[i]]--

(d) A: count[A%10]++ and B: count[b[i]]++

[2006 : 2 Marks]

1.57 What is the output printed by the following program?

```
#include <stdio.h>
int f(int x, int *y)
{
    if (x == 0) return 0;
    else if (x % 2 == 0) return f(x/2, 2*y);
    else return f(x/2, 2*y + *y);
}
int main()
{
    printf("%d", f(80, 1));
    return 0;
}
```

(a) 5 (b) 6

(c) 9 (d) 80

[2005 : 2 Marks]

1.58 What is the choice given below should be given when the following program is executed?

```
#include <stdio.h>
struct tag
{
    int i;
    char *c;
} st[] = {0, "Bravo", 4, "Hotel", 8, "Jungle", 9,
"Lemon", 7, "Mango"};
main()
{
    struct tag *p = st;
    p = p + ...
        ... p = ...
    printf("%d", p->i + ...);
    printf("%s", *(p->c + ...));
    printf("%d", p[0].i);
    printf("%s", p[1].c);
}
```

(a) Jungle, n, 5, mango

(b) Hotel, l, 8, mangle

(c) Lemon, k, 6, jungle

(d) Hotel, m, 9, mangle

[2006 : 2 Marks]

1.58 [illegible] choices given below would be printed when the following program is executed?

```
#include <stdio.h>
void swap(int *x, int *y)
{
    int temp;
    temp = *x;
    *x = *y;
    *y = temp;
}
void permute(int i, int n)
{
    int j, k = [illegible];
    if (i == n)
    {
        [illegible];
        [illegible];
        [illegible];
    }
    else
    {
        for (j = i; j <= n; j++)
        {
            swap(&a[i], &a[j]);
            permute(i+1, n);
            swap(&a[i], &a[j]);
        }
    }
}
int main()
{
    int a[4] = {1, 2, 3, 4};
    permute(0, 3);
}
```

(a) 1 2 3 4
(b) 4 3 2 1
(c) 1 2 4 3
(d) 1 3 4 2

[2006 : 2 Marks]

1.59 Which one of the choices given below would be printed when the following program is executed?

```
#include <stdio.h>
int a[] = {86, 1, 28, 15, 54, 63};
int b[] = {35, 86, 29, 74};
int [illegible] = {1, 2, 3, 4};
int *[illegible] = {a1, a2, a3};
void main()
{
    int *[illegible];
    p = a3;
    printf("%d", *p);
    [illegible];
    printf("%d", *p);
    [illegible];
}
```

(a) 63 15
(b) 63 28
(c) 63 54
(d) 63 86

[2006 : 2 Marks]

1.60 The following pseudocode [illegible]

```
[illegible]
void [illegible]
{
    if (*[illegible] == [illegible])
    {
        [illegible];
        [illegible] = [illegible] + 1;
        [illegible]
    }
}
```

In the above function, which one of the following is the correct calling sequence?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

1.61 Match the following components and their locations possible across lines.

Component	Description
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[2006 : 8 Marks]

3.63 Consider the following code written in a procedure reference language similar to FORTRAN and these statements about the code.

```
S1: subroutine swap (ix,iy)
        ... =
L:      ix = iy
L2      iy = it
    end
    ...
    x = 5
    y = 6
    call swap (x, y + 5)
    ...
    end
```

S1: The compiler will generate code to evaluate y + 5 in a temporary, and pass the address of this temporary to swap.

S2: On execution, the code will generate an infinite loop at line L1

S3: On execution, the code will generate a run-time error at line L2

S4: The program will print 11 and 6

S5: The program will print 11 and 5

Identify the following set of statements that are correct:

(a) S1 and S4 (b) S1 and S5

(c) S4 (d) S1 and S3

[2006 : 2 Marks]

3.64 Consider the following C function in which a[n] and b[m] are two sorted integer arrays and c[n+m] is another integer array.

```
void merge(int a[], int n, int b[], int m, int c[])
{
    int i, j, k;
    i = j = k = 0;
    while (i < n && j < m)
      if (a[i] < b[j]) c[k++] = a[i++];
      else c[k++] = b[j++];
}
```

Which of the following statements is true when the terminating condition of the loop is correct?

(a) i == n && j == m && k == n + m

(b) i == n || j == m || k == n + m

(c) i == n && j == m && k == n + m

(d) i == n || j == m || k == n + m

[2006 : 2 Marks]

3.65 Consider two functions and two statements S1 and S2 shown below.

```
int work1(int *, int, int);
{
    int x = 0;
    x = 2*x + y;
    return x;
}

int work2(int *, int, int);
{
    int x = 0;
    ... = 2*x + y;
    x = 2*x + 1;
    return x;
}
```

S1: If the transformation from work1 to work2 is valid, then for any program and input, both work1 and work2 will compute the same output and have the same effect on the program state variables.

S2: All the transformations suggested in work2 are valid if the variables x and y are dead at the point where the function is called.

(a) S1 is false and S2 is true

(b) S1 is true and S2 is false

(c) S1 is true and S2 is true

(d) S1 is false and S2 is false

[2006 : 2 Marks]

3.66 Consider the C code fragment given below. Two statements S1 and S2 are given.

```
void swap(int *px, int *py)
{
    *px = *px - *py;
    *px = *px + *py;
    *py = *px - *py;
}
```

S1: will generate a compilation error

S2: may generate an error during execution depending on the arguments passed

S3: Correctly swaps the contents of the two integers passed to it

[illegible]

(a) [illegible] correctly for some but not all valid input pointers.

(b) [illegible]

(c) P1 (d) S3 and S5

(e) S6 and S4 (f) S8 and S7

[2006 : 2 Marks]

1.07 Consider the following C function:

```
int fun(n)
{
    static int x = 0;
    if(n <= 0) return 1;
    if(n > 3)
    {
        x = x + n;
        return fun(n - 3) + 2;
    }
    return fun(n - 1) + x;
}
```

What is the value of fun(5)?

(a) 8 (b) 7

(c) 9 (d) 18

[2007 : 2 Marks]

1.08 The function f is defined as follows:

```
int f(int n)
{
    if (n <= 1)
    return n;
    else if (n % 2 == 0) return f(n/2);
    else return f(n - 1);
}
```

When the function f is invoked with a positive integer as the parameter, consider the following statements.

(i) The function f terminates for every positive integer value of n.

(ii) The function f terminates for every positive integer value of n > 1.

(iii) The function f terminates for every positive integer value of n > 1 which is a power of 2.

(iv) The function f terminates for every positive integer value of n > 1 which is odd.

Which one of the following options is true? The stack

(a) (i) and (iii) (b) (ii) and (iv)

(c) (iii) and (iv) (d) (ii) and (iii)

[2007 : 2 Marks]

1.09 Consider the C program given below:

```
#include stdio.h
int main()
{
    int count = 0, numcom = 0, i, n = 9;
    int a[9] = {0, -3, -2, 0, 4, 0, 9};
    for(i = 0; i < n; i++)
    {
        count = count + (a[i] < 0) * (a[i] < a[i + 1]);
        if
        {
            if(count > numcom)
                numcom = count;
            count = (a[i] < 0) ? (a[i] : 0;
        }
        else count = (a[i]);
        /* Update numcom if a longer run was found */
        printf("%d", numcom);
    }
}
```

What is the value printed when the above program is executed?

(a) 5 (b) 2

(c) 7 (d) 6

[2008 : 2 Marks]

1.10 Consider the following C program:

```
#include stdio.h
#define EOF -1
void push(int);
int pop();
int stack[100];
int top = -1;
void FlagError();
int main()
{
    int a, b, r;
    while (a = getchar() != EOF)
    {
        if (a == '+' || a == '*')
        {
            b = pop();
            a = pop();
            r = (a == '+') ? a + b : a * b;
            push(r);
        }
        else if (a != '\n')
            FlagError();
    }
    printf("%d", pop());
```

What is the output of the program for the following input?

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(2007 : 2 Marks)

1.71 Consider the program below in a hypothetical language which allows global variables and static scoping. Assume call by value and call by reference of procedure parameters.

```
int i;
procedure main()
{
    i = 10;
    i = 20;
    call f(i, j);
    print i, j;
}
procedure f(x, y)
{
    i = 100;
    x = 10;
    y = x + i;
```

[illegible]

(a) Call by value : i = [illegible], j = 10; Call by reference : i = 110, j = [illegible]

(b) Call by value : i = [illegible], j = [illegible]; Call by reference : i = 110, j = 70

(c) Call by value : i = 20, j = 70; Call by reference : i = 110, j = 60

(d) Call by value : i = 60, j = [illegible]; Call by reference : i = 10, j = 20

(2007 : 2 Marks)

1.72 Consider the program below in a hypothetical programming language which allows global variables and static scoping.

```
int i;
procedure main()
{
    i = 10;
    call f(i);
}
procedure f(x)
{
    int i = 20;
    call g(i);
}
procedure g(y)
{
    print i;
}
```

[illegible]

Choose the correct option.

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(2007 : 2 Marks)

1.73 Late binding or dynamic binding is performed at run-time and late binding occurs in some particular order of execution time. Consider the following statements.

(i) [illegible]

(ii) Dynamic scoping requires a stack [illegible]

(iii) Early binding and static scoping [illegible]

(iv) Late binding [illegible]

The right choice of (i), (ii), (iii) and (iv) (in that order) is

(a) Early, late, dynamic, static

(b) Late, early, static, dynamic

(c) Late, early, dynamic, static

(d) Early, late, static, dynamic

(2007 : 2 Marks)

1.74 Match the programming paradigms and languages given in the following lists.

List I (Paradigms)

I. Imperative

II. Object oriented

III. Functional

IV. Logic

List II (Languages)

A. Prolog

B. Lisp

C. C, Fortran 77, Pascal

D. C++, Smalltalk, Java

Codes:

(a) I-C, II-D, III-B, IV-A

(b) I-A, II-B, III-C, IV-D

(c) I-B, II-C, III-D, IV-A

(d) I-C, II-D, III-B, IV-A

(2008 : 2 Marks)

1.75 What is the value of the integer variable x after the following expression is evaluated?

```
x = (x = y, y = 1, x = 2) ? x : 3 : ((y = 2) ? y : 0)
```

(a) x = 0, y = 1, x = 2
(b) x = 0, y = 2, x = 2
(c) x = 0, y = 1, x = 3
(d) x = 2, y = 1, x = 2 [2003 : 1 Mark]

1.76 What is the output generated by the following C code?

```
#include <stdio.h>
int main()
{
    char s[] = "abcd";
    int i;
    for (i = 0; i < 4; i++)
    printf("%s\n", s);
}
```

(a) abcd (b) Runtime error
(c) dbcd (d) abcd
abcd
abcd
abcd
[2003 : 2 Marks]

1.77 Consider the C program below. What does it print?

```
#include <stdio.h>
#define swap(a,b) tmp = a; a = b; b = tmp
void swap1(int a, int b)
{
    int tmp;
    tmp = a; a = b; b = tmp;
}
void swap2(int *a, int *b)
{
    int tmp;
    tmp = *a; *a = *b; *b = tmp;
}
int main()
{
    int num1 = 5, num2 = 9, tmp;
    if (num1 > num2)
    swap1(num1, num2);
    if (num1 > num2)
    swap2(&num1, &num2);
    printf("%d, %d", num1, num2);
}
```

(a) 5, 9 (b) 9, 5
(c) 9, 9 (d) 5, 5
[2003 : 2 Marks]

1.78 Consider the C program given below. What does it print?

```
#include <stdio.h>
int main()
{
    int i, j;
    int a[9] = {1, 2, 3, 4, 5, 6, 7, 8, 9};
    for(i = 0; i < 9; i++)
    {
        a[i] = a[i+2];
        i++;
    }
    for (i = 0; i < 9; i++)
    {
        int j = i[a];
        a[j] = a[j] - 1;
    }
    printf("%d %d %d", i, a[i], j);
}
```

(a) 2, 3, 3 (b) 2, 4, 4
(c) 4, 5, 5 (d) 3, 5, 5
[2003 : 2 Marks]

1.79 A C program is given below.

```
#include <stdio.h>
int main(int,
{
    int i, j;
    char *x[2][3] = {{"a", "b", "c"}, {"d", "e", "f"}};
    char b[10][2];
    char *p = *b;
    for (i = 0; i < 3; i++)
    {
        for (j = 0; j < 2; j++)
            *p++ = *x[i][j] + 1 [ a [ ] [j];
        }
    }
}
```

What is the value of the elements of the array b at the end of the program?

(a) a c
a e
a f
(b) a d
a e
a f
(c) a e
b d
b f
(d) a e
b d
b f
[2003 : 2 Marks]

Direction for Question 1.80 to 1.81:
Consider the recursive function in C below

```
void f(int n)
{
    if(n ≤ 1)
    {
        printf("%d", n);
    }
    else
    {
        f(n/2);
        printf("%d", n%2);
    }
}
```

1.80 What is the output of f(9)?

(a) 1001 (b) 1001 (c) 1001001 (d) 1001001

[2008 : 2 Marks]

1.81 Which of the following implementations will produce the same output for f(n) as the above function for all values of n?

```
P1:                 P2:
void f(int n)       void f(int n)
{                   {
  int i = 0;          int i = 0;
  while(n > 0)        while(n > 0)
  {                   {
    i = i*10 + n%2;     i = i*10 + n%2;
    n = n/2;            n = n/2;
  }                   }
  printf("%d", i);    printf("%d", i);
}                   }
```

(a) Both P1 and P2 (b) P2 only

(c) P1 only (d) Neither P1 nor P2

[2008 : 2 Marks]

1.82 What is printed by the following C program on a little endian machine?

```
int f(int *, int *);
int main() {
  int a, b, c = 0;
  int *pc = &c, *pb = &b, *pa = &a;
  *pc = 2[pc];
  *pb = 3[pc];
  a = f(pa, pb);
  printf("%d", a + b + c);
}
int f(int *x, int *y) {
  int a = *x, b = *y;
  *x = 4;
  *y = 5;
  return a + b;
}
```

(a) 19 (b) 16

(c) 21 (d) 26

[2009 : 2 Marks]

1.83 Choose the correct option to fill ? so that the program prints the input string in reverse order. Assume that the input string is terminated by a newline character.

```
void reverse(void);
int main() {
  int c;
  printf("Enter Text: ");
  ?;
  printf("\nReversed Text: ");
  reverse();
  return 0;
}
void reverse(void) {
  int c = getchar();
  if (c != '\n') {
    reverse();
  }
  putchar(c);
}
void reverse(void) {
  int c = getchar();
  if (c != '\n') {
    putchar(c);
    reverse();
  }
}
void reverse(void) {
  int c = getchar();
  if (c != '\n') {
    reverse();
  }
  putchar(c);
}
```

[2009 : 2 Marks]

Linked Answer Q.1.84 and Q.1.85

Consider the following C program that attempts to locate the nth occurrence of a character in an array S[], using binary search. The program is erroneous.

```
1.  int BS(char A[], int n, char x) {
2.      int i, j, k;
3.      i = 0; j = n;
4.      do {
5.          k = (i + j) / 2;
6.          if (A[k] < x) i = k + 1; else j = k;
7.      } while (i < j);
8.      if (i == n || A[i] != x)
9.          printf("%c not in A[0..%d]", x, n);
10.     else
11.         printf("%c is at A[%d]", x, i);
12. }
```

1.84 On which of the following statements of BS and x does the program fail?

(a) S is [1, 2, 3, 4, 6, 8, 9, 10] and x = 6

(b) S is [1, 4, 6, 7, 9, 11, 15, 17, 18] and x = 1

(c) S is [3, 3, 3, 3, 3, 3, 3, 3] and x = 3

(d) S is [2, 4, 6, 8, 10, 12, 14, 16, 18, 20] and x = 20 and n is even

[2009 : 2 Marks]

125. The [illegible] in the preceding [illegible] were processing is

(a) change [illegible] j = 3 - i

(b) change [illegible]

(c) change [illegible]

(d) change line 7 [illegible]

[2008 : 2 Marks]

126. Consider the program shown

```
#include <stdio.h>
int [illegible]
{
  int *[illegible];
  if (n == 0)
    [illegible] = 1;
    return [illegible]
  [illegible]
  [illegible] = [illegible] * f(n-1);
  [illegible] = [illegible];
  return f;
}

int main()
{
  int x = 10;
  printf("%d", fun(x));
  return 0;
}
```

The value printed is

(a) 6 (b) 8

(c) 11 (d) [illegible]

[2008 : 2 Marks]

127. What does the following program print?

```
#include <stdio.h>
void f(int *p, int q) {
  q = 16;
  *p = 12;
}
int i = 8, j = 16;
int main()
{
  f(&i, &j);
  printf("%d %d\n", i, j);
  return 0;
}
```

(a) 12 16 (b) 12 12

(c) 8 16 (d) 8 12

[2010 : 1 Mark]

128. What is the value printed by the following C program?

```
#include <stdio.h>
int f(int *p, int n) {
  int r = 0, s = 1;
  while (n > 0) {
    s = s * *p;
    r = r + s;
    n = n - 1;
  }
  return r;
}
int main() {
  int a[] = {3, 5, 2};
  printf("%d", f(a, 3));
  return 0;
}
```

(a) 15 (b) 7

(c) 10 (d) 19

[2010 : 2 Marks]

129. The following program [illegible] for reversal of a string:

```
begin
  [illegible] = by [illegible]
  [illegible]
  [illegible]
  [illegible]
end
```

The best choice for [illegible] is

(a) expression: int used for the pointer variable

(b) [illegible]

(c) [illegible] = d

(d) [illegible] = d

[2010 : 2 Marks]

130. What does the following program print?

```
[illegible]
```

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2010 : 2 Marks]

[illegible]

[illegible] [2011 : 1 Mark]

Common Data Question 1.31 and 1.32

Consider the following [illegible] function that takes two [illegible].

[illegible]

```
{
  [illegible]
  else return 0;
}
```

1.31 What [illegible] value [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2011 : 2 Marks]

1.32 [illegible] value of [illegible]

(a) 8 (b) 6

(c) 5 (d) 2

[2011 : 2 Marks]

1.33 What [illegible] output [illegible] program segment?

```
char [illegible] = A
switch(ch) {
  case 'A' : printf("Choice A");
  case 'B' : printf("Choice B");
  default : printf("Invalid");
}
```

(a) Invalid

(b) Choice A

(c) Choice A

Choice B Invalid

(d) Program gives no output as it is erroneous

[2013 : 1 Mark]

1.34 Consider the program given below in a hypothetical procedural language with static scoping and nested procedures permitted.

```
Begin proc
var i;
Procedure A1;
  var i;
  call A2;
End A1
Procedure A2
  var i;
  Procedure A3;
    var i;
    Call A1;
  Begin A3
  Call A3;
End A2
Begin A2
Call A2;
End proc
```

Consider [illegible] Main [illegible] → A

The [illegible] along with [illegible] access links is given by

(a)

(b)

(c)

(d)

[2013 : 2 Marks]

Common Data for Questions 1.95 and 1.96:
Given for the following C code segment

```
int x, y, z = 1
void f(int *y, int);
main()
{ register int x = 1;/* Line 1*/
  or Fun();
  z=1;
  printf("%d", x);
  printf("%d %d %d", y, z);
  extern int Fun(int);
  printf("%d", Fun(z));
  x += z;
  printf("%d %d %d", x, y, z);
}
```

1.95 What current value is printed on line 1 of the given code sequence?

(a) 8 (b) 1
(c) 4 (d) 1
(e) 1 (f) 2
(g) 1 (h) 2
(i) 2 (j) 0
(k) 4 (l) 1
(m) 6 (n) 2
(o) 6 (p) 2

[2012 : 2 Marks]

1.96 What output will be generated by the given code sequence?

"x" is assigned by memory;
"y" is assigned by register type;

(a) 3 (b) 1
(c) 4 (d) 1
(e) 1 (f) 4
(g) 4 (h) 1
(i) 6 (j) 2
(k) 2 (l) 0
(m) 4 (n) 1
(o) 4 (p) 2
(q) 2 (r) 0

[2012 : 2 Marks]

1.97 Consider the return-value of fun(p). The value of p is initialized to 0 before the call. Note that the first parameter is passed by reference, whereas the second parameter is passed by value.

```
int f(int &x, int y)
{   y = y - 1;
    if (y == 0) return 1;
    x = x - 1;
    return x * f(x, y);
}
```

(a) 1 (b) 0
(c) 2 (d) 1
(e) 3 (f) 1

[2013 : 2 Marks]

1.98 Consider the following program in C language.

```
#include <stdio.h>
int f()
{
  int x;
  int *p = &x;
  x = 10;
  printf("%d", *p);
}
```

Which one of the following statements is TRUE?

(a) Compilation fails.
(b) Execution results in a run-time error.
(c) Execution prints a garbage value.
(d) Execution prints the value assigned to x.

[2014 (Set-1) : 1 Mark]

1.99 Consider the following pseudo code. What is the total number of multiplications to be performed?

```
D = 2
for i = 1 to n do
    for j = i to n do
        M[i, j] = D * M[i, j]
        D = 2 * D
```

(a) Half of the product of n – 1 consecutive integers.
(b) Half of the product of the consecutive integers.
(c) Cannot be predicted as it depends on the consecutive integers.
(d) None of the above.

[2014 (Set-1) : 2 Marks]

1.100 Consider the function fun, shown below.

```
int fun(int n)
{   int count = 0;
    while (n)
    {   count = count + (n & 1);
        n >>= 1;
    }
    return (count);
}
```

The value returned by fun(14650) is _____.

[2014 (Set-3) : 1 Mark]

1.101 Suppose a pointer is assigned to variable in a C program. Which one of the following statements is most likely to cause a problem?

(b) [illegible]
(d) [illegible]
(c) [illegible]
(d) [illegible]

[2014 (Set-3) : 1 Mark]

1.103 [illegible]

```
double f(double x);
  [illegible]
  [illegible]
}
```

[illegible] value of in 2 decimals is such that f(x) will [illegible] 0

[2014 (Set-2) : 2 Marks]

1.105 Consider the C function given below.

```
int f(int, )
    int [illegible] = 5;
    int [illegible]
      [illegible]
      printf("%d\n", [illegible]);
      [illegible] = f(a);
      [illegible]
    }
    [illegible]
}
```

While compiling this following is TRUE?

(a) The function [illegible]
(b) The [illegible] the string [illegible] for [illegible] values of [illegible]
(c) The function [illegible] when [illegible] = 30
(d) The function will [illegible] when [illegible] = 30

[2014 (Set-2) : 2 Marks]

1.104 The output of the following C program is

```
void f(int, int [illegible]) {
    int [illegible];
    [illegible]
}

void [illegible](int [illegible], int [illegible]) {
    int [illegible];
    [illegible]; /* [illegible] */
}

int main() {
    int [illegible];
    f(a, b);
    if (b[illegible]) [illegible]
    printf("%d\n", [illegible]);
}
```

[2013 (Set-1) : 1 Mark]

1.106 Consider the following pseudocode, where [illegible] are positive integers.

```
begin
    [illegible]
    [illegible]
    while [illegible]
      begin [illegible] = q + 1; end
end
```

The pair [illegible] is the quotient and remainder, respectively, of the division [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2016 (Set-1) : 3 Marks]

1.108 Consider the following C function.

```
int [illegible](int [illegible]) {
    int [illegible];
    [illegible];
    for (i=1; [illegible]; i++) {
        [illegible] = [illegible] * [illegible];
    }
    return [illegible];
}
```

The return value of the function [illegible]

[2015 (Set-2) : 1 Mark]

1.109 Consider the following function written in the C [illegible]

```
void [illegible] {
    if (a && [illegible]) {
        [illegible];
        [illegible]
    }
}
```

[illegible] the same function is written in Pascal [illegible]

(a) [illegible] (c) [illegible]
(b) [illegible] (d) [illegible]

[2015 (Set-3) : 1 Mark]

1.108 A Young tableau is an array of integers increasing from left to right and from top to bottom. Any unfilled entries are marked by [illegible] and hence there can be no entry to the right of or below a [illegible]. The following Young tableau [illegible]

1	2	5	14
3	4	6	22
10	12	18	27
24	[illegible]		[illegible]

When an element is removed from a heap, the last element in the heap is placed in place of the removed element and then it is moved up or down to restore the heap property. The minimum number of swaps required to remove 1 from the given heap is _____.

[2013 (Set-2) : 2 Marks]

1.108 Consider the following C program segment

```
#include <stdio.h>
int main() {
    char s[7] = "1234";
    p = (int *) s;
    *p = 30;
    printf("%d", s[1]);
}
```

What will be printed on the screen?

(a) 48 (b) 128 or 48
(c) 254 (d) 1024

[2013 (Set-3) : 1 Mark]

1.110 Consider the following C program

```
#include <stdio.h>
int main() {
  static int a[] = {10, 20, 30, 40, 50};
  static int *p[] = {a, a+2, a+3, a+4};
  int **ptr = p;
  printf("%d%d", ptr[0][1], *ptr[1]);
}
```

The output of the program is

[2013 (Set-3) : 2 Marks]

1.111 Consider the following recursive C function.

```
void get(int n) {
    if (n < 1) return;
    int x = 1;
    get(n-1);
    printf("%d", x);
}
```

If get(3) is called, it will print 1 1 1. How many times will the get() function be called before returning to the main()?

(a) 4 (b) 2
(c) 3 (d) 1

[2013 (Set-3) : 2 Marks]

1.112 Consider the following C program

```
#include <stdio.h>
int main() {
    int i, j, k = 0;
    j = 8 * (3 / 4) + 6[1] + 5[2] / 5;
    k = ++j - j--;
    for(i = 0; i < 5; i++) {
        switch(k + 2) {
            case 7: printf("%d%d", i + k);
            case 8: printf("%d%d", i + k);
            default: printf("%d%d", i + k);
        }
    }
    return 0;
}
```

The number of times printf statement is executed is _____.

[2013 (Set-4) : 2 Marks]

1.113 Consider the following C program

```
#include <stdio.h>
int fun(int);
int fun(int);
int x = 10;
int main() {
    int x = 1;
    x = fun(x) + fun(x) + x;
    printf("%d", x);
    return 0;
}
int fun(int n) { static int x = 5; x += n; return x;
int fun(int n) { x *= 10; return x; }
```

The output of the program is _____.

[2013 (Set-3) : 2 Marks]

1.114 Consider the following C program

```
void f(int, char*);
void main() {
    int i = 100;
    char *p = "abc";
    f(i, p);
}
```

Which one of the following expressions, when placed in the blank space, will NOT result in a syntax/linking error?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2018 (Set-1) : 1 Marks]

1.115 Consider the following C code.

```
void swap(int *pint1, int *pint2)
{
    int *temp;
    temp = pint1;
    pint1 = pint2;
    pint2 = temp;
}
int main()
{
    int a = 10, b = 20, c = 30, d = 40;
    swap(&a, &b);
    swap(&c, &d);
    printf("%d %d %d %d", a, b, c, d);
}
```

The output of the program is _____

[2018 (Set-1) : 1 Mark]

1.116 The following function computes the [illegible] value contained in [illegible] array p[0] ... p[size - 1].

```
int [illegible] (int p[], int size)
{
    int [illegible] = 0, [illegible] = [illegible] - 1;
    while ([illegible])
    {
        if (p[i] <= p[j])
        {
            [illegible] = [illegible] + 1;
        }
        else
        {
            [illegible] = [illegible] - 1;
        }
    }
    return p[i];
}
```

The missing [illegible] are

(a) [illegible] = 0 (b) [illegible] = 0

(c) [illegible] = [illegible] (d) [illegible] = 0

[2018 (Set-1) : 2 Marks]

1.117 What will be the output of the following C program?

```
#include <stdio.h>
void count(int n)
{
    static int d = 1;
    printf("%d", n);
    printf("%d", d);
    d--;
    if (d > 0)
    count(n - 1);
    printf("%d", d);
}
int main()
{
    count(3);
}
```

(a) 3 2 2 1 1 0 0 0 (b) 3 2 1 0 1 2

(c) 3 2 2 1 1 0 (d) 3 2 1 2

[2016 (Set-1) : 2 Marks]

1.118 What will be the output of the following pseudo-code when parameters are passed by reference and dynamic scoping is assumed?

```
a = 10;
proc f()
{
    a = a + x;
}
proc g()
{
    int a = 5;
    f();
    print(a);
}
proc main()
{
    a = 1; x = y = 2;
    g();
}
```

(a) 5, 3 (b) 5, 6

(c) 11, 3 (d) 11, 4

[2016 (Set-1) : 2 Marks]

1.119 The value printed by the following program [illegible]

```
void f(int *p, int n)
{
    n = n + 5;
    *p = *p + n;
    return;
}
void main()
{
    int i = 5, j = 10;
```

[illegible]

[illegible] (Set 2) : 1 Mark]

1.130 The following [illegible] for [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[2014 (Set-2) : 2 Marks]

1.131 [illegible] program:

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[2016 (Set-2) : 2 Marks]

■■■■

Answers Programming

1.1	(a)	1.2	(d)	1.6	(d) & 1.7	(b)	1.8	(c)	1.9	(a)	1.10	(a)	1.11	(a)	1.12	(a)			
1.13	(b)	1.14	(d)	1.15	(b)	1.16	(c)	1.17	(d)	1.18	(b)	1.19	(c)	1.20	(a)	1.21	(d)		
1.22	(c)	1.23	(a)	1.24	(d)	1.25	(a)	1.26	(c)	1.27	(a)	1.28	(b)	1.29	(c)	1.30	(c)		
1.31	(b)	1.32	(d)	1.33	(b)	1.34	(a)	1.35	(d)	1.36	(a)	1.37	(a)	1.38	(d)	1.39	(b)		
1.40	(a)	1.41	(a)	1.42	(b)	1.43	(a)	1.44	(a)	1.45	(d)	1.46	(a)	1.47	(d)	1.48	(c)		
1.49	(c)	1.50	(c)	1.51	(b)	1.52	(d)	1.53	(d)	1.54	(a)	1.55	(c)	1.56	(d)	1.57	(c)		
1.58	(b)	1.59	(d)	1.60	(a)	1.61	(c)	1.62	(a)	1.63	(b)	1.64	(d)	1.65	(b)	1.66	(b)		
1.67	(d)	1.68	(d)	1.69	(a)	1.70	(b)	1.71	(d)	1.72	(c)	1.73	(d)	1.74	(c)	1.75	(a)		
1.76	(b)	1.77	(c)	1.78	(b)	1.79	(b)	1.80	(c)	1.81	(c)	1.82	(b)	1.83	(c)	1.84	(c)		
1.85	(a)	1.86	(b)	1.87	(d)	1.88	(c)	1.89	(b)	1.90	(a)	1.91	(a)	1.92	(d)	1.93	(a)		
1.94	(d)	1.95	(a)	1.96	(d)	1.97	(b)	1.98	(d)	1.99	(c)	1.100	(a)	1.101	(d)	1.102	(b)		
1.103	(d)	1.104	(c)	1.105	(b)	1.106	(d)	1.107	(d)	1.108	(c)	1.109	(d)	1.110	(d)	1.111	(a)		

Explanations: Programming

1.1 (a)

Following is the execution sequence of [illegible] main program

[illegible]

[illegible]

(2) [illegible] = output is 0

So the correct answer is option (b)

1.2 (d)

[illegible] array element by value [illegible] Because like [illegible] structure, there may be different [illegible] call by reference [illegible] by name [illegible] passing the [illegible] array elements.

1.3 (b)

- [illegible] implicit [illegible] explicitly changing [illegible] data type [illegible]
- [illegible] dynamic [illegible]
- The [illegible] is called [illegible] second. So [illegible] used [illegible]
- Dynamic [illegible] is used for [illegible] data type.

[illegible] b, c, d, p

1.4 (b)

[illegible]

1.5 (b)

A, B, T=T1 (C=1, D=4)

1.6 (a) & (c)

[illegible]

In pascal, in [illegible] integer variable is called [illegible] integer variable is called [illegible]

Procedure P(var x, y: integer) [illegible] are [illegible] variables, and [illegible] called by reference for x and y.

Procedure Q(x: integer; var y: integer) [illegible] integer variable and y is pointer [illegible] call in the sequence for y.

[illegible]

1.8 (a)

[illegible] developed [illegible] and its [illegible] and Q [illegible] accessible in P because [illegible] variable with same name.

1.9 (a)

Given program [illegible] mapping

1.10 (c)

FORTRAN [illegible] static allocation for variables and [illegible] supports [illegible]

1.11 (b)

A [illegible] to [illegible] with the [illegible] on the [illegible] assignment [illegible] because it makes program more difficult to verify.

1.12 (b)

Call by name and call by reference parameter passing techniques may differ when passed an array element as a parameter.

1.13

Because using static scoping, global variable value can not be changed.

1.14

Active memory → Subroutines
Heap memory → Assembler
Reference counts → Garbage collection
Address Binding time → Linking phase

1.15

Heap memory is required when part dynamic data structure is used. Memory can be allocated and deallocated at run time.

1.16

B_1 is independent of B_2's variable.

1.17

Activation tree helps to assess the non local variables and keep an array of pointers to activation records.

1.18

Any binary goto, because on executing calls and repeat loops in program may cause the program counter in infinite loop, i.e., program may not terminate.

1.19

Pass by value of a name is global variable.
Value = 0.5 and after call pass value according to question.

1.20

Assembler cannot be a compiler if the processor supports only immediate and direct addressing modes. Compiler requires at least indexing mode.

1.21

Take some values and verify the options 10, 6, 7.

1.22

Indirect addressing → Pointer:
MOV A, 100
Immediate addressing → Constants.
MOV A, 10
Array access → Index
for (int i = 10; i++);

1.23

Indirection array each element of which is a pointer to a char type node.

1.24

Dangling pointers:

1.25

Infinite recursions.

1.26

Here structure size is the minimum of array size multiplied by the size of the data type.
So total minimum size required = 20.

1.27

Static variable becomes time when value is changed it becomes the new value of counter.
So first count value of this 1, then 2, then 3 and last 10.

1.28

1.29

It is a single dimensional array type is declared and initialized.
Type wrong, variable pointer can't work like pointer to function.

1.30

1.81 (b)

Call-by-value result and call by reference may differ in the presence of aliasing (sending the same parameter more than once).

1.82 (b).

Recursion may use activation records, but not necessarily.

1.83 (b)

Address of a[2] = Base address of a + 2 × element size
= 0 + 4000 × 1 + 2 × 1 = 4002
Therefore option (b) is correct.

1.84 (a)

Here int A[10] is an array of 10 elements.
Weight of the A[2] is 2 bytes.
Hence the A[2] is stored in 2 bytes.
Hence the base address of A[2] is 2000 and hence B[2][3] can obviously be stored at 2000. Hence option (a) is correct. Declaration of B[10][10] indicates B[2] can not be used because it is a 2-dimensional array.

1.85 (d)

T[i] A[i] translates
T[A[i]] = –0;
Prints the value of the last element of the array.

1.86 (d)

```
x = 5;
p (&x);
x = 4; y = 3 = 2 = 1 = 0;
Goto
x = 7
x = x + 1 = 1 + 2 =>    print(x) = 12
y = x = 1 = 8 =>    print(x) = 0
printf(y) = 8
```

1.87 (d)

The pointer-driven programming paradigm provides control flow based on the program state. Structures exist that execute the code according to this requirement. Petri nets are good examples of structures programming. In structures programming, programs are formed from structures connected in such a manner that sequential manner.

1.88 (d)

Consider the given C function int
void swap (int a, int b)
{ int temp;
temp = a;
a = b;
b = temp;
}
The main program for given code is
#include <stdio.h>
void main ()
{ int x, y;
x = 1;
y = 2;
swap(x, y);
printf("%d", x);
printf("%d", y);
}
Here all variables are global from where it will be seen. In the value of x and y are passed by value. There is interchange in actual parameters x and y but no change in formal parameters a and b because the scope of a and b lies within the function but not in the main program. In the value of x and y are interchanged. If we will call it by reference then the value of x and y will be interchanged in the case
void swap (int *a, int *b)
{ int temp;
temp = *a;
*a = *b;
*b = temp;
}
Swapping of the variables.

1.89 (b)

Then x = 0
Then x = 1
So the output is x = 1
Then x = 1
Then x = 0
So the output is x = 1 x
= 1 – 1 = 0
So option B is correct.

1.40 [illegible]

[illegible] if j equals zero, [illegible] swapping M[i][j] with M[i][i] [illegible] M[i][i] and M[i][j] so that matrix M will remain unchanged. So to avoid the double swapping we need to [illegible] only upper triangular matrix or lower triangular matrix.

```
for (i = 1; j < n; ++j)
{   Swap the swapping M[i][j] with M[j][i]
    t = M[i][j];
    M[i][j] = M[j][i];
    M[j][i] = t;
}
```

1.41 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

In first call of [illegible] = 16.

In second call of [illegible] = 12 and it [illegible] f(+12) = [illegible].

So, [illegible] is incremented by [illegible] = 36 [illegible] = 55 = 55 [illegible]

[illegible] incremented by 12 + 12 = 37 [illegible] = 20

1.42 [illegible]

A [illegible] call by value [illegible] applicable only for local.

[illegible] is printed.

[illegible] is printed.

[illegible] is printed.

1.43 [illegible]

```
    int f(int n)
2.  {       static int [illegible]
3.          if (n == 0) return n;
4.          [illegible]
5.          [illegible];
6.          return n;
7.  }
```

Execution	n	
[illegible]	1	[illegible]
[illegible]	[illegible]	[illegible]
	2	[illegible]
	3	[illegible]

For [illegible] the value [illegible] = 4 and [illegible] = 7. [illegible] is 7 [illegible] returns the value 7.

1.44 [illegible]

The given C program [illegible]

[illegible]

```
    if m = [illegible] then [illegible]
2.  if m > n then m = m - n
4.  if m < n then n = n - m
```

Two example [illegible] = 60 and [illegible] = 82

[illegible] = 60 and [illegible] = 38

[illegible]

[illegible]

[illegible] = 22 = 22 = 6

[illegible] = 6 and [illegible] = 6

[illegible] = 6

1.45 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

1.46 [illegible]

[illegible]

```
[illegible]
[illegible] = 0;
while (n > 0)
    rev = rev * 10 + n%10;
    n = n/10;
}
```

Let input [illegible]

where d_1 = [illegible], d_2 = [illegible], d_3 = [illegible] and d_4 = [illegible]

[illegible]

[illegible]

1.47 [illegible]

[illegible]

1.48 [illegible]

int (*fp)(int);

[illegible] is a pointer to a function that accepts an integer argument and returns an integer.

1.49 [illegible]

The abstract data type (ADT) refers to a programming data type with a set of operations that can be performed on the data.

1.50 [illegible]

[illegible]

1.51 [illegible]

[illegible]

1.52 [illegible]

[illegible]

1.53 [illegible]

```
[illegible]
    [illegible]
    [illegible]
    [illegible]
    sum = sum + [illegible]
    [illegible]
```

```
    printf("%d", [illegible]);
}
int main()
    [illegible]
    [illegible]
    [illegible]
}
```

[illegible]

When [illegible] function

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

1.54 [illegible]

It [illegible] the adjacency [illegible] will contain all the [illegible] [illegible] of that vertex. [illegible] the time complexity.

[illegible] part of the [illegible] in adjacency matrix. [illegible] F2, which [illegible] from [illegible] this is done through

[illegible]

[illegible] that there is no edge. [illegible] A[i][j] [illegible] A[i][j] [illegible] sequence [illegible] vertices. Now, the [illegible] candidate for sink is [illegible] i = j

1.55 [illegible]

[illegible] adjacency matrix [illegible]

[illegible]

1.56 (d)

A: [illegible] that is equal to the ASCII value of the character to its [illegible]

B: [illegible] that is equal to the ASCII value of the character in a pointing [illegible]

If one string is [illegible] there would [illegible] increment [illegible] and [illegible] that is what the loop checks for [illegible] element is found, it [illegible] strings are [illegible]

1.57 (d)

The [illegible] has to be [illegible]

[illegible]

1.58 (d)

[illegible]

1.59 (b)

[illegible] swap function [illegible] and [illegible] [illegible]

[illegible]

[illegible] and [illegible]

[illegible]

1.60 (b)

[illegible]

[illegible] will print [illegible] = 5.

[illegible]

[illegible] will print [illegible] = 9

[illegible]

[illegible] pointer [illegible] address of an integer [illegible] [illegible] will print [illegible]

[illegible] pointer [illegible] [illegible] will be [illegible] and [illegible] will be the first element [illegible] which is 20

printf("%d", [illegible])

[illegible] will subtract a size of pointer from the base address [illegible] Actually this means [illegible] memory address, but since we have [illegible] a pointer to [illegible] will [illegible]

So [illegible] will be [illegible] which was the value [illegible] and [illegible] [illegible] and 20.

1.61 (a)

[illegible]

1.63 (b)

1. Overloading is defined only [illegible] functions
2. [illegible] keyword [illegible] members [illegible] access to the private data of an object
3. Constructor is a function which is automatically called when an object is created.
4. Friend keyword allows a function to access the private parts of another class
5. Friend is a property of the object associated with the current function.
6. Inheritance allows a new class to have properties of another class.

1.62 (a):

So: The compiler will generate code to allocate memory for the array, initialize the 10, and place the address of the array in a variable.

So: The program will print 8 [illegible]

1.64 (d):

If array [illegible] contains the element, then [illegible] will [illegible].

1.65 (b):

It will give the following [illegible]

Assume [illegible]

Work1 output is 16

Work2 output is 10

Compiler [illegible]

1.66 (a):

So [illegible]

1.67 (c):

The [illegible]

[illegible]

1.68 (d):

The [illegible] for all positive [illegible] is infinite, hence [illegible] is [illegible] (a) or (b).

Let n = 5.

Now, recursive [illegible] will go like 5 + 4 + 3 + 2 + 1 + 0 and then [illegible]. And if we multiply 5 with [illegible] power [illegible] also result will be infinite recursion. Hence [illegible] is more [illegible] possible, [illegible] are [illegible] infinite possible recursion, [illegible] this case only).

For [illegible] is true.

For [illegible] false.

1.69 (d):

[illegible] right [illegible] given below

```
#include <stdio.h>
int main ()
{
    int num = 5, [illegible] = 0, [illegible] = 6;
    int a[ ] = {2, 4, [illegible], 3, [illegible]};
    for (i = 0; [illegible]; i++)
    {
        if (i == 0 || [illegible] > a[i - 1])
        {
            if (num > [illegible] && [illegible] > maximum)
            maximum = (a[i] > 0) ? a[i] : 0;
        }
        i--
        num = a[i];
    }
    if (num > maximum) maximum = num
    printf ("%d\n", maximum);
}
```

[illegible] and [illegible] value of num [illegible] maximum for a = 3, [illegible]

i	[illegible]
num	[illegible]
maximum	[illegible]

So maximum = [illegible]

1.70 (c):

push 5

push 4

push 5* = 10 (pop 5 and 4)

push 3

push 5

push 2

push 6 + 2 = 8 (pop 6 and 2)

push 3 * 8 = 15 (pop 3 and 8)

push 10 + 15 = 25 (pop 10 and 15)

1.71 (d):

Call by value: [illegible] passed [illegible]. Then [illegible] local variable [illegible] integer [illegible] and global variable [illegible] and [illegible].

[illegible]

1.74 [illegible]

[illegible]

1.75 [illegible]

[illegible]

1.76 (b)

$$[illegible] = \begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix}$$

When 1 step will, [illegible]

[illegible]

[illegible]

1.77 [illegible]

[illegible]

[illegible]

[

1.58 (b) :

Block Scope: A block is a set of statements enclosed within a pair of braces (curly brackets). Blocks may be nested. A C program may contain other blocks inside it. A variable declared inside a block is not visible outside the block. Its visibility is limited to that block, but it is visible within the block.

If an inner block declares a variable with the same name as the variable declared in the outer block, then the visibility of the outer block variable ceases to exist. Inside the inner block, the variable of the outer block is not visible. Thus the scope of this block variable and outer block variable is mutually exclusive.

```
i = 7            j = 5 (true)
  = 7/2 = 3
a[3] = 5[3]      j = 7 -   = 3
  = 3            j = 3 (true)
i = 6/2 = 3
a[3] = a[3] - 1 - 3 -   = 2
j = 5            j = 4 (true)
i = 3/2 = 1
a[2] = a[2] -   - 4   1 = 3
j = 4            j = 3 (false)
```

Now outside the for loop and the inner block, the scope is also end and the outer block variable now visible.

So the output is 3, 2, 2.

1.59 (b) :

First integer type two variables are declared and initialized. Then an integer type 2-D array a[3][4] is declared and initialized. Pointer variable p[2] is created and initialized.

Address	Value	Address	Value	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Now the array type pointer is declared and the base address of array b is put in it. Now the for loop is started where i is initialized to 0, as:

```
While i = 0 ; i < 2 (true)
    i = 0 ; j < [illegible] (true)
        *(p[illegible] + 2*i + j) = a[i][j]
        *(b[illegible]) = a
        j++
    j = 1 ; j < 3 (true)
        *(p[illegible] + 2*i + j) = a[illegible]
        *(b[illegible]) = [illegible]
        j++
    j = 2 ; j < 3 (false)
        j++
While i = 1 ; i < 2 (true)
    j = 0 ; j < 3 (true)
        *(p[illegible] + 2*i + j) = a[illegible]
        *(b[illegible]) = a
        j++
    j = 1 ; j < 3 (true)
        *(p[illegible] + 2*i + j) = a[illegible]
        *(b[illegible]) = [illegible]
        j++
    j = 2 ; j < 3 (true)
        *(p[illegible] + 2*i + j) = a[illegible]
        *(b[illegible]) = [illegible]
        j++
    j = 3 ; j < 3 (false)
        i++
```

Now the values in array b is

b[0][0]	[illegible]	[illegible]
b[0][1]	[illegible]	[illegible]
b[1][0]	[illegible]	[illegible]
b[1][1]	[illegible]	[illegible]
b[2][0]	[illegible]	[illegible]
b[2][1]	[illegible]	[illegible]

Hence the output will be (b) above.

Note: *(p + 2*i + j)

[illegible] Hence with this statement we can access the [illegible] in the array b. Hence pointer p

1.60 (d) :

[illegible]

[illegible] will be

The function call [illegible] = 880 in part only.

[illegible]

1.81 [illegible]

[illegible]

1.82 [illegible]

[illegible]

```
int y, z
[illegible]
x = *pp = 6;
*px = *py = 8;
y = *py = 7
z = [illegible]
[illegible] = y + z = 19
```

1.83 [illegible]

There are two [illegible] operator which has [illegible] priority. [illegible] operator.

[illegible] = [illegible] has to be in brackets and after [illegible] the value we can [illegible] as [illegible] for [illegible] the [illegible]

1.84 [illegible]

When 2 is [illegible] and [illegible] power [illegible] search can [illegible]

1.85 [illegible]

There will [illegible] [illegible] = k + 1, [illegible] j = k = [illegible] for all the [illegible]

1.86 [illegible]

```
x = 17
[illegible]
[illegible]
[illegible]
x          [illegible] = 4,   [illegible] = 5 = 8, [illegible]
           return 5
fun(1, 100,
[illegible]
           [illegible] = 4,   [illegible] = 5 [illegible] = 2
           return 5
fun(8, 100)
           [illegible] = 1,   [illegible] = 2 [illegible] = 2
           return 9
[illegible]
x          [illegible] = 1,   [illegible] = 1 [illegible]
           return 8
fun(1, 100,
           [illegible]
           return [illegible]
```

[illegible] will return [illegible] will be printed.

1.87 [illegible]

In the given program [illegible] initialized by 0 and 1. [illegible] function [illegible] the address of [illegible]

When p = [illegible] and *p = 2 [illegible]

*q = 1, so value of *q is [illegible]

[illegible] value of *q [illegible] j

[illegible] expected.

[illegible]

1.88 [illegible]

[illegible]

[illegible] first choice

The [illegible] numbers stored in the array are [illegible] by [illegible]

[illegible]

1.70 (d):

[illegible]

1.71 (b):

Consider the following [illegible]

```
int fun(int n, int m)
{
    if (n > 0) return fun(n-1, m);
    else return m;
}
```

Value returned by the function fun(540, 100)

The return value is 0 + [illegible] = 12

1.72 (d):

[illegible]

1.73 (c):

[illegible]

Hence the switch [illegible] will execute A and execute every statement from there.

[illegible]

[illegible]

1.94 (d):

Circuit will be equivalent to the program:

[illegible]

1.95 (b):

1.96 (d):

1.97 (b):

Value of x is passed by reference so any time changes are reflected in original value

Option (b) is correct.

1.98 (d):

[illegible]

1.99 (c):

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} 1 = O(n+n^2+n^3) = O(n^3)$$

1.100 (c):

[illegible]

Statement is incorrect. 8 times.

1.101 (c):

[illegible]

Option (c) is correct.

1.102 Ans.

[illegible]

if [illegible], the [illegible]

Therefore, [illegible]

Time [illegible] $O(\log n)$ [illegible].

1.103 (d)

[illegible] seconds [illegible]

1.104 (d)

[illegible]

[illegible]

Output: [illegible]

1.105 (b)

[illegible]

```
while [illegible] do
    begin
        [illegible]
        [illegible]
    end
```

Post conditions: [illegible]

[illegible]

[illegible]

[illegible]

1.106 (c)

[illegible]

```
[illegible]
[illegible]
    [illegible]
    [illegible]
```

[illegible]

[illegible]

(a) [illegible]

[illegible]

Computation [illegible]

[illegible]

1.107 (c)

[illegible]

[illegible] DCBA

1.108 Ans.

The minimum number of movements are 5. The following sequences will give minimum movements after 1 movement.

Seq 1: Move 2 left, move 3 left, move 6 [illegible] move 19 [illegible] move 32 left

Seq 2: Move 2 left, move [illegible] move 3 left, move 19 [illegible] move 26 left

[illegible]

1.109 (b)

[illegible]

[illegible]

[illegible]

1.110 [illegible]

[illegible]

1.111 [illegible]

Number of times invoked

[illegible]

∴ Number of times get function invoked on get (3) = 23

1.112 [illegible]

[illegible]

∴ Total 10 times print is executed.

1.113 [illegible]

[illegible]

1.114 [illegible]

[illegible]

1.115 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Option (d): $X^2 = 2x + x^2$

$x^2 = (1 \times 5)^2 = 125 \times 125$

After iteration 1

res = 6; p = 5; k = 2; x = 5; Y = 5

$X^2 = (res \times p)^k = (6 \times 5)^2 = 30^2 = 900$

125 × 827 Hence option (d)

Option (d) cannot be answer.

Option (d): $X^2 = 2x + x^2$

$x^2 = 125 \times 125 = 125$

After iteration 1

res = 5; p = 5; k = 2; x = 5; Y = 5

$X^2 = (res \times p)^k = (5 \times 5)^2 = 25^2 = 625 = 125$

After iteration 2

res = 25; x = 5; k = 1; X = 5

$X^2 = (res \times p)^k = (25 \times 5)^1 = 125 = 125$

So, all conditions satisfied

So option (d) will be answer.

1.121 [illegible]

[illegible]

1. [illegible] number of elements in the array.
2. Every time [illegible] is less than equal to [illegible] and stop the program execution continue with recursive function call.

1. (a, b)

*P = [illegible] P pointed to some address contained in M.

x = 5; x value greater than 1.

So, max(*P = [illegible] 6, 10, 3, [illegible]) or max(*P + 1, 4) = 12;

2. p(P + 1, 4)

*P = P + 1; P is pointed to next element of array i.e., 3

x = 4; x value previous [illegible]

So, max(max(*P + 1, 2, [illegible] 5 = 25, 2) or max(max(*P + 1), 3, 5, −2)

3. p(P + 1, 3)

*P = P + 1; P is pointed to next element of array i.e., 5

x = 3; x value greater than 1

So, max(max(max(*P + 1, 2, [illegible] 3), 2, [illegible] 5) −2) or max(max(max(*P+1, 2), [illegible] 5) −2),

4. p(P + 1, 2)

*P = P + 1, P is pointed to next element of array i.e., 6

x = 2, x value greater than 1

So, max(max(max(max(*P + 1, 2) = 11, 2), 4), 3), [illegible] 10

max(max(max([illegible]*P + 1, 11, 2), 4), 3), 6

5. p(P + 1, 1);

*P = P+1, P is pointed to next element of array i.e., 4

x = 1, x value equal to 1 so return 0.

So, max(max(max([illegible] 11, 2), 4), 3), 2)

max(max(max([illegible] 11, 2), 4), 3)

max(max([illegible] 11, 2), 4)

max(11, 4) = 11

So the value should be given [illegible]

■■■■

2 Arrays

2.1 In a computer, a single-dimensional array representation of a lower triangular matrix of size n elements can be accessed by a formula in n. Let a_{ij} be the element of the lower triangular matrix and the array stores the matrix row-wise, the address of the $(i,j)^{th}$ element of the lower triangular matrix is [illegible] and the corresponding index is

(a) $i(i-1)/2 + j$ (b) $i(i-1)/2 + j - 1$

(c) $j(j-1)/2 + i$ (d) $i + j(j-1)/2$

[1994 : 2 Marks]

2.2 Let A be a two dimensional array declared as follows:

A: array [1..10] [1..15] of integer;

Assuming that each integer takes one memory location. The array is stored in row major order and the first element of the array is stored at location 100, what is the address of the element A[i][j]?

(a) 15i + j + 84 (b) 15j + i + 84

(c) 10i + j + 99 (d) 10j + i + 99

[1998 : 2 Marks]

2.3 An n × n array A is defined as follows:

A[i, j] = i + j for all i, j, 1 ≤ i ≤ n, 1 ≤ j ≤ n.

The sum of the elements of the array is

(a) n^2 (b) $n^2 + 3n + 2$

(c) $n^2 - 3n - 2$ (d) $n^2 + n - 1$

[2000 : 1 Mark]

2.4 Suppose we are given an array A[1..n] and a procedure reverse(A, i, j) which reverses the order of the elements between positions i and j (both inclusive). What does the following sequence do, where 1 ≤ k ≤ n:

```
reverse(A, 1, k);
reverse(A, k+1, n);
reverse(A, 1, n);
```

(a) Rotates A left by k positions

(b) Leaves A unchanged

(c) Reverses all elements of A

(d) None of the above

[2000 : 1 Mark]

2.5 An array of integers of size n can be converted into a heap by adjusting the heap property for each internal node, beginning with the last internal node (root node is at index 0) in the order from $\lfloor n/2 \rfloor - 1$ to 0. The time required for this operation is

(a) $\Theta(n)$ (b) $O(n)$

(c) $\Theta(n \log n)$ (d) $O(n \log n)$

[2001 : 2 Marks]

2.6 A program P reads in 500 integers in the range [0..100) representing the scores of students in an examination. P prints the number of students scoring above 50. What would be the best way to P read the integers?

(a) An array of 50 numbers

(b) An array of 100 numbers

(c) An array of 500 numbers

(d) A dynamically allocated array of 500 numbers

[2005 : 1 Mark]

2.7 Consider an array containing n numbers in increasing order. The following algorithm determines whether there are two distinct numbers in the array whose sum is a specified number S > 0.

```
i = 0; j = n-1;
while (i < j)
    if (A[i] + A[j] == S)
        return (A[i], A[j]);
    else if (A[i] + A[j] < S)
        i++;
    else
        j--;
}
return (printf("not found"));
else return (printf("n<2");
```

Choose the correct expression for R

(a) A[i] + A[j] < S (b) A[i] + A[j] > S

(c) A[i] + A[j] < S (d) A[i] + A[j] > S

[2005 : 2 Marks]

2.8 Let A and B be two sorted arrays of size m and n integers each, in non-decreasing order. Let C be a sorted array of size m + n integers obtained

[illegible] sorting the two arrays a and b. Assuming the arrays are indexed starting from zero, consider the following two statements:

I. [illegible] a[i] ≥ b[i] ⇒ c[2i] ≥ a[i]

II. [illegible] a[i] > b[i] ⇒ c[2i] > b[i]

III. [illegible] c[i] ≥ a[i] ⇒ c[2i] ≥ a[i]

IV. [illegible] c[i] ≥ a[i] ⇒ c[2i] > a[i]

Which of the following is TRUE?

(a) only I and II (b) only I and IV

(c) only II and III (d) only III and IV

[2005 : 2 Marks]

2.9 Consider the following C function to search a key in the sorted array A:

```
int MyX(int *A, int n, int key) {
    int L = 0;
    int R;
    int i, j, k;
    int *[illegible];
    [illegible] = (int *) malloc(n * sizeof(int));
    [illegible] = n - 1;
    for (i = 0; i < size; i++)
        for (j = 1; j < size; j++) {
            [illegible]
            int [illegible] = j + 1;
            [illegible] = [illegible] - k[k];
            if (key > [illegible]);
                [illegible] = [illegible];
        }
    return [illegible];
}
```

The value returned by the function MyX is the

(a) maximum index of elements in array A smaller than key

(b) maximum element in array A

(c) number of the maximum elements in array A smaller than key

(d) [illegible] of the elements in the array A

[2014 (Set-1) : 2 Marks]

2.10 The [illegible] matrix multiplication is performed using the following pseudocode. What is the [illegible] output?

```
C = 0;
for i = 1 to n do
    for j = 1 to n do
        {
            Temp = A[i, j] + C;
            A[i, j] = A[j, i];
            A[j, i] = Temp - C;
        }
for i = 1 to n do
    for j = 1 to n do
        output A[i, j];
```

(a) The sum of the matrix

(b) Twice the sum of the matrix

(c) Assuming 0.01 is the upper diagonal elements and subtracting 100 from [illegible] elements of the matrix

(d) Zero at the [illegible]

[2014 (Set-3) : 1 Mark]

2.11 What is the output of the following C code? Assume that the address of a [illegible] and that integers are [illegible] bytes of memory.

```
int main()
{
    unsigned int x[4][3] =
    {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}, {10, 11, 12}};
    printf("%u, %u, %u, %u", x+3, *(x+3), *(*(x+3)), [illegible]);
}
```

(a) 2000, 2006, 2006, [illegible] (b) 2000, 2006, 2006, [illegible]

(c) 2006, 10, 10, [illegible] (d) [illegible], 4, 6

[2015 (Set-1) : 2 Marks]

2.12 Consider the following two C code segments. X and Y and X[i][j] are integers, and X and Y are two-dimensional arrays of size n × n, where 2 ≤ n ≤ 10. Assume that [illegible] both code segments are stored in row-major order, and each element X[i, j] of array X is initialized to i + j. Further assume that [illegible] memory cache. Initially all elements of Y are zero and cache is empty.

Code segment 1:

```
// initialize elements of Y to 0
// initialize elements X[i, j] of X to i+j
    for i = 0; i < n; i++
        Y[i, i] = X[i, i];
```

Code segment 2:

```
// initialize elements of Y to 0
// initialize elements X[i][j] of X to i+j
    for i = 0; i < n; i++
        for j = 0; j < n; j++
            Y[i, j] = X[i, j];
```

Which of the following statements is correct?

S1: Elements of array X can be computed in linear time.

S2: Contents of array X can be used to find the longest increasing subsequence in linear time.

S3: Elements of array X can be used to find the longest increasing subsequence in linear time using additional memory.

(a) Only S2 is correct.

(b) Only S3 is correct.

(c) Only S1 and S2 are correct.

(d) Only S1 and S3 are correct.

[2015 (Set 3) : 2 Marks]

2.15 Suppose $a = [a[0], \ldots, a[n-1]]$ is an array of length n where all the entries are from the set $\{0, 1\}$. For any positive integer n, consider the following pseudocode.

```
DOSOMETHING(a, n)
s = 1
for i = 0 to n-1
    do s = s * a[i]
    if s = 0
        then return 0
return 1
```

If $n = 1$ or $n = 2$ or $n = 3$, then the value returned by DOSOMETHING is 1 if and only if

[2015 (Set-3) : 2 Marks]

■■■■

Answers Arrays

2.1 (c) 2.2 (c) 2.3 (a) 2.4 (a) 2.5 (b) 2.6 (a) 2.7 (b) 2.8 (c) 2.9 (c)

2.10 (a) 2.11 (c) 2.12 (c)

Explanations Arrays

2.1 (c)

[illegible] is just [illegible] in the lower triangular matrix and

$= i(i-1)/2 + j$ [illegible]

2.2 (c)

[illegible] the number of elements in a row.

Address of the element A[i][j] =

$= \text{Base address} + (i \times (i-1)/2 + j - 1)$

$= 100 + (10 \times (10-1)/2 + 5 + 3 - 1)$

$= 100 + (10 \times 9/2 + 7)$

$= 100 + (45 + 7)$

$= 152$

2.3 (a)

Array is

0	-1	2	...		n
1	0	-1			
2		0			
3	2	1			
		3			
-1	n-1	2n-3			0

Sum = 0 + 1 + 2 + ... + (2n-3) + ... + 0.

2.4 (a)

When [illegible] is represented in the array [illegible] is represented in the array [illegible] by [illegible].

Let, [illegible]

1	2	3	...	n-1	n

[illegible] $n = 7, k = 5$

reverse(0, [illegible]) we get [2, 1, 3, ..., 5, 6, 7]

reverse(5, [illegible]) we get [2, 1, 3, 6, 5, [illegible]]

reverse(0, [illegible]) we get [3, 1, 2, 6, 7, 1, 3,

2.5 (b)

There are 300 students that score any type in C in 100. Now the frequency of those students who have scored 50 to 100 to an array of all students is available in [illegible] using the frequency.

2.6 (a)

For case-1 it was found that difference of [4][j] – A[i][j] was zero and [illegible] difference was [illegible] and [illegible] equal to 15. Total time

[illegible] Then [illegible] greater than [illegible] so we know that no more [illegible] further [illegible] same [illegible] and so we [illegible]

We do it [illegible] if not [illegible] in [illegible] however, [illegible] = n.

2.8 [illegible]

a[1:2:1:1]: [illegible] are [illegible] in [illegible]. In pivoting, [illegible] elements smaller than [illegible] [illegible] and [illegible] were smaller than [illegible].

So a[1] > a[j] means there are [illegible] elements smaller than a[1]. [illegible] in the merged array a[1] [illegible] only after [illegible] elements. [illegible] will be [illegible]

So a[1] [illegible] [illegible] only [illegible] [illegible] elements.

Similarly a[j] > a[j] exists for b that [illegible] more than 2 elements smaller than a[j] in the sorted array. So [illegible] from b, and maximum number of elements from b.

So, b[1] < a[1]

So T and T[illegible] are correct [illegible]

2.9 [illegible]

[illegible] The value [illegible] returned by function is the minimum [illegible] number of [illegible] in [illegible] array of range E.

2.10 [illegible]

[illegible]

[illegible]

[illegible]

[illegible] swaps [illegible] elements.

For example

[illegible] and [illegible] elements are exchanged

[illegible] and [illegible] are in swapped

[illegible] For the given [illegible] matrix A, [illegible] will be the output.

2.11 [illegible]

[illegible] of two-dimensional array address.

[illegible] is the base address of the [illegible] row

[illegible] is the base address of the [illegible] row

[illegible] is the base address of the [illegible] row

Summary [illegible] is [illegible] row. So [illegible] the address of the [illegible] element in the [illegible] row the printing [illegible]

The output printed by second statement [illegible]

[illegible] 2088 [illegible] address of [illegible]

[illegible] 2088 [illegible] element in [illegible]

[illegible] 2088 [illegible] address in [illegible] row [illegible]

[illegible] make the [illegible] address [illegible] [illegible]

[illegible] by 3 × 4 = 12

[illegible] 2088.

2.12 [illegible]

[illegible] dimensional array

[illegible] two dimensional array

Code segment 1:

[illegible]

[illegible]

Code segment 2:

```
for (i = 0; i < n; i++)
    Y[i] = X[i][0];
```

S1: First column of array X will be stored in code segment

S2: Xerrors are stored row wise. Operations of code segment row wise accessed, optimum cache memory.

S3: In code segment 2, column of X are not accessed in contiguous

Sol 18 Hint:

```
X = 4
```

```
n = 9, a = 5
Z = 1;
for (i = 0; i < n; i++) mod 3 = 1
    Y[i][0] = ... X = 1 × 21 mod 3 = 2
    i = ... Z = ... mod 3 = 4
    i = ... Z = 4 mod 3 = 1
    Y[i][2] = ... X = 2 × 21 mod 3 = 0
    i = ... Z = 12 mod 3 = 7
    Z[i][2] = ... Z = 3 × 21 mod 3 = 1
    Return 0
```

■■■■

3 Stacks & Queues

3.1 The following sequence of operations is performed on a stack:
PUSH(10), PUSH(20), POP, PUSH(10), PUSH(20), POP, POP, POP, PUSH(20), POP
The sequence of values popped out is:
(a) 20, 10, 20, 10, 20 (b) 20, 20, 10, 10, 20
(c) 10, 10, 20, 10, 20 (d) 20, 10, 10, 20, 10

[1991 : 2 Marks]

3.2 Which of the following permutations can be obtained in the output (in the same relative order) using a stack, assuming that the input is the sequence 1, 2, 3, 4, 5 in that order?
(a) 3, 4, 5, 1, 2 (b) 5, 4, 3, 1, 2
(c) 1, 5, 2, 4, 3 (d) 4, 3, 5, 1, 2

[1994 : 2 Marks]

3.3 The minimum number of stacks required to implement a queue is:
(a) 1 (b) 2 (c) 3 (d) 4

[1995 : 2 Marks]

3.4 Consider the following statements:
(i) First-in-first-out type of computations are efficiently supported by stacks.
(ii) Implementing lists on linked lists makes it possible to insert an element into the middle of the list in O(1) operations.
(iii) Implementing queues on circular lists allows all queue operations to be carried out in constant time.
(iv) Last-in-first-out type of computations are efficiently supported by queues.
(a) (i) & (iii) are true (b) (ii) & (iv) are true
(c) (i), (ii) & (iii) are true (d) (i) & (iii) are true

[1996 : 1 Mark]

3.5 Which of the following is essential for converting an infix expression into its postfix form efficiently?
(a) An operator stack
(b) An operand stack
(c) An operand stack and an operator stack
(d) A parse tree

[1997 : 1 Mark]

3.6 A queue is defined as a linear data structure. "USE" (x) is an implemented as "INSERT (Q, x)" where Q is an queue pointer. "USE" (x) is implemented as DELETE(Q, x). Which sequence of operations has been carried out in a queue initially empty?
(a) INSERT(Q, 1), INSERT(Q, 2), INSERT(Q, 3), USE(3), USE(2), USE(1)
(b) INSERT(Q, 1), INSERT(Q, 2), USE(1), INSERT(Q, 3), USE(2), USE(3)
(c) INSERT(Q, 1), INSERT(Q, 2), USE(1), INSERT(Q, 3), USE(3), USE(2)
(d) INSERT(Q, 1), INSERT(Q, 2), USE(1), USE(2), INSERT(Q, 3), USE(3)

[1997 : 2 Marks]

3.7 Convert the prefix expression into its equivalent postfix expression.

$$+^*\ln(y)-(y)-\frac{a}{z}$$

[1998 : 2 Marks]

3.8 What value would be returned by the following function when the input is 40?
Function f(int x, int *pres);
Begin
If x > 100 [pres = x - 10]
Else [pres = f(x + 11, pres) + 11]
End;
(a) 90 (b) 91
(c) 92 (d) 93

[1998 : 2 Marks]

3.9 The Fibonacci sequence is defined as 1, 1, 2, 3, 5, 8, 13, 21, ... Starting with the numbers stored in two variables, the function shown below uses only one variable to perform the computation. Assume that PUSH and POP operations take X seconds each, and Y seconds elapse between successive PUSH and POP operations of the stack. If the stack is initially empty, the time (in seconds) elapsed from the start of computing the sum of the first 10 Fibonacci numbers, to the end of computing the sum of the first 20 Fibonacci numbers, is:
(a) n(2X + Y) (b) 2X + 2Y
(c) n(2X + 2Y) − X (d) 2Y + 8X

[2003 : 1 Mark]

8.10 A single array A[0 ... MAXSIZE-1] is used to implement two stacks. The two stacks grow from opposite extremities of A. Variables top1 and top2 (top1 < top2) point to the locations of the topmost elements in each of the stacks. While pushing an element in stack 1, the value of top1 is incremented, but while pushing an element in stack 2, the value of top2 is decremented. Initially, top1 = -1 and top2 = MAXSIZE. The condition for stack full is

(a) top1 = MAXSIZE/2 and top2 = MAXSIZE/2 + 1

(b) top1 + top2 = MAXSIZE

(c) top1 = MAXSIZE/2 and top2 = MAXSIZE/2

(d) top1 = top2 - 1

[2004 : 1 Mark]

8.11 The best data structure to check whether an arithmetic expression has balanced parentheses is

(a) queue (b) stack

(c) tree (d) list

[2004 : 1 Mark]

8.12 A circularly linked list is used to represent a queue. A single variable is used to access the queue. The queue should be processed such that after the execution of enQueue and deQueue, the queue should be left unchanged. Assuming no other operations are allowed,

(a) enQueue

(b) deQueue

(c) both possible with a single pointer

(d) neither enQueue nor deQueue

[2001 : 2 Marks]

8.13 Evaluate the postfix expression 10 5 + 60 6 / * 8 - where the binary operator + is right associative and the other binary operators are left associative. All operators have the same precedence. The value of the postfix expression corresponding to the infix expression a + b * c - d / e is

(a) 10 (b) 15

(c) 30 (d) 120

[2004 : 2 Marks]

8.14 A program accepts as input a string of lowercase alphabets. Consecutive occurrences of the same character are replaced by the character followed by the count of consecutive occurrences. For example, the string aaabccddeeeeff is replaced by a3bc2d2e4f2. The minimum length of the string that can be produced as output is

(a) 3 (b) 4

(c) 5 (d) 6

[2004 : 2 Marks]

8.15 A queue is defined as a linear data structure in which insertion is done at rear end and deletion at front end. Let Q be a queue with front = 0 and rear = 6 (front points to the first element and rear points to the last element). If queue Q contains the integers 3, 5, 7, 9, 11, 13, 15, in order from front to rear, what will be the value of front and rear after two successive deletions?

(a) 2, 6 (b) 4, 6

(c) 2, 5 (d) 4, 5

[2005 : 1 Mark]

8.16 An implementation of a queue Q using two stacks S1 and S2 is given below:

```
void insertQ(int x) {
    push(S1, x);
}
int deleteQ() {
    int x, y = pop(S2), z;
    if (stack_empty(S2)) {
        while (!stack_empty(S1)) {
            z = pop(S1);
            push(S2, z);
        }
        x = pop(S2);
    }
    return x;
}
```

Let n insertQ and m deleteQ operations be performed in an arbitrary order on an empty queue Q. Let x and y be the number of push and pop operations performed respectively in the process. Which of the following is true for all n ≥ 0 and m ≥ 0?

(a) x = 2n and 2m ≤ y ≤ n + m

(b) x = n + m and 2m ≤ y ≤ 2n

(c) 2n ≤ x ≤ 2n and 2m ≤ y ≤ n + m

(d) 2n ≤ x ≤ 2n and 2m ≤ y ≤ 2m

[2004 : 2 Marks]

8.17 The following postfix expression with single-digit operands is evaluated using a stack:

[illegible]

Note that '^' is the exponentiation operator. The top two elements of the stack after the first '*' is evaluated are:

(a) 6, 1 (b) 6, 7
(c) 3, 2 (d) 1, 2

[2007 : 2 Marks]

8.18 Suppose you are given an implementation of a queue of integers. The operations that can be performed on the queue are:

(i) isEmpty(Q) — returns true if the queue is empty, false otherwise.
(ii) delete(Q) — deletes the element at the front of the queue and returns its value.
(iii) insert(Q, i) — inserts the integer i at the rear of the queue.

Consider the following function:

```
void Q(queue Q) {
    int i;
    if (!isEmpty(Q)) {
        i = delete(Q);
        Q(Q);
        insert(Q, i);
    }
}
```

What operation is performed by the above function Q?

(a) Leaves the queue Q unchanged
(b) Reverses the order of the elements in the queue Q
(c) Deletes the element at the front of the queue Q and inserts it at the rear of the queue Q leaving the other elements in the queue in the same order
(d) Empties the queue Q

[2008 : 2 Marks]

8.19 Suppose a circular queue of capacity (n – 1) elements is implemented with an array of n elements. Assume that the insertion and deletion operations are carried out using REAR and FRONT as array index variables, respectively. Initially, REAR = FRONT = 0. The conditions to detect queue full and queue empty are

(a) FRONT = (REAR + 1) mod n == FRONT
respectively REAR == FRONT
(b) FRONT = (REAR + 1) mod n == FRONT
respectively (FRONT + 1) mod n == REAR
(c) FRONT == REAR
respectively (FRONT + 1) mod n == REAR
(d) FRONT == (REAR + 1) mod n
respectively FRONT == REAR

[2010 : 2 Marks]

8.20 Consider the following operation along with enqueue and dequeue operations on queues, where k is a global parameter.

```
MultiDequeue(Q) {
    m = k;
    while (Q is not empty) and (m > 0) {
        dequeue(Q);
        m = m - 1;
    }
}
```

What is the worst case time complexity of a sequence of n queue insertions and MultiDequeue operations on an initially empty queue?

(a) $O(n)$ (b) $O(n + k)$
(c) $O(nk)$ (d) $O(n^2)$

[2015 : 2 Marks]

8.21 Consider a stack machine that supports an instruction DUP that duplicates the top element of the stack in addition to the PUSH and POP instructions. Which of the following statements is/are true? Evaluation of the following postfix expression on the stack:

(a) A program cannot be implemented using this stack.
(b) A program can be implemented using this stack where LOAD pushes a single instruction and STORE pops a sequence of two instructions.
(c) A program can be implemented where LOAD pushes two instructions and STORE pops a single instruction.
(d) A program can be implemented where both LOAD and STORE push a single instruction each.

[2014 (Set-1) : 2 Marks]

3.22 Consider the C program below.

```
#include <stdio.h>
int stack[100];
int stackTop;
int stackPop(int, int);
int main()
{
    int i, int *stackTop = 0;
    switch (i) {
        case 1: stackTop = stack;
        stackPush(stackTop, 1);
        case 2: stackPush(stackTop, stack[stackTop - 1]
        - stackPop(stack);
        default: printf("Stack Not Found\n");
    }
    return 0;
}
int stackPop(int *stack, int *stackTop)
{
    int i[10] = {0};
    int *temp = i;
    int j = 0;
    int k = 10;
    printf("%d", stack[*stackTop - 1]);
    return stack[*stackTop - 1];
}
```

The output produced by the above program is _____.

[2015 (Set 2) : 2 Marks]

3.23 The result of evaluating the postfix expression 10 5 + 60 6 / * 8 - is

(a) 142 (b) 272

(c) 149 (d) 71

[2015 (Set 3) : 1 Mark]

3.24 Consider a simple linear queue of size 5 with the following sequence of enqueue and dequeue operations. Assume that the queue is empty initially. The value of the following statement is EMPTY to refer to the number of items in the queue.

(a) Half-queue can be performed in O(1) time

(b) A normal enqueue operation in the worst case is O(1) time but the worst case time for the other operation is O(n) time.

(c) The worst case time complexity for both operations will be O(n).

(d) Worst case time complexity for both operations will be O(1).

[2018 (Set-1) : 1 Mark]

3.25 Let S and Q be an empty stack and queue. Push(S, x) returns the element x to the head of the queue Q without removing it. Enqueue(Q, x) adds the element x to the tail of the queue Q. Similarly Top(S) returns the element at the top of the stack S without removing it. Pop(S) removes the element at the top of the stack S. Consider the algorithm given below.

```
while (Queue Empty) do
if (Stack Empty OR Top(S) < Head(Q)) then
    x = Dequeue(Q);
    Push(S, x);
else
    x = Top(S);
    Enqueue(Q, x);
end
end
```

The maximum possible number of iterations of the while loop in the algorithm is _____.

[2016 (Set 1) : 2 Marks]

3.26 Consider the following New-order traversal for binary trees.

- Visit the root;
- Visit the right subtree using New-order;
- Visit the left subtree using New-order.

The New-order traversal of the expression tree corresponding to the postfix expression 5 4 + 3 – 2 * 6 7 + – is given by

(a) + – 4 3 5 * 2 – 7 6 +

(b) – 4 3 + 5 * 6 7 + 2 – 3 4

(c) – + 4 3 * 5 6 – 2 + 7 3

(d) 4 7 3 5 4 2 5 6 + – * + –

[2018 (Set-3) : 2 Marks]

■■■■

Answers Stacks & Queues

3.1	(b)	3.2	(b)	3.4	(b)	3.5	(a)	3.6	(a)	3.8	(d)	3.9	(c)	3.10	(d)	3.11	(b)	3.12	(c)
3.13	(a)	3.14	(c)	3.15	(a)	3.16	(d)	3.17	(a)	3.18	(a)	3.19	(d)	3.20	(b)	3.21	(c)	3.22	(c)
3.23	(b)	3.24	(c)	3.25	(c)														

Explanations : Stacks & Queues

8.1 (b)

∴ The sequence of stack operations is 20, 20, 10, 10, 30.

So option (b) is correct.

8.2 (b)

PUSH operation is used to add an element into the stack and POP operation is used to remove an element from the stack.

8.3 (b)

Total = 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36

Postfix: ABCD – * E/ + F – G/ –

Prefix: – + – * A B C D / E F / G

8.4 (b)

Stack is LIFO operation so All O is not supported by stack hence statement I is incorrect and given in queue FIFO so statement II is correct.

8.5 (b)

Operations on stack are push and pop. Pop is an expensive operation and only operations on stack is stack.

8.6 (d)

In stack the elements will be deleted first but according to question, it should be work like reversed list. Element which is last, first that we give lower priority to last element. So all elements should be in stack in reverse order.

8.7 (d)

Stack is stepping-in-in-out pattern

= 2 * log 4 + 1 log 4 + 2

= 8 + 4 + log 4 + 2

= 2x1 + log 4 + 2

= 4x1 + log 4 + 2

8.8 (c)

So the result value is 11.

8.9 (c)

Hint: Refer the answer using few examples.

8.10 (c)

Consider the following diagram for the given problem

Let the two stacks are S_1 and S_2 and push and pop are for S_1 and S_2 respectively. The implementation of two stacks in a single array is called multi-stack array. According to the question push and pop operations are taken place in S_1 grows from initial location of A and S_2 grows from last location of A and both grows opposite direction. Assume the array size limit is n. Stack S_1 and S_2 are in consistent.

In this strategy there is a requirement of keeping track of top and bottom of an array stack of size n. For the condition to "stack full" we can easily check by using relation

$$top1 - top2 = 1$$

8.12 (b):

[illegible]

8.13 (a)

[illegible]

8.14 (d):

[illegible]

8.15 (b):

[illegible]

8.16 (c):

[illegible]

8.17 (c):

[illegible]

8.18 (a):

[illegible]

8.19 (b):

[illegible]

8.19 (a)

[illegible]

Full : Rear = 1, front = 1

Empty : front = rear

8.20 (d)

Since there are 3 queue operations [illegible] sequence operation will take [illegible] time. Multi-queue operations will take $O(n)$ time. [illegible] for worst case when [illegible] is maximum [illegible].

8.21 (b)

[illegible]

8.22 (c)

[illegible]

[default case executed]

[default case executed]

[default case executed]

[default case executed]

∴ Process names 10 + 5 = 15

8.23 (c)

[illegible]

Result is 1.2

8.24 (c)

Implement a queue using stacks:

ENQUEUE Operation:

Check if stack1 is full

if stack1 is full

swap

else push the element on the stack1 array

which will take $O(1)$ time

DEQUEUE Operation:

Check if the empty stack

if stack2 is [illegible]

[illegible]

else pop the element from stack2 array and assume this popped value [illegible] in the element of array

which will take $O(1)$ time.

So, the array implementation of queue ENQUEUE and DEQUEUE operations take $O(1)$ time.

8.25 (b)

[illegible]

Loop in algorithm when we take [illegible] elements in descending order [illegible] is 15

Last maximum number of iterations of which loop in algorithm [illegible] 15, 14, 13, ..., 1.

First 8 will push all elements in stack [illegible] in the end of [illegible]. This process [illegible] till we get 1 as top element. When top element is 1 then simply pop the 1 in stack. In this manner we have to pop all elements in stack in descending order, until queue is empty it will take 200 iterations.

Alternate:

Sequence of operations now in while loop is

1 dequeue()
2 push()
3 dequeue()
enqueue()
push()
4 pop()
5 dequeue()
6 enqueue()
push()
push()
7 push()
8 dequeue()
enqueue()
enqueue()
push()

So, Queue = {4, 1, 5, 6, 8}, Stack = {8, 6, 5, 1, 4} (top element is at the top of stack)

So, Total no. of elements in queue and stack = 5 + 5 = 10

Total no. of operations in while loop = 8

3.26 (c).

The expression is given in reverse polish notation (RPN) or postfix notation.

Expression is 2 3 * 6 4 / + 7 5 * 8 + -

Expression tree can be formed using postfix expression and it is

From the tree expression in NLR (PREORDER) traversal is - + 1 / 7 6 * 2 3 + 5 8.

■■■■

4 Linked List

4.1 In a circular linked list organization, insertion of a record involves modification of

(a) One pointer (b) Two pointers

(c) Three pointers (d) Four pointers

[1997 : 2 Marks]

4.2 Linked lists are not suitable data structures of which one of the following applications?

(a) Implementation of stacks

(b) Implementation of queues

(c) Dictionary

(d) Polynomial manipulation

[1998 : 2 Marks]

4.3 In the worst case, the number of comparisons needed to search a singly linked list of length n for a given element is

(a) $\log_2 n$ (b) $n/2$

(c) $\log_2 n - 1$ (d) n

[2002 : 1 Marks]

4.4 Consider the function f defined below.

```
struct item
{
  int value;
  struct item *next;
};

int f(struct item *p)
{
  if (p == NULL || p->next == NULL)
    return 1;
  q = f(p->next);
  return (p->value == q->value) && p->value);
}
```

For a given linked list p, the function f returns 1 if and only if

(a) The list is empty or has exactly one element

(b) The elements in the list are in non-decreasing order of data value

(c) The elements in the list are in non-increasing order of data value

(d) All elements in the list have the same data value

[2006 : 2 Marks]

4.5 Let P be a singly linked list. Let Q be the pointer to an intermediate node x in the list. What is the worst case time complexity of the best known algorithm to delete the node x from the list.

(a) $O(n)$ (b) $O(\log n)$

(c) $O(\log \log n)$ (d) $O(1)$

[2004 : 1 Mark]

4.6 The following C function takes a singly linked list of integers as a parameter and reverses the elements of the list. The function is called with this list containing the integers 1, 2, 3, 4, 5, 6, 7 in this order. What will be the contents of the list after the function completes execution?

```
struct node {int value; struct node *next;};

struct node *f(struct node *p) {
  struct node *q, *r;
  if (p == NULL || p->next == NULL)
    return p;
  q = f(p->next);
  r = p->next->next;
  p->next->next = p;
  p->next = r;
  return q;
}
```

(a) 1, 2, 3, 4, 5, 6, 7 (b) 2, 1, 4, 3, 6, 5, 7

(c) 4, 3, 2, 1, 7, 6, 5 (d) 2, 3, 4, 5, 6, 7, 1

[2005 : 2 Marks]

4.7 The following C function takes a singly linked list of integers as a parameter and reverses the elements of the list. The function is called with this list containing the integers 1, 2, 3, 4, 5, 6, 7 in this order. What will be the contents of the list after the function completes execution?

```
struct node
{
    int value;
    struct node *next;
};
void rearrange(struct node *list)
{
    struct node *p, *q;
    int temp;
    if (!list || !list->next)
    return;
    p = list; q = list->next;
    while(q)
    {
        temp = p->value;
        p->value = q->value;
        q->value = temp; p = q->next;
        q = p ? p->next : NULL;
    }
}
```

(a) 1, 3, 2, 4, 5, 6, 7
(b) 2, 1, 4, 3, 6, 5, 7
(c) 1, 3, 2, 5, 4, 7, 6
(d) 2, 3, 4, 5, 6, 7, 1

[2008 : 2 Marks]

41.8 The following C function implements a singly linked list. In this list, each node stores a character and a pointer to the next node. The function should reverse the linked list and return the pointer to the head of the reversed list.

```
struct node
{
    char value;
    struct node *next;
};
struct node *
reverse(struct node *list)
{
    struct node *p, *q;
    if (!list || !list->next)
        return list;
    q = NULL;
    p = list;
    while (p->next != NULL)
    {
        r = p;
        p = p->next;
    }

    return r;
}
```

Choose the correct alternatives to replace the blank line.

(a) q = NULL; p->next = q; head = p;
(b) q = p->next; p->next = NULL; head = p;
(c) head = p; p->next = q; q->next = NULL;
(d) q->next = NULL; p->next = head; head = p;

[2010 : 2 Marks]

■■■■

Answers Linked List

4.1 (b) 4.2 (b) 4.3 (d) 4.4 (b) 4.5 (a) 4.6 (b) 4.7 (b) 4.8 (a)

Explanations Linked List

4.1 (b)

4.2 (b)

Using linked list binary search is not possible. Binary search is possible with array. So binary search is not possible with linked list.

4.3 (d)

Linear search takes n number of comparisons. O(n).

4.4 (b)

[illegible] linked list [illegible] and [illegible] of the elements in the list are stored in non decreasing order. Insertion: insert at last data value

[illegible] NULL; p → Next = NULL; [illegible] = [illegible] = [illegible] =

5 Trees

5.1 It is possible to construct a binary tree uniquely whose pre order and post order traversals are given. (True/False)

[1997 : 1 Mark]

5.2 If the number of leaves in a tree is not a power of 2, then the tree is not a complete tree. (True/False)

[1997 : 1 Mark]

5.3 In the binary tree in figure, is assigned in order the values in which the values will be visited is

[1999 : 2 Marks]

5.4 A complete binary tree has 31 nodes. The number of leaves in it is _____

(a) 15 (b) 16

(c) 8 (d) 32

[1998 : 1 Mark]

5.5 In the binary tree in figure, the number of leaves is

(a) 1 (b) 2

(c) 7 (d) 8

[1998 : 1 Mark]

5.6 Which of the following sequences denotes the post order traversal sequence of the tree of above question?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[1998 : 1 Mark]

5.7 A binary search tree is generated by inserting in order of the following integers 50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24. The number of nodes in the left subtree and right subtree of the root respectively is

(a) (4, 7) (b) (7, 4)

(c) (8, 3) (d) (3, 8)

[1998 : 2 Marks]

5.8 A 3-way search tree of height 2 can hold 8 keys. What is the maximum number of keys that can be stored in a 3-way search tree of height 3? Explain.

[1998 : 2 Marks]

5.9 A binary search tree has nodes with the values 1, 2, 3, 4, 5, 6, 7, 8. The tree is traversed in pre order and the values are printed out. Which of the following sequences is a valid output?

(a) 5 3 1 2 4 6 7 8 (b) 5 3 1 2 4 6 8 7

(c) 5 3 1 2 4 7 6 8 (d) None of these

[1997 : 2 Marks]

5.10 Which of the following statements is false?

(a) A tree with n nodes has (n – 1) edges.

(b) A labeled rooted binary tree can be uniquely constructed given its pre order and post order traversal results.

(c) A complete binary tree with n internal nodes has (n + 1) leaves.

(d) The maximum number of nodes in a binary tree of height h is $(2^{h+1} - 1)$.

[1998 : 1 Mark]

5.11 A complete binary tree in which every internal node has two children. If k is the number of internal nodes of the complete binary tree, the number of leaves in it is given by

(a) $k(k-1)/2$ (b) $2k-1$

(c) $2k+1$ (d) $2(k+1)$

[1998 : 2 Marks]

5.12 Consider the following statements pertaining to binary trees: [illegible] and [illegible] are the left and right subtrees, respectively, of node X. Note that Y and X may be NULL. [illegible] used. Which of the following expressions is valid binary tree?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2000 : 1 Mark]

5.13 LASTPOST, LASTIN and LASTPRE denote the last vertex visited in a postorder, inorder and preorder traversal. Consider the following statements for a binary tree T. Which of the following is always true?

(a) LASTIN = LASTPOST
(b) LASTIN = LASTPRE
(c) LASTPRE = LASTPOST
(d) None of the above.

[2000 : 2 Marks]

5.14 The [illegible] of the root node of a rooted tree of n nodes, with each node having [illegible]. The [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2002 : 2 Marks]

5.15 In a heap with n elements with the smallest element at the root, the 7th smallest element can be found in time

(a) $\Theta(n \log n)$
(b) $\Theta(n)$
(c) $\Theta(\log n)$
(d) $\Theta(1)$

[2003 : 1 Mark]

5.16 In "[illegible]" a recursive binary search [illegible]

Then $T(n) = \sum_{k=1}^{n} T(k-1) + T(n-1)$, where $n > 1$

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2003 : 1 Mark]

5.17 Suppose the numbers 7, 5, 1, 8, 3, 6, 0, 9, 4, 2 are inserted in that order into an initially empty binary search tree. The binary search tree uses the usual ordering on natural numbers. What is the inorder traversal sequence of the resultant tree?

(a) 7 5 1 0 3 2 4 6 8 9
(b) 0 1 2 3 4 5 6 7 8 9
(c) 9 8 6 4 2 3 0 1 5 7

[2003 : 1 Mark]

5.18 A data structure is required for storing a set of integers such that each of the following operations can be done in $O(\log n)$ time, where n is the number of elements in the set.

1. Deletion of the smallest element.
2. Insertion of an element if it is not already present in the set.

Which of the following proposed data structures can be used for this purpose?

(a) A balanced search tree
(b) A hash table
(c) A heap
(d) A linked list in sorted order

[2006 : 2 Marks]

5.19 The following sequence of keys are inserted into an empty binary search tree in the given order: 8, 1, 3, 7, 10, 12, 20. What is the height of the binary search tree (the height is the maximum distance of a leaf node from the root)?

(a) 2
(b) 3
(c) 4
(d) 6

[2001 : 1 Mark]

5.20 Level order traversal of a rooted tree can be done by starting from the root and performing

(a) breadth-first search
(b) depth-first search
(c) depth-first search
(d) breadth-first search

[2001 : 1 Mark]

5.21 The elements 32, 15, 20, 30, 12, 23, 35, 18 are inserted one by one in the given order into a 2-3 tree. The resulting tree is

(a)

(b)

(a)

(b)

[2004 : 2 Marks]

5.22 Consider the following C program segment

```
struct CellNode
{
  struct CellNode *LeftChild;
  int element;
  struct CellNode *RightChild;
};
int DoSomeThing(struct CellNode *p)
{
  int value = 0;
  if (p != NULL)
  {
    if (p->LeftChild != NULL)
      value = DoSomething(p->LeftChild);
    if (p->RightChild != NULL)
      value = max(value, 1 + DoSomething(p->
        RightChild));
  }
  return(value);
}
```

The value returned by the function DoSomething, when p points to the root of a non-empty tree, is the maximum of

(a) The number of leaves in the tree

(b) The number of nodes in the tree

(c) The number of internal nodes in the tree

(d) The height of the tree

[2004 : 2 Marks]

5.23 Consider the labelled rooted tree shown in the following picture. Its nodes are labelled in the order in which they are visited during a depth first traversal. Which of the given pairs of vertices are lowest common ancestors (LCA) of the pair (7, 9)?

(a) 1 and 2

(b) 2 and 3

(c) 3 and 4

(d) 4 and 5

[2004 : 2 Marks]

5.24 Which one of the following binary trees has its inorder and preorder traversals as IDFAGCB and ABDFIGCE respectively?

(a)

(b)

(c)

(d)

[2004 : 2 Marks]

5.25 The numbers 1, 2, ..., n are inserted in a binary search tree in some order. In the resulting tree, the right child of the root contains the number n. The first number to be inserted in the tree must be

(a) 1

(b) n

(c) n/2

(d) n − 1

[2005 : 1 Mark]

5.26 In a binary tree, the number of leaf nodes is one more than the number of nodes of degree 2. If the height of the tree is h, then the maximum number of nodes in the tree is

(a) $2^h - 1$

(b) $2^h + 1$

(c) $2^{h+1} - 1$

(d) 2^{h+1}

[2005 : 2 Marks]

5.27 A binary search tree contains the numbers 5, 3, 4, 6, 7, 9. When traversed in preorder, the values in each node are printed. The values in each node are also printed out when the tree is traversed in postorder. The sequence of values printed out is 4, 3, 9, 7, 6, 5. The tree is traversed in postorder. The sequence of values printed out is

(a) 6, 7, 9, 3, 4, 5

(b) 4, 3, 9, 7, 6, 5

(c) 3, 4, 5, 6, 7, 9

(d) 4, 3, 5, 9, 7, 6

[2005 : 2 Marks]

5.28 Postorder traversal of a given binary search tree, T, produces the following sequence of keys

[illegible]

Which one of the following sequences of keys can be the result of an inorder traversal of the tree?

(a) 9, 19, 15, 28, [illegible]
(b) 5, 10, [illegible], 35, 40, 50, 60, 95, 80, [illegible]
(c) [illegible], 13, 9, 19, 15, [illegible]
(d) 35, 50, 60, 40, 27, [illegible], 15, 29

[2005 : 2 Marks]

5.20 A priority-queue is implemented as a Max Heap. Initially, it has 5 elements. The level-order traversal of the heap is the following.

10, 8, 5, 3, 2

Two more elements '7' and '1' are inserted in the heap in that order. The level-order traversal of the heap after the insertion of the elements is:

(a) 10, 8, 7, 5, 3, 2, 1
(b) 10, 8, 7, 3, 2, 5, 1
(c) 10, 8, 7, 2, 3, 5, 1
(d) 10, 8, 7, 5, 3, 2, 1

[2005 : 2 Marks]

5.21 How many distinct binary search trees can be created out of 4 distinct keys?

(a) 5 (b) 14
(c) 24 (d) 35

[2006 : 2 Marks]

5.22 In a complete binary tree, internal nodes are numbered from 1 onwards. The number of leaves in such a tree with n internal nodes is:

(a) n (b) $n + 1$, if $n = 1$
(c) $n + k - 1$ (d) $n + k - 2$

[2006 : 2 Marks]

5.23 In a binary tree having n nodes, the smallest element can be found in time

(a) $O(n)$ (b) $O(\log n)$
(c) $O(\log \log n)$ (d) $O(1)$

[2008 : 1 Mark]

5.24 In a binary tree, the number of internal nodes of degree 1 is 5 and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is

(a) 10 (b) 11
(c) 12 (d) 13

[2006 : 1 Mark]

5.25 Which of the following sequences of keys represents a valid heap?

(a) 23, 17, 14, 6, 13, 10, 1, 5, 7, 12
(b) 23, 17, 14, 6, 13, 10, 1, 5, 7, 12
(c) 23, 17, 14, 7, 13, 10, 1, 12, 5, 8
(d) 23, 17, 14, 7, 13, 10, 1, 12, 5, 8

[2006 : 2 Marks]

5.26 Suppose that we have numbers between 1 and 1000 in a binary search tree and we want to search for the number 363. Which of the following sequences CANNOT be the sequence of nodes examined?

(a) 910, 70, 64, 400, 60, 57, 363
(b) 400, 12, 69, 94, 82, 63, 77
(c) 98, 65, 87, 66, 42, 90, 363
(d) 970, 87, 72, 90, 116, 56, 363

[2008 : 2 Marks]

Common Data for Q. 5.27 & Q. 5.27:

A 3-ary max heap is like a binary max heap, but instead of 2 children, nodes have 3 children. A 3-ary heap can be represented by an array as follows: The root is stored in the first location, a[0], nodes in the next level, from left to right, is stored in a[1] to a[3]. The nodes from the second level of the tree from left to right are stored from a[4] onwards. An item can be inserted into a 3-ary heap containing n items by placing it in the next available location and then moving it up the tree to restore the heap property.

5.27 Which of the following is a valid 3-ary max heap stored in an array representing 9 keys?

(a) 7, 3, 2, 6, 5, 8 (b) 9, 7, 1, 3, 2
(c) 7, 2, 6, 3, 5, 1 (d) 9, 7, 6, 3, 2, 1

[2009 : 2 Marks]

5.27 Suppose the keys 7, 2, 10, and 4 are inserted in an empty 3-ary max heap. The resulting heap will have the above properties. Which one of the following is the sequence of items in the array representing the resulting heap?

(a) 7, 2, 10, 4
(b) 10, 4, 2, 7
(c) 10, 7, 4, 2
(d) 10, 4, 7, 2

[2009 : 2 Marks]

5.28 The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is:

(a) $2h$ (b) $2^{h+1} - 1$
(c) $2^{h+1} - 1$ (d) 2^h

[2007 : 1 Mark]

6.39 The maximum number of binary trees that can be constructed with five unlabeled nodes is

(a) 1 (b) 5

(c) 6 (d) 9

[2009 : 1 Mark]

6.40 The inorder and preorder traversal of a binary tree are [illegible] and [illegible] respectively. The postorder traversal of the binary tree is

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2009 : 2 Marks]

6.41 A complete binary tree is a tree in which each node has either zero or two children. Let l be the number of internal nodes and L be the number of leaves in a complete binary tree. If $L = 11$ and $l = 10$, what is the value of n

(a) [illegible] (b) [illegible]

(c) [illegible] (d) 6

[2007 : 2 Marks]

6.42 Consider the process of inserting an element into a Max-heap, where the Max-heap is represented by an array. Suppose we perform a binary search on the path from the new leaf to the root to find the proper place for the new element, and then insert the element. In the worst case, the number of comparisons performed is

(a) $\Theta(\log_2 n)$ (b) $\Theta(n \log_2 n)$

(c) $\Theta(n)$ (d) $\Theta(n^2 \log_2 n)$

[2007 : 2 Marks]

6.43 Consider the following C program segment where CellNode represents a node in a binary tree:

```
struct CellNode {
struct CellNode *leftChild;
int element;
struct CellNode *rightChild;
};
int GetValue(struct CellNode *ptr) {
  int value = 0;
  if (ptr != NULL) {
    if (ptr->leftChild == NULL &&
        ptr->rightChild == NULL)
      value = 1;
    else
      value = value +
              GetValue(ptr->leftChild)
            + GetValue(ptr->rightChild);
  }
  return value;
}
```

The value returned by GetValue when a pointer to the root of a binary tree is passed as its argument is

(a) the number of nodes

(b) the number of internal nodes in the tree

(c) the number of leaf nodes in the tree

(d) the height of the tree

[2007 : 2 Marks]

6.44 When searching for the key value 60 in a binary search tree, nodes containing the key values 10, 20, 30, 40, 50, 60, 70 are encountered, not necessarily in the order given. How many different orderings are possible in which the key values can occur in the search path from the root to the node containing the value 60?

(a) 45 (b) 64

(c) 128 (d) 5040

[2007 : 2 Marks]

6.45 Which of the following is/are true?

(a) The cost of searching an AVL tree is $\Theta(\log n)$ but that of a binary search tree is $O(n)$

(b) The cost of searching an AVL tree is $\Theta(\log n)$ but that of a binary search tree is $\Theta(\log n)$ on average

(c) The cost of searching a binary search tree is $O(\log n)$ but that of an AVL tree is $O(n)$

(d) The cost of searching an AVL tree is $\Theta(\log n)$ but that of a binary search tree is $O(n)$

[2002 : 1 Mark]

Directions for Question 6.46 to 6.48:

A binary Search Tree (BST) stores values in the range 37 to 732. Consider the BST search sequences:

I. 83, 237, 132, 99, 287, 732, 305

II. 97, 87, 121, 135, 149, 141, 139

III. 146, 246, 398, 328, 362, 370, 370

IV. 300, 270, 307, 305, 308, 302, 370

6.46 Suppose this BST has been constructed by inserting values in the order 237, 83, 732. Which of the above sequences lists values in the order in which they could have been encountered during the search?

(a) II and III only (b) I and IV only

(c) III and IV only (d) I and II only

[2008 : 2 Marks]

5.47 Which of the following statements is TRUE?

(a) I, II and III are inorder sequences of three different BSTs

(b) I is a preorder sequence of some BST with 450 as the root

(c) I is an inorder sequence of some BST with 121 as the root and 32 as a leaf

(d) III is a postorder sequence of some BST with 121 as the root

[2006 : 2 Marks]

5.48 How many distinct BSTs can be constructed with 3 distinct keys?

(a) 4 (b) 5

(c) 6 (d) 9

[2006 : 2 Marks]

Linked Questions 5.49 & 5.50:

A binary tree with $n > 1$ nodes has n_0 and n_2 nodes of degree zero and two respectively. Which of the following is/are the number of the nodes of degree one?

5.49 n_1 can be expressed as:

(a) $n_2 + n_0 - 1$ (b) $n_0 - 2$

(c) $|n_0 - n_2|$ (d) $n_2 - 1$

[2008 : 2 Marks]

5.50 Starting with the above tree, while traversing it in preorder, a number of degree two nodes will be visited before the first degree one node is visited. The last degree two node visited will be the parent of the leftmost leaf in the tree.

How many nodes will be visited at the end of the process?

(a) $2 * n_0 - 3$ (b) $n_0 + 2 - n_1 - 2$

(c) $n_0 - n_2$ (d) $n_0 + n_2 - 2$

[2008 : 2 Marks]

5.51 You are given a min-heap stored in an array. If the binary search tree on the elements 1, 3, ..., ... You have to determine the unique binary search tree that has the same preorder traversal. What is the time complexity of the most efficient algorithm for doing so?

(a) $O(\log n)$

(b) $O(n)$

(c) $O(n \log n)$

(d) None of the above as the tree cannot be uniquely determined

[2008 : 2 Marks]

5.52 We have a binary heap on n elements and we want to insert n more elements (not necessarily one at a time) into the heap. The worst time complexity of this is:

(a) $O(\log n)$ (b) $O(n)$

(c) $O(n \log n)$ (d) $O(n^2)$

[2008 : 2 Marks]

5.53 Which of the following statements is/are true? Consider a binary tree in which each internal node stores one integer. The sum of the integers stored in the left subtree and the integers stored in the right subtree differ by at most one.

I. MIN HEAPS II. BINARY SEARCH TREES

III. MAX HEAPS

Pick the correct statement from the following:

(a) I and II are provided with preorder sequences, respectively

(b) I and III are provided with postorder sequences, respectively

(c) I is provided with inorder sequence, and nothing can be said about the other two sequences

(d) II and III are provided with inorder and postorder sequences, respectively

[2008 : 2 Marks]

5.54 What is the minimum height of any AVL tree with 7 nodes? Assume that the height of a tree with a single node is 0.

(a) 2 (b) 3

(c) 4 (d) 5

[2008 : 2 Marks]

Linked Answer Questions 5.55 and 5.56:

Consider a binary max-heap implemented using an array.

5.55 Which one of the following array represents a max-heap?

(a) {25, 12, 16, 13, 10, 8, 14}

(b) {25, 14, 13, 16, 10, 8, 12}

(c) {25, 14, 16, 13, 10, 8, 12}

(d) {25, 14, 13, 16, 10, 8, 12}

[2008 : 2 Marks]

5.56 What is the array after two deletions operations on the array in the previous question?

(a) {14, 13, 12, 16, 8} (b) {14, 13, 16, 10, 8}

(c) {14, 16, 8, 12, 10} (d) {14, 13, 12, 8, 10}

[2008 : 2 Marks]

5.57 A max-heap is a binary tree in which the value of each parent is greater or at least equal to the value of its children. Which of the following is a max-heap?

(a)

(b)

(c)

(d)

[2011 : 1 Mark]

5.58 We are given a set of n distinct elements and an unlabeled binary tree with exactly n nodes. In how many ways can we place the elements in the tree so that it becomes a max-heap?

(a) 0 (b) 1

(c) n! (d) $\frac{n!}{(n-1)}$

[2011 : 2 Marks]

5.59 The height of a tree is defined as the number of edges on the longest path in the tree. The function shown below takes the root of a tree as the argument and returns its height. Assume that the height of an empty tree is -1 and that the height of a tree with a single node is 0.

```
int height(struct node *p) {
    int h, lh, rh;
    if (p == NULL)
        return -1;
    lh = height(p->left);
    rh = height(p->right);
    if (lh == rh)
        return lh + 1;
    else if (lh > rh)
        return lh + 1;
    else
        return rh + 1;
}
```

The appropriate recurrence relation for the number of calls to the function height, as a function of n, is

(a) $B(n) = 1 + \text{height}(n-1) + \text{height}(n-2)$
$B(0) = 1$, $B(1) = \max(B(0), B(-1))$

(b) $B(n) = 1 + \text{height}(n-1)$
$B(0) = 1$, $B(1) = \max(B(0), B(-1))$

(c) $B(n) = 1 + \text{height}(n-1) + \text{height}(n-2)$
$B(0) = 1$, $B(1) = \max(B(0), B(-1))$

(d) $B(n) = 1 + \text{height}(n-1) + \text{height}(n-2)$
$B(0) = \max(B(-1), B(-2))$

[2013 : 2 Marks]

5.60 The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the postorder traversal sequence of the same tree?

(a) 10, 20, 15, 23, 25, 35, 42, 39, 30
(b) 15, 10, 25, 23, 20, 42, 35, 39, 30
(c) 15, 20, 10, 23, 25, 42, 35, 39, 30
(d) 15, 10, 25, 23, 20, 42, 35, 39, 30

[2013 : 2 Marks]

5.61 Consider a max-heap, organized as an array, and containing elements in the range 1 to 100 in any order. The minimum number of elements that must be examined in the array to find the element with the value 50 is _____.

[2014 (Set-1) : 1 Mark]

5.62 A priority queue implemented as a max-heap is initially empty. It has 5 elements. The level order traversal of the heap is 10, 5, 6, 3, 2. Two new elements 1 and 7 are inserted in the heap in that order. The level order traversal of the heap after the insertion of the elements is

(a) 10, 5, 7, 6, 2, 1, 3 (b) 10, 5, 7, 2, 3, 1, 6
(c) 10, 5, 7, 1, 2, 3, 6 (d) 10, 5, 7, 6, 3, 2, 1

[2014 (Set-3) : 1 Mark]

5.63 Consider the following complete binary tree with the vertices labeled P to V as shown below.

The order in which the vertices are visited during an in-order traversal of this tree is

(a) PQRTSUV (b) SRPUQVT
(c) SQPTRVU (d) PQSRTUV

[2014 (Set-3) : 1 Mark]

5.64 Consider the [illegible] given below. The function DoSomething() takes a binary tree and a pointer to an integer as parameters. A binary tree can be represented as the [illegible] data structure. Root node of the tree is a struct treeNode.

```
struct treeNode* DoSomething(struct treeNode* root, int* count)
{
    struct treeNode* leftChild = NULL, *rightSibling = NULL;
    int temp = 0;
    int value = 0;
    if (root != NULL)
    {
        if (root->leftChild == NULL)
            value = 1;
        else
            value = DoSomething(root->leftChild, count);
        if (root->rightSibling == NULL)
            temp = 1;
        else
            temp = DoSomething(root->rightSibling, count);
        *count = value + temp + 1;
        return root;
    }
    return NULL;
}
```

When the pointer to the root of a tree is passed as the first argument to DoSomething, the value returned by the function will be equal to the

(a) number of leaf nodes in the tree.
(b) height of the tree.
(c) number of nodes without a right sibling in the tree.
(d) number of leaf nodes in the tree.

[2014 (Set-3) : 2 Marks]

5.65 Consider the C function given below. Assume that the array list A contains n elements sorted in ascending order.

```
int Prog(int A[], int n, int x)
{
    int i, j, k;
    i = 0;
    j = n - 1;
    do
    {
        k = (i + j)/2;
        if (x <= A[k])
            j = k - 1;
        else
            i = k + 1;
    }while (i <= j);
    if (A[k] == x)
        return(k);
    else
        return -1;
}
```

We invoke the function Prog() as follows: Prog(A, n, x). Which of the following statements is true?

(a) It will return -1 if x is not in the list A.
(b) It will return -1 if x is greater than the largest element in the list A.
(c) It will return -1 if x is smaller than the smallest element in the list A.
(d) It will return -1 if x is present in the list A.

[2014 (Set-2) : 2 Marks]

5.66 Which of the following is/are correct inorder traversal sequence(s) of a binary search tree?

1. 3, 5, 7, 8, 10, 12
2. 5, 6, 9, 12, 15, 18, 25
3. 4, 7, 12, 15, 18, 20, 25
4. 4, 6, 7, 9, 18, 20, 33

(a) 1 and 4 only (b) 2 and 3 only
(c) 2 and 4 only (d) 2 only

[2015 (Set-1) : 1 Mark]

5.67 The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are

(a) 63 and 6 respectively
(b) 31 and 6 respectively
(c) 32 and 6 respectively
(d) 31 and 5 respectively

[2015 (Set-1) : 1 Mark]

5.68 Consider a max-heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4.

Array index	1	2	3	4	5	6	7	8	9
value	40	30	20	10	15	16	17	8	4

Now a new element 35 is inserted into the heap. After insertion, the new heap is

(a) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
(b) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
(c) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
(d) 40, 35, 20, 10, 30, 16, 17, 8, 4, 30

[2015 (Set-1) : 2 Marks]

5.69 A binary tree T has 20 leaves. The number of nodes in T having two children is _____.

[2015 (Set-2) : 1 Mark]

5.70 While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is

(a) 65 (b) 67
(c) 69 (d) 83

[2015 (Set-3) : 1 Mark]

5.71 Consider an AVL tree with following integer keys inserted in the order: 30, 15, 50, 10, 18, 35, 2, 5, 11, 3, 8, 10.
The minimum number of rotations required to restore AVL property is _____.

(a) 4 (b) 5
(c) 2 (d) 0

[2015 (Set-3) : 1 Mark]

5.72 Consider the following rooted tree.

The number of distinct topological orderings of the vertices of the graph is _____.

[2016 (Set-1) : 1 Mark]

5.73 An operation delete(i) for a binary heap data structure takes as input an integer i and deletes the i th node of the heap. If the heap has depth d (number of edges on the path from the root to the leaf node), then which of the following is the time complexity to perform this delete operation?

(a) $O(1)$
(b) $O(d)$ but not $O(1)$
(c) $O(2^d)$ but not $O(d)$
(d) $O(d2^d)$ but not $O(2^d)$

[2016 (Set-1) : 2 Marks]

5.74 A complete binary min-heap is made by including each integer in $[1, 1023]$ exactly once. The depth of a node is the length of the path from the root of the heap to the node. The root is at depth 0. The maximum depth at which integer 9 can appear is _____.

[2016 (Set-2) : 2 Marks]

5.75 The minimum number of comparisons required to determine if an integer appears more than once in a sorted array of size 1024 is _____.

[2016 (Set-2) : 2 Marks]

■■■■

Answers Keys

5.4	(b)	5.5	(b)	5.6	(c)	5.7	(b)	5.8	(d)	5.9	(b)	5.10	(a)	5.11	(b)	5.12	(b)
5.13	(c)	5.14	(b)	5.15	(d)	5.16	(c)	5.17	(a)	5.18	(d)	5.19	(b)	5.20	(d)	5.21	(a)
5.22	(d)	5.23	(b)	5.24	(d)	5.25	(c)	5.26	(b)	5.27	(d)	5.28	(a)	5.29	(d)	5.30	(b)
5.31	(a)	5.32	(d)	5.33	(c)	5.34	(c)	5.35	(c)	5.36	(c)	5.37	(a)	5.38	(b)	5.39	(d)
5.40	(a)	5.41	(a)	5.42	(b)	5.43	(c)	5.44	(a)	5.45	(a)	5.46	(d)	5.47	(a)	5.48	(b)
5.49	(b)	5.50	(a)	5.51	(c)	5.52	(a)	5.53	(d)	5.54	(a)	5.55	(b)	5.56	(c)	5.57	(b)
5.58	(a)	5.59	(a)	5.60	(d)	5.61	(a)	5.62	(d)	5.63	(b)	5.64	(a)	5.65	(a)	5.66	(b)
5.67	(a)	5.68	(b)	5.69	(a)	5.70	(b)	5.71	(d)	5.72	(b)						

Explanations Trees

5.1 [illegible]

False

- [illegible] processing [illegible]

5.2 [illegible]

False

[illegible] given [illegible]

5.3 [illegible]

[illegible]

5.4 [illegible]

[illegible] 2 leaf [illegible]

5.5 (b)

[illegible]

[illegible]

5.6 [illegible]

[illegible]

5.7 [illegible]

[illegible]

[illegible]

5.8 [illegible]

Possible sequence

[illegible]

Not possible

[illegible]

5.9 [illegible]

(a) [illegible]

[illegible]

(b) [illegible]

[illegible]

(c) [illegible]

[illegible]

5.10

A labelled rooted binary tree can be constructed uniquely if in addition to preorder and inorder traversal.

5.11

In any tree the total number of vertices are

$$n = \sum x + 1$$

Where x is the number of nodes with degree 1. Let l be the number of leaf nodes

$$x + l = \sum x + 1$$

$$l = \sum_{i=0}^{n-1} i x_i + 1$$

So number of leaf nodes in any tree is $l = \sum_{i=0}^{n-1} i x_i + 1$

5.12

(1, 2, 3), (5, 6, 7)

5.13

Because it is complete binary tree, space for its array representation

5.14

Binary tree

Total no. of nodes = internal nodes + leaf nodes

Here, $n = x + l$

Total number of internal nodes = leaf nodes

$$x = l - 1$$

$$n = 2l - 1$$

$$n = \min\{2i + 1 : i \text{ is a leaf node}\}$$

$$n = \min\{2i + 1 : i \text{ is a leaf node}\}$$

$$n = 2n - 1$$

$$l = \frac{2n - 1}{3}$$

Number of leaf nodes.

Space complexity is $O(n)$.

5.15

In a heap with n elements, we find the smallest element at the root. Hence, data structure is known as min heap or priority queue. We can implement priority queue with the help of 2–3 tree. We can find the smallest element, minimum search. Time complexity is $O(\log n)$. To find the second smallest element, in 2–3 tree we can find the 2nd smallest element in $O(\log n)$ time. We can delete member, insert or delete instruction can be done in $O(\log n)$ steps as a 2–3 tree with n leaves.

5.16

Left = Right = 1 + x

Left = 1 + x + 1 = n

x = n − 1

5.17

Level sequence: 7, 5, 1, 2, 6, 9, 8, 10

Hence, we get the BST in which the value of left sub-tree is less than its root and the value of right sub-tree is greater than its root as shown in the next.

In order traversal of 1, 2, 5, 6, 7, 8, 9, 10

Note: The in order traversal of a BST is in ascending order.

5.18

Since in multiplication of AVL tree, Max heap will take O(log n) time. But in this case, in order to access the leaf node of the Max heap, we have to access the root node. In this Max heap will take O(n). AVL tree will take O(log n).

5.19 (b) :

So the maximum height of BST = [illegible]

5.20 (d) :

[illegible] algorithm [illegible] distance [illegible] reaches only vertices [illegible] due to that it is called breadth-first search.

5.21 (a) :

[illegible]

5.22 (d) :

[illegible] subtree [illegible] and the [illegible] subtree [illegible] [illegible]

5.23 (b) :

[illegible] expression [illegible] expression and [illegible] expression [illegible] [illegible] subtree node [illegible] [illegible] similarly [illegible] [illegible] numbers assigned [illegible] [illegible] [illegible]

5.87 (b):

5.88 (a):

The inorder traversal of a binary search tree is always in increasing order. [illegible] The inorder traversal of the given tree is 10, [illegible] 20, 30, 40, 50.

5.89 (b):

Initially heap [illegible] Max Heap is

So the sequence of level order traversal is 10, 9, 7, 8, 5, 1, 6.

5.90 (a):

[illegible] number of distinct nodes in search tree with n nodes is $\frac{1}{n+1}\binom{2n}{n}$

$$T_n = \frac{1}{n+1}\binom{2n}{n}$$

$$T_4 = \frac{1}{4+1}\binom{8}{4} = \frac{1}{5}\cdot 70$$

$$T_4 = \frac{1\cdot 8\cdot 7\cdot 6\cdot 5}{5\times 4\times 3\times 2\times 1} = 14$$

5.91 (b):

The complete binary tree has two number of nodes exactly [illegible] leaf nodes. They contain [illegible] Where [illegible] are internal nodes. [illegible] $n = 2l - 1$.

5.92 (a):

MAX Heap could be used to identify max element in O(1) time but [illegible] O(1) min-heap [illegible] O(log n) to apply binary search.

5.93 (b):

Let number of vertices in [illegible] tree = n

Number of edges = n − 1

[illegible]

[illegible]

⇒ [illegible] = 26

Number of leaf nodes = 26 − 6 − 10 = 10

5.94 (d):

For a complete binary tree should be greater [illegible] in the left child of [illegible] node. If has 2i + 1 th position and [illegible] 2i + 1 [illegible] optimal we can verify it.

Option (d) [illegible] follows the property.

5.95 (b):

In optimal binary search tree [illegible] 40, 42, ...

As 37 [illegible] that [illegible] will be on right side of 15, so in further comparison [illegible] will not come.

[illegible] optimal binary tree.

5.96 (d):

The key must be [illegible]

So the valid sequence is 9, 5, 11, 4, 8, 7.

5.97 (b):

After inserting 6, 8, 20, the structure has become

10, 5, 15, 2, 8, [illegible] 5, 2, 8, 3, 4

5.38 (d)

The height of a binary tree is the maximum number of edges in any root to leaf path. Apply Dijkstra's algorithm.

Distance of Vertices

0 [illegible]

1 [illegible]

[illegible]

So maximum number of edges in a binary tree of height k is [illegible]

5.39 (b)

The maximum number of [illegible] nodes

$$= \frac{1}{n-1}\binom{2n-2}{n-1}$$

Given $n = 8$

$$= \frac{1}{8-1}\binom{2\times 8-2}{8-1} = \frac{1}{7}\binom{14}{7}$$

$$= \frac{1}{7}\cdot\frac{14\times 13\times 12\times 11\times 10\times 9\times 8}{7\times 6\times 5\times 4\times 3\times 2\times 1} = 429$$

5.40 (a)

The inorder traversal [illegible] and the postorder traversal sequences is a b d c e f g.

So the tree is

In the postorder traversal the root is the last node. So the root is a.

5.41 (d)

[illegible]

$$n = \frac{2l+1}{l+1}$$

$$n = \frac{2\times 4+1}{4+1} = \frac{9}{5} = 1$$

$$n = 4 + 1 = 5$$

5.42 (b)

After inserting 70 elements of certain max heap we perform binary search on 2 to get 1 from 2 to n. Call to the root to find the correct location for the newly inserted element. Comparisons will be [illegible] but it will be $O(\log n)$ in most cases.

5.43 (b)

In given 15 [illegible] sequence [illegible] = 0 [illegible] moves to 0 values matches to zero. Consider the requirement [illegible] [illegible] [illegible]

5.46 (a):

Only IV is the valid sequence.
Sequence I is wrong after 2nd.

5.47 (a):

Inorder sequences are sorted. In preorder traversal [illegible] is processed before [illegible] and [illegible] is processed before traversal [illegible] left.

5.48 (b):

For distance DST we apply [illegible]
[illegible]
[illegible] and [illegible] = 5

5.49 (b):

Minimum height of binary tree is 3 $n_2 = 1$ and $n_1 = 3$
Hence $n = 2$ gives n_0.

5.50 (a):

From above tree we will get [illegible]

There [illegible] in the tree.
[illegible] = 6

5.51 (b):

We have post order traversal and the tree is binary search tree so in order traversal of binary search tree is ascending order.

Using [illegible] binary search tree [illegible]

5.52 (d):

It was how a binary search tree [illegible] which is insert in more elements than [illegible] recursive call build [illegible] is [illegible] to construct the [illegible].
The time spent by build [illegible] is [illegible] the sum [illegible] of the heights of the vertices. But at most [illegible] vertices in a tree have [illegible]. So the total time spent by build heap is

$$\sum_{i=1}^{\lfloor n/2 \rfloor} i = O(n).$$

5.53 (a):

I. MAXCRYPTH (Preorder)
II. MAXCRYPTH (Inorder)
If I is preorder and II is inorder then it is possible to construct binary tree.

→ I, MAXCRYPTH is a unique

5.54 (b):

Maximum height of an AVL tree is 1.5

There may be [illegible] ways to draw AVL with 7 nodes.

5.55 (d):

[illegible] A[i] = 2i, A[2i] = i and A[i] = A[2i] = 0 for [illegible] where i is the left child and 2i = 1 right child of node i of a max heap.

Binary max-heap

3.56 (a)

Deleting 25 : Replace it by 2

After heapifying

Deleting 16 : Replace it by 5

After heapifying

3.57 (b)

3.58 (a)

3.59 (a)

e.g.

$h_1 = 1 + \max(0, 0), h_2 = 0$

return $1 + \max(h_1, h_2)$

3.60 (d)

Given a complete binary tree with preorder traversal given as 40, 30, 10, 15, 25, 70, 50, 35, 42

The postorder traversal of the complete binary tree is 10, 25, 15, 30, 35, 42, 50, 70, 40

3.61 (b)

Given the value of the root node, we can determine the number of nodes in the left subtree. Let L_n denote the number of nodes in the left subtree of a tree with n nodes.

$L_n = 2^{\lfloor \log_2 n \rfloor} - 1$

$L_{100} = 2^{\lfloor \log_2 100 \rfloor} - 1$

3.62 (a)

5.65 (b)

Inorder: [illegible]

∴ Traversal is: [illegible]

5.66 (c)

Given program computes the [illegible] in a binary tree.

5.67 (b)

Given program is the pseudo-code of binary search. It increases key. A is in middle position of the array. [illegible] may change based on [illegible] key with a comparison at A[i] element.

5.68 (a)

The INORDER traversal of BST will give the sorted sequence (ascending order). Hence the sequence 1, 2, 3, 4, 5, 6, 7, 8 is in increasing order.

5.69 (b)

Height is defined as path length from root (root = 1) will give number of levels.

Total binary tree of height h = 5

Maximum number of nodes = $2^h - 1 = 2^5 - 1$

= 63 − 1 = 62

Minimum number of nodes = 6 i.e. one node at every level.

5.70 (a)

The resulting max heap is:

To insert 25 into the tree.

Now swap 15 and 25 to maintain the max heap property.

Next swap 25 and 20 to maintain the max heap property. Finally we get 25

Array elements are: 25, 20, 20, 10, 15, 17, 9, 6, 15

5.71 (c)

n_0 : Number of leaf nodes (degree 0)

n_1 : Number of internal nodes with degree 1

n_2 : Number of internal nodes with degree 2

We have a binary tree, so in a binary tree $n = n_0 + n_1 + n_2$ (where n is the total number of nodes in the tree)

Assume in a full binary tree, assume that there are n_1 leaf nodes and the root node is not a child of any node.

We get

$= n_0 \times 1 - n_2 \times 2 + 1 \quad n_0 + 2n_2 + 1 \quad (2)$

Combining equations (1) and (2) we get

$n_1 = n_2 + 1 \Rightarrow n_2 = n_1 - 1 \Rightarrow n_2 = n_0 - 1$

Therefore the number of internal nodes of degree two in any binary tree = number of leaf nodes − 1

5.72 (b)

71, 65, 84, 68, 67, 83 are inserted into empty BST. Binary search tree

∴ 83 is present in the last level.

5.73 (d)

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12
100	90	70	40	30	35	60	50	80	15	20	85	95

[illegible]

Max heap array:

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	2	3	4	5	6	7	8	9	10	11	12

5.17 [illegible]

Number of binary trees = 6

a	b	c	d	e	f
a	b	f	e	c	d
a	d	b	c	e	f
a	d	b	f	c	e
a	d	e	b	c	f
a	d	e	f	c	b

5.18 [illegible]

Heap is implemented using array. If it is [illegible] element then it is [illegible] child. If an element is deleted from the last level of tree, it will take $O(1)$ time. Since element can be deleted from any level of heap, in worst case [illegible] is deleted, then it takes level one element is [illegible].

Example:

Minimum $O(1)$ time will take if [illegible] is deleted in heap [illegible] $O(1)$.

5.19 [illegible]

Min heap is a complete binary tree [illegible] than from its [illegible] child [illegible] right child. Max heap is a complete binary tree [illegible] than its right child [illegible].

Note: [illegible] complete binary tree have max number of levels.

[illegible] element can be present at 1st level, 2nd min number elements can be present at 2nd level, [illegible] so 5th [illegible] number of elements can be present at 5th level. When level is 1 – height is 0. So when level is 5 – height is 4.

Example: With 7 nodes, element [illegible] level 3 in worst case but 2nd level [illegible]

So integer 5 can be present at height 4.

5.20 [illegible]

Long [illegible] restriction on BST is height should be [illegible]. Total [illegible] given. For [illegible] height [illegible] of BST we have to take [illegible] elements [illegible].

At 1 level = We have 2 choices i.e. either take 1 or 6.

At 2 level = We have 4 choices [illegible] 7. Now if we take 4 [illegible] 2. If 3 is more than 2 choose will be [illegible] 6.

At 3 level = [illegible] 3 and 2 at 2nd level. [illegible] choose i.e. [illegible] level. [illegible] level 4 we have 2 [illegible] element.

So till 3 level we have two choice in every level and for last [illegible] with weight element. So number of BST with height 6

$= 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$.

■■■■

6 Graphs

6.1 Which [illegible] representation of the graph given in figure.

[2006 : 2 Marks]

6.2 Consider the following graph.

Among the following sequences

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

Which are depth first traversals of the above graph?

(a) 2 and 3 only (b) 1 and 3 only
(c) 2, 3 and 4 only (d) 1, 2 and 4 only

[2008 : 1 Mark]

6.3 Let $G = (V, E)$ be an undirected graph with a subgraph $G_1 = (V_1, E_1)$. Weights are assigned to edges of G as follows:

$$w(e) = \begin{cases} 0 & \text{if } e \in E_1 \\ 1 & \text{otherwise} \end{cases}$$

A single source shortest path algorithm is executed on the weighted graph (V, E, w) with an arbitrary vertex v_1 of V_1 as the source. Which of the following statements about the shortest paths is correct?

(a) The number of edges in the shortest path from v_1 to all vertices in V_1 is minimum
(b) G_1 is connected
(c) V_1 forms a clique in G
(d) G_1 is a tree

[2008 : 2 Marks]

6.4 Let $G = (V, E)$ be a directed graph with n vertices. A path from v_i to v_j in G is a sequence of vertices $(v_i, v_{i+1}, \ldots, v_j)$ such that $(v_k, v_{k+1}) \in E$ for $i \le k < j$. A simple path is a path in which no vertex appears more than once. Let A be an $n \times n$ array initialized as follows:

$$A[j, k] = \begin{cases} 1 & \text{if } (j, k) \in E \\ 0 & \text{otherwise} \end{cases}$$

Consider the following algorithm

for $i = 1$ to n
for $j = 1$ to n
for $k = 1$ to n
$A[j, k] = \max(A[j, k], A[j, i] + A[i, k])$;

Which of the following statements is/are correct?

S1: If $A[j, j] = 1$ then G has a Hamiltonian cycle
S2: If there exists a path from j to k, $A[j, k]$ contains the length of the longest path from j to k
S3: If there exists a path from j to k, every simple path from j to k contains at most $A[j, k]$ edges

[2008 : 3 Marks]

6.5 In a depth-first traversal of a graph G with n vertices, k edges are marked as tree edges. The number of connected components in G is

(a) k (b) $n - k + 1$
(c) $n - k - 1$ (d) $n - k$

[2005 : 1 Mark]

6.6 Let G be a directed graph whose vertices are the integers from 1 to 100. There is an edge from vertex i to vertex j if either $j = i + 7$ or $j = 3i$. The minimum number of edges in a path in G from vertex 1 to vertex 100 is

(a) 4 (b) 7
(c) 20 (d) 99

[2006 : 3 Marks]

Q.7 Let T be a depth first search tree in a connected graph G. Vertex u is an ancestor of vertex v in T. The degree of both u and v in G is at least 2. Which one of the following statements is true?

(a) There must be a vertex w in G such that u is an ancestor of w and w is a leaf in T.

(b) There must be a vertex w in G such that u is an ancestor of w and w is a leaf in G.

(c) There must exist a cycle in G containing u and all its descendants in T.

(d) There must exist a cycle in G containing u and all its ancestors in T.

[2006 : 2 Marks]

Q.8 What is the largest integer n such that every simple connected graph with n vertices and n edges contains at least 2 distinct spanning trees?

(a) 1 (b) 2

(c) 3 (d) 4

[2007 : 2 Marks]

Q.9 The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges takes time

(a) $O(n)$ (b) $O(m)$

(c) $O(m + n)$ (d) $O(mn)$

[2008 : 1 Mark]

Q.10 The adjacency list representation of a graph has been implemented using the given data structures. One pointer, and edges are shown by the nodes of the following graph.

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 1 Mark]

Q.11 Let G be a graph with n vertices and m edges. What is the tight upper bound on the running time of Depth First Search on G, when G is represented as an adjacency matrix?

(a) $O(n)$ (b) $O(n + m)$

(c) $O(n^2)$ (d) $O(n^3)$

[2011 : (Set 1) : 1 Mark]

Q.12 Suppose depth first search is executed on the graph below starting at vertex a. Assume that the neighbours of a vertex are visited in alphabetical order. The finishing time of the vertex b is

[2014 (Set-3) : 1 Mark]

Q.13 Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There are vertices at distance five from the root. What is the maximum possible distance of a vertex from the root in this BFS traversal?

[2018 (Set-2) : 1 Mark]

Q.14 In an adjacency list representation of an undirected simple graph $G = (V, E)$, each node stores a list of its neighbours. Let vertices $\{u\}$ be the adjacency list of $\{v\}$ and $\{v\}$ be the adjacency list of $\{u\}$. These are called [illegible] and [illegible] respectively. Assume memory access time is constant. What is the time complexity of the most efficient algorithm for finding all pairs of vertices that are mutual neighbours?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2016 (Set-2) : 2 Marks]

Answers Graphs

6.2 (d) 6.4 (b) 6.4 (d) 6.5 (d) 6.6 (b) 6.7 (a) 6.8 (c) 6.9 (c) 6.10 (c)

6.11 (c) 6.12 (b)

Explanations Graphs

6.1 (c):

Adjacency Matrix

	B_1	B_2	B_3	B_4	B_5
B_1	0	1	0	0	0
B_2	0	1	1	1	0
B_3	1	0	0	0	0
B_4	0	1	1	0	1
B_5	1	1	0	0	0

6.2 (d):

Consider each case

I. Search (a) = b add (b, a)
Search (b) = a add (a, b)
Search (c) = a add (a, b, c)
Search (g) = b add (b, a, b, g)
Search (h) = f add (f, a, b, g, f)

II. Search (a) = b add (b, a)
Search (b) = c add (b, a, c)
Search (c) = b add (c, a, b)
Search (d) = g add (g, a, b, c, g)
Search (g) = e add (e, a, b, c, d, g, e)

III. Search (a) = b add (b, a)
Search (b) = c add (c, a, b)
Search (c) = d add (d, a, b, c, d)
Search (d) = e add (e, a, b, c, d, e)
Search (e) = a add (a, a, b, c, d, e, a)

6.3 (b):

Let $G = (V, E)$ be an undirected graph. $G_1 = (V, E_1)$ such that $E_1 = E$ and $V_1 = V$.

Consider an example

Now we add a vertex v_1 in E_1. It means deleted of V_1 by v_1 and other edges leaving out. It is obvious that G_1 is connected. V_1 does not form of any cycle in G_1 as well as G_1 is not a tree. So option (b) is correct.

6.4 (d):

Let $A[i, j]$ = weight of edge (i, j)
$A[i, j] = \min(A[i, j], A[i, k] + A[k, j])$
$A[i, j]$ is initially and $A[i, k] = \infty$ if (i, k) is not an edge.
Either exists a path from i to k and with intermediate vertices $A[i, k]$ exists.

6.5 (d):

Number of vertices = n
Number of edges = k
Number of connected components = n − k
As it is a tree, n − 1 edges

So connected components = n − (n − 1) = 1.

6.6 (b):

If we remove k edges from the graph using either DFS or BFS, we get k + 1 components.
The edge sequence with minimum number of edges is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 which consists of 100 edges.

6.7 (a):

Consider the following graph:
G: DFS path: Start Search: C

[illegible]

Consider each statement separately

Statement (I): There must exist a vertex in G which adjacent to every other vertex in G is a cut vertex.

This implies there must exist a vertex v which on removal disconnects the graph. It is not true because when we remove v from G graph is connected

Statement (II): There must exist a cycle in G containing all vertices of G. It is false because in given graph there are two cycles [illegible] different cycles

Statement (III): There must exist a cycle in G containing any two vertices of G. It is false because [illegible] vertex u & v are not in a single cycle of G

6.8 [illegible]

[illegible]

6.9 [illegible]

The most efficient algorithm for finding the [illegible] in a connected graph G is Kruskal and [illegible] using depth first search takes O(E+V) time. Assume [illegible]

6.10 [illegible]

The DFS using Queue data structure is QUEUEDFS

6.11 [illegible]

Tight upper bound on the number of DFS is $2n^2$

6.12 [illegible]

Max = [illegible] DFS traversal depth.

[illegible] $P \to Q \to R \to S \to T$

6.13 [illegible]

It were a complete binary tree with height h then distance from root to leaf is h.

Suppose we have to visit node A.

[illegible]

6.14 [illegible]

By using DFS (Depth First Search) traversal we can [illegible] in adjacency list.

[illegible] time (since [illegible] adjacency list are changed).

■■■■

7 Hashing

7.1 A hash table with 10 slots with one slot per bucket is used to hold the figures of the numbers 81 to 87. A hash function using modulo hashing function with linear probing. The total number of comparisons required when the symbol being searched is not in the table is

[1999 : 2 Marks]

7.2 An advantage of chained hash table (open hashing) over the open addressing scheme is

(a) worst-case complexity of searching is less

(b) space requirement is less

(c) deletion is easier

(d) hashing is faster

[1998 : 1 Mark]

7.3 Given the following hash table of size 10, 92, 101, 1171, 2672, 572, 811, 476, 305, 1609 and the hash function mod 10, which of the following statements are true?

1. 305, 1609, 572 hash to the same value
2. 1672, 811 hash to the same value
3. All elements hash to the same value
4. Each element hashes to a different value

(a) 1 only (b) 2 only

(c) 1 and 3 only (d) 2 and 4 only

[2004 : 1 Mark]

7.4 A hash table contains 10 buckets and uses linear probing to resolve collisions. The keys are integers and the hash function used is key % 10. If the values 43, 165, 62, 123, 142 are inserted in the table, in what order are the keys values 142 are inserted?

(a) 5 (b) 3

(c) 4 (d) 6

[2006 : 1 Mark]

7.5 Which of the following statements is/are true?

I. A hash function takes a message of fixed length and produces a fixed length code.

II. A hash function takes a message of fixed length and produces a code of variable length.

III. A hash function may map two distinct messages to the same hash value for distinct messages.

(a) I only (b) I and II only

(c) I and III only (d) II only

[2006 : 1 Mark]

7.6 Consider a hash function that distributes keys uniformly. The hash table size is 20. After hashing of how many keys will the probability that any new key hashed collides with existing one exceed 0.5.

(a) 5 (b) 6

(c) 4 (d) 10

[2007 : 2 Marks]

7.7 Consider the hash table of size seven, with starting index zero, and a hash function $(3x + 4) \bmod 7$. Assume the table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 9, 5 is inserted into the table using closed hashing? Assume that collisions are resolved by taking consecutive locations in the table.

(a) 2, -, -, -, -, -, 10 (b) 1, -, 10, -, -, -, 5

(c) 1, -, -, -, -, -, 5 (d) -, 1, 5, -, -, -, 9

[2007 : 2 Marks]

7.8 Consider a hash table of size seven that uses open addressing with linear probing. Let $h(k) = k \bmod 7$ be the hash function used. A sequence of records with keys 43, 90, 65, 42, 23, 67, 90 is to be inserted in an initially empty hash table, the bins of which are indexed from zero onwards. What is the bin in which the last record is inserted?

(c) 4 (d) 7

(e) 5 (f) 7

[2008 : 1 Marks]

7.9 The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function $h(k) = k \bmod 10$ and linear probing. What is the resulting hash table?

(a)

0	
1	
2	12
3	23
4	
5	5
6	
7	
8	18
9	15

(b)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	15

(c)

0	
1	
2	12
3	13
4	2
5	3
6	
7	
8	18
9	15

(d)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

[2009 : 2 Marks]

Linked Answer Questions 7.10 and 7.11

Consider a hash table of size 10 using open addressing with hash function $h(k) = k \bmod 10$, and quadratic probing. After inserting 6 values into an empty hash table, the table is as shown below.

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

7.10 Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

(a) 46, 42, 34, 52, 23, 33

(b) 34, 42, 23, 52, 33, 46

(c) 46, 34, 42, 23, 52, 33

(d) 42, 46, 33, 23, 34, 52

[2010 : 2 Marks]

7.11 How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

(a) 10 (b) 20

(c) 30 (d) 40

[2010 : 2 Marks]

7.12 Consider a hash table with 9 slots. The hash function is $h(k) = k \bmod 9$. The collisions are resolved by chaining. The following 9 keys are inserted in the order: 18, 19, 20, 30, 36, 12, 10, 9, 27. The maximum, minimum, and average chain lengths in the hash table respectively are

(a) 3, 0 and 1 (b) 3, 0 and 3

(c) 4, 0 and 1 (d) 3, 0 and 2

[2014 (Set-1) : 2 Marks]

7.13 Consider a hash table with 9 slots. The hash function is $h(k) = k \bmod 9$. The collisions are resolved by chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?

(a) $(8/9) \times (8/9) \times (7/9) = 56/243$

(b) $(8/9) \times (7/9) \times (6/9) = 56/243$

(c) $(8/9) \times (8/9) \times (8/9) = 512/729$

(d) $(8/9) \times (8/9) \times (8/9) \times (7/9) = 3584/6561$

[2014 (Set-3) : 2 Marks]

7.14 Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for the keys 0 to 2020?

(a) $h(x) = x \bmod 10$

(b) $h(x) = (x \times 10) \bmod 10$

(c) $h(x) = (11 \times x) \bmod 10$

(d) $h(x) = (12 \times x) \bmod 10$

[2015 (Set-3) : 2 Marks]

7.15 Given a hash table T of 25 slots that stores 2000 elements, the load factor α for T is _____.

[2015 (Set-4) : 1 Mark]

■■■■

Answers Hashing

7.1 (a) 7.2 (c) 7.3 (c) 7.4 (b) 7.5 (a) 7.6 (d) 7.7 (b) 7.8 (d) 7.9 (c) 7.10 (c)
7.11 (c) 7.12 (a) 7.13 (c) 7.14 (b)

Explanations Hashing

7.1 [illegible]

Maximum number of elements in the hash table
Search time with O(L) collision [illegible]

7.2 [illegible]

A hash table is faster because it uses a hash table which has [illegible]

7.3 [illegible]

Total = [illegible]
Given: Hash function,
h(x) = x mod 10
h(1) = h(1)
= [illegible] hash to the same value
h(3) = h(3) = [illegible]
h(5) = h(5) = [illegible]
h(7) = h(7)
= [illegible] table is [illegible]
So, h(1), h(3), h(5), h(7) are simply for given input [illegible] statement [illegible]

7.4 [illegible]

43, 137, 62, 123 and 142 are inserted in the table.
43 mapped to location 3, 137 mapped to location 7, 62 mapped to location 2, 123 mapped to location 3 (collision), with given collision [illegible] mapped to location 5 (since slots 3, 4 and 5 are occupied), with given collision [illegible]

7.5 [illegible]

A hash function takes a message of arbitrary length and generates a fixed length code by taking some (some value).
A hash function never gives the same hash value for different messages.

7.6 [illegible]

Probability of [illegible] = 0.5
h(20) = 0.5
→ [illegible]
After 10 keys, the probability of any exceed [illegible]

7.7 [illegible]

Size of hash table = 7
h(a) = (22 × 5) mod 7
h(b) = (5 × 4) mod 7 = [illegible] mod 7
= [illegible] location
h(c) = (5 × 4) mod 7 = 20 mod 7
= [illegible] location
h(d) = (5 × 4) mod 7 = 20 mod 7
= [illegible] is already filled by element [illegible] Second [illegible] Linear [illegible] 2 position
h(e) = (5 × 4) mod 7 = 54 mod 7 = 6 but [illegible] position [illegible] already filled with the element [illegible] so insert [illegible] location which is [illegible] position

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	1	2	[illegible]	3	5	6

7.8 [illegible]

4, 36, 82, [illegible], 11, 4, 51, 19, 14
h(x) = x mod 11

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	[illegible]	2	3	4	5	6	7	8	9	10

[illegible] 14 is inserted.

7.9 [illegible]

12 mod 10 = 2
19 mod 10 = 9
10 mod 10 = 0
2 mod 10 = 2 [illegible]
(2 + 1) mod 10 = [illegible] [illegible] linear probing
[illegible] mod 10 = [illegible]
8 mod 10 = 8 collision
(8 + 1) mod 10 = 9 collision using linear probing
(9 + 1) mod 10 = 0
[illegible] mod 10 = 8 collision
(8 + 1) mod 10 = 9 collision
(9 + 1) mod 10 = [illegible] again collision
(0 + 1) mod 10 = [illegible]
5 mod 10 = 5 [illegible]

[illegible] mod 10 = 6 [illegible]
[illegible] mod 10 = 7
[illegible] mod 10 = 2 [illegible]
[illegible] mod 10 = 3 [illegible]
[illegible] mod 10 = 7 [illegible]
[illegible] mod 10 = 8 [illegible]
[illegible] mod 10 = 9
So resulting hash table

2	[illegible]
3	[illegible]
4	[illegible]
5	[illegible]
6	[illegible]
7	[illegible]
8	[illegible]
9	[illegible]

3.10 [illegible]

In [illegible] checking [illegible] time [illegible] hashing

Option (a): 46, 42, 54, 63, [illegible]

0	
1	
2	42
3	23
4	[illegible]
5	53
6	46
7	
8	
9	

According [illegible] probing this [illegible] hash table

Option (b): 54, 42, 35, 12, [illegible], 26

0	
1	
2	42
3	23
4	54
5	35
6	46
7	
8	

Hash table

Option (c): 46, 54, 42, 23, [illegible]

0	
1	
2	42
3	23
4	54
5	46
6	53
7	
8	
9	

Hash table

Therefore [illegible] is correct.

3.11 [illegible]

Different [illegible] using same hash [illegible] probing

46, 34, 42, 23, 52, 33

No problem to enter 46, 34, 42, 23

0	
1	
2	42
3	23
4	[illegible]
5	
6	46
7	
8	
9	

or

0	
1	
2	42
3	23
4	[illegible]
5	52
6	46
7	
8	
9	

The [illegible] 53 [illegible] are 6 possible options. After [illegible] 52 we get hash table. Now to enter 33 there are 6 [illegible] options.

$= 6 \times 6 = 36$ [illegible] options

3.12 [illegible]

Maximum length = 5

Minimum length = 0

Avg length = [illegible]

7.13 Ans.

[illegible]

$$\text{Load factor} = \frac{[illegible]}{[illegible]} = \frac{[illegible]}{[illegible]} = \frac{[illegible]}{[illegible]} \text{ [illegible]}$$

[illegible]

7.14 [illegible]

$h(i) = i^2 \bmod 10$

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$h(0) = 0^2 \bmod 10 = 0 = h(10) = h(20) = h(30)$
$= \ldots$

$h(1) = 1^2 \bmod 10 = 1 = h(11) = h(21) = h(31)$
$= \ldots$

$h(2) = 2^2 \bmod 10 = 4 = h(12) = h(22) = h(32)$
$= \ldots$

$h(3) = 3^2 \bmod 10 = 9 = h(13) = h(23) = h(33)$
$= \ldots$

$h(4) = 4^2 \bmod 10 = 6 = h(14) = h(24) = h(34)$
$= \ldots$

$h(5) = 5^2 \bmod 10 = 5 = h(15) = h(25) = h(35)$
$= \ldots$

$h(6) = 6^2 \bmod 10 = 6 = h(16) = h(26) = h(36)$
$= \ldots$

$h(7) = 7^2 \bmod 10 = 9 = h(17) = h(27) = h(37)$
$= \ldots$

$h(8) = 8^2 \bmod 10 = 4 = h(18) = h(28) = h(38)$
$= \ldots$

$h(9) = 9^2 \bmod 10 = 1 = h(19) = h(29) = h(39)$
$= \ldots$

7.15 Ans.

$$\text{Load factor} = \frac{\text{Number of elements}}{\text{Number of slots}}$$

$$= \frac{2000}{25} = 80$$

■■■■

Unit - VI

Algorithms

Contents

Sl.	Topic	Page No.
1.	Algorithm Analysis and Asymptotic Notation	342
2.	Divide and Conquer	355
3.	Greedy Method	363
4.	Dynamic Programming	374
5.	P & NP Concepts	380
6.	Miscellaneous Topics	385

UNIT VI Algorithms

Syllabus : Analysis, Asymptotic notation, Notions of space and time complexity, Worst and average case analysis; Design: Greedy approach, Dynamic programming, Divide-and-conquer; Tree and graph traversals, Connected components, Spanning trees, Shortest paths; Hashing, Sorting, Searching, Asymptotic analysis (best, worst, average cases) of time and space, upper and lower bounds, Basic concepts of complexity classes – P, NP, NP-Hard, NP-Complete.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Mark Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	–	1			2
1991	–	4			8
1992	–	3			6
1993	–	1			2
1994	–	8		2	16
1995	5	–			5
1996	1	5		9	41
1997	2	1		2	14
1998	2				2
1999	4				4
2000	1				2
2001	3				6
2002	2	5			12
2003	1	6		–	13
2004	1	7		–	15
2005	4			–	18
2006	7	5		–	17

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2007	2	8	18
2008		11	22
2009	2	5	12
2010	1	3	7
2011	1	3	7
2012		4	8
2013	4	3	10
2014 Set 1	2	2	6
2014 Set 2	2	3	8
2014 Set 3	1	2	5
2015 Set 1	3	3	9
2015 Set 2	1	4	9
2016 Set 1	2	4	10
2016 Set 2	2	2	6

1 Algorithm Analysis and Asymptotic Notations

1.1 Solve the recurrence equations
$T(n) = T(n-1) + n$
$T(1) =$

[1997 : 2 Marks]

1.2 What is the generating function (GF) for the sequence of Fibonacci numbers?

[1998 : 2 Marks]

1.3 $\sum_{i=1}^{n} O(n)$, where $O(n)$ stands for order n is

(a) $O(n)$ (b) $O(n^2)$
(c) $O(n^3)$ (d) $O(3^n)$

[1998 : 2 Marks]

1.4 Consider the following functions:

$$g(n) = \begin{cases} n^3 & \text{for } 0 \le n \le 10{,}000 \\ n^2 & \text{for } n \ge 10{,}000 \end{cases}$$

$$h(n) = \begin{cases} n & \text{for } 0 \le n \le 100 \\ n^3 & \text{for } n > 100 \end{cases}$$

Which of the following is true?
(a) $g(n)$ is $O(h(n))$ (b) $g(n)$ is $O(n^3)$
(c) $h(n)$ is $O(g(n))$ (d) $h(n)$ is $O(n)$

[1999 : 2 Marks]

1.5 Which of the following statements?

(a) $100n \log(n) = O\left(\frac{n^2}{100}\right)$
(b) $100n + \log(n) = O(n \log(n))$
(c) $100n + n^2 = O(n^2)$
(d) $2^n = O(n^2)$

[1996 : 1 Mark]

1.6 The worst case number of comparisons to be performed for $O(n \log n)$ time. Which of the following statements is/are false?
(a) Binary linked list
(b) Doubly linked list
(c) Circular doubly linked list
(d) Array implementation of list

[1997 : 1 Mark]

1.7 Let $f(n) = n^2 \log n$ and $g(n) = n(\log n)^{10}$ be two positive functions of n. Which of the following statements is correct?
(a) $f(n) = O(g(n))$ and $g(n) = O(f(n))$
(b) $g(n) = O(f(n))$ but $f(n) \ne O(g(n))$
(c) $f(n) = O(g(n))$ and $f(n) \ne O(g(n))$
(d) $f(n) \ne O(g(n))$ and $g(n) \ne O(f(n))$

[2001 : 1 Mark]

1.8 In the worst case, the number of comparisons needed to search a singly linked list of length n for a given element is

(a) $\log n$ (b) $n/2$
(c) $\log n - 1$ (d) n

[2002 : 1 Mark]

1.9 Consider the following functions:

$f(n) = 2n^{\sqrt{n}}$
$g(n) = 2^{0.5n \log n}$
$h(n) = n!$

Which of the following is true?
(a) $f(n)$ is $O(g(n))$ (b) $h(n)$ is $O(f(n))$
(c) $g(n)$ is $O(h(n))$ (d) $h(n)$ is $O(g(n))$

[2002 : 2 Marks]

1.10 Consider the following algorithm for searching for a given number x in an unsorted array $A[1 \ldots n]$ having n distinct values:

Choose an index i uniformly at random from $[1, n]$
If $A[i] = x$ then Stop else Goto 1

Assume that x is present in A at least once. The expected number of times A is accessed by the algorithm before it terminates?
(a) 1 (b) $n - 1$
(c) $2n$ (d) n^2

[2009 : 2 Marks]

1.11 The time complexity of the following algorithm

Fibonacci(n)

if n = 0 then return 0; else if n = 1 then return 1; else return (Fibonacci(n-1) + Fibonacci(n-2));

is best described by
(a) $O(n)$ (b) $O(\log n)$
(c) $O(n \log n)$ (d) $O(n^2)$

[2002 : 2 Marks]

1.15 Consider the [illegible] recurrence
[illegible] where [illegible] are constants
[illegible]
[illegible]
Which of the following statements is/are true?
(a) [illegible] (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3
[2008 : 1 Mark]

1.16 Consider the following function.
```
float fun(int n)
{
  int p, i, k;
  for (p = 1; p = 1; i = 1; i < n; i++)
  {
    p *= n;
    k = p;
  }
  return k;
}
```
For large values of n, the return value of the function fun() is most approximately
(a) n^2 (b) n^3
(c) $n!$ (d) 2^n
[2004 : 1 Mark]

1.17 The worst case time complexity of the quicksort algorithm is
(a) $O(n)$ (b) $O(n^2)$
(c) $O(n \log n)$ (d) $O(\log n)$
[2004 : 1 Mark]

1.18 What does the following recursive function approximate? Assume $x > 1$, $\epsilon > 0$.
```
x = n;
y = 1;
while (x - y > ε)
{
  x = (x + y)/2;
  y = n/x;
}
return x;
```
(a) $\log n$ (b) n^2
(c) n^3 (d) $n^{1/2}$
[2001 : 2 Marks]

1.19 Let $S = \{1, 2, \ldots, n\}$. In an array storing a list of numbers from S in random order, find the number whose value is one less than the number in the list. Consider the following program fragment written in a C-like language.
```
counter = 0;
for (i = 1; i < n; i++)
{
  if (a[i] == i) counter++;
  else
  {
    break;
    count = 0;
  }
}
```
The complexity of the program fragment is
(a) $O(n^2)$ (b) $O(n \log n)$ and $\Omega(n^2)$
(c) $O(n)$ (d) $O(\log n)$
[2004 : 2 Marks]

1.17 The time complexity of the following C function is (assume n > 0)
```
int recurse(int n)
{
  if (n == 1)
    return (1);
  else
    return (recurse(n-1) + recurse(n-1));
}
```
(a) $O(n)$ (b) $O(n \log n)$
(c) $O(n^2)$ (d) $O(2^n)$
[2004 : 2 Marks]

1.18 The recurrence equation
$T(1) = 1$
$T(n) = 2T(n-1) + n, \; n \geq 2$
evaluates to
(a) $2^{n+1} - n - 2$ (b) $2^n - n$
(c) $2^{n+1} - n - 3$ (d) $2^n + n$
[2004 : 2 Marks]

1.19 Let f(n), g(n), and h(n) be functions from the positive integers to the positive integers such that $f(n) = O(g(n))$, $g(n) = O(h(n))$, and $h(n) = O(g(n))$. Which one of the following is a valid relationship?
(a) $f(n) = O(h(n))$ (b) $f(n) = \Omega(h(n))$
(c) $h(n) = O(f(n))$
(d) $f(n) = \Theta(g(n))$
[Econ. 2004] 2 Marks

1.20 The time complexity of finding the kth smallest element of a binary search tree with n elements is known to be
(a) $O(n)$ (b) $O(\log n)$
(c) $O(n^2)$ (d) $O(n^3)$
[2003 : 1 Mark]

1.21 Let $T(n)$ be a function defined by the recurrence relation $T(n) = 2T(n/2) + \sqrt{n}$ and $T(1) = 1$. Which of the following statements is TRUE?

(a) $T(n) = O(\log n)$ (b) $T(n) = \Theta(\sqrt{n})$

(c) $T(n) = \Theta(n)$ (d) $T(n) = \Theta(n \log n)$

[2005 : 2 Marks]

1.22 Suppose $T(n) = 2T(n/2) + n$, $T(0) = T(1) = 1$. Which one of the following is TRUE?

(a) $T(n) = O(n^2)$ (b) $T(n) = O(n \log n)$

(c) $T(n) = \Theta(n)$ (d) $T(n) = \Omega(n \log n)$

[2006 : 2 Marks]

Common Data for Q. 1.23 & Q. 1.24

Consider the following C function:

```
double fun(int n)
{
  i = 1
  ...
  while (i <= n)
  {
    sum = 0.0;
    for (j = 1; j <= i; j++)
    sum = sum + j;
    printf("%f", sum);
    i = i * 2;
  }
}
```

1.23 The space complexity of the above function is

(a) $O(1)$ (b) $O(n)$

(c) $O(n^2)$ (d) $O(n^3)$

[2006 : 2 Marks]

1.24 The space complexity of the above function is $O(n)$ and stores the values in $(0, 0, 0, \ldots, n)$ in sum when they are computed. With this modification, the time complexity for function fun() is observed to be $O(n)$. The space complexity of the modified function would be

(a) $O(1)$ (b) $O(n)$

(c) $O(n^2)$ (d) $O(n^3)$

[2006 : 2 Marks]

1.25 Consider the following C program fragment in which i, j and n are integer variables.

for (i = n, j = 0; i > 0; i = i/2, j = j + i)

printf("%d", j); // prints the value of j on the screen

Let $T(n)$ represent the number of times the statement j = j + i is executed. For simplicity, ignore floor. Which of the following is true?

(a) $T(n) = \Theta(\log \log n)$ (b) $T(n) = \Theta(n \log n)$

(c) $T(n) = \Theta(n^2)$ (d) $T(n) = \Theta(n)$

[2006 : 2 Marks]

1.26 Consider the following statement:

$T(n) = 2T(\sqrt{n}) + \log n$, $T(1) = 1$

Which one of the following is true?

(a) $T(n) = \Theta(\log \log n)$ (b) $T(n) = \Theta(\log n)$

(c) $T(n) = \Theta(\sqrt{n})$ (d) $T(n) = \Theta(n)$

[2006 : 2 Marks]

1.27 Given an unsorted array sequence of n elements

```
...
...
while (j <= n)
...
```

The number of comparisons made in the worst case is

(a) $\lceil \log_2 n \rceil + 1$ (b) ...

(c) $\lceil \log_2 n \rceil$ (d) ...

[2007 : 1 Mark]

1.28 What is the time complexity of the following recursive function?

```
int DoubleSum(int n)
{
  if (n <= 0)
    return 1;
  else
    return DoubleSum(n-1) + DoubleSum(n-1);
}
```

(a) $O(n^2)$ (b) $O(n \log n)$

(c) $O(2^n)$ (d) $O(n)$

[2007 : 2 Marks]

1.29 An array of n numbers is given, where n is an even number. The median of these n numbers is defined as the $(n/2)$th smallest element. Which of the following statements is true about the number of comparisons needed?

(a) At least $2n - 2$ comparisons are needed.

(b) At least $n \log_2 n$ comparisons are needed.

(c) At least n^2 comparisons are needed.

(d) None of the above

[2007 : 2 Marks]

1.61 Consider the following C-like pseudocode:

```
int IsPrime(n)
{
  for (i=2;
  for (i = 2; i <= sqrt(n); i++)
    {
      if (n % i == 0)
        printf("Not a prime");
      return 0;
    }
  return 1;
}
```

Let $T_{pr}(n)$ denote the number of times the function is executed by the above program. Which of the following is true?

(a) $T_{pr}(n) = O(\sqrt{n})$ and $T_{pr}(n) = \Omega(\sqrt{n})$

(b) $T_{pr}(n) = O(\sqrt{n})$ and $T_{pr}(n) = O(n)$

(c) $T_{pr}(n) = O(n)$ and $T_{pr}(n) = \Omega(\sqrt{n})$

(d) None of these

[2007 : 2 Marks]

1.62 Arrange the following functions in increasing order of asymptotic growth:

A. 2^n B. n^3

C. $n^{0.99}$ D. $n \log n$

E. $n(\log n)^2$

(a) A, B, C, D, E (b) D, B, C, E, A

(c) A, C, D, E, B (d) A, C, D, B, E

[2008 : 1 Mark]

1.63 When $n = 2^k$ for some $k \geq 0$, the recurrence relation $T(n) = 2T(n/2) + \sqrt{n}$, $T(1) = 1$ evaluates to

(a) $\sqrt{n}(\log n + 1)$ (b) $\sqrt{n}\log n$

(c) $\sqrt{n}\log\sqrt{n}$ (d) $n \log \sqrt{n}$

[2006 : 2 Marks]

1.64 Consider the following functions:

$f(n) = n^2$

$g(n) = n^3$

$h(n) = n^n$

Which of the following statements about the asymptotic behaviour of $f(n)$, $g(n)$, and $h(n)$ is true?

(a) $f(n) = O(g(n))$; $g(n) = O(h(n))$

(b) $h(n) = O(g(n))$; $g(n) = O(f(n))$

(c) $g(n) = O(f(n))$; $h(n) = O(f(n))$

(d) $h(n) = O(f(n))$; $g(n) = O(h(n))$

[2006 : 3 Marks]

1.65 The minimum number of comparisons required to determine both the minimum and the maximum of n integers is

(a) $n/2$ (b) $n-1$

(c) $n \log n$ (d) $2n-2$

[2006 : 1 Mark]

Common Data Questions Q.1.66 and Q.1.67

Consider the following C function:

```
int Fun(int n)
{
  int x = 1, k = 2;
  while (k < n) {
    x = x + 4*k;
    k = k + 2;
  }
  return x;
}
```

```
int Fun(int n)
{
  int x, k;
  x = 1; k = 2;
  while (k < n) {
    x = x + 4*k;
    k = k + 2;
  }
  return x;
}
```

1.66 The asymptotic time complexity of Fun(n) is

(a) $O(n^2)$ (b) $O(n^3)$

(c) $O(n)$ (d) $O(\log n)$

[2006 : 2 Marks]

1.67 If the statement "k = k + 2" is replaced by "k = k * 2", then the asymptotic time complexity of Fun(n) is

(a) $O(n^2)$ (b) $O(n)$

(c) $O(\log n)$ (d) $O(1)$

[2006 : 2 Marks]

1.68 The recurrence relation $T(1) = 2$, $T(n) = 4T(n/2) + n$, $n > 1$, evaluates to

(a) $n^2 \log n$ (b) $n \log n$

(c) n^2 (d) $n^2 \log n$

[2006 : 2 Marks]

1.69 The recurrence relation $T(1) = 2$, $T(n) = 3T(n/4) + n$ evaluates to

(a) $O(n)$ (b) $O(\log n)$

(c) $O(n^{0.75})$ (d) $O(n^2)$

[2006 : 2 Marks]

1.70 The minimum number of comparisons required to determine both the minimum and the maximum of n integers is

(a) $n/2$ (b) $n-1$

(c) $n \log n$ (d) $2n-2$

[2006 : 1 Mark]

1.38 The runtime of an algorithm is represented by the following recurrence relation:

$$T(n) = \begin{cases} c & n \le 5 \\ T(n-1) + T\left(\frac{n}{5}\right) + n & \text{otherwise} \end{cases}$$

Which one of the following represents the time complexity of the algorithm?

(a) $O(n)$ (b) $O(n \log n)$

(c) $O(n^2)$ (d) $O(n^2 \log n)$

[2009 : 2 Marks]

1.39 Let $W(n)$ and $A(n)$ denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is ALWAYS TRUE?

(a) $A(n) = \Omega(W(n))$ (b) $A(n) = \Theta(W(n))$

(c) $A(n) = O(W(n))$ (d) $A(n) = o(W(n))$

[2010 : 1 Mark]

1.40 The recurrence relation capturing the optimal execution time of the Towers of Hanoi problem with n discs is

(a) $T(n) = 2T(n-2) + 2n$

(b) $T(n) = 2T(n-1) + n$

(c) $T(n) = 2T(n/2) + 1$

(d) $T(n) = 2T(n-1) + 1$

[2014 : 1 Mark]

1.41 Consider the following function

```
int f(int n)
{
  int i, j, k = 0;
  for (i = n/2; i <= n; i++)
    for (j = 2; j <= n; j = j * 2)
      k = k + n/2;
  return (k);
}
```

The time complexity of the function is

(a) $O(n^2)$ (b) $O(n^2 \log n)$

(c) $O(n)$ (d) $O(n \log n)$

[2018 : 2 Marks]

1.42 Which one of the following correctly determines the solution of the recurrence relation with $T(1) = 1$?

$$T(n) = 2T\left(\frac{n}{2}\right) + \log n$$

(a) $O(n)$ (b) $O(n \log n)$

(c) $\Theta(\log n)$ (d) $O(\log n)$

[2014 (Set-2) : 1 Mark]

1.43 Suppose we have a sorted array of distinct integers that has been circularly shifted to the right by an unknown amount. For example, {10, 15, 20, 25, 1, 3, 5, 7} is such an array, but {3, 5, 7, 10, 15, 20, 25, 1} is not. We wish to find the index of the element 1 in the array. We require the worst-case complexity to be $O(\log n)$. If the highest possible value of n is 2^{16}, the value of n is _____.

[2014 (Set-3) : 2 Marks]

1.44 Consider the following C function

```
int fun(int n)
{
  int i, j, k, p, q = 0;
  for (i = 1; i < n; i++)
    {
      p = 0;
      for (j = n; j > 1; j = j/2)
        {
          p = p + n;
        }
      for (k = n; k > 1; k = k/2)
        {
          q = q + p;
        }
    }
  return q;
}
```

Which one of the following most closely approximates the return value of the function fun?

(a) n^3 (b) $n^3 \log n$

(c) $n \log n$ (d) $n \log(\log n)$

[2015 (Set-1) : 2 Marks]

1.45 An operation on a data structure takes $O(1)$ time for the first time and $O(n)$ time for the second time. The first time operation is performed on a set of data items with keys drawn from a universe U. The second time operation is performed on the same data structure and the same set of data items, but with keys drawn from $U - \{x\}$, where $x \in U$. The expected time complexity of the second operation is $O(1)$. Which one of the following data structures and operations is consistent with the above description?

(a) Unsorted array

(b) Max heap

(c) Stack or queue

(d) Sorted array or sorted list

[2015 (Set-1) : 2 Marks]

1.46 Consider a complete binary tree where the left and the right subtrees of the root are max-heaps. The lower bound for the number of operations to convert the tree to a heap is

(a) $\Omega(\log n)$ (b) $\Omega(n)$

(c) $\Omega(n \log n)$ (d) $\Omega(n^2)$

[2015 (Set-2) : 1 Mark]

1.47 An unsorted array of distinct elements. Then number of comparisons to find minimum in this list that in worst case maximum is

(a) $\Theta(n \log n)$ (b) $O(n)$

(c) $\Theta(\log n)$ (d) $\Theta(1)$

[2015 (Set-3) : 1 Mark]

1.48 Consider the equality $\sum_{i=0}^{n} i^3 = X$ and the following choices for X

I. $\Theta(n^4)$ II. $\Theta(n^3)$

III. $O(n^4)$ IV. $\Omega(n^4)$

The correct combination of X is exactly

(a) I, II

(b) III, IV

(c) I and II and IV but not III

(d) II and III and IV but not I

[2015 (Set-3) : 1 Mark]

1.49 Let $f(n) = n \sin(g(n))$ and $g(n) = n^2 + n^3$, where n is a positive integer. Which of the following statements is/are correct?

I. $f(n) = O(g(n))$ II. $f(n) = \Omega(g(n))$

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

[2015 (Set-3) : 1 Mark]

1.50 The given [illegible] shows the flow chart for a recursive function. Assume that all statements except for the recursive call take $O(1)$ time complexity. If the [illegible] time complexity of this function is $O(n^k)$, then the least possible value (correct up to nearest integer) of k is _____.

The flow chart for Recursive function Stat

[2016 (Set-3) : 2 Marks]

1.51 Consider a binary tree stored in a 1-D array. Every data element [illegible] the tree is to be deleted. For a deletion, a data element is provided to the procedure. [illegible] operation is to be performed. The algorithm performs [illegible] in-order traversal [illegible] $O(\log N)$ time and to $O(1)$ decrease key. What is the time complexity of all these operations put together?

(a) $O(\log N)$ (b) $O(N)$

(c) $O(N^2)$ (d) $O(N \log N)$

[2016 (Set-2) : 1 Mark]

■■■■

Answers: Algorithm Analysis and Asymptotic Notations

1.8	(b)	1.9	(a)	1.10	(a, d)	1.11	(c)	1.12	(b)	1.13	(d)	1.14	(d)	1.15	(a)	1.16	(a)	1.17	(c)
1.18	(a)	1.19	(b)	1.20	(c)	1.21	(c)	1.22	(c)	1.23	(b)	1.24	(b)	1.25	(c)	1.26	(b)	1.27	(d)
1.28	(d)	1.29	(b)	1.30	(b)	1.31	(a)	1.32	(a)	1.33	(a)	1.34	(d)	1.35	(b)	1.36	(c)	1.37	(a)
1.38	(c)	1.39	(d)	1.40	(b)	1.41	(b)	1.42	(a)	1.43	(d)	1.44	(a)	1.45	(a)	1.46	(a)	1.47	(d)
1.48	(c)	1.49	(d)	1.50	(c)														

Explanations: Algorithm Analysis and Asymptotic Notations

1.1 Sol:

By writing recurrence relation in the form of [illegible] series

$c = 1 + 2 + 3 + \ldots (n-2) + (n-1) + n \ldots 3 + 2 + 1$

While assuming the natural numbers.

$$= \frac{n(n+1)}{2} \rightarrow O(n^2)$$

1.2 Sol:

The generating function for the [illegible] sequence is

$$G(x) = \frac{x}{1-x-x^2}$$

1.8 (b)

$$\sum_{i=1}^{n} O(i) = O(1) + O(2) + O(3) \quad \ldots O(n)$$

$$= O\left(\frac{n(n+1)}{2}\right) = O(n^2)$$

1.9 (b)

Therefore,

$$n^2 = n^2 \times 2 \times 1000$$

$$n = O(n\sqrt{n})$$

Option (b) is correct.

1.10 (b)

(a) We know that

$$\sin x = O(\log(x)) \ldots$$

Since $\log(x)$

k, x is a positive integer and $n \geq x_0$

$$\text{Pr. } n \log n \leq 1000 \times \frac{n \log n}{100}$$

$$\log n = 100 \log n$$

$$\therefore \log n = O\left(\frac{n \log n}{100}\right)$$

(b) $\sqrt{\log n}$ = [illegible]

$$\therefore \sqrt{\log n} = O(\log \log n)$$

(c) [illegible]

$$\ldots n^2 = O(n^3)$$

(d) [illegible]

$$\ldots n^{\log n} = O(n^k)$$

1.11 (b)

Selection sort is an inplace sorting algorithm. It is not stable. It usually makes fewer swaps than bubble sort. Quicksort is a complexity [illegible] sorting algorithm. Quicksort [illegible]

1.12 (b)

$$\text{fine} = \text{[illegible]}$$

$$\text{[illegible]}$$

$$n(\log n)^2 = n^2 \log n$$

$$\therefore \text{[illegible]} = O(n \log n)$$

Whereas

$$\text{[illegible]}$$

$$n^{\log n} = n^{\log n}$$

1.13 (b)

Whereas [illegible] strongly linked [illegible] system [illegible] doesn't [illegible] at all [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

1.14 (d)

$$\text{[illegible]}$$

$$= \text{[illegible]}$$

$$= \text{[illegible]}$$

1.15 (a)

[illegible] [illegible] [illegible] [illegible] [illegible] [

Substituting [illegible] in [illegible] we get

$$\frac{1}{n} = \frac{1}{n} + \frac{n-1}{n}\cdot\frac{1}{n} + \frac{n-1}{n}\cdot\frac{n-1}{n}\cdot\frac{1}{n} + \cdots$$

The expected weight of the MST with infinite elements

So apply the geometric series $\left(\frac{1}{1-r}\right)$

$$\frac{1}{n} = \frac{1}{1-\left(\frac{n-1}{n}\right)} = n$$

$X = n$

Therefore option (a) is correct.

1.11 (b)

Recursive relation is given as follows

$$T(n) = 7T\left(\frac{n}{2}\right) + cn^2 \quad \text{for } n > 2$$

Let $n = 2^k$

$T(1) = 7T(2^{k-1})$

$T(2^k) = 8(n)$

$T(n) = 8(n^2)$

$$T(n) = 7T\left(\frac{n}{2}\right) + cn^2 = 7\left(7T\left(\frac{n}{4}\right) + c\left(\frac{n}{2}\right)^2\right) + cn^2$$

$$T(n) = 7^2T\left(\frac{n}{2^2}\right) = 7^3T\left(\frac{n}{2^3}\right) + cn^2 \quad \text{for } n > 2$$

$$\therefore \quad T(n) = 7^kT\left(\frac{n}{2^k}\right) + \cdots + cn^2\log n^2$$

$= O(n^{\log_2 7})$ [$\because 2^k = n$, $k = \log_2 n$]

$T(n) = O(n^2\log n)$

$\therefore$ Optimal solution.

1.12 (a)

Consider each statement separately

I. $f(n) = (n+k)^m$

$\Rightarrow f(n) = O(n^m)$

Assume $k = 1$ for simplicity

$f(n) = (n+1)^m = C_0n^m + C_1n^{m-1} + \cdots C_mn^0$

$f(n) = O(n^m)$

II. $f(n) = n^{\log n}$

$f(n) = 2^{\log n \log n}$

$f(n) = 2^{\log^2 n}$

$f(n) = O(2^{\log^2 n})$

III. $f(n) = 2^n$

$f(n) = 2^{n+1}$

$f(n) = 2\cdot 2^n$

$f(n) = O(2^n)$

Therefore I and III are correct.

1.13 (b)

Given function is continuous, so consider the following recurrence method.

1	2	3	...
	$p = p^2$	$x = x^2$	
Initial: 1	1	1	
1	$p = x$	$s = 1 + x$	
2	$p = x\cdot\frac{x}{2}$	$s = 1 + x + \frac{x^2}{2}$	
3	$p = \frac{x^2}{2}\cdot\frac{x}{3}$	$s = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$	
4	$p = \frac{x^3}{6}\cdot\frac{x}{4}$	$s = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$	
5	$p = \frac{x^4}{24}\cdot\frac{x}{5}$	$s = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$	

For large value of x the value of s tends to infinite in second iteration of for loop and hence to overflow. In the given function we only are changing in p but not in s. The value of s is the function itself.

$$s = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots$$

$$s = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \infty$$

$s = e^x$

1.14 (b)

Any decision tree that sorts n distinct elements has height at least $\log n!$. So the decision tree lower bound is

The number of comparisons required is at least $\log n!$

Stirling approximation gives

$$n! = \left(\frac{n}{e}\right)^n$$

Taking log both sides

$\log n! = \log\left(\frac{n}{e}\right)^n$

$\log n! = n\log\left(\frac{n}{e}\right)$

$\log n! = n(\log n - \log e)$

$\log n! = n(\log n - 1.44)$

$\log n! = n\log n - 1.44n$

So $\log n! = \Theta(n\log n)$

1.15 (c) :

[illegible]

1.16 (c) :

[illegible]

1.17 (d) :

[illegible]

1.18 (b) :

[illegible]

1.19 (c) :

[illegible]

120 (d)

[illegible]

121 (c)

$T(n) = 2T(n/2) + 1 + \sqrt{n}$

By substitution method $T(n) = O(n)$

$T(n) = 2T(n/2) + 1 + \sqrt{n}$ ...(1)

$= 2\left[2T(n/4) + 1 + \sqrt{n/2}\right] + \sqrt{n}$

$= 4T(n/4) + 2 + \sqrt{n/2} + \sqrt{n}$ ...(2)

$= 8T(n/8) + 4 + \sqrt{n/4} + \sqrt{n/2} + \sqrt{n}$ ...(3)

$= 2^k T(n/2^k) + 2^{k-1} + \sqrt{n/2^{k-1}} + \sqrt{n/2^{k-2}} + \sqrt{n/2^{k-3}} + \ldots + \sqrt{n}$

$= 2^k T(n/2^k) + 2^{k-1} + \sqrt{n}\sum_{i=1}^{k} (2^{1/2})^{i-1}$

$= O(n)$

122 (a)

$T(0) = T(1) = 1$

$T(n) = 2T(n/2) + n$

$T(n)$ is computed as follows

$T(n) = n\sum_{i=1}^{k} (n/2^i)^i$

$T(n) = n\sum_{i=1}^{\log n} (n/2^i)^i$

$T(n) = 8n\log n$

[$T(n)$ can also be solved by Master theorem]

If $T(n) = aT(n/b) + f(n)$ then $f(n) = n^{\log_b a}$ and $f(n)$ is $n \times O(\log n)$

123 (b)

[illegible]

$\log(n) = O(\log n) + O(\log n) + \ldots O(\log n)$

Therefore space complexity $= O(n)$

[Note: Time complexity $= O(n^2)$]

124 (b)

[illegible]

Time complexity $O(n^2)$, [illegible] the space complexity is $O(n)$.

[Note: Time complexity $= O(n)$]

125 (b)

The height of the [illegible] $= O(n)$ [illegible] $T(n) = O(n)$ [illegible]

[illegible]

126 (b)

[illegible]

$T(n) = T(n/m) + 1$

[illegible] $T(n) = S(m)$

[illegible] $S(m) = 2S\left(\frac{m}{2}\right) + 1$

[illegible]

$S(m) = O(m)$

$= O(\text{where no. of nodes } n = 2^m)$

Now, $T(n) = T(2^m) = S(m)$

$= O(\log n)$

Therefore, [illegible]

127 (d)

[illegible] according to the quicksort, while loop [illegible] $\log_2 n$

- Once [illegible] comparisons required [illegible] total number of comparisons $= n - 1 = 1 + \log_2 n - 1$

128 (d)

The given function is [illegible] recurrence relation is

$T(n) = 1; \quad n \le 2$

$T(n) = \sqrt{n} + n, n > 2$

All the leaves same as height. The problem size at level k is $n^{1/2^k}$ and we stop when the problem size reaches 2.

So, $n^{1/2^k} = 2$, solving for k we get

$$2^k \log_2 n = 1$$
$$\Rightarrow 2^k = \log_2 n$$
$$\Rightarrow k = \log_2 \log_2 n$$
$$\therefore T(n) = \Theta(\log_2 n \log_2 n)$$

1.39 (b)

When we apply the divide and conquer method such that divide n into two parts at the root then recurrence equation is

$$T(n) = \text{constant}, n = 2$$
$$T(n) = 2T(n/2) + \text{constant}$$

Using master method

$$\text{Cost} = \frac{3}{2}n - 2$$
$$= 1.5n - 2 \text{ comparisons}$$

1.40 (d)

The inner loop number of times is for i is executed by for loop in n times.

$$T(n) = \sqrt{n} + 1$$

When $n = 1$ then loop is executed 1 times and stops constant time, i.e., $T(n) = \Theta(1)$.

Therefore,

$$T(n) = \sqrt{n} + 1 \text{ or } T(n) = \Theta(\sqrt{n}).$$

1.41 (d)

$A \times C$ and $A \times D$

$C \times B$ and $C, D \times B \times F$ exponential functions.

Now we must consider n^2 in D $\times$ largest. In C, term n^2 and n are $\times$ only in B we have $\log^2 n$.

$n^2 \times$ largest than $\log^2 n$.

So, D $\times$ C.

1.42 (a)

$$T(n) = \sqrt{2}\, T(n/2) + \sqrt{n} \quad \ldots(i)$$
$$= \sqrt{2}\, T(n/4) + \sqrt{2} + \sqrt{n} \quad \ldots(ii)$$
$$= [\sqrt{2}]^2\, T(n/2^2) + 2\sqrt{2} \quad \ldots(iii)$$
$$= [\sqrt{2}]^3\, T(n/2^3) + 3\sqrt{2} \quad \ldots(iv)$$
$$\vdots$$
$$= [\sqrt{2}]^k\, T(n/2^k) + k\sqrt{2}$$
$$= [\sqrt{2}]^{\log n}\, T(1) + \log n\sqrt{2} \quad (\because \text{Put } k = \log n)$$
$$= \sqrt{n}\, T(1) + \sqrt{n}\log n$$
$$= \sqrt{n}(\log n + 1)$$

1.43 (d)

$$\text{fun} = n^2 \Rightarrow O(n) = O(n^2)$$
$$g(n) = n^2 \Rightarrow g(n) = O(n^2)$$
$$h(n) = 2^n \Rightarrow h(n) = O(2^n)$$

n is n^2 $\ldots$ n is n $\ldots$ n is 2^n

The computation time of fun $\ldots$

$\therefore$ $\log_2 n = \log_2 n^2 = n^2 \ldots$

$\ldots$

$\ldots$

$= n^{k+1} \ldots$

$n^{k+1} \ldots$

$\ldots = 2$

$h(n) = O(h(n)) = \text{cost}$

$h(n) = O(2^n) = h(n)$ is k $h(n) \ldots$ time

$g(n) = O(n^2) = g(n)$ is n^2 is also true

1.44 (d)

Counting sort assumes that input elements are in a small range. We can sort an array of integers in linear time by using the fact that we know the range of the integers in the array. Counting sort uses this fact and performs sorting as follows:

binary search that is optimal worst case complexity is at least as fast as $\log_2(n+1)$. But if integer is [illegible] times more than array and array is already sorted [illegible] time for non-recursive binary search. [illegible] of comparisons is at greater than $\log_2(n+1)$.

1.36 (b) :

The given recurrence relation is:

The recurrence equation for $f(n)$ is:

$T(0) = 0$

$T(1) = 1$

$T(n) = 2T(n-1) + 2T(n-2)$

The solution of this linear equation is non-linear polynomial [illegible] the number has [illegible] of [illegible]

1.37 (a) :

$T(0) = 0$

$T(1) =$

$T(n) = 2T(n-1) + 2T(n-2)$

$T(2) = 2T(1) + 2T(0) = 2 \times 1 + 2 \times 0 = 2$

$T(3) = 2T(2) + 2T(1) = 2 \times 2 + 2 \times 1 = 4 + 2 = 6$

$T(4) = 2T(3) + 2T(2) = 2 \times 6 + 2 \times 2 = 12 + 4 = 16$

$T(5) = 2T(4) + 2T(3) = 2 \times 16 + 2 \times 6 = 32 + 12 = 44$

$T(6) = 2T(5) + 2T(4) = 2 \times 44 + 2 \times 16 = 88 + 32 = 120$

$T(7) = 2T(6) + 2T(5) = 2 \times 120 + 2 \times 44 = 240 + 88 = 328$

$T(8) = 2T(7) + 2T(6) = 2 \times 328 + 2 \times 120 = 656 + 240 = 896$

So [illegible] is the recurrence relation for the number of $f(n)$ is $O(2^n)$.

1.38 (a) :

[illegible]

1.39 (b) :

Complexity of [illegible] is [illegible]

$T(n) = T(\log n) + n$ for $n > 1$

Using master theorem

Here $a = 1$, $b = 2$, $\log_b a = \log_2 1 = 0$

$n^{\log_b a} = n^0 = 1$, n^ϵ

Since $n^\epsilon = n^1 > n^{\log_b a} = n^0$

Case 3 applies to case III of master theorem

where $af(n/b) = T(n/2) = n/2 < n$

1.40 (c) :

[illegible]

$T(n) = 2T(n/2) + n$

$T(n) = 2T(n/2) + 2T(n/4) + n$

1.41 (d) :

[illegible]

$= \frac{n}{2} \times \frac{n}{2} = \frac{n^2}{4} = O(n^2)$

$= O(n^2 \log n)$

1.42 (d) :

$T(n) = 2T(n/2) + \log n$

[illegible]

$= O(n)$

1.43 (c) :

$T(n) = \log n + 2(n^2 \log n + n^2 \log n)$

$= a = 0, b = 1, c = 1, d = 0$

$\Rightarrow$ [illegible]

$= O(n^2 \log n) = O(n)$

1.44 (d) :

```
int fun(int n)
{
    int i, j, k, p, q = 90
    for (i = 1; i < n; i++)
    {
```

[illegible]

1.46 [illegible]

[illegible]

1.47 [illegible]

Take three elements. The middle of the three elements will be the element that [illegible] minimum or maximum in the array.

Hence O(1) time to compare [illegible] elements.

Assume the array is [illegible]. Then 1 is the middle element of 1, 2 and 3. Which is neither minimum nor maximum.

1.48 (c):

[illegible]

$$x = \frac{[illegible]}{2} \cdot 3^2 - \frac{[illegible]}{4} \cdot 1^2 = 6 \text{ [illegible]}$$

$x = \Theta(n^2)$

$y = O(n^2)$

$z = \Omega(n^2)$

1.49 (B):

When every [illegible] is 1 then [illegible] value stays. Even [illegible] is 1.

Hence [illegible] = Digit [illegible] is [illegible]. Hence [illegible] is false.

[illegible] is 1, then [illegible] = n

Hence [illegible] is violated. Hence II is false.

1.50 (A):

[illegible]

[illegible]

By using Master theorem [illegible] which is equivalent to $n^{2.32}$. So the value of c is 2.32.

1.51 (d):

[illegible]

Sorted doubly linked list takes [illegible] number of of O(N log(N)) time but it [illegible] complexity

So [illegible] = O(N²)

■■■

2 Divide and Conquer

2.1 Let [illegible] be equivalent programs [illegible] according to the recurrence [illegible] and [illegible] respectively. Which of the following statements is true?

(a) [illegible] for [illegible]
(b) [illegible] for [illegible]
(c) [illegible] for [illegible]
(d) [illegible] for [illegible]

[1997 : 1 Marks]

2.2 Find a solution to the following recurrence equation

$$T(n) = T\left(\frac{n}{2}\right) + n$$

$$T(1) = 1$$

[1997 : 2 Marks]

2.3 Assume that the last element in the array is used as pivot in partition. Quicksort is performed on the array [illegible] in which the pivot is chosen, given an input for which Quicksort takes maximum time.

[1998 : 2 Marks]

2.4 Following sorting algorithms can be used to sort n integers in the range [1, n^k] in O(n) time

(a) Heapsort
(b) Quicksort
(c) Mergesort
(d) Radixsort

[1998 : 2 Marks]

2.5 The recurrence relation [illegible] describes some particular recursive algorithm. The complexity of this algorithm is

(a) $T(n) = T(n-1) + n$ = [illegible] constant
(b) $T(n) = T(n-1) + T(n-2)$ = [illegible]
(c) $T(n) = T(n/2) + \log n$
(d) $T(n) = T(n-1) + \log n$

[1999 : 2 Marks]

2.6 Which of the following statements is false?

(a) Optimal binary search tree can be constructed efficiently using dynamic programming
(b) Breadth first search can be used to find connected components of a graph
(c) In an undirected graph with unique edge costs, the minimum cost spanning tree is unique
(d) Depth first search can be used to find connected components of a graph

[1999 : 2 Marks]

2.7 Merge sort uses

(a) Divide and conquer strategy
(b) Backtracking approach
(c) Heuristic search
(d) Greedy approach

[1999 : 1 Mark]

2.8 For merging two sorted lists of sizes m and n into a sorted list of size m + n, we require comparisons of

(a) O(m)
(b) O(n)
(c) O(m + n)
(d) O(min(m, n))

[1999 : 1 Mark]

2.9 Which of the following statements is false?

I. As the number of entries in a hash table increases, the number of collisions increases
II. Binary search programs are efficient
III. The worst case complexity for quicksort is O(n²)
IV. Prim's algorithm using adjacency matrix is O(n²)

(a) I and II
(b) II and III
(c) III and IV
(d) I and IV

[1999 : 1 Mark]

2.10 The recurrence relation

$$T(1) = 2$$

$$T(n) = 3T\left(\frac{n}{4}\right) + n$$

has the solution T(n) equal to

(a) O(n)
(b) O(log n)
(c) O(n^{0.75} − n)
(d) None of these

[1999 : 2 Marks]

2.11 Quicksort is run on two inputs shown below to sort in ascending order

(i) [illegible]
(ii) [illegible]

C_1 and C_2 denote the number of comparisons made in the computation on (i) and (ii) respectively. Then,

(c) [illegible]
(d) [illegible]
(e) [illegible]
(f) [illegible] for a binary tree

[1996 : 2 Marks]

2.18 Let $T(n)$ be the function defined by $T(1) = 1$, $T(n) = 2T\left(\frac{n}{2}\right) + n$ for $n \geq 2$. Which of the following statements is true?

(a) $T(n) = O(\log n)$ (b) $T(n) = O(n)$
(c) $T(n) = O(n \log n)$ (d) None of these

[1999 : 2 Marks]

2.19 A sorting technique is called stable if it

(a) takes $O(n \log n)$ time
(b) keeps unique elements in the relative order they appear in the list to be sorted
(c) uses divide and conquer paradigm
(d) [illegible]

[1999 : 1 Mark]

2.20 If one uses straightforward recursion to merge sort a given list of quantities, the following [illegible] in descending order:

20, 47, 15, 8, 9, 4, 40, 30, 12, 17

Then the order of these numbers after the second pass of the algorithm is

(a) 8, 9, 15, 20, 47, 4, 12, 17, 30, 40
(b) 8, 15, 20, 47, 4, 9, 30, 40, 12, 17
(c) 15, 20, 47, 4, 8, 9, 12, 30, 40, 17
(d) 4, 8, 15, 20, 47, 9, 12, 17, 30, 40

[1999 : 1 Mark]

2.21 Let S be a sorted array of n integers. Let $t(n)$ denote the time taken for the most efficient algorithm to determine if there are two elements with sum less than 1000. Which of the following statements is true?

(a) $t(n)$ is $O(n)$
(b) $t(n)$ is $O(\log n)$
(c) $t(n) = \Omega(n)$
(d) [illegible] [2000 : 1 Mark]

2.22 Consider a randomized quicksort in which the pivot is chosen randomly. What is the worst case complexity of sorting n numbers using this randomized quicksort?

(a) $O(n)$ (b) $O(n \log n)$
(c) $O(n^2)$ (d) $O(n^3)$

[2001 : 1 Mark]

2.23 Consider the following implementation of binary search on a sorted array. It recursively divides the array into two halves and calls itself on one of the halves depending on the key. Let the length of the array be n. Let the number of calls to the binary search function be $T(n)$. Which of the following is a tight bound on $T(n)$?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 1 Mark]

2.24 Consider the following algorithm and a list of recurrences or recursions as given below. Each recurrence relation corresponds to a recursive algorithm and it is required to derive the time complexity of the algorithm.

List-I (Recursive Algorithm)

P. Binary search
Q. Merge sort
R. Quick sort
S. Tower of Hanoi

List-II (Recurrence Relations)

I. $T(n) = T(n-1) + T(1) = T(n) + n$
II. $T(n) = 2T(n-1) + 1$
III. $T(n) = 2T(n/2) + n$
IV. $T(n) = T(n/2) + 1$

Which of the following pairs of recurrence matches between the algorithms and their recurrence relations?

Codes:

	P	Q	R	S
(a)	I	III	IV	II
(b)	IV	III	I	II
(c)	III	I	IV	II
(d)	IV	II	I	III

[2001 : 2 Marks]

2.25 Suppose there are $\log n$ sorted lists of n elements each. The time complexity of merging the sorted lists into a single sorted list of $n \log n$ elements is

(a) $O(\log \log n)$ (b) $O(\log n)$
(c) $O(n \log n)$ (d) $O(n^2)$

[2006 : 2 Marks]

2.26 The median of elements can be found in $O(n)$ time. Which one of the following is correct about the complexity of quick sort, in which median is chosen as pivot?

(a) $O(n)$ (b) $O(n \log n)$
(c) $O(n^2)$ (d) $O(n^3)$

[2008 : 2 Marks]

2.23 Which of the following sorting algorithms has the lowest worst-case complexity?

(a) Merge sort (b) Insertion sort
(c) Quick sort (d) Selection sort

[2007 : 1 Mark]

2.24 In the following function, [illegible]

```
int [illegible](n, m)
{
    if (n == 0) return m;
    [illegible]
    return [illegible](n, m);
}
```

How many recursive calls are made by this function?

(a) $O(\log_2 n)$ (b) $O(n)$
(c) $O(\log_2(n!))$ (d) $O(n^2)$

[2007 : 2 Marks]

2.25 Consider the following algorithm. Assume that [illegible] is a procedure that returns an element of [illegible] at random. [illegible] at least one. [illegible] the element of [illegible] has [illegible] comparisons required in the worst case. Then

(a) $T(n) = 2T(n/2) + n$
(b) $T(n) = T(n-1) + T(1) + n$
(c) $T(n) = 2T(n-1) + n$
(d) $T(n) = 2T(n/2) + 1$

[2008 : 2 Marks]

2.26 [illegible] that 240 [illegible] 100 times per hour. What is the worst-case time complexity of the quicksort?

(a) [illegible] (b) [illegible]
(c) $O(n^2)$ (d) [illegible]

[2008 : 2 Marks]

2.27 [illegible] M_1, M_2, and M_3 [illegible] respectively can be [illegible] of total [illegible] multiplications. Two [illegible] when [illegible] the total number of scalar multiplications is [illegible] per [illegible]. When [illegible] the total number of scalar multiplications is [illegible] per [illegible]

[illegible] the minimum number of scalar multiplications [illegible]

(a) 248000 (b) [illegible]
(c) 24000 (d) 20000

[2011 : 2 Marks]

2.28 The worst case running time to search for a given element in a balanced binary search tree with n elements is

(a) $O(n \log n)$ (b) $O(n^2)$
(c) $O(n)$ (d) $O(\log n)$

[2013 : 1 Mark]

2.29 A [illegible] is defined as a graph in which [illegible] topological order is [illegible] the longest path in the graph. The [illegible] of this algorithm is

(a) $O(\log E)$ (b) $O(E \log E)$
(c) $O(E + \log V)$ (d) $O(E)$

[2012 : 2 Marks]

2.30 Which one of the following [illegible] and upper bound [illegible] expressions for the number of [illegible] in an object-oriented [illegible] of size n?

(a) $O(n)$ (b) $O(n \log n)$
(c) $O(n^2)$ (d) $O(n^2 \log n)$

[2013 : 1 Mark]

2.31 [illegible] quicksort [illegible] according to [illegible] the [illegible] to [illegible] by [illegible] [illegible] 2 [illegible]. What [illegible]?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2014 (Set-1) : 1 Mark]

2.32 The minimum number of comparisons required to find the minimum and the maximum of 100 numbers is

[2014 (Set-1) : 2 Marks]

2.33 You have an array of n elements. Suppose you implement quicksort by always choosing the middle element of the array as the pivot. Then the tightest upper bound on the worst-case performance is

(a) $O(n^2)$ (b) $O(n \log n)$
(c) $O(n \log n)$ (d) $O(n^2)$

[2014 (Set-3) : 1 Mark]

3.82 Which of the following is the recurrence equation for the worst case time complexity of the Quick sort algorithm for sorting n elements? In the recurrence equations given in the options, c is a constant.

(a) $T(n) = 2T(n/2) + cn$

(b) $T(n) = T(n-1) + T(1) + cn$

(c) $T(n) = 2T(n-1) + cn$

(d) $T(n) = T(n/2) + T(n/2) + cn$

[2013 (Set-1) : 1 Mark]

3.83 What is the worst case complexity for inserting a node in a doubly linked list?

(a) O(n) for both insertion and deletion

(b) O(n) for both insertion and deletion

(c) O(1) for insertion and O(n) for deletion

(d) O(1) for insertion and O(1) for deletion

[2013 (Set-1) : 1 Mark]

3.84 Suppose we are using the following function for the Quick sort in the C programming language

```
int partition (int a[], int n);
```

That returns the index of the pivot element after partitioning the array, such that all elements less than the pivot are to the left and all elements greater than the pivot are to the right. In addition, it moves the pivot element to its final position. The return value is the number of elements in the left part.

The following partially given function in the C programming language is intended to find the median of an array of integers by using the partition function.

```
int median(int a[], int n, int k)
{
    int left, pos;
    while (1)
    {
        pos = partition(a, n);
        if (pos == k)
            return a[pos];
        if (pos > k)
            n = pos;
        else
            {
                k = k - pos;
                a = a + pos;
                n = n - pos;
            }
    }
}
```

The missing argument lists are respectively

(a) (a, left, right), (a, left, pos - 1), (a, pos + 1, right)

(b) (a, left, right), (a, left, pos), (a, pos + 1, right)

(c) (a, left, right), (a, left, pos - 1), (a, left, pos - 1)

(d) (a, left, right), (a, left, pos), (a, left, pos)

[2015 (Set-3) : 2 Marks]

3.85 Assume that a merge sort algorithm in the worst case takes 30 seconds for an input of size 300. Which of the following most closely approximates the worst case running time for inputs of size 600?

(a) 60 s (b) 90 s

(c) 120 s (d) 180 s

[2015 (Set-3) : 2 Marks]

3.86 The worst case running time to search for an element in a balanced in-order binary search tree with n nodes is $O(n \log n)$. Which of the following is true?

(a) The tree root is always the median element

(b) The left and right subtrees of the root are always balanced

(c) The left and right subtrees of every node are always balanced

(d) The height of the tree is $O(\log n)$

[2016 (Set-1) : 1 Mark]

3.87 Assume that the quicksort implementation uses the rightmost element as the pivot. If the input sequence is already in ascending order, which of the following is true?

I. Quicksort runs in $\Theta(n^2)$ time

II. Quicksort runs in $\Theta(n \log n)$ time

III. Merge sort runs in $\Theta(n \log n)$ time

IV. Insertion sort runs in $\Theta(n^2)$ time

(a) I and II only (b) I and III only

(c) II and IV only (d) I and IV only

[2016 (Set-3) : 1 Mark]

Answers: Divide and Conquer

2.1	(c)	2.4	(d)	2.5	(a)	2.6	(b)	2.7	(a)	2.8	(c)	2.9	(d)	2.10	(a)	2.11	(c)
2.12	(b)	2.13	(b)	2.14	(b)	2.15	(a)	2.16	(c)	2.17	(a)	2.18	(b)	2.19	(a)	2.20	(b)
2.21	(a)	2.22	(a)	2.23	(b)	2.24	(a)	2.25	(c)	2.26	(c)	2.27	(b)	2.28	(b)	2.29	(c)
2.30	(a)	2.31	(b)	2.32	(b)	2.33	(a)	2.34	(a)	2.35	(c)	2.36	(d)	2.37	(d)		

Explanations: Divide and Conquer

2.1 (c)

$c1 = 2$, $c2 = 1$

Sorting {1, 2, 3, 4} takes less time compared to sorting {4, 3, 2, 1} since. First half has 2 elements to sort, second half has 2 elements to sort. So, worst case of quick sort.

2.3 (d)

By using master theorem we get following series

$$\Rightarrow T(n) = \frac{n}{2^0} + \frac{n}{2^1} + \frac{n}{2^2} + \frac{n}{2^3} + \cdots + \frac{n}{2^{\log_2 n}} + 1$$

$\Rightarrow$ So put $2^{\log_2 n} = n$ in above equation

$$T(n) = \frac{n}{2^0} + \frac{n}{2^1} + \frac{n}{2^2} + \frac{n}{2^3} + \cdots + \frac{n}{2^{\log_2 n}} + 1$$

$= O(n)$

2.4 (a)

Quick sort is a non-comparative sorting algorithm that performs better than any comparison based sorting algorithm which has a lower bound of $n \log n$ and so it will take $O(n)$ time.

2.5 (a)

Binary search is a half of the array

So, $T(n) = T\left(\frac{n}{2}\right) + K$

2.6 (b)

Connected components of a graph can be computed in linear time by using breadth first search or depth first search.

So, option (b) is false.

2.7 (a)

A sorting algorithm is a comparison based sorting algorithm and divides the input algorithm.

2.8 (c)

The number of comparisons required in the worst case is $O(n \log n)$.

2.9 (d)

I. In the binary search of the number of elements in a sorted array is n, the number of elements in the array.

II. In merge sort, the number of elements in the array is n.

III. Quick sort has a comparison sort and in the worst case makes $O(n^2)$ comparisons.

IV. Binary search can be used with linked list, however, it is inefficient.

So option (d) is correct.

2.10 (a)

$T(n) = 2T(n/2) + n$

So, $T(n) = O(n \log n)$

2.11 (c)

Both of the statements are

Quicksort: Worst case $O(n^2)$ comparisons are required.

2.12 (b)

Master Theorem $n^{\log_2 2} = n\sqrt{n}$

2.13 (b)

A sorting algorithm is said to be stable if it keeps elements with equal keys in the same order as in the input array. Merge sort is the least

[illegible]

2.14 (b)

Given: 20, 4, 10, 5, 8, 1, 30, 3, 12, 17

[illegible]

[illegible]

The order of elements after second pass of the algorithm is 5, 10, 20, 17, 1, 4, 30, 8, 3, 12

2.15 (d)

Take starting 3 elements only, if some of these two are less than 1000 then return [illegible]

2.16 (c)

Karp's method [illegible] worst case time complexity = $O(n^2)$

2.17 (a)

The [illegible] array using binary search to denote the position the worst case [illegible] $O(n^2)$. [illegible] elements [illegible] binary search [illegible] times the number [illegible] Thus [illegible] time in terms of the number of comparisons [illegible] [illegible] the worst case adding [illegible] binary search [illegible] requires $O(n)$ [illegible] time

2.18 (b)

Binary search: $T(n) = T(n/2) + 1$

Merge sort: $T(n) = 2T(n/2) + cn$

Quick sort: $T(n) = T(n-1) + c$ [illegible]

Tower of Hanoi: $T(n) = 2T(n-1) + 1$

2.19 (a)

When we [illegible] the minimum data structure [illegible] which each [illegible] $\lceil n/\log n \rceil$ [illegible] Data structure of [illegible] Fibonacci heap [illegible] it will take $\lceil n/\log n \rceil$ [illegible] time complexity $O(n \log \log n)$.

2.20 (b)

In the [illegible] elements can be found in $O(n)$ time and we select the median as pivot. Then [illegible] array $A[i]$ [illegible] $A[i]$ [illegible] so that [illegible] all the [illegible] less than [illegible] $A[i]$ and all those with [illegible] greater than $A[i]$ [illegible] $A[i]$. With [illegible] only in $A[i]$, $A[i]$ [illegible] $A[i+1]$ [illegible] $A[n]$.

The recurrence relation becomes in [illegible]

$$T(n) = \frac{1}{n}\sum_{i=1}^{n} T(i-1) + T(n-i)$$

[illegible]

Given we find median $O(n)$ time, [illegible]

2.21 (a)

Sorting Algorithm	Worst case complexity (which is superior)
Merge sort	$O(n \log n)$
Bubble sort	$O(n^2)$
Quick sort	$O(n^2)$
Selection sort	$O(n^2)$

2.22 (a)

[illegible] $T(m, n)$ for the total [illegible]

Let $T(m, 0) = 0$, $T(m, n) = T(n, m)$ [illegible] average

$$T_n = \frac{1}{n}\sum_{i=1}^{n} T(i, n)$$

$T_1 = 1 + \frac{1}{n}(T_1 + \ldots + T_{n-1})$

$T_2 = F_n$

$S_1 = 1 + \frac{1}{n}(S_1 + \ldots + S_{n-1})$

$S_1 = 1 + \frac{1}{n}(n S_1 - S_n + S_n)$

$= 1 + \frac{1}{n+1}(n S_{n-1}) + S_n$

$= 1 + \frac{1}{n+1} + S_n + \frac{1}{n+1}$

So $S_n = O(\log_2 n + 1)$

$= O(\log_2 n)$

2.50 (b)

[illegible]

[Note: $n = \frac{n}{5} + \frac{2n}{5} + \frac{3n}{5}$]

2.51 (b)

The relation [illegible]

The given recurrence [illegible] will double [illegible] which [illegible] always [illegible] [illegible]

2.52 (c)

We get minimum = [illegible] comparisons using [illegible]

[illegible] number of multiplication

= [illegible]

= 1000

2.53 (c)

We know that in sorted [illegible] binary search tree [illegible] is [illegible]

$\log_2(n+1) = \Theta(\log n) = \log 2^n$

[illegible] $\log_2^n = \log_2 n + n = \Theta(n)$

[illegible]

2.54 (b)

[illegible]

2.55 (b)

In the worst case [illegible]

[illegible]

2.56 (c)

Input 1: [illegible]

Input 2: [illegible]

So, [illegible] elements [illegible]

[illegible] $\frac{3n}{2}$ comparisons [illegible]

$= \Theta(n)$

[illegible] Thus [illegible] Choosing [illegible] will give [illegible] comparisons [illegible]

[illegible]

2.57 Sol.

$n = 1000$

[illegible] minimum [illegible]

[illegible]

[illegible]

2.58 (b)

[illegible] always can not make the [illegible] equal parts

[illegible] complexity [illegible]

2.59 (b)

[illegible] either [illegible] elements are always [illegible] proportion

The [illegible] would be

$T(n) = T(1) + T(n-1) + O(n) = O(n^2)$

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

2.30 (a):

In worst case the BST may be skewed BST. The worst case occurs when we insert the elements in increasing order or decreasing order.

Extreme case: [illegible] elements in order 1, 2, 3, 4, 5, 6, ...

The BST would be

[illegible] we may have to traverse the entire tree which takes $O(n)$ time.

Hence for both methods [illegible] worst case is $O(n)$.

2.31 (d):

Let_partition pivot = partition(A, p, r);

Make k^{th} smallest to same as the element at index k. This [illegible]

if (k == l_pivot+1) — k^{th} smallest found.

return A[k]; // return pivot element

if (k < l_pivot+1) // k^{th} smallest lies before pivot

[illegible] k^{th} smallest lies after pivot (right side)

[illegible]

2.32 (b):

[illegible]

$$c_1 = \frac{[illegible]}{[illegible]} \quad ...(1)$$

Now [illegible]

$$c \log n = \frac{[illegible]}{[illegible]}$$

$$c \log n = \frac{[illegible]}{5} \times [illegible]$$

$$c \log n = 4002$$

$$c \log n = 512 \times [illegible] = 512$$

2.33 (b):

The worst case time complexity of a given array

Insertion sort = $O(n^2)$

Merge sort = $O(n \log n)$

Quick sort = $O(n^2)$

2.34 (d):

Quick sort takes $O(n^2)$ time to sort the [illegible] in descending order by using first/last as pivot element.

Insertion sort take $O(n^2)$ time [illegible] in descending order [illegible] time and to compare.

Merge sort [illegible] take $O(n^2)$ time.

So option (d) is most appropriate.

■■■■

3 Greedy Method

3.1 The minimum cost spanning tree of the given graph is

[1991 : 2 Marks]

3.2 Kruskal's algorithm for finding a minimum spanning tree of a weighted graph G with n vertices and e edges has time complexity of

(a) $O(n^2)$ (b) $O(ne)$

(c) $O(e^2)$ (d) $O(e \log e)$

[1991 : 2 Marks]

3.3 Complexity of Kruskal's algorithm for finding minimum spanning tree of a connected graph containing n vertices and m edges if the edges are sorted is ____

[2002 : 2 Marks]

3.4 Let G be an undirected connected graph with distinct edge weights. Let e_{max} be the edge with maximum weight and e_{min} the edge with minimum weight. Which of the following statements is false?

(a) Every minimum spanning tree of G must contain e_{min}

(b) If e_{max} is in a minimum spanning tree, then its removal must disconnect G

(c) No minimum spanning tree contains e_{max}

(d) G has a unique minimum spanning tree

[2002 : 2 Marks]

3.5 What is the weight of a minimum spanning tree of the following graph?

(a) 29 (b) 31

(c) 36 (d) 41

[2003 : 2 Marks]

3.6 The following table gives the data for a set of activities to be scheduled on a single machine. Each activity has a release time and a processing time. The machine can be idle between the release time of one activity and the start time of the next activity. The objective is to minimize the total idle time. What is the minimum idle time achievable?

(a) 3	(b) 4
(c) 6	(d) 9

[2003 : 2 Marks]

3.7 Suppose we run Dijkstra's single source shortest path algorithm on the following edge weighted directed graph with vertex P as the source.

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

Q.8 Consider [illegible] graph below

[illegible] minimum spanning tree?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2004 : 2 Marks]

Q.9 [illegible] following table [illegible] algorithm [illegible] algorithm [illegible] algorithm [illegible] with its [illegible] complexity.

List-I

1. [illegible]
2. [illegible]
3. Floyd-Warshall algorithm
4. [illegible]

List-II

A. [illegible]

B. [illegible]

C. [illegible]

D. [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2004 : 1 Mark]

Q.10 [illegible] adjacency matrix [illegible]

- [illegible]
2. [illegible]

Which [illegible] PRIM'S

(a) Graph G has no [illegible]

(b) Graph G has unique MST [illegible]

(c) Graph G has multiple [illegible] different [illegible]

(d) Graph G has multiple spanning trees of different [illegible]

[2005 : 1 Mark]

Q.11 Let G be a weighted connected graph and let T be a minimum spanning tree of G. Let P be a path connecting the vertices u and v in T. Let [illegible] be an edge in G not in T. Let [illegible] be the edge in P [illegible]

(a) There [illegible] of maximum weight

(b) There [illegible] of minimum weight

(c) Every [illegible] cycle.

(d) All edges in G have the same weight

[2006 : 2 Marks]

Q.12 The G [illegible] graph with positive edge [illegible] path algorithm [illegible] binary heap [illegible]

(a) [illegible]

(b) [illegible]

[2006 : 2 Marks]

Common Data for Q.13 & Q.14

We are [illegible] tasks $T_1, T_2, \ldots, T_n$. [illegible] time [illegible]

Task	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Q.13 [illegible] tasks completed in [illegible] profit?

(a) All tasks are completed

(b) T_1 [illegible] T_7 are left out

(c) T_1 and T_7 are left out

(d) T_7 and T_8 are left out

[2006 : 2 Marks]

8.14 What is the maximum profit earned?

(a) 147 (b) [illegible]

(c) 167 (d) 178

[2006 : 2 Marks]

8.15 Consider the polynomial [illegible] The [illegible] polynomial [illegible] is

(a) [illegible] (b) [illegible]

(c) 3 (d) 8

[2006 : 1 Mark]

8.16 Consider a connected bipartite graph [illegible] The weight of a minimum spanning tree of G is

(a) $n - 1$ (b) $2n - 2$

(c) $\frac{n}{2}$ (d) [illegible]

[2006 : 1 Mark]

8.17 To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, the data structure to be used is

(a) Queue (b) Stack

(c) Heap (d) [illegible]

[2006 : 1 Mark]

8.18 Consider the following graph

Which edges will be in the edges of minimum spanning tree using Kruskal's algorithm?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

8.19 The binary search tree is used to [illegible] The [illegible] is used to represent the [illegible] corresponding to the [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

8.20 In an unweighted, undirected connected graph, the shortest path from a node S to every other node is computed most efficiently by

(a) Dijkstra's shortest path algorithm

(b) Warshall's algorithm

(c) performing DFS starting from S

(d) performing BFS starting from S

[2007 : 2 Marks]

8.21 [illegible] maximum weight among all edge weights in a minimum spanning tree of a connected graph. Which of the following is FALSE?

(a) There is a minimum spanning tree containing [illegible]

(b) [illegible] in a minimum spanning tree T, [illegible] edges have the same weight.

(c) Every minimum spanning tree has exactly [illegible] edges with weight [illegible]

(d) [illegible] minimum spanning tree.

[2007 : 2 Marks]

Common Data for Q. 8.22 & Q. 8.23

[illegible] $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{8}$ respectively.

8.22 What is the [illegible] Huffman code for the alphabets in [illegible]

(a) [illegible]

(b) [illegible]

(c) 1, 0.5, 0.25, 0.25

(d) 2.00, 1.00, 0.50, 0.50

[2007 : 2 Marks]

3.23 What is the average length of the shortest path between any pair of vertices?

(a) 8 (b) 2.877

(c) 2.25 (d) 1.561

[2007 : 2 Marks]

3.24 Consider a job set $\{J_1, J_2, \ldots, J_n\}$ such that each job takes one unit of time to finish and a non-negative integer weight w_i. The weighted mean completion time of the jobs is $\sum_{i=1}^{n} w_i C_i / \sum_{i=1}^{n} w_i$ where C_i is the completion time of job J_i. Assuming that there is only one processor available, in what order must the jobs be executed in order to minimize the weighted mean completion time of the jobs?

(a) Non-increasing order of w_i

(b) Non-increasing order of w_i/t_i

(c) Non-increasing order of $w_i t_i$

(d) Non-decreasing order of w_i

[2007 : 2 Marks]

3.25 For the connected weighted graph given below, which of the following sequences of edges represents a minimal spanning tree (MST)? (Edges are represented as ordered pairs of vertices. Edge weights are not shown.)

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2008 : 2 Marks]

3.26 G is a graph on n vertices and $2n - 2$ edges. The edges of G are coloured red and blue such that G has $n - 1$ red edges and $n - 1$ blue edges. Let a red-blue spanning tree be a spanning tree in G with exactly k red edges. The minimum k for which the induced subgraph of G is a red-blue spanning tree is

(a) 1 (b) 2

(c) 3 (d) 4

[2008 : 2 Marks]

3.27 Consider the following graph:

Which one of the following properties of G is false?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2008 : 2 Marks]

3.28 Consider the following graph:

(a) only [illegible]

(b) only [illegible]

(c) [illegible] a, b, c, d

(d) [illegible]

[2009 : 2 Marks]

3.29 Which of the following properties does a shortest path spanning tree have?

(a) Always finds a minimum cost spanning tree if one exists

(b) Finds a shortest path spanning tree if one exists

(c) Finds a minimum cost spanning tree if one exists

(d) Finds a shortest path spanning tree if one exists

[2009 : 1 Mark]

3.30 Consider the following graph:

Which one of the following properties does the edge added to the minimum spanning tree using Kruskal's algorithm have?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2009 : 2 Marks]

3.31 The number of sequences $a_1, a_2, \ldots, a_{2n-1}$ of -1 and $+1$ such that $a_1 + a_2 + \ldots + a_{2n-1} = 1$ and $a_1 + a_2 + \ldots + a_k \geq 0$ for $k = 1, 2, \ldots, 2n-1$ is equal to the number of sequences of n $+1$s and $n-1$ -1s such that

By deleting some elements from the sequence, keeping the order of the remaining elements, we obtain a subsequence. Let X denote the maximum possible weight of a subsequence of $a_1, a_2, \ldots, a_n$, and Y the maximum possible weight of a subsequence of $a_1, a_2, \ldots, a_{n-1}$. Then X is equal to

(a) $\max(Y, a_n + Y)$ (b) $\max(Y, a_n + Y/2)$

(c) $\max(Y, a_n + 2Y)$ (d) $a_n + Y/2$

[2010 : 2 Marks]

Common Data for Questions 8.81 and 8.82

Consider a complete undirected graph with vertex set $\{0, 1, 2, 3, 4\}$. Let W_{ij} be the weight of the edge (i, j). Let W be the matrix shown below. For each pair of vertices i, j, $W_{ij} = W_{ji}$ and $W_{ii} = 0$.

$$W = \begin{vmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 6 \\ 4 & 9 & 3 & 6 & 0 \end{vmatrix}$$

8.81 What is the minimum weight of a spanning tree of this graph? [illegible] that vertex 0 is a leaf in the MST?

(a) 7 (b) 8

(c) 9 (d) 10

[2010 : 2 Marks]

8.82 What is the [illegible] possible weight of a path from vertex 1 to vertex 2 in this graph such that no vertex is visited more than once?

(a) 7 (b) 8

(c) 9 (d) 10

[2010 : 2 Marks]

Linked Data Questions 8.83 and 8.84

Consider a connected graph $G(V, E)$ with [illegible] vertices numbered $v_1, v_2, \ldots, v_n$. Two vertices v_i and v_j are connected if $|i - j| \le 2$. Each edge (v_i, v_j) has a weight $|i - j|$. A sample graph with $n = 5$ is shown below.

8.83 What will be the cost of the Minimum Spanning Tree (MST) of such a graph with n vertices?

(a) $\frac{1}{2}(3n^2 - 5n)$ (b) [illegible]

(c) [illegible] (d) [illegible]

[2011 : 2 Marks]

8.84 The length of the path from v_1 to v_n in the MST of the previous question with $n = 10$ is

(a) 11 (b) 28

(c) 34 (d) 45

[2011 : 2 Marks]

8.85 Let G be a weighted graph with edge weights greater than one and G' be the graph constructed by squaring the weights of edges in G. Let T and T' be the minimum spanning trees of G and G' respectively, with total weights w and w'. Which of the following statements is TRUE?

(a) $T' = T$ and $w' = w^2$ (b) $T' = T$ and $w' < w^2$

(c) $T' \neq T$ and $w' = w^2$ (d) $T' \neq T$ and $w' < w^2$

[2012 : 2 Marks]

8.86 Consider the weighted graph shown in the figure below. There are two shortest paths between vertices S and T. Each edge will be assigned a unit weight by Dijkstra's shortest path algorithm. Assume that, in case of a tie, the shortest path is chosen to be the path which has the fewer number of edges. The shortest path from S to T is recovered.

(a) SUT (b) SVUT

(c) SxUT (d) SxUVT

[2012 : 2 Marks]

8.87 What is the [illegible] following is the correct upper bound that represents the number of edges required to [illegible] in the worst case?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2013 : 1 Mark]

8.88 Consider that we have a DFS traversal for a connected graph $G(V, E)$ in which [illegible] connected.

[illegible]

(a) the shortest path from [illegible] to every other vertex in the graph

(b) the shortest path from [illegible] to every other vertex in the graph

(c) the shortest path from [illegible] to every other vertex in the graph

(d) the longest path from [illegible] to every other vertex in the graph

[2014 (Set-2) : 1 Mark]

5.33 Suppose [illegible] are stored in an array of integers [illegible] 25, 24, 30, 35, 26. [illegible] are to be merged into a single sequence by successively merging pairs of numbers. The total number of comparisons that will be needed in the worst case by the optimal algorithm [illegible] is _____.

[2014 (Set-3) : 2 Marks]

5.34 The graph shown below has edge weights as indicated. Assume that the edges are processed in the order specified by the weights. The minimum number of edges of the edge-weighted graph that will be processed in Kruskal's algorithm to find a minimum spanning tree is _____.

[2015 (Set-1) : 2 Marks]

5.35 Consider the weighted undirected graph with positive edge weights and distinct edge weights. Let [illegible] be a minimum spanning tree of [illegible]. Which of the following statements is/are true?

I. Minimum spanning tree of [illegible] is also a minimum spanning tree of [illegible] with the same edge weights.

II. Shortest path between any pair of vertices in [illegible] is also a shortest path between the same pair of vertices in [illegible].

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

[2015 (Set-1) : 1 Mark]

5.36 Let [illegible] be a connected and weighted graph with vertices [illegible] and [illegible]. The maximum possible weight that a minimum spanning tree of [illegible] can have is _____.

[2016 (Set-1) : 2 Marks]

5.37 Consider the undirected complete graph [illegible] with vertices [illegible] and edge weight [illegible]. Which of the following statements is/are true? [illegible] is a minimum spanning tree of [illegible].

I. [illegible]

II. [illegible] is the minimum spanning tree of [illegible] with [illegible].

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

[2016 (Set-1) : 2 Marks]

■■■■

Answers Greedy Method

5.1	(d)	5.2	(c)	5.3	(b)	5.4	(b)	5.5	(c)	5.6	(d)	5.7	(a)	5.8	(c)	5.9	(a)	5.10	(c)
5.11	(a)	5.12	(a)	5.13	(a)	5.14	(a)	5.15	(b)	5.16	(c)	5.17	(c)	5.18	(d)	5.19	(a)	5.20	(b)
5.21	(b)	5.22	(a)	5.23	(b)	5.24	(d)	5.25	(c)	5.26	(d)	5.27	(d)	5.28	(b)	5.29	(d)		
5.30	(d)	5.31	(d)	5.32	(b)	5.33	(b)	5.34	(c)	5.35	(d)	5.36	(d)	5.37	(b)	5.38	(b)		
5.39	(a)	5.40	(a)																

Explanations: Greedy Method

8.1 (a)

Sol for with [illegible] = 32.

8.2 (d)

Kruskal's algorithm time complexity

$= O(E \log E)$

$= O(E \log V)$

8.3 Sol.

Optimal subproblems are [illegible] solution [illegible] solved.

8.4 (c)

All edges are distinct weights.

Removal of any may [illegible] other edges [illegible] weight.

8.5 (b)

Number of vertices in the graph [illegible] adding [illegible] edges with [illegible] vertices [illegible] to the minimum spanning tree of the.

Total weight $= \sum_{i=1}^{n-1} w_i$

$= 1 + 2 + 2 + 2 + 3 + 3 + 4 + 5 + 6 + 9 = 31$

8.6 (a)

The set of vertices are A, B, C, D, E, F, G and H. The [illegible] edges are [illegible]

Three phase [illegible] algorithm [illegible] vertices. Connect the edges [illegible] between [illegible] in which two activities [illegible] and [illegible] minimum [illegible] time [illegible].

8.7 (b)

In [illegible] the second [illegible] following sequence.

Total [illegible] = 9

Node sequence: P, Q, R, S, T

8.8 (d)

[illegible] connected components with the CLOSE [illegible] algorithm.

For each connected component every [illegible] edge added MST where [illegible] vertices [illegible] randomly. [illegible] edge [illegible] minimum weight [illegible] is chosen. Therefore (a) [illegible] be chosen BETWEEN (a, b). Therefore (a) is false.

8.9 (a)

Bellman-Ford algorithm [illegible]

Dijkstra's algorithm : $O(m \log n)$

Floyd-Warshall algorithm : $O(n^3)$

Topological sorting : $O(n + m)$

8.10 (c)

The adjacency matrix representation is as follows

$$\begin{array}{c|cccc} & 1 & 2 & \cdots & n \\ \hline 1 & 0 & 1 & 1 & \cdots & 1 \\ 2 & 1 & 0 & & \cdots & 1 \\ 3 & 1 & 1 & 0 & \cdots & 1 \\ \vdots & & & & & \\ n & 1 & 1 & 1 & & 0 \end{array}$$

In undirected graph G, the maximal connected component is one such that there is exactly 1 edge from every vertex to every other vertex. Such a connected component of weight = n − 1 represents a complete graph. This complete graph applies Kelly's theorem. The total number of minimum spanning trees is n^{n-2}. Given the number of vertices is n, so we have n vertices. Thus we will must use the algorithm after solution. Hence the algorithm each connected component takes $O(n^2)$ time.

8.11 (a)

Option (a) is always true.

Option (b) is false when it is not part of a cycle.

Option (c) is not true when it is part of a cycle and a lesser weight edge exists in that cycle.

Option (d) is need not be true when it is not part of a cycle.

Option (a) is always true as only the minimum edge in a cut set will be part of any minimum spanning tree.

8.12 (a)

Dijkstra's algorithm is a single source shortest path

(i) Using binary heap takes
$O(|E| + |V| \log |V|)$

(ii) Using Fibonacci heap takes
$O(|E| + |V| \log |V|)$

8.13 (b)

The given problem is job scheduling problem with deadline $T_1, T_2, \ldots, T_n$.

Set initially empty then according to deadline and increasing $T_1, T_4, T_3, T_5, T_6, T_2$.

So, total 4 jobs can be scheduled.

8.14 (d)

Total profit is given by algorithm
= 15 + 20 + 18 + 16 + 28 + 15 + 20 = 132

8.15 (b)

$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Apply Horner's rule to evaluate $p(x)$

$$\begin{aligned} p(x) &= a_0 + x(a_1 + x(a_2 + a_3 x)) \\ &= a_0 + x \\ &= t = a_3 \\ &= t = t \times x \\ &= t = t + a_2 \\ &= t = t \times x \\ &= t = t + a_1 \end{aligned}$$

So, 3 minimum, when multiplication needed are required to compute.

8.16 (b)

Given $G = (V, E)$, $V = \{v_1, v_2, \ldots, v_n\}$

Weight of the edge $= w(v_i, v_j) = |i - j|$

$E = \{(v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n)\}$

So, the sum of minimum spanning tree weight

Given, total graph = Overhead, so the weight of minimum spanning tree of G

$$\begin{aligned} &= \sum_{i=1}^{n-1} |v_i - v_{i+1}| \\ &= 2\sum_{i=1}^{n-1} |v_i - v_{i+1}| - \sum_{i=1}^{n-1} 1 \\ &= 2(n-1) \\ &= 2n - 2 - n + 1 \\ &= 2n - 1 = 2n - 2 \end{aligned}$$

8.17 (c)

Heap and priority queue are very best data structures for Dijkstra

- Insert element into heap with an associated priority
- Remove element from the heap with the highest priority and return it
- Remove the element with higher priority without removing it.

A simple array implementation gives a queue data structure in which elements are stored. Dijkstra's shortest path algorithm is a greedy algorithm.

3.15 (b) :

[illegible] Kruskal's algorithm we will add the edge of minimum weight which does not form a cycle. Consider the edges in order

[illegible]

In this case, we add the edges of weight 6, 7, 8 and 9 [illegible] the minimum spanning tree.

3.19 (c) :

The number of [illegible] edges to be added is equal to the number of connected components.

The [illegible] connected components are shown in the figure.

3.20 (d) :

In an undirected graph [illegible] Dijkstra's algorithm only [illegible] the graph contains [illegible] edges. Dijkstra's algorithm is not good for negative weight graph. Warshall's algorithm for all pair shortest path is best for dense graph.

3.21 (b) :

If we have negative weight cycle in all the edges in an undirected graph [illegible] edge in the graph. Thus, [illegible] minimum spanning tree [illegible] minimum spanning tree is the [illegible] minimum spanning tree.

3.24 (d) :

The probabilities are $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}$

Using Huffman coding we will get the following codes respectively

3.25 (d)

The average length of the code is

$$= 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + 4 \times \frac{1}{16} + 5 \times \frac{1}{16}$$

$$= \frac{1}{16} = 1.9375$$

3.26 (d) :

The [illegible] must follow SJF. Since we have [illegible] we can only maximize the [illegible] decreasing the [illegible] processes that follow.

3.27 (b) :

[illegible] we can use the Prim's algorithm [illegible] edges [illegible] connected components [illegible] the [illegible]

Second thing [illegible] process [illegible] the [illegible] edges [illegible] the [illegible] check the vertices.

[illegible] connected [illegible] edges. Hence [illegible] is eliminated.

(b) [illegible] will straight away go to the point [illegible] moving along [illegible] we can provide edges [illegible] Hence [illegible] is adjacent [illegible] them. So (a) [illegible]. You can do this by marking the graph as well.

(c) This [illegible] equal [illegible]

(d) [illegible] find the graph [illegible] and [illegible] greedy approach. Because [illegible] [illegible] Hence [illegible].

8.46 (d).

Given graph G contains [illegible] $2n - 1$ edges. [illegible] $\sum_{i=1}^{n} d_i$ [illegible] $4 = 2(n-2)$. So here [illegible] possible between [illegible] pair of vertices.

8.47 (d)

[illegible] single source shortest path [illegible] work for graph with negative weight edges, but [illegible] graph. For the [illegible]

[illegible]

After 1st pass distance array [illegible]

[illegible]

After 2nd pass [illegible]

[illegible]

After 3rd pass distance array [illegible]

After 4th pass distance [illegible]

[illegible]

8.48 (b)

[illegible] Bellman-Ford shortest path algorithm always finds any negative weighted cycle which is reachable from the source. If the negative weighted cycle is not reachable from, Bellman-Ford algorithm [illegible].

8.49 (d).

In Kruskal's algorithm edges are added in non-decreasing order, starting from [illegible] weight, as long as they do not form a cycle with already added edges.

[illegible] weight 2 [illegible] first. This step is [illegible] edge weight. Edge [illegible] added [illegible] edge weight 3, [illegible] is added.

[illegible]

[illegible] Shortest path [illegible] would be allowed by Kruskal's algorithm.

8.50 (d)

	A	B	C	D	E
A	0	1	8	[illegible]	[illegible]
B	1	0	12	1	4
C	[illegible]	12	0	[illegible]	5
D	[illegible]	[illegible]	[illegible]	0	8
E	4	6	5	2	0

[illegible]

We have to make minimum possible spanning tree having [illegible]

So Minimum possible weight

8.58 (b)

So weight

8.59 (d)

Total number of comparisons in worst case

$$= \sum_{i=1}^{n-1} i = \sum_{i=1}^{n-1} i = n^2 - n$$

8.60 (a)

8.61 (d)

Step-I:

Step-II:

... [illegible]

... [illegible] is correct

3.36 (d)

- The given [illegible] path [illegible] = 59.3%.

3.37 (b)

[illegible] and [illegible] from where we start. Then we find the maximum element in the array and swap it with marked index. Now we mark the next index and repeat and the minimum in the array and swap it with marked index.

In every iteration there is a swapping to bring the worst case to [illegible] swap.

Option (b) is correct.

3.38 (d)

BFS traversal for [illegible] path from [illegible] is [illegible]

3.39 Sol:

Given array: 26, 34, 30, 46, 50

Step-I:

Step-II:

Step-III:

Step-IV:

- Total 80 [illegible] 358 comparisons

3.40 Sol:

MST = 38 and [illegible] edges [illegible] only [illegible].

[illegible]

J.12. [illegible]

Statement 1: Since every edge weight is positive, [illegible] edge [illegible] minimum spanning tree of G does not change.

Statement 2: Taking an example:

[illegible] have positive [illegible]. But here [illegible] since [illegible] is [illegible] can be changed.

So statement is wrong.

8.12. [illegible]

[illegible] complete graph [illegible] 4 [illegible] edge weight [illegible]

[illegible] edge weight [illegible]. From the [illegible] we have to choose two minimum weight minimum [illegible] So MST will be

So max weight will be [illegible]

8.48 [illegible]

Since 'BD' is high weight in cycle BCD, but here MST is not [illegible] edge 'BD' [illegible]

In second statement, if in any graph 'G' is the heaviest edge in the cycle, every MST [illegible]

■■■■

4 Dynamic Programming

4.1 A [illegible] is given as the integer sequence under the following integer set, [illegible] 5, 6, 9, 10, 11, 12. The number of ways in the [illegible] subsets and equal subsets of the set respectively are

(a) 14, 7 (b) [illegible]

(c) 14, 6 (d) [illegible]

[1996 : 2 Marks]

4.2 Which one of the following problems can be solved using dynamic programming in polynomial time on a directed acyclic graph?

(a) Dynamic programming

(b) Backtracking

(c) Greedy

(d) [illegible]

[1998 : 1 Mark]

4.3 A Fibonacci number is defined as the sum of the previous two Fibonacci numbers. What is the maximum number of recursive calls required for computing the 5th Fibonacci number?

(a) 0 (b) 4

(c) 5 (d) 6

[2008 : 2 Marks]

4.4 The subset sum problem is defined as follows. Given a set of positive integers, S and a value integer W, determine whether there is a subset of S whose elements sum to W.

An algorithm Q solves the problem in O(nW) time. Which of the following statements is false?

(a) Q solves the subset sum problem in polynomial time when the input is encoded in unary

(b) Q solves the subset sum problem in polynomial time when the input is encoded in binary

(c) The subset sum problem belongs to the class NP

(d) The subset sum problem is in P

[2005 : 2 Marks]

Linked Answer Questions 4.5 and 4.6

The subset sum problem is defined as follows. Given a set of positive integers, S and a value integer W, is there a subset of S whose elements sum to W? A dynamic programming solution for the subset sum problem uses a boolean array X[ ], where X[i] is true if and only if there is a subset of the first i elements of S that sums to i. Initially, X[0] is true and X[i] = false for all other values of i. The array X is updated by the following pseudocode.

4.5 Which of the following is a valid for-loop to update X?

(a) X[j] = X[j] or X[j-S[i]]

(b) X[j] = X[j] or X[j-S[i]] or X[j-1]

(c) X[j] = X[j-1] or X[j-S[i]]

(d) X[j] = X[j-S[i]] or X[j-1]

[2008 : 2 Marks]

4.6 Which entry in the array X, if TRUE, implies that there is a subset whose elements sum to W?

(a) X[0, W] (b) X[1, 0]

(c) X[n, W] (d) X[n-1, W]

[2008 : 2 Marks]

Common Data for Questions 4.7 and 4.8

A subsequence of a given sequence is obtained by deleting zero or more elements from the given sequence. For example, if the given sequence is <B, C, D, A, E>, then <B, D, E> is a subsequence. Let X[m] and Y[n] be two sequences and L(m, n) be the length of the longest common subsequence of X[m] and Y[n]. The function L satisfies the following recurrence relation.

L(m, n) = 0 if m = 0 or n = 0

= L(m-1, n-1) + 1 if m, n > 0 and X[m-1] = Y[n-1]

= max(L(m, n-1), L(m-1, n)) if m, n > 0 and X[m-1] ≠ Y[n-1]

4.7 Which of the following is a valid recurrence for the length of the longest common subsequence of X[m] and Y[n]?

(a) expr1 = L(m-1, n) + 1

(b) expr2 = L(m, n-1)

(c) expr3 = max(expr1, expr2)

(d) expr4 = L(m-1, n) + L(m, n-1)

[2009 : 2 Marks]

4.8 [illegible] dynamic programming based on the recurrence relation defined in [illegible] the longest common subsequence of array L [illegible] where M = [illegible] and N = [illegible] and L [illegible].

Which one of the following statements would be TRUE regarding the [illegible] minimum [illegible] of L [illegible] according to the definition of L[i, j]?

(a) All elements of L should be initialized to 0 for the values of L[i, j] to be properly computed.

(b) The values of L[i, j] may be computed in a row major order or column major order of L.

(c) The value of L[i, j] may be computed in a column major order only if L [illegible] M, N.

(d) L[M, N] [illegible] the length of the longest common subsequence [illegible]

[2009 : 2 Marks]

4.9 [illegible] the length of the longest monotonically increasing subsequence of the sequence [illegible]

Let L[i] be the length of the longest monotonically increasing subsequence ending at index i.

Let L[0] = 1.

Then L[i] is characterized by [illegible]

$$L[i] = \begin{cases} 1 + \max\{L[j] : 0 \le j < i \text{ and } A[j] < A[i]\} \\ \text{Otherwise} \end{cases}$$

Which the statements of the correct monotonically increasing sequence [illegible]

Which of the following statements is TRUE?

(a) The algorithm [illegible] dynamic programming paradigm.

(b) The algorithm [illegible] greedy paradigm.

(c) The algorithm [illegible] polynomial time [illegible] paradigm.

(d) The algorithm [illegible] divide and conquer paradigm.

[2011 : 1 Mark]

4.10 What is the time complexity of Bellman Ford single source shortest path algorithm on a complete graph of n vertices?

(a) $\Theta(n^2)$ (b) $\Theta(n^2 \log n)$

(c) $\Theta(n^3)$ (d) $\Theta(n^3 \log n)$

[2013 : 1 Mark]

4.11 [illegible] common subsequences between A and B. Let the length of the longest common subsequence between A and B is [illegible]. The number of such longest common subsequences between A and B is [illegible] = _____.

[2014 (Set-2) : 2 Marks]

4.12 Consider the weighted undirected graph with positive edge weights and vertices P, Q, R, S, T as shown below. The weight of the minimum spanning tree of the graph is _____.

[2016 (Set-1) : 2 Marks]

4.13 The Floyd-Warshall algorithm for all-pairs shortest path computation is based on

(a) Greedy paradigm

(b) Divide-and-conquer paradigm

(c) Dynamic Programming paradigm

(d) Greedy paradigm [illegible] Dynamic Programming paradigm.

[2016 (Set-2) : 1 Mark]

4.14 Let A_1, A_2, A_3 be four matrices of dimensions 10×5, 5×20, 20×10 and 10×5, respectively. The minimum number of scalar multiplications in the product $A_1 A_2 A_3 A_4$ using the matrix multiplication method is _____.

[2016 (Set-3) : 2 Marks]

■■■■

Answers **Dynamic Programming**

4.1	(b)	4.2	(a)	4.3	(a)	4.4	(d)	4.5	(d)	4.6	(c)	4.7	(c)	4.8	(b)	4.9	(a)	4.10	(c)
4.11	(c)	4.12	(a)																

Explanations **Dynamic Programming**

4.1 (b)

The [illegible] as follows

The minimum-cost [illegible] path is [illegible] and the cost is [illegible]. Therefore option (b) is correct.

4.2 (a)

Bellman-Ford algorithm is used to find all pairs shortest distances in a graph with [illegible] weights. [illegible] technique.

4.3 (a)

A Huffman tree [illegible]

A Huffman tree [illegible] contains 3 leaves. [illegible] leaves. [illegible] node [illegible]

Height of Huffman tree [illegible]

$$h = \log_2 n$$

In a complete binary tree [illegible]

$$h = \log_2 (n+1) - 1$$

$$h \le 1 + \log_2 n$$

$$h \le 1 + 3$$

$$h \le 4$$

4.4 (d)

If an algorithm [illegible] subset sum problem in $O(n^2 W)$ time where W is an integer which is [illegible] the subset sum problem in $O(n)$ [illegible] input [illegible] unary [illegible]

4.5 (d)

The subset sum problem is [illegible] problem.

For example [illegible] $S = \{-7, -3, -2, 5, 8\}$ [illegible] the sum of [illegible] is zero [illegible] $W \ge 0$. For example if $S = \{-3, -2, 5\}$ [illegible]

In general [illegible] two [illegible] W [illegible] W is a positive weight [illegible] sum

$$x[i, j] = x[i-1, j] \vee x[i-1, j - w_i] \quad j \le W$$

The [illegible] depends on [illegible] of S and [illegible] weight

If $S = \{-7, -3, -2, 5, 8\}$ and $W = 14$

[illegible] $W = 14$ [illegible] $a_1, a_2, \ldots, a_n$ [illegible] W.

So $X[i, j] = X[i-1, j] \vee X[i-1, j - a_i]$

[illegible] from [illegible] [illegible] $i = 1$ [illegible] generation starts from j [illegible] $j \ge a_i$

4.6 (c)

The entry $X[n, W]$ is true because [illegible] subset [illegible] W [illegible]

4.7 (d):

```
int
= 0; n either = 0 or j = 0
= [1][1] = 1;
    if = j > 0 && x[i-1] == Y[j-1]
= = max(i = j && i j = 0):
    if = > 0 and i = = Y[i-1]
= = max(i = j, i, j = 0)
```

4.8 (b):

Array values can be computed in row major order or column major order.

4.9 (b):

The key to the dynamic programming solution.

4.10 (a)

Total number of edges

$$= \frac{n(n-1)}{2}$$

Time complexity of Bellman-Ford algorithm is $O(|V| \cdot |E|)$. Dijkstra's algorithm takes $O(|V|^2)$ for a complete graph time complexity

$$O\left(n \cdot \frac{n(n-1)}{2}\right) = O(n^3)$$

So, option (a) is correct.

4.11 Not.

```
A = 9999
B = 9999
A[i][j] B[i] = 9999
[i] = 9999
[i] = 9999
= = length[A] = length[B]
= = 4, B[i] = 9
= + 10 = 4 + 10 × = 44
```

4.12 (c):

By drawing we get a multistage graph with minimum cost path as matrix is

By search in a given graph, we see that the cost two vertices, one is C and D and C & D having weight 4 and 1 respectively to reach C or D.

Since we have to find atleast 3 vertices more than. Let 3 vertices path between C to D. So we have to consider 12 as edge weight to reach C to D. So we have atleast 4 path between C to D i.e. 1 each edge and each edge weight 12 each.

4.13 (a):

Floyd Warshall Algorithm is "Dynamic programming" approach. The based on dynamic programming approach.

4.14 (d):

The number of multiplication required using binary matrix multiplication method will be 100.

■■■■

5 P & NP Concepts

5.1 A problem in NP is NP complete if

(a) it can be reduced to the 3-SAT problem in polynomial time

(b) the 3-SAT problem can be reduced to it in polynomial time

(c) it can be reduced to any other problem in NP in polynomial time

(d) some problem in NP can be reduced to it in polynomial time

[2009 : 1 Mark]

5.2 Two problems X and Y are NP complete and X reduces to Y in polynomial time. Which of the following statements is/are true?

(a) If X can be solved in polynomial time then so can Y

(b) Y is NP complete

(c) Y is NP hard

(d) X is NP hard, but not necessarily NP complete

[2008 : 1 Mark]

5.3 Let π_1 is a decision problem and it belongs to class NP. Which one of the following statements is FALSE?

(a) There is a polynomial time algorithm for π_1

(b) π_1 can be solved deterministically in polynomial time if $P = NP$

(c) π_1 is NP hard if π_1 is NP complete

(d) π_1 may be undecidable

[2008 : 1 Mark]

5.4 Which of the following statements is FALSE?

1. The problem of determining whether a given vertex is in a maximal clique is in P.
2. The problem of determining whether there exists a clique of size k in a given graph is in NP.
3. If a problem is NP Complete, there exists a non-deterministic polynomial time algorithm to solve it.

(a) 1, 2 and 3 (b) 1 and 2 only

(c) 2 only (d) 3 only

[2013 : 1 Mark]

5.5 Suppose a polynomial time algorithm is found for a problem in NP. Which of the following statements is true?

(a)

(b)

(c)

(d)

[2014 (Set-II) : 2 Marks]

5.6 Consider the satisfiability problem 2SAT(SAT with at most two literals per clause).

[illegible]

The given formula is in 2CNF. The decision problem 2CNFSAT is

(a) NP-Complete

(b) solvable in polynomial time by reduction to directed graph reachability

(c) solvable in linear time using depth first search

(d) NP-hard, but not NP-complete

[2011 (Set-I) : 2 Marks]

5.7 Consider two decision problems Q_1, Q_2 such that Q_1 reduces to polynomial time to 3-SAT and 3-SAT reduces in polynomial time to Q_2. Which one of the following statements is inconsistent with the above statement?

(a) Q_1 is in NP, Q_2 is NP hard
(b) Q_1 is in NP, Q_2 is NP hard
(c) Both Q_1 and Q_2 are in NP
(d) Both Q_1 and Q_2 are NP hard

[2015 (Set 3) 1 Mark]

5.8 Language L_1 is polynomial time reducible to language L_2. Language L_2 is polynomial time reducible to language L_1. Which of the following statements is true?

I. If $L_1 \in P$, then $L_2 \in P$
II. If $L_1 \in NP$, then $L_2 \in NP$
III. $L_1 \in NP$ if and only if $L_2 \in NP$
IV. If $L_1 \in P$, then $L_2 \in P$ and $L_1 \in P$

(a) I and II (b) I only
(c) I and IV only (d) I only

[2016 (Set 1) 2 Marks]

■■■■

Answers: P & NP Concepts

5.1 (a) 5.2 (b) 5.3 (c) 5.4 (d) 5.5 (d) 5.6 (b) 5.7 (a) 5.8 (c)

Explanations: P & NP Concepts

5.1 (a):

3-SAT belongs to NPC problem, reducing NP problem to 3-SAT would mean that NP problem is NPC.

5.2 (b):

It is possible to NP-complete problem is NPC.

5.3 (c):

Given $L_1 \in NP$

L_1 is NP-hard and it is given that $L_1 \in NP$, it means that L_1 is NP-complete.

So statement (c) is correct.

Notice that choice (a) is incorrect since P ⊆ NP, so some NP problems are actually P, and hence reduces in linear time to P problem.

Choice (b) is incorrect since L_1 can be solved exponentially in polynomial time, which means $L_1 \in NP$. In this case, it is additionally known that L_1 is NP-complete.

Choice (d) is incorrect since

All problems belonging to P or NP have to be decidable.

5.4 (d):

1. By depth first search we can find whether graph is cyclic or not in linear time $O(V+E)$ time. So it is P problem.
2. But the NP-hard in this problem will also be exponential so it is NP problem.
3. TSP is the definition of NP complete so it is trivially true.

So option (d) is correct.

5.5 (d):

Clique is NP complete problem.

If clique is P then P = NPC

So P = NP = NPC

5.6 (b):

SAT is P problem.

The reduction is possible in polynomial time so reduction is not needed for reachability.

6.7 [illegible]

Given $Q_1 \le$ 3-SAT and 3-SAT $\le Q_2$

3-SAT is NP-[illegible]

[illegible] NP and [illegible] NP-[illegible]

$Q_1 \le$ 3-SAT and 3-SAT is NP — Q_1 is NP-problem

3-SAT $\le Q_2$ and 3-SAT is NP hard — Q_2 is NP hard problem

So the correct statements is Q_1 is in NP and Q_2 is NP hard.

6.8 [illegible]

$L_1 \le_p L_2$, $L_2 \le_p L_3$ and $L_3 \le_p L_4$

I. If $L_1 \in P$, $L_3 \in P$ is true

$L_1 \le_p L_4$

II. If $L_1 =$ [illegible] is false

$L_1 \le_p L_3$ and $L_3 \le_p L_4$

III. $L_1 \in P$, NP and $L_4 \in P$ is false

If $L_1 \in P$ we cannot say anything about L_3 from $L_1 \le_p L_3$

Since we cannot say anything about L_3 then we cannot say something about L_4 either from $L_3 \le_p L_4$

So statement III is false.

IV. If $L_4 \in P$ then $L_3 \in P$ and $L_2 \in P$ is true.

If $L_4 \in P$ then from $L_3 \le_p L_4$ we can prove that $L_3 \in P$

and from $L_2 \le_p L_3$ we can prove that $L_2 \in P$

also from $L_1 \le_p L_2$ we can prove that $L_1 \in P$

So statement IV is true.

■■■■

6 Miscellaneous Topics

6.1 Match the following:

List-I

A. [illegible]

B. [illegible]

C. [illegible]

D. Floyd's shortest path algorithm

List II

P. [illegible]

Q. Dynamic programming

R. [illegible]

S. Depth first search [1999 : 2 Marks]

6.2 [illegible]

[1998 : 2 Marks]

6.3 Following [illegible] can be used to store n integers in the range [illegible] in time O(n)

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[1998 : 2 Marks]

6.4 [illegible] number of interchanges needed to convert [illegible] is

(a) 0 (b) 1

(c) 2 (d) 3

[1998 : 2 Marks]

6.5 [illegible] number of key comparisons [illegible] sorted sequence of size n

(a) [illegible] (b) $n-1$

(c) [illegible] (d) $n+1$

[1998 : 2 Marks]

6.6 The correct match for the following pairs is

List-I

A. [illegible]

B. [illegible]

C. [illegible]

D. Connected components

List II

1. [illegible]

2. [illegible]

3. [illegible]

4. [illegible]

Codes

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1997 : 1 Mark]

6.7 Match the following:

List-I	List-II
A. [illegible]	P. [illegible]
B. [illegible]	Q. [illegible]
C. [illegible]	R. Binary search
D. [illegible]	S. [illegible]

Codes

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1998 : 1 Mark]

6.8 Suppose we want to arrange n [illegible] on a straight line such that no two [illegible] overlap. Minimum number of [illegible] required in the worst case is

(a) n (b) n

(c) [illegible] (d) [illegible]

[1998 : 1 Mark]

6.9 The number of [illegible] in the following graphs

(a) 0 (b) 1

(c) [illegible] (d) 8

[1998 : 1 Mark]

6.10 [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) [illegible] (f) [illegible]

(g) [illegible]

(h) [illegible]

(i) [illegible]

(j) [illegible]

(k) [illegible]

(l) [illegible]

[1991 : 2 Marks]

6.11 In [illegible] graph. Consider a [illegible] [illegible] be the [illegible] [illegible] [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2000 : 2 Marks]

6.12 [illegible] $\frac{\pi}{2}$

(c) [illegible] (d) [illegible]

[2001 : 1 Mark]

6.13 [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) [illegible] (f) [illegible]

[2001 : 2 Marks]

6.14 [illegible]

(a) 2 (b) [illegible]

(c) 4 (d) [illegible]

[2002 : 1 Mark]

Common Data for Q. 6.15 & Q. 6.16

[illegible]

6.15 [illegible] what is the [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2003 : 2 Marks]

6.16 What [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2003 : 2 Marks]

6.17 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2008 : 2 Marks]

6.18 [illegible]

[illegible]

(a) [illegible] row major and column major order
(b) Upper triangular matrix in row major order
(c) Lower triangular matrix in column major order
(d) Indexed list of the storage in the [illegible]

[2004 : 2 Marks]

6.19 Suppose each array element requires 2 bytes. A two dimensional array stores elements in row order. Which of the operations, accessing elements, inserting new elements, randomly will be faster?

(a) Accessing
(b) Inserting in middle
(c) Inserting in random
(d) Inserting, in middle

[2004 : 2 Marks]

6.20 A program takes as input a balanced binary search tree with n leaf nodes and computes the value of a function g(x) for each node x. If the time to compute g(x) is minimum of the values in its children's nodes plus 1, then the worst case time complexity of the program is

(a) $O(n)$ (b) $O(n \log n)$
(c) $O(n^2)$ (d) $O(2^n)$

[2004 : 2 Marks]

6.21 A heap for storing binary trees in an array is defined as follows. The root of T is at 1. The left child of node corresponding to $T[i]$ is at node corresponding to $T[2i]$ and the right child, if any, is at $T[2i+1]$. Assuming that the tree is stored in an array, the maximum size of T when it has n nodes is

(a) $\log_2 n$ (b) n
(c) $2n-1$ (d) 2^n

[2006 : 1 Mark]

6.22 Which one of the following implementations of a stack can never result in stack overflow?

(a) Queue (b) Linked list
(c) Selection sort (d) Heap sort

[2006 : 1 Mark]

6.23 A complete binary tree T is defined as follows. If it is empty, then it is also a complete binary tree. Otherwise, let L and R be the left and right children of the root of T. Then L is a complete binary tree and R is a complete binary tree.

(a) Solve in linear time using depth first traversal of the tree
(b) Solve in linear time using breadth first traversal of the tree
(c) Solve in linear time using divide and conquer in $O(n \log n)$
(d) Solve in [illegible]

[2006 : 1 Mark]

6.24 A set X can be represented by an array x[n] as follows:

$$x[i] = \begin{cases} 1, & \text{if } i \in X \\ 0, & \text{otherwise} \end{cases}$$

Consider the following algorithm in which x, y and z are boolean arrays of size n:

```
Algorithm ABC(x[], y[], z[], n)
{
  for i
    z[i] = x[i] | y[i];
  for i
    z[i] = x[i] & y[i];
}
```

The set Z computed by the algorithm is

(a) $X \cup Y$ (b) $X \cap Y$
(c) $(X - Y) \cup (Y - X)$ (d) $X \cup (Y - X)$

[2006 : 2 Marks]

6.25 [illegible] arrays of n numbers, $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ respectively. Each number in the arrays is at most n. The fastest algorithm to find the number of pairs (a_i, b_j) such that $a_i + b_j = n$ is

(a) Takes $O(n^2)$ and $\Omega(n^2)$ time in the worst case
(b) Takes $O(n^2)$ and $\Omega(n \log n)$ time in the worst case
(c) Takes $O(n \log n)$ time and space
(d) Takes $O(n)$ time and space
(e) Takes $O(\sqrt{n})$ time and gives the number of pairs in the worst case in $O(n)$ time

[2006 : 2 Marks]

6.26 What is the following in the current discussion and the directed graph given below represents a connected component?

(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

8.27 Consider a directed graph with three vertices P, Q, and R. Let discovery time d(u) represent the time instant when the vertex u is first visited, and finish time f(u) represent the time instant when the vertex u is last visited. Given that

d(P) = 5 units, f(P) = 15 units
d(Q) = 6 units, f(Q) = 10 units
d(R) = 1 unit, f(R) = 18 units

which one of the following statements is always true for all vertices u, v in the graph?

(a) There is only one connected component
(b) There are two connected components, one containing P and R and the other containing Q
(c) There are three connected components, one containing P and Q, and R and Q are connected
(d) There are two connected components, one containing P and R and the other containing Q

[2007 : 2 Marks]

Directions for Questions 8.28 to 8.30:
An array X[i] = 0 indicates that the i-th element of the array is empty. The indexes of the array elements are 0 to n-1.

8.28 The index of the parent of the element X[i] is
(a) i/2
(b) (i-1)/2
(c) [illegible]
(d) [illegible]

[2008 : 2 Marks]

8.29 The worst case time complexity for inserting an element into the array is
(a) O(n)
(b) O(n log n)
(c) O(log n)
(d) O(n log log n)

[2008 : 2 Marks]

8.30 If the array is organized as a max heap, the number of elements in X[i] where X[i] = 0 is
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2008 : 2 Marks]

8.31 A depth first search is performed on a directed acyclic graph. Let d[u] be the time at which vertex u is visited for the first time and f[u] be the time at which the DFS call to the vertex u terminates. Which of the following statements always true for any two vertices u and v in the graph?
(a) d[u] < d[v]
(b) d[u] < f[v]
(c) f[u] < d[v]
(d) f[u] < f[v]

[2007 : 2 Marks]

8.32 Two fair dice are rolled. Let X and Y be the random variables representing the outcomes of the first and second dice respectively. Let Z = X + Y. Let H(X), H(Y), H(Z) be the entropies of X, Y, Z respectively. Which one of the following statements is true?
(a) H(X) = H(Y)
(b) H(X) > H(Y)
(c) H(Z) = H(X)
(d) H(Z) = H(Y)

[2008 : 2 Marks]

8.33 Consider the following sequence of numbers for the undirected graph given below.

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

All such sequences are possible. The number of distinct sequences is ____. Which all of the sequences is/are possible as output?
(a) 1 and 2 only
(b) 2 and 3 only
(c) 2 and 4 only
(d) 1, 2 and 3 only

[2008 : 2 Marks]

8.34 Two software packages A and B are available. Package A requires 2000 units of memory and 300 units of processor time. Package B requires 4000 units of memory and 60 units of processor time. What is the minimum number of units of memory required if the processor time available is 1800 units?
(a) 12
(b) 10
(c) 8
(d) 6

[2010 : 1 Mark]

8.35 The number of edges that can be created in a bipartite graph with 7 vertices is ____.

(a) $O(n)$

(b) $\Theta(\log n)$

(c) $\Theta\left(\frac{\log n}{\log \log n}\right)$

(d) $O(\log n)$

[SO 8 : 2 Marks]

8.48 Consider the directed graph given below.

Which one of the following is TRUE?

(a) The graph does not have any topological ordering.

(b) Both PQRS and QSRP are topological orderings.

(c) Both PSRQ and SPRQ are topological orderings.

(d) PSRQ is the unique topological ordering.

[2014 (Set-I) : 1 Mark]

8.49 Suppose we want to move from 0 to 100. In each step, you either move ahead by a unit distance or you take a shortcut. A shortcut is simply a pre-specified pair of integers (i, j). Given a shortcut (i, j) if you are at position i on the number line, you may choose to move to j. Suppose we want to find the smallest number of steps needed to move from 0 to 100. Suppose further that there are no shortcuts that involve going past 100, and that no shortcut from 0 has i value equal to 1.

Let g and h be such that $g(0) = 1 + \max(g(i), g(j))$. Then the value of $h(0)$ is

[2014 (Set-I) : 2 Marks]

8.50 Match the following

List-I

A. Dijkstra's shortest path algorithm

B. Floyd-Warshall algorithm for all pairs shortest paths

C. Mergesort

D. Hamilton's circuits

List-II

1. Backtracking
2. Dynamic programming
3. Greedy method
4. Divide and conquer

Codes:

	A	B	C	D
(a)	3	2	4	1
(b)	1	2	4	3
(c)	2	4	3	1
(d)	2	3	4	1

[2015 (Set-I) : 1 Mark]

8.51 Given lists of algorithms and some data structures, match the algorithm with the data structure it most naturally uses.

List-I

A. Dijkstra's Shortest Path

B. Floyd-Warshall algorithm to compute all pairs shortest paths

C. Binary search on a sorted array

D. Backtracking search on a graph

List-II

1. Queue and Dequeue
2. Dynamic Programming
3. Greedy design
4. Breadth-first search
5. Depth-first search

Match the above algorithms (List-I) to the corresponding data structures (List-II) they will use.

Codes:

	A	B	C	D
(a)	1	2	1	5
(b)	2	2	1	5
(c)	4	3	1	4
(d)	3	3	1	5

[2016 (Set-I) : 2 Marks]

Answers : Miscellaneous Topics

6.1	(d)	6.2	(c)	6.3	(a)	6.4	(b)	6.5	(b)	6.6	(d)	6.7	(d)	6.8	(d)	6.9	(c)
6.10	(b)	6.11	(c)	6.12	(c)	6.13	(b)	6.14	(d)	6.15	(a)	6.16	(d)	6.17	(a)	6.18	(a)
6.19	(b)	6.20	(c)	6.21	(b)	6.22	(b)	6.23	(d)	6.24	(c)	6.25	(c)	6.26	(c)	6.27	(b)
6.28	(c)	6.29	(a)	6.30	(a)	6.31	(a)	6.32	(b)	6.33	(b)	6.34	(c)	6.35	(c)	6.36	(c)

Explanations : Miscellaneous Topics

6.1 (d):

A B, C D, E D C

6.2 (c):

Since for selection sort the number of comparisons is independent of the input, number of comparisons is $\frac{n(n-1)}{2}$.

6.3 (a):

Median cut takes O(n) time and other two are O(n log n).

6.4 (b):

The representation of 4 pixels is

So two exchanges are required.

6.5 (b):

$$\text{Average time} = \frac{O(n^2)}{n} = \frac{n+1}{2}$$

6.6 (d):

All pairs shortest path → Dynamic programming
Quick sort → Divide and conquer
Matrix multiplication → Dynamic programming
Numerical integration → Depth-First Search

6.7 (d):

Insertion sort → O(n log n)
Insertion and Selection sort → O(n²)
Merge sort → O(n log n)
Insertion sort → O(n²)

6.8 (d):

In worst case, minimum number of exchanges required = $\frac{n}{2}$.

i. [illegible]
ii. [illegible]

Sorting is not used in hospital scheduling problem. Median cut is used in where median is used as middle element.

We require $\left\lceil \frac{n}{2} \right\rceil$ exchanges in worst case. First and last elements are exchanged with middle elements.

6.9 (d):

Consider a graph having maximum edges. Graph must be in diameter 1 and 2 and 3 vertices disjoint points and all other in one line.

6.10 (b):

[illegible] → O(n²)
[illegible] → O(n)
[illegible] → O(n log n)
[illegible] → O(n log n)

6.11 (c):

Draw some random graph and try to verify the options.

6.13 (b)

Partition $\beta = \frac{3}{2}$

Therefore

Expected $(S_1) = \frac{3}{2} = 1.5$

Expected $(S_2) = \frac{3}{2} = 1.5$

6.14 (d)

Cycle sort is an in-place, unstable, sorting algorithm that is asymptotically optimal in terms of total number of writes to the original array.

(i) $L_1 = \{1, 2, 3, 4, 5\}$

(ii) $L_2 = \{1, 2, 3, 4, 5\}$

Total number of required swaps in the cycle sort is 4.

Minimum number of indices = $n - \frac{n}{2} + 1$.

6.15 (b)

Let $a_1, a_2, a_3, \ldots, a_n$ be a permutation of $1, 2, 3, \ldots, n$. Then (i, j) is an inversion of the permutation if $i < j$ but $a_i > a_j$. For example, in the permutation 3 1 2 there are two inversions (3, 1), (3, 2). Let us assume that all permutations are equally likely. Let X be the random variable denoting the number of inversions in a random permutation.

$$= \frac{n(n-1)}{2 \cdot 2} = \frac{1}{4} n(n-1)$$

$$= \frac{1}{4} \frac{n!}{(n-2)!}$$

$$= \frac{1}{4} \frac{n(n-1)(n-2)!}{(n-2)!}$$

$$= \frac{n(n-1)}{4}$$

6.16 (d)

Trapezoid rule is a technique for approximating the definite integral of a function by treating it as a trapezoid.

The time complexity is $O(n^2)$.

6.17 (b)

The cube root of a natural number n is defined as the largest natural number m such that $m^3 \le n$. Complexity of computing the cube root of n is $O(\log n)$ by binary search method.

6.18 (b)

Average case equal to worst case and best case is assumed in order to access any element takes constant time.

6.19 (d)

Graphs can be represented as a linked list with elements in an array. Vertices in a graph are usually stored in an array. Adjacency lists are usually stored in linked lists. Adjacency matrix is a 2D array.

In the case of intersection, let A and B be two sets. An operation $A \cap B$ requires time $O(|A| + |B|)$ if both the sets are represented as a hash table or a direct access table. If one of the sets is represented as a linked list, then the other set must still be scanned in linear time.

For the operation $A \cup B$, we first insert all elements of A into a hash table. Then we insert all elements of B into the same hash table. Finally, we scan the hash table and output all elements in the hash table. This takes $O(|A| + |B|)$ time.

We will know a graph contains a cycle if we find a back edge. It takes $O(|E| + |V|)$ time to find a back edge in a graph. Finding intersection operation is the slowest operation.

6.30 (d):

[illegible] left subtree [illegible] right subtree [illegible]. If balanced binary search tree is used, and comparing in order then in worst case time complexity will be [illegible]

6.31 (b):

[illegible]

Let [illegible] stored at [illegible]

Let [illegible] stored at [illegible]

[illegible]

[illegible]

6.32 (a):

In selection sort, minimum [illegible] swap [illegible]

Number of swaps = $O(n)$

Number of comparisons = $O(n^2)$

[illegible] only once in every pass

6.33 (c):

[illegible] right and [illegible] the leader.

[illegible]

[illegible]

6.34 (b):

[illegible] expression.

[illegible]

[illegible]

6.35 (d):

[illegible] and the [illegible] in $O(1)$ time because there is only fixed [illegible]

6.36 (c):

In strongly connected component, every vertex [illegible] from every other vertex and it is maximal in [illegible]

[illegible]

6.37 (d):

Since $d(G) = n - 1$ and $\lambda(G) = k(G)$ which means I and G are connected [illegible]

6.38 (a):

Left child of [illegible] and right child is $2i+1$. The children of the parent of element $X[i]$ are $[illegible]$

6.39 (b):

[illegible]

6.40 (a):

Index [illegible] Every [illegible] as shown in the [illegible] [illegible] elements [illegible] level in the [illegible] to serve as a [illegible]

6.41 (a):

Since DFS is performed [illegible] graph [illegible] back edges.

Since [illegible] forward edges [illegible]

6.42 (b):

[illegible]

Q.32 (b)

DFS orders:

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

Q.33 (a)

A requires 0.001 s time to [illegible]
[illegible] time [illegible] processor [illegible]
processor B would be preferred over A.

$$\frac{n^2}{[illegible]} = 1000 \log_{10} n$$

$$n^2 = 1000 \log_{10} n$$

[illegible] with [illegible]
So [illegible] = 8 for which [illegible]
will be preferred over A.

Q.34 (a)

If we have [illegible] elements [illegible] time
complexity [illegible]
We [illegible] time complexity
to [illegible] elements [illegible] $O(\log n)$ [illegible]
divide [illegible] $O(n)$ and $O(1)$
Now [illegible]

$$= O\left[\frac{\log n}{\log \log n} \log \frac{\log n}{\log \log n}\right]$$

$$= O\left[\frac{\log n}{\log \log n} [\log \log n - \log \log \log n]\right]$$

$$= O\left[\log n \left[\frac{\log \log n - \log \log \log n}{\log \log n}\right]\right]$$

[illegible]

Option [illegible] is best.

Q.35 (b)

[illegible]

Q.36 Sol.

Given that there is shortest from [illegible]

$$\text{GTF}$$

[illegible]

$\Rightarrow$ [illegible] = 10 × 16

[illegible] = 10 × 16 = 160

Q.37 (b)

Primary [illegible] Kruskal's algorithm [illegible]
algorithm. [illegible] dynamic programming.
Merge sort is a Divide and Conquer algorithm.
Dijkstra's [illegible] problem
and [illegible] by using [illegible].

Q.38 (b)

Dijkstra's shortest path → Greedy design

Floyd-Warshall algorithm [illegible] dynamic programming

[illegible] Divide and Conquer [illegible]

[illegible] Depth First Search [illegible]
[illegible] graph.

■■■

Unit - VII

Compiler Design

Contents

Sl.	Topic	Page No.
1	Lexical Analysis	391
2	Parsing Techniques	397
3	Syntax Directed Translation	407
4	Code Generation and Optimization	413

UNIT VII Compiler Design

Syllabus: Lexical analysis, Parsing, Syntax directed translation, Runtime environments, Intermediate and target code generation, Basic code optimization

Analysis of Previous GATE Papers

Exam Year	1 Mark Questions	2 Marks Questions	3 Marks Questions	5 Marks Questions	Total Marks
1990	2			–	2
1991				–	–
1992		1		–	2
1993				–	–
1994	1			–	1
1995	3	1			5
1996	1	–			1
1997	1	–			1
1998	2	–		1	7
1999	–	1			2
2000	3	–			3
2001	2	–			2
2002		–			–
2003	2	5			13
2004	3	1			5
2005		1			8
2006	1	3			11

Exam Year	1 Mark Questions	2 Marks Questions	Total Marks
2007		5	11
2008	2	1	4
2009		–	1
2010	2	1	4
2011	2	1	5
2012		2	4
2013		3	7
2014 Set-1		1	1
2014 Set-2	3	2	8
2014 Set-3		1	4
2015 Set-1	1	2	5
2015 Set-2	2	1	4
2015 Set-3	1	1	3
2016 Set-1	1	2	8
2016 Set-2	1	1	3

1 Lexical Analysis

1.1 [illegible] module that checks every [illegible] character is allowed

(a) The code optimizer

(b) The code generator

(c) The lexical analyzer

(d) The syntax analyzer

[1988 : 1 Mark]

1.2 Match the following:

Group-I

A. [illegible] analysis

B. Code optimization

C. Code generation

D. Algebraic groups

Group II

P. [illegible]

Q. Syntax tree

R. [illegible]

S. Finite automata

[1990 : 1 Mark]

1.3 Which of the following grammars can adequately be used to [illegible] the longest possible input character when compiling a Pascal program?

I. Regular II. Context III. [illegible]

(a) I (b) II

(c) III (d) All of the above

[1990 : 1 Mark]

1.4 In a programming language, an identifier is permitted to be a letter followed by any number of letters or digits. If L and D denote the sets of all letters and digits respectively, which of the following regular expressions defines an identifier?

(a) $(L+D)^*$ (b) $(L+D)^*L$

(c) $(L+D)^*D$ (d) $L(L+D)^*$

[1995 : 1 Mark]

1.5 [illegible] is not a phase of a compiler?

(a) Lexical analysis

(b) Syntax analysis

(c) Syntax directed translation

(d) Code optimization

[1996 : 1 Mark]

1.6 The number of tokens in the following C statement is [illegible]

printf("i = %d, &i = %x", i, &i);

[1998 : 2 Marks]

1.7 The [illegible] of tokens in the following C statement printf("i = %d, &i = %x", i, &i) is

[2000 : 1 Mark]

1.8 The number of tokens in the following C statement

printf("i = %d, &i = %x", i, &i)

(a) 5 (b) 6

(c) 10 (d) 11

[2000 : 1 Mark]

1.9 When a file is opened in READ mode, an old file with the same name is destroyed and a new file is created. A dynamically linked library is loaded only when a program calls functions in the library. Which of the following statements is/are correct?

(a) Lexical analyzer can be implemented as a finite state machine

(b) Lexical analyzer generates syntax tree

(c) Parser generator can automatically generate parsers from context free grammars

(d) Lexical analyzer can be implemented using regular expressions and finite automata

[2006 : 2 Marks]

1.10 Consider a lexical analyzer for a language in which all keywords are reserved words. Let M_1 and M_2 be two regular expressions representing identifiers and keywords respectively. Let M_1 be a regular expression for an identifier defined as M_2, the regular expression for keywords. The lexical analyzer uses longest match to resolve conflicts between identifiers and keywords. Which of the following is/are true?

(a) Finite time (b) Compile time

(c) Link time (d) Load time

[2007 : 1 Mark]

1.11 Consider the following number 8 in the following C program.

```
int main()
{
  int i;
  for (i = 0; i < 8; i++)
  {
    printf("%d", i);
  }
}
```

Identify the correct statement about this line.

(a) Lexeme in the lexical analyzer

(b) Not a compilation error

(c) Only syntactic error

(d) Only semantic error

(e) Both syntactic and semantic error

[2008 : 2 Marks]

1.12 Which of the following statements are TRUE?

I. Chomsky's parsing algorithm is used for some programming languages whose syntax is context-free.

II. A programming language which allows recursion can be implemented with static storage allocation.

III. Non-terminals in a definition can be eliminated by the transformation rules.

IV. Code improving transformations can be performed at both source language and intermediate code level.

(a) I and II (b) I and IV

(c) III and IV (d) I, II and IV

[2009 : 1 Mark]

1.13 Which data structure in a compiler is used for managing information about variables and their attributes?

(a) Abstract syntax tree

(b) Symbol table

(c) Semantic stack

(d) Parse table

[2010 : 1 Mark]

1.14 In a compiler, keywords of a language are recognized during

(a) parsing of the program

(b) code generation

(c) lexical analysis of the program

(d) syntax analysis

[2011 : 1 Mark]

1.15 Match the following

List I

P: Lexical analysis

Q: Top-down parsing

R: Semantic analysis

S: Intermediate code generation

List II

i: Leftmost derivation

ii: Type checking

iii: Regular expressions

iv: Intermediate records

Codes:

(a) P – i, Q – ii, R – iv, S – iii

(b) P – iii, Q – i, R – ii, S – iv

(c) P – i, Q – iii, R – ii, S – iv

(d) P – iii, Q – i, R – ii, S – iv

[2018 (Set-3) : 1 Mark]

■■■■

Answers: Lexical Analysis

1.1 (c) 1.2 (c) 1.3 (c) 1.4 (a) 1.5 (a) 1.6 (c) 1.7 (c) 1.8 (a) 1.9 (c) 1.10 (c) 1.11 (a) 1.12 (b)
1.13 (b) 1.14 (c) 1.15 (b)

Explanations: Lexical Analysis

1.1 (a)

Lexical analysis is a process of scanning the source code in which stream of characters are converted into meaningful sequences. A program or information which performs lexical analysis is called a lexical analyzer. So lexical analyzer checks every character of the source code.

1.2 (c)

(A → (S), B → P, C → R, D → T) (Q)

1.3 (c)

I. Regular expressions can be followed by any phase to form identifier, keyword, number, etc.

II. Regular expressions can be used by any phase to form identifier, keyword, number, etc.

III. Regular definitions can be used in a lexical analyzer without looking at the rest of the input character.

1.4 (a)

L : Set of letters

D : Set of digits

Identifier = Letter (Letter | Digit)*

Digit Star = D(D | D)*

1.5 (a):

Type checking verifies that a type of a construct matches that expected by its context.

[illegible]

1.6 (b):

In Pascal, an identifier consists of an upper-case letter followed by zero or more letters and digits. Thus, the identifier consists of letters and digits. Whitespace is not permitted. Comments are delimited by (* and *). Whitespace is not permitted following the opening (*.

[illegible]

∴ The number of tokens in the C statement is 6.

1.7 (a):

[illegible]

∴ The number of tokens in the C statement is 11.

1.8 (c):

Inside the parser, the parser between the lexical analyzer.

1.9 (b):

Statically linked libraries require more disk space, because each executable file has its own copy of the library, and loaded into memory at compile time. Dynamically linked libraries require less disk space, because only one copy of the library is required, regardless of how many programs are running.

1.10 (c):

Machine code is compiled sequentially and all linking is done at load time, so it is more efficient.

1.11 (b):

Error is underlined. Exception 3.4

Where is more serious error rather than lexical or syntax error.

1.12 (b):

Recursive descent can be implemented with static analysis of syntax.

Attribute definitions can be evaluated in any order that satisfies the dependencies of the syntax.

Set and TV are correct.

1.13 (b):

[illegible]

1.14 (c):

Semantic analysis of the program.

1.15 (b):

Lexical analysis → Syntax analysis → Semantic analysis → Intermediate code generation → Machine independent code optimization → Code generation → Machine dependent code optimization.

■■■■

2 Parsing Techniques

2.1 Consider the [illegible] parsing tables be correct for a grammar. Which of the following statements is true?

(a) The grammar of the table may be different

(b) The [illegible] are identical in both the tables

(c) The reduce entries in the table may be different

(d) The shift entries in the table may be different

[1992 : 2 Marks]

2.2 A shift-reduce parser carries out the actions specified on [illegible] based on its [illegible] and the next symbol. Consider the following grammar:

S → aS | aSbS | ε

Let ... be the sets of states of the shift-reduce parser for this grammar. Which of the following statements is true?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

What is the translation of the sentence aaabab using the syntax-directed translation scheme described by the above rules?

(a) 2[illegible]0 (b) [illegible]

(c) [illegible] (d) [illegible]

[1995 : 2 Marks]

2.3 The parser in the compiler consists of the following stages

(i) Lexical analysis

(ii) Syntax analysis

(iii) Code generation

(iv) Address resolution

(a) I, II, III, IV (b) I, II, III

(c) I, II, IV (d) I, II, III, IV

[1998 : 1 Mark]

2.4 Which of the following statements is true?

(a) SLR parser is more powerful than LALR

(b) LALR parser is more powerful than canonical LR parser

(c) Canonical LR parser is more powerful than LALR parser

(d) The parsers for Canonical LR and LALR have the same power

[1999 : 1 Mark]

2.5 The process of assigning load addresses to the various parts of the program and adjusting the code and data in the program to reflect the assigned addresses is called

(a) Assembly (b) Parsing

(c) Linking (d) Symbol resolution

[2003 : 1 Mark]

2.6 Which of the following statements is false?

(a) An unambiguous grammar has some ambiguous strings

(b) [illegible] parser is a top down parser

(c) LALR is more powerful than SLR

(d) An unambiguous grammar can never be left recursive

[2001 : 1 Mark]

2.7 Which of the following is sufficient to convert an arbitrary CFG to an LL(1) grammar

(a) Removing ε-productions alone

(b) Removing the [illegible] productions

(c) Removing left recursion and factoring the grammar

(d) None of the above

[2008 : 1 Mark]

2.8 Assume that the SLR parser for a grammar G has n1 states and the LALR parser for G has n2 states. The relationship between n1 and n2 is

(a) n1 is necessarily less than n2

(b) n1 is necessarily less than or equal to n2

(c) n1 is necessarily greater than n2

(d) None of these

[2009 : 1 Mark]

2.9 Consider the grammar defined below

S → [illegible]

S → (S)S | ε

S → a | b

In the predictive parser table, M, of this grammar, the entries M[S, (] and M[S, a] respectively are

(a) S → (S)S and S → a

(b) S → (S)S and S → ε

(c) S → ε and S → a

(d) S → ε, S → (S)S and S → a

[2004 : 2 Marks]

2.10 Consider the grammar shown below

S → C C

C → c C | c

The grammar is

(a) LL(1)

(b) SLR(1) but not LL(1)

(c) LALR(1) but not SLR(1)

(d) LR(2) but not LALR(1)

[2003 : 2 Marks]

2.11 Which of the following grammar rules violate the rules for a context free grammar? (A, B, C are non-terminals and a, b are terminals)

(i) A → B C (ii) A → a B b

(iii) A → a (iv) A → B | b

(a) (i) only (b) (i) and (iii)

(c) (i) and (iii) only (d) (ii) and (iv) only

[2004 : 1 Mark]

2.12 The grammar A → A + A | A * A | a is not suitable for predictive parsing because the grammar is

(a) ambiguous (b) left recursive

(c) right recursive (d) an operator grammar

[2003 : 1 Mark]

2.13 Consider the grammar

E → E + n | n

Let the number of shift moves and reduce moves in the LR(0) parser for the grammar be n_s and n_r respectively. The sum $n_s + n_r$ is

(a) $n_s = n_r + 1$ (b) $n_s = n_r + n$

(c) $n_s = n_r + 2$ (d) $n_s = 2n_r + 1$

[2006 : 2 Marks]

Linked Answer for Q. 2.14 & Q. 2.15:

Consider the context free grammar

E → E + T | T

T → T * F | F

F → a

where E is the starting symbol, the terminal symbols are {a, +, *} and the non-terminal symbols are {E, T, F}.

2.14 Which of the following strings is a valid sentence in the language generated by the above grammar?

(a) a + a + a (b) a + a * a

(c) a * a * a (d) a + a + a + a

[2005 : 2 Marks]

2.15 For the grammar in the previous question, how many parse trees are there for the string a + a * a?

(a) 1 (b) 2

(c) 3 (d) 4

[2005 : 2 Marks]

2.16 Consider the following grammar.

S → a S b | b

S → B

A → B | F

B → b

F → nil

Consider the following LR(0) items corresponding to the grammar above.

S → a . S b

S → B .

S → a S . b

Given the input string "abab" and the LR(0) parser, which of the items appear in the initial state in the LR(0) parsing table for the grammar?

(a) S → a . S b (b) S → a S . b

(c) S → B . (d) None of these

[2004 : 1 Mark]

2.17 Consider the following grammar

S → (S)

S → *S | -

F → nil

In the predictive parsing table, M, the grammar rule for the entries M[S, *] and M[S, -] respectively are

(a) S → *S and S → -S (b) S → *S and S → (S)

(c) S → (S) and S → -S (d) S → (S) and S → *S

[2006 : 2 Marks]

Linked Data Questions 2.18 & 2.19

2.18 Which one of the following grammars generates the language $L = \{a^i b^j c^k \mid i = j + k\}$?

(a) S → aSc | aAc

A → aAb | ε

(b) S → aS | bSc | A

A → aAb | ε

(c) S → aSc | A

A → aAb | ε

(d) S → aSb | bSc | A

A → aAb | ε

(c) S → aSb | ε
(d) [illegible]
A → [illegible]
T → [illegible] [2004 : 2 Marks]

2.18 [illegible] length [illegible] number of [illegible] strings of [illegible]
(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
[2006 : 2 Marks]

2.19 Which [illegible] top down parser?
(a) Recursive descent parser
(b) [illegible]
(c) [illegible]
(d) [illegible]
[2007 : 1 Mark]

2.21 Consider the grammar [illegible] and [illegible]
[illegible]
S → [illegible]
[illegible] → b
[illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
[2007 : 2 Marks]

2.22 Consider the following [illegible]
P: [illegible]
Q: [illegible]
Which of the following [illegible]?
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
[2007 : 2 Marks]

Common Data for Q. 2.23 & Q. 2.24
Given a context free grammar [illegible] and a [illegible] [illegible] the start symbol [illegible] [illegible]

[illegible]	S → [illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

2.23 Which of the following strings is [illegible] by [illegible]?
(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
[2007 : 2 Marks]

2.24 [illegible] above question [illegible]?
(a) 1 (b) 2
(c) 3 (d) 4
[2007 : 2 Marks]

2.25 Which of the following [illegible] applicable to LR parsing [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
[2006 : 1 Mark]

2.26 [illegible] grammar [illegible] shift reduce (S-R) conflicts [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
[2006 : 2 Marks]

2.27 [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
[2006 : 2 Marks]

Linked Data Questions 2.28 and 2.29
[illegible] grammar [illegible] [illegible] are [illegible] the empty string [illegible] [illegible] right hand side [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

2.45 The [illegible] and FOLLOW sets for the non-terminals A and B are

(a) FIRST(A) = {a, b, ε}, FIRST(B) = {[illegible]}
FOLLOW(A) = {a, b}
FOLLOW(B) = {a, b, $}

(b) FIRST(A) = {a, b, ε}
FIRST(B) = {b, ε}
FOLLOW(A) = {a, b}
FOLLOW(B) = {a}

(c) FIRST(A) = {a, b, ε} = FIRST(B)
FOLLOW(A) = {a, b}
FOLLOW(B) = {a}

(d) FIRST(A) = {a, b, ε} = FIRST(B)
FOLLOW(A) = {a, b}
FOLLOW(B) = {a, b}

[2012 : 2 Marks]

2.46 The appropriate entries for F1, F2 and F3 are

(a) F1 : S → [illegible], B → S
F2 : S → [illegible], B → S
F3 : B → S

(b) F1 : S → aAbB, S → ε
F2 : B → bAaB, B → ε
F3 : S → ε

(c) F1 : S → aAbB, S → ε
F2 : S → bAaB, B → ε
F3 : B → S

(d) F1 : A → ε, S → ε
F2 : A → ε, S → ε
F3 : B → S

[2012 : 2 Marks]

2.47 What is the maximum number of reduce moves that can be taken by a bottom-up parser for a grammar with no ε-productions and unambiguous productions of type A → aA and A → a to parse a string with n tokens?

(a) n/2 (b) n−1
(c) n+1 (d) 2n

[2012 : 1 Mark]

2.48 Consider the following grammar G (with S as the start symbol) and the following precedence and associativity declarations for the operators + and *.

S → S + S | S * S | a

%left + %left *

Which of the following statements is/are true?

1. The grammar is unambiguous since the precedence and associativity declarations resolve all conflicts.
2. The grammar is ambiguous and will result in S/R conflicts.
3. The grammar is ambiguous and will result in R/R conflicts.
4. The grammar is unambiguous since precedence and associativity declarations are used.

(a) 1 only (b) 2 only
(c) 1 and 4 (d) 2 and 4

[2013 : 2 Marks]

2.49 Consider the following items in a given LR(0) parser

S → L = R
R → L

Which of the following statements is/are true?

(a) shift-reduce conflict and reduce-reduce conflict
(b) shift-reduce conflict but no reduce-reduce conflict
(c) reduce-reduce conflict but no shift-reduce conflict
(d) neither shift-reduce nor reduce-reduce conflict

[2014 (Set 1) : 2 Marks]

2.50 Consider the following grammar G with two operators + and *

E → E + E
E → E * E
E → (E) | E
E → id

Which one of the following is TRUE?

(a) + has higher precedence than * if + is left associative
(b) + has higher precedence than * if * is left associative
(c) Both + and * have the same precedence if both are left associative
(d) Both + and * have the same precedence if both are right associative

[2014 (Set-3) : 1 Mark]

2.51 Which one of the following is a TRUE statement in a shift-reduce parser?

(c) Variable protection approximately at the bottom of the stack does not include
(d) Variable protection approximately at the bottom of the stack does include
(e) The [illegible] variable
(f) The [illegible] variable exclusive

[2015 (Set-II) : 1 Mark]

2.56 Match the following.

List-I
A. Lexical analysis
B. Parsing
C. Register allocation
D. Dependence analysis

List-II
1. Graph coloring
2. DFA minimization
3. Parse tree traversal
4. Production tree

Options

	A	B	C	D
(a)	2	3	[illegible]	[illegible]
(b)	3	1	4	2
(c)	2	1	[illegible]	3
(d)	2	3	4	1

[2015 (Set-II) : 1 Mark]

2.57 Among simple LR (SLR), canonical LR and look-ahead LR (LALR) which of the following is/are true? Which method is the most powerful, in terms of order?
(a) SLR, LALR
(b) Canonical LR, LALR
(c) SLR, canonical LR
(d) LALR, canonical LR

[2016 (Set-3) : 1 Mark]

2.58 Consider the following grammar G.

$S \rightarrow F \mid H$
$F \rightarrow p \mid c$
$H \rightarrow d \mid c$

Where S, F and H are non-terminal symbols, p, d and c are terminal symbols. Which of the following statements is/are correct?
S1: LL(1) parsing for all strings that are generated using grammar G
S2: LR(1) parsing for all strings that are generated using grammar G
(a) Only S1 (b) Only S2
(c) Both S1 and S2 (d) Neither S1 nor S2

[2015 (Set-III) : 2 Marks]

■■■■

Answers Parsing Techniques

2.1	(b), (c), (d)			2.2	(a)	2.3	(b)	2.4	(a)	2.5	(a)	2.6	(a)	2.7	(d)	2.8	(b)		
2.9	(c)	2.10	(a)	2.11	(b)	2.12	(a)	2.13	(a)	2.14	(a)	2.15	(a)	2.16	(d)	2.17	(a)		
2.18	(a)	2.19	(a)	2.20	(a)	2.21	(c)	2.22	(c)	2.23	(c)	2.24	(b)	2.25	(d)	2.26	(b)		
2.27	(c)	2.28	(a)	2.29	(a)	2.30	(c)	2.31	(d)	2.32	(c)	2.33	(b)	2.34	(c)	2.35	(a)		
2.36	(c)	2.37	(d)																

Explanations Parsing Techniques

Q.1 (b)

2. (b), (c), (d)

Hint: The syntax parsing phase in compiler uses context free grammar. Statement parsing is done using finite automata. Code optimization and code generation are based on syntax directed translation. Lexical analyzer is used in compiler.

Q.2 (a)

Syntax tree for the given expression is as follows:

So translation grammar is (a).

[The given translation scheme is a top down parser. The parser uses the top down parsing with left recursion.]

Q.3 (b)

A macro processor is a program that accepts as input a stream of characters, recognizes macro calls, expands the calls, and produces a stream of characters as output. The macro processor may be a separate program or part of the assembler. The second pass is possible only if the program statements are in machine language.

(i) Object code generation – 2

(ii) Literals added to literal table – 1

(iii) Lists printed – 2

(iv) Symbols added to symbol table – 1

(v) Intermediate code generation – 2

Q.4 (b)

Lexical analyzer time complexity

SLR > LALR > CLR > LR(0)

Q.5 (c)

Recursive descent parser is a top down parser. It is used to parse an input string. It uses a set of recursive procedures to match the input with the grammar rules.

Q.6 (a)

(i) Ambiguous grammar may have two different leftmost and rightmost derivations. However, an unambiguous grammar has only one derivation tree. So statement is false.

(ii) LALR is a top down parser.

(iii) LALR is a top down parser. False statement.

(iv) Parser uses context free grammar. False statement.

Q.7 (d)

A multiple type grammar can be ambiguous. Removing left recursion and factoring the grammar does not always produce the unambiguous grammar.

Therefore, option (d) is correct.

Q.8 (b)

SLR parser has a subset of grammars of LALR parser. LALR parser has a subset of grammars of CLR. Thus, the number of corresponding states in the parser is minimum in LALR while it is maximum in CLR. Thus, LALR parser uses less space and time than CLR parser.

$n_{LALR} = n_{SLR}$

Q.9 (b)

The given grammar is

$S \rightarrow iE\,t\,S\,S'$

$S' \rightarrow e\,S \mid \epsilon$

$E \rightarrow b$

The predictive parse table is

Non-terminal	a	b	e	i	t	$\$$
S	$S \rightarrow a$			$S \rightarrow iE\,t\,S\,S'$		
S'			$S' \rightarrow e\,S$, $S' \rightarrow \epsilon$			$S' \rightarrow \epsilon$
E		$E \rightarrow b$				

So, $S' \rightarrow \epsilon$ is the entry for S' and $\$$.

Multiple entry = 5 entries

3.10 (a) :

Consider the grammar

$S \to (S)$

$S \to \epsilon$

The above grammar is LL(1)

It is unambiguous, left recursive and left factored.

Hence it is SLL(1), TAB(1) and CTB(1)

Therefore option for matches correctly

3.11 (a) :

The given grammar is unambiguous. The grammar has the property that the production rule is not in form we it doesn't contain [illegible]

$S \to T \to (S)$ and $T \to \epsilon$ eliminate the very first member and [illegible] the grammar.

3.12 (a)

The string [illegible] can be generated by the given grammar.

Since grammar is ambiguous, there is more than one parse tree possible for the same input string.

3.13 (b)

Given :

Parser	Number of states
SLR(1)	n_1
TB(1)	n_2
LALR(1)	n_3

If n_1 = number of states in SLR(1) parser, n_2 = number of states in TB(1) parser and n_3 = number of states in LALR(1) parser then $n_1 = n_3 = n_2$. The number of states of deterministic finite automata in LR(0) is greater than or equal to number of states of deterministic finite automata in SLR(1), LALR(1) and TB(1)

So $n_1 = n_3 = n_2 \geq n_0$

3.14 (a)

id+id*id has more than one parse tree.

3.15 (a) :

[illegible] id+id*id

2.18 (a):

Construct the DFA, L = [illegible] L. Since both of these two states are processing the same type of above [illegible]

2.19 (a):

Construct Predictive parser table as follows

Non-terminal	id	[illegible]	[illegible]
S	S → TR		
R		R → +S	R → ε
T	T → id		

Rule 2 [illegible] = {+, ε, $}

and Rule 3 [illegible] = {id, $}

2.20 (d):

$$S \to AC \mid CB$$
$$C \to aCb \mid \varepsilon$$
$$A \to aA \mid a$$
$$B \to bB \mid b$$

From the above grammar,

[illegible] → L(C) = {a^n b^n}

[illegible] → L(A) = a

[illegible] → L(B) = b

Resultant language L(A) and L(B) [illegible]

S → AC ⇒ L(S) = {a^n b^n | m ≥ n}

S → CB ⇒ L(S) = {a^n b^n | m ≥ n}

L(S) = {a^n b^n | m ≥ n or m ≥ n} = {a^n b^n | n ≥ 1}

2.21 (d):

The minimum length is 2 [illegible] generated after initially we will [illegible] the total length is [illegible].

2.22 (d):

Predictive parser and recursive descent parser are example of top down parser. Whereas LR(k) and LALR(k) parser are bottom up parser.

2.23 (d):

$$S \to aTbA \mid \varepsilon$$
$$S \to bSaS \mid \varepsilon$$
$$T \to Sa$$

Consider the string = [illegible] there are two parse trees under this string.

Parse Tree 1

Parse Tree 2

For the same string [illegible] present in above two parse trees. Hence the given grammar is ambiguous.

2.24 (a):

- A regular language is unambiguous. So [illegible] grammar [illegible] the [illegible] Therefore, it is true that every regular grammar is [illegible].
- Every regular language is [illegible], but converse is false. A grammar is ambiguous if there exists some grammar which is unambiguous. [illegible] that every regular [illegible] unambiguous.

∴ P is false and Q is true.

So option (d) is correct option.

2.25 (c):

$$S \to aB \to aaSB \to aabSS$$
$$\to aabbS \to aabbbSS$$

So [illegible] is generated by the grammar.

2.24 (b):

Two parse trees are possible for the string abbcd:

2.25 (d):

Definition of a CFG is given in chapter.

2.26 (b):

LL(1) Parser is a top-down parser for a grammar. Here S, A and B are all non-terminals. The LR(1) Parser for a CFG, S, A and B are all

2.27 (c):

S → aSbS | ε

Terminals are a, b, ε

Since [aSbS] → first [aSbS] = {a} = a

&

First [bS] → first [S] = {b, ε} = b

&

First [ε] → first [ε] = {ε} = ε

So the given grammar is LL(1). Also S → ε is a grammar.

2.28 (a) & 2.29 (c)

S → aAB | bBA | ε

A → S

B → S

First of S = {a, b, ε}

First of B = {a, b, ε}

First of A = {a, b, ε}

Follow of S = {$, a, b}

Follow of A = {b, a}

Follow of B = {$, a, b}

LL(1) Parsing table:

	a	b	$
S	S → aAB S → ε	S → bBA S → ε	S → ε
A	A → S	A → S	error
B	B → S	B → S	B → S

2.30 (b):

Unit production: A → b

Note: There is a typo error in question. The unit production was given as A → b

Null production: A → ε

Production:

A → aAb

A → ε

B → b

[Consider the case ab ⇒ Ab ⇒ aAb ⇒ ab. It means ab is a valid production]

2.31 (d)

1. Cannot be merged since accept & reject states differ in FA1 & FA2. Merging accept & reject states destroys DFA.
2. Can be merged because in both FA1 & FA2 the two states are non accepting states. Merging non accepting states does not affect DFA.
3. Can be merged because in both FA1 & FA2 F → F. No DFA states are merged.
4. Cannot be merged since accepting state in FA1 is merged with non accepting state in FA2. Merging of states destroys DFA.

2.32 (d):

S → [illegible]

Q → [illegible]

[illegible]

2.34 (b):

S → T a P

T → b T | b

P → Q | T Q

Q → b d

T → b

2.35 (b):

Minimal set of LR(0) items is given above.

2.36 (c):

(a) Lexical Analysis → DFA minimization (T)

(b) Parsing → Top down parsing (F)

(c) Register allocation → Graph coloring (T)

(d) Runtime environment → Post order traversal (F)

2.36 (b):

Among SLR, CLR, LALR parsers

(i) SLR parser is simple and very easy to implement and speed is their property.

(ii) CLR parser is most powerful. But it is very [illegible] and memory [illegible] LALR parser.

2.37 (d):

S → A | B

A → [illegible]

B → [illegible]

Given grammar is not ambiguous since strings [illegible] can be generated in [illegible] way.

An ambiguous grammar must have at least two LR(0) items since every LR(0) [illegible] is a viable prefix.

[illegible] Minimum 2 LR(0) items.

■■■■

3 Syntax Directed Translation

3.1 Generation of intermediate code in the base machine model is useful because

(a) it makes implementation of lexical analysis and syntax analysis easier

(b) syntax directed translation can be used for intermediate code generation

(c) it makes retargeting possible by replacing the front end for different source languages

(d) it is not possible to generate code for real machines directly from syntax directed languages

[1991 : 1 Mark]

3.2 A parser is given a stream of tokens as input and produces a parse tree as output. Which of the following statements is true about the parser?

(a) Top down parser

(b) Bottom up parser

(c) Number of nodes in the parse tree depends on the input string only

(d) Absolute [illegible]

[1993 : 1 Mark]

3.3 In the following grammar

$X \to X \& Y \mid Y$

$Y \to Y \# Z \mid Z$

$Z \to id$

Which of the following is true?

(a) '#' is left associative while '&' is right associative

(b) Both '&' and '#' are left associative

(c) '&' is for right associative while '#' is left associative

(d) None of the above

[1997 : 1 Mark]

3.4 In a bottom up evaluation of a syntax directed definition, inherited attributes can

(a) always be evaluated

(b) be evaluated only if the definition has only synthesized attributes

(c) be evaluated only if the definition has only inherited attributes

(d) be evaluated

[2008 : 1 Mark]

3.5 Consider the translation scheme shown below.

$S \to T\ \{ \}$

$T \to T_1 * F\ \{ T.val = T_1.val \times F.val \}$

$T \to F\ \{ T.val = F.val \}$

$F \to (S)\ \{ F.val = S.val \}$

$F \to num\ \{ F.val = num.val \}$

Here num is a token that represents an integer and num.val denotes the corresponding integer value. For a string 2 * (3 * 4), this translation scheme will [illegible]

(a) 2 3 4 * * (b) 2 3 * 4 *

(c) [illegible] (d) [illegible]

[2009 : 2 Marks]

3.6 Consider the [illegible] given below:

$S \to id := E$ { gen(top.place '=' E.place); }

$E \to E_1 + E_2$ { [illegible] }

$\quad$ | $E_1 * E_2$ { [illegible] }

$\quad$ | $-E_1$ { [illegible] }

$\quad$ | (E_1) { [illegible] }

$\quad$ | id { E.place = id.place; }

Here gen is a function that generates [illegible] and top.place [illegible] a temporary variable name generator. [illegible]

For the statement $X := Y + Z$, the 3-address code sequence generated by this definition is

(a) $X = Y + Z$

(b) $t_1 = Y + Z;\ X = t_1$

(c) $t_1 = Y;\ t_2 = t_1 + Z;\ X = t_2$

(d) $t_1 = Y;\ t_2 = -Z;\ t_3 = t_1 + t_2;\ X = t_3$

[2010 : 2 Marks]

3.7 Consider the grammar rule $E \to E_1 - E_2$ for arithmetic expressions. The code generation is done in a single user defined function. The [illegible] function requires the first operand to be in the register [illegible] and the second operand in any register. [illegible] the following code:

[illegible]

(a) [illegible]
(b) [illegible]
(c) Evaluation of E_1 and E_2 should be done by [illegible]
(d) Order of evaluation of E_1 and E_2 should be [illegible]

[2001 : 1 Mark]

8.8 Consider the grammar with the following production rules and the start symbol E.

$E \rightarrow E_1 + T$ {E.value = E.value + T.value}
| T {E.value = T.value}
$T \rightarrow T_1 * F$ {T.value = T_1.value * F.value}
| F {T.value = F.value}
$F \rightarrow num$ {F.value = num.value}

Compute E.value for the root of the parse tree for the expression 2 * 3 + 6 * 4.

(a) 30 (b) [illegible]
(c) 108 (d) 40

[2006 : 2 Marks]

8.9 Consider the [illegible] $S \rightarrow a \mid b \mid (T)$, $T \rightarrow T, S \mid S$. Here a, b, (,), , are terminal symbols. The [illegible] for right recursion free and left recursion free versions of the grammar are [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible] and [illegible]
(d) [illegible]

[2006 : 2 Marks]

Common Data for Q. 8.10 & Q. 8.11

Consider the following expression grammar. The semantic rules for arithmetic operations are as usual. Assume a recursive descent parser for the grammar.

$T \rightarrow num = num \mid E$ {T.val = num.val}
| $E_1 + E_2$ {T.val = E_1.val + E_2.val}
| $E_1 * E_2$ {T.val = E_1.val × E_2.val}

8.10 The above grammar and the semantic rules are to be implemented in a recursive descent parser. Which one of the following is true about the action of the parser for the given grammar?

(a) It performs a left-to-right scan and a left-most derivation
(b) It performs a left-to-right scan and a right-most derivation
(c) It detects a shift-reduce conflict and resolves the conflict in favour of a shift over a reduce action
(d) It detects a shift-reduce conflict and resolves the conflict in favour of a reduce over a shift action

[2006 : 2 Marks]

8.11 Assume the precedence in decreasing order is * then +. Which of the following is a valid parse tree for the expression 2 * 3 + 6 * 4 + 1? Which associativity property does the corresponding parser follow?

(a) Input precedence and left associativity for expression is evaluated in T
(b) Input precedence and right associativity for expression is evaluated in T
(c) Precedence of '+' is higher than that of '*' and left associativity for expression is evaluated in T
(d) Precedence of '+' is higher than that of '*' and both operators are left associative for expression is evaluated in T

[2006 : 2 Marks]

8.12 Consider the following translation scheme.

$S \rightarrow aAb$
$A \rightarrow Ra \mid a$
$R \rightarrow a$

Here S is the start symbol, a and b are terminal symbols and R and A are non-terminal symbols. The number of reduce moves taken by a shift-reduce parser on input aaaab is _____.

[2008 : 2 Marks]

8.13 Consider the following translation scheme.

$S \rightarrow bA$
$A \rightarrow Ra \mid a$
$R \rightarrow a$

Here S is the start symbol, a and b are terminal symbols and R and A are non-terminal symbols. The number of shift moves taken by a shift-reduce parser on input baaaa is _____.

[2008 : 2 Marks]

8.14 The simplified [illegible] instruction set of a hypothetical processor is given below.

MOV R, S — Register R ← Register S
ADD R, S — Register R ← Register R + Register S
MOV R, M — Register R ← Memory location M
ADD R, M — Register R ← Register R + Memory location M

The instruction set architecture has exactly [illegible] registers. Each register can hold any integer value. Initially, register R0 stores the address of the first memory location in the array A, R1 stores the address of the last memory location in the array A, and R2 stores the integer 0. The array A contains [illegible] integers. The [illegible] instruction is executed [illegible] times.

[illegible]

Consider the following statements

$t_1 = a + b$

$t_2 = c + d$

$t_3 = e - t_2$

$t_4 = t_1 - t_3$

Assume that all operands are stored in memory. The first two of the computations are held in memory. What is the minimum number of [illegible] instructions in the code generated for the above block?

(a) 2 (b) 3

(c) 6 (d) 9

[2007 : 2 Marks]

3.14 Consider an infix binary operator '*' and '/' with the precedence of operators being equal. Let the left associativity of the operator '*'. Let the right associativity of '/'. Which of the following represents the parse tree for expression $a * b / c$ / d?

(a)

(b)

(c)

(d)

[2011 : 2 Marks]

3.15 Consider the [illegible] shown below. Each leaf represents a number and internal node represents an operator. Consider all possible binary trees with the above three nodes. How many of these trees represent expressions which evaluate to 10?

[2014 (Set-3) : 2 Marks]

3.16 One of the phases of the intermediate code generation is

(a) [illegible]

(b) [illegible]

(c) [illegible] the machine independent optimizer in other compilers

(d) [illegible]

[2014 (Set-3) : 1 Mark]

3.17 The evaluation of [illegible] expression in some programming language are given below.

Operator	Precedence	Associativity
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

The value of the expression [illegible] is

[2014 (Set-1) : 2 Marks]

3.18 Consider the following syntax directed translation scheme (SDTS), with non-terminals S, A and terminals a, b.

$S \rightarrow aA$ {print 1}

$S \rightarrow a$ {print 2}

$A \rightarrow Sb$ {print 3}

Using the SDTS, the output printed for the input aab is

(a) [illegible] (b) 2 2 3

(c) 3 3 1 (d) syntax error

[2016 (Set-1) : 2 Marks]

Answers: Syntax Directed Translation

3.	(a)	3.7	(d)	3.8	(a)	3.9	(b)	3.10	(d)	3.11	(b)	3.12	(b)	3.13	(c)	3.14	(d)
3.15	(d)	3.16	(a)	3.17	(b)	3.18	(b)	3.19	(b)	3.20	(c)	3.21	(c)				

Explanations: Syntax Directed Translation

3.1 (d)

[illegible] abstract machine model to model the [illegible] because it [illegible] implementation of lexical analysis and syntax analysis phases.

3.2 (d)

[illegible] generated by compiler and combined from [illegible] in program address space. Therefore absolute [illegible] are included in object module.

3.3 (a)

Grammar syntax tree for [illegible]

Therefore [illegible] is left associative.

Grammar syntax tree for [illegible]

Therefore [illegible] is right associative.

3.4 (b)

Every (synthesized) attributed definition can be implemented with bottom-up parsing extended to some [illegible] parameters. Consider the following example

Production	Semantic Rule
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

In the example above for nonterminal [illegible] we indicate the number of [illegible] generated by [illegible]. Since the production [illegible] is the last that a [illegible] would reduce by, the value at this [illegible] is the number of [illegible] from the input. [illegible] inherited attributes can be evaluated in the [illegible] but [illegible] in bottom-up parsing [illegible] simply [illegible] implement [illegible] inherited attributes [illegible] inherited attributes [illegible]

3.5 (b)

The [illegible] 8 + 5 = 3 [illegible] as shown below:

3.6 (b)

[illegible] Y + Z and only one [illegible] is [illegible]

$\therefore L_1 = Y + X, X = _$

8.7 (b):

[illegible]

8.8 (b):

[illegible]

[illegible]

8.9 (d):

[illegible]

8.10 (d):

[illegible]

8.11 (b):

[illegible]

8.12 (d):

[illegible]

8.13 (b):

[illegible]

3.14 Sol:

3.15 Sol:

4.16 Sol:

Intermediate code generation is machine independent code optimization. Machine independent code optimization improves the intermediate code.

3.17 Sol:

Since given expression is infix expression. So we convert postfix form.

[illegible]

So [illegible] digit [illegible]

[illegible]

Postfix Evaluation [illegible]

[illegible]

So [illegible] = 9.

So [illegible] evaluation is 9.

3.18 Sol:

Event is [illegible]

[illegible] for given input

So output will be 1, 2, 3 and 1 because process order will be 1, 2, 3.

So [illegible]

4 Code Generation and Optimization

4.1 Consider the following C code segment

```
int i = 0, j = 0, x = 0, y;
while (i < 10)
{
    int j = 0;
    while (j < 10)
    {
        if (j % 2)
        {
            x = x + (4 * j) + 5 * i;
            y = y + (7 + i * j);
        }
        j = j + 1;
    }
    i = i + 1;
}
```

Which one of the following statements is false?

(a) The code contains loop invariant statements

(b) There is scope for common sub-expression elimination in this code

(c) There is scope for strength reduction in this code

(d) There is scope for loop unrolling in this code

[2008 : 2 Marks]

4.2 Consider the following statements about compiler intermediate code optimizations

(a) They increase the portability of the compiler from one target processor to another

(b) Dataflow analysis is more accurate on intermediate code than on machine code

(c) The information from data flow analysis cannot be used for machine dependent optimizations

(d) The information from data flow analysis cannot be used for machine independent optimizations

[2008 : 1 Mark]

4.3 Which of the following statements is/are true about allocation in for loop in a compiler?

(a) It is the compiler's responsibility

(b) It is done at the compiler time

(c) It is done at run time

(d) It is done at link time

[2010 : 1 Mark]

Common Data for Questions 4.4 and 4.5

The following code segment is executed on a processor which has two general purpose registers R1 and R2. Instruction cost is 1 cycle if operands are in registers, 2 cycles if one operand is in memory, and 3 cycles if both operands are in memory. Each operand fetch, instruction execution, and write back takes one cycle.

```
a = 2 + a;
b = a * b;
c = 2 + a;
x = c * c;
if (x > 8)
    x = x + a;
d = a * a;
e = d * d;
```

4.4 Suppose the intermediate code statements are executed in a machine with two registers. The optimal code generation results in the minimum number of cycles. What is the minimum number of cycles required?

(a) 11 (b) 12

(c) 13 (d) 14

[2013 : 2 Marks]

4.5 What is the minimum number of registers needed in the instruction set architecture of the processor to generate code that results in no spill/reload? Do not consider any instructions other than those required for operands allocation.

(a) 1 (b) 2

(c) 3 (d) 4

[2013 : 2 Marks]

4.6 What is/are true about LALR(1)?

(a) It accepts a superset of LR(1) languages

(b) Similar to canonical LR(1) parser, it can be used for automatic parser generation

(c) Type checking can be done in the parser

(d) $x = 4 * 5 - x$ is an example of common subexpression elimination

[2014 (Set-1) : 1 Mark]

4.7 [illegible] is NOT performed during compilation?

(a) Dynamic [illegible]
(b) Type checking
(c) [illegible]
(d) [illegible]

[2014 (Set-2) : [illegible] Marks]

4.8 [illegible] the following intermediate code is generated by a compiler. Assume that the variables x, y and z are integers, and their values are stored in locations x, y and z, respectively.

[illegible]

Which one of the following statements about the optimal code for the given statement is CORRECT?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2014 (Set-2) : 2 Marks]

4.9 Which of the following statements is/are TRUE?

I. [illegible] by a compiler [illegible]
II. [illegible]
III. [illegible]
IV. [illegible]

(a) I and II only
(b) [illegible] only
(c) I and I only
(d) [illegible]

[2014 (Set-3) : [illegible] Marks]

4.10 Consider the [illegible] basic block.

[illegible]

The number of [illegible] and edges present in the DAG representation of the [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2014 (Set-3) : 2 Marks]

4.11 A variable x can be [illegible] in the following [illegible]

I. [illegible]
II. [illegible]
III. [illegible]

Choose the [illegible] in which the statement in block 2 and in the [illegible] is dead.

(a) [illegible] only
(b) [illegible]

[2015 (Set-1) : 2 Marks]

4.12 [illegible] required to [illegible]

[2015 (Set-1) : 2 Marks]

4.13 [illegible] (AST) and CFG [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2015 (Set-2) : 1 Mark]

4.14 Consider the intermediate code given below

```
1.  i = 1
2.  j = 1
3.  t1 = 5 * i
4.  t2 = t1 + j
5.  t3 = 4 * t2
6.  t4 = t3
7.  a[t4] = -1
8.  j = j + 1
9.  if j <= 5 goto (3)
10. i = i + 1
11. if i < 5 goto (2)
```

The number of nodes and edges in the control flow graph constructed for the above code respectively are

(a) 5 and 6 (b) 6 and 7

(c) 5 and 7 (d) 7 and 8

[2013 (Set-2)] 2 Marks

4.15 Consider the following statements

```
x = 10 - 5;
z = x * 2;
y = y + 10;
z = z - x;
y = y * y;
```

The minimum number of variables required to represent the above code segment in static single assignment form is _____.

[2016 (Set-1)] 1 Mark

4.16 A grammar is said to be in Greibach normal form (GNF) if all production rules are of the form $A \to a\alpha$ where A is a non-terminal, a is a terminal symbol and α is a string of non-terminals. The dimension of the grammar is the maximum number of non-terminals in the right side of any production. For example,

$$S \to aAB \mid bA \mid a$$

The grammars $G1$ and $G2$ are in Greibach normal form and use the terminal symbols set, id [] num.

Grammar G1	Grammar G2
$D \to id\ [E]$	$E \to id\ [A$
$E \to id\ [E$	$A \to id$
$E \to num$	$A \to A\ num$
$E \to num\ [E$	$A \to num$

Which of the grammars is correctly generates the declaration mentioned above?

(a) Only G1

(b) Only G2

(c) Both G1 and G2

(d) Neither G1 nor G2

[2016 (Set-3)] 2 Marks

■■■■

Answers Code Generation and Optimization

4.1	(d)	4.2	(b)	4.3	(c)	4.4	(b)	4.5	(b)	4.6	(d)	4.7	(a)	4.8	(a)	4.9	(d)		
4.10	(a)	4.11	(c)	4.12	(c)	4.13	(b)	4.14	(a)	4.15	(3)	4.16	(a)						

Explanations Code Generation and Optimization

4.1 (d)

(a) 15 is a loop invariant and can be moved before the loop.

(b) 4*j is common sub-expression and can be replaced by a temporary.

(c) 4*j can be replaced by a temporary in strength reduction.

(d) There is no dead code in given code segment. So there is no scope of dead code elimination in this code.

Hence only option (d) is correct.

4.2 (b)

Dead code and common sub-expressions are carried out on the intermediate code because program analysis is more accurate on intermediate code than on machine code.

4.3 (c)

Run-time environment means we deal with dynamic memory allocation and stack is a dynamic data structure.

[illegible] dynamic code structure necessarily need heap allocation when structure increases in size.

4.4 (b)

[illegible]

4.5 (b)

[illegible]

4.6 (d)

[illegible]

4.7 (a)

[illegible]

4.8 (d)

[illegible]

4.9 (a)

[illegible]

4.10 (a)

[illegible]

4.11 (d)

[illegible]

(i) [illegible] Block 2 and Block 3. There is a path from S_1 & S_2 to S_3, but S_3 has no assignment [illegible]

(ii) [illegible] Block 2 and Block 3. There is a path from S_1 & S_2 to S_3, but S_3 has no assignment to [illegible]

(iii) [illegible] Block 2 and Block 3. There is a path from S_1 & S_2 to S_3. S_3 [illegible] assignment to [illegible]

(iv) [illegible] Block 2 and Block 3. There is a path from S_1 and S_2 to S_3 [illegible] both S_1 and S_2 [illegible] but S_3 has no assignment [illegible] this path.

(v) Similarly [illegible] for [illegible] Block 2 and Block 3

(vi) Only [illegible] Block 2 and Block 3.

4.12 [illegible]

In static single assignment, every variable assigned only once and that [illegible] number of times without assignment.

Expression [illegible]

$t_1 = 10$

$t_2 = [illegible]$

$t_3 = t_1 + t_2$

$t_4 = [illegible]$

$t_5 = [illegible]$

$t_6 = [illegible]$

$t_7 = t_5 + t_6$

$t_8 = t_3 + t_7$

So temporary variables are [illegible]

4.13 [illegible]

Maximum number of [illegible] code in an AST and a DAG depends on input grammar. DAG does contain a cycle.

[illegible] node may correspond [illegible] expression

5.11 [illegible]

Number of nodes = [illegible]

Number of edges = [illegible]

5.15 [illegible]

$t_1 = [illegible]$

$t_2 = [illegible]$

$t_3 = [illegible]$

$t_4 = [illegible]$

$t_5 = [illegible]$

[illegible] assignment form [illegible] variable and [illegible] have [illegible] one of [illegible] the code [illegible] the code [illegible] are needed.

4.16 [illegible]

Both G1 and G2 [illegible] DDG [illegible] as follows:

G1 [illegible]

G2 [illegible]

Unit - VIII

Operating System

■ **Contents**

Sl.	Topic	Page No.
	Process Management-I	
	Introduction, Process, Threads & CPU Scheduling	420
2.	Process Management II	
	CPU Synchronization and Concurrency	435
3.	Deadlock	447
4	Memory Management and Virtual Memory	454
5.	File System and Device Management	470

UNIT VIII Operating System

Syllabus : [illegible] CPU scheduling, [illegible] memory [illegible] I/O systems, Protection [illegible]

Analysis of Previous GATE Papers

Exam Year	1 Marks Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1987	–	7		2	24
1988	–	4			12
1989	1	2			10
1990	–	5		2	20
1991	–	2		2	14
1992	1	5		1	20
1993	1	4		2	22
1994	7				11
1995	9	4		2	26
1996	2	4		2	19
1997	[illegible]	1			12
1998	10	5		1	1
1999	[illegible]	5		4	31
2000	3	5		[illegible]	14
2001	[illegible]	[illegible]		[illegible]	11
2002	–	2		–	4
2003	7	2		4	17

Exam Year	1 Marks Ques.	2 Marks Ques.	Total Marks
2007	[illegible]	9	14
2008	2	5	12
2009	2	5	12
2010	2	3	7
2011	1	3	8
2012	1	4	8
2013	2	4	10
2014 (Set-1)	2	[illegible]	8
2014 (Set-2)		3	7
2014 (Set-3)		3	7
2015 (Set-1)	2	4	10
2015 (Set-2)	2	3	5
2015 (Set-3)	2	2	6
2016 (Set-1)	1	4	9
2016 (Set-2)	1	3	7

Process Management-I

(Introduction, Process, Threads & CPU Scheduling)

1.1 Resource sharing, Elasticity, Scalability, which of the following is not increasing performance of a computer system?

(a) Turnaround time

(b) Waiting time

[1998 : 2 Marks]

1.2 The [illegible] is the [illegible] time [illegible] the [illegible] time [illegible]

[1998 : 2 Marks]

1.3 Assume that the following jobs to be executed on a single processor system are [illegible]

Job Id	CPU Burst Time
[illegible]	4
[illegible]	1
[illegible]	8
[illegible]	1
[illegible]	2

Jobs are assumed to have arrived at time 0 in the order [illegible]. Calculate the average turnaround time [illegible] if scheduling is done with same slice 1.

(a) [illegible] (b) 10

(c) 11 (d) [illegible]

[1998 : 2 Marks]

1.4 Which of the following policy is most suitable for a time sharing operating system?

(a) Shortest Job First

(b) Round Robin

(c) First Come First Serve

(d) None of

[1998 : 1 Mark]

1.5 The sequence [illegible] is a priority based non preemptive scheduling sequence for the following jobs which leave the CPU idle for [illegible] unit of time.

Job	Arrival time	Burst Time
1	0.0	7
2	0.5	3
3	[illegible]	1

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[1998 : 2 Marks]

1.6 The process state transition diagram figure is represented

(a) [illegible]

(b) an operating system with a preemptive scheduler

(c) an operating system [illegible] preemptive scheduler

(d) a multiprocessor operating system

[1996 : 1 Mark]

1.7 Which of the following is an example of spooled device?

(a) A line printer used to print the output of a number of jobs

(b) A terminal used to enter input data to a running program

(c) A secondary storage device in a virtual memory system

(d) A video display device

[1996 : 1 Mark]

1.8 Four jobs to be executed on a single processor system arrive at time 0 in the order A, B, C, D. Their burst CPU time requirements are 4, 1, 8, 1 time units respectively. The completion time of A under round robin scheduling with time slice of one time unit is

(a) 10 (b) 4

(c) 9 (d) 5

[1998 : 2 Marks]

1.9 Which of the following is an example of spooled device?

(c) [illegible]
(d) [illegible]
(e) [illegible]
(f) [illegible]

[1995 : 1 Mark]

1.10 [illegible] what must be the [illegible] quantum size [illegible] so that [illegible] is [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[1998 : 1 Mark]

1.11 [illegible] the average response time?

[1995 : 2 Marks]

1.12 [illegible]
(a) [illegible]
(b) polling
(c) [illegible]
(d) [illegible]

[1999 : 1 Mark]

1.13 [illegible] multiprocessing [illegible]
(a) [illegible]
(b) address translation
(c) [illegible]
(d) At least [illegible] CPU [illegible] privileged and non privileged
(e) Demand paging

[1998 : 2 Marks]

1.14 [illegible]
(a) [illegible]

(b) Changing [illegible]
(c) [illegible]
(d) [illegible]

[1998 : 2 Marks]

1.15 [illegible]
(a) [illegible]
(b) [illegible]
(c) [illegible] privileged instructions
(d) [illegible]

[2001 : 1 Mark]

1.16 [illegible]
(a) [illegible]
(b) [illegible]
(c) a privileged instruction [illegible] interrupts is needed
(d) a non-privileged instruction [illegible] interrupts is needed

[2001 : 1 Mark]

1.17 [illegible] throughput?
(a) Round Robin
(b) Shortest Job First
(c) [illegible]
(d) [illegible]

[2001 : 1 Mark]

1.18 [illegible] scheduling algorithm is non-preemptive?
(a) Round Robin
(b) First in first out
(c) Multi Level Queue Scheduling
(d) Multi Level Queue Scheduling with feedback

[2001 : 1 Mark]

1.19 [illegible] not interrupt a running process?
(a) Address (b) Timer
(c) Scheduler process (d) Power failure

[2001 : 2 Marks]

1.20 [illegible] subject to characteristics of OS as a multi-[illegible] [illegible] propagation by

[illegible]

(a) (i) (b) (i) and (ii)
(c) (ii) and (iii) (d) [illegible]

[2002 : 2 Marks]

1.27 [illegible]

[2002 : 2 Marks]

1.28 [illegible]

[illegible]

(a) [illegible]
(b) Static priority scheduling with different [illegible] for the two processes
(c) Round-robin scheduling with a time quantum of 5 sec.

[2001 : 2 Marks]

1.29 Consider the following statements with respect to user-level threads and kernel-supported threads

(i) Context switching is faster with kernel-supported threads
(ii) Blocking a user-level thread causes the kernel to block the entire process
(iii) Kernel-supported threads can be scheduled independently
(iv) User-level threads are managed entirely at the kernel

Which of the above statements are true?

(a) (i), (ii) and (iii) only (b) (ii) and (iii) only
(c) (iii) and (iv) only (d) (i) and (iii) only

[2001 : 1 Mark]

1.30 [illegible]

(a) Stack
(b) Address space
(c) File Descriptor Table
(d) Message Queue

[2004 : 2 Marks]

1.31 [illegible]

There are several two parameter instructions in the [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

1.32 Consider the following set of processes with the arrival times and the CPU-burst times given in milliseconds

Process	Arrival time	Burst time
[illegible]	[illegible]	5
[illegible]	[illegible]	3
[illegible]	[illegible]	3
[illegible]	[illegible]	[illegible]

What is the average waiting time for these processes with the preemptive shortest remaining time first (SRPT) algorithm?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

1.33 A multiprocessor computer system has [illegible] CPU's. Computation has [illegible] scheduling [illegible] appropriate [illegible] terminating the process. [illegible] which the overhead of handling contains [illegible]?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 1 Mark]

1.34 Given the following [illegible] processes P1, P2 and P3 on a uniprocessor system [illegible] CPU time [illegible] and arrival times are [illegible] below.

Process	Priority	CPU burst required	Arrival time (milliseconds)
P1	[illegible] (highest)	[illegible]	[illegible]
P2	5	[illegible]	[illegible]
P3	7 (lowest)	[illegible]	[illegible]

[illegible] preemptive or non-preemptive scheduling. In preemptive scheduling, [illegible] assuming higher priority process can preempt a running process with lower priority. In non-preemptive scheduling, higher priority [illegible] process must wait until the currently executing process completes before it can be scheduled on the processor. Which of the [illegible] time [illegible] arrival [illegible] completion of P2 in a preemptive and non-preemptive scheduling respectively?

(a) 20 sec, 30 sec (b) 30 sec, 10 sec

(c) 12 sec, 18 sec (d) 20 sec, [illegible] sec

[2006 : 2 Marks]

129. Two shared resources R_1 and R_2 are used by processes P_1 and P_2. Each process accesses a pair of resources as follows: P_1 first locks the resource R_1 and then locks R_2. Similarly, P_2 locks resource R_2 and then locks R_1.

Given the following:

I. $T_{P1} > T_{P2}$ II. $T_{P1} < T_{P2}$

III. $T_{P1} = T_{P2}$ IV. $T_{P1} \neq T_{P2}$

Which of the following conditions ensure that P_1 and P_2 never deadlock?

(a) (I) and (IV) (b) (II) and (III)

(c) (I) and (III) (d) None of these

[2006 : 2 Marks]

130. Consider the following [illegible]

[illegible] link [illegible]

[illegible] a pointer [illegible]

[illegible] a pointer [illegible]

Let [illegible] be the values returned by the [illegible] and [illegible] in the [illegible] pointer [illegible] be dereferenced [illegible] Which one of the following is TRUE?

(a) [illegible] and [illegible]

(b) [illegible] and [illegible]

(c) [illegible] and [illegible]

(d) [illegible] and [illegible]

[2006 : 2 Marks]

131. Consider three CPU-intensive processes, which require 10, 20 and 30 units of time and arrive at times 0, 2 and 6, respectively. How many context switches are needed if the operating system implements a shortest remaining time first scheduling algorithm? Do not count the context switches at time zero and at the end.

(a) 1 (b) 2

(c) 3 (d) 4

[2006 : 1 Mark]

132. The process state transition diagram of an operating system is given below.

Which of the following may be the state of the above operating system?

(a) Uniprocessor multiprogrammed operating system

(b) Time sharing preemptive scheduling

(c) Batch uniprogrammed scheduling

(d) Uniprocessor operating system

[2006 : 1 Mark]

133. The execution time, priority and duration of CPU and I/O bursts for each of three processes P_1, P_2 and P_3 are given in the table below. Each process has a CPU burst followed by an I/O burst. Assume that only one CPU burst can be allocated at a time. Assume that each process uses only one I/O resource.

Process	Arrival time	Priority	Burst duration (CPU, I/O, CPU)
P_1	0	2	1, 5, 1
P_2	2	1 (Highest)	3, 3, 1
P_3	4	3 (Lowest)	4, 1, 1

The multiprogrammed operating system uses preemptive priority scheduling. What are the finish times of the processes P_1, P_2 and P_3?

(a) 11, 15, 8 (b) 11, 7, 8

(c) 11, 15, 10 (d) 12, 7, 10

[2006 : 2 Marks]

134. Consider three processes, all arrive at time zero, with total execution time of 10, 20 and 30 units, respectively. Each process spends the first 20% of execution time doing I/O, the next 70% of time doing computation, and the last 10% time doing I/O again. The operating system uses a shortest remaining time first scheduling algorithm and schedules a new process either when the running process gets blocked on I/O or when the current process finishes. A new process is picked [illegible]

when the mainly operations [illegible] occur jobs [illegible]. Assume that all I/O operations can be overlapped as much as possible. For what percentage of time does the CPU remain idle?

(a) [illegible] (b) [illegible]
(c) 50.0% (d) [illegible]

[2006 : 2 Marks]

1.36 Consider three processes (process id 0, 1, 2) with compute time bursts 2, 4, and 8 time units. All processes arrive at time zero. Consider the gantt chart resulting from FIFO scheduling of these jobs. If [illegible] is used by giving priority to the processes in [illegible] order, the average turnaround time is

(a) 12 units (b) [illegible]
(c) [illegible] units (d) 16 units

[2006 : 2 Marks]

1.37 Let [illegible] be four blocks of a cache. [illegible] The cache is initially empty. The following sequence of accesses to blocks [illegible] occurs in the cache. The [illegible] in the cache at the end of the sequence is

Cache Block	Required access	Time
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

The [illegible] which has remains for [illegible] will be completed will be

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2007 : 1 Mark]

1.37 Match the following CPU scheduling algorithms with List-II to name some applications. Match correctly between List-I and List-II.

List-I

A. Long Term Scheduling
B. Short Term Scheduling
C. Medium Term Scheduling

List-II

1. Decision to add to the pool of processes to be executed
2. Deciding which process to run next
3. Deciding which process to be swapped out

Codes:

	A	B	C
(a)	[illegible]	2	1
(b)	[illegible]	2	3
(c)	[illegible]	3	1
(d)	[illegible]	3	2

[2007 : 1 Mark]

1.38 Consider the following statements about user level threads and kernel level threads. Which one of the following statements is FALSE?

(a) Context switch time is longer for kernel level threads than for user level threads
(b) User level threads can be used on an operating system that does not support multiprocessor threads
(c) Kernel level threads can be used to exploit the parallelism available in multiprocessor systems
(d) Blocking one kernel level thread blocks all related threads

[2007 : 1 Mark]

1.39 An operating system uses Shortest Remaining Time first (SRT) process scheduling algorithm. Consider the arrival times and execution times for the following processes:

Process	Execution time	Arrival time
[illegible]	[illegible]	0
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

What is the total waiting time for process P2?

(a) 5 (b) 15
(c) 40 (d) 50

[2007 : 2 Marks]

1.40 If the time taken by the scheduler is greater than the maximum time required for any process, then the preferred is

(a) Long term scheduler
(b) Medium term scheduler
(c) Short term scheduler
(d) None of the above

[2005 : 2 Marks]

1.41 Which of the following statements is true regarding synchronous and asynchronous I/O?

(a) An ISR is invoked in response to I/O in asynchronous I/O but not in synchronous I/O
(b) In both synchronous and asynchronous I/O an ISR (Interrupt Service Routine) is invoked after completion of the I/O

(c) A process making a synchronous I/O is blocked until I/O is completed but a process making an asynchronous I/O call does not wait for completion of the I/O

(d) In the case of synchronous I/O, the process waits until the I/O is completed but in asynchronous I/O, the I/O is initiated after the completion of I/O

[2008 : 2 Marks]

1.45 A process requires the following resources in order to complete its execution:

R1 : 2 units
R2 : 1 unit
R3 : 1 unit

The minimum number of units required in R1 to avoid deadlock is

(a) 4 (b) 5
(c) 6 (d) 7

[2008 : 2 Marks]

1.46 In the following process state transition diagram for a uniprocessor system, assume that there are always some processes in the ready state:

Now consider the following statements:

I. If a process makes a transition D, it would result in another process making a transition A immediately

II. A process P2 in blocked state can make transition E while another process P1 is in running state

III. The OS uses preemptive scheduling

IV. The OS uses non-preemptive scheduling

Which of the statements are true?

(a) I and II (b) I and III
(c) II and III (d) I and IV

[2009 : 2 Marks]

1.47 Which of the following statements are true?

I. Shortest remaining time first scheduling may cause starvation

II. Preemptive scheduling may cause starvation

III. Round robin is better than FCFS in terms of response time

(a) I only (b) I and II only
(c) II and III only (d) I, II and III

[2010 : 1 Mark]

1.48 Two threads share a variable between each other. Each thread has a critical section of code to access the variable. While one thread is in its critical section, the other thread cannot enter its critical section.

Which of the following is TRUE?

(a) $t_1 < t_2$
(b) $t_1 = t_2$
(c) $t_1 > t_2$
(d) nothing can be concluded about the relative ordering of t_1 and t_2

[2011 : 1 Mark]

1.49 A computer executes several interrupt service routines. The following events occur in the order shown:

- Interrupt from CPU temperature sensor (assume interrupt CPU temperature is too high)
- Interrupt from Mouse (assume mouse interrupt if the mouse is moved or button is pressed)
- Interrupt from Keyboard (assume interrupt when a key is pressed or released)
- Interrupt from Hard Disk (assume interrupt when a disk read is completed)

Which one of these will be handled as the HIGHEST priority?

(a) Interrupt from Hard Disk
(b) Interrupt from Mouse
(c) Interrupt from keyboard
(d) Interrupt from CPU temperature sensor

[2011 : 1 Mark]

1.50 A task is normally allocated as a light weight process by a Operating System (OS) multiple threads can share data structures for a task. Which one of the following is TRUE?

(a) On per thread basis, the OS maintains only CPU register state

(b) The OS does not maintain a separate stack for each thread

(c) On per thread basis, the OS does not maintain virtual memory state

(d) On per thread basis, the OS maintains only scheduling and accounting information

[2011 : 1 Mark]

1.51 Consider the following table of arrival time and burst time for three processes P0, P1 and P2

Process	Arrival time	Burst time
P0	0 ms	9 ms
P1	1 ms	4 ms
P2	2 ms	9 ms

The non-preemptive shortest job first scheduling algorithm would be a better choice than the round robin scheduling if we are interested in minimizing the average waiting time for the given processes.

(a) 6.0 ms (b) 5.0 ms
(c) 7.75 ms (d) 7.25 ms

[2011 : 2 Marks]

Q.48 A process executes the code

```
fork();
fork();
fork();
```

The total number of child processes created is

(a) 3 (b) 4
(c) 7 (d) 8

[2012 : 1 Mark]

Q.49 Consider three processes P1, P2 and P3 given in the table.

Process Name	Arrival Time	Time Units Required
P1	0	5
P2	1	7
P3	3	4

The completion order of the processes under the policies FCFS and RR2 (round robin scheduling with CPU quantum of 2 time units) are

(a) FCFS: P1, P2, P3 RR2: P1, P2, P3
(b) FCFS: P1, P3, P2 RR2: P1, P3, P2
(c) FCFS: P1, P2, P3 RR2: P1, P3, P2
(d) FCFS: P1, P3, P2 RR2: P1, P2, P3

[2012 : 2 Marks]

Q.50 A scheduling algorithm assigns priority proportional to the waiting time of a process. Every process starts at priority zero (the lowest priority). The scheduler re-evaluates the process priorities every T time units and decides to run the next process in the ready queue based on the priorities. In ties, FCFS is used. Priorities are not adjusted while a process is waiting in the ready queue. All processes are non-preemptive.

(a) This algorithm is equivalent to the first-come-first-serve algorithm
(b) This algorithm is equivalent to the round-robin algorithm
(c) This algorithm is equivalent to the shortest-job-first algorithm
(d) This algorithm is equivalent to the shortest-remaining-time-first algorithm

[2013 : 1 Mark]

Q.51 Which one of the following is FALSE?

(a) User level threads are not scheduled by the kernel
(b) When a user level thread is blocked, all other threads of its process are blocked
(c) Context switching between user level threads is faster than context switching between kernel level threads
(d) Kernel level threads cannot share the code section

[2014 (Set-1) : 1 Mark]

Q.52 Consider the following set of processes that need to be scheduled on a single CPU. All the times are given in milliseconds.

Process Name	Arrival Time	Processing Time
A	0	7
B	1	5
C	2	4
D	3	2
E	10	1

Using shortest remaining time first scheduling algorithm, the average process turnaround time (in milliseconds) _____

[2014 (Set-3) : 2 Marks]

Q.53 Three processes A, B and C are to be scheduled on a single CPU. Each process alternates between using the CPU and waiting for a fixed period of time on a single I/O device. The processes are ready to start at time zero in the order A, B, C. The CPU time and I/O time required by each process are given below.

Process	CPU time	I/O time
A	30 ms	300 ms
B	150 ms	300 ms
C	300 ms	300 ms

The processes A, B and C are started at time zero and the OS uses round robin scheduling with a time slice of 50 ms. The total time (in ms) taken by the CPU to complete the execution of all the processes is _____

[2014 (Set-3) : 2 Marks]

1.86 An operating system uses shortest remaining time first scheduling algorithm for preemptive scheduling of processes. Consider the following set of processes with the arrival times, burst CPU burst times (in milliseconds).

Process	Arrival Time	Burst Time
P_1	0	12
P_2	3	4
P_3	5	6
P_4	8	2

The average waiting time (in milliseconds) of the processes is _____.

[2014 (Set-I) : 2 Marks]

1.87 Consider a uniprocessor system executing three tasks T_1, T_2 and T_3, each of which is composed of an infinite sequence of jobs (or instances) which arrive periodically at intervals of 3, 7 and 20 milliseconds, respectively. The priority of each task is the inverse of its period and in a given schedule task preemption is allowed. Jobs of T_1 and T_2 have execution times of 1 millisecond and 2 milliseconds, respectively. The first job of T_3 begins execution at the beginning of the 5th millisecond and no more preemptions are allowed. The first instance of T_3 completes its execution at the end of _____ milliseconds.

[2015 (Set-I) : 2 Marks]

1.88 The maximum number of processes that can be in "Ready" state for a computer system with 7 CPU cores is

(a) 6 (b) n^2

(c) 7 (d) Task dependent

[2015 (Set-II) : 1 Mark]

1.89 The 5 processes identified in the following table are to be scheduled using the following scheduling schemes. Which one has the lowest average turnaround time?

Process	Arrival Time	Processing Time
A	1	7
B	2	5
C	3	1
D	4	2
E	5	8

(a) First Come First Serve

(b) Non preemptive Shortest Job First

(c) Shortest Remaining Time

(d) Round Robin with Hard Time Quantum

[2015 (Set-II) : 2 Marks]

1.90 Consider an uniprocessor system executing three tasks with unique CPU burst lengths scheduled at the same time in a computer system. Which one of the following process scheduling algorithm would minimize the average waiting time in the system?

(a) Shortest Job First

(b) Round Robin with time quantum less than the shortest CPU burst

(c) FIFO scheduling

(d) Highest priority first with priority proportional to CPU burst length

[2016 (Set-1) : 1 Mark]

1.91 Consider the following processes, with the arrival time and burst length of the CPU burst given in milliseconds. The scheduling algorithm used is preemptive shortest remaining time first.

Process	Arrival time	Burst time
P_1	0	12
P_2	4	6
P_3	7	1
P_4	8	3

The average turnaround time of these processes is _____ milliseconds.

[2016 (Set-II) : 2 Marks]

Answers Process Management

1.3	(c)	1.4	(b)	1.5	(a)	1.6	(b)	1.7	(a)	1.8	(d)	1.9	(b)	1.10	(c)	1.11	(a)
1.12	(c)	1.13	(c)	1.14	(c)	1.15	(a)	1.16	(d)	1.17	(b)	1.18	(a)	1.19	(d)	1.20	(d)
1.21	(a)	1.22	(a)	1.23	(c)	1.24	(a)	1.25	(a)	1.26	(d)	1.27	(c)	1.28	(d)	1.29	(b)
1.30	(b)	1.31	(b)	1.32	(b)	1.33	(a)	1.34	(c)	1.35	(a)	1.36	(d)	1.37	(b)	1.38	(c)
1.39	(c)	1.40	(b)	1.41	(a)	1.42	(a)	1.43	(d)	1.44	(c)	1.45	(a)	1.46	(c)		

Explanations Process Management

1.1 (c)

Underlying characteristics of

(a) Turn around time: Given time of the process should not be changed.

(b) Waiting time: Arrival time of the processes should not be same.

1.2 (c)

Statement, [illegible]

1.3 (c)

[illegible]

1.4 (b)

Round Robin scheduling [illegible] quantum, after [illegible] processes get [illegible] CPU in a circular manner and for [illegible] time slices [illegible] time sharing system. So Round Robin is best for time sharing system.

1.5 (a)

[illegible]

1.6 (b)

[illegible]

1.7 (a)

Blocked devices are those which [illegible] data [illegible] memory [illegible] processing [illegible] online [illegible] capacity [illegible].

1.8 (d)

[illegible] scheduling [illegible]

[illegible]

[illegible]

1.9 (b)

[illegible] input [illegible] output jobs.

1.10 (c)

[illegible]

[illegible]

So [illegible]

$n = 1$

1.11 (a)

The average response time depends on the value of α.

If value of $\alpha = 0$

then order of execution [illegible]

x_1, x_2, x_3

If value of $\alpha = 0$ & $x_1 > 0$

then order of execution [illegible]

x_1, x_2, x_3

If value of $\alpha = 0$ & $x_1 < 0$

then order of execution [illegible]

x_1, x_2, x_3

If value of $\alpha > 0$

then order of execution [illegible]

x_1, x_2, x_3

Note: Expected time [illegible] increasing order.

1.12 (a):

[illegible] are invoked by using [illegible] instructions.

1.13 (d):

The [illegible] is [illegible] the [illegible]. [illegible] is [illegible] the [illegible]. Otherwise [illegible] program and [illegible] system will be [illegible]. Thus, it is the [illegible] to [illegible] the [illegible] and protection have [illegible].

1.14 (b):

Swapping and [illegible] memory management [illegible] are performed by OS [illegible] [illegible] processes [illegible].

1.15 (a):

[illegible] provides [illegible] instructions, [illegible] [illegible] and [illegible] memory.

1.16 (d):

[illegible] the mode from [illegible] to user [illegible] the [illegible] instructions [illegible] [illegible] privileged [illegible].

1.17 (b):

[illegible] processes [illegible] [illegible] [illegible] in the case of SJF.

1.18 (b):

[illegible] is not preemptive algorithm.

1.19 (b):

[illegible] can [illegible] [illegible] but [illegible] [illegible] [illegible] [illegible] [illegible] processes.

1.20 (b):

(i) Virtual memory by [illegible] [illegible] program.

(ii) [illegible] [illegible] [illegible] can execute [illegible] [illegible].

(iii) [illegible] complicated [illegible] [illegible] [illegible] [illegible].

1.21 (a):

[illegible] the [illegible] by using all criteria as below

1.22 (d):

$TPU = 10$ ms

$TIO = 50$ ms

FCFS:

[illegible]

Round Robin:

[illegible]

Thus, [illegible] [illegible] scheduling with a time quantum [illegible] [illegible] FCFS [illegible].

1.23 (a):

Statement for each statement

(i) Statement is false because there is no [illegible] [illegible] [illegible] [illegible] [illegible].

(ii) Statement is true because [illegible] threads blocking system call [illegible] [illegible] [illegible].

(iii) Statement is true because kernel-supported threads [illegible] [illegible] and scheduled independently by the operating system.

(iv) Statement is true because [illegible] [illegible] [illegible] [illegible] [illegible]

1.25 (a) :

Process	Arrival Time	Burst Time
P_1	0	9
P_2	1	3
P_3	2	5
P_4	4	

The Gantt chart for SRPT (Preemptive shortest remaining time first) algorithm is

0	1	2	3	4	5	6	7	8	9	10	12
P_1	P_2	P_3	P_2	P_3	P_1	P_1	P_1	P_1	P_1	P_1	P_4

Process P_1 arrives at time 0. Since there is no other in process, P_1 is scheduled. Assuming time of P_1 is not preempted before 1 time unit. The remaining time of P_1 is 8 and P_2 arrives. P_2 is not preempted for time units the remaining time of P_2 is 2 so... P_2 is scheduled. The process now is scheduled upon arrival 12.

The waiting time is the interval between submission and completion of a given process.

The waiting time of each process is:

$W_1 = 12 - 0 = 12$

$W_2 = 4 - 1 = 3$

$W_3 = 8 - 2 = 6$

Average waiting time is

$$= \frac{12 + 3 + 6 + 0}{4} = \frac{21}{4} = 5.25$$

1.27 (a) :

When more local processes triggering the Cache signal, then the bus signal is given to bus arbiter and bus arbiter grants access to current master. Now master and slave the sequence handling.

Overwriting cycle and write operations are serviced in current mode operation mode.

1.28 (b) :

TAT = Completion Time − Arrival Time

The Gantt Chart for Non Preemptive scheduling will be (P1P2, (P3P4), (P5P6P7)).

From above table we can calculate average turnaround time for P3 is 16, for P7 is 35 and P5 is 15.

Gantt Chart for Preemptive (P1P2, (P1P3, (P1P4, (P1P2, (P1P5, (P1P6, (P1P7)).

Similarly we can calculate turnaround time for P3, P7 and P5 and then we can find average turnaround time.

Average time is 20.57.

1.29 (b) :

In following Type of Thread, some be scheduled P_1 will get the CPU cycle case K and K.

If K_1 and K_2 are mapped to the process P_1 then P_1 will execute as go and relinquish. After that process P_1 will get both the resources and complete its task.

1.30 (b) :

Here process child is a process that process TD child process is process process.

Text segment, size = 6

Data segment, size = 3

Stack segment, size = 0

Thus, the virtual addresses of parent and child processes will be the same.

1.31 (b)

In this process we have P_1, P_2 and P_3.

Process	Arrival Time	Burst Time
P_1	0	10
P_2	4	3
P_3	8	50

The Gantt chart for SRTF scheduling process is

0	10	13	63
P_1	P_2	P_1	P_3

So there is only two context switches at time unit 10 switches from P_1 to P_2 and at time unit 13 switches from P_2 to P_1.

1.32 (b) :

The time at which a process is scheduling that flow would have been a transition state from running state to ready state. So it is non preemptive scheduling.

1.33 (b) :

At t = 0 CPU is present so process it for time remaining CPU time of P1 = 5 now it will enter into I/O.

At t = 1 process is waiting for CPU but CPU will be in idle and P1 is still in I/O transmitting I/O state of P1 = 1.

At t = 5 P2 and P3 are available for CPU and P3 will be allocated for CPU because priority of P3 will be 20 (remaining CPU time of P2 = 1 & remaining I/O time of P3 = 8).

At t = 8 P3 is executed and it is having highest priority among all processes available for CPU so processor is allocated to P3 for 1 unit of CPU time. At t = 9 P3 will be at priority 20 (remaining CPU time of P3 = 0, now P3 will perform I/O & remaining I/O time of P3 = 1).

At t = 9 only P2 is available for CPU so it will execute for 1 unit and P2 (t = 10) will perform I/O (remaining CPU time of P2 = 1, remaining I/O time of P2 = 2, remaining I/O time of P1 = 4, now P1 will perform CPU).

At t = 9 P2 and P1 are available for CPU but P1 will be selected because it has higher priority (P1 = 15, P2 = 20) (remaining CPU time of P1 = 2 and remaining I/O time of P1 is 3).

At t = 7 again P1 will be executed for 1 unit (remaining CPU time of P1 = 1 & remaining I/O time of P1 = 0, now P1 will perform CPU).

At t = 9 now P1, P2 & P3 are available for CPU so P1 will be selected because it has higher priority (P1 = 14, P2 = 20, P3 = 21) and P1 is executed for 2 units (remaining CPU time of P1 = 0 & P1 completed at t = 10).

At t = 10 only P2 is available for CPU so it will execute for remaining CPU burst of 1 unit (remaining CPU time of P2 = 0, now P2 will perform I/O).

At t = 11 P3 will perform I/O for 5 units and CPU of 1 unit (t = 17).

At t = 17 P3 will perform CPU burst and P3 is completed at t = 18.

Process	Arrival time	I/O burst	CPU burst	Priority
P1	0	4	1	15
P2	5	3	1	20
P3	3	7	5	21

135. (B)

Process	Arrival time	I/O burst	CPU burst	Priority
P1	0	4	1	15
P2	5	3	1	20
P3	3	7	5	21

The Gantt chart is

P1	P3	P2	P3	P1
0	3	5	6	17	18

Total time spent = 18

Idle time = 3 + 1

$$\text{Net CPU time} = \frac{2}{18} \times 100 = 10.5\%$$

136. (D) :

Process	Arrival time	CPU Burst
P_0	0	8
P_1	1	4
P_2	2	9

Remaining Time
P_0 3
P_1 2
P_2 8

The Gantt chart for LRPT CPU scheduling algorithm.

P0	P1	P0	P1	P0	P2	P1	P2	P2
0	1	2	3	4	5	6	10	17	26

Turnaround time for P_0 = 13 − 0 = 13

Turnaround time for P_1 = 6 − 1 = 5

Turnaround time for P_2 = 17 − 2 = 15

Average turnaround time

= 33/3 = 11 ms

137. (D) :

138. (B) :

Listed

A. Rate Monotonic Scheduling

B. Earliest Deadline Scheduling

C. Shortest Slack Scheduling

[illegible]
3. [illegible]
2. [illegible]
1. [illegible]

1.38 [illegible]

In Kernel level threads, the scheduling is done by the kernel, so the kernel can schedule another thread in the application. Statement (i) is false and kernel knows about threads.

1.39 [illegible]

Process	Processing Time	Arrival Time
P_1	25	0
P_2	[illegible]	10
P_3	[illegible]	[illegible]
P_4	[illegible]	[illegible]

The Gantt chart for SRT Scheduling algorithm is

$$P_1 \quad P_2 \quad P_3 \mid P_2 \quad P_4 \mid$$
$$0 \quad 20 \quad 30 \quad 40 \quad 55 \quad 70$$

So the waiting time for

$$W_2 = [70 - 10] - [10 - 10]$$
$$= 60 - 20 = 40$$

1.40 [illegible]

When the quantum used is very small, which schedule gives more than maximum throughput, response time and fairness then it behaves like First Come First Serve.

1.41 [illegible]

Statement (ii) is not true because in LFR (Longest Service Received) scheduling, the complexity of DO in synchronous is not less than asynchronous DO.

1.42 [illegible]

Total [illegible] processes initially. After first fork(), it creates a single process. After second fork(), it creates one process and two child processes. After third fork() the total number of processes is 8. So, without the one in [illegible] the total number of processes is 7.

1.43 [illegible]

Transition Condition from [illegible] to preemptive scheduling.

[illegible]

1.44 [illegible]

Since another remaining time from scheduling process will be less than the remaining time of the high remaining time so [illegible] alternative [illegible]

Shortest remaining time first scheduling will cause a novel [illegible] because of a large CPU burst requires processes in between. [illegible] this will make processes which are coming have less CPU burst time. [illegible] all single executable program [illegible]

[illegible] scheduling [illegible] process priority due to low priority. [illegible] due to life starvation [illegible]

Response time = Time response - process arrival time

In [illegible] time [illegible] process [illegible] time [illegible] this it gives more better than [illegible] response time.

1.45 (d)

Process [illegible] and deadlock.

Continue [illegible] in [illegible] mode.

1.46 [illegible]

[illegible] CPU [illegible] system processes. When CPU [illegible] interrupt the Operating System OS gives top priority to this interrupt and [illegible] link to [illegible] interrupt.

1.47 [illegible]

(a) False: On per thread basis, OS maintains both CPU [illegible] and [illegible]. Hence option (a) is false, [illegible] OS [illegible] state maintained.

(b) False: OS maintains a separate stack for each thread.

(c) True: [illegible]

(d) False: OS [illegible] scheduling and [illegible] information.

1.45 (a):

Average waiting time

$$= \frac{\text{[illegible]}}{3} = \text{[illegible]}$$

$$= \frac{[\text{illegible}] + [\text{illegible}] + [\text{illegible}]}{3} = \text{[illegible]} \text{ ms}$$

1.46 (d):

The number of child processes created

$= 2^n - 1 = 2^3 - 1 = 7$

(where n are the number of fork() statements)

1.50 (a):

RR Queue

P1 P2 P1 P3 P4 P1 P5 P3

0 1 2 3 4 5 6 10

P1	P2	P1	P3	P4	P	P5	P3

0 2 4 6 8 10 11 13 15

→ P3, P4, P5

FCFS: P1, P2, P3

Priority scheduling is preemptive

SJFS: P1, P4, P5

RR: P1, P3, P2

1.51 (b):

Let us take an example

Process	Execution time	Arrival time
1	10	0
2	8	1
3	5	2
4	6	3

Consider a scheduler set to give priority over all other processes according to the order in which they arrive. The P1, P2, P3, P4 processes in the order arrive, which is exactly same as round robin so answer is (b)

1.53 (d):

Kernel, I/O, Unix and Windows are the code segments

1.53 Sol.:

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Processing Sequence: [illegible]

T.A.T(P1) = 8 − 0 = 8

T.A.T(P2) = 3 − 2 = 1

T.A.T(P3) = 15 − 6 = 9

T.A.T(P4) = 21 − 7 = 14

T.A.T(P5) = 16 − 10 = 6

Average T.A.T = $\frac{8 + 1 + 9 + 14 + 6}{5} = 7.8$

1.54 Sol.:

[illegible]

1.55 (Sol):

SRTF: [illegible]

Waiting time of P1 = 17 − 3 = [illegible]

Waiting time of P2 = 2 − 2 = 0

Waiting time of P3 = 6 − 4 = 2

Waiting time of P4 = 12 − 9 = 3

Average waiting time =

$$= \frac{[illegible] + 0 + 2 + 3}{4} = [illegible]$$

1.56 (Sol):

Periodic arrival times of T1: 0, 3, 6, 9, 12, 15, 18, 21....

Priority of T1 = 1/3 = 0.33, service time of T1 = 1

Periodic arrival times of T2: [illegible]

Priority of T2 = 1/5 = 0.2, service time of T2 = [illegible]

Periodic arrival times of T3: 0, 20, 40....

Priority of T3 = 1/20 = 0.05, service time of T3 = [illegible]

T1 has highest priority and T3 has lowest priority.

Last instance of T3 is finally completed at the end of 22 ms.

1.57 (a):

Maximum number of [illegible] ready queue is independent of [illegible] scheduling (C).

1.58 (a):

Process	Arrival Time	B.T
A	0	5
B	1	3
C	2	1
D	3	2

(a) FCFS:

$$\text{Avg TAT} = \frac{5 + 6 + 7 + 9}{4} = \frac{28}{4} = [illegible]$$

(b) Non-preemptive SJF:

$$\text{Avg TAT} = \frac{5 + 8 + [illegible]}{4} = [illegible]$$

(c) SRTF:

$$\text{Avg TAT} = \frac{[illegible] + 1 + [illegible] + 1}{4} = \frac{26}{4}$$

$$= 6.5$$

(d) RR:

$$\text{Avg TAT} = \frac{[illegible]}{4} = \frac{[illegible]}{4} = 8.25$$

∴ SRTF has lowest turnaround time.

1.59 (a):

To minimize the average waiting time we need to reduce the elapsed time. [illegible] First because of the preemptive [illegible] and [illegible] have an equal CPU burst time.

All the processes will be [illegible] waiting time. So, the average is SRTF [illegible]

1.60 (Sol):

Gantt chart:

Average turnaround time is [illegible]

■■■■

2 Process Management-II

(IPC, Synchronization and Concurrency)

2.1 A critical region is

(a) One which is invoked by a set of P and V operations on a semaphore

(b) A program segment that has not been proved to be safe

(c) A program segment that allows concurrent access to shared variables

(d) A program segment where shared resources are accessed.

[1987 : 2 Marks]

2.2 Semaphore operations are atomic which means they are implemented as a single instruction.

[1990 : 2 Marks]

2.3 An uniprocessor system of computer in the values of a counting semaphore is 7. Then 20 P operations and 15 V operations are completed on this semaphore. The resulting value of the semaphore is:

(a) 12 (b) 2

(c) 8 (d) 12

[1992 : 2 Marks]

2.4 A critical section is a program segment

(a) Which avoid deadlock or starvation

(b) Which avoids busy waiting

(c) Where shared resources are accessed

(d) Which may be enclosed by a pair of semaphore operations P and V

[1996 : 1 Mark]

2.5 [illegible] Dining Philosophers Problem [illegible] avoid deadlocks

(a) [illegible] pick up the left fork before the right fork

(b) [illegible] pick up the right fork before the left fork

(c) [illegible] pick up the right fork and then the left fork

(d) None of the above

[1998 : 2 Marks]

2.6 Mutex [illegible] is defined as follows

[illegible]

Mutex

Process

The code for P is defined as follows: P(mutex); [illegible] V(mutex); [illegible] P(mutex); What is the maximum number of processes that can be inside the critical section at any moment?

(a) 1 (b) 2

(c) 3 (d) None of these

[1997 : 2 Marks]

2.7 When a result of a computation depends on the speed of the processes involved, there is said to be

(a) Cycle stealing (b) Race condition

(c) [illegible] (d) Deadlock.

[1999 : 1 Mark]

2.8 A counting semaphore was initialized to 10. Then 6 P (wait) operations and 4 V (signal) operations were completed on this semaphore. The resulting value of the semaphore is:

(a) 0 (b) 8

(c) 10 (d) 12

[2006 : 1 Mark]

2.9 Let m[0]...m[4] be mutexes (binary semaphores) and P[0]...P[4] be processes. Suppose each process P[i] executes the following:

wait (m[i]); wait (m[(i+1) mod 4]);

.....

release (m[i]); release (m[(i+1) mod 4]);

This could cause

(a) Deadlock

(b) Starvation

(c) Mutex, but not deadlock

(d) None of the above

[2000 : 2 Marks]

2.10 Suppose a processor [illegible] two stack pointer registers SP1 and SP2 and the following instructions are added:

(c) It must have enough memory for the stack
(d) It can have unbounded number of recursive calls, as long as no nested procedure calls
(e) Recursive subroutines can be possible only in interpreted languages
(f) Global variables cannot be used in recursive subroutines

[2001 : 1 Mark]

2.11 Which of the following is a greedy algorithm?
(a) PERT (b) Dijkstra
(c) KMP (d) Binary search

[2001 : 1 Mark]

2.12 Consider Dijkstra's algorithm for finding the shortest path between two given vertices in a graph. The program uses a boolean variable [illegible] to store the vertices for which the shortest path has been found.

```
Dijkstra
    flag[i] = false
    count = 1
    while (flag[i] = false) do
        Select a vertex i such that flag[i] = false and
        is a minimal vertex
        flag[i] = true;
        Update the distance of other vertices from the
        selected vertex
    Until flag[i] = true
```

To the program to generate minimal spanning tree, the predicate P in the above algorithm is

(a) flag[i] = true and count = i
(b) flag[i] = true and count = j
(c) flag[i] = true and count = j
(d) flag[i] = true and count = i

[2001 : 2 Marks]

Common Data for Q. 2.13 & 2.14

Suppose we want to synchronize two concurrent processes P and Q using binary semaphores S and T. The code for the processes P and Q is shown below.

```
Process P:          Process Q:
while (1) {         while (1) {
    W                   Y
    print('0')          print('1')
    print('0')          print('1')
    X                   Z
}                   }
```

Synchronization statements can be inserted only at points W, X, Y and Z.

2.13 Which of the following ensures that 0 and 1 are always printed alternately?
(a) P(S) at W, V(S) at X, P(T) at Y, V(T) at Z, S and T initially 1
(b) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S initially 1, T initially 0
(c) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S and T initially 1
(d) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S initially 1, and T initially 0

[2003 : 2 Marks]

2.14 Which of the following ensures that 0 and 1 are printed alternately and starting with 0?
(a) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S and T initially 1
(b) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S and T initially 0
(c) P(S) at W, V(S) at X, P(T) at Y, V(T) at Z, S initially 1
(d) V(S) at W, P(T) at X, V(T) at Y, P(S) at Z, S and T initially 1

[2003 : 2 Marks]

2.15 Consider two processes P_1 and P_2 accessing the shared variables X and Y protected by two binary semaphores S_1 and S_2 respectively, both initialized to 1. P and V denote the usual semaphore operations, where P decrements the semaphore value, and V increments the semaphore value. The pseudocode of P_1 and P_2 is as follows:

```
P1:                 P2:
while (true) {      while (true) {
    L1: .....           L3: .....
    L2: .....           L4: .....
    X = X + 1;          Y = Y + 1;
    Y = Y - 1;          X = X - 1;
    V(S1);              V(S2);
    V(S2);              V(S1);
}                   }
```

In addition to mutual exclusion, the above operations ensure that L_1 precedes L_2, and L_3 precedes L_4. Which of the following is a valid sequence of P and V operations?
(a) P(S_1), P(S_2), P(S_1), P(S_2)
(b) P(S_1), P(S_2), P(S_2), P(S_1)
(c) P(S_2), P(S_1), P(S_2), P(S_1)
(d) P(S_2), P(S_1), P(S_1), P(S_2)

[2004 : 2 Marks]

Q.16 The [illegible] variables full, empty and mutex are initialized to 0, 5 and 1, respectively. Process P_1 repeatedly produces items to insert in a buffer of size 5, and process P_2 repeatedly removes one item at a time from the buffer. In the code given below, E, F, M and N are constant integers.

```
P1 : while (1)
{
    X
    P(mutex);
    Add an item to the buffer.
    V(mutex);
    Y;
}
P2 : while (1)
{
    M;
    P(mutex);
    Remove an item from the buffer.
    V(mutex);
    N;
}
```

The values of X, Y, M and N are respectively

(a) P(full), V(empty), P(empty), V(full)

(b) P(full), P(empty), V(empty), V(full)

(c) P(empty), V(full), P(full), V(empty)

(d) P(empty), V(full), V(full), P(empty)

[2004 : 2 Marks]

Q.17 Given below are two [illegible] processes:

semaphore S = 0;

/* Initialize S to zero */

Process P1	Process P2
while (1) {	while (1) {
Compute;	Compute;
V(S);	V(S);
}	}

Consider the following statements about processes P1 and P2.

I. It is possible for process P1 to starve.

II. It is possible for process P2 to starve.

Which of the following is/are true?

(a) Both I and II are true

(b) I is true but II is false

(c) II is true but I is false

(d) Both I and II are false

[2005 : 2 Marks]

Q.18 Two counting semaphores P and Q are initialized to 1 and 0 respectively. The operations P(x) and V(x) are defined as follows:

P(x):	V(x):
Continue	Continue
Loop forever	Loop forever
Wait (x);	Wait (x);
Critical section	Critical section
Signal (x);	Signal (x);
Remainder section	Remainder section

Each process accesses the critical section three times. Which sequence can be observed by any process at any time? The following [illegible] represent the sequence of events by the processes:

- P1 enters critical section of P1 before P2 gets access to P1
- P2 enters critical section of P2 before P1 gets access to P2
- P1 enters critical section of P1 before P2 gets access to P1
- P2 enters critical section of P2 before P1 gets access to P2

Which are the other scheduling constraints [illegible] for a binary semaphore [illegible] the minimum number of binary semaphores needed?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

Q.19 The atomic fetch and set [illegible] instruction is used to implement mutual exclusion and bounded waiting. Consider the following [illegible] and V [illegible] binary semaphores.

```
void P(binary_semaphore *S) {
    unsigned y;
    unsigned *x = &(S->value);
    do {
        fetch_and_set(x, y);
    } while (y);
}
void V(binary_semaphore *S) {
    *(S->value) = 0;
}
```

Which one of the following [illegible]

(c) The implementation [illegible] work [illegible] not [illegible] in P
(d) [illegible] and [illegible], a process [illegible] [illegible] can be used
(e) The [illegible] in V is [illegible]
(f) The code does [illegible] mutual exclusion [illegible] busy waiting

[2006 : 2 Marks]

Common Data for Q. 8.30 & 8.31:

Barrier is a synchronization mechanism where a set of concurrent processes must all reach a certain point before any of them can proceed. Consider a set of processes P0, P1, P2 and P3 and a barrier implemented using a shared variable count and a binary semaphore mutex. Initially, count is 0 and mutex is 1. Each process calls the barrier function when they reach a certain point in their execution. The pseudocode of the barrier function is shown below:

```
void barrier (void) {
        P(S);
  2     count = count + 1;
  3     V(S);
  4     while (count < 4);
  5     P(S);
  6     count = count - 1;
  7     if (count == 0)
  8         process_released = 1;
  9         process_released = 0;
 10     }
 11     V(S);
 12 }
```

The variables count and process_released are shared by all processes and are initialized to zero. In a concurrent program all the four processes call the barrier function when they reach a certain point in their execution.

8.30 [illegible]
Which one of the following statements?
(a) The [illegible] implementation [illegible] the mutex binary semaphore S
(b) The [illegible] implementation may lead to a deadlock if two [illegible] are [illegible] in immediate succession
(c) Busy [illegible] is used [illegible] variables [illegible]
(d) The [illegible] ensures that [illegible] processes [illegible] [illegible]

[2006 : 2 Marks]

8.31 Which one of the following [illegible] the implementation?
(a) [illegible]
(b) [illegible] processes to enter the barrier wait [illegible] processes [illegible] becomes zero [illegible]
(c) [illegible] deadlock at the beginning of the barrier and so [illegible]
(d) [illegible] process [illegible] [illegible] inserted [illegible]

[2006 : 2 Marks]

8.32 Two processes P1 and P2 need access to a critical section of code. Consider the following synchronization mechanism [illegible] shared variable turn initized to 0. Mutual [illegible] the main process [illegible]

```
process P1 {
    while (turn != 0);
    critical section
    turn = 1;
}

process P2 {
    while (turn != 1);
    critical section
    turn = 0;
}
```

Consider the following statements:
(i) It is possible [illegible] P1 and P2 [illegible] within [illegible]
(ii) [illegible] lead to a deadlock
Which of the [illegible] is valid?
(a) (i) is true [illegible]
(b) Both (i) and (ii) are false
(c) (i) is true and (ii) is false
(d) Both (i) and (ii) are true

[2007 : 1 Mark]

8.33 Two processes P1 and P2 need to access critical section of code. Consider the following synchronization mechanism [illegible] shared variable [illegible]

```
process P1 {
    while (true) {
        [illegible]
        while (turn == 0);
        critical section
        [illegible]
        turn = 0;
    }
    [illegible]
}
```

```
/* P[i] */
  wait(mutex);
    [illegible] = true;
    while (flag[1] && turn == 1)
    {   flag[i] = false;
        Busy wait();
    }
    critical section
    turn = 2;
    flag[i] = false;
/* Remainder section */
```

Here, turn and flag are shared variables, which are initialized to false. Which of the following statements is TRUE about the above construct?

(a) It does not ensure mutual exclusive

(b) It does not ensure bounded waiting

(c) It ensures that processes enter their critical sections in strict alternation

(d) It does not prevent deadlock but ensures mutual exclusive

[2007 : 2 Marks]

Q.25 Synchronization in the classical readers and writers problem can be achieved through using semaphores. Include in your complete code for readers-writers problem, two binary semaphores mutex and wrt and a shared variable readcount.

```
wait(mutex);
[illegible] is performed
signal(wrt);
wait(mutex);
readcount = readcount +
if readcount = 1 then Gi
Si
reading is performed
Si
readcount = readcount - 1
if readcount = 0 then Si
signal(mutex);
```

The values of S1, S2, S3, S4, Gi are:

(a) wait(wrt), signal(wrt), wait(mutex), signal(mutex)

(b) wait(wrt), signal(mutex), wait(mutex), signal(wrt)

(c) wait(mutex), wait(wrt), signal(mutex), signal(wrt)

(d) wait(mutex), wait(mutex), signal(mutex), signal(mutex)

[2007 : 2 Marks]

Q.26 The [illegible] semaphores are used to provide mutual exclusion and synchronization. Consider the following:

```
P(S):  S = S - 1;
       Busy Wait
V(S):  S = S + 1;
       Busy Wait
```

Assume that P_b and V_b the wait and signal operations on binary semaphores are provided. Two binary semaphores X_b and Y_b are used to implement the semaphore variable S and V as follows:

```
P(S):   P_b(X_b);
        S = S - 1;
        if (S < 0)
          V_b(X_b);
          P_b(Y_b);
        else
        V_b(X_b);
V(S):   P_b(X_b);
        S = S + 1;
        if (S <= 0)
        V_b(Y_b);
        V_b(X_b);
```

The initial values of X_b and Y_b are respectively

(a) 0 and 0 (b) 0 and 1

(c) 1 and 0 (d) 1 and 1

[2006 : 2 Marks]

Q.27 The [illegible] have CS (critical section) [illegible] critical section is given [illegible] Consider interleaving in [illegible]

```
void lock() {CS}(S)
{
  while (test and set (S))
}
void lock() {CS}(S)
{
  S = 0;
}
```

In the above solution, S is a memory location associated with the CS and is initialized to 0. Now consider the following statements:

I. The [illegible] solution to CS problem is deadlock-free.

II. The solution is starvation free.

III. The processes are in CS in FIFO order.

IV. More than one process can be in CS at the same time.

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2014 : 2 Marks]

2.27 Given two binary [illegible] processes P1 and P2 [illegible] shared variables [illegible] as shown below. The initial values of shared boolean variables [illegible] are assigned.

Method used by P1	Method used by P2
while (S == 2) ;	while (S == 1) ;
S = S - 1;	S = S - 1;
Critical Section	Critical Section
S = S + 2;	S = S + 1;

Which of the following statements describes properties of the above [illegible]

(a) Mutual exclusion but not progress
(b) Progress but not mutual exclusion
(c) Neither mutual exclusion nor progress
(d) Both mutual exclusion and progress

[2013 : 1 Mark]

2.28 The following program consists of concurrent processes [illegible] binary semaphores. The semaphores are initialized as [illegible] = 0, [illegible] = 0.

Process P1	Process P2	Process P3
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]		
[illegible]		
[illegible]		

How many times will process P3 print '0'?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2013 : 2 Marks]

2.29 [illegible] Add(X, i) [illegible] and [illegible] Write [illegible] the value [illegible] and return the old value of X. [illegible] used in the pseudocode shown below [illegible] shared variable initialized to 0. The value of [illegible] is lost [illegible] available.

```
Acquire(lock):
    while (Test_and_Set(lock) == 1)
        ;
;
Release(lock):
    lock = 0;
;
Thread_count(lock)
lock = 0;
Acquire(lock);
    [illegible] value of [illegible]
    [illegible] available
[illegible] may [illegible]
    [illegible]
[illegible] correctly with [illegible]
```

[2012 : 2 Marks]

2.30 The [illegible] processes X, Y and Z [illegible] to [illegible] code segments that [illegible] and [illegible] shared variables. Process X [illegible] for Process Y [illegible] Process Z [illegible] the P [illegible] completing the [illegible] code segments, each [illegible] the following [illegible] the processes?

(a) X: P(a), P(b), P(c)
    Y: P(b), P(c), P(a)
    Z: P(c), P(a), P(b)
(b) X: P(a), P(b), P(c)
    Y: P(b), P(c), P(a)
    Z: P(c), P(a), P(b)
(c) X: P(a), P(b), P(c)
    Y: P(b), P(c), P(a)
    Z: P(c), P(a), P(b)
(d) X: P(a), P(b), P(c)
    Y: P(b), P(c), P(a)
    Z: P(c), P(a), P(b)

[2012 : 1 Mark]

2.31 [illegible] processes W, X, Y, Z are [illegible] Each of the processes W and X [illegible] Exam memory [illegible] memory [illegible] Each of the

processes X and Y reside in [illegible] memory. Semaphores are used to [illegible] signals to the semaphores [illegible] memory. Semaphore S is initialized to [illegible] is the maximum possible value of S after all processes complete execution?

```
{ [illegible]                    [illegible] = 1
[illegible] 1                    [illegible] 2
```

[2018 : 2 Marks]

2.52 A [illegible] and P each have [illegible] decomposed [illegible] Y and Y copy the data back. The [illegible] 2 units of time. The [illegible] 1 unit of time. The [illegible] are shown below.

```
Process-X                    Process-Y
[illegible]                  [illegible]
[illegible]                  [illegible]
{                            {
    [illegible]                  [illegible]
    [illegible]                  [illegible]
                             }
```

Which one of the following represents the CORRECT implementation of Process-X and Process-Y?

```
(a) Process-X() {            (b) Process-Y() {
      P(S);                        V(S);
      V(S);                        V(S);

    Process-Y() {                Process-X() {
      P(S);                        P(S);
      V(S);                        P(S);
    }                            }

(c) Process-X() {            (d) Process-Y() {
      P(S);                        V(S);
      V(S);                        V(S);
    }                            }

    Process-Y() {                Process-X() {
      V(S);                        P(S);
      P(S);                        P(S);
    }                            }
```

[2013 : 2 Marks]

2.53 Processes P and Q share a binary semaphore for the following [illegible] which uses semaphores [illegible]

```
[illegible]                  [illegible]
{                            {
[illegible]                  [illegible]

    [illegible]                  [illegible]
    [illegible]                  [illegible]
    [illegible]                  [illegible]
    [illegible]                  [illegible]
    [illegible]                  [illegible]
}                            }
```

While [illegible] PROC Q

(a) The processes will be [illegible] the [illegible] and the consumer will never consume

(b) The consumer will consume more than one [illegible]

(c) [illegible] access if the producer [illegible] acquiring the semaphore when the buffer is empty

(d) The starting value for the semaphore must be [illegible] deadlock between [illegible]

[2014 (Set 4) : 2 Marks]

2.54 The following [illegible] P1 and P2 [illegible] a variable [illegible] initial value 2 [illegible]

```
P1()                         P2()
{                            {
    [illegible]                  [illegible]
    [illegible]                  [illegible]
}                            }
```

[illegible]

[2013 (Set 1) : 1 Mark]

2.55 Two processes A and B [illegible] critical section. Consider [illegible] protocol used by both the processes.

```
Process A
[illegible]
[illegible]
    [illegible]
    [illegible]
    [illegible] critical section [illegible]
    [illegible]
    }
}
[illegible] process P2
```

Process 5

/* other code for process 5 */

while true,

{ wants[5] = true;

while (turn != 5)

{ /* Critical Section */

turn = 5;

}

}

/* other code for process 5 */

Here, turn is a shared variable and wants[i] and wants[j] are initialized to false. Which one of the following statements is true?

(a) The proposed solution prevents deadlock but fails to guarantee mutual exclusion.

(b) The proposed solution guarantees mutual exclusion but fails to prevent deadlock.

(c) The proposed solution guarantees mutual exclusion and prevents deadlock.

(d) The proposed solution prevents deadlock but fails to guarantee mutual exclusion.

[2015 (Set-4) : 1 Mark]

238 Consider the following proposed solution for the critical section problem. There are n processes $P_0 \ldots P_{n-1}$. In the code, function pmax returns an integer not smaller than any other argument. Variable a[i] is initialized to zero for each i.

Entry(i):

{

a[i] = 1 + pmax(a[0], a[1], ..., a[n-1]);

for (j = 0; j < n; j++)

{ while (a[j] != 0);

while (a[j] != 0 && a[i] < a[j]);

}

Critical Section(i);

Remainder Section;

while (true);

Which one of the following is TRUE about the above solution?

(a) It ensures mutual exclusion but fails to prevent deadlock.

(b) The bounded wait condition is satisfied.

(c) The progress condition is satisfied.

(d) It uses test-and-set-lock.

[2016 (Set-1) : 2 Marks]

239 Consider the following two-process synchronization solution.

Process 0

Entry: loop while (turn == 1);

Critical section

Exit: turn = 1;

Process 1

Entry: loop while (turn == 0);

Critical section

Exit: turn = 0;

The shared variable turn is initialized to zero. Which one of the following is TRUE?

(a) This is a correct two-process synchronization solution.

(b) This solution violates mutual exclusion requirement.

(c) This solution violates progress requirement.

(d) This solution violates bounded wait requirement.

[2016 (Set-3) : 2 Marks]

240 Consider a non-pipelined machine M. The operation DIV decrements S, and JNZ(S) increments S. During an execution, 20 JNZ(S) operations are executed. The initial value of S for which at least one JNZ(S) operation will remain unexecuted is

[2016 (Set-2) : 2 Marks]

■■■■

Answers Process Management-II

2.1	(d)	2.2	(b)	2.3	(c)	2.4	(c)	2.5	(c)	2.6	(d)	2.7	(b)	2.8	(b)	2.9	(b)
2.10	(a)	2.11	(a)	2.12	(b)	2.13	(b)	2.14	(c)	2.15	(d)	2.16	(d)	2.17	(c)	2.18	(b)
2.19	(c)	2.20	(a)	2.21	(b)	2.22	(c)	2.23	(d)	2.24	(a)	2.25	(a)	2.26	(c)	2.27	(c)
2.28	(c)	2.29	(a)	2.30	(b)	2.31	(d)	2.32	(c)	2.33	(c)	2.34	(a)	2.35	(c)	2.36	(c)
2.37	(c)																

Explanations Process Management-II

2.1 (d)

A critical region is a piece of code that accesses a shared resource (data structure or device) that must not be concurrently accessed by more than one thread of execution.

2.2 (b)

False

2.3 (b)

[illegible] and up-gradation [illegible] shared up-gradation [illegible]

2.4 (a)

Critical section is an area of a program where the resource can be accessed.

2.5 (c)

Dining philosophers problem [illegible] the [illegible] the [illegible]

2.6 (d)

Every time when process P_i enters its critical section, the TSL operation is performed on a variable lock. [illegible] critical section.

2.7 (b)

True condition

2.8 (b)

The priority number is 10. Then 6 down operations are performed [illegible] up operations [illegible]

[illegible]

2.9 (b)

All processes are identical in [illegible] critical section.

2.10 (a)

[illegible] resource [illegible] available then [illegible] from the process [illegible] be executed. [illegible] busy waiting [illegible] instruction.

2.11 (b)

Disk [illegible]

2.12 (b)

[illegible] the [illegible] case [illegible] process [illegible] which [illegible]

[illegible]

2.13 (b)

[illegible] the [illegible] process [illegible] $S = 1$ and $T = 0$

2.14 (c)

[illegible] never contains a [illegible] process [illegible]

$S = 1$

Process T:	Process Q
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

2.15 (d)

[illegible]

2.16 (d)

[illegible]

2.17 (b)

[illegible]

2.18 (a)

[illegible]

```
P1:                 P2:
P(X)                .
P(Y)                P(X)
V(X)                P(Y)
                    V(X)
                    V(Y)
P(Y)                P(X)
V(Y)
V(X)
P(X)                P(Y)
                    V(X)
                    V(Y)
V(X)                V(Y)
V(Y)
```

2.19 (c)

The P and V statement is as follows

```
1   void P(binary semaphore * S)
2       unsigned j;
3       unsigned *p = (unsigned *) S;
4       int i;
5       while (1)
6           if (*p == 1)
7               *p = 0;
8               return;
9           else if (*p == 0)
10              for (i = 0; i < 100; i++)
11              ;
```

Both and semaphore operations allow access for multiple processes and threads. Binary semaphore is a semaphore that only two values 0 and 1. When one process enters CS it decrements the value to 0. Then other processes and threads wait by busy waiting. The value is changed to 1 when the process exits CS. In this implementation busy waiting is done properly and don't spin indefinitely.

2.20 (d)

The busy implementation may lead to a deadlock if two busy-waiting processes are in [illegible] which [illegible] = 5 and 7.

2.21 (b)

[illegible]

2.22 (b)

[illegible]

The busy [illegible] and it [illegible] just before entrance and before exit [illegible] also gets entrance since the flag is still [illegible].

Now both processes are [illegible] critical section!!

So [illegible], mutual exclusion is broken.

There is a case that [illegible] processes are made out of choice, if someone enters the critical section, it will [illegible] flag = false. So [illegible]

2.32 (d)

In process P_1 of variant 1 from Peterson's algorithm, P_1 enters critical section and in process P_2 of variant 2 from Peterson's algorithm, P_2 also enters critical section but in this case a conflict occurs and it violates the mutual exclusive.

2.33 (b)

S2: If condition is true, semaphore is released, wait on write or read write wait.
S3: After mechanism has been updated, mutex mutex is applied.
S4: Wait on mutex is applied in modified condition.
S1: If mechanism is in the middle of a condition, signal on mutex is released to allow writer to write.

2.35 (b)

In given code, P_1 decrements the value of S_1 by 1 and P_2 increment the value of semaphore P_2 by 1. Now wait and signal operations of binary semaphore S_1 and S_2 are operations applied to avoid the mutual exclusion of the value of S_1 and S_2 be equal to 0 when P_1 and P_2 work properly.

2.36 (d)

Only I is true.

2.37 (b)

P1	P2
While (S == 0);	While (S == 0);
Critical section	Critical section
S = 0;	S = 0;

We initially take $S_1 = 1$ and $S_2 = 1$.
When P1 execute S1 = 0 it stands for entering and process P1 enter critical section after some time it release the value of S1 = 1 and when P1 execute then P2 will execute. So the mutual exclusion condition is met so the mutual exclusion that processes are blocked by these mutexes so in fact one process can be delayed indefinitely.
Deadlock occurs when both P1 make a wait and block from (S1 = S2 and S2 = and S1);.

2.38 (b)

Wait(S_1) = 1, S = 0
Initially the value of S_1 and S_2 = 1. P_1 and P_2 Not execute. The value of S_1 = 1. P_1 is execute process P_1 Now value of S_1 = 0
And in print statement time when it execute (S) still the value of S_1, S_2 increase by 1. So P_1 consumer P_2 makes value of S_1 equal to zero. S_2 = 1. after release (S_2) make value of S_2 = 2 or applying wait and S_2 = 0. So step in P_1 process present. and wait (S) means it decrements the mutex. Print statement. Both process consumer so in S_2 get increment in 1 time.

2.39 (b)

Assume P_1 enters in until while condition and consumption increasing, L = 1. Now P_2 executes and enter critical section L = 0. Then P_1 without checking L it enters L = 1. Now P_2 enters and also L = 1 and when it was preempted.
So L value can never value L = 0 when this task is executed L = 0)

2.40 (b)

Trick: The given code conflict in the given code process parallely i.e. first executed the functions of all the processes then executed the second line of all processes and so on.

(a)

X		Y		Z	
1.	P1(a)	1.	P2(a)	1.	P3(a)
2.	P1(b)	2.	P2(b)	2.	P3(b)
3.	P1(c)	3.	P2(c)	3.	P3(c)

Values of a, b, c are initially

Execution: First line of all processes i.e. X_1 → Y_1 → Z_1 after the execution of X_1, Y_1, Z_1 value of y is by = 0. Now no process is allowed to enter into the critical section. So deadlock

(b)

X	Y	Z
1. P1(a)	1. P2(a)	1. P3(a)
2. P1(b)	2. P2(b)	2. P3(b)
3. P1(c)	3. P2(c)	3. P3(c)

1. Now at X_1 value of b gets 0.
2. At Y_1 value of b = 0 when we reach to Y_1 it gets blocked. Y_1 is blocked.

8. Now execute X_1 and finally set 0.
9. Now try to execute the second line of code. Since mutex lock is already acquired by the process Y, process X will wait to execute x_1 at get_mutex() and X is blocked.
10. Now execute Y_2, value of x get 0.
11. So process X and Y mutex lock and unlock can easily execute X_2.

So correct sequence. Last three are mutex lock and unlock operations of process Y.

2.83 (d)

	X	W	Y	Z
1.	P(s)	A(s)	P(s)	P(s)
2.	V(s)	V(s)	V(s)	V(s)
3.	[illegible]	[illegible]	[illegible]	[illegible]
4.	Signal(s)	Signal(s)	Signal(s)	Signal(s)
	MM	MM	MM	MM
5.	V(s)	V(s)	V(s)	V(s)

1. Start with X. First [illegible] Now before [illegible] process Y.
2. Y will [illegible] then [illegible] and V(s) [illegible]
3. X will perform P(s) [illegible] and [illegible]
   then [illegible] and V(s) [illegible]
4. Now process [illegible] value of [illegible] and V(s) [illegible]
5. Now process W performs P(s) [illegible] V(s) [illegible]
   Option (d) is correct.

2.84 (d)

The condition is unique [illegible] X and Y. Y and W two [illegible] process [illegible] the values [illegible] by the [illegible] the array that will be used by process Y [illegible]

Option (a): [illegible] because [illegible] a deadlock.

Option (b): It is a simple [illegible] process X can [illegible] but process Y can [illegible] only [illegible] values.

Option (c): It is correct, because [illegible] value [illegible] process X. [illegible] immediately consumed by process Y.

Option (d): It is wrong, because [illegible] Y can insert two values. [illegible] but the process Y can enter [illegible] only [illegible] value if the preceptor [illegible] the entry counter of process Y.

2.85 (a)

[illegible] from waiting and [illegible] by the [illegible] value.

After consumer sleep, producer [illegible] by consuming [illegible].

2.86 (d)

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

- [illegible]
- These different values are possible.

2.87 (b)

(i) The mutual exclusion fails in the given sequence. [illegible] process Y [illegible] statements of [illegible] process Y resumes all statements till [illegible] time. Now both process X and process Y [illegible] while [illegible] at same time. Thus mutual exclusion [illegible] simultaneously. Therefore [illegible] is violated.

(ii) Deadlock is [illegible] by the given [illegible]

2.88 (a)

It satisfies [illegible] mutual exclusion [illegible] only one process [illegible] critical section at [illegible]

2.89 (b)

Given problem is [illegible] which [illegible] process [illegible] critical section [illegible] is described by [illegible] section of another [illegible].

So, process properly [illegible] satisfied.

2.90 (b)

[illegible]

So the initial value of the semaphore should be 2.

■■■■

3 Deadlock

Q.1 A computer system has five tape drives and five processes competing for them. Each process may need two tape drives. The maximum value of the available tape drives is minimum value for which deadlock will not occur is

(a) 2 (b) 3
(c) 4 (d) 5

[1992 : 2 Marks]

Q.2 Consider a system having m resources of the same type. These resources are shared by three processes A, B and C, which have peak demands of 3, 4 and 6 respectively. For what value of m deadlock will not occur?

(a) 7 (b) 9
(c) 10 (d) 12

[1998 : 2 Marks]

Q.3 A solution of the critical section problem satisfies the following properties. Mutual exclusion, progress and bounded waiting. The minimum number of shared variables needed to ensure that no deadlock will ever occur is

(a) 2 (b) 3
(c) 4 (d) 8

[1997 : 2 Marks]

Q.4 A computer has six tape drives, with n processes competing for them. Each process may need two drives. What is the maximum value of n for the system to be free from deadlock?

(a) 7 (b) 5
(c) 4 (d) 3

[1999 : 1 Mark]

Q.5 Which of the following is NOT a valid deadlock prevention scheme?

(a) Release all resources before requesting a new resource
(b) Number the resources uniquely and never request a resource with a number lower than the last one requested
(c) Never requests a resource after a lock-in-progress resource
(d) Request and all required resources must be allocated before execution

[2000 : 2 Marks]

Q.6 In a distributed computing system, deadlock prevention is attempted using the following scheme. Each process is assigned a unique time stamp and a resource will be allocated to the process with the lowest time stamp. Let P_1 be the process holding a resource R. P_2 has a pending request for this same resource. Let $TS(P_1)$ and $TS(P_2)$ be the time stamps respectively. The decision to wait or preempt one of the processes is based on the following identities.

```
if TS(P1) < TS(P2)
    kill P1
else
    wait
```

Which one of the following is TRUE?

(a) The scheme is deadlock-free, but not starvation-free
(b) The scheme is neither deadlock-free nor starvation-free
(c) The scheme is free from deadlock and starvation-free
(d) The scheme is free from deadlock but not starvation-free

[2001 : 2 Marks]

Q.7 Suppose n processes, $P_1, \ldots, P_n$ share m identical resource units which can be reserved and released one at a time. The maximum resource requirement of process P_i is s_i, where $s_i > 0$. Which one of the following is a sufficient condition for ensuring that deadlock does not occur?

(a) $\sum_{i=1}^{n} s_i < m + n$ (b) $\sum_{i=1}^{n} s_i < m + n$

(c) $\sum_{i=1}^{n} s_i < m + n$ (d) $\sum_{i=1}^{n} s_i < m + n$

[2006 : 2 Marks]

Q.8 Consider the solution to the bounded buffer producer consumer problem by using general semaphores S, E and F. The semaphore S is the mutual exclusion semaphore initialized to 1. The semaphore E corresponds to the number of free slots in the buffer. The semaphore F is initialized to 0.

The semaphore variables full, empty and mutex shared by the producer and consumer processes are initialized to 0, 5 and 1 respectively.

Producer Process	Consumer Process
Produce an item in	Wait(full)
Wait(empty)	Wait(mutex)
Wait(mutex)	Remove an item from buffer
Add the item to the buffer	Signal(mutex)
Signal(mutex)	Signal(empty)
Signal(full)	Consume the item

Which of the following interchange operations may result in deadlock?

(i) Interchanging Wait(full) and Wait(mutex) in the producer process.

(ii) Interchanging Signal(full) and Signal(mutex) in the Consumer process.

(a) I only (b) II only

(c) Neither I nor II (d) Both I and II

[2006 : 2 Marks]

Q.10 Consider the following snapshot of a system running n processes. Process i is holding x_i instances of a resource R for $i = 1, 2, \ldots, n$. Currently, all instances of R are occupied. Further, for $i \neq j$, process i is waiting for a number of instances of R held by process j. Process i can complete execution only if it is granted y_i more instances of R and then releases all instances of R it holds. Deadlock is a situation where no process can complete execution. The minimum number of additional instances of R that must be made available so that the following condition is a necessary condition for deadlock to occur?

$y_1 = y_2 = \ldots = y_n = 0$. Which of the following condition is a necessary condition to indicate that the system is in deadlock?

(a) $\min(x_1, x_2, \ldots, x_n) \geq \max(y_1, y_2, \ldots, y_n)$

(b) $x_i = x_j \; \forall i, j$ $\max(y_1, y_2, \ldots, y_n) \geq x_1$

(c) $\min(y_1, y_2, \ldots, y_n) \geq x_1$

(d) $\min(y_1, y_2, \ldots, y_n) \geq \min(x_1, x_2, \ldots, x_n)$

[2006 : 2 Marks]

Q.11 A single processor system has three resource types X, Y and Z, which are shared by three processes. The maximum number of units of each resource type required by a process at any time is given in the table. These processes are initiated at different times, but they are independent of each other and execute concurrently. The system allocates the requested number of units of each resource type to each process, and the column correct demand is the number of units of each resource type requested by a process at any time. The system has 10 units of X, 10 units of Y and 10 units of Z.

	allocation			maximum		
	X	Y	Z	X	Y	Z
P0	1	2	1	3	4	3
P1	2	0	1	4	2	2
P2	2	2	1	3	3	3

(a) P0

(b) P1

(c) P2

(d) None of the above, since the system is in a deadlock

[2007 : 2 Marks]

Q.12 The following program consists of two concurrent processes and a binary semaphore variable S initialized to 1. Further, P and Q are binary variables shared by the processes and initialized to 0 and 1 respectively.

```
cobegin P = 1, Q = 0;
integer x;
process P;                    process Q;
while (true) do               while (true) do
begin                         begin
P(S);                         P(Q);
x = produce();                consume(x);
V(Q);                         V(P);
end                           end
```

Which of the following is TRUE about the program above?

(a) The program results in deadlock

(b) Only alternate items produced by the producer may be lost

(c) Some of the items produced by the producer may be lost

(d) Values generated and stored in x by the producer are consumed in order and no produced value is lost

[2009 : 2 Marks]

Q.13 An operating system implements a policy that requires a process to release all resources before making a request for another resource.

Select the TRUE statement from the following:

(a) The system is free from deadlock and starvation

(b) Starvation can occur but deadlock cannot occur

(c) Deadlock cannot occur but starvation can occur

(d) Neither deadlock nor starvation can occur

[2008 : 2 Marks]

Q.14 Which of the following is NOT true about deadlock prevention and deadlock avoidance schemes?

(a) In deadlock prevention, the request for resources is always granted. The system remains safe.

(b) In deadlock avoidance, the request for resources is always granted if the resulting state is safe.

(c) Deadlock avoidance is less restrictive than deadlock prevention.

(d) Deadlock avoidance requires knowledge of resource requirements apriori.

[2009 : 3 Marks]

3.14 Consider a system with 4 types of resources R1 (3 units), R2 (2 units), R3 (3 units), R4 (2 units). Six processes P0, P1, P2, P3, P4, P5 compete for these resources as follows (resources indicated in brackets):

Process P0	Process P1	Process P2
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Which one of the following statements is TRUE if all four processes execute concurrently starting at time t = 0.

(a) All processes will finish without any deadlock

(b) Only P1 and P2 will be in a deadlock

(c) Only P1 and P2 will be in a deadlock

(d) All four processes will be in deadlock

[2009 : 2 Marks]

3.15 A system has n resources $R_0, R_1, \ldots, R_{n-1}$ and k processes $P_0, P_1, \ldots, P_{k-1}$. The implementation of the resource request logic of each process P_i is as follows:

```
if (i % 2 == 0) {
    if (i < n) request R_i
    if (i+1 < n) request R_{i+1}
}
else {
    if (i < n) request R_i
    if (i+1 < n) request R_{i+1}
}
```

In which choice of n and k the following statements is a deadlock free?

(a) n = 10, k = 20 (b) n = 21, k = 5

(c) n = 20, k = 10 (d) n = 11, k = 19

[2010 : 2 Marks]

3.16 An operating system uses the Banker's algorithm for deadlock avoidance when managing the allocation of three resource types X, Y and Z to three processes P0, P1, and P2. The table given below presents the current system state. Here, the Allocation matrix shows the current number of resources held; the Max matrix shows the maximum number of resources each process may request; and the Available vector shows the number of free resources.

	Allocation			Max		
	X	Y	Z	X	Y	Z
P0	0	1	1	6	1	3
P1	4	1	1	6	2	3
P2	2	1	1	4	1	3

There are 3 units of type X, 2 units of type Y and 3 units of type Z still available. The system is currently in a safe state. Consider the following independent requests for additional resources in the current state:

REQ1: P0 requests 0 units of X, 1 unit of Y and 2 units of Z

REQ2: P1 requests 1 unit of X, 1 unit of Y and 1 unit of Z

Which one of the following is TRUE?

(a) Only REQ1 can be permitted.

(b) Only REQ2 can be permitted.

(c) Both REQ1 and REQ2 can be permitted.

(d) Neither REQ1 nor REQ2 can be permitted.

[2014 (Set-1) : 2 Marks]

3.17 A system contains three programs and each requires three units of a resource. The minimum number of units of the resource must be such that deadlock-free is ___

[2014 (Set-3) : 2 Marks]

8.8 A system has 6 identical resources and N processes competing for them. Each process can request at most 2 resources. Which one of the following values of N can't lead to deadlock?

(a) 1 (b) 2

(c) 3 (d) 4

[2013 (Set-2) : 1 Mark]

8.9 Consider the following protocols for preventing deadlock in a system with mutually exclusive resources.

I. Processes must request all their resources at the beginning of execution. If any one of them is not available, all resources are released and retrieved.

II. The resources are ordered uniquely, and processes are allowed to request for resources only in increasing resource ordering.

III. The resources are ordered uniquely and processes are allowed to request for resources in an arbitrary resource number.

IV. The resources are ordered uniquely. A process is allowed to request only for a resource with a resource number larger than its currently held resources.

Which of the above protocols can be used for preventing deadlock?

(a) Any one of I and III but not any of II and IV

(b) Any one of I and IV but not any of II and III

(c) Any one of I and III but not any of IV

(d) Any one of I, II, III and IV

[2013 (Set-3) : 2 Marks]

■■■■

Answers : Deadlock

8.1	(a)	8.2	(d)	8.3	(c)	8.4	(b)	8.5	(c)	8.6	(a)	8.7	(a)	8.8	(a)	8.9	(b)		
8.10	(c)	8.11	(d)	8.12	(b)	8.13	(a)	8.14	(a)	8.15	(d)	8.16	(b)	8.17	(d)				

Explanations : Deadlock

8.1 (a)

There are 8 resources and N processes will hold 7 resources and there are 8 resources for which last resource is requested and deadlock has occurred.

8.2 (d)

Everyone seeks a more satisfied life or more happy and successful.

8.3 (c)

There are 3 processes and 4 resources. Each process requires 2 resources to complete. Deadlock won't occur and assuming that they complete if resources are enough.

8.4 (b)

Each process needs 2 resources. With 3 resources atleast one can get whole 2 and complete. So deadlock occurs if each 2 resources which each is used by 1 but requested

P1 P2 P3 R1 R2

1 1 1

8.5 (c)

It is not a valid deadlock avoidance scheme.

8.6 (a)

When the process wakes up from sleep it has been killed. Its existence is revoked. Now someone else is killed first time and the sleeping can never be aware of that process that was killed. So it creates a new process but it may have arrived.

So everyone when the killing process wakes up again. It will always find a new process that will say "your time stamp is less than me". When A receives we know and C gets we will signal back to B. This may lead to infinite loop of messages requesting and telling that "I am alive". Every one is try to access B. So the system is possible. Deadlock is not possible.

8.7 [illegible]

There are n processes $P_1, P_2, \ldots P_n$

Number of identical resources = m

The maximum [illegible] demand of any process P_i is M_i where $M_i > 0$.

If n processes are in state of deadlock, [illegible] allocation [illegible] processes [illegible] deadlocked. Worst case [illegible] each P_i [illegible] resources [illegible] available resources plus the resources held by processes P_i [illegible]

$$\text{i.e. } \sum_{i=1}^{n} M_i - n \text{ [illegible] this [illegible] minimum [illegible] } m.$$

$$\therefore \sum_{i=1}^{n} M_i - n < m$$

8.8 [illegible]

Suppose [illegible] $P = 0$

[illegible] Wait1 and Wait2 are [illegible] and Wait2 succeeds. The producer will wait for Wait1 which is never going to [illegible]

Consumer will be waiting for Wait1.

So deadlock [illegible]

It signal1 and signal2 are interchanged [illegible]

Consumer deadlock [illegible] trigger.

It will just give [illegible] in the [illegible] number waiting.

8.9 [illegible]

Total no. of resources = 6

1 resource is holding [illegible] and B [illegible]

Process [illegible] holding [illegible] additional required x_i. There are two processes [illegible] such that $x_1 = x_2 = 3$

For no deadlock condition

$$x_1 + x_2 \le [illegible]$$

8.10 [illegible]

Consider the matrix

	allocation	required
	XYZ	XYZ
P_1	[illegible]	1,0
P_2	[illegible]	[illegible]
P_3	2	2

So the available resources are (0, 1, 2) and the requirement of process P_1 is (0, 1, 2) and P_2 can use the available resources after releasing the resources for matrix [illegible]

	allocation	required
	XYZ	XYZ
P_1	121	103
P_2	0,0	1,1
P_3	231	120

In this case available matrix is (2, 1, 3) and resources required P_1 is (1, 0, 3) so P_3 gets the resources of for releasing the resources. The available matrix is (3, 3, 1) so P_1 will use the resources P_2 will finish first.

8.11 [illegible]

Consumer [illegible] the producer [illegible] the item, and producer [illegible] process the first time only [illegible] the consumer [illegible] the system.

8.12 [illegible]

Release [illegible] time [illegible] the number [illegible] is a technique of deadlock avoidance [illegible] deadlock cannot occur.

However, we [illegible] starvation [illegible] may occur.

8.13 [illegible]

In deadlock prevention, the request for resources may not be granted even if the resource is free.

8.11 [illegible]:

Initially available resources

R1	R2	R3	R4
[illegible]	[illegible]	3	2

Process		Allocate				Available			
		R1	R2	R3	R4	R1	R2	R3	R4
t = 0									
P_1 : requests 2 units of R3	P1	0	0	0	0	3	0	1	2
P_2 : requests 2 units of R3	P2	0	0	2	0				
P_3 : requests 1 unit of R2	P3	0	0	0	1				
t = 1									
P_1 : requests 1 unit of R3	P1	0	0	1	0	0	0	0	2
	P2	0	0	2	0				
	P3	0	0	0	1				
t = 2									
P_1 : requests 1 unit of R4	P1	0	0	1	0	0	0	0	0
P_2 : requests 1 unit of R3	P2	0	0	2	1				
	P3	0	0	0	1				
t = 3									
P_1 : requests 1 unit of R1	P1	1	0	1	0	0	0	0	0
release 1 unit of R3	P2	1	0	2	1				
	P3	0	0	0	1				
t = 4									
P_1 : requests 1 unit of R2									
[illegible]									
t = 5									
P_1 : release 1 unit of R1	P1	0	1	1	0	1		0	0
and [illegible] of P1	P2	0	0	2					
	P3	0	0	0					
P_2 : release 2 units of R3									
now request of P_1 1 unit of T1 will be granted and request of P_2 1 unit of T1 will be granted									
t = 6									
P_2 : release 1 unit of R3	P2			1	0	1	1	1	0
	P1	0	0	1	1				
	P3	0	0	0	1				
t = 7									
P_1 : release 1 unit of R3	P1	1	1	0	0	1	0	2	0
P_2 : requests 1 unit of T1	P2	1	0	1	1				
	P3	0	1	0	1				
t = 8									
P_1 : finish	P1	1	1	0	0	2	0	2	1
	P3	0	1	0					
P_1 requests 2 units of R1 initially		1	1	0	0	2	0	2	1
P_3 : request 1 unit of R3		0	1	1	1				
t = 9									
P_2 : finish	P1	1	1	0	0	2	1	0	2
request of 2 units of R1 by P1 will be granted									
	P1	1	1	0	0	2	1	0	0
t = 10									
P_1 : finish						2	2	2	2
All processes will finish without any deadlock.									

3.15 [illegible]

[illegible]

3.16 [illegible]

Allocation	Max	Need	Available additional resources
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

3.17 [illegible]

[illegible]

3.18 [illegible]

[illegible]

3.19 [illegible]

[illegible]

4 Memory Management and Virtual Memory

4.1 Match the pairs in the following exercises.

Pair-I	Pair-II
A. Virtual Memory	P. Temporal locality
B. Paged memory	Q. Spatial locality
C. Loop [illegible]	R. [illegible]
D. Training [illegible]	S. [illegible]

[1989 : 8 Marks]

4.2 Under paged memory management scheme [illegible] memory may still be required if the [illegible] mapping hardware.

[1990 : 2 Marks]

4.3 Match the [illegible] Pair-II [illegible]

(a) [illegible]	(f) Fragmentation
(b) [illegible]	(g) [illegible]
(c) Working set	(h) Principle of locality
(d) [illegible]	(e) [illegible]

[1990 : 2 Marks]

4.4 State whether [illegible] memory access [illegible] memory [illegible] memory access [illegible] main memory [illegible] maximum speed.

[1990 : 2 Marks]

4.5 State whether the following statements are TRUE or FALSE with reason. The [illegible] and [illegible] require [illegible] only items.

[1990 : 2 Marks]

4.6 Match the pairs in the following questions by writing the corresponding letters only.

(a) Buddy system	(f) Dynamic type specification
(b) [illegible]	(g) Segmentation
(c) Pointer type	(h) Memory fragmentation
(d) Virtual memory	(e) Buddy allocation

[1991 : 2 Marks]

4.7 The total [illegible] in a virtual memory system is limited by

(a) the [illegible]
(b) the available secondary storage
(c) the available main memory
(d) All of these

[1991 : 2 Marks]

4.8 A link editor is a program which

(a) Matches the parameters of the macro definition with those of the parameters of the macro call
(b) Binds [illegible] program and [illegible] their location in other programs
(c) Matches the parameters of subroutine definition with the actual parameters of [illegible]
(d) [illegible]
(e) Acts as a link between compiler and [illegible] program

[1991 : 2 Marks]

4.9 [illegible] the following statements given below.

(a) The amount of virtual memory available is limited by the availability of secondary storage
(b) Any [illegible] of a critical section requires the use of an indivisible machine instruction such as test and set.
(c) The LRU page replacement policy may cause thrashing in some type of programs.
(d) The best fit technique for memory allocation results in memory will never be fragmented.

[1993 : 2 Marks]

4.10 When page replacement policy sometimes leads to more page faults when size of memory is increased?

(a) [illegible]	(b) LRU
(c) [illegible]	(d) [illegible]

[1992 : 1 Mark]

4.11 A [illegible] two-pass assembler. In the following in the first pass

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[1994 : 2 Marks]

4.12 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) None of these

[1991 : 2 Marks]

4.14 Consider the following [illegible] blank [illegible] hatched region [illegible]

The [illegible] for [illegible] 700, 20, 120, [illegible] is

The moving [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[199[illegible] : 2 Marks]

4.14 The principle of locality justifies [illegible]

(a) Interrupts (b) [illegible]
(c) [illegible] (d) Virtual Memory

[1995 : 1 Mark]

4.15 In a paged [illegible] memory management [illegible] page table [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[1995 : 1 Mark]

4.16 A [illegible] given [illegible] that were [illegible] separately. What information need to be [illegible] in an object module?

(a) Opcodes
(b) Relocation bits
(c) Name and [illegible] of all external symbols [illegible]
(d) Absolute address of internal symbols

[1991 : 1 Mark]

4.17 The [illegible] memory unit is defined by the number of words multiplied by the number of bits/word. How many address and data lines are needed for a memory of 4 K × 16?

(a) 12 address, 16 data lines
(b) 11 address, 8 data lines
(c) 12 address, 16 data lines
(d) 12 address, 12 data lines

[1996 : 2 Marks]

4.18 The address [illegible] generated by [illegible] program consisting of [illegible] page with 1 [illegible] memory. Name is [illegible] address. What is the number of page faults [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[1995 : 2 Marks]

4.19 In a virtual memory system the address space specified by the address lines of the CPU must be ____ than the physical memory size [illegible]

(a) smaller, smaller (b) smaller, larger
(c) larger, smaller (d) larger, larger

[1991 : 2 Marks]

4.20 A [illegible] bit [illegible] word length. The size of [illegible] requires is

(a) 256 × 16 (b) [illegible]
(c) [illegible] (d) [illegible]

[1994 : 1 Mark]

4.21 A [illegible] memory is [illegible] using [illegible] parallel [illegible] respectively. The [illegible] Write [illegible]

(a) 161 (b) 201
(c) 801 (d) 31

[1998 : 2 Marks]

4.22 Locality of reference implies that the page references being made by a process
(a) will always be to the page used in the previous page reference
(b) is likely to be one of the pages used in the last few page references
(c) will always be to one of the pages residing in main memory
(d) will always be to a new page not in main memory

[1997 : 1 Mark]

4.23 Thrashing
(a) Reduces page I/O
(b) Decreases the degree of multiprogramming
(c) Increases the page I/O
(d) Increases the degree of multiprogramming

[1997 : 1 Mark]

4.24 Dynamic linking of a program is possible
(a) If the program is stored on magnetic device
(b) If the program is in EXE format
(c) If the program is in a virtual page
(d) None of the above

[1997 : 1 Mark]

4.25 In a multiuser OS, computation which of the following system software is provided in the main memory under all circumstances
(a) Assembler (b) Linker
(c) Loader (d) Compiler

[1995 : 1 Mark]

4.26 The memory map for a program is as shown below

What will be the size of the partition (in K bytes) required in main memory to load and run this program?
(a) 12 KB (b) 14 KB
(c) 10 KB (d) 8 KB

[1998 : 2 Marks]

4.27 Two interacting processes communicate by passing messages through kernel. In addition, processes exchange some synchronization information. The minimum number of messages required per message exchange is
(a) 1 (b) 2
(c) 3 (d) 4

[1996 : 2 Marks]

4.28 Which of the following is a secondary storage and not memory?
(a) Flash memory is an example.
(b) Processor can be programmed to read and write.
(c) Address space is independent of whether the program will fit in physical memory.
(d) Programs larger than the physical memory size can be run.

[1999 : 2 Marks]

4.29 Suppose the Linux kernel runs at 100 Hz. This means that the kernel wakes up every 10 milliseconds, which is known as a kernel tick. Then, a delay of 5 ticks results in actual delay of time between
(a) 50 milliseconds
(b) 1 millisecond
(c) 5 milliseconds
(d) 0.05 milliseconds

[2000 : 2 Marks]

4.30 Consider a machine with 64 MB physical memory and a 32-bit virtual address space. If the page size is 4 KB, what is the approximate size of the page table?
(a) 16 MB (b) 8 MB
(c) 2 MB (d) 24 MB

[2001 : 2 Marks]

4.31 Which of the following statements is false?
(a) Virtual memory implements the translation of a program's address space into physical memory address space
(b) Virtual memory allows each program to exceed the size of the primary memory
(c) Virtual memory increases the degree of multiprogramming
(d) Virtual memory reduces the context switching overhead

[2001 : 1 Mark]

4.84 The process of assigning load addresses to the various parts of the program and adjusting the code and data in the program to reflect the assigned addresses is called

(a) assembly (b) parsing
(c) relocation (d) symbol resolution

[2001 : 1 Mark]

4.85 Consider a virtual memory system with FIFO page replacement policy. For a certain page trace, it is observed that increasing the number of page frames in main memory will

(a) always decrease the number of page faults
(b) always increase the number of page faults
(c) sometimes increase the number of page faults
(d) never affect the number of page faults

[2001 : 1 Mark]

4.86 Which of the following is not a form of memory?

(a) Instruction cache
(b) Instruction register
(c) Instruction opcode
(d) Translation look-aside buffer

[2002 : 1 Mark]

4.87 Which of the following requires a device driver?

(a) Disk drive with the root file system
(b) Disk partition for the swap area
(c) Hard disk used as a raw device
(d) Hard disk used for mounting a file system

[2002 : 1 Mark]

4.88 Demand paging can cause thrashing to occur because

(a) Permit is slow
(b) The page fault rate is very high
(c) Page tables are not known at run time
(d) Page faults occur
(e) Cryptographic procedures are used for dynamic linking

[2002 : 2 Marks]

4.89 In a system with 32 bit virtual address and 2 KB page size, use of one level page table for virtual to physical address translation is not practical because of

(a) the large amount of internal fragmentation
(b) the large amount of internal fragmentation
(c) the large memory overhead in maintaining page tables
(d) the large computation time required to access the translation tables

[2003 : 1 Mark]

Common Data for Q. 4.90 & 4.91

A processor uses 2-level page tables for virtual to physical address translation. Page tables for both levels are stored in the main memory. Virtual and physical addresses are both 32 bits wide. The memory is byte-addressable. For virtual to physical address translation, the first 10 bits of the virtual address are used to index into the first level page table, the next 10 bits are used to index into the second level page table. The 12 least significant bits of the virtual address are used as offset within the page. Assume that the page table entries occupy exactly one word each. Further, assume that the processor has a TLB with a hit rate of 96%. The TLB caches recent virtual page number to physical page number translations. The processor also has a private cache with a hit rate of 90%. Assuming zero access time to the cache, the cache access time is 1 ns, and TLB access time is also 1 ns.

4.90 Assuming that a page is in the cache, the average time taken to access a virtual address is approximately (in nanoseconds)

(a) 1.5 ns (b) 2 ns
(c) 3 ns (d) 4 ns

[2008 : 2 Marks]

4.91 Suppose a process has exactly the following pages in its virtual address space: two contiguous code pages starting at virtual address 0x00000000, two contiguous data pages starting at virtual address 0x00400000, and a stack page starting at virtual address 0xFFFFF000. The amount of memory required for storing the page tables of this process is

(a) 8 KB (b) 12 KB
(c) 16 KB (d) 20 KB

[2008 : 2 Marks]

4.92 Consider an operating system capable of loading and executing a single sequential user process at a time. The disk head scheduling algorithm used is First Come First Served (FCFS). If CPU is engaged in I/O with one user process at a time, which of the disk scheduling algorithms does not result in starvation?

(a) First Come First Serve (FCFS)
(b) Shortest Seek Time First (SSTF)
(c) Elevator (SCAN)
(d) Circular SCAN (C-SCAN)

[2003 : 1 Mark]

[illegible] performance results, which is the expected improvement in the CPI performance of the program?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 1 Mark]

4.41 [illegible] page frames [illegible] be allocated to a running process in a virtual memory system. [illegible] determined by

(a) the instruction set architecture

(b) paging

(c) physical memory size

(d) number of processes in memory

[2004 : 1 Mark]

4.42 Consider a system with a two-level paging scheme in which a regular memory access also [illegible] and accessing a page in the page table [illegible] milliseconds. The average instruction time is 1 [illegible] seconds of CPU time and [illegible] memory accesses. The TLB hit ratio is 90%, and the page fault rate is one in every 10,000 instructions. What is the [illegible] average instruction time in [illegible]?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

4.43 Consider a fully associative cache with 8 cache blocks (0–7) and the following block requests: 4, 3, 25, 8, 19, 6, 25, 8, 16, 35, 45, 22, 8, 3, 16, 25, 7.

If LRU replacement policy is used, which cache block will have memory block 7?

(a) 4 (b) 6

(c) 3 (d) 7

[2004 : 2 Marks]

4.44 In a virtual memory system, size of virtual address is 32-bit, size of physical address is 30-bit, page size is 4 Kbyte and size of each page table entry is 32 bit. The virtual memory is organized as 8-way set associative cache. Which one of the following is the minimum number of bits that can be used for a tag field in each cache entry?

(a) 18 (b) 20

(c) 22 (d) 24

[2004 : 2 Marks]

4.45 [illegible] 2-way set-associative cache memory with 4 sets and LRU [illegible] cache blocks (0–7) and a [illegible] memory with 128 blocks (0–127). What memory blocks will be present in the cache after the following sequence of memory block references [illegible] LRU replacement policy? [illegible] assuming [illegible] the cache did not have any memory block loaded [illegible] job?

0 5 3 9 7 0 16 55

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

4.46 Consider [illegible] processes P, Q, R, and T. The total [illegible] is 100 [illegible] the number of [illegible] given below.

Process	Total time in KB	Number of segments
P	147	4
Q	45	6
R	48	2
T	96	3

The page size is 1 KB. The [illegible] in the page table is 4 bytes. The size of [illegible] in the segment table is 8 bytes. The maximum [illegible] is 256 KB. If paging method [illegible] memory management unit [illegible] paging, the [illegible] page table [illegible]. The [illegible] required for the segments [illegible] method is [illegible]. The [illegible] required for the segments [illegible] and paging [illegible] is [illegible]. What is the relationship among the overheads for [illegible] methods in the memory management unit [illegible] the [illegible] processes?

(a) P < Q < R (b) S < P < T

(c) R < T < Q (d) [illegible] < S < P

[2005 : 2 Marks]

4.47 A CPU generates 32-bit virtual address. The page size is 4 Kb. The memory [illegible] 16 Mb which [illegible] 128 page [illegible]. The minimum size of the TLB tag is

(a) 11 bits (b) 13 bits

(c) 15 bits (d) 23 bits

[2006 : 2 Marks]

4.48 A computer system supports 32-bit virtual addresses and 24-bit physical addresses.

Since the virtual address space is of the same size as the physical address space, the operating system designer decides to get rid of the entire memory unit. Which one of the following is true?

(a) Efficient implementation of multiprocessor support is no longer possible.
(b) The processor-cache-memory interface can be made more efficient now.
(c) The view supported by memory management unit no longer needed.
(d) CPU scheduling can be made more efficient now.

[2006 : 3 Marks]

4.49 The following sequence represents the trace of a particular program running on a pure demand paging system with 100 bytes per page. (1, 0), (2, 0), (3, 0), (4, 0), (5, 0), (6, 0), (7, 0), (8, 0), (9, 0), (10, 0), (11, 0), (12, 0), (13, 0)

Suppose that the memory can accommodate one page and LRU is the address replacement policy. Then the page fault rate from address 0 to address 99 are located in the memory. How many page faults will occur?

(a) 0 (b) 1
(c) 7 (d) 8

[2007 : 1 Mark]

4.50 A demand paging system takes 100 time units to service a page fault and 300 time units to replace a dirty page. Memory access time is 1 time unit. The probability of page fault is p. In case of a page fault, the probability of page being dirty is also p. It is observed that the average access time is 3 time units. Then the value of p is

(a) 0.194 (b) 0.333
(c) 0.514 (d) 0.001

[2007 : 2 Marks]

Common Data for Q. 4.51 & Q. 4.52

A process has been allocated 4 page frames. Assume that none of the pages of the process are available in the memory initially. The process makes the following sequence of page references (1 time unit each): 1, 2, 3, 4, 7, 1, 2, 5, 3, 1.

4.51 If demand paging is used, how many page faults occur for the above reference string?

(a) 7 (b) 8
(c) 9 (d) 10

[2007 : 2 Marks]

4.52 Least Recently Used (LRU) page replacement policy is a good approximation to optimal page replacement. For the above reference string how many more page faults occur with LRU than with the optimal page replacement policy?

(a) 0 (b) 1
(c) 2 (d) 3

[2007 : 2 Marks]

4.53 A paging scheme uses a translation look-aside buffer (TLB). A TLB access takes 10 ns and a main memory access takes 50 ns. What is the effective access time (in ns) if the TLB hit ratio is 90% and there is no page fault?

(a) 51 (b) 60
(c) 65 (d) 70

[2008 : 1 Mark]

4.54 A processor uses 36-bit physical addresses and 32-bit virtual addresses, with a page frame size of 4 Kbytes. Each page table entry is 32 bits long. The first level page table is tailored to fit in one page frame. Virtual address is divided as follows:

- Bits 31-30: These are used to index into the first level page table.
- Bits 21-29: These are used to index into the second level page table.
- Bits 0-12: These are used as the offset within the page.

The number of bits required for addressing the next level page table (or page frame) in the page table entry of the first, second and third level page tables are respectively:

(a) 20, 20 and 20 (b) 24, 24 and 24
(c) 24, 24 and 20 (d) 20, 20 and 24

[2008 : 2 Marks]

4.55 Assume that a main memory with only 4 page frames and 20 bytes is initially empty. The CPU generates the following sequence of virtual addresses and uses the Least Recently Used (LRU) page replacement policy.

0, 4, 5, 20, 21, 39, 44, 10, 85, 90, 91, 84, 23, 22, 39, 40

[illegible]

(a) 6 and 1, 3, 4, 5, 7 and 1, 2, [illegible]
(b) 7 and 1, [illegible] (d) 8 and 2, 3, 5

[2005 : 2 Marks]

4.58 Match the following lists based on the concept of virtual memory management in the context of the list I. Choose the correct answer from the list II. Choose the correct answer from the list II. Choose the correct answer from the list II

4.65 A computer has twenty physical page frames which contain pages numbered 101 through 120. Now a page fault occurs for page number 150. Which page should be replaced if the page replacement policy is:

(a) Least recently used

(b) First-in-first-out

(c) Least frequently used

(d) Most recently used

[2014 (Set-2) : 2 Marks]

4.66 Assume that 8 page frames in a virtual memory system are initially empty. Pages are loaded in these frames in response to the following sequence of page references: 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6. Which page replacement policy results in the least number of page faults?

(a) First-In-First-Out (FIFO)

(b) Least Recently Used (LRU)

[2014 (Set-3) : 1 Mark]

4.67 Consider a paging hardware with a TLB. Assume that the entire page table and all its contents are in the physical memory. Each memory access takes 100 nanoseconds. It takes another 100 nanoseconds to access a page table entry in the physical memory. The TLB hit ratio is 0.6. The effective memory access time (in nanoseconds) is _____

[2014 (Set-3) : 2 Marks]

4.68 Consider a system with byte-addressable memory, 32 bit logical address, a page size of 4 KB and page table entry size of 4 bytes. The maximum size of the page table in the system in megabytes is _____

[2015 (Set-1) : 1 Mark]

4.69 Consider a main memory with five page frames and the following sequence of page references: 2, 3, 2, 1, 5, 2, 4, 5, 3, 2, 5, 2. Which one of the following is true with respect to page replacement policies First-In-First-Out (FIFO) and Least Recently Used (LRU)?

(a) Both cause the same number of page faults

(b) FIFO incurs 2 more page faults than LRU

(c) LRU incurs 2 more page faults than FIFO

(d) LRU incurs 1 more page fault than FIFO

[2015 (Set-1) : 2 Marks]

4.70 A computer system implements 8 kilobyte pages and a 32-bit physical address space. Each page table entry contains a valid bit, a dirty bit, three permission bits, and the translation. If the maximum size of the page table for a process is 4 megabytes, then the length of the virtual address supported by the system is _____ bits.

[2015 (Set-2) : 2 Marks]

4.71 Consider a demand paged virtual memory system with 40-bit virtual addresses and 16-KB pages. Virtual pages are mapped onto 256-MB physical memory. The computer system has a two-level page table per process and each page table entry requires 8 bytes. The size of the per process page table is _____ pages.

[2016 (Set-1) : 2 Marks]

■■■■

Answers Memory Management and Virtual Memory

4.7	(b)	4.8	(c)	4.9	(b, d)	4.10	(c)	4.11	(a)	4.12	(b)	4.13	(b)	4.14	(d)	4.15	(c)
4.16	(c)	4.17	(c)	4.18	(c)	4.19	(c)	4.20	(d)	4.21	(d)	4.22	(b)	4.23	(a)	4.24	(c)
4.25	(c)	4.26	(b)	4.27	(c)	4.28	(d)	4.29	(d)	4.30	(c)	4.31	(c)	4.32	(a)	4.33	(c)
4.34	(c)	4.35	(d)	4.36	(b)	4.37	(c)	4.38	(c)	4.39	(a)	4.40	(d)	4.41	(a)	4.42	(a)
4.43	(b)	4.44	(d)	4.45	(c)	4.46	(b)	4.47	(a)	4.48	(a)	4.49	(a)	4.50	(a)	4.51	(a)
4.52	(c)	4.53	(c)	4.54	(b)	4.55	(b)	4.56	(d)	4.57	(a)	4.58	(b)	4.59	(a)	4.60	(b)
4.61	(b)	4.62	(c)	4.63	(c)	4.65	(d)	4.69	(a)	4.71	(a)						

Explanations Memory Management and Virtual Memory

4.1 (b)

A. C, D B. C D. D E.

4.2 (d)

[illegible]

4.3 (c)

[illegible]

4.4 (b)

[illegible] data in the form of blocks to cache and [illegible] [illegible] memory unit to [illegible] memory [illegible]

4.5 (b)

This because in [illegible] some part [illegible] which is simply wastage of memory. As this [illegible] number of assemblies and [illegible] [illegible]

4.6 (d)

[illegible]

4.7 (b)

Since virtual memory is implemented [illegible] secondary storage.

4.8 (b)

[illegible] a program which [illegible] Theory.

4.9 (b, d)

(a) Without [illegible] instruction [illegible] can be [illegible] using [illegible].

(b) [illegible]

4.10 (c)

[illegible] Memory [illegible] in TLB. More [illegible]

4.11 (a)

[illegible]

4.12 (b)

[illegible]

4.13 (b)

[illegible] 30, 33, 36, 38

[illegible]

[illegible]

4.14 (d)

The principle [illegible] the [illegible] memory.

4.15 (a)

[illegible] must have a page [illegible] in a page segmented scheme.

4.16 (b)

Refer to definition: Thrashing is a process in which [illegible] with high page faults.

4.17 (c)

Size of memory = [illegible] × 16 bits
= [illegible] × 16 bits
⇒ 2 address lines are [illegible]

4.18 (c)

Given LAS is eight times as PAS, which can hold 16 records of size [illegible] each.

100	270	[illegible]	[illegible]	[illegible]	[illegible]
590	190	[illegible]	[illegible]	[illegible]	[illegible]
470					

4.19 (c)

The virtual memory system, Address space is usually [illegible] address [illegible] GB. [illegible] lower than the [illegible] memory size [illegible] than the secondary storage size.

4.20 (d)

$2^{16} \times 8^{2} = 2^{18} \times 16$

4.21 (d)

(a) 150 can fit in any of [illegible]
(b) 350 can fit in any of [illegible]
(c) 250 can fit in [illegible]
(d) 500 can fit in [illegible] only.
∴ This requires 54% unused.

4.22 (b)

Locality of reference is a phenomenon describing the same value or related storage locations being frequently accessed.

4.23 (c)

Thrashing: Refer to text using the maximum number of the allowed in main memory. CPU utilization and time [illegible] this context is the Thrashing.

The thrashing graph is given in page 150.

4.24 (a)

Which page has been in the [illegible] with a page fault [illegible] many, so it needs to be removed [illegible].

4.25 (a)

Loader is used to load the program into main memory.

4.26 (b)

A maximum stack that can be accessed at a time is 16 [illegible] using 20 bits. The maximum space to hold the program is decided by the maximum stack taken by [illegible] from the [illegible] = 2^20 = 1 MB.

4.27 (a)

[illegible] × 10
page fault rate
page fault service time
memory access time

$$= \frac{(1-p)}{2} + \left(1 - \frac{p}{R}\right) \cdot 1 + \frac{(1-p)}{R} = 1 + \frac{p}{R}$$

4.28 (a)

Virtual memory allows the user to run the [illegible] larger than the physical memory size.

4.29 (a)

Average memory access time = (hit ratio × time [illegible] + (1 − hit ratio) × (time [illegible] + memory access time))

If page [illegible] memory access time is [illegible] then

[illegible]
= [illegible]
= [illegible] = [illegible]

4.80 [illegible]

Number of entries in page table = $\frac{2^{32}}{2^{12}} = 2^{20}$

Frame size = $\frac{2^{32}}{2^{20}}$ = [illegible]

∴ PT has [illegible] entries, total size of page table = [illegible] entries × size of PT

$= 2^{20} \times 2 = 2^{21} = 2$ MB

4.81 [illegible]

In a system with virtual memory [illegible] switching includes [illegible] switching of address spaces.

4.82 [illegible]

Relocation can change the assigned address of a memory location in the program.

4.83 [illegible]

Binary [illegible] number [illegible] page table

4.84 [illegible]

[illegible] = [illegible], but [illegible] increased with [illegible] that [illegible] perform

4.85 [illegible]

Dirty [illegible] if any data has been used [illegible]

4.86 [illegible]

[illegible] the process [illegible]

4.87 [illegible]

Given, Page size = [illegible] KB = $2^{[illegible]}$ [illegible]

Virtual address = [illegible] bits long

Number of pages

$= \frac{[illegible]}{[illegible]} = [illegible]$

$2^{[illegible]}$ is a very large [illegible] memory overhead in maintaining page tables.

4.88 [illegible]

Average time taken to access a virtual address

$= [0.[illegible] \times 0.9 + 0.9 \times 1 + 0.1 \times 0.1 + 0.1 \times 1]$

$= [0.[illegible] \times 0.9 + 0.9 \times 1 + 0.1 \times 0.1 + 0.1 \times 1]$

$= [0.[illegible] + 0.9 + 0.1 \times 0.1 + 0.1 \times 1] = [0.[illegible]]$

$= 2.[illegible]$ [illegible]

4.89 [illegible]

Logical address = 32 bits

P = [illegible]

D = [illegible]

[illegible]

First virtual address given

Address will be found in [illegible] page table [illegible]

Second virtual address given

Address will be found in [illegible] page table [illegible]

Third virtual address given

Address will be found in [illegible] page table [illegible]

[illegible] the process [illegible] page table in main memory

$= 2^{[illegible]} \times 4 B = 16$ kB

4.90 [illegible]

[illegible] performance [illegible] program dominated by [illegible] instructions [illegible] reducing [illegible] disk. When we uses DCIS and replace it by SSD [illegible] improve [illegible] performance [illegible] the [illegible] performance. So SSD improve [illegible] process [illegible]

4.41 (b)

Minimum number of page frames allocated to a running process in order to avoid excessive page faults.

4.42 (b)

Page fault rate $P = 10^{-6}$

Effective memory access time (EMAT)

$= (1-P) \times 150 + P \times 8 \times 10^{6} = (1 - 10^{-6}) \times 150 + 10^{-6} \times 8 \times 10^{6} = 165$ ns

Average total instruction time = CPU time + $P \times$ SM = 1 + $P \times$ DMA

$= 100 \times 10^{-9} + 10^{-6} \times 8 \times 10^{6} \times 10^{-9} = 100 \times 10^{-9} + 8 \times 10^{-3}$

$= 8100.00 \times 10^{-9}$ sec

4.43 (b)

When 4 frames are allocated, the number of faults are

4 8 27 8 19 6 16 17

LRU using Global replacement policy, page fault occurring with

[1 3 4 2 5 6 7 8 9 10 11 12]

So, replacement 4

46 8 27 8 19 6 16 46

[2 18 3 25 5 16 27 35]

Similarly, 7 replaces 1 for page

43 28 73 8 19 6 16 35

[19 6 27 8 16 35 43 22]

9 replaces 2

16 22 28 9 19 6 39 35

[10 8 35 18 7 19 44 9]

8 replaces 19

46 28 28 9 8 6 16 46

[9 25 39 35 42 28 8 3]

18 replaces 35 which is in block 6

43 28 73 8 7 8 18 73

[8 26 47 22 8 16 27 8]

Finally 7 replaces 16, which is in block 2.

4.44 (b)

Page table entry must contain bits for representing frames and a few bits for keeping information like dirty bit, reference bit, etc.

Number of frames = number of possible physical memory / Page size = $2^{32}/2^{12}$

$= 2^{20}$

18 bits + 3 bits (PT entry size) = 21 bits

$= 3$ bytes

4.45 (d)

Last three memory blocks are mapped to cache in cache. So number of tag bits = 32 − 2 − 3 − 2 = 25 bits. Also, 32 blocks going to one cache memory tag bits. Now, these 5 bits identify a memory block, and 3 bits are used for index. Bits reference to cache memory references, so total locality will hamper cache performance.

So, based on the two index bits (2 bits) 4 blocks will be mapped in cache.

Set Number	Block Numbers
0	0, 16
1	1, 8
2	2
3	3, 7, 35

Since, cache set has only 2 places, 3 will be fine, so we have to consider only set 3. Block 35, final numbers of misses will be 0, 8, 7, 8, 15, 35.

4.46 (b)

Overhead is considered as memory wasted in maintaining the page table.

So, we need to find the overhead in every case in the given question.

Case-1: Overhead using one level paging,

Total page table = 1 GB

Total pages = 1GB / 4KB = $2^{30}/2^{12} = 2^{18}$ = 256 KB pages.

Each page table entry = 4 bytes.

Size of page table = 256 × 4 = 1 Mbyte

Size of page table = 256 × 8 = 2048 bytes

Size of page table = 256 × 8 = 2048 bytes

Overhead in paging (B) = 2 × 2048 + 2062

= 6158 bytes.

Case-2: Overhead using segmentation.

Number of segments = 4 = 2 + 8 + 8 = 20 bits for segment number

Maximum [illegible] segment size

[illegible]

[illegible] = [illegible] bytes (or [illegible] KB)

Number of bits required to find the limit segment offset = [illegible]

Size of segment table [illegible]

[illegible] = [illegible] bytes

∴ Overhead using segment size of [illegible] = [illegible] bytes

Hence B: [illegible] segment size and segment

Number of segments [illegible] bits are required for segment number.

Maximum [illegible] = [illegible] bytes → [illegible] bits required for page offset

[illegible] page table entries required.

[illegible] segment needed for page table.

Size of segment table = [illegible] × [illegible] = [illegible] bytes.

Size of each page table = [illegible] × [illegible] = [illegible] bytes.

∴ Segmentation and paging overhead (B)

= [illegible] B + [illegible] × [illegible] B = [illegible] B

A: Simple segmentation makes less overhead.

D: Two level paging gives little more overhead because [illegible] two page tables.

B: Segmented paging [illegible] more overhead because [illegible] page number for segment and segment offset.

So B < D < C

1.19 [illegible]

Size of virtual address = [illegible] bits

Page size = [illegible] KB

Block size = [illegible] = 32

Number of bits used for indexing

= [illegible] − [illegible] − [illegible] = [illegible]

Number of sets = [illegible] = [illegible] = [illegible] sets, [illegible] bits

Total tag bits = [illegible] × [illegible] = [illegible] bits

1.18 [illegible]

The computer system supports 32-bit virtual address and [illegible] 28-bit physical address space. The [illegible] translation from virtual to physical address is done by hardware called Memory Management Unit (MMU). This MMU [illegible] supports [illegible] memory management [illegible] required.

1.19 [illegible]

0100 page fault, address still 100 in memory

0200 page fault, address still 200 in memory

0300 page fault, address still 300 in memory

0400 no page fault

0100 no page fault

0500 page fault, address still 500 in memory

0200 no page fault

0300 page fault, address still 300 in memory

0400 page fault, address still 400 in memory

0100 no page fault

0200 no page fault

0300 page fault, address still 300 in memory

0400 no page fault

1.20 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

1.21 [illegible]

Number of page faults = [illegible]

Consider the page table entries as [illegible] algorithm as follows

[illegible]

Total number of page faults = 7

4.32 (d)

Consider LRU page replacement policy

Page Reference	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Page frames	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Page faults	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

So, total number of page faults = 9

Number of page faults in LRU = Number of page faults in optimal + [illegible] more page faults.

4.33 (c)

Effective access time = Hit ratio × (TLB access time + Memory access time) + (1 − hit ratio) × (TLB access time + Memory access time + Memory access time)

= 0.9 × (1 + 100) + 0.1 × (1 + 100 + 100)

= 90 + 11 = 101

4.34 (b)

Size of page table at first level = 24

Size of page table at second level = 24

Size of page table at third level = 24

4.35 (b)

1. Page fault = 1
   Pages in memory = 0
2. Page fault = 2
   Pages in memory = 0
3. Page fault = 3
   Pages in memory = 0
4. Page fault = 4.
   Pages in memory = 0, 1
5. Page fault = 5
   Pages in memory = 0, 1, 2
6. Page fault = 6.
   Pages in memory = 0, 1, 3
7. Page fault = 7,
   Pages in memory = 1, 2, 0
8. Page fault = 8,
   Pages in memory = 1, 2, 0, 4
9. Page fault = 9
   Pages in memory = 1, 2, 0, 4
10. Page faults = 10.
    Pages in memory = 2, 0, 4, 3
11. Page fault = 11,
    Pages in memory = 2, 0, 4, 6
12. Page fault = 12,
    Pages in memory = 0, 4, 3
13. Page fault = 13,
    Pages in memory = 0, 1, 3
14. Page fault = 14,
    Pages in memory = 1, 3, 5
15. Page fault = 15,
    Pages in memory = 3, 5, 6

4.36 (d)

Belady's anomaly is well known. Reference [illegible] is a variation of FIFO called second chance (SC) policy, in which [illegible] page replacement [illegible] page [illegible] and [illegible] page [illegible] is used in [illegible] system [illegible] which is a page replacement policy [illegible] is [illegible] page [illegible].

Note, but is not used for page replacement. It is not used in the page replacement policy. It [illegible] the page [illegible] memory is [illegible] or not. If [illegible] is [illegible] page [illegible].

So basically [illegible] page [illegible] replacing, it is [illegible] we know [illegible] page [illegible] we are filling empty spaces in [illegible] locations.

4.37 (d)

In Belady's Anomaly if number of frames are increased then number of page faults increases. This is observed only with FIFO.

4.38 (b)

Page frame numbers are not important, virtual page numbers are [illegible] page [illegible].

4.39 (d)

LRU policy for page replacement.

Access [illegible] distinct pages by taking some example [illegible]

Access leading to get

and now access these pages in reverse order

4 page faults

Allocated = 4 + 4 = 8 = 12 page frames

So 2 pages = 2 × 4 = 12

So for 3 pages = 3 × 4 = 12

So for 100 pages = 2 × (100) + 4 = 104

1.60 (b)

[Effective access time = [(1 − page fault rate) × access time when no page fault] + [page fault rate × access time during page fault]]

$$= \left[\left(1 - \frac{1}{10^7}\right) \times 30\,ns\right] + \left[\frac{1}{10^7} \times 10\,ms\right]$$

$$= \left(\frac{10^7 - 1}{10^7}\right) \times 30\,ns + \left[\frac{1}{10^7} \times 10\,ms\right]$$

$$= 30\,ns$$

1.61 (b)

FIFO

→ 6 MISS

So optimal = 1 + 2 + 1 = 4

1.62 (c)

Logical address space is 32 bit = 4 GB

$\frac{4\,GB}{8\,KB} = 2^{19}$

→ $2^{19} \times 2^{13} = 2^{32}$ KB

1.63 (c)

Since each [illegible] 1024 [illegible] hence [illegible]

→ No [illegible] = 8.

1.64 (c)

Optimal page replacement process:

[illegible] page faults

1.65 (b)

Pages:

[illegible]

Optimal page replacement:

Most recently used:

4.66 Sol:

4.67 Sol:

Effective memory access time:

$= 0.8 \times 100 + 0.2 \times 100 + 90 = 80 + 20 + 90 = 190$ ns

4.68 (b)

Page table size = Number of page table entries × Entry size

= Number of pages × 4 bytes

$= \frac{2^{32}}{2^{12}} \times 4$ bytes $= 4$ MB

4.69 (a)

FIFO (Page fault)

Here LRU also incurs 8 page faults.

Both FCFS and LRU incur the same number of page faults.

4.70 Sol:

Page size = 8 kB = 2^{13} bytes

→ 13 bits required for offset.

Number of sets in cache = 32

→ 5 bits required for index field offset

22 tag bits are required

4.71 (b)

The best Time of Day = Dual Time + the system efficient partition.

∴ The memory partitions 200 KB and 500 KB are allocated.

4.72 Sol:

Page size = 8 KB → 13 bit offset

Number of bits = 32 − 13 = 19 bits

Page table entry = Valid + Dirty + Physical address bits + Protection + Reference bits

= 1 + 1 + 19 + 2 + 1 + 2

= 26 bits

Page table size = 64 entries

Number of pages = Number of page table entries

$= \frac{2^{32}\ \text{Blocks}}{2^{13}\ \text{Blocks}} = 2^{19} = 524$ pages

∴ 26 bits required for page and 5 bits for Total length of frame number = 26 − 16 = 10 bits.

4.73 Sol:

Page table size = Number of entries in page table × Page table entry size

$\left\lceil \frac{2^{64}}{2^{13}} \right\rceil \times 48$ bits $= 2^{51} \times 6$ bytes

$= 2^{54} \times 6$ B $= 288$ PB

■■■■

3 File System and Device Management

5.1 On receiving an interrupt from an I/O device, the CPU:

(a) [illegible] time

(b) stands over writes of address and data [illegible] interrupting device.

(c) [illegible] interrupt service routine [illegible] immediately

(d) transfers control to interrupt service routine after completion of the current instruction.

[1993 : 2 Marks]

5.2 Disk storage consists of disk drives for cylinders 10, 22, 20, 2, 40, 6 and 38 in that order at some time when the disk arm is moving towards cyl 20. The arm can move 6 cylinders in a single move. Compute the total seek time for the disk arm scheduling algorithms:

(a) First come first served

(b) Closest cylinder next

[1991 : 2 Marks]

5.3 State whether for the following statements are TRUE or FALSE with reason. The data counter between memory and I/O devices [illegible] transferred [illegible] in [illegible] [illegible]

[1990 : 2 Marks]

5.4 The real dimension of disk could be [illegible]

(a) at a fixed address in main memory

(b) at a fixed location on the disk

(c) anywhere on the disk

(d) at a fixed location in the system disk.

[1998 : 2 Marks]

5.5 The correct matching for the following pairs:

List-I	List-II
A. [illegible]	1. High speed RAM
B. Cache	2. Disk
C. Interrupt [illegible]	3. Monitor
D. Centralized [illegible]	4. [illegible]

Codes:

(a) A-1 B-2 C-3 D-4

(b) A-4 B-1 C-3 D-2

(c) A-1 B-3 C-2 D-4

(d) A-2 B-1 C-3 D-4

[1997 : 1 Mark]

5.6 The correct matching for the following pairs is:

List-I	List-II
A. Disk scheduling	1. Round robin
B. Batch processing	2. Seek
C. Time sharing	3. JCL
D. [illegible] processing	4. FIFO

Codes:

(a) A-1 B-2 C-3 D-4

(b) A-2 B-1 C-4 D-3

(c) A-3 B-4 C-1 D-2

(d) A-4 B-3 C-2 D-1

[1997 : 1 Mark]

5.7 DMA stands for

(a) [illegible]

(b) [illegible] The [illegible]

(c) [illegible] 2 processors through a proc

(d) None of the above [1997 : 1 Mark]

5.8 [illegible]

(a) [illegible]

(b) average change rate of interrupted [illegible]

(c) [illegible] interrupt

(d) [illegible]

[1997 : 1 Mark]

5.9 Which of the following devices should get higher priority in assigning interrupts?

(a) Hard disk (b) Printer

(c) Keyboard (d) Floppy disk

[1996 : 1 Mark]

5.10 Which of the following is true?

(a) Unless enabled, a CPU will not be able to process interrupts

(b) Interrupt vectors are not affected if they [illegible]

(c) A processor [illegible] before executing a new instruction

(d) Code level instructions are possible in microprocessors

[1995 : 1 Mark]

5.11 [illegible] below are some [illegible] operating system [illegible] in the [illegible] hardware components [illegible]

A. [illegible] 1. Interrupt
B. [illegible] 2. Memory
C. [illegible] 3. CPU
D. Signal 4. [illegible]

[1999 : 1 Mark]

5.12 Which of the following [illegible] is [illegible] throughput?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[1999 : 1 Mark]

5.13 Which of the following [illegible] is the [illegible]?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2001 : 2 Marks]

5.14 In the index of a [illegible] file, the [illegible] of the [illegible]

(a) [illegible] and the [illegible] of the [illegible]
(b) [illegible] and the [illegible] of the [illegible]
(c) the size of the [illegible] and the [illegible] of the [illegible]
(d) None of the above

[2002 : 2 Marks]

5.15 Using a larger block size in a file system leads to

(a) better disk throughput but poor disk space utilization
(b) better disk throughput and better disk space utilization
(c) poor disk throughput but [illegible] disk space utilization
(d) poor disk throughput and poor disk space utilization

[2003 : 1 Mark]

5.16 A Unix-style i-node has 10 direct pointers and one [illegible] indirect pointer. A block [illegible] units. Disk block size is 1 Kbyte. Disk block address [illegible] bits, and 48 bits are used. What is the maximum size of a file?

(a) 2^{32} bytes (b) 2^{38} bytes
(c) 2^{42} bytes (d) 2^{48} bytes

[2004 : 3 Marks]

5.17 Access to a 200 [illegible] numbered 0 through 199. At a given time, it was serving the request of reading data from track 120. The previous request was for track 90. The seek [illegible] are for track numbers

80, 30, 1, [illegible], 190, 25.

How much [illegible] will be saved by [illegible] scheduling [illegible] Disk Scheduling [illegible] Track Time Period and [illegible] Disk Scheduling?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

5.18 In a [illegible] OS, [illegible] data [illegible] are [illegible] in a [illegible] database [illegible] simple [illegible] and [illegible] indirect [illegible] [illegible] of the [illegible] approximately the maximum size of a file in the [illegible]?

(a) [illegible] MB (b) [illegible] GB
(c) [illegible] (d) 16 GB

[2004 : 2 Marks]

5.19 A [illegible] in a computer system running Unix. The [illegible] "File1", "File2" and "File3" exist in the current working directory. [illegible] and write permissions for all three files. Assume that the [illegible] bit is [illegible] and the umask is [illegible]. The [illegible] information about the [illegible] is [illegible]. The [illegible] the following sequence of [illegible] commands in an empty directory:

[illegible] File1 File2
[illegible] File2 File3

What will be the following [illegible] in the [illegible] system?

I. [illegible] II. [illegible]
III. [illegible]

(a) I and II only (b) I and III only
(c) [illegible] (d) II and III only

[2005 : 1 Mark]

5.20 The chmod command

[illegible]

is equivalent to [illegible] [illegible] the above command was executed in the same directory:

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible] [2005 : 1 Mark]

5.21 In a computer system, four files of size 600 bytes, 400 bytes, 270 bytes and 1640 bytes are to be stored in a disk. Disk storage is divided into blocks of 512 bytes each. Blocks are numbered 0, 1, 2, 3, ... and the first block of the disk is reserved for storing the directory. Each directory entry consists of the file name, the block number of the first block of the file, and the file size. The directory is stored in block 0 and the directory entry for each file is 32 bytes. The free space on the disk is managed using a bit map. The bit map is stored in block 1. Bit 0 represents block 0, bit 1 represents block 1, and so on. Bit value 1 indicates a free block and bit value 0 indicates a used block. Assume that the directory and the bit map are kept in memory. Which of the following is the minimum number of disk blocks that must be accessed to store the contents of the files in the disk?

(a) 3 (b) 4

(c) 5 (d) 6

[2005 : 2 Marks]

5.22 In a certain operating system, which of the following is done by the scheduling routine to prevent starvation?

I. It migrates another process if there is enough extra time.

II. It allows the process to suspend itself and save the state of the working area.

III. Increases the priority of a waiting process.

(a) I only (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 1 Mark]

Directions for Questions 5.23 to 5.24:

The [illegible] page. The [illegible] [illegible]

5.23 Which of the following [illegible] [illegible] [illegible]

5.37 Consider a disk queue with requests for I/O to blocks on cylinders 98, 183, 37, 122, 14, 124, 65, 67. Assuming that the head is currently at cylinder 53, and the time taken to satisfy all requests is proportional to the number of cylinders moved, what is the total seek time for the following disk scheduling algorithms?

(a) FCFS (b) SSTF

(c) SCAN (d) C-SCAN

[2009 : 2 Marks]

5.38 A multiprocessor system supports 32 bit physical address and 48 bit virtual address. The page size is 8 KB and each page table entry requires 8 bytes. The size of the page table in the worst case is _____ GB.

[2014 (Set-2) : 2 Marks]

5.39 Suppose the following disk request sequence (track numbers) for a disk with 100 tracks is given: 45, 20, 90, 10, 50, 60, 80, 25, 70. Assume that the initial position of the R/W head is on track 50. The additional distance that will be traversed by the R/W head when the Shortest Seek Time First (SSTF) is used compared to the SCAN (Elevator) algorithm (assuming that SCAN algorithm moves towards 0 initially) is _____ tracks.

[2015 (Set-1) : 2 Marks]

5.40 Consider a disk queue with requests to I/O blocks on cylinders 98, 183, 37, 122, 14, 124, 65, 67. The head is initially at cylinder 53. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is _____.

[2016 (Set-3) : 2 Marks]

5.41 Consider a system with 2 user and 2 kernel processes. Each process is provided with an infinite sequence of pages, numbered 0, 1, 2, ... where each page is a distinct virtual page number. The OS uses the least recently used (LRU) page replacement policy. How many page faults occur in the following sequence, assuming that the system has 2 page frames and all frames are initially empty?

0, 1, 2, 0, 1, 3, 0, 1, 2, 3, 0, 1, 2, 3

[2016 (Set-3) : 2 Marks]

5.42 Consider a system with 2 user and 2 kernel processes. Each process is provided with an infinite sequence of pages, numbered 0, 1, 2, ... where each page is a distinct virtual page number. The OS uses the least recently used (LRU) page replacement policy. How many page faults occur in the following sequence, assuming that the system has 3 page frames and all frames are initially empty?

0, 1, 2, 0, 1, 3, 0, 1, 2, 3, 0, 1, 2, 3

[2016 (Set-3) : 2 Marks]

5.43 Consider a system with 2 user and 2 kernel processes. Each process is provided with an infinite sequence of pages, numbered 0, 1, 2, ... where each page is a distinct virtual page number. The OS uses the least recently used (LRU) page replacement policy. How many page faults occur in the following sequence, assuming that the system has 4 page frames and all frames are initially empty?

0, 1, 2, 0, 1, 3, 0, 1, 2, 3, 0, 1, 2, 3

[2016 (Set-3) : 2 Marks]

5.44 Consider a system with 2 user and 2 kernel processes. Each process is provided with an infinite sequence of pages, numbered 0, 1, 2, ... where each page is a distinct virtual page number. The OS uses the least recently used (LRU) page replacement policy. How many page faults occur in the following sequence, assuming that the system has 5 page frames and all frames are initially empty?

0, 1, 2, 0, 1, 3, 0, 1, 2, 3, 0, 1, 2, 3

[2016 (Set-3) : 2 Marks]

5.45 Consider a disk queue with requests for I/O to blocks on cylinders 98, 183, 37, 122, 14, 124, 65, 67. The head is initially at cylinder 53. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is _____.

[2016 (Set-3) : 3 Marks]

5.46 Consider a system with 2 user and 2 kernel processes. Each process is provided with an infinite sequence of pages, numbered 0, 1, 2, ... where each page is a distinct virtual page number. The OS uses the least recently used (LRU) page replacement policy. How many page faults occur in the following sequence, assuming that the system has 3 page frames and all frames are initially empty?

0, 1, 2, 0, 1, 3, 0, 1, 2, 3, 0, 1, 2, 3

[2016 (Set-3) : 2 Marks]

5.47 Consider a disk queue with requests for I/O to blocks on cylinders 98, 183, 37, 122, 14, 124, 65, 67. The head is initially at cylinder 53. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is _____.

[2016 (Set-3) : 3 Marks]

5.48 In which one of the following page replacement algorithms it is possible for the page fault rate to increase even when the number of allocated frames increases?

(a) LRU (Least Recently Used)

(b) OPT (Optimal Page Replacement)

(c) MRU (Most Recently Used)

(d) FIFO (First In First Out)

[2016 (Set-3) : 1 Mark]

■■■■

Answers: File System and Device Management

5.1	(d)	5.4	(d)	5.5	(b)	5.8	(c)	5.7	(b)	5.8	(d)	5.9	(a)	5.10	(a, c)	5.12	(b)
5.13	(d)	5.14	(c)	5.15	(a)	5.16	(c)	5.17	(c)	5.18	(b)	5.19	(b)	5.20	(d)	5.21	(c)
5.22	(a)	5.23	(b)	5.24	(b)	5.25	(a)	5.26	(d)	5.27	(c)	5.28	(b)	5.29	(b)	5.30	(d)

Explanations: File System and Device Management

5.1 (d):

After getting the interrupt CPU [illegible] interrupt handler [illegible]

5.2 (c):

[illegible]

$6 \times 10 = 12 + 2 \times 16 = 32 + 17 = 49$

(c) Closest cylinder next

$6 \times 32 + 12 = 32 + 36 = 68 = 196$

5.3 (b):

[illegible] DMA [illegible] DMA [illegible] or transmission [illegible] DMA [illegible] interrupt driven I/O. CPU is [illegible] interrupt [illegible] and in interrupt driven I/O, [illegible] between memory and I/O [illegible] CPU.

5.4 (d):

The root directory of a disk is stored at a fixed location [illegible] disk.

5.5 (a):

[illegible] I/O → Disk-

Cache [illegible]

[illegible] I/O → [illegible]

Character code [illegible] ASCII

5.6 (c):

Disk scheduling → SCAN

Keyboard [illegible] → I/O [illegible]

Time [illegible]

Interrupt processing → [illegible]

5.7 (b):

DMA [illegible] helps [illegible] file [illegible] input [illegible]

5.8 (d):

[illegible] blocked [illegible] process.

5.9 (a):

Hard disk should be higher priority compared to printer, keyboard and floppy disk. Because of [illegible] programs and OS [illegible] hard disk.

5.10 (a, c):

(a) When interrupt is enabled, a CPU [illegible] able to process the interrupt

(c) CPU [illegible] before executing [illegible]

5.11 (b):

Thread → CPU

Virtual address space → Memory

File system → Disk

Signal → Exception

5.12 (b):

[illegible] cylinder next [illegible]

5.13 (d):

[illegible] and device [illegible]

5.14 (d)

$\frac{[illegible]}{[illegible]}$ = Number of [illegible] in [illegible]

[illegible]

5.15 (b)

[illegible] The [illegible] largest block size gives better [illegible] [illegible] disk space utilization.

5.16 (d)

The number of disk block pointers that will be [illegible] in one block

$$= \frac{[illegible] \text{ bytes}}{[illegible]} = \frac{2^{[illegible]} \text{ bytes}}{[illegible] \text{ bytes}} = 256$$

Maximum file size if [illegible] single indirect pointer

= 256 × 1 = 256 KB

Maximum = [illegible] × [illegible] direct pointer

= [illegible] × 1 = [illegible] KB

Maximum file size if single indirection pointer

= 256 × [illegible] = [illegible] MB

Maximum file size if double indirection pointer

= 256 × 256 × 256 × 1 KB

Maximum file size

= 256 × 256 × 256 × [illegible] KB

= $2^{[illegible]}$ × $2^{[illegible]}$ × $2^{[illegible]}$ × $2^{[illegible]}$ bytes

= $2^{[illegible]}$ bytes

= [illegible] bytes

5.17 (c)

SSTF: Direction [illegible] 120, [illegible], 30

FCFS: Direction [illegible] 20, 80, [illegible]

5.18 (b)

Maximum file size = [Direct DBA + Number of Data blocks in [illegible] + Number of [illegible] block × [illegible] + Number of [illegible] block × [illegible]] × Data block size.

Data Block size = 1024 byte.

Disk Block size [illegible] = [illegible] Disk Block address stored in the Disk Block.

Maximum file size = [illegible] × 1024 = [illegible] × 1024 Byte

= Approx. [illegible] GB.

5.19 (b)

[illegible] Files → Files → Level → [illegible]

[illegible] and Tree[illegible] [illegible]

5.21 (c)

[illegible] Block

[illegible] = [illegible] Blocks [illegible]

[illegible] which [illegible] [illegible] Disk Blocks. So, totally 111 + 6 = 117 disk [illegible]

Estimate [illegible]

[illegible] = 60 = (20 × 100) = 62

[illegible] = 62 + (25 × 100) = 65

[illegible] = [illegible] + (19 × 100) = [illegible]

[illegible] = [illegible] = [illegible] bytes

[illegible] bytes [illegible]

56 = [illegible] = 29

25 = [illegible] = 25

[illegible] = [illegible] = [illegible]

[illegible] = [illegible] = 53

[illegible] × 220 = [illegible]

5.23 (b)

[illegible] technique [illegible] in [illegible] [illegible] of the working [illegible] [illegible] total [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

5.24 (d)

When the head [illegible] from 100 [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

5.24 [illegible]

[illegible] requires 1 bit / control signal.

For 125 control signals we need 125 bits.

Total number of microinstructions [illegible] = 1017 = 990 that requires 10 bits.

5.25 (b):

[illegible] the number of page frames allocated to a process [illegible] page fault rate [illegible] replacement policy.

5.26 (d):

The data blocks of a very large file [illegible] allocated using an extension of linked allocation in NTFS file system.

5.27 (b):

This is correct [illegible] under fifo.

[illegible] = 10
[illegible] = 7
[illegible] = 8
[illegible] = 4
[illegible] = 5
[illegible] = 9
[illegible] = 1
[illegible] = 5
[illegible] = 8
[illegible] = 11

Total misses = 13 + [illegible] + 1 + 5 + 8 + 1 + 5 = 51

= 120

5.28 (b):

Multilevel page table is preferred to reduce the size of page table because it implement hierarchical address space of a process.

5.29 (d):

Maximum possible size

= [illegible] divided by [illegible] where block [illegible]

= address provided by [illegible] / size of page table entry

= [illegible]

= 128 B

= 32 × 8 = 2 kB + 1 kB

= 3 kB

5.30 (b):

[illegible]

5.31 (b):

Overhead of each entry in TLB = 4 bytes

Block size = 1 kbytes

Total size of each entry = 1004 bytes

Maximum [illegible]

$$= \frac{[illegible] \times 10^6}{1004} = 0.0[illegible]$$

Maximum size of a file

= [illegible] × 10^9 bytes

= [illegible] × 10^9 bytes

5.32 (a):

Seek distance = [illegible]

= [illegible]

SCAN distance = [illegible]

= [illegible]

∴ Seek distance = SCAN distance

= [illegible]

3.88 Sol:

[illegible]

[illegible]

3.91 Sol:

The question is [illegible], but [illegible]. You [illegible] the pattern by taking smaller examples [illegible] smaller values.

As example, 1, 2, 3, 4, 5, 6, 7, 8 with [illegible]

[illegible]

Total page faults = 7

Optimal [illegible]

Total page faults = 6

7 − 6 = 1.

3.96 Sol:

Because of [illegible] anomaly [illegible] will happen in LRU phase replacement algorithm.

■■■■

Unit . IX

Databases

■ **Contents**

Sl.	Topic	Page No.
1.	ER-Model	460
2.	Database Design: Functional Dependencies and Normalization	480
3.	Structured Query Language (SQL)	493
4.	Relational Model: Relational Algebra and Tuple Calculus	505
5.	Transactions & Concurrency Control	518
6.	File Structures	527

UNIT IX Databases

Syllabus: [illegible] relational model, relational algebra, tuple and domain relational calculus, SQL, [illegible] normalization, [illegible] transaction and concurrency control.

Analysis of Previous GATE Papers

Exam Year	1 Mark Questions	2 Marks Questions	3 Marks Questions	5 Marks Questions	Total Marks
1990	–	–			–
1991	–	–			–
1992	–	–			2
1993		–			–
1994	2				3
1995		1			2
1996		1			2
1997	–	2			4
1998	2	3			8
1999		5			10
2000	2	3			8
2001	2	2			6
2002	2	1			3
2003	2	0			2
2004	2	5			12
2005	1	4			11
2006	1	–			2

Exam Year	1 Mark Questions	2 Marks Questions	Total Marks
2007	–	6	12
2008	1	5	11
2009	[illegible]	5	10
2010	2	3	8
2011	–	2	4
2012	2	5	12
2013	[illegible]	4	11
2014 Set-1	2	3	8
2014 Set-2	2	3	8
2014 Set-3	2	3	8
2015 Set-1	2	3	8
2015 Set-2	2	2	6
2015 Set-3	2	2	6
2016 Set-1	1	1	4
2016 Set-2	2	2	6

ER-Model

1.1 [illegible]

[illegible]

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(iv) [illegible]

(v) [illegible]

(vi) [illegible]

Choose the correct answer

(a) (i), (ii), (iv), (vi)

(b) [illegible]

(c) [illegible]

(d) [illegible] [2006 : 2 Marks]

1.2 Consider the following entity relationship diagram (ERD) where two entities E1 and E2 are related in an 1:N relationship. Total 1 on

The [illegible] of E1 are A11, A12 and A13 where A11 is the key attribute and A12 is a multi valued attribute. Relation R does not have any attribute. A normalized database management system [illegible] of tables with each table satisfying the [illegible] normal form. The minimum number of tables in the 3NF [illegible] is

(a) 2 (b) 3

(c) 4 (d) 5

[2006 : 2 Marks]

1.3 Consider [illegible] relationship diagram with [illegible] relationship 'owning' [illegible]

[illegible] payment [illegible] different [illegible] attribute of

(a) Person (b) Head Count

(c) Income (d) None of these

[2006 : 1 Mark]

1.4 [illegible] entities [illegible] attributes T_1 and T_2 are two relationships between E1 and E2, where R1 is many-many and R2 is many-1. [illegible] attribute of [illegible] relationship. What is the minimum number of tables required in a normalized database for this situation in the relational model?

(a) 2 (b) [illegible]

(c) [illegible] (d) 5

[2006 : 2 Marks]

1.5 [illegible] following table [illegible] A [illegible] C where A is the primary key and B is a foreign key referencing A [illegible]

A	B
2	4
[illegible]	[illegible]
1	6
3	2
7	8
9	5
6	1

[illegible] tuples [illegible] integrity [illegible] the tuple (2, 4) is deleted is

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

Common Data for Q.1.6 & Q.1.7

Consider the following ER diagram

1.6 The minimum number of tables required to represent E, M, F, D, B is

(a) [illegible] (b) 8
(c) 4 (d) 5

[2006 : 2 Marks]

1.7 Which of the following pairs of attributes for one of the tables that represents F cannot be NULL?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

1.8 Given the basic ER and relational models, which of the following statements is false?

(a) An attribute of an entity can have more than one value
(b) An attribute of an entity can be composite
(c) In a row of a relation, if one attribute can have more than one value, then
(d) In a row of a relation, if one attribute can have more than one value, then a NULL can be

[2013 : 1 Mark]

1.9 Consider an Entity Relationship (ER) model in which entity sets E_1 and E_2 participate in a many-to-many relationship R_1, and E_1 and E_3 participate in a one-to-many relationship R_2 and E_3 participate in a one-to-many relationship R_3.

E_1 has two simple attributes a_1 and a_2, of which a_1 is the key attribute. E_2 has two simple attributes a_3 and a_4, of which a_3 is the key attribute. E_3 has two simple attributes a_5 and a_6, of which a_5 is the key attribute. The minimum number of tables required to represent this ER model in a relational database is

[illegible]

[2013 (Set-1) : 2 Marks]

■■■

Answers : ER-Model

1. (a) 1.8 (b) 1.11 (c) 1.1 (d) 1.3 (e) 1.6 (f) 1.7 (g) 1.9 (h)

Explanations : ER-Model

1.1 (d)

[illegible]

1.2 (b):

[illegible]

1.3 (a)

[illegible]

1.4 (b)

[illegible]

1.5 (c):

[illegible]

1.6 (a)

The minimum number of tables needed for each [illegible]

1.7 (b)

[illegible]

1.8 (c)

[illegible]

1.9 (b)

[illegible]

2 Database Design: Functional Dependencies and Normalization

2.1 [illegible]
[illegible] normal form (BCNF) [illegible] and dependency preservation.

[1991 : 1 Mark]

2.2 (a) Consider the relation schema R(A, B, C, D). Which of the following functional dependencies imply the [illegible]

$AB \rightarrow C$

$C \rightarrow A$

Based on the above, [illegible] Third Normal Form (3NF) [illegible] Boyce-Codd Normal Form (BCNF).

(b) [illegible]

[1993 : 2 Marks]

2.3 For a database [illegible] dependencies [illegible] is in [illegible] functional dependencies and [illegible] can be inferred from the [illegible]

$a \rightarrow bc$

$b \rightarrow d$

This relation is

(a) [illegible] normal form

(b) [illegible] normal form

(c) [illegible] normal form

(d) None of the above

[1995 : 2 Marks]

2.4 Let R and S be two relations [illegible] primary key of R. Consider the [illegible] operations on R and S

(i) [illegible]

(ii) [illegible]

(iii) Delete tuple from S

Which of the following is true [illegible]

(a) Some of (i), (ii), (iii) [illegible] violations

(b) All of (i), (ii), (iii) [illegible] violations

(c) Deleting [illegible]

(d) Deleting [illegible]

[1995 : 1 Marks]

2.5 Which normal form is concerned with [illegible] normalization design?

(a) 2NF (b) 3NF

(c) 1NF (d) BCNF

[1996 : 1 Mark]

2.6 Three tables are given below.

Student	Age	Course	Course Category
Ramu	27	CO21	A
Abdul	26	EE01	A
Jennifer	25	DD01	B
Mayo	32	MA01	D
[illegible]	24	MD03	D

There is no index. The [illegible] the fields in the [illegible]

(a) Age (b) Ramu

(c) Course (d) Category

[1998 : 1 Mark]

2.7 Consider the following [illegible] containing information [illegible]

Book id

Subject Category [illegible]

Name of Author

Nationality of Author

[illegible]

(a) [illegible] normal form [illegible]

(b) Suppose the [illegible] and Author [illegible] and the [illegible] key is changed to Name_of_Author, Book_id, [illegible] the highest normal form satisfied by the relation.

[1998 : 2 Marks]

2.8 Let R = {A, B, C, D, E, F} be a relation schema with the following dependencies: C → F, F → A, E → A, D → A, A → B. Which of the following is a key for R?

(a) CD (b) EC
(c) AE (d) AF

[1999 : 1 Mark]

2.9 Consider the schema R = (S, T, U, V) and the dependencies S → T, T → U, U → V and V → S. Let R = (R1) and (R2) be a decomposition such that R1 ∩ R2 = φ. The decomposition is

(a) not in 2NF
(b) in 2NF but not in 3NF
(c) in 3NF but not in 2NF
(d) in both 2NF and 3NF

[1999 : 2 Marks]

2.10 Given the following relation instance

X	Y	Z
1	4	2
	5	3
1	6	2
4	2	3

Which of the following functional dependencies is satisfied by the instance?

(a) XY → Z and Z → Y
(b) YZ → X and Y → Z
(c) YZ → X and X → Z
(d) XZ → Y and Y → X

[2000 : 2 Marks]

2.11 Consider a relation R(A, B, C, D) and functional dependencies A → B and C → D. Then the decomposition of R into R1(AB) and R2(CD) is

(a) dependency preserving and lossless join
(b) lossless join but not dependency preserving
(c) dependency preserving but not lossless join
(d) not dependency preserving and not lossless join

[2001 : 1 Mark]

2.12 R(A, B, C, D) is a relation. Which of the following does not have a lossless join, dependency preserving BCNF decomposition?

(a) A → B, B → C, C → D
(b) A → B, B → C, A → D
(c) AB → C, C → A, C → D
(d) A → BCD

[2001 : 2 Marks]

2.13 Relation R is in second normal form but not in third normal form. E/F is a minimal cover for the functional dependencies in R. Which of the following statements is true?

(a) Zero
(b) More than zero but less than that of an equivalent 3NF decomposition
(c) Exactly one
(d) Two or more

[2002 : 1 Mark]

2.14 Relation R is decomposed into a set of relations that are in third normal form. E and relation S is the original relation with functional dependencies, C. Does decomposition in third normal form (3NF) preserve dependencies? Assume that the attribute of E and C are mutually exclusive.

(a) Dependency preservation
(b) Lossless-join
(c) 3NF definition
(d) 2NF definition

[2002 : 2 Marks]

2.15 Consider a relation R(A, B, C) satisfying the following functional dependencies:

	B	C
	1	1
1	1	1
1	1	1
1	1	1
1	1	1

(a) A functionally determines B and B functionally determines C
(b) A functionally determines B and B does not functionally determine C
(c) B does not functionally determine C
(d) A does not functionally determine B and B does not functionally determine C

[2002 : 2 Marks]

2.16 Consider the following functional dependencies in a database:

Name → Birthday, Age
Age → Eligibility
Name → Social_security_number
Department_number → Name
Department_number → Department_location

[illegible]

[illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2003 : 2 Marks]

2.17 Consider the following [illegible] to a student database:

student (roll no, name, address)

[illegible]

where [illegible] primary key [illegible]. The number of tuples in the student relation are 150 and 6 respectively. What is the minimum and maximum number of tuples that can be present in the [illegible] relation?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 1 Mark]

2.18 [illegible] relations [illegible] and course [illegible] with primary keys [illegible] respectively. The following [illegible] relations

name (course, No, grade)

[illegible]

name (No, [illegible])

[illegible]

The minimum normal form of this relation schema is

(a) 2 NF (b) 3 NF

(c) BCNF (d) 1 NF

[2004 : 2 Marks]

2.19 A relation [illegible] is defined with attributes [illegible] and functional dependencies [illegible]. Let [illegible] be a candidate key. Also [illegible] is a key, then it is a composite key. Which of the following statements is true?

(a) [illegible]

(b) [illegible] and hence also in 2NF

(c) [illegible] and hence also in 3NF

(d) [illegible] and hence also in BCNF

[2004 : 2 Marks]

2.20 [illegible] relations [illegible] with the following functional dependencies

[illegible]

Identify the [illegible] key.

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[2003 : 2 Marks]

2.21 Which one of the following statements about normal forms is FALSE?

(a) BCNF is stricter than 3NF

(b) Lossless, lossless, dependency preserving decomposition into 3NF is always possible

(c) Lossless, lossless, dependency preserving decomposition into BCNF is always possible

(d) Any relation with two attributes is in BCNF

[2006 : 1 Mark]

2.22 Consider a relation scheme R = (A, B, C, D, E, H) on which the following functional dependencies hold:

[illegible]

What are the candidate keys of R?

(a) AB, DB (b) AD, BE, DE

(c) ACH, BCH, BCH (d) ABH, BGH, DGH

[2005 : 2 Marks]

2.23 [illegible] relations [illegible] and [illegible] with the following functional dependencies [illegible]

[illegible] → B

[illegible] → C

[illegible] → D

D → E

E → A

Which of the following functional dependencies [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

2.24 Consider [illegible] with primary keys [illegible] respectively. The relation [illegible] contains 200 tuples and [illegible] contains 500 tuples. The maximum size of the join [illegible]

(a) [illegible] (b) 200

(c) [illegible] (d) 500

[2006 : 1 Mark]

2.25 [illegible] relations [illegible] with attributes V, W, X, Y, and Z. The following functional dependencies are [illegible] and [illegible]. What is the [illegible] candidate key for R?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

2.26 The following functional dependencies are given: AB → CD, AF → D, DE → F, C → G, F → E, G → A

Which one of the following is a candidate key?
(a) CF (b) BCEF (c) ADF (d) ABEF

[2006 : 2 Marks]

2.27 Which one of the following statements is FALSE?
(a) Every relation with two attributes is in BCNF
(b) A relation in which every key has only one attribute is in 2NF
(c) A prime attribute can be transitively dependent on a key in a 3NF relation
(d) A prime attribute cannot be transitively dependent on a key in a BCNF relation

[2007 : 2 Marks]

2.28 Consider the following statements pertaining to the functional dependencies of an instance of a relational database that is in 3NF:
(i) It is in 2NF.
(ii) It is in BCNF.
(iii) It is in 4NF.
(iv) It is in 5NF.

Choose the correct option from the following:
(a) I (b) II
(c) III (d) IV

[2007 : 2 Marks]

2.29 Let R(A, B, C, D, E, F) be a relation scheme with the following functional dependencies:

A → B, B → C, C → D, D → E, D → F

The decomposition of R into (A, B), (B, C), (B, D), (D, E), (D, F) is
(a) Lossless join and dependency preserving
(b) Lossless join but not dependency preserving
(c) Dependency preserving but not lossless join
(d) Does not have a lossless join and is not dependency preserving

[2008 : 2 Marks]

2.30 Let R(A, B, C, D, E, F) be a relation scheme with the following functional dependencies:

A → BCD, BC → DE, B → D, D → A

The relational schema R is
(a) 1NF but not in 2NF
(b) 2NF but not in 3NF
(c) 3NF but not in BCNF
(d) BCNF

[2008 : 2 Marks]

2.31 Consider the following statements about the Binary Relation:

Book (Book_ID, Title, Author_name, Publisher, Year_pub)

Which the following functional dependencies:

I. Title → Author_name → Year_pub
II. Publisher → Year_pub
III. Author_name, Title → Publisher

Assume (Book_ID, Title) is the key for the book relation. Which of the following statements is true?
(a) 2NF holds and Collection is in 1NF
(b) 3NF holds and Collection is in 2NF
(c) Both 3NF and Collection is in 2NF
(d) 3NF holds and Collection is in 1NF

[2008 : 2 Marks]

2.32 The following functional dependencies hold for relations R(A, B, C) and S(B, D, E):

B → A
A → C

The relation R contains 200 tuples and the relation S contains 100 tuples. What is the maximum number of tuples possible in the natural join R ⋈ S?
(a) 100 (b) 200
(c) 300 (d) 600

[2010 : 2 Marks]

2.33 Consider a database that has an instance of the student registration table as shown below:

Student_ID	Student_name	Course_name	Course_ID	Course_credit
1	Ram	Unix	101	3
2	Ram	DBMS	102	3
3	Ram	Unix	101	3
4	Sam	Unix	101	3
5	Sam	Compiler	103	4

Student_ID, Course_ID, Course_name, Course_credit

1. Name Sam [illegible]
2. Model [illegible]

[illegible] which of the following statements is [illegible]

(a) [illegible]

(b) Registration [illegible] is a primary key

(c) [illegible] uniquely identifies a tuple in the [illegible]

(d) [illegible] SQL [illegible]

[2011 : 1 Mark]

2.84 Which of the following are TRUE?

(a) Every relation in 2NF is also in 3NF

(b) A relation R is in 3NF if every non prime attribute of R is fully functionally dependent on every key of R

(c) Every relation in BCNF is also in 3NF

(d) No relation can be in both BCNF and 3NF

[2012 : 1 Mark]

Linked Answer Questions: 2.85 and 2.86

Relation R has eight attributes ABCDEFGH. Fields of R contain only atomic values.

F = {A → CD, E → FGH, D → EG, C → H, G → A}

is a set of functional dependencies (FDs) so that F+ is exactly the set of FDs that hold for R.

2.85 How many candidate keys does the relation R have?

(a) 3 (b) 4

(c) 5 (d) 6

[2013 : 2 Marks]

2.86 The relation R is

(a) in 1NF but not in 2NF

(b) in 2NF, but not in 3NF

(c) in 3NF but not in BCNF

(d) in BCNF [2013 : 2 Marks]

2.87 Consider the relation schema R = (A, B, C, D, E, F, G, H, I, J) and functional dependencies {E, F} → {G}, {F, G} → {I}, {A, B} → {C}, {C, D} → {E}, {B, C} → {D}, {D} → {A}. What is the key for R?

(a) {A, B} (b) {E, F, B}

(c) {A, B, C, D, E, F} (d) {A}

[2014 (Set 1) : 1 Mark]

2.88 Given the following statements

S1: A foreign key declaration can always be replaced by an equivalent check assertion in SQL.

S2: Given the table sailor(sid, sname, rating, age) sid is a key for the table. The following is a valid table definition

CREATE TABLE S (
sid INTEGER,
sname TEXT(10),
rating INTEGER,
age REAL,
PRIMARY KEY(sid),
FOREIGN KEY(rating) REFERENCES R)

Which of the statements is/are correct?

(a) S1 is TRUE and S2 is FALSE.

(b) Both S1 and S2 are FALSE.

(c) S1 is FALSE and S2 is TRUE.

(d) Both S1 and S2 are TRUE.

[2014 (Set 2) : 1 Mark]

2.89 Given the following statements:

S1: [illegible] is decidable in [illegible] in DN1, 2N1, 3N1 and BCNF.

S2: AB → C, D → A, B → C is a minimal cover for a set of functional dependencies {AB → C, B → C, D → A, AD → B, B → C}.

Which one of the statements is CORRECT?

(a) S1 is TRUE and S2 is FALSE.

(b) Both S1 and S2 are FALSE.

(c) S1 is FALSE and S2 is TRUE.

(d) Both S1 and S2 are TRUE.

[2014 (Set 3) : 2 Marks]

2.90 The maximum number of candidate keys for a relation R(A, B, C, D) with R.A → R.B is

[2014 (Set 3) : 1 Mark]

2.91 Given an empty instance of the STUDENT(S) relation, we insert the following:

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	Smith	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	Smith	[illegible]		[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]		[illegible]

Insert (StudentName, StudentAge) [illegible] key. Provide instances [illegible] X should NOT be [illegible] is ___.

[2014 (Set 3) : 1 Mark]

2.92 A prime attribute is an attribute that is an attribute that belongs to

(a) the [illegible] of R.

(b) some candidate key of R

(c) the [illegible] of R.

(d) only in the primary key of R

[2014 (Set 3) : 1 Mark]

2.43 Consider the relation $R(P, Q, R, S, T)$ with the following set of functional dependencies:

$P \rightarrow Q$

$QR \rightarrow S$

$S \rightarrow T$

Which of the following is the minimal key in R on the basis of F?

(a) PR (b) PQR (c) $PQRS$ (d) $PQRST$

[2016 (Set-1): 1 Mark]

2.44 Which of the following is NOT a superkey in a relation schema with attributes V, W, X, Y, Z and primary key V?

(a) V (b) VW (c) $VWXZ$

(d) $VWXY$ (e) $VWXYZ$

[2016 (Set-1): 1 Mark]

2.45 Consider the following relation schemas for a library database:

BOOK(Book_id, Title, City, Price)

BOOK_OUT(Book_id, Number, Date_of_issue)

The primary keys are underlined. The following functional dependencies exist in the schemas:

Book_id → Title

Book_id, Number → Date_of_issue

Number → Name, Address

Which of the following is a lossless-join decomposition of BOOK_OUT?

(a) BOOK_OUT1(Book_id, Number), BOOK_OUT2(Number, Name, Address)

(b) BOOK_OUT1(Book_id, Number), BOOK_OUT2(Number, Date_of_issue)

(c) BOOK_OUT1(Book_id, Number), BOOK_OUT2(Book_id, Date_of_issue)

(d) BOOK_OUT1(Book_id, Number), BOOK_OUT2(Book_id, Name, Address)

[2016 (Set-1): 2 Marks]

2.46 Consider a relation scheme $R(A, B, C, D)$ with the following functional dependencies:

$AB \rightarrow C$

$C \rightarrow A$

The decomposition of R into $R_1(A, B)$ and $R_2(A, C, D)$ satisfies the following functional dependencies:

$AB \rightarrow C$

$C \rightarrow A$

$C \rightarrow D$

Which of the following is true about the above decomposition?

(a) It is in BCNF and lossless-join decomposition

(b) It is in BCNF but not a lossless-join decomposition

(c) It is not in BCNF but is a lossless-join decomposition

(d) It is not in BCNF and not a lossless-join decomposition

[2016 (Set-1): 2 Marks]

■■■■

Answers Database Design: Functional Dependencies and Normalization

2.8	(a)	2.9	(d)	2.5	(d)	2.6	(c)	2.6	(b)	2.9	(a)	2.10	(b)	2.11	(c)	2.12	(c)
2.13	(a)	2.14	(c)	2.15	(b)	2.16	(d)	2.17	(c)	2.18	(b)	2.19	(b)	2.20	(a)	2.21	(c)
2.22	(d)	2.23	(b)	2.24	(a)	2.25	(b)	2.26	(a, d)	2.27	(d)	2.28	(c)	2.29	(c)	2.30	(d)
2.31	(c)	2.32	(a)	2.33	(a)	2.34	(c)	2.35	(b)	2.36	(a)	2.37	(b)	2.38	(c)	2.39	(a)
2.40	(b)	2.41	(d)	2.42	(b)	2.43	(b)										

Explanations Database Design: Functional Dependencies and Normalization

2.1 (a)

False

Sometimes, there is a choice as to which functional dependencies we preserve.

For example, if we have a relation scheme $R(A, B, C)$, and we have the FDs

$AB \rightarrow C$

$C \rightarrow B$

Then R is in BCNF. We can decompose R into $R_1(A, B)$ and $R_2(A, C)$ and preserve the dependency $AB \rightarrow C$.

2.2 (a)

(a) Relation schema $R(A, B, C, D)$ has the following functional dependencies:

$AB \rightarrow C$

$C \rightarrow A$

The decomposition of relation schema R into $R_1(A, B)$, $R_2(A, C, D)$

Functional dependency

$AB \rightarrow C$

$C \rightarrow A$

$C \rightarrow D$

Lossless join decomposition: BCNF & 3NF

Functional dependency

$C \rightarrow A$

[illegible] is a proper subset of [illegible]
key. Therefore functional dependency C → A
is allowed in 2NF, however, not allowed in
3NF, 4NF.
Therefore R is in 2NF but not in 3NF, 4NF.
(c) The minimal key of relation R are (A, B, C).

2.3 (b):

Relation R(A, B, C, D, E) has following functional
dependencies.
A → B
B → C
The minimal key of relation R is (A, D)
All non-key attributes are prime attributes because
they are part of prime attribute.
Since the FDs of R is of partial
Proper subset of any candidate key → Non-prime
attribute
Above type of FD's are not allowed in 2NF therefore
relation R is in 1NF but not in 2NF.

2.4 (d):

Transitive dependency occurs when non-key attribute
is functionally determined from another non-key
attribute.
Transitive functional dependency violates the
principle of keys and foreign key.

2.5 (d):

The most appropriate decomposition of the R(A, B, C, D)
that is lossless and dependency preserving is
3NF. Because it satisfies all properties of
database design.

2.6 (c):

The index is built on the search key.
(a) OR → Index value
TNO → Index value
DNO → Index value
SAL → Index value
MGR → Index value

2.7 (a):

(a) BCNF → 3NF

2.8 (a):

We can choose the primary key for R from the
candidate keys.
(a) CD
(BD)⁺ = {C, D, B}
So CD is a candidate key for R.

(b) (ACD)⁺ = {A, B, C, D, E}
So ACD is a candidate key for R
(c) (ABC)⁺ = {A, B, C, D}
So ABC is a candidate key for R
(d) (ABD)⁺ = {A, B, C, D}
So ABD is a candidate key for R
Therefore, CD is a key for R.

2.9 (a):

R = (ABCDEF)
S = {A → B, C → D, C → E, B → F}
Assume that two R₁ and R₂ such that R₁ ∩ R₂ = ∅
is lossless join decomposition which is not allowed
in 2NF, 3NF, BCNF.

2.10 (b):

(a) X → Z is valid if and only when X → Y and Y → Z are
satisfied.
(b) Y → X is not satisfied.
(c) Y → X is satisfied when X → Y and X → Z are not
satisfied.
(d) X → Y is satisfied when X → Z and Y → Z are not
satisfied.

2.11 (d):

R(A, B, C, D)
F = {A → B and B → D}
F₁ = {A → B and F₂ = {C → D}}
So the given functional dependency can be
represented in F₁ and F₂, hence it is lossless
dependency preserving. The attribute B is functionally
determined by either A or C.
Therefore, it will be uniquely determined
either A or B or C.
So F is lossless and dependency preserving.

2.12 (c):

(a) {R₁, R₂} is 3NF decomposition but not
lossless and dependency preserving
(b) {R₁, R₂, R₃} is BCNF decomposition is
lossless join and dependency preserving
(c) {R₁, R₂, R₃} is BCNF decomposition is
lossless join but not dependency preserving
(d) Given functional dependencies → BCNF.

2.14 (a):

Degree of redundancy every transitional
dependencies (and included BCNF is zero).

2.14 (c)

[illegible]

2.15 (b)

[illegible]

2.16 (d)

[illegible]

2.17 (a)

[illegible]

2.18 (a)

[illegible]

2.19 (b)

[illegible]

2.20 (c)

[illegible]

2.21 (a)

[illegible]

2.22 (b)

[illegible]

2.23 (b)

[illegible]

2.24 (b)

[illegible]

2.25 (b)

[illegible]

2.26 (d)

[illegible]

2.87 (d)

[illegible]

2.88 (a)

[illegible]

2.89 (b)

[illegible]

2.90 (b)

[illegible]

2.91 (d)

[illegible]

2.92 (a)

[illegible]

2.93 (b)

[illegible]

2.94 (b)

[illegible]

2.95 (b)

[illegible]

2.96 (b)

[illegible]

R2: CREATE TABLE S(
a INTEGER,
c INTEGER
e INTEGER,
PRIMARY KEY(a),
FOREIGN KEY (a) reference R)

All referenced attributes are part of primary key in the table used for the reference and referencing attributes are the primary key of their respective tables so same.

2.30 (a)

S is 3rd normal form relation in BCNF

$S = \{A, B, C, D, E\}$

Set FD = $\{AB \to C, C \to D, AB \to E, E \to C\}$

$\{AB\}^+ = \{A, B, C, D, E\}$ = AB is candidate key.

AB is candidate key

$\therefore$ All is PRIME and S is in 3rd NF

2.31 (b)

Super keys:

= {F, RD, ED, ED, EBD, FED, FED, EBD}

2.32 Sol.

Attribute arrangement, Student Agent is a key

Shankar 3

Shankar 19, Rest should not be 19

2.33 (b)

A non-prime attribute is an attribute that appears in some candidate key of a relation R.

2.34 (a)

$R \cap S$ is trivial MVD, because $R \cup S = R$

2.35 (d)

In T(W,X,Y,Z) ... complete combination for each ... is the super key.

2.36 (b)

Journal(Volume, Number, Start_page, End_page, Title, Year, Issue)

Primary key: Volume, Number, Start_page, Endpage

FD's: Volume, Number, Start_page, End_page → Title

Volume, Number, Start_page → Year

Since relation NF has not 2NF, then DB is redundance. Following schema

R_1(Volume, Number, Start_page, Endpage, Title, Year) which has 2NF

Volume, Number, Start_page, Endpage → Title

Volume, Number, Start_page, Endpage → Year

Which is in BCNF

R_2(Volume, Number, Issue)

Volume, Number → Issue

Which is also in BCNF.

Journal = $R_1 \bowtie R_2$

$T = R_1 \bowtie R_2$

Whenever NF violation is in R_1 and R_2 and tuples for T are lost in 2NF.

Structured Query Language (SQL)

4.1 Suppose we have a database consisting of the following three relations:

[illegible]

[illegible]

[illegible]

Consider the query to find the names of students who have taken all courses offered in the Computer Science (CS) department.

Express this query in SQL:

[illegible]

[2000 : 2 Marks]

4.2 [illegible]

(a) An SQL query can manipulate [illegible] relations

(b) The SQL query will not work if there are [illegible]

(c) SQL queries [illegible]

(d) None of the above

[1999 : 2 Marks]

4.3 Consider the [illegible]

EMP (Employee, Dept, Employee_name, Salary)

[illegible]

Write an SQL query [illegible]

(a) Find the [illegible] departments located in Calcutta and whose salary is greater than Rs. [illegible]

(b) Find the [illegible] employees with [illegible]

[1999 : 2 Marks]

4.4 In SQL [illegible] and the predicate [illegible] Suppose [illegible] Which of the following predicates is not equivalent?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) None of the above

[2000 : 2 Marks]

4.5 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2000 : 2 Marks]

4.6 [illegible]

select [illegible]

from [illegible]

where [illegible]

[illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2004 : 1 Mark]

4.7 Consider [illegible] relations [illegible] for SQL queries that follow:

Students:

(Roll_number, Name, Date_of_birth)

Courses:

(Course_number, Course_name, Instructor)

Grades:

(Roll_number, Course_number, Grade)

select distinct Name

from Students, Courses, Grades

where

Students.Roll_number = Grades.Roll_number

[illegible]

Courses(Course_number, Course_name)

Teaches(Teacher_number, Course_number)

Teachers(Teacher_number, Teacher_name)

Which of the following options is correct for the above relations?

(a) Names of students who have got grade A in all courses taught by Smith

(b) Names of students who have got grade A in all courses

(c) Names of teachers who have given grade A to all students in the courses they teach

(d) None of the above

[2003 : 2 Marks]

Q.8 The employees of firms in the company have salaries as follows.

Employee(emp_id, emp_name, emp_salary, emp_dept)

Consider the following SQL query

select dept, Name

from Employee

where salary > 10000

group by dept name

having count(emp_id) > 1

order by count(emp_id) desc limit 1;

If the name of the department is the department with the highest number of employees earning more than 10000, then

(a) the query returns the department with the highest average salary of employees earning more than 10000

(b) the average salary of those employees is more than the average salary of all employees in the company

(c) the average salary of those employees is more than the average salary of employees in the same department

(d) the average salary of those employees is more than the average salary of employees in the company

[2001 : 2 Marks]

Q.9 A relational database contains two relations student and department in which student table has [illegible] department table has [illegible] department_id and dept_name. [illegible] empty tables.

Insert into department values (1, 'Mechanical');

Insert into department values (2, 'Electrical');

Insert into student values (1, 'Steven', 1);

Insert into student values (2, 'Steve', 2);

Insert into student values (3, 'Gaurav', 1);

How many rows are in the result of the following SQL query when it is executed on the database?

select * from student, department

(a) 3 rows and 4 columns

(b) 2 rows and 4 columns

(c) 3 rows and 3 columns

(d) 6 rows and 5 columns

[2003 : 2 Marks]

Q.10 A table T1 in a relational database has the following rows and columns.

roll no.	marks
1	10
2	20
3	30
4	NULL

The following sequence of SQL statements was executed on the relation table T1.

Update T1 set marks = marks + 10

Select avg(marks) from T1

What is the output of the select statement?

(a) 15.75 (b) 20

(c) 20 (d) NULL

[2004 : 2 Marks]

Q.11 Consider the following relations and relational algebra operations.

Table : Student

Roll no	Name	Dept Id
1	ABC	1
2	DEF	1
3	GHI	2
4	JKL	3

Table : Department

Dept Id	Dept name
1	A
2	B
3	C

Roll no is the primary key of the Student table and Dept Id is the primary key of the Department table. Dept Id is a foreign key from Department Dept Id.

What will happen if we try to execute the following update statement?

(a) update Student set Dept Id = NULL where Roll no = 1

(b) update Department set Dept Id = NULL where Dept Id = 1

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) Both (a) and (b) are correct

[2004 : 2 Marks]

Q.18 A company maintains a database of sales made by its salesperson in various cities. Commission based on sales is calculated and sales made are recorded. The data is maintained in a table. Consider the table:

[illegible]

In addition, the company decides to increase commission by 1% for salespersons who have sales of more than Rs. 100,000. The following commission structure is used:

Commission = 1.00% of sales if sales ≤ Rs. 100,000

Commission = 1.00% of sales + 1.00% of sales in excess of Rs. 100,000 if sales > Rs. 100,000

The IT manager has written the following two update statements to implement the new commission structure. Assume that these scripts are run on the table given above.

T1: Update sales set
[illegible]
Where [illegible]

T2: Update sales set
Set commission = commission * 1.00
Where commission > 100000;

T3: Update sales set
Set commission = commission * 1.00
Where commission > 100000;

Which of the following statements is/are true when the above updates are executed?

(a) Execute T1 followed by T2 will leave the table in a consistent state.
(b) Execute T2 followed by T1 will leave the table in a consistent state.
(c) Execute T3 alone will leave the table in a consistent state.
(d) Execute T3 followed by T2 followed by T1 will leave the table in a consistent state.

[2005 : 2 Marks]

Q.19 In an inventory management system implemented in a relational database, suppose we have the following two tables and some sample data. The following SQL query has been written to list the total number of different items supplied by each supplier who supplies at least ten different items.

Supplier (supplier_id, supplier_name)
Inventory (item_id, supplier_id, price)

Select supplier_id, count(*) from inventory group by supplier_id having count(*) >= 10;

Select distinct S.supplier_id
From Supplier S, Inventory I1, Inventory I2
Where S.supplier_id = I1.supplier_id
And I1.supplier_id = I2.supplier_id
And I1.item_id <> I2.item_id
Group by S.supplier_id, S.supplier_name
Having count(*) >= 10;

For the sample data given, the query will list the supplier_ids of

(a) supplier 1 only
(b) supplier 1 and 2 only
(c) supplier 1 and 3 only
(d) supplier 1, 2 and 3 only

[2005 : 2 Marks]

Q.20 The relation R(A, B) is the result of a natural join between two tables. Assume that the primary key of the first table is A. The following SQL query is executed:

Select count(*)
From R
Group by A
Having count(*) > 1;

(a) The number of tuples returned is 0.
(b) The number of tuples returned is 1.
(c) The number of tuples returned is the number of distinct values of A in R.
(d) The number of tuples returned is the number of tuples in R.

[2005 : 2 Marks]

Directions for Questions Q.21 to Q.26:

Consider a database with three relations mentioned below. The primary keys are underlined. Foreign keys are shown in the following order of the primary key attributes in the table. No indexes are available in the database.

D : Directors relation

did	dname	rating	age
22	Kochhar	7	35
29	Sriman	1	38
31	Bose	8	50
32	Amrish	9	35
39	Santosh	10	35
64	Sachin	-	55
71	Sonia	10	41
74	Sachin	-	63
85	Pitt 2	9	35
95	Rajib	7	53

R : Reserves relation

did	sid	day
22	101	10/10/96
22	102	10/10/96
22	103	08/10/96
22	104	07/10/96
31	102	10/11/96
31	103	06/11/96
31	104	12/11/96
74	101	12/08/98
74	102	25/08/98
74	103	25/08/98

C : Class relation

cid	cname	cmanager
101	Pascal	Alex
102	Algorithm	Ali
103	Network	James
104	Jigsaw	Ram

Q.15 What is the output of the following SQL query?

select Dnames

from Directors D

where D.did in (

select R.sid

from Reserves R

where R.day like '%/10/%'

group by R.sid

having count(distinct R.day) = (select count(distinct day) from Reserves

)

(a) Kochhar, Santosh

(b) Kochhar, Sriman

(c) Kochhar, Santosh, Sachin

(d) Kochhar, Santosh, Sachin, Sonia

[2008 : 2 Marks]

Q.16 Let R, S, T be three relations in a database. Let the primary key of R be a single attribute. Let the primary key of S be a single attribute. Let the primary key of T be a single attribute. Let the attribute in R be a foreign key in S. Let the attribute in S be a foreign key in T. Let the attribute in T be a foreign key in R. Let there be no null values in any attribute. Let the cardinality of R be 100, of S be 200, and of T be 300. Let the cardinality of the cross product of R, S, T be 6000000. Let the cardinality of the cross product of R and S be 20000. Let the cardinality of the cross product of S and T be 60000. Let the cardinality of the cross product of T and R be 30000. Let the cardinality of the cross product of R, S, T be 6000000. Let the cardinality of the cross product of R and S be 20000. Let the cardinality of the cross product of S and T be 60000. Let the cardinality of the cross product of T and R be 30000. Let the cardinality of the cross product of R, S, T be 6000000.

(a) 100 (b) 101 (c) 200

(d) 600 (e) 604 (f) 1000 (g) 1004

[2006 : 2 Marks]

Q.17 Consider a database that has two relations where each relation has a primary key and no null values. We want to make changes according to a certain scenario. The database rules that must be kept intact. The rules must be kept intact. The rules must be kept intact. The rules

Query 2: [illegible]

Query 3: [illegible]

Query 4: [illegible]

(a) [illegible]

(b) Query 2 and Query 3 [illegible] for which Query 3 [illegible]

(c) [illegible]

(d) [illegible]

[2006 : 2 Marks]

8.19 Consider the table employee [illegible] the two queries Q_1, Q_2 [illegible]

Q_1 : [illegible]

Q_2 : [illegible]

(a) Q_1 [illegible]

(b) Q_2 is the correct query

(c) Both Q_1 and Q_2 produce the same answer

(d) Neither Q_1 nor Q_2 [illegible]

[2007 : 2 Marks]

Directions for Question 8.20 to 8.21:

Consider the following relations [illegible]

8.20 [illegible]

SELECT [illegible]

FROM [illegible]

WHERE [illegible]

AND

[illegible]

AND

[illegible]

GROUP BY [illegible]

HAVING COUNT [illegible]

Choose the correct statement

(a) for each school with more than 50 students, [illegible] the name of the school and the number of [illegible] students

(b) for each school [illegible] the name of the school and the number of students

(c) for each school [illegible] the school and the number of students [illegible]

(d) [illegible]

[2008 : 2 Marks]

8.21 [illegible] following [illegible] query

[illegible]

Choose the correct statement [illegible]

(a) [illegible]

(b) schools with more than [illegible] students [illegible]

(c) schools with a [illegible]

(d) schools with a [illegible]

[2009 : 2 Marks]

8.22 Consider a database table T containing two [illegible] the contents of the table are recorded [illegible]

[illegible] and MY [illegible] maximum [illegible] Sand [illegible] [illegible] in the [illegible] [illegible] [illegible] and MY [illegible] [illegible] in the table [illegible] [illegible] [illegible]

[illegible] SELECT [illegible]
FROM employee
WHERE [illegible]
AND [illegible]
employee [illegible]
[illegible]
[illegible]
GROUP BY [illegible]

What is the output?

(a) It executes but [illegible] the correct result.
(b) [illegible] and gives the correct result.
(c) It [illegible] because of [illegible] comparison.
(d) It generates an error because [illegible] GROUP BY [illegible] table [illegible]

[2014 (Set-1) : 2 Marks]

328. SQL allows duplicate tuples in relations and in the results of queries. [illegible] the following queries [illegible] the table [illegible] shown below.

select * from R where a in (select a from S)

(a) select R.* from R, S where R.a = S.a
(b) select distinct R.* from R, S where R.a = S.a
(c) select R.* from R where R.a = S.a and [illegible]
(d) select R.* from R where R.a = S.a and [illegible]

[2014 (Set-3) : 2 Marks]

329. [illegible]

[2014 (Set-3) : 1 Mark]

330. Consider the following relational schema:

employee(emp_id, emp_name, emp_type)
[illegible]

[illegible] employee [illegible]

What is the following query [illegible]

SELECT emp_name
FROM employee R
WHERE NOT EXISTS (SELECT * FROM [illegible]
[illegible]
[illegible]
[illegible]

(a) Names of all the employees with [illegible] the [illegible] having [illegible]
(b) Names of all the employees with [illegible] the [illegible] having [illegible]
(c) Name of all the employees [illegible] the [illegible] having [illegible]
(d) Names of all the employees with all their [illegible] having [illegible]

[2014 (Set-3) : 2 Marks]

331. Consider the following relations:

[illegible]

Consider the following SQL query.

SELECT [illegible] Product_Name, [illegible]
FROM [illegible]
WHERE [illegible] Bill_No = [illegible]
[illegible]

The number of rows that will be returned by the SQL query is ____.

[2016 (Set-1) : 2 Marks]

332. [illegible] operations in SQL [illegible]

(a) For selection [illegible]
(b) [illegible] operations [illegible] SQL [illegible]
(c) [illegible] operations [illegible]
(d) [illegible] except that [illegible] SELECT [illegible] SQL [illegible]

[2015 (Set-1) : 1 Mark]

8.24 Consider the following [illegible]
Cinema ([illegible])
Which [illegible] will be [illegible] the [illegible] of the SQL query

- SELECT [illegible]
- [illegible] FROM [illegible]

[illegible] way to find the [illegible] of [illegible]

(a) [illegible] = [illegible] from Cinema P2
(b) WHERE [illegible] = Any (select [illegible] from Cinema P2)
(c) WHERE [illegible] > All (select max [illegible] from Cinema P2)
(d) WHERE [illegible] > any (select max [illegible] from Cinema P2)

[SC-3 Mark-By: 1 Mark]

8.25 Consider the following database [illegible] as shown

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

The [illegible] is given by the following SQL query

```
with total_[illegible] as
    select [illegible]
    from [illegible]
    group by [illegible]
with total_avg(salary) as
    select avg(salary)
    from temp
select [illegible]
    from [illegible]
    where [illegible]
```

2016 (Set-B: 2 Marks)

■■■■

Answers: Structured Query Language (SQL)

8.2	(d)	8.3	(c)	8.5	(a)	8.6	(a)	8.7	(c)	8.8	(d)	8.9	(c)	8.10	(c)	8.11	(a)
8.12	(b)	8.13	(d)	8.14	(d)	8.15	(c)	8.16	(c)	8.17	(c)	8.18	(b)	8.19	(b)	8.20	(b)
8.21	(c)	8.22	(a)	8.23	(a)	8.24	(c)	8.25	(a)	8.26	(b)	8.27	(b)	8.28	(c)	8.29	(a)
8.30	(d)	8.31	(d)	8.32	(a)												

Explanations: Structured Query Language (SQL)

8.1 (d)

Select Student
From [illegible]
Where [illegible] in (select [illegible] from [illegible] where [illegible] in (select [illegible] from [illegible] where [illegible] = [illegible] and [illegible] = null))

8.2 (d)

[illegible] SQL [illegible] known as with [illegible] an SQL query.
SQL [illegible] the same solution. Therefore option (d) is correct.

8.8 (d)

(a) [illegible]
From [illegible]
Where [illegible]
and [illegible]

or

Select [illegible]
From [illegible]
Where [illegible]
and [illegible]

(b) Since: [illegible]
Zoom: [illegible]
Where: [illegible]
[illegible]

3.4 (b)

[illegible]

3.5 (c)

[illegible]

3.6 (d)

[illegible]
select student$_{id}$, [illegible]
from [illegible]
where [illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

3.7 (b)

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

3.8 (d)

[illegible]
[illegible]
[illegible]
[illegible]

[illegible]

[illegible]

3.9 (b)

[illegible]
[illegible]
[illegible]

3.10 (b)

[illegible]
[illegible]
[illegible]
[illegible]

3.11 (b)

(i) [illegible]
(ii) [illegible]

3.12 (c)

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

3.13 (b)

[illegible]

3.14 (c)

[illegible]

3.15 (a):

[illegible]

3.16 (d):

Subqueries:

[illegible]

(ii) Select R.Bid

[illegible]

Where [illegible] = 'Red'

Where R.Bid = S.Bid

Number of comparisons for above query:

= [illegible] comparisons for Reserves selection.

+ 20 comparisons for R.Sid = S.Sid.

(iii) [illegible]

[illegible]

Where [illegible] = 'Red'

Where R.Sid = S.Sid

Number of comparisons for above query:

= [illegible] comparisons for Reserves selection.

+ 20 comparisons for R.Sid = S.Sid.

(iv) [illegible]

[illegible]

(v) [illegible]

[illegible]

Total [illegible] = 2+4+8+20 = 34

3.17 (d):

Consider the [illegible]

Query 1:

Select A.customer [illegible]

[illegible]

Where A.balance > B.balance

group by A.customer

The query computes the group of customers where the balance of first account is more than the average balance of other group.

Query 2:

Select A.customer [illegible]

[illegible]

Where A.balance < B.balance group by A.customer

[illegible] all customers.

Both Query 1 and Query 2 doesn't compute the required problem rank the customers according to the number of accounts and the customer = [illegible] largest number of accounts get rank 1.

3.18 (b):

[illegible] student, student_id, student_name, student_address, [illegible] student relation.

Query 1: [illegible] student [illegible] student_id, student_name, student_address [illegible] student relation.

Query 2: [illegible] student_id [illegible] student relation.

Query 3: Retrieve student_id [illegible] student_id [illegible] student_id [illegible]

X	Y
[illegible]	[illegible]
[illegible]	[illegible]

Now VX = MY = 0

Now we calculate Q = [illegible] = 10.71

X	Y
[illegible]	[illegible]
[illegible]	[illegible]

Rounding the [illegible] Table 129 times and looks like as follows.

X	Y
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Therefore, when X = 7, Y = 12.

So option (a) is correct.

Q.23 (a):

Directly using the given query.

SELECT [illegible] Branch_name, Bank_Manager FROM [illegible]

Branch_name	Bank_Manager
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

SELECT Bank_Manager, [illegible] FROM [illegible]

Bank_Manager	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Natural join T

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Therefore, the output of the SQL query contains [illegible] tuples.

Q.24 (d):

As per SQL standard, HAVING clause allowed only if GROUP BY clause is [illegible]

[illegible]

SELECT DISTINCT [illegible]

FROM [illegible]

[WHERE [illegible]]

[GROUP BY [illegible]]

[HAVING [illegible]]

All attributes used in the GROUP BY clause must appear in the SELECT clause.

Q.25 (a):

[illegible]	ID	Name	Age
[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]

[illegible]

ID	Name	Age	ID	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Q.26 (b):

```
[illegible]
    [illegible]
    [illegible]
```

[illegible]

Q.27 (b):

Given SQL query will correctly display the [illegible] and hire date of all [illegible] in this [illegible] department [illegible] ID 1700.

Q.28 (b):

```
Select * from [illegible]
[illegible]

[illegible]
[illegible]
```

8.29 (a):

[illegible]

8.30 (b):

[illegible]

8.31 Hint:

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

8.32 (d):

[illegible]

8.33 (b):

The following query finds the [illegible] [illegible] from Customer [illegible]

[illegible]

8.34 Hint:

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	

[illegible]

[illegible]

[illegible]

[illegible]

4 Relational Model; Relational Algebra and Tuple Calculus

4.1 [illegible] relations [illegible] distinct [illegible] instances. Which [illegible] statements are true? [illegible] candidate key for R?

[1991 : 1 Mark]

4.2 [illegible] relational algebra expression [illegible] the minimum [illegible] relation [illegible] to [illegible] A → B.

[1992 : 1 Mark]

4.3 A Boyce-Codd relational database [illegible] the following schemas

[illegible]

Explain [illegible] which [illegible] to assign to determine

(a) [illegible]

(b) [illegible]

[1996 : 2 Marks]

4.4 Which of the following [illegible] are [illegible] the RHS [illegible] R_1 and R_2 are relations. C_1, C_2 are selection conditions and A_1, A_2 are attributes of R_1?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1998 : 2 Marks]

4.5 Consider the [illegible] relation R with attributes [illegible] and suppose that it has [illegible] tuples. The maximum and minimum [illegible] of the join respectively are

(a) [illegible]

(b) [illegible]

(c) [illegible] and [illegible]

(d) [illegible] and [illegible]

[1999 : 1 Mark]

4.6 The tuple [illegible] expression equivalent to the following [illegible] is

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[1999 : 1 Mark]

4.7 Given the [illegible]

employee [illegible]

department [illegible]

Which of the following queries cannot be expressed using the basic relational algebra operations [illegible]?

(a) [illegible]

(b) [illegible]

(c) Employees who do not work in the same [illegible]

(d) [illegible]

(e) [illegible]

[2000 : 1 Mark]

4.8 Which of the following relational calculus expressions [illegible]?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2001 : 2 Marks]

4.9 With respect to the expressive power of [illegible] relational query languages, which of the following statements is true?

(a) Relational algebra is more [illegible] than [illegible]

(b) Relational calculus has the same power as relational algebra

(c) Relational calculus has [illegible]

(d) None of the above

[2003 : 1 Mark]

4.10 Let $R_1(A, B, C)$ and $R_2(D, E)$ be two relations where A is the primary key and D is a foreign key in R_1 referring to R_2. Suppose both the relations initially satisfy the referential integrity constraint. The transaction T_1 deletes the tuple in R_2 which matches with the tuple in R_1, which was the last tuple in R_1. Assume that the following statements would successfully produce an empty solution?

(a) T_1; T_2; T_3

(b) T_1; T_3; T_2

(c) T_2; T_1; T_3

(d) T_2; T_3; T_1

[2004 : 1 Mark]

4.11 Consider the relation STUDENT(name, sex, marks, hobby) where the primary key is (name, hobby). Assuming no student has more than one hobby, which of the following is/are equivalent to this SQL query?

SELECT name, sex, MAX(marks) FROM STUDENT GROUP BY name;

(a) name of girl students with the highest marks

(b) name of girl students with more marks than some boy student

(c) name of girl students with marks greater than some boy student

(d) name of girl students with more marks than all the boy students

[2001 : 2 Marks]

4.12 Let $R(A, B, C, D)$ be a relation. Consider the following decomposition of R into $R_1(A, B)$ and $R_2(A, C, D)$. Assuming R satisfies the functional dependency $A \rightarrow B$, which of the following is TRUE?

(a) R_1 is in 1NF but not in 2NF

(b) R_2 is in 2NF but not in 3NF

(c) R_1 is in 2NF but not in 3NF

(d) R_1 and R_2 are in 3NF

[2001 : 1 Mark]

4.13 Consider the table STUDENT with schema (name, hobby) and the table HOBBY with schema (hobby, hobbyname). Hobby is the primary key in HOBBY. The contents of the tables are shown below.

Table: Student

Roll	Name	Hostel	Marks
1085	Manoj Rathod	[illegible]	85
[illegible]	[illegible]	[illegible]	65
2580	[illegible]	7	94
2581	[illegible]	6	92
2582	[illegible]	5	76
2611	[illegible]	6	[illegible]
2612	[illegible]	5	69
2625	[illegible]	5	94
2629	[illegible]	7	86
3125	[illegible]	5	85

Table: Hobby

Roll	Hobbyname
1085	chess
1085	[illegible]
2584	[illegible]
2629	swimming
2581	cricket
[illegible]	chess
[illegible]	[illegible]
2611	[illegible]
[illegible]	football
2625	cricket
2629	photography
[illegible]	[illegible]
3125	[illegible]

The following SQL query is executed on the above tables.

SELECT hostel,

FROM student natural join hobby

WHERE hobbyname = 'football' GROUP BY hostel HAVING COUNT(*) > 1;

Relational Bag Algebra allows multiple copies of tuples in a relation. A new relation R is created by the following relational algebra expression:

$R = \Gamma_{\text{[illegible]}}(\sigma_{\text{[illegible]}}(\text{STUDENT} \bowtie \text{HOBBY}))$

The difference between the number of tuples in the SQL output and the number of tuples in R is

(a) 0 (b) 1

(c) 2 (d) 3

[2004 : 2 Marks]

4.14 Which of the following relational query languages are [illegible]?

I. Relational algebra.

II. Tuple relational calculus restricted to safe expressions.

III. Domain relational calculus restricted to safe expressions.

(a) II and III only (b) I and II only

(c) I and III only (d) I, II and III

[2006 : 1 Mark]

4.15 Consider the [illegible] and the common pool "student" amount [illegible] is the primary key. Assume no null values and no foreign key [illegible] constraints. [illegible] 6000, 7000, 9000, 8000 [illegible]. However each [illegible] [illegible]. Consider the query plan [illegible] [illegible] the [illegible] and [illegible] [illegible]

A disk seek takes [illegible] and checking a tuple [illegible] takes [illegible]. Which of the following statements is correct?

(a) Plan 1 and Plan 2 [illegible] [illegible]

(b) [illegible] output of Plan 1 [illegible]

(c) [illegible] = 9000, Plan 1 [illegible] Plan 2 [illegible]

(d) [illegible] = 6000, [illegible] Plan 2 [illegible]

[2006 : 2 Marks]

4.16 Consider a relation [illegible] with [illegible] tuples. Assume that the attribute values [illegible] randomly distributed [illegible] 0, 500. Which one of the following statements is true [illegible] the number of tuples returned by the given range query?

(a) [illegible] (b) [illegible]

(c) 100 (d) [illegible]

[2007 : 2 Marks]

4.17 Consider the following relation [illegible]

[illegible] = [illegible]

[illegible] = [illegible]

[illegible] = [illegible]

[illegible] of the above schemas. Assume that [illegible] [illegible] that [illegible]

Consider the following query.

[illegible]

Which one of the following gives the most efficient evaluation of the above query?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2007 : 2 Marks]

4.18 Information about a collection of students and courses they have taken is represented in a relational database. The following relations are given which [illegible] each student [illegible] and the courses they have taken. Assume that every course is taken by at least one male and one female student. What [illegible] relational algebra expression represents [illegible]

(a) [illegible]

(b) Courses in which all the female students are enrolled

(c) Courses in which a proper subset of female students are enrolled

(d) Courses in which only male students are enrolled

(e) None of the above

[2007 : 2 Marks]

4.19 Consider the relation employee(name, sex, salary, supervisor, dno) where the attribute name is the key. Assume that each department has at least one employee and that the supervisor is also an employee in the same department. What does the following Tuple Relational Calculus query produce?

{t | ∃ employee(t) ∧ ∃ employee(s) ∧ t.sex = 'male' ∧ t.salary > s.salary ∧ t.supervisor = s.name ∧ s.sex = 'male'}

(a) Names of employees who are male supervisors

(b) Names of employees who are male and earn more than their male supervisors

(c) Names of employees who are male and earn more than some male subordinates

(d) Names of employees who are male supervisors and have at least one male subordinate

[2007 : 2 Marks]

4.20 Let R and S be two relations with the following schemas:

$R(P, Q, R_1, R_2, R_3)$

$S(P, Q, S_1, S_2)$

Let T be the result of $R \bowtie_{P=S} S$. Which of the following expressions is equivalent to $\pi_{P,Q,R_1}(T)$?

I. $\pi_{P,Q,R_1}(R) \bowtie \pi_{P,Q}(S)$

II. $\pi_{P,Q,R_1}(R) \bowtie \pi_{P,Q,R_1}(R \bowtie_{P=S} S)$

III. $\pi_{P,Q,R_1}(R \bowtie_{P=S} S)$

IV. $\pi_{P,Q,R_1}(R \bowtie_{P=S} \pi_{P,Q,S_1,S_2}(S))$

(a) I and II (b) I and III

(c) II and IV (d) III and IV

[2008 : 2 Marks]

4.21 Let R and S be two relations as defined in question 4.20. Let $R \bowtie_{P=S} S = T$. Let $U = \pi_{P,Q,R_1}(T)$. Now consider the following queries on the database:

I. $\pi_{P,Q,R_1}(R) - \pi_{P,Q,R_1}(U)$

II. $\pi_{P,Q,R_1}(R) - \pi_{P,Q,R_1}(R \bowtie_{P=S} \pi_{P,Q,S_1,S_2}(S))$

III. $\pi_{P,Q,R_1}(R \bowtie_{P=S} \pi_{P,Q,S_1,S_2}(S)) - \pi_{P,Q,R_1}(R \bowtie_{P=S} S)$

IV. $\pi_{P,Q,R_1}(R) - \pi_{P,Q,R_1}(T)$

Which of the above queries are equivalent?

(a) I and III (b) I and II

(c) II and IV (d) III and IV

[2008 : 2 Marks]

Common Data for Questions 4.22 and 4.23

Consider the following relations for a database that keeps track of business trips of employees:

Trip(trip#, date, duration, description)

Stay(hotel#, trip#, nights, rate)

4.22 Consider the following relational query on the above database.

SELECT S.hotel#

FROM Stay S

WHERE S.rate IN (SELECT T.rate

FROM Stay T

WHERE T.hotel# = S.hotel#

FROM Trip T

WHERE T.description = 'conference');

Assume that relations are not empty. Which of the following is the correct interpretation of the above query?

(a) Find the hotels that have supplied rooms to people on conference trips at the same rate they have supplied rooms to other people.

(b) Find the hotels that have supplied rooms to people on conference trips at a rate lower than they have supplied rooms to other people.

(c) Find the hotels that have supplied rooms to people on conference trips at a rate higher than they have supplied rooms to other people.

(d) Find the hotels that have never supplied rooms to people on conference trips.

[2008 : 2 Marks]

4.23 Assume that in the relations used in questions 4.20 and 4.21, each trip and each hotel stay has a unique stay and trip#, respectively. There are no other candidate keys. Neither trip# nor hotel# can be supplied for other trips. These must be foreign keys and candidate keys. Which one of the following is TRUE about the above schemas?

(a) The schema is in 3NF but not in BCNF

(b) The schema is in 2NF but not in 3NF

(c) The schema is in 1NF but not in 2NF

(d) The schema is not in 1NF

[2008 : 2 Marks]

4.24 Consider the Dept(DeptNo, DeptName, ManagerID, ...) and Emp(EmpNo, EmpName, ...) relations. Dept.ManagerID and Emp.EmpNo are the corresponding primary keys of the relations. Based on the above relations, which of the following is a valid integrity constraint?

(a) $\pi_{ManagerID}(Dept) - \pi_{EmpNo}(Emp) = \phi$ (b) $\pi_{EmpNo}(Emp) - \pi_{ManagerID}(Dept) = \phi$

(c) $\pi_{ManagerID}(Dept) = \pi_{EmpNo}(Emp)$ (d) $\pi_{ManagerID}(Dept) \cap \pi_{EmpNo}(Emp) = \phi$

[2012 : 2 Marks]

4.25 Consider the following relation schemas:

[illegible]

Course(courseno, instructor, [illegible])

Register(regno, [illegible])

Which of the following statements are equivalent to the query in English?

"Find the [illegible] of students who are registered in more than 10 courses."

(a) [illegible]

(b) $\Pi_{[illegible]}$ [illegible]

(c) [illegible]

(d) [illegible]

(e) [illegible] and IV

(f) [illegible] only (g) [illegible]

[2013 : 2 Marks]

4.26 Consider a join $(r \bowtie s)$ of two relations r and s [illegible] hash-based join method. [illegible]

Assuming $|r| \le |s|$, the join $r \bowtie s$ [illegible] disk accesses is

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2014 (Set-2) : 2 Marks]

4.27 Consider the following [illegible] relations, where child of the dependent relation [illegible] referring to empID of the primary employee. Assume that every employee has at least one associated dependent in the dependent relation.

employee (empID, empName, empAge)

dependent (childID, depName, depAge)

Consider the following relational algebra query.

$\Pi_{[illegible]}$ employee − $\Pi_{[illegible]}$ employee

(a) [illegible]

The above query returns the names of all employees who satisfy one of the following conditions:

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2014 (Set-1) : 2 Marks]

4.28 Consider the relations $r(A, B)$ and $s(B, C)$ with the tuples $r = \{(1, 2), (1, 3), (2, 3)\}$ and $s = \{(2, 4), (3, 4)\}$.

Assume that $r \div s$ is the tuples that remain in r after removing tuples that match every tuple in s. Consider the following tuples of the form (A, B, C): $t_1 = (1, 2, 4)$, $t_2 = (1, 2, 3)$, $t_3 = (1, 3, 4)$, $t_4 = (1, 3, 3)$, $t_5 = (2, 3, 4)$. Which of the following statements are correct?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2013 (Set-2) : 1 Mark]

END

Answers: Relational Model: Relational Algebra and Tuple Calculus

4.1	(b)	4.2	(b)	4.3	(c)	4.4	(d)	4.5	(a)	4.6	(c)	4.7	(b)	4.8	(d)	4.9	(c)
4.10	(b)	4.11	(c)	4.12	(c)	4.13	(d)	4.14	(b)	4.15	(b)	4.16	(c)	4.17	(d)	4.18	(c)
4.19	(a)	4.20	(b)	4.21	(a)	4.22	(a)	4.23	(a)	4.24	(d)	4.25	(c)				

Explanations | Relational Model: Relational Algebra and Tuple Calculus

4.1 (Sol):

(a) So

For given instance 'A' values uniquely identify tuples in relation R.

So, A can be candidate key of R. Since, the instance of the DB does not hold on particular instance.

4.2 (Sol):

[illegible]

4.3 (Sol):

(a) The minimum [illegible] is 5.

(b) Determine name of students of those who are [illegible] who have [illegible]

4.4 (d).

[illegible]

If the [illegible] then we can express the expression as RHS respectively. The RHS first projection is on [illegible] condition, will [illegible]

4.5 (b):

Whenever there is no foreign key [illegible] between two relations then the max & min [illegible] tuples in the join are [illegible] respectively.

4.6 (c):

[illegible] expression given by [illegible] & B = 30 [illegible]

Therefore, [illegible] expression to given [illegible]

4.7 (c):

[illegible]

4.8 (a):

Given $R_1(A, B)$ and $R_2(B)$ respectively [illegible]. However, it takes all tuples such that [illegible] $i = 2$.

The [illegible] tuples is divide [illegible] in the [illegible] all integers. Therefore, this is [illegible] query.

4.9 (d):

Every query that can be expressed in tuple calculus can also be expressed in relational algebra query.

[illegible] has the same power as relational calculus.

4.10 (b):

$R_1(A, B, C)$

$R_2(D, E)$

C is the common [illegible] and is [illegible] primary key of R_2.

D is the primary key of R_2.

The relation R_1 [illegible] relation R_2 [illegible] of R_1 is [illegible] of tuples in [illegible]

4.11 (d):

For given query [illegible] the [illegible] of qui [illegible] with more marks than all the boy students.

4.12 (c):

Given

$R = (A, B, C, D)$

$r_1 = \pi_{A,B,C}(r)$

$r_2 = \pi_{A,D}(r)$

$S = r_1 \bowtie r_2$

All tuples [illegible] decomposed from r R.

4.13 (b):

SQL query will return the following [illegible]

Rol.	Hostel
[illegible]	7
[illegible]	8
[illegible]	5

2048 ? Duplicate Row is present in Holder table
4069 ?
4098 ?
2040 ?

Total 7 rows are selected

In Relation Algebra, only distinct values of tuples are selected i.e. 5,8,7

Therefore SQL row count RA row count = 7 − 3 = 4.

1.1 [illegible]

Given three relational query languages have same expressive power.

1.13 [illegible]

There are two plans mainly plan1 and plan2. In plan 1 first all the tuples pair is created then they are given to plan 2 all pair are checked for second time and then the records are checked. The second time it takes 4 more disk transfer so it is 9 0 MBytes. So the = 30.00 but plan 1 cost is lower than plan 2 for all databases

1.16 [illegible]

Number of tuples in relation = 1000

Each value of A is unique. Hence, tuples are uniformly distributed in [0, 999]

$\sigma_{A<10}(r)$ returns 100 tuples.

1.17 [illegible]

Join is more small compared to product which is also seen in statement.

$\sigma_{[illegible]}$ which will already filter city as agra making it small, and below that $\sigma_{[illegible]}$ accounts for expensive filter and gives accurate tables so, between them will give some results and better cost.

1.18 [illegible]

Let the Relations are:

Student table

sid	Name	Sex
[illegible]	Sam	F
2	Ram	M
3	Geeta	F
4	Sujoy	M
5	Radha	F

Enroll

Student	Course
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Now the query $\pi_{[illegible]}(\sigma_{[illegible]}(\text{Student} \bowtie \text{Enroll}))$ results in

[illegible]
[illegible]
[illegible]
[illegible]

$\pi_{[illegible]}(\sigma_{[illegible]}(\text{Student} \bowtie \text{Enroll}))$

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

$\sigma_{[illegible]}(\text{Student} \bowtie \text{Enroll})$

Student_Id	Course_Id
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

T

Student_Id	Course_Id
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Executing [illegible] will [illegible] relations

[illegible]

D

C

A

A, C, D are key attributes which is given by the [illegible] relation of female children.

1.19 (c) :

[illegible]

1.20 (d) :

[illegible]

1.21 (c)

[illegible]

1.22 (c)

[illegible]

1.23 (b)

[illegible]

1. [illegible] supplier and [illegible] unique name

   [illegible]

2. [illegible]

   [illegible]

3. [illegible]

   [illegible]

   Each [illegible] only [illegible]

1.24 (a) :

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

1.25 (a) :

[illegible]

1.26 (d) :

[illegible]

1.27 (d) :

[illegible]

Where,

[illegible]

Means

[illegible]

[illegible]

Means

[illegible]

1.28 (d) :

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

5 Transactions & Concurrency Control

5.1 For the schedule given below, which of the following is correct?

1	Read A	
2		Read B
3	Write A	
4		Read A
5		Write A
6		Write B
7	Read B	
8	Write B	

(a) This schedule is serializable and can be implemented in a strict 2PL protocol.
(b) This schedule is serializable but cannot be implemented in a strict 2PL protocol.
(c) This schedule is not serializable but can be implemented in a strict 2PL protocol.
(d) This schedule is not serial and also cannot be implemented in a strict 2PL protocol.

[1999 : 2 Marks]

5.2 Which of the following statements is/are true in relation to concurrency control in database?

(a) A transaction writes a data item only after it has acquired a lock on the item.
(b) A transaction reads a data item only after it has acquired a lock on the item.
(c) A transaction releases a lock on a data item only after it has written the item back to the database permanently.
(d) A transaction releases a lock on a data item only after it has read the item.

[2003 : 1 Mark]

5.4 Consider the schedule of three transactions T_1, T_2 and T_3 and the following precedence graph of conflicts between T_1, T_2 and T_3. In the graph, R(A) and W(A) denote the read and write operations on the data item A respectively.

(a) The schedule is serial and serializable.
(b) The schedule is serial but not serializable.
(c) The schedule is not serial but is serializable.
(d) The schedule is neither serial nor serializable.

[2002 : 2 Marks]

5.5 Which of the following provides the maximum concurrency in accessing B-tree indexes?

(a) 2PL
(b) Tree locking
(c) 2PC
(d) Page locking is the level of granularity gives the maximum concurrency.

[2001 : 1 Mark]

5.6 Consider the following two transactions T1 and T2.

Which of the following is true about the schedule S?

(a) S is a serial schedule as all T1 precedes T2.
(b) S is a serial schedule as all T2 precedes T1.
(c) S is a serializable schedule as T1 and T2 are independent.
(d) S is not a serializable schedule as T1 and T2 are independent.

[2001 : 2 Marks]

5.8 Consider the following properties of transaction. Two-phase locking protocol guarantees the properties of transaction in

(a) Except in the case of Deadlock, Consistency
(b) Except in the case of Deadlock, Isolation
(c) Except in case of a power failure
(d) Always, even if a system failure occurs

[2003 : 1 Mark]

5.7 [illegible] following for accounts of [illegible] balance 1,000 [illegible] 2000 [illegible] mortgage payments and then apply 5% [illegible].

1. [illegible]
2. T1: R(A) = 2000; now = 1000
3. T2: R(A) = 0; now = 2000
4. [illegible]
5. [illegible]
6. T2: R(A) = 1000; now = 10500
7. T2: [illegible]

Suppose the database [illegible] before [illegible]. When the [illegible] which one statement is true of the resulting [illegible]?

(a) We must [illegible]

(b) We must undo [illegible] 1000 and then redo [illegible] 2 and 3

(c) We [illegible] because [illegible]

(d) We can apply redo and undo operations in [illegible]

[2006 : 1 Marks]

5.8 Consider the [illegible] transactions. Which one of the following statements is TRUE?

S_1: [illegible]

S_2: [illegible]

(a) Both S_1 and S_2 are conflict serializable

(b) S_1 is conflict serializable and S_2 is not conflict serializable

(c) S_1 is not conflict serializable and S_2 is conflict serializable

(d) Both S_1 and S_2 are not conflict serializable

[2007 : 2 Marks]

5.9 Consider the [illegible] transactions T1 and T2.

T1:	T2:
read(A);	read(A);
write(A);	write(A);
read(B);	read(B);
write(B);	write(B);
commit;	commit;

Which of the following schedules is conflict serializable and also satisfies the two phase locking protocol (2PL) [illegible] transactions?

(a)

S1 :	S2 :
lock-S(A);	lock-S(B);
read(A);	read(B);
lock-S(B);	lock-S(A);
read(B);	read(A);
if A = 0	if B = 0
then B = B+1;	then A = A+1;
write(B);	write(A);
commit;	commit;
unlock(A);	unlock(B);
unlock(B);	unlock(A);

(b)

S1 :	S2 :
lock-X(A);	lock-X(B);
read(A);	read(B);
lock-X(B);	lock-X(A);
read(B);	read(A);
if A = 0	if B = 0
then B = B+1;	then A = A+1;
write(B);	write(A);
unlock(A);	unlock(B);
commit;	commit;
unlock(B);	unlock(A);

(c)

S1 :	S2 :
lock-S(A);	lock-S(B);
read(A);	read(B);
lock-X(B);	lock-X(A);
read(B);	read(A);
if A = 0	if B = 0
then B = B+1;	then A = A+1;
write(B);	write(A);
unlock(A);	unlock(B);
commit;	commit;
unlock(B);	unlock(A);

(d)

S1 :	S2 :
lock-S(A);	lock-S(B);
read(A);	read(B);
lock-X(B);	lock-X(A);
read(B);	read(A);
if A = 0	if B = 0
then B = B+1;	then A = A+1;
write(B);	write(A);
unlock(A);	unlock(B);
unlock(B);	unlock(A);
commit;	commit;

[2007 : 2 Marks]

5.10 Consider the following three schedules of transactions T1, T2 and T3. [illegible] following NOTA represents the [illegible] read, W for write [illegible] by transaction [illegible] on data item X.

S1: [illegible]

S2: [illegible]

S3: [illegible]

Which of the following statements is CORRECT?

(a) S1, S2 and S3 are all conflict serializable schedules

(b) None of S1, S2 and S3 are conflict equivalent to each other

(c) S2 is conflict equivalent to S3, but not to S1

(d) S1 is conflict equivalent to S2, but not to S3

[2006 : 2 Marks]

5.11 Consider two transactions T_1 and T_2 and two schedules S_1 and S_2 of T_1 and T_2 as given below

T_1: $R_1[X]\ W_1[X]\ W_1[Y]$

T_2: $R_2[X]\ R_2[Y]\ W_2[X]$

S_1: $R_1[X]\ R_2[Y]\ R_1[Y]\ W_1[X]\ W_1[Y]\ W_2[X]$

S_2: $R_1[X]\ R_2[Y]\ W_2[X]\ R_1[Y]\ W_1[X]\ W_1[Y]$

Which of the above schedules is/are conflict serializable?

(a) S_1 and S_2 (b) S_1 only

(c) S_2 only (d) S_2 only

[2009 : 2 Marks]

5.12 Which of the following concurrency control protocols ensure both conflict serializability and freedom from deadlock?

I. 2 phase locking

II. Time stamp ordering

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

[2010 : 1 Mark]

5.13 Consider the following schedule for transactions T1, T2 and T3

T1	T2	T3
Read(X)		
	Read(X)	
		Read(X)
	Write(X)	
Write(X)		
		Write(X)
	Read(X)	
	Write(X)	

Which one of the schedules below is the correct serialized schedule for the above?

(a) T1 → T3 → T2 (b) T2 → T1 → T3

(c) T2 → T3 → T1 (d) T3 → T2 → T1

[2010 : 2 Marks]

5.14 Consider the following transactions with data items P and Q initialized to zero:

T_1: read(Q);
read(P);
if (Q = 0) then P = P + 1;
write(P);

T_2: read(P);
read(Q);
if (P = 0) then Q = Q + 1;
write(Q);

Any non-serial interleaving of T_1 and T_2 for concurrent execution is acceptable.

(a) is a serializable schedule

(b) is a schedule that is not conflict serializable

(c) leads to a deadlock

(d) is a schedule for which a precedence graph cannot be drawn

[2012 : 2 Marks]

5.15 [illegible]

(a) [illegible] but [illegible] multiple keys

(b) [illegible] the order of the primary key

(c) the data [illegible] the file [illegible] the contents of the index

(d) the [illegible] of the file are [illegible] the same order as the [illegible] of the index

[2015 : 1 Mark]

5.16 Consider the following [illegible] transactions [illegible] and [illegible] respectively. [illegible] serializable?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2014 (Set 1): 2 Marks]

5.17 Consider the [illegible] – [illegible] transactions T_1, T_2, T_3 and T_4.

Which [illegible] is/are serializable?

(a) [illegible] but not [illegible]
(b) [illegible] but not [illegible]
(c) [illegible] but not [illegible]
(d) [illegible] but not [illegible]

[2014 (Set 2): 2 Marks]

5.18 Consider the [illegible] T_1, T_2, T_3 and T_4 and [illegible] Schedule S [illegible]

T_1: [illegible]
T_2: [illegible]
T_3: [illegible]
S1: [illegible]
S2: [illegible]

S' is [illegible] the [illegible] TRUE?

(a) Only S1 is conflict serializable
(b) Only S2 is conflict serializable
(c) Both S1 and S2 are conflict serializable
(d) Neither S1 nor S2 is conflict serializable

[2014 (Set 2): 2 Marks]

5.19 Consider the following [illegible] for [illegible] and y.

[illegible] = 50 write(y)

This [illegible] that the [illegible] and should remain [illegible]

(a) [illegible]
(b) [illegible]
(c) Isolation
(d) Durability

[2015 (Set-4): 1 Mark]

5.20 Consider [illegible] transactions [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2016 (Set-2): 1 Mark]

5.21 Consider [illegible] Schedule S [illegible] transactions T_1 and T_2. Order [illegible] transactions [illegible] Schedule S [illegible] Schedule S [illegible]

S [illegible] transaction T_1 [illegible] time instance 9. Which one of the following statements is correct?

(a) [illegible] memory
(b) [illegible] transaction [illegible]
(c) [illegible] transaction [illegible]
(d) Schedule S is recoverable and cascadless [illegible]

[2017 (Set-2): 2 Marks]

5.22 Consider the following directed graph:

The number of different topological sorting of the vertices of the graph is _____

[2016 (Set-1) : 2 Marks]

5.23 Which one of the following is NOT a part of the ACID properties of database transactions?

(a) Atomicity (b) Consistency

(c) Isolation (d) Scalability

[2016 (Set-1) : 1 Mark]

5.24 Consider the following two phase locking protocol. Suppose a transaction T accesses a fixed set of locks $l_1, l_2, \ldots, l_k$. This protocol works in the following manner:

Step 1: T requests exclusive locks on $l_1, l_2, \ldots, l_k$ in increasing order of the indices.

Step 2: The regular operations are performed.

Step 3: All locks are released.

This protocol is:

(a) prevent deadlocks and deadlock-free

(b) prevent deadlocks but not deadlock-free

(c) prevent deadlocks but not deadlock-free

(d) neither prevent deadlocks nor deadlock-free

[2016 (Set-1) : 1 Marks]

5.25 Consider a database schedule S involving transactions $T_1, \ldots, T_n$. Construct the precedence graph of S with vertices representing the transactions and edges representing the conflicts. S is serializable, which one of the following orderings of the vertices of the precedence graph represents a valid serial schedule?

(a) Topological order

(b) Depth-first order

(c) Breadth-first order

(d) Ascending order of transaction indices

[2016 (Set-2) : 1 Mark]

5.26 Consider the following relational schema with two relations, V_1 and V_2.

$S = r_1(A, B) \bowtie r_2(B, C) \bowtie r_3(C, D) \bowtie r_4(D, E) \bowtie r_5(E, A)$;

$S = V_1 \bowtie V_2$

where r_i denotes a relation by attribute T_i, on a variable X_i, and V_i denotes a view defined by T_i, on a variable Z and V_i denotes a view defined by transaction T_i.

Which one of the following statements about the above schema is **TRUE**?

(a) S is acyclic

(b) S is acyclic, but S is not a chain

(c) S does not have a join tree

(d) S is a chain

[2016 (Set-2) : 2 Marks]

■■■

Answers Transactions & Concurrency Control

5.1	(d)	5.2	(d)	5.3	(d)	5.4	(c)	5.5	(d)	5.6	(d)	5.7	(c)	5.8	(c)	5.9	(c)
5.10	(d)	5.11	(b)	5.12	(b)	5.13	(a)	5.14	(b)	5.15	(c)	5.16	(d)	5.17	(c)	5.18	(a)
5.19	(b)	5.20	(a)	5.21	(b)	5.22	(d)	5.23	(a)	5.24	(a)	5.25	(c)				

Explanations: Transactions & Concurrency Control

5.1 (a)

[illegible]

5.2 (b)

[illegible]

5.3 (d)

[illegible]

5.4 (c)

[illegible]

5.5 (d)

[illegible]

5.6 (d)

[illegible]

5.7 (c)

[illegible]

5.8 (c)

[illegible]

5.9 (c)

[illegible]

5.10 (c)

(i) [illegible]

[illegible]

(ii) [illegible]

[illegible]

S3 is not conflict serializable

Thus schedule S1 and S2 are conflict equivalent. Schedule for S3 is not conflict equivalent to S1 and S2.

5.11 (b):

Schedule S3:

Dependency graph:

S3 has no cycle.

Schedule S4:

Dependency graph:

S4 also has no cycle.

S3 and S4 are conflict serializable.

5.12 (a):

Two-phase locking protocol ensures conflict serializability in the context of a multiuser database system. It may cause deadlock.

Deadlock

T_1 is waiting for T_2 to release lock on B and T_2 is waiting for T_1 to release lock on A.

Timestamp based locking protocol ensures conflict serializability and free from deadlock.

5.13 (a):

By using the precedence graph we solve the problem.

1. Read(x) in T_1 is followed by write(x) in T_2
2. Read(y) in T_2 is followed by write(y) in T_3
3. Read(z) in T_3 is followed by write(z) in T_1

So this schedule has precedence graph:

5.14 (b):

Cycle is present so schedule is not serializable.

5.15 (c):

Secondary indexes store the data in the form of pairs on their key attribute. The values are which contained index in memory.

Secondary indexes are dense.

5.16 (d):

Conflict cycle. Not conflict serializable.

Conflict cycle. Not conflict serializable.

Conflict cycle. Not conflict serializable.

No conflict cycle. So conflict serializable.

5.17 (a):

Conflict serializable and view serializable.

5.18 (a):

No cycle → Conflict serializable.

Cycle → Not conflict serializable.

5.19 (b):

Conflict serializable.

5.20 (a):

The transactions that commited before checkpoint can be ignored in the updations stored in data base.

Those transactions that are committed after checkpoint can be redone at recovery. The transactions that are not committed after checkpoint have to be UNDONE. UNDO : T3, T4. Redone : T1, T2. T1 was committed before checkpoint.

REDO : T2 was committed after checkpoint.

5.21 (b):

T_1 is blind write.

T_2 is performing read operation on x, and T_3 commits before T_2.

Hence it is DIRTY Reader Uncommitted Read.

Hence it is non-recoverable as T_2 is committed before T_3 commits it.

It does not cause transaction inconsistency.

5.22 Sol.

Number of topological orders = 6

a	b	c	e	f	d
a	b	d	e	f	c
a	e	b	c	d	f
a	e	b	d	c	f
a	d	e	b	c	f
a	d	b	e	c	f

5.23 (d):

ACID stands for

A: Atomicity

C: Consistency

I: Isolation

D: Durability

So, the blind write is not the ACID property.

Ans. (b)

[illegible] objects [illegible]

Step 1: [illegible]

Step 2: [illegible]

Step 3: [illegible]

Because of 2PL [illegible]

Ans. (a)

[illegible]

Ans. (c)

[illegible]

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

6 File Structures

6.1 Which of the following statements?

(a) Primary and secondary indexes are B+ trees stored in main memory.

(b) Range queries are faster on B+ trees.

(c) Deletion in primary indexes and B+ trees requires secondary indexes.

(d) The height of a B+ tree is independent of the number of records.

[1999 : 1 Mark]

6.2 B+ trees are used in binary search trees in database because

(a) Disk capacities are greater than memory capacities.

(b) Disk access is much slower than memory access.

(c) Disk does not support random access but does support sequential access.

(d) Data cannot be stored in more than one memory location.

[2000 : 1 Mark]

6.3 A B+ tree is used to index the Name attribute of the relation STUDENT. Assume that all student names are of length 8 bytes, disk blocks are of size 512 bytes, and indexes point to records stored in the heap file. Given this scenario, what is the best choice of the order (i.e., the number of pointers per node) of the B+ tree?

(a) 16 (b) 32

(c) 48 (d) 64

[2002 : 2 Marks]

6.4 Consider a file of 16384 records. B+ tree indexes are built on the name field. Assume that a block pointer is 6 bytes long, the name field is 34 bytes long and the block size is 512 bytes. What is the order of the leaf node?

(a) 24 (b) 25

(c) 26 (d) 27

[2004 : 2 Marks]

6.5 Consider a database [illegible] relational database [illegible] a key field [illegible] B+ tree [illegible] Assuming that [illegible] maximum number of tree pointers in a B+ tree index node. Assume that disk block size is 512 bytes, each block pointer is 6 bytes long and each block pointer is 6 bytes long. In worst case in B+ tree, index node should contain at least two pointers, what is the minimum value of p?

(a) 20 (b) 22

(c) 24 (d) 26

[2004 : 2 Marks]

6.6 What one of the following is a key factor for preferring B+ trees to binary search trees for indexing database relations?

(a) Insertions take a large number of records

(b) Database is accessed on the primary key

(c) B+ trees require less memory than binary search trees

(d) Data can be stored in disk in blocks

[2005 : 1 Mark]

6.7 A B+ tree used as an index for a large dictionary is stored on disk. Assuming that each node fits in one disk block and that it takes 20 msec to read a node from disk, the maximum number of nodes that could be newly created in the process is

(a) 3 (b) 4

(c) 5 (d) 2

[2005 : 1 Mark]

Linked Answer Questions 6.8 and 6.9:

A database [illegible] has 20000 records and occupies 80 disk blocks. Another table [illegible] 40000 records and occupies 80 disk blocks. Each tuple in either table has a unique integer attribute. Two joins are to be [illegible] the two tables. The memory buffer space available is [illegible] 11 and one block is reserved for storing the join result. Cost is measured in disk block read/write. Nested loop join is available as a join method.

6.8 If Nested loop join algorithm is employed to perform the join, what is the minimum cost of the

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

6.9 [illegible] is used with the most appropriate [illegible] number of seeks accessing [illegible] the pending [illegible] will be

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

6.10 [illegible] is by traversing [illegible] pointer in the second block pointer [illegible] B-tree [illegible] memory pointer. The [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

6.11 [illegible] number of [illegible] block size [illegible] block pointer [illegible] block pointer [illegible] block pointer [illegible] of the leaf node?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

Directions for questions 6.12 to 6.16.

Consider the [illegible] relations [illegible] and [illegible] shown.

6.12 Tuples [illegible] and [illegible] are inserted into [illegible] in the order shown. How many of the following [illegible] will be present in the tree after the insertions?

(a) 1 (b) 2

(c) 3 (d) 4

[2007 : 2 Marks]

6.13 Now the tuple [illegible] is deleted from [illegible]. Consider the following statements about the [illegible] resulting structure.

(i) The height of the tree remains the same.

(ii) The root [illegible] contains [illegible] key.

(iii) The leaf node [illegible] and [illegible] disappears [illegible].

Which one of the [illegible] statements is true?

(a) Statements (i) and (ii) are true

(b) Statements (i) and (iii) are true

(c) Statements (ii) and (iii) are true

(d) All the statements are false

[2007 : 2 Marks]

6.14 [illegible]

(a) Sparse key-sequence ordering

(b) Dense key-sequence ordering

(c) Key-sequence ordering

(d) Key-sequence ordering

[2008 : 1 Mark]

6.15 [illegible] The file is [illegible] 1024 bytes, and the size of a block pointer is 10 bytes. The [illegible] index is built on the key field of the file and a block-on-block index is used to [illegible] the index. The [illegible] respectively

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

6.16 The following [illegible] in which order the index [illegible] 2, and [illegible] 1. The [illegible] index [illegible] 2 [illegible] 1. The [illegible] index [illegible] 2 [illegible] 1. The [illegible] index [illegible] 2

20, 12, 10, 8, 4, 2, 1

The maximum number of [illegible] in a [illegible] is

(a) 2 (b) 3

(c) [illegible] (d) [illegible]

[2009 : 2 Marks]

Q.17 [illegible] B-tree in which the maximum number of keys in a node is 5. What is the minimum number of keys in any non-root node?

(a) [illegible] (b) 2

(c) [illegible] (d) [illegible]

[2010 : 1 Mark]

Q.18 A file is organized so that the average of data records is the same as a binary search tree with 7 data entries in sorted order. Which that such a file is [illegible]

(a) Heap (b) Sparse

(c) Clustered (d) Unclustered

[2013 (Set-1) : 1 Mark]

Q.19 [illegible] in the B+ tree [illegible] shown below, [illegible] minimum number of nodes including the root node that must be fetched in order to satisfy the following query "Get all [illegible] greater than or equal to 7 and less than 15" is [illegible]

[2013 (Set-2) : 1 Mark]

Q.20 Consider a B+ tree in which the search key is 12 bytes long, block size is 1024 bytes, record pointer is 10 bytes long and block pointer is 8 bytes long. The maximum number of keys that can be accommodated in each non-leaf node of the tree is _____.

[2016 (Set-3) : 2 Marks]

Q.21 B+ trees are considered balanced because

(a) [illegible] and the path from the root to all leaf nodes is of equal length

(b) the lengths of the paths from the root to all leaf nodes differ from each other by at most 1.

(c) the number of children of any two non-leaf sibling nodes is the same.

(d) the number of records in any leaf node differs by at most 1.

[2016 (Set-2) : 1 Mark]

■■■■

Answers Transactions & Concurrency Control

Q.1	(b)	Q.2	(b)	Q.3	(d)	Q.4	(a)	Q.5	(c)	Q.6	(d)	Q.7	(a)	Q.8	(c)	Q.9	(b)
Q.10	(c)	Q.11	(a)	Q.12	(a)	Q.13	(c)	Q.14	(a)	Q.15	(c)	Q.16	(a)	Q.17	(b)	Q.18	(c)
Q.21	(a)																

Explanations File structures

6.1 (b)

Most database systems [illegible]

6.2 (b)

Disk access is slow and B+ tree [illegible] in less [illegible] disk hits. This is [illegible] B+ tree, [illegible] have very high fan-out [illegible] which reduces the [illegible] tree [illegible]

6.3 (a)

Given:

Given, k, k [illegible] the number of the [illegible] attributes of students

Block size B = [illegible]

Index pointer size = 4 [illegible]

[illegible] the maximum number of keys [illegible]

Let the first is

[illegible]

[illegible] = 320

$$= \frac{320}{12} = 13$$

So [illegible]

6.4 (c)

Size of a child [illegible]

Size of search field value [illegible]

Block size = 512

Therefore [illegible]

[illegible] = 18 [illegible] 4 + [illegible] = 518

[illegible] = 9

6.5 (d)

[illegible] key [illegible] + pointer size) +

pointer size) [illegible] 2

[illegible] = 25

6.6 (d)

The [illegible] from disk to primary memory [illegible] of data access at a date [illegible] is [illegible] than [illegible]

6.7 (b)

Suppose [illegible] are completely full that [illegible] every node has [illegible] keys.

[illegible]

[illegible]

So answer should be [illegible]

6.8 (c)

[illegible] block accesses.

Let be T1 can be T1 or T2

If T1 = T2 then total number of block access is

9000+90 = 90 = 40090

If R = T2 then total number of block access is

4000+90 + 90 = 44090

Better [illegible] total number of block access = 1820000.

6.9 (b)

[illegible] through [illegible] second table

Here we will take table T2 as the outer table in nested loop join algorithm. The number of block accesses then will be

[illegible] = 24020.

[illegible] in memory [illegible] memory and [illegible] tuples.

[illegible]

The difference is [illegible] = 100 ms

Q.10 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Q.12 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Q.13 [illegible]

[illegible]

[illegible]

Q.13 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Q.14 [illegible]

[illegible]

[illegible]

Q.15 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Q.16 [illegible]

[illegible]

[illegible]

[illegible]

Insert 11 :

Insert 14 :

Insert 2

one more node

Insert 1 :

So overall splits = 2.

6.17 (b) :

In a B+ tree the non-root node has minimum two block pointer and maximum P block pointer where P = order. Let n be the minimum number of keys in a leaf node.

$$\left\lceil \frac{P}{2} \right\rceil - 1$$

So in the question key = 8, order = 6

So minimum number of keys in a leaf node =

$$\left\lceil \frac{6}{2} \right\rceil - 1 = 2$$

6.18 (b) :

File is sequential so that the structure of data records is the same as or close to the order of data entries in the index. Thus the index is called clustered index.

6.19 (a) :

We need three node accesses to search the key 7.

1. B+ tree all neighbouring leaf nodes are connected (via block pointers) and is possible to access sequence of records by traversing the leaf level. So we can move to next leaf node and search records here. So 16.

Nodes A, B, D, E and G are leaf nodes and so they are connected. So we can move to next leaf node. So 7.

6.20 Sol:

For Non-leaf nodes

$p \times 20 + (p-1) \times 8 \le 4096$

$p \times 28 \le 4104$

$p \le 146.6$

$p = 146$

Number of keys = $p - 1 = 145$

6.21 (a) :

A tree is balanced because the length of the path from root to all leaf nodes are all equal and every internal node must be half full.

■■■■

Unit X

Computer Networks

Contents

Sl	Topic	Page no.
1.	ISO/OSI Stack and TCP/IP	38
2.	LAN	58
3.	TCP, UDP and IP	214
4.	Routing, Application Layer and Network Security	541

UNIT X

Computer Networks

Syllabus : [illegible] Ethernet, [illegible], [illegible], Routing algorithms, [illegible] TCP/UDP and [illegible] IPv4, [illegible] concepts of hubs, switches, gateways and routers.

Network Security: [illegible] concepts of [illegible] key and private key cryptography, digital signatures, firewalls.

Analysis of Previous GATE Papers

Exam Year	1 Mark Questions	2 Mark Questions	Total Marks
2002	2	2	8
2004	3	4	11
2005	6	7	15
2006	[illegible]	6	11
2007	5	6	14
2008	[illegible]	4	9
2009	–	5	10
2010	3	3	8
2011	3	2	8
2012	3	2	8
2013	3	2	7
2014 Set-1	3	2	8
2014 Set-2	3	2	7
2014 Set-3	3	2	8
2015 Set-1	4	2	8
2015 Set-2	2	3	8
2015 Set-3	2	3	8
2016 Set-1	2	4	10
2016 Set-2	3	4	11

1 ISO/OSI Stack and SWP

1.1 Hosts A and B are sending data to each other on full duplex links. A and B are using the sliding window protocol for flow control. The sender and receiver windows are 5 packets each. Data packets are of fixed length. Time is slotted, with a slot equal to the transmission time for one data packet. Acknowledgments are very small and require negligible transmission time. The propagation delay between A and B is 250 ms. What is the maximum achievable throughput in the steady state?

(a) 7.68 × 10^6 bps (b) 11.52 × 10^6 bps

(c) 15.36 × 10^6 bps (d) 15.36 × 10^6 bps

[2003 : 2 Marks]

1.2 Choose the correct matching between List I and List II

List I

P. Data link layer

Q. Network layer

R. Transport layer

List II

1. Responsible for reliable process-to-process delivery of a message

2. Responsible for delivery of frames between two processes

3. Responsible for delivery of packets across multiple hops

4. Responsible for error detection

	A	B	C
(a)	4	3	1
(b)	4	1	3
(c)	3	2	1
(d)	1	2	3

[2003 : 1 Mark]

1.3 How many 8-bit characters can be transmitted per second over a 9600 baud line if one start bit, one stop bit, and one parity bit are used with each character?

(a) 960 (b) 480

(c) 768 (d) 1920

[2001 : 1 Mark]

1.4 A serial transmission of 7-bit characters takes place with one start bit and one parity bit. Transmission is asynchronous. The maximum bit rate is 9600 bits/sec. The minimum time (in msec) required to transmit 1000 characters is

(a) 100 characters/sec, 15 characters/sec

(b) 60 characters/sec, 38 characters/sec

(c) 100 characters/sec, 135 characters/sec

(d) 60 characters/sec, 68 characters/sec

[2004 : 2 Marks]

1.5 Frames of 1000 bits are sent over a 10 Mbps link. The propagation delay is 10 μsec. Assuming flow control, the frame delivery time is given by

(a) 100 μsec (b) 200 μsec

(c) 300 μsec (d) 400 μsec

[2004 : 2 Marks]

1.6 Consider a TCP connection with a 32 KB receiver window. The sender window is 64 KB and the receiver window size is 32 KB. The maximum number of distinct sequence numbers required to avoid overlap between sequence numbers and acknowledgements is

(a) 32 (b) 64

(c) 128 (d) 256

[2004 : 2 Marks]

1.7 ABC Inc. is a satellite link using a convolution coding scheme. The receiver employs the Viterbi algorithm for decoding. The code rate is 1/3. Assuming that the frame size is 100 bytes, what is the minimum buffer size required?

(a) 5 kbps (b) 10 kbps

(c) 15 kbps (d) 30 kbps

[2004 : 2 Marks]

1.8 Consider a parity check code with three 2-bit bytes and two parity check bits. Three of the code words are 111111, 110011 and 111100.

Which of the following are also code words?

I. 110000 II. 111111

III. 101010 IV. 000000

(a) I and III (b) I, II and III
(c) II and IV (d) I, II, III and IV

[2006 : 2 Marks]

1.9 [illegible] using the selective repeat protocol [illegible] sequence [illegible] is

(a) [illegible] (b) [illegible]
(c) $2^n - 1$ (d) 2^{n-1}

[2006 : 1 Marks]

1.10 Consider the following message $M = 1010001101$. The cyclic redundancy check (CRC) for this message using the divisor polynomial $x^5 + x^2 + 1$ is

(a) 01110 (b) 00011
(c) [illegible] (d) [illegible]

[2005 : 2 Marks]

1.11 A channel has a bit rate of 4 kbps and a one-way propagation delay of 20 ms. The channel uses stop-and-wait protocol. The transmission time of the acknowledgement is negligible. To give efficiency of at least 50%, the minimum frame size should be

(a) 80 bytes (b) 80 bits
(c) 160 bytes (d) 160 bits

[2005 : 2 Marks]

1.12 In an 802.11 [illegible] network, [illegible] frames are transmitted successfully. The sender and receiver are 1 hop apart. The sender sends RTS and receives CTS. How many [illegible] are exchanged?

(a) 11 (b) 16
(c) 18 (d) 20

[2005 : 2 Marks]

1.13 Consider a [illegible] network [illegible] and stop-and-wait is used to send data [illegible] over the link. The [illegible] sender sends [illegible] packets. What is the average number of transmissions required to send 100 packets?

(a) 110 (b) 120
(c) 130 (d) 200

[2005 : 2 Marks]

1.14 Between A and B, 100 bits per second are to be sent using a sliding window protocol. The round trip delay between A and B is 80 milliseconds. Assuming that frames are always full, what is the optimal window size for this protocol?

(a) 10 (b) 20
(c) 100 (d) 200

[2005 : 2 Marks]

1.15 [illegible] packets [illegible] 9 [illegible] packets [illegible] 3 [illegible] packets [illegible] 7?

(a) 12 (b) 14
(c) 16 (d) 18

[2005 : 2 Marks]

1.16 The message 1001101 is to be sent with cyclic redundancy check (CRC) using the generator polynomial $x^3 + x + 1$. The message transmitted is

(a) 1001101000 (b) 1001101001
(c) 1001101010 (d) 1001101011

[2005 : 2 Marks]

1.17 [illegible] 0101010101010101 [illegible] 1010101010101010 [illegible] 1111000011110000 [illegible] the maximum number of [illegible] (in [illegible])?

(a) 0 (b) 1
(c) 2 (d) 3

[2005 : 2 Marks]

1.18 In the waveform shown below, a bit stream is encoded by Manchester encoding scheme. The [illegible] encoded in a different encoding scheme [illegible] (a) [illegible] (b) [illegible] which scheme are

(a) 1000, 1011 and Differential Manchester respectively
(b) 01[illegible] and Differential Manchester respectively
(c) 1000, 1011 and Integral Manchester respectively
(d) 011[illegible] and Integral Manchester respectively

[2007 : 2 Marks]

1.19 Two [illegible] multiplexers of packets. The number of [illegible] is distributed uniformly between 1 and 6. The number of packets [illegible] the first [illegible] 4, 1, 3, 6, 2, 5 respectively. The [illegible] capacity of multiplexer is 6000 packets per [illegible]. The [illegible] packets [illegible]

(a) 3 (b) 46
(c) [illegible] (d) 4

[2007 : 2 Marks]

1.20 Two [illegible] given the following [illegible]

[illegible]

Which of the following are substrings of the bit [illegible] resulting from the above [illegible]?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

1.21 The [illegible] hypothetical [illegible] consists of 15 [illegible] over a [illegible] link with bit rate of 9600 bps. [illegible] made with odd parity and two stop bits [illegible]

(a) [illegible] (b) 12,000 [illegible]
(c) [illegible] (d) 27,000 [illegible]

[2008 : 1 Mark]

1.22 A [illegible] satellite [illegible] transmission system. The distance of the satellite is 36,000 km and speed of the [illegible] 3×10^8 m/s. What should be the packet size for a data [illegible] of 256 [illegible] link [illegible] 37 [illegible] and propagation?

Assume that the acknowledgement packets are negligible [illegible] and [illegible] transmission.

(a) [illegible] bytes (b) [illegible] bytes
(c) [illegible] bytes (d) [illegible] bytes

[2008 : 2 Marks]

1.23 [illegible] link [illegible] parity scheme [illegible]

[illegible] sequence of 56 bits is arranged in a 7 × 8 [illegible] parity bits [illegible] of parity bits computed using the even parity scheme. Each bit of [illegible] is independently [illegible] the parity of the [illegible]. These 48 bits are [illegible] data link [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	0	1	0	1	0	[illegible]		
[illegible]			0	0	1	1		0
[illegible]	0	0	0	1	0	1	0	0
[illegible]	0			0	1	0		0
[illegible]	0	1	0	0	0	1	1	0

The [illegible] received [illegible] and [illegible]. What is the minimum possible value of n^2?

(a) [illegible] (b) 2
(c) 4 (d) [illegible]

[2008 : 2 Marks]

1.24 Let [illegible] be the [illegible] polynomial [illegible] GF(2) [illegible]. What is the condition that should be satisfied by [illegible] to be a primitive polynomial?

(a) [illegible]
(b) [illegible] does not divide [illegible] for any [illegible] including the [illegible]
(c) 1 is a root of [illegible]
(d) [illegible] multiplied by [illegible] terms

[2008 : 2 Marks]

Linked Answer for Q.1.25 and Q.1.26

[illegible] 100 km [illegible]. The propagation time is [illegible] ms. Frames are to be transmitted on this link [illegible] with no error.

1.25 What is the minimum number of bits [illegible] that will be required to represent the sequence [illegible] time? Assume [illegible] are to be given between transmission of two frames.

(a) [illegible] = 2 (b) [illegible] = 3
(c) [illegible] = 4 (d) [illegible] = 5

[2008 : 2 Marks]

1.86 Suppose [illegible] with the sender [illegible] number [illegible] in the earlier part and [illegible] piggybacked [illegible] what is the minimum time the sender will have to wait before [illegible] transmission of the next packet? Ignore the [illegible] ignoring the frame processing time?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2008, 2 Marks]

1.87 [illegible] 7 layers of the ISO/OSI protocol [illegible] the INCORRECT pairing?

(a) Network layer and Frames
(b) Data Link layer and Binary synchronization
(c) Transport layer and End-to-end process communication
(d) Application layer and Data encoding

2014 (Set-1) 1 Mark]

1.88 A bit stuffing based framing protocol uses 8-bit [illegible] flag pattern 01111110. If the input [illegible] 0111111111110, then the output bit stream is

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2014 (Set-3) : 1 Mark]

1.89 Suppose the [illegible] propagation delay of a link with a [illegible] and a wait-and-stop protocol [illegible] transmission time for a [illegible] packet is negligible. The [illegible] is by selectively [illegible] is

[2015 (Set-1) : 2 Marks]

1.90 A link has a transmission speed of [illegible] bits/sec. [illegible] packets of size 1000 bytes each. Assume that the [illegible] are negligible. [illegible] that the propagation delay [illegible] the same as the transmission delay. [illegible] that for successive packets, the [illegible] are negligible. The efficiency of the stop and wait protocol [illegible] is closest to [illegible]

[2015 (Set-2) : 1 Mark]

1.91 Consider a network connecting two [illegible] separated by [illegible] kilometers each. The bandwidth of the [illegible] is [illegible] bits per second. The propagation speed in the medium is [illegible] m/s. [illegible] processor. It needs to transmit [illegible] using a stop-and-wait protocol across the network. The average packet size is [illegible] bits. The network is [illegible] and [illegible] for [illegible]

[illegible] processing delays at nodes are negligible. [illegible] in bits/sec of the [illegible] link is [illegible]

[2015 (Set-3) : 2 Marks]

1.92 Two hosts are connected via a packet switch with 10 bits per second links. Each link has a propagation delay of [illegible] microseconds. The switch begins [illegible] 156 microseconds after it receives the packet. It takes 1000 bits of data and [illegible] between the two hosts using a packet size of 3000 bits. The [illegible] between the first bit sent and the first bit received is [illegible]

[2016 (Set-3) : 2 Marks]

■■■■

Answers : ISO/OSI Stack and UDP

1.1	(b)	1.2	(a)	1.3	(b)	1.4	(c)	1.5	(d)	1.6	(c)	1.7	(b)	1.8	(b)	1.9	(a)
1.10	(a)	1.11	(d)	1.12	(c)	1.13	(b)	1.14	(b)	1.15	(a)	1.16	(b)	1.17	(d)	1.18	(c)
1.19	(c)	1.20	(c)	1.21	(c)	1.22	(a)	1.23	(a)	1.24	(d)	1.25	(d)	1.26	(b)	1.27	(b)

Explanations ISO/OSI Stack and SNP [illegible]

1.1 (b)

Given

Window size

= [illegible] packets

Packet size = 1000 bytes

Total packet size = 6 × 1000 = 5 [illegible] bytes

Ping Time = Transmission Time

+ Propagation Time

= [illegible]

= [illegible] + [illegible]

= [illegible] μs

= [illegible]

Maximum [illegible]

$$= \frac{\text{Data Size}}{\text{Total Time}}$$

$$= \frac{[illegible]}{[illegible] \times 10^{-6}}$$

$$= \frac{[illegible] \times 10^{6}}{[illegible]}$$

= 11.11 × 10⁶ [illegible]

1.2 (a)

I. Data Link layer

1. Detection and correction of data over a physical point-to-point link. Reassembling error correction & detection done by Data Link layer

II. Network layer

4. Transmission [illegible] packet switching

III. Transport layer

8. [illegible] between two processes with the help of TCP and UDP port [illegible]

1.3 (b)

Total number of bits = 12

[illegible]

Number of [illegible] bit [illegible]

$$\text{Transmitted} = \frac{[illegible]}{[illegible]} = [illegible]$$

1.4 (c)

T_1 : 3 characters [illegible]

Transfer Rate = 1200/12 = 100 [illegible]

T_2 : Transfer [illegible] = [illegible] = 254 [illegible]

[illegible] for 80 characters

Data 1200 size = [illegible]

254/80 = 12.075

Therefore X = [illegible]

1.5 (d)

[illegible] are used for [illegible] [illegible] 0.110 [illegible] and [illegible] 0.110 0.110.

1.6 (c)

S.133: Describe W_1, W_2 × Sequence [illegible] number of [illegible] packets [illegible] W_1 [illegible] sequence [illegible] W_1 is size of sender window and W_2 is receiver [illegible]

1.7 (b)

[illegible]

T_0 = 400 ms

[illegible]

[illegible] = 10 = [illegible] = 1000

[illegible]

[illegible]

1.8 (b)

In case of the selective Reject Protocol the maximum [illegible] $= \frac{N}{2} = 2^{n-1}$

In [illegible] will be half.

1.10 (a)

Remainder polynomial is of degree [illegible] and [illegible] by generator polynomial [illegible]

Remainder is 011110

1.11 (d)

Efficiency of stop and wait = $\frac{L}{L + a_p}$

$\frac{L}{L+(2a_p)} = 1/3$

$\Rightarrow 2a_p = 2L$

$\Rightarrow L = a_p = 270 \text{ bits}$

1.12 (c)

It says we have 5 bits [illegible] and [illegible] can't have two consecutive zeros in first and second [illegible] and also can't have two consecutive zeros in last [illegible]

Codeword with first [illegible] = C(10) = [illegible] = 8

Codeword with last two bits zero = [illegible] = 8

Codeword with first [illegible] and last two bits zero = C(8) = [illegible] = 5

Codeword with first [illegible] and last two bits zero = [illegible] = 14

Therefore possible codewords = 32 − 14 = 18

1.13 (0.8):

Error rate is 0.2

In stop and wait protocol sender will retransmit 100 × (1 + 0.2) + (0.2)² + (0.2)³ + ... packets.

= 100 × (1/(1 − 0.2)) = 100/0.8

= 125 [illegible] of size 0.1 [illegible]

1.14 (66):

Queuing [illegible] = 80 ms

= 80 × 10⁻³ sec

$T_{[illegible]} = 1/3$ kbps

= 1000/3 bps

$L = 1k$

= 1.2 × 10⁶ × 80 × 10⁻³

= 96 × 10³

So, optimal size [illegible]

$= \frac{[illegible] \times 96}{[illegible] \times 3} = \frac{[illegible]}{[illegible]}$

n = 10

1.16 (b):

1.16 (b)

P(x) = 1101 001

divisor D(x) = 1001 and CRC remainder is 011.

So the transmitted message is 11001001011.

1.17 (d):

The minimum hamming distance among the [illegible] is 5, i.e.

10101
11111
11010

The maximum number of bit errors that can be detected is 4.

The hamming distance to detect all errors up to 11 bits is

d − 1 = 11

d = 8

∴ Maximum 4 bit errors can be corrected.

1.19 (b):

'0' = Low to high

'1' = High to low.

Pulse is in middle of each bit period.

∴ The bit pattern is 100 00 10011.

1.19 [illegible]

In synchronous TDM, the bandwidth is divided into a stream of [illegible] time slots for each source [illegible] bandwidth. STDM does not reserve a time slot for each source [illegible] time slots for each source [illegible] terminal [illegible] to send its data.

Multiplexed bandwidth = [illegible] bps

[illegible] packet [illegible]

[illegible] = 7 + 8 + 10 + 7 + 6 + 9 + 20

= 7.4

1.22 [illegible]

Efficiency for a sliding window protocol is

$$\eta = \frac{N}{1+2a}$$

where N = the window size

$\Rightarrow N = 127 \times 0.8 = 126$

Efficiency given as 0.25

$$\eta = 0.25 = \frac{126}{1+2a} \quad \text{(1)}$$

$$a = \frac{1}{2}$$

$$a = \frac{\text{propagation time}}{\text{Transmission time}}$$

$$a = \frac{9 \times 3600 \times 10^{3}}{2 \times 10}$$

[illegible]

1.24 [illegible]

[illegible] generator polynomial [illegible] must be divisible at least [illegible] consecutive [illegible]

1. It should be not divisible by x.
2. It should be divisible by [illegible]

Therefore [illegible]

1.26 [illegible]

The link [illegible] for sending [illegible] is longer than the window. The sender window is of size N.

Transmitting [illegible] = [illegible]

$$N \times 1000 \text{ bits sequence} = \frac{1}{10} \times N \times 10^{3}$$

$$= N \times 10^{2} \text{ bits} = N \text{ m sec}$$

Min size = 25 m sec, N = 25 × 8

∴ Minimum number of bits required [illegible] represent sequence numbers distinctly.

1.28 [illegible]

[illegible]

[illegible]

frame = 1000 [illegible]

Time taken for the [illegible] acknowledged = 33 × 2 = 30 ms

Time [illegible] = 30 − 22 = 36 ms

1.27 [illegible]

[illegible] layer

1.29 [illegible]

Input sequence: [illegible]

Output sequence: [illegible] 0.010

After the [illegible] in inserted.

1.30 [illegible]

[illegible] 20% efficiency [illegible]

[illegible] = 81 kbps × 40 ms = [illegible] = 320 bytes.

1.30 [illegible]

$$\text{Efficiency of Stop & Wait} = \frac{1}{1+2a}$$

$$\therefore 0.25 = \frac{1}{1+2a}$$

$$a = \frac{P_d}{t_d} \cdot t_d = 8 \text{ ms}$$

$$\therefore 0.25 = \frac{1}{1+2\times\frac{P_d}{t_d}} \quad \Rightarrow P_d = 32 \text{ ms}$$

1.81 Sol:

v = [illegible]

Bandwidth = [illegible]

Propagation delay

$$t_p = \frac{[illegible]}{[illegible]} = [illegible]$$

Packet size = [illegible]

Transmission delay

$$t_t = \frac{[illegible]}{[illegible]}$$

$$= \frac{1}{[illegible]} [illegible]$$

$$s = [illegible]$$

$$[illegible] = \frac{[illegible]}{[illegible]}$$

$$a = \frac{t_p}{t_t} = \frac{[illegible]}{[illegible]} [illegible]$$

$$[illegible] = \frac{[illegible]}{[illegible]}$$

$$[illegible] = [illegible]$$

$\Rightarrow$ Number of packets = [illegible]

$\therefore$ [illegible] are transmitted for stop-and-wait protocol

1.82 Sol:

Source-destination delay = [illegible] seconds per packet

Size = [illegible] bits

$$\text{Number of packets} = \frac{[illegible]}{[illegible]} = [illegible] \text{ packets}$$

Transmission delay of one packet = [illegible] sec

At t = [illegible] sec, first bit of packet 1 is placed on link 1 by A and Transmission of packet begins

At t = [illegible] sec, first bit of packet 1 reaches switch 1.

At t = [illegible], first bit of packet 1 is received at link 2 by [illegible]

At t = [illegible], first bit of packet 2 is placed on link 1 by A

At t = [illegible], first bit of packet 2 reaches switch. Switch [illegible] 2 [illegible] packet [illegible] switch [illegible] when transmitting it.

First bit of packet 1 will be placed on link [illegible] switch at [illegible] sec.

Hence [illegible] 2 [illegible]

At t = [illegible] second, first bit is placed on Link 2.

At t = [illegible] first bit of packet 2 reaches B.

[illegible] is required.

■■■■

2 LAN

2.1 A [illegible] LAN has 10 [illegible] bandwidth and uses CSMA/CD. The propagation delay [illegible] 2.5 μs. What is the minimum packet size that can be used on this network?

(a) [illegible] bytes (b) 100 bytes
(c) 200 bytes (d) None of these

[2003 : 2 Marks]

2.2 A host is connected to a Department network which is part of a University network. The University network is part of the Internet. The largest network in which the Ethernet address of the host is unique is

(a) the subnet to which the host belongs
(b) the Department network
(c) the University network
(d) the Internet

[2004 : 1 Mark]

2.3 Consider a [illegible] time-slotted [illegible] MAC protocol, where each host always has data to send and transmits with probability $p = 0.2$ in each time slot. [illegible] workstations and one frame can be transmitted in one data slot. [illegible] successfully transmitted data frames. What is the maximum [illegible] which this protocol can support, if each host [illegible] provided a minimum throughput of 0.15 frames per time slot?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

2.4 A CSMA/CD network operates at a rate of 5 Mbps. The [illegible] propagation delay equal 25 μs. Both A and B attempt to transmit at time t. [illegible] A wins the contention and transmits a 512 bit packet. After the end of this successful transmission by A, both A and B attempt to transmit at the first slot time. The probability that A wins the second contention is

(a) 0.5 (b) 0.625
(c) 0.75 (d) 1.0

[2004 : 2 Marks]

2.5 In a network of LANs connected by bridges, packets are sent from one LAN to another through intermediate bridges. Which of the following statements is/are true?

(a) The shortest path between LANs
(b) The spanning tree in the routing graph
(c) Loop free routing
(d) Loop initialization

[2005 : 1 Mark]

2.6 Which of the following statement is TRUE about CSMA/CD?

(a) IEEE 802.11 wireless LAN runs CSMA/CD protocol
(b) Ethernet uses Token Ring MAC protocol
(c) CSMA/CD is [illegible] High propagation delay networks work like satellite networks
(d) There is no contention in a CSMA/CD network

[2006 : 1 Mark]

2.7 Which of the following statements is/are FALSE regarding bridges?

(a) Bridge is a layer 2 device
(b) Bridges reduce collision domain
(c) Bridges [illegible] two or more LAN segments
(d) Bridges reduce broadcast [illegible]

[2006 : 1 Mark]

2.8 A network with CSMA/CD protocol in the MAC layer is running at 1 Gbps over a 1 km cable with no repeaters. The signal speed in the cable is 2 × 10⁸ m/sec. The minimum frame size for this network should be

(a) 10000 bits (b) 10000 bytes
(c) 5000 bits (d) 5000 bytes

[2006 : 2 Marks]

2.8 [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

2.10 [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 1 Marks]

2.11 [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 1 Mark]

2.12 [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

2.13 [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

2.14 The distance between two [illegible] and [illegible] is [illegible] km. All links are [illegible] bits/sec. The propagation speed per kilometer is [illegible] seconds. Let [illegible] be the channel capacity. Assuming that processing delay is negligible, the minimum number of bits for the sequence number field in a [illegible] window protocol is used is:

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

2.15 The minimum [illegible] required for a [illegible] computer network transmitting at 1 Gbps over a [illegible] with a link speed of [illegible] is

(a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[2008 : 2 Marks]

2.16 Consider a [illegible] computer network [illegible] [illegible] and [illegible] [illegible] [illegible]

(a) 1 (b) 2
(c) [illegible] (d) [illegible]

[2018 : 2 Marks]

2.15 Consider a LAN with four nodes S_1, S_2, S_3 and S_4. Time is slotted in the network and nodes can begin transmission only at the beginning of a slot. A collision occurs if two or more nodes transmit in the same slot. The probabilities of transmission in a time slot for S_1, S_2, S_3 and S_4 are 0.1, 0.2, 0.3 and 0.4, respectively. The probability of exactly a frame is the first slot without any collision by any of these four stations is _____.

[2015 (Set-1) : 2 Marks]

2.16 Consider a CSMA/CD network that operates at a rate of 10 Mbps. The propagation delay between two stations is 50 μs. The minimum frame size required for the network to be 1250 bytes. What is the maximum speed (in km/s) at which the station can be located?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2015 (Set-3) : 1 Mark]

2.20 Which one of the following protocols is NOT used between a mail server and the mail client?

(a) IMAP (b) [illegible]
(c) TELNET (d) HTTP

[2016 (Set 1) : 1 Marks]

2.21 Which of the following protocol(s) of stack (Application layer protocol)?

(a) HTTP (b) FTP
(c) [illegible] (d) [illegible]
(e) [illegible] (f) [illegible] only
(g) [illegible] (h) [illegible]

[2016 (Set 1) : 1 Marks]

2.22 A [illegible] uses the Stop-and-Wait ARQ protocol for reliable transmission of frames. Frames are of size 1000 bytes and the propagation delay between the sender and the receiver is 50 ms. The bit rate of the channel is 20 kbps. The minimum time (in seconds) after which a frame can be transmitted successfully is _____.

Assuming no frame is lost or corrupted, [illegible] acknowledgment [illegible] is negligible.

[2016 (Set-1) : 2 Marks]

2.23 In an Ethernet network, which one of the following statements is TRUE?

(a) It uses slotted ALOHA to access the channel even in case of no contention.
(b) The protocol of the jamming signal is used to inform the other hosts that a collision has occurred.
(c) A station must wait for a fixed time after the previous transmission before it can start a new transmission.
(d) The size of the minimum data frame must be at least twice the maximum propagation delay.

[2016 (Set-2) : 2 Marks]

2.24 A network [illegible] a bandwidth of 20 × 2^[illegible] bits per second. It uses CSMA/CD on the MAC layer. The maximum signal propagation time between any two stations is 49 microseconds. The minimum size of a frame in the network is _____ bytes.

[2016 (Set-2) : 2 Marks]

2.25 Consider a 128 × 10 [illegible] network. [illegible] Kbits. The minimum number of bits required for the sequence number field in the header is _____.

[2016 (Set-2) : 2 Marks]

■■■■

Answers: LAN

2.1	(d)	2.2	(d)	2.3	(b)	2.4	(b)	2.5	(b)	2.6	(c)	2.7	(d)	2.8	(a)	2.9	(c)
2.10	(c)	2.11	(b)	2.12	(a)	2.13	(a)	2.14	(a)	2.15	(a)	2.16	(a)	2.17	(b)	2.18	(a)
2.19	(a)	2.20	(c)	2.21	(c)												

Explanations: LAN

2.1 (d):

[illegible]

$$= \frac{d}{v} = \frac{\text{distance}}{\text{velocity}}$$

[illegible]

$$t = \frac{d}{v} = \frac{[illegible]}{2 \times 10^{8}} = [illegible]$$

[illegible]

2.2 (d):

[illegible]

2.3 (b):

[illegible]

So the throughput [illegible]

[illegible]

It implies $S = 1$, so $b = a$

2.4 (b):

[illegible]

Expected availability

$$= \frac{1}{1+a} + \frac{1}{1+\frac{1}{a}} = \frac{1}{1+a} + \frac{1}{a+1} = \frac{2}{1+a} = 0.667$$

2.5 (b):

[illegible] which reduces automatically delay routing [illegible] changing topology. The algorithm consists of [illegible] main [illegible]

1. Frame Forwarding
2. [illegible]
3. [illegible]

2.6 (c):

[illegible] 2.2.12.

[illegible]

2.7 (d):

[illegible]

2.8 (a):

Let [illegible] minimum packet size

$$T_t = (1 \text{ km}/2 \times 10^{8} \text{ m/s})$$

[illegible]

Minimum frame size can be found by formula

[illegible]

$$S = 10^{7} \times 10^{-6} = 10^{1} \text{ bits.}$$

2.9 (c):

So [illegible] is equal to [illegible] + propagation delay.

[illegible]

$T_t = 100/10$ Mbps $= 10$ μs.

[illegible]

Bandwidth efficiency is 100/1000

[illegible]

[illegible]

2.10 (a) :

[illegible] minimum the speed of forwarding packets i.e. 100 kbps.

Processing time given will be not more than its minimum transmission time.

[illegible] = 3.75 milli seconds

2.11 (b) :

In Manchester encoding, the bit period is divided into two equal halves. Therefore, [illegible] signal element = half the number of bits per sec.

Hence, [illegible] is 2 kbit/sec.

2.12 (a) :

[illegible] = 100 kbps

Delay [illegible]

Efficiency of channel = transmission time / total delay

[illegible]

Maximum throughput is

= [illegible] kbps = [illegible] kbps.

2.13 (a) :

[illegible]

Exchange [illegible] is a packet switch [illegible] packet switch [illegible] the medium. This [illegible] possible [illegible] Ethernet [illegible] Token Ring [illegible] the event of [illegible]

[illegible]

2.14 (a) :

[illegible]

Frame [illegible]

Propagation delay [illegible]

[illegible]

[illegible]

2.15 (b) :

The minimum [illegible]

[illegible]

2.16 (b) :

[illegible]

Propagation delay [illegible] from S to R

[illegible]

Total propagation delay [illegible] from S to D = [illegible]

[illegible]

Total transmission time for 1 packet

[illegible]

[illegible]

[illegible] packet will take [illegible] to reach D. [illegible] packets [illegible] D after [illegible] packet [illegible] 2000 packets [illegible] 999 packets will take 999 [illegible]

2.17 (b) :

[illegible]

2.13 Sol.:

Probability of frame being generated by station S.

Required probability $P_{S_1} = \text{Only one station generates frame}$

$= P_1(1-P_2)(1-P_3)(1-P_4) + (1-P_1)P_2(1-P_3)(1-P_4) + (1-P_1)(1-P_2)P_3(1-P_4) + (1-P_1)(1-P_2)(1-P_3)P_4$

$= 0.1 \times 0.9 \times 0.3 \times 0.6 + 0.9 \times 0.1 \times 0.3 \times 0.6 + 0.9 \times 0.9 \times 0.7 \times 0.6 + 0.9 \times 0.9 \times 0.3 \times 0.4$

$= 0.0162 + 0.0162 + 0.0378 + 0.0108 = 0.1080$

2.14 (d):

Bandwidth $= 100 \times 10^6$ bps

$n = 1000$

For CSMA/CD, $t_{prop} = 25\mu s$

$$\frac{1500 \times 8}{100 \times 10^6} = 2 \times \frac{1000}{v \times 10^6}$$

$$v = \frac{2 \times 1000 \times 100 \times 10^6}{1500 \times 8}$$

$$2 \times 10^8 = 200000 \text{ km/sec}$$

2.15 (a):

- DNS is used for mapping host name to IP address.
- ARP is a protocol for finding physical address of a host when its IP address is known.
- RARP is a protocol which is used to map MAC address with IP address.
- DHCP is also used for mapping MAC address with IP address.

Since all options are used to find mac address, so mac address is unique address and option (d) is more appropriate answer.

2.16 (c):

TCP has a mechanism of acknowledgement and retransmission. HTTP is a stateless protocol. TCP is a connection oriented protocol but not stateful. TCP is implication protocol and is generally used with help of TCP.

2.17 Sol.:

$$\text{Sender throughput} = \frac{\text{Data}}{\text{Total time}}$$

$$= \frac{100 \text{ bytes}}{1 + 100 \times 0.1 + 0.1} = 95.00 \text{ bytes/sec}$$

2.18 (d):

Exponential backoff algorithm is used for persistent collisions in network.

2.19 Sol.:

Bandwidth $= 20 \times 10^6$ bps

Propagation time $= 40$ μsec

Use ethernet.

Transmission time $= 2 \times$ Propagation time

$$\frac{\text{Packet size}}{20 \times 10^6 \text{ bits/sec}} = 2 \times 40 \times 10^{-6}$$

$$\text{Transmission time} = 80 \times 10^{-6} \times 2 \times 40 \times 10^{-6}$$

$$= 160 \text{ bits}$$

$$= \frac{160}{8} = 20 \text{ bytes}$$

2.20 Sol.:

$$\text{Transmission time} = \frac{\text{Packet size}}{\text{Bandwidth}}$$

$$\text{CPU} = \frac{1024 \times 8}{128 \times 10^3} = \frac{1024}{16} \text{ msec}$$

$= 64$ msec [$1 \text{ KB} = 8^{10}$]

$$\text{Efficiency} = \text{Work} \times \frac{\text{CPU}}{\text{CPU} + \text{Overhead}}$$

$$100\% = \text{Work} \times \frac{1}{1 + \frac{\text{Overhead}}{\text{CPU}}}$$

$$1 + \frac{\text{Overhead}}{\text{CPU}} = \text{Work}$$

$$1 + 2\frac{\text{Overhead}}{\text{CPU}} = \text{Work}$$

$$= 2 \times 2 \times 4 = \text{Work}$$

$$\text{Work} = 16 \text{ bits} = 2$$

In worst case space ABCQ.

Total window size $= \log_2(\text{Sequence number}) + \text{Sender window} = \text{Receiver window}$

$$= \log_2[6 + 8] = \log_2[12] = 4$$

■■■■

3 TCP, UDP and IP

3.1 [illegible]

(a) [illegible] IP address

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2008 : 1 Mark]

3.2 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2008 : 1 Mark]

3.3 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2008 : 2 Marks]

3.4 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2004 : 1 Mark]

3.5 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2001 : 1 Mark]

3.6 [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 1 Mark]

3.7 [illegible]

(a) byte (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 1 Mark]

3.8 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2001 : 3 Marks]

3.9 [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2004 : 2 Marks]

3.10 [illegible] packets consists of [illegible] bytes. Assuming propagation time [illegible] is 12000 [illegible]. Each packet

8.19 Given TCP Reno sender's congestion window size is Congestion Window = 16 KB. The window size advertised by the receiver is Advertised Window = 24 KB. The last byte sent by the sender is Last Byte Sent = 50 KB and the last byte acknowledged by the receiver is Last Byte Acked = 31 KB. The current window size at the sender is

(a) 25 KB (b) 19 KB
(c) 16 KB (d) 8 KB

[2006 : 2 Marks]

8.20 In a communication network, a packet of length L bits takes link L1 with a probability of p_1 or link L2 with a probability of p_2. Links L1 and L2 are known to introduce delays of d_1 and d_2 respectively. The probability that the packet will be received within a time delay less than D is

(a) $(1 - p_1)d_1 + (1 - p_2)d_2$
(b) $p_1 d_1 + p_2 d_2$
(c) $(1 - p_1)d_1 + (1 - p_2)d_2 + p_1 d_1 + p_2 d_2$
(d) $1 - (p_1 d_1 + p_2 d_2)$

[2006 : 2 Marks]

8.21 A communication channel with a transmission rate of [illegible] bits/sec is shared by two sender nodes, one sends 300 bits/sec and the other sends 500 bits/sec. Which of the following mechanisms will be the most suitable to share the channel without any loss of data?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2006 : 2 Marks]

8.22 Which of the following statements is TRUE?

(a) Both Ethernet frame and IP packet include checksum fields
(b) Ethernet frame includes a checksum field but IP packet includes a CRC field
(c) Ethernet frame includes a CRC field and IP packet includes a checksum field
(d) Both Ethernet frame and IP packet include CRC fields

[2006 : 1 Mark]

8.23 For which one of the following protocols Time To Live (TTL) is a mandatory field in the IP datagram header?

(a) Protocol for carrying network control information
(b) Protocol for carrying routing information
(c) Internet control message protocol
(d) Protocol for carrying data packets

[2006 : 1 Mark]

8.24 A router uses the following routing table:

Destination	Mask	Interface
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

A packet arrives at the router with the address 144.16.68.117. On which interface will it be forwarded?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2006 : 2 Marks]

8.25 Suppose a TCP connection has a propagation delay of 50 ms and a bottleneck link of 1 Mbps. The bottleneck link has a buffer that can hold 20 packets. Each packet is 1000 bytes. The TCP connection is in the congestion avoidance phase and the congestion window size is 10 Mbytes. Consider the time at which the sender receives the first ACK. If only one packet was lost, what is the minimum time (in seconds) after which the sender can send the next packet?

(a) 1 (b) 2
(c) 3 (d) 4

[2006 : 2 Marks]

8.26 A multicast router (MC) has two ports connected to two multicast trees. Both trees generate multicast traffic at 10 Mbps and when the same is off, the rate of traffic is zero. For duration of one second, which is the ratio between the minimum and maximum bandwidth that can be allocated on the link so that both trees can send multicast traffic?

[illegible] assume to be [illegible] assuming that all connections are synchronized and the packet sizes are received at the large buffer, the maximum number of connections that can be multiplexed so that no data loss occurs is S_2. The values of S_1 and S_2 are respectively

(a) 20 and 80 (b) [illegible] and 80
(c) [illegible] and [illegible] (d) 15 and 25

[2006 : 3 Marks]

8.37 A [illegible] machine X [illegible] TCP connection to port 5678 on a machine Y and a TCP [illegible] port 8088 on machine Z.

However, there is no application listening on the port number port 8088 on Z. An [illegible] Port Unreachable error" is generated by

(a) Y [illegible] (b) Z [illegible]
(c) Neither Y nor Z (d) Both Y and Z

[2006 : 2 Marks]

8.38 A subnet mask has the following address : 144.16.68.255. The subnet mask

(a) is necessarily 255.255.255.0
(b) is necessarily 255.255.240.0
(c) is necessarily 255.255.248.0
(d) [illegible] 255.255.240.0 or 255.255.248.0

[2006 : 2 Marks]

8.39 Two computers C1 and C2 are on Ethernet LAN follows. C1 has IP address 192.168.2.50 and netmask 255.255.255.0. C2 has IP address 192.168.2.60 and netmask 255.255.255.0. Which one of the following statements is true?

(a) C1 and C2 belong to same network
(b) C1 assumes C2 is on same network, but C2 assumes C1 is on a different network
(c) C2 assumes C1 is on same network, but C1 assumes C2 is on a different network
(d) C1 and C2 both assume they are on different networks

[2006 : 2 Marks]

8.40 Consider the following statements about the [illegible] in TCP.

(i) The initial value is sent to the RTT [illegible] and Trip time. [illegible] during TCP connection establishment for the [illegible] of the connection.
(ii) Appropriate RTT estimation is possible only if [illegible] the Timestamp value of a TCP segment.
(iii) [illegible] to RTT estimation is possible [illegible] data [illegible] sender to the receiver.

Which of the following statements is true?

(a) (i) is false, but (ii) and (iii) are true
(b) (i) and (ii) are false, but (iii) is true
(c) (i) and (iii) are false, but (ii) is true
(d) (i), (ii) and (iii) are false

[2006 : 1 Mark]

8.41 Two-hundred TCP connections are active [illegible] receiver sending at 50 Kbps. The sender sends segments each 1 sec. The sequence numbers of the first and second segments are 250 and 500 respectively. The first segment was lost, but the second segment was received correctly by the receiver.

Let X be the amount of data received in the first segment. Let Y be the ACK number sent by the receiver. The values of X and Y are respectively

(a) 50 and 251 (b) 250 and 251
(c) 50 and 251 (d) 250 and 500

[2007 : 1 Mark]

8.42 The address of a class B network is to be divided into subnets. [illegible] subnet number. What is the maximum number of subnets and the maximum number of hosts in each subnet?

(a) 62 subnets and 2046 hosts
(b) 64 subnets and 2044 hosts
(c) 62 subnets and 1022 hosts
(d) 64 subnets and 1024 hosts

[2007 : 2 Marks]

8.43 What is the minimum overhead to send TCP/IP datagram in an Ethernet LAN, assuming IP header has no options and TCP header has no options?

(a) 40 bytes (b) 54 bytes
(c) 58 bytes (d) 64 bytes

[2008 : 1 Mark]

8.44 Which of the following protocols verify the integrity of the received IP packets?

(a) ICMP (b) ARP
(c) TCP (d) UDP

[2009 : 1 Mark]

3.35 Which of the following statements is/are TRUE?

S1: [illegible] and flow control

S2: [illegible] and flow control

S3: Transport layer deals with [illegible] and flow control

S4: Sliding window mechanism decides which bits [illegible] and flow control

(a) S1, S2 and S3 only

(b) S1 and S2 only

(c) S1 and S4 only

(d) S3 and S4 only

(e) S1, S2, S3 and S4 only

[2010 : 2 Marks]

3.36 The Ethernet header has the following fields as shown below.

Which of the following protocols are used in the [illegible]

S1: [illegible] from the source MAC address to the destination address

S2: [illegible] address to MAC address which identifies the connection

S3: The source port number [illegible] SYN_SENT → ESTABLISHED [illegible] the source port number [illegible]

S4: The source port number [illegible] SYN_RCVD → ESTABLISHED in the client [illegible] connection

(a) S2 and S4 only (b) S1 and S3 only

(c) S1 and S4 only (d) S2 and S3 only

[2009 : 2 Marks]

Directions for Question 3.37 to 3.38:

[illegible] address 176.168.[illegible] and a connection through two routers R1 and R2 [illegible] network [illegible] 168.155.[illegible] and 168.1.210.[illegible] and IP address 192.168.[illegible] and 192.168.[illegible]. The network mask in the network is 255.255.255.[illegible]

3.37 [illegible] the minimum number of bits required to uniquely address all hosts in the network?

(a) 1 (b) 2

(c) 3 (d) 4

[2009 : 2 Marks]

3.38 Which of the following is the network address of the network containing the host?

(a) [illegible] (b) 192.168.1.0

(c) [illegible] (d) 192.168.1.255

[2009 : 2 Marks]

3.39 In the context of the IEEE 802.11 CSMA/CA protocol, the size of the contention window

(a) Does not increase

(b) Increases linearly

(c) Increases exponentially

(d) Increases randomly

[2009 : 2 Marks]

3.40 In the Ethernet connected LAN, a host sends an ARP query when it has a packet to send to another host. What is the maximum number of ARP queries per minute?

(a) 1000 (b) 600

(c) 2000 (d) 2047

[2009 : 2 Marks]

3.41 A client process P needs to interact with a time server process Q. Consider the following statements, where [illegible] is a list of system calls in that order, following which it is ready to receive a response from Q. The client process P [illegible] followed by connect [illegible] system call. The server process Q [illegible] accept [illegible] system call. Which one of the following events could take place?

(a) [illegible] call returns successfully

(b) connect [illegible]

(c) connect [illegible] system call [illegible]

(d) [illegible] system call [illegible]

[2009 : 2 Marks]

3.42 Which [illegible] TCP [illegible] the initial sequence number [illegible] the clock [illegible] The low order 32 bits of the counter [illegible] the clock [illegible] increment [illegible] The maximum [illegible] time is given to be [illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2008 : 2 Marks]

3.43 [illegible] Time-To-Live (TTL) [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible] spoofing

[2010 : 1 Mark]

3.44 Suppose computers A and B have IP addresses 10.105.1.113 and 10.105.1.120 [illegible] they belong to the same IP [illegible] A. Which of the [illegible] and B should belong to the same network?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2010 : 2 Marks]

3.45 [illegible] protocol [illegible] up to [illegible] port [illegible], GABRIOT

(a) Block all [illegible] 6.0.0.0/8
(b) Block all [illegible]
(c) Stop incoming traffic from a specific IP address by blocking the IP address
(d) [illegible] TCP to [illegible] a specific port [illegible]

[2011 : 1 Mark]

3.46 [illegible] protocol is used to support conference calls?

(a) SMTP (b) TP
(c) FTP (d) [illegible]

[2012 : 1 Mark]

3.47 In the IPv6 addressing [illegible] number of [illegible] under Class C addresses is

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2013 : 1 Mark]

3.48 An Internet Service Provider (ISP) has the following chunk of CIDR-based IP addresses available with it: 245.248.128.0/22. The ISP wants to give half of this chunk of addresses to Organization A, and a quarter to Organization B, while retaining the remaining with itself. Which of the following is the allocation to Organization A?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2012 : 2 Marks]

3.49 Consider a link using TCP Selective Acknowledgment (SACK) [illegible] Assume that a [illegible] the 3rd [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2012 : 2 Marks]

3.50 In a [illegible] datagram, the [illegible] value of [illegible] is [illegible] the fragment offset [illegible] The [illegible] of the fragments [illegible] the first and [illegible] of the original datagram respectively are

(a) First fragment [illegible] and [illegible]
(b) Third fragment [illegible] and [illegible]
(c) [illegible] and [illegible]
(d) Middle fragment [illegible] and [illegible]

[2013 : 2 Marks]

3.51 Let the size of congestion window of a TCP connection be 32 KB when a timeout occurs. The round-trip time of the connection is 150 ms and the maximum segment size used is 2 KB. The time taken (in seconds) by the TCP connection to get back to a congestion window of 32 KB is [illegible]

[2011 (Set 1) : 2 Marks]

8.65 Consider a selective repeat sliding window protocol that uses 2-bit sequence numbers. Frames are of fixed size and contain 1000 bits in addition to the sequence number. Acknowledgements are sent on separate frames and are very small. Propagation delay between the two stations is 20 msec. Frames are lost with probability 0.01. For minimum number of bits per second, the sender should use _____.

[2014 (Set 1): 8 Marks]

8.66 Which one of the following protocols is used to convert an IP address into a MAC address?

(a) ARP (b) RARP

(c) [illegible] (d) [illegible]

[2014 (Set-2): 1 Mark]

8.67 Two hosts are connected via a router. The router receives IP packets from the first host and forwards them to the second host. The router has a maximum IP packet size of 1500 bytes. The router receives a TCP segment of 1500 bytes from the first host. The IP header is 20 bytes long. The router fragments the TCP segment into two IP packets. The size of the IP payload in the second packet is _____ bytes.

[2014 (Set-2): 1 Mark]

8.68 Consider the above network. Assume that each switch uses a table to forward packets to the next hop. Packets are addressed by a pair (x, y) where x is the host number and y is the port number. The table at switch S1 is shown below.

Destination	Output port
(1, 1)	1
(1, 2)	2
(1, 3)	3
(1, 4)	4
(2, 1)	1
(2, 2)	2
(2, 3)	3
(2, 4)	4
(3, 1)	1
(3, 2)	2
(3, 3)	3
(3, 4)	4

The table at switch S2 is shown below.

Destination	Output port
(1, 1)	1
(1, 2)	2
(1, 3)	3
(1, 4)	4
(2, 1)	1
(2, 2)	2
(2, 3)	3
(2, 4)	4
(3, 1)	1
(3, 2)	2
(3, 3)	3
(3, 4)	4

Which of the following is the sequence of switches traversed by a packet with destination address (3, 3) if the packet is sent to port 1 of switch S1?

(a) S1, S2, S2, S1 (b) S1, S2, S2, S2

(c) S1, S2, S1, S1 (d) S1, S2, S1, S2

[2013 (Set 2): 2 Marks]

8.69 Consider a LAN with four nodes S1, S2, S3, S4. Node S1 transmits a frame to node S2. Assume that no other node senses the transmission of S1. When S2 receives the frame, it waits for a duration equal to the transmission time of a frame and then transmits a frame to S3. The frame from S2 is received by S3 when S3 is also transmitting a frame. The collision is detected by S2 and S3. Which of the following is TRUE?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2013 (Set 2): 2 Marks]

8.70 Consider a LAN with four nodes S1, S2, S3, S4. Node S1 transmits a frame to node S2. Assume that no other node senses the transmission of S1. When S2 receives the frame, it waits for a duration equal to the transmission time of a frame and then transmits a frame to S3. The frame from S2 is received by S3 when S3 is also transmitting a frame. The collision is detected by S2 and S3. Which of the following is TRUE?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2013 (Set-2): 1 Mark]

8.71 A router receives an IPv4 packet with a payload of 1000 bytes. The router does not fragment the packet. The router adds a header to the packet. The total length of the packet is _____ bytes.

[2013 (Set-2): 2 Marks]

8.72 Consider the above network. Assume that each switch uses a table to forward packets to the next hop. Packets are addressed by a pair (x, y) where x is the host number and y is the port number. The table at switch S1 is shown below.

Destination	Output port
(1, 1)	1
(1, 2)	2
(1, 3)	3
(1, 4)	4
(2, 1)	1
(2, 2)	2
(2, 3)	3
(2, 4)	4
(3, 1)	1
(3, 2)	2
(3, 3)	3
(3, 4)	4

The table at switch S2 is shown below.

Destination	Output port
(1, 1)	1
(1, 2)	2
(1, 3)	3
(1, 4)	4
(2, 1)	1
(2, 2)	2
(2, 3)	3
(2, 4)	4
(3, 1)	1
(3, 2)	2
(3, 3)	3
(3, 4)	4

Which of the following is the sequence of switches traversed by a packet with destination address (3, 3) if the packet is sent to port 1 of switch S1?

(a) S1, S2, S2, S1 (b) S1, S2, S2, S2

(c) S1, S2, S1, S1 (d) S1, S2, S1, S2

[2013 (Set 2): 2 Marks]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2015 (Set-2) : 2 Marks]

3.60 Suppose two hosts use a TCP connection to transfer a large file. Which of the following statements is/are true with respect to the TCP sequence numbers?

1. If the sequence number of a segment is m, then the sequence number of the subsequent segment is always m+1.
2. The initial round trip time is one second and after the first loss, the value of the retransmission timeout is always set to twice the most recent round trip time.
3. The size of the advertised window never changes during the lifetime of a TCP connection.
4. The maximum congestion window size never exceeds the advertised window size.

(a) 1 only (b) 2 only
(c) 1 and 4 only (d) 2 and 4 only

[2015 (Set-2) : 1 Mark]

3.61 Which one of the following field in an IP header is NOT modified by a router?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2015 (Set-1) : 1 Mark]

3.62 Identify the correct order in which a server process must invoke the functions accept, bind, listen, and multiple instances of read/write when using TCP sockets.

(a) bind, listen, accept, read/write
(b) bind, listen, accept, read/write
(c) [illegible]
(d) accept, bind, listen, read/write

[2015 (Set-2) : 1 Mark]

3.63 Assume that the round-trip time (RTT) between a client and a server is 100 ms. Assume that the TCP window size is 8 kB and that the TCP minimum retransmission timeout is 1 sec. Assuming no loss and no window scaling option, what is the maximum throughput (in kB/sec) that can be achieved?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2015 (Set-2) : 2 Marks]

3.64 Consider the following routing table at an IP router:

Network No.	Subnet Mask	Next Hop
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	R
[illegible]	[illegible]	R
[illegible]	[illegible]	R

Packets IP address in the range [illegible] will be routed to the next hop [illegible] from the routing table above.

	List-I		List-II
A.	[illegible]	1.	[illegible]
B.	[illegible]	2.	[illegible]
C.	[illegible]	3.	[illegible]
D.	[illegible]	4.	[illegible]
		5.	[illegible]

Codes:

	A	B	C	D
(a)	1	[illegible]	3	4
(b)	[illegible]	1	2	5
(c)	2	[illegible]	[illegible]	3
(d)	2	4	[illegible]	[illegible]

[2015 (Set-3) : 2 Marks]

3.65 Host A sends a TCP datagram containing 880 bytes of data to Host B over an Ethernet LAN. Ethernet frames may be of maximum length 1500 bytes. The header length of all IP datagrams is 20 bytes. The header length of all TCP segments is 20 bytes. The maximum number of TCP data bytes in each Ethernet frame sent between A and B is:

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2015 (Set-3) : 2 Marks]

3.66 Consider the following statements:

I. TCP connections are full duplex

II. TCP [illegible] network layer

III. TCP connections are message oriented

(a) Only I is correct

(b) Only I and II are correct

(c) Only I and III are correct

(d) All of I, II and III are correct

[2015 (Set-3) : 1 Mark]

3.67 [illegible] network 200.15.11.[illegible]27 [illegible] which can be merged to a block is

[2015 (Set-4) : 2 Marks]

3.68 An IP datagram of size 1000 bytes arrives at a router. The router has to forward the datagram on a link whose MTU (Maximum transmission unit) is 400 bytes. Assume that the size of the IP header is 20 bytes. The number of fragments that the datagram will be divided into for transmission is _____.

[2014 (Set-1) : 2 Marks]

3.69 [illegible] algorithm for congestion control. Let the token bucket size be 2 megabytes and the maximum output rate be 20 megabytes per second. Tokens are added to the token bucket at a rate of 2 megabytes per second. The time (in seconds) required to empty the token bucket to zero, from the instant the token bucket is completely full, is _____ seconds.

[2014 (Set-1) : 2 Marks]

■■■■

Answers : TCP, UDP and IP

3.1	(d)	3.2	(d)	3.3	(c)	3.4	(b)	3.5	(a)	3.6	(c)	3.7	(a)	3.8	(d)	3.9	(c)
3.10	(b)	3.11	(a)	3.12	(d)	3.13	(b)	3.14	(b)	3.15	(b)	3.16	(d)	3.17	(c)	3.18	(c)
3.19	(b)	3.20	(a)	3.21	(d)	3.22	(c)	3.23	(c)	3.24	(c)	3.25	(a)	3.26	(a)	3.27	(a)
3.28	(d)	3.29	(c)	3.30	(c)	3.31	(d)	3.32	(c)	3.33	(a)	3.34	(a)	3.35	(d)	3.36	(c)
3.37	(b)	3.38	(a)	3.39	(d)	3.40	(c)	3.41	(c)	3.42	(b)	3.43	(d)	3.44	(d)	3.45	(d)
3.46	(c)	3.47	(a)	3.48	(a)	3.49	(a)	3.50	(a)	3.51	(c)	3.52	(d)	3.53	(d)	3.54	(a)
3.55	(b)	3.56	(b)	3.57	(b)	3.58	(c)	3.59	(a)	3.60	(c)	3.61	(a)				

Explanations : TCP, UDP and IP

3.1 (d)

[illegible]

Choice (a): [illegible] computers have multiple IP addresses. Multihoming is the [illegible] computer [illegible] network. In IP address must change [illegible] IP address for computer [illegible] the host computer [illegible] changed.

Choice (b): IP packets from the same source to the same destination may take different paths in the network. In packet switching network [illegible] different routing algorithms [illegible] time.

Choice (c): IP guarantees that a packet is discarded if it is [illegible] that it will arrive [illegible]

Choice (d): IP [illegible] packet [illegible] source [illegible] the [illegible]

[illegible] only in the routing tables in the [illegible]. The name [illegible] is the [illegible] machines on internet routing to [illegible] each machine (computer or router) [illegible] machines. [illegible] Device in [illegible] and [illegible] the [illegible] computer and [illegible] destinations with [illegible] IP address in the [illegible] table.

8.2 (d)

This managed protocol provides [illegible] connectivity that enables network layer protocols from the data link layer [illegible] networks. A strategy protocol can be either [illegible] such as TCP or [illegible] TCP.

8.3 (d)

The subnet mask is
255.255.[illegible].0
The given IP address belongs to class B network. [illegible] the physical network. The third octet is used to [illegible] the local physical network connection and the [illegible] address of class B network is 128.[illegible].0.0 and the highest address is 191.255.[illegible]. So IP address 191.[illegible] belongs to a valid network.

8.4 (b)

Choice (b) is correct.
A bridge operates at layer 2 (Data Link Layer) so it uses MAC addresses [illegible] layer 3 (Network Layer) [illegible].

8.5 (d)

Given in the question. For TCP does not provide [illegible] guarantee a minimum transmission [illegible] is not permitted to transmit at any rate to middle, the actual rate is regulated by IP. [illegible] and, [illegible] for sender to send at a low average rate.
Hence statement (d) [illegible] all the statements are false.

8.6 (d)

Since you have 8 subnet bits [illegible] Hosts.

8.7 (d)

TCP requires [illegible] byte in the stack. Assuming [illegible] number to [illegible] byte [illegible] a single byte packet does exist. The second packet will have a sequence number equal to [illegible] sequence number plus the number of bytes in the first packet.

8.8 (d)

Destination Port number is 5001 present in TP header.
Because the TP header has [illegible] the port number.
Port numbers are used by the transport layer to ensure process to process delivery.

8.9 (c)

In the given the header is only given the [illegible] 1200 and 900 is not a multiple of 8 so we have to add 4 bits in order to make it a multiple of 8.
Now the new payload is 1208
8 × 20 the size packet will be divided into 400, 400, 400 with each having 20 header and 200 will be divided into 400, 400 and 1208 bits added each having 20 IP header.
So total overhead = 60 × 3 = 180 B

8.10 (b)

In slow start phase, increase 2 MSS, but is not increase the current congestion window by Maximum Segment Size (MSS).
In the question it is given a packet size of 2000 so we see that in the MSS.
So after the 2 RTT the current congestion window = 4000 + 2000 = 6000 B

8.11 (a)

Solution for IP address 128.13.13.19
10000000.00001101.00001011.00010011
11111111.11111111.11111111.00000000
10000000.00001101.00001011.00000000
[illegible]
128.13.[illegible].0
255.255.255.255
128.13.[illegible].255
But number of Hosts [illegible] is more than 2550 so router will [illegible].

8.12 (d)

Network A has a maximum packet size of [illegible] bytes. Network B having the maximum packet size [illegible] times as large as network A. Since the process combines incoming packets from network A to G

So the packet size

$= 100 + 100 = 60 = 980$ bytes

8.13 (b)

[illegible]

8.14 (b)

TCP and UDP are [illegible] protocols. TCP and UDP are [illegible] protocols. Following protocols are [illegible] in connection [illegible] and UDP are [illegible]

8.15 (d)

The sliding window protocol [illegible] allows [illegible] to send [illegible] packets [illegible] before receiving [illegible] acknowledgement. [illegible]

8.16 (c)

[illegible] degree [illegible] 8 bits of [illegible]

8.17 60

Let [illegible] be the source station and [illegible] be the destination station [illegible] and [illegible]

[illegible]

[illegible] bytes [illegible] the [illegible]

[illegible] = [illegible] = 1 [illegible] 8 bytes header [illegible] 8 bytes header [illegible] Consider the [illegible] choices

(a) Packet size = 6 then [illegible] size

$= 6 - 8 = -2$ bytes [illegible]

(b) Packet size = 7 [illegible] size

$= 7 - 8 = -1$ bytes [illegible]

(d) [illegible] size

$= 9 - 8 = 1$ [illegible] 1 [illegible]

[illegible]

8.18 (b)

[illegible]

Frame size $= 2T_p \times$ (Bandwidth)

$= 8 \times 10^{-6} \times 10$ Mbps

$= 80$ kbits

8.19 (b)

[illegible]

8.20 (a)

[illegible] = [illegible] + [illegible]

[illegible]

8.21 (d)

[illegible]

[illegible]

[illegible]

[illegible]

Subnet mask for 2nd subnet and 8th subnet

[illegible]

Subnet address for 8th subnet is [illegible] **and 3rd subnet is** [illegible]

335. [illegible]

[illegible]

336. [illegible]

[illegible]

337. [illegible]

[illegible]

338. [illegible]

[illegible]

339. [illegible]

[illegible]

340. [illegible]

[illegible]

341. [illegible]

[illegible]

342. [illegible]

[illegible]

3.29 (c)

The two computers C_1 and C_2 are connected as follows:

	C_1	C_2
Hardware Port Number	[illegible]	[illegible]
	[illegible]	[illegible]
	[illegible]	[illegible]
	[illegible]	[illegible]
	[illegible]	[illegible]
	[illegible]	[illegible]
	[illegible]	[illegible]

C_1 [illegible] C_2 [illegible] but C_2 [illegible] C_1 [illegible]

3.30 (c)

(i) [illegible] TCP [illegible] time [illegible] propagation delay [illegible] time depending on the traffic.

(ii) [illegible] propagation delay [illegible] to the [illegible].

[illegible]

3.31 (d)

In IPv6 fragment number is assigned to each unit by [illegible]

Number of bytes in fragment is

200 – [illegible] = 8 bytes

In UDP 20K [illegible] fragment is [illegible] which is 240 bits. [illegible]

[illegible]

[illegible] fragment is [illegible] connected, [illegible] with sequence number [illegible] byte of first segment [illegible] fragment [illegible].

3.32 (c)

The bits Bit Pattern is as follows

Maximum [illegible]

= 2[illegible] = 64 = [illegible]

Maximum [illegible]

[illegible] = 8[illegible] = [illegible]

3.33 (a)

[illegible]

3.34 (b)

Connection [illegible] in [illegible] packet.

3.35 (d)

[illegible] TCP [illegible] congestion control and flow control mechanism.

[illegible] flow control or congestion control mechanism [illegible] data [illegible]

[illegible] TCP [illegible] time. The [illegible]

[illegible]

The [illegible] flow control [illegible] window [illegible] flow control [illegible]

[illegible]

3.36 (a)

[illegible] SYN [illegible] SYN [illegible] SYN [illegible] state. [illegible] state.

[illegible] is [illegible] state.

The loss of SYN-ACK [illegible] will not allow the client [illegible] ESTABLISHED state.

[illegible]

3.37 (b)

[illegible] ARP [illegible]

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible] 011 [illegible]

So answer should be option (b).

8.38 (a):

[illegible]

8.39 (b):

[illegible]

8.40 (c):

[illegible]

8.41 (c):

[illegible]

8.42 (d):

[illegible]

8.43 (d):

[illegible]

8.44 (b):

[illegible]

3.46 (d)

[illegible]

3.47 (c)

[illegible]

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

3.48 (d)

[illegible]

$\Rightarrow$ [illegible]

[illegible]

3.49 (d)

Threshold = 5 MHz

Window size for [illegible]

Window size for [illegible]

Window size for [illegible]

[illegible]

[illegible]

Window size for [illegible] = 8 MHz

Window size for [illegible] = 4 MHz

[illegible]

Window size for [illegible] = 6 MHz

[illegible]

3.50 (d)

[illegible]

3.51 (d)

[illegible]

Payload : [illegible]

Fragment offset = [illegible]

[illegible]

[illegible]

[illegible]

3.52 (d)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

3.53 (c)

[illegible]

3.54 (d)

[illegible]

8.53 (d)

Bandwidth = 10 Mbps

Packet size = 1000 bytes

1st transmission: 1000 + 100 = 1100 bytes transmitted at a time from A

10 bits = 1 sec

1100 bytes ?

Transmission time = 1100 × 8 × 10⁻⁶ = 8.8 msec

From A, B1 and B2 it takes 8.8 + 8.8 + 8.8 = 26.4 msec to reach B

2nd transmission: 900 + 100 = 1000 bytes transmitted 10 times from A

10 bits = 1 sec

1000 bytes ?

Transmission time at A for 1000 bytes = 1000 × 8 × 10⁻⁶ = 8 msec

From A to B = 8 + 8 + 8 = 24 msec

Note: Transmission time at B1 and B2 when 1st packet is transmitted from A. It receives at 8.8 msec.

From A, B1 and B2 it takes 8 + 8 + 8 = 24 msec to reach B

3rd transmission: 800 + 100 = 900 bytes transmitted 20 times from A

10⁶ bits = 1 sec

900 bytes ?

Transmission time = 900 × 8 × 10⁻⁶ for 900 bytes = 7.2 × 8 × 10⁻⁶ = 7.2 msec

From A to destination = Time for entire packet = 7.2 × 3 = 21.6 msec.

Note: Time when entire 3rd transmission is over, 1st packet is transmitted from A, B receives at 7.2 msec.

From A, B1 and B2 it takes 7.2 + 7.2 + 7.2 = 21.6 msec to reach B.

T1 = 7.2 + 7.2 = 14.4 msec

T2 = 19.2 msec; T2 > T1

8.54 (d)

1. TTL check can not be done at the next hop router.
2. Check sum = 0 must be made at each hop as TTL changes.
3. Fragment offset may be computed again.
4. TTL check sum and fragment offset must be changed in the transmission.

8.55 (a)

[illegible]

[illegible]

[illegible]

Time of preformatted is [illegible]

[illegible] packet

8.56 (a)

[illegible] bits.

[illegible]

[illegible]

[illegible]

[illegible]

Number of packets will be [illegible]

Number of packets [illegible]

= [illegible] × 2000

We have to add [illegible] unique ID [illegible] number of [illegible] = 25.

[illegible]

$$= \left\lceil \frac{2^{[illegible]}}{[illegible]} \right\rceil = 262.14 \approx 262$$

8.57 (d)

MTU = 1500 bytes

Header length = 20 bytes

Data size = 4404 bytes

Fragment 1: 1480 + 20

Fragment 2: 1480 + 20

Fragment 3: 1444 + 20

[illegible] length in the IP fragments = [illegible]

MF bit = 1

[illegible]

8.58 (b)

Statement 1 is false because the seq. [illegible] number of the subsequent segment is equal to the number of bytes characters in the current segment.

Statement 2 is true because [illegible] is always [illegible] Delayed ACK.

Statement 3 false. Receiver window is used to give the sender an idea of how much free buffer space is available at the receiver.

Statement 4 is true. [illegible] is not [illegible] at the receiver buffer.

Hence when [illegible] and [illegible] The [illegible] would [illegible].

8.84 (b)

[illegible] modified by a router.

8.85 (a)

[illegible] Amount Used Based.

8.86 (c)

Maximum [illegible] amount of data. [illegible] in IPTT.

$$\therefore \text{IPTT} = \frac{[illegible]}{[illegible]} = [illegible]$$

Following [illegible] Option (c) is correct.

8.87 (a)

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

8.88 (a)

Over case in [illegible] = [illegible] bytes

[illegible] = [illegible] bytes

[illegible] = [illegible] bytes

We need to break [illegible] fragments.

Ethernet frame size [illegible] = [illegible] = [illegible] packets [illegible]

1st fragment [illegible] (1480 bytes)

2nd fragment [illegible] (1480 bytes)

3rd fragment [illegible] (1480 bytes)

4th fragment [illegible] (1480 bytes)

5th fragment [illegible] (1480 bytes)

6th fragment [illegible] (1480 bytes)

7th fragment [illegible] (1480 bytes)

[illegible] Option (a) is correct.

8.89 (a)

TCP is [illegible] Hence [illegible] Cumulative Acknowledgment Hence [illegible].

TV Transmission [illegible]

[illegible]

8.90 Sol.

[illegible]

[illegible]

[illegible]

4 Routing, Application Layer and Network Security

4.1 Which one of the following statements is false?

(a) Packet switching results in better bandwidth utilization than circuit switching

(b) Packet switching results in less predictable delay than circuit switching

(c) Packet switching requires more sophisticated processing than circuit switching

(d) Packet switching can lead to congestion, unlike circuit switching

[2004 : 1 Mark]

4.2 A sender is to employ a public key cryptosystem to send a secret message to a receiver. Which one of the following statements is TRUE?

(a) Sender encrypts using his own public key

(b) Sender encrypts using his own private key

(c) Receiver decrypts using the sender's public key

(d) Receiver decrypts using his own public key

[2005 : 1 Mark]

4.3 Consider the protocols HTTP, HTTPS, FTP, SMTP, POP3 and IMAP. Which of the following statements is true?

(a) HTTPS uses TCP, port 443

(b) HTTP, HTTPS, SMTP, FTP

(c) HTTPS, HTTP, SMTP, POP3

(d) HTTPS, HTTP, FTP, IMAP

[2005 : 1 Mark]

4.4 The internet protocol suite provides a connectionless service by packets moving from source host to destination host. Which of the following options represents the fields used by routers to decide on the next hop?

(a) By looking only at the destination address in the packet and finding the longest matching prefix in the routing table associated with the destination address

(b) By looking only at the source address in the packet and finding the longest matching prefix in the routing table associated with the source address

(c) By looking only at the destination address in the ICMP reply packet returned to the sending host because the destination was unreachable

(d) By looking only at the source port in the TCP header of the packet

[2005 : 1 Mark]

4.5 Consider the following statements about the Internet Control Message Protocol (ICMP).

(i) ICMP is a routing protocol

(ii) ICMP is a transport layer protocol

(iii) ICMP is used to report errors in UDP messages

(iv) ICMP is used to perform congestion control

[2005 : 1 Mark]

4.6 Consider a DNS query message for the domain name www.iitb.ac.in. Which of the following statements is most appropriate, in a non-recursive query in a DNS query message?

Background: The DNS root name server knows the IP address of the name server for the top level domain "in", which knows the IP address of the name server for "ac.in", which knows the IP address of the name server for "iitb.ac.in", which knows the IP address of "www.iitb.ac.in".

(a) Query is sent to the root DNS server and then DNS server for "in" and then DNS server for "ac.in"

(b) Query is sent to DNS server for "in" and then DNS server for "ac.in"

(c) Query is sent to DNS server for "ac.in" and then DNS server for "iitb.ac.in"

(d) Query is sent to DNS server for "iitb.ac.in"

[2006 : 2 Marks]

4.7 Suppose that two nodes A and B each have a pair of public keys (PU, PR) and they have exchanged their public keys. Diffie-Hellman key exchange is used to establish a session key between A and B. The session key is used to encrypt all messages between A and B. Which of the following statements is true?

(a) A

(b) B

(c) C

(d) D

[2006 : 2 Marks]

4.8 [illegible] and [illegible] respectively belong to the protocols

(a) [illegible] and [illegible]

(b) [illegible] and [illegible]

(c) [illegible] and [illegible]

(d) [illegible] and [illegible]

[2006 : 2 Marks]

Linked Data for Q.4.9 and Q.4.10

Consider the diagram shown below where [illegible] LAN segments are connected by bridges. [illegible] packets are exchanged between the hosts. Assume that the bridges are self-learning type. First, the network topology is identified as the packets move through the bridges. Assume that [illegible] hosts are connected to each LAN segment. Assume that [illegible] possible. The receiving port for spanning tree of frames sent by each host is [illegible] bridges.

Each bridge identifies a port for forwarding through which it will forward frames to the root bridge. This port is identified in favour of the port with the lowest root value. When there is a possibility of multiple ports having the same LAN segment, the port with the lowest port number is preferred.

4.9 [illegible] connected [illegible] by bridges, which one of the following shows correct the [illegible] of the spanning tree of bridges?

(a) B1 – H6, B2 – H4

(b) B1 – H3, B2 – H5

(c) B1 – H5, B2 – H3

(d) B1 – H3, B2 – H4

[2006 : 2 Marks]

4.10 [illegible] spanning trees for the [illegible] ping packets. Which of the following options represents the correct forwarding tables?

(a)

Hosts	Port
1	2 H2, H4
[illegible]	H3, H10
H7	[illegible], H12

(b)

Hosts	Port
[illegible]	2
H3, H5	[illegible]
H4, H6	
H7, H8, H9, H10	[illegible]
H11, H12	[illegible]

(c)

Hosts	Port
H2, H4	[illegible]
H5, H6, H7, H8, H9	1
H1, H3	4
[illegible], H10, H11, H12	3

(d)

Hosts	Port
H2, H3, H4	2
H5, H6, H7, H8	1
H9, H10, H11, H12	4

[2006 : 2 Marks]

4.11 [illegible] following uses UDP as the transport protocol?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 1 Mark]

4.12 Consider the following two statements

(S1) A Boolean function of [illegible] variables can be realized by a multiplexer with [illegible] data inputs and [illegible] digital logic minimization techniques.

(S2) [illegible] techniques such as TCBS perform a permutation on the truth table of input variables.

Which one of the following options is valid for the above two statements?

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2007 : 1 Mark]

1.13 The minimum number of bits required to uniquely identify all 17 – 1 [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 1 Mark]

1.14 [illegible] is a commonly used public key cryptosystem. Which of the following statements is true? [illegible] computation of the [illegible] modulo n, 0 < b, n < m

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2007 : 1 Mark]

1.15 A firewall is a device configured to allow or deny access to a private network based on TCP connections and UDP packets. [illegible] it will simply allow or deny access to UDP packets on a [illegible] basis. [illegible] that are being sent to internal hosts. [illegible] TCP connections to internal hosts. [illegible] network. To achieve this, the firewall must be capable of [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible] with one state

(d) [illegible] with two states

[2007 : 1 Mark]

1.16 For the network given in the figure below, the routing tables of the nodes A, B, D and C are shown. Suppose that the link delay [illegible] 8, C, 2 and [illegible] respectively and updated routing tables are exchanged. [illegible]

Routing Table at A	
[illegible]	[illegible]
[illegible]	[illegible]
C	14
D	17
[illegible]	31
[illegible]	[illegible]
[illegible]	[illegible]

Routing Table at B	
A	24
B	2[illegible]
C	[illegible]
D	23
[illegible]	[illegible]
[illegible]	[illegible]
D	[illegible]

Routing Table at D	
A	[illegible]
B	[illegible]
C	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
G	[illegible]

Routing Table at C	
A	21
B	[illegible]
C	[illegible]
D	19
E	22
F	10
G	

(a)

[illegible]	[illegible]
[illegible]	[illegible]
C	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

A	[illegible]
B	[illegible]
C	[illegible]
D	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

(b)

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

A	[illegible]
B	[illegible]
C	[illegible]
D	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[2007 : 2 Marks]

1.17 A group of 10 routers are interconnected in a centralized complex topology with a number of [illegible] routers. [illegible] messages [illegible] the state of the [illegible] The root then sends the message back down to router 1. The mean number of hops per message, assuming all possible communication [illegible]

(a) [illegible] (b) [illegible]
(c) 4.55 (d) 3.5

[2007 : 2 Marks]

4.16 Consider the following statements

(a) [illegible] is suitable for client-server [illegible]
(b) [illegible] is a connectionless protocol
(c) [illegible] is the protocol [illegible]
(d) [illegible] suitable for [illegible]
(e) May need [illegible] for proper operation

The option that has the correct combination of correct statements

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2007 : 2 Marks]

4.17 Match the following

P.	SMTP	1.	Application layer
Q.	RIP	2.	Transport layer
R.	TCP	3.	Network layer
S.	PPP	4.	Data link layer
		5.	Physical layer

Codes

(a) P-2, Q-1, R-4, S-5 (b) P-1, Q-3, R-2, S-4
(c) P-1, Q-3, R-4, S-5 (d) P-2, Q-3, R-4, S-5

[2007 : 2 Marks]

4.18 Two popular routing protocols are Routing Information Protocol (RIP) and Link State Protocol (LSP). Which of the statements is/are true?

S1: Convergence is a problem only with DV and not with LS routing
S2: In LS, the shortest path calculation is run only at one node
S3: In DV, the shortest path calculation is run only at one node
S4: DV routing uses broadcast to send messages from LS

(a) S1, S2 and S4 only
(b) S2 and S3 only
(c) S1, S2 and S4 only
(d) S1 and S3 only

[2008 : 2 Marks]

4.19 The total number of hosts required for a class B subnet with a mask of 255.255.248.0 is

(a) [illegible] and 2n
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2008 : 2 Marks]

Linked Answer Questions 4.20 and 4.21

Consider a simple graph with [illegible] vertices. Each node maintains a [illegible] containing the next hop router [illegible] packets to [illegible] and the cost of the path to each destination. [illegible] initially [illegible] the routing tables [illegible] the distance vector routing protocol. [illegible] are performed.

(i) A node transmits a vector to its neighbours in the graph whenever its table is updated. Upon receiving a vector from a neighbour, a node updates its table. Upon receiving the vector, the node updates its table.

(ii) Each node transmits its vector to its neighbours at periodic intervals (independently of the other neighbours) the next hop.

(iii) Each node transmits its vector to its neighbours only if its table is updated, and the information in the vector is sufficient to update the table. Upon receiving a vector, the node updates its table.

4.20 Which of the following is the correct statement for routing protocols for vertices nodes after they have exchanged routing tables shown in the following figure? [illegible] is the source node.

4.21 Which of the following is the possible routing table for vertices node after they have exchanged routing tables shown in the following figure? [illegible] is the source node.

Table for node A

	A	–	–
	B	D	1
(a)	C	C	2
	D	D	3
	E	C	4
	F	C	5

Q.: Table for node C

A	A	1
B	B	1
C	-	
D	D	2
E	-	
F	F	3

(b): Table for node D

A	A	2
B	B	1
C	C	3
D		
E	E	1
F	F	1

(c): Table for node E

A	A	3
B	B	2
C	C	1
D	D	2
E		
F	F	3

[2005 : 8 Marks]

4.26 Consider using the link state routing algorithm in a network with 5 nodes. Assume that each link cost is 1. Let the routing table at node A be as shown below.

(a) 1	(b) 2
(c) 3	(d) 4

[2003 : 2 Marks]

4.27 A computer on a 10 Mbps network is regulated by a token bucket. The token bucket is filled at a rate of 2 Mbps. It is initially filled to capacity with 16 Megabits. The token bucket can hold at most 16 Megabits. What is the maximum time (in seconds) a host can transmit at the full speed without violating the token bucket?

(a) 1 seconds	(b) 2 seconds
(c) 3 seconds	(d) 4 seconds

[2008 : 2 Marks]

4.28 In the RSA public key cryptosystem, the public and private keys are functions of two distinct large prime numbers. Let M be an integer such that $0 < M < n$ where $n = pq$ and p and q are large primes. Let d be an integer such that $0 < d < (p-1)(q-1)$ and $ed = 1 \bmod (p-1)(q-1)$. Now consider the following statements:

I. $M^{ed} = M^{e} \bmod n$

II. $M^{ed} = M^{d} \bmod n$

III. $M^{ed} = M \bmod n$

IV. $M^{ed} = M^{e} \bmod n$

Which of the following statements is/are true?

(a) I and II	(b) I and III
(c) II and IV	(d) III and IV

[2009 : 2 Marks]

4.29 Which one of the following statements is true for a type 0 routing?

(a) Distance vector routing	(b) Link state routing
(c) Flooding	(d) None of these

[2010 : 1 Mark]

Linked Data Questions - Q.4.30 and Q.4.31

Consider a network with 6 routers R1 to R6 connected with links as shown in the following diagram.

4.30 All routers use the distance vector based routing algorithm to update their routing tables. Each router advertises its routing table initially with a cost of 1 to all its neighbours with its own cost of 0 to itself and cost of infinity to all other routers. After all the routing tables stabilize, how many entries in the routing table of router R1 will have non-infinite cost?

(a) 4	(b) 5
(c) 2	(d) 3

[2010 : 2 Marks]

4.31 Suppose the weights of all connected links in the previous question are changed to 2 and the distance vector routing algorithm is used to stabilize routing tables everywhere. How many link-state packets are required?

(a) 6	(b) 7
(c) 2	(d) 3

[2010 : 2 Marks]

4.80 Consider the following statements related to email [illegible]

S1: [illegible] email [illegible] secure

S2: [illegible] email [illegible] a mailboxed

S3: Checking email [illegible] web browser

(a) [illegible] TP [illegible] POP

(b) [illegible] TP [illegible] TP

(c) [illegible] TP [illegible] TP

(d) [illegible]

[2011 : 1 Mark]

Linked Data Question Q. 4.80 and Q. 4.81

Consider a network with five nodes, N1 to N5 as shown below.

The routing tables of the nodes N1 to N5 are shown below. Each table lists the distance to other nodes in the following format.

N1: (-, 1, 4, 3, 1)

N2: (1, 0, 6, 4, 3)

N3: (4, 6, 0, 2, 3)

N4: (3, 4, 2, 0, 4)

N5: (1, 3, 3, 4, 0)

Each table entry contains the distance to the best known path to the destination node, N1 to N5, where the distance to itself is '-'. Assume all links are symmetric and the cost is identical in both directions. In a distance vector routing protocol, nodes exchange their distance vectors with their respective neighbors. Then all nodes update their distance vectors. In the next iteration, any node may update its table if it finds a shorter path to any destination. Assume that the link between N1 and N2 goes down.

4.80 After the routing tables in the previous question, the node N1 and N2 need to exchange routing tables. The minimum number of iterations required to reach the stable routing state is _____ (up to two decimal places)

(a) 2 (b) 3

(c) 4 (d) 5

[2011 : 2 Marks]

4.82 The Protocol Data Unit (PDU) for the application layer in the Internet stack is

(a) Datagram (b) Segment

(c) Message (d) Frame

[2018 : 1 Mark]

4.83 The transport layer protocols used for real time multimedia, conferencing and streaming, SMS and mail respectively are

(a) TCP, UDP, TCP and TCP

(b) UDP, TCP, TCP and TCP

(c) UDP, TCP, UDP and TCP

(d) TCP, UDP, TCP and TCP

[2018 : 1 Mark]

4.84 Using public key cryptography, X sends a digital signature to Y on message M. Assume that H is a secure hash function and that encryption and decryption are carried out using the following sequence of steps in some order. For the signature to be valid, which of the following sequences of steps is sufficient for the verification?

(a) Decryption with X's public key followed by H with the received text. Decryption with X's public key followed by the received text

(b) Decryption with Y's private key followed by X's private key. Decryption with X's public key followed by Y's private key

(c) Decryption with X's public key followed by Y's private key. Decryption with Y's public key followed by X's private key

(d) Decryption with X's private key followed by Y's public key. Decryption with Y's private key followed by X's private key

[2018 : 1 Mark]

4.85 Assume that source and destination [illegible] connected through two intermediate routers as shown in Fig. 1. Determine how many times each packet has to be processed in the network layer and the data link layer in order to reach from S to D.

(a) Network layer – 2 times and Data link layer – 4 times
(b) Network layer – 3 times and Data link layer – 3 times
(c) Network layer – 4 times and Data link layer – 2 times
(d) Network layer – 2 times and Data link layer – 3 times

[2013 : 1 Mark]

4.86 Consider the following statements about link state routing protocols, for a topology with 200 routers and 4000 links.

S1: The link state routing protocol is higher than distance vector routing

S2: A link state routing protocol uses Dijkstra's algorithm to compute routing tables, and is not suitable for large networks.

S3: After a topology change, a link state routing protocol will converge faster than a distance vector routing protocol.

Which one of the following is correct about S1, S2, and S3?

(a) S1, S2 and S3 are all true.
(b) S1, S3 and S2 are false.
(c) S1 and S3 are true, but S2 is false
(d) S1 and S2 are true, but S3 is false.

[2014 (Set-1) : 1 Mark]

4.87 Which of the following is used to convert a message digest into a fixed length encrypted message?

(a) RSA (b) SHA-1
(c) DES (d) MD5

(e) C and D only (f) A and C only
(g) A and B only (h) B and D only

[2014 (Set-1) : 1 Mark]

4.88 Which one of the following is TRUE about intra-domain routing protocols RIP and OSPF?

(a) RIP uses distance vector routing and OSPF uses link state routing
(b) OSPF uses distance vector routing and RIP uses link state routing
(c) Both RIP and OSPF use link state routing
(d) Both RIP and OSPF use distance vector routing

[2014 (Set-3) : 1 Mark]

4.89 A IP datagram is to be sent from a host to another host in the same LAN. The three IP addresses are P1, P2 and P3.

$Q \quad R \quad S \quad P1 \quad P2 \quad P3$

Q is connected to the LAN and has two IP addresses. R is a router with two IP addresses, one for each LAN. S is a router with only one IP address. Which of the following fields of the IP datagram will never change while it is sent from P1 to P3?

(a) IP Identification
(b) Time to live
(c) Fragment offset
(d) Source IP address

Which of the IP header fields, and header checksum, will be modified when the datagram is forwarded by a router?

(a) Only TTL and Checksum (b) Only Checksum
(c) Only TTL and IP (d) Only Checksum and IP

[2014 (Set-2) : 2 Marks]

4.90 Consider the following protocols. Which of the protocols given below, uses multiple TCP connections between the same client and server? Which one is the correct option?

(a) HTTP 1.0, FTP (b) HTTP 1.1, FTP, TELNET
(c) FTP, SMTP (d) HTTP 1.1, HTTP 1.0, SMTP

[2014 (Set-1) : 1 Mark]

4.91 Suppose that we want to distribute a message to a group of N people where each person has a unique message that is destined for the other N-1 people. If we use public key cryptography, then the number of keys required to ensure that only the intended recipient can read each message is

(a) N (b) N-1
(c) 2(N-1) (d) N(N-1)

[2013 (Set-1) : 1 Mark]

4.92 Consider the Diffie-Hellman key exchange scheme. Let the pair of private and public keys for A and B be denoted by (a, A) and (b, B) respectively. Let q be a prime and α be a primitive element. Which one of the following is the correct expression for the shared secret key?

(a) $A^b \bmod q$ (b) $B^a \bmod q$
(c) $\alpha^{ab} \bmod q$ (d) $\alpha^{(a+b)} \bmod q$

[2013 (Set-2) : 1 Mark]

[illegible]

[20[illegible]-1] : 2 Marks]

4.41 [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2018 (Set-3) : 1 Mark]

4.42 Identify the correct sequence in which the following protocols are mentioned in the network layer by a web browser when a user requests a webpage from a remote server, assuming that the server has not been contacted before.

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2018 (Set-2) : 1 Mark]

4.43 [illegible]

[2016 (Set-2) : 2 Marks]

4.45 [illegible] "[illegible] i-ace [illegible]"

I. [illegible]

II. [illegible]

III. [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[2016 (Set-1) : 2 Marks]

■■■■

Answers: Routing, Application Layer and Network Security

4.1	(c)	4.2	(a)	4.3	(a)	4.4	(a)	4.5	(b)	4.6	(c)	4.7	(d)	4.8	(d)	4.9	(a)
4.10	(c)	4.11	(c)	4.12	(c)	4.13	(d)	4.14	(c)	4.15	(d)	4.16	(c)	4.17	(d)	4.18	(d)
4.19	(b)	4.20	(d)	4.21	(a)	4.22	(c)	4.23	(b)	4.24	(b)	4.25	(b)	4.26	(b)	4.27	(c)
4.28	(b)	4.29	(c)	4.30	(a)	4.31	(a)	4.32	(c)	4.33	(c)	4.34	(d)	4.35	(c)	4.36	(d)
4.37	(c)	4.38	(a)	4.39	(c)	4.40	(a)	4.41	(c)	4.42	(b)	4.43	(c)	4.44	(c)	4.45	(b)

Explanations: Routing, Application Layer and Network Security

4.3 (a)

In public key cryptography, sender uses receiver's public key for encryption and receiver uses his own private key for decryption. [illegible]

[illegible] sender encrypts using its own private key. Even receiver knows the sender can decrypt and only sender knows his private key [illegible]

4.4 (a)

[illegible]

4.8 (d)

RCPT: [illegible] MAIL: [illegible] DATA: [illegible]

Prompt: [illegible] prompting [illegible]

[illegible]

4.9 (a)

[illegible]

The first number is the type of the message. The number 8 in this case indicates that the packet is an ICMP echo request.
The sender sends an ICMP Time Exceeded message back to the sender.
The next set of packets are given a TTL value of 2 in the IP header. The second router sends back and replies with ICMP Time Exceeded.
Proceeding in this way, eventually a packet with a TTL value of 30 will reach the destination host and receive an ICMP Echo Reply message.

4.5 (b):

Initially the router sends ICMP redirect packets to the source host, which contains the IP address of the router, which is closer to the destination host. Thus, the host sends packets, thanks to the unreachable network in a hop away.
Meanwhile, the routing table has been updated. The next time, the host sends packets to the destination network, it sends it to a hop from the second router, which now is a hop away from the unreachable network.
The multicast routing in the mentioned RIP has been used. The unicast routing table is fully updated.

4.6 (c)

The aggregate sequence number is 4000 + 1000 = 5000.
Step 2: Query a Name server for the address of the host.
For such information, querying the IP address 145.125.16.34 is located at Amazon.com.
145.125.16.34 is the IP address.
Step 3: Send a TCP request to the server for establishing a connection.
The client sends a request to the IP address. The server responds with an acknowledgement. This forms the fully qualified domain name (FQDN) which is known as a URL. Lookup ends.

4.7 (d)

DHCP (Dynamic Host Configuration Protocol) is a network protocol that enables a server to automatically assign an IP address to an computer or other device from a defined range of numbers configured for a given network.

4.8 (d):

HELLO: Initiates a session with the mail server. When many [illegible] specific [illegible] done in [illegible] server receive who you are.
The DHCP messages [illegible] to establish a secondary connection (address and port) [illegible]

4.9 (a):

[illegible]

4.10 (d):

[illegible] speed.

4.11 (b):

Domain Name System (DNS) maps a name to an IP address, so a packet from a source calls a DNS server to translate which the destination. [illegible] the destination address. The [illegible] packet [illegible] DNS [illegible] to the same [illegible] IP address [illegible] which [illegible] DNS [illegible] DNS.

4.12 (d):

Joining both networks [illegible] the same [illegible] value.
[illegible] function is [illegible]

4.13 (d):

By [illegible] modulo 17 = 1.
8 × 9 = 16

4.14 (d):

[illegible]

4.15 (d):

A passive [illegible] a general purpose computer.
The [illegible] internal hosts and external hosts are bounded, and [illegible] [illegible] [illegible]

1.16 (b) :

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]					
[illegible]					
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Total [illegible] the given [illegible] only [illegible] [illegible] [illegible].

1.17 (d) :

Consider [illegible] :

Forward packet to the [illegible] at level 3: Total [illegible] required [illegible] [illegible] [illegible] required if D wants to communicate with [illegible] level 1 node

Hops required if D wants to communicate with all other nodes

$= (8 \times 1) + (4 \times 2) + (2 \times 3) + (1 \times 4) = 18$.

Total [illegible] communication with all other nodes then hops required $= 1.5 \times 18 = 27$.

Similarly, Hops required if D wants to communicate with [illegible] all other nodes [illegible]

$= 1 \times 4 + 2 \times 8 + 1 \times 2 = 26$.

Total hops required if communication with all other nodes then hops required $= 0.5 \times 1 = 0.5$

Hops required if D wants to communicate with all other nodes

$= 8 \times 4 + 4 \times 8 + 2 \times 12 + 1 \times 8 + 1 \times 4 = 47$.

Total hops required if communication with all other nodes then hops required $= 47 \times 1 = 47$.

Hops required if A wants to communicate with all other nodes $= 8 \times 1 + 4 \times 2 + 2 \times 3 + 1 \times 4 = 27$

Total hops required if we will consider [illegible] with all other nodes

$= 551 + 240 + 18 + 47 = 856$.

Total number of messages

$= 8 \times 7 = 56$ [illegible] $= 0.5 \times 56 = 28$.

[illegible] is [illegible] communication between A and B. A [illegible] message to D and D sends message to A.

Mean number of hops per message

$= 856/28 = 30.57$

1.18 (d) :

DHCP : [illegible] multicast [illegible] server.

FTP : Provides two ports 20 and 21 for FTP [illegible] and FTP [illegible].

HTTP : HTTP is a stateless protocol. [illegible] a connectionless protocol.

DNS : It is primarily [illegible] a distributed [illegible] a structured and [illegible].

SMTP : [illegible] and not suitable for client server [illegible].

1.19 (b) :

P. SMTP – Application Layer protocol

Q. DCP – Network Layer protocol

R. TCP – Transport layer protocol

S. PPP – Data Link layer protocol

1.20 (d) :

Count to infinity problem [illegible] distance vector routing.

Theory S1 is true

In link state routing [illegible] to share updates [illegible].

S2 and S4 are both false [illegible] algorithm [illegible] to be executed [illegible] each and every node

1.21 (d) :

For [illegible] graph [illegible] individual [illegible] will be required.

If an individual wants to communicate [illegible] individuals he should have [illegible] keys. [illegible]

[illegible] key encryption [illegible] individual [illegible] have a public and private key, the total keys required is $2n$.

4.34 (b)

Data transfer rate = [illegible] = 10 Mbps
Initially fixed time slot = 15 Microsec
Maximum duration = [illegible] seconds

4.35 (b)

[illegible] equations [illegible] RSA [illegible]

4.36 (b)

[illegible] application. [illegible] authentication required [illegible]

4.37 (c)

Routing table

Node F

Destination	Next hop	Distance
R2	[illegible]	R8
R1	[illegible]	R1
R4	12	R8
R5	7	R1
R6	15	R6

Node R8

Destination	Next hop	Distance
R2	[illegible]	R8
R1	8	R1
R4	7	R4
R5	6	R5
R6	12	R6

Node R5

Destination	Next hop	Distance
R1	5	R1
R2	2	R2
R1	8	R1
R4	9	R4
R6	10	R6

Node R7

Destination	Next hop	Distance
R1	R2	R2
R2	[illegible]	R2
R1	8	R8
R4	[illegible]	R4
R6	[illegible]	R6

Node R4

Destination	Next hop	Distance
R1	R2	R2
R2	5	R5
R1	8	R1
R5	[illegible]	R5
R6	1	R6

Node R6

Destination	Next hop	Distance
R1	R6	R6
R2	12	R6
R1	13	R5
R4	[illegible]	R4
R5	[illegible]	R5

So [illegible] routing table [illegible] between [illegible] and [illegible]

So [illegible] packet [illegible]

4.38 (b)

As in the previous question

So we get [illegible] apply.
So [illegible] only [illegible] R5-R8 [illegible]

4.39 (c)

SMTP is typically used by mail [illegible] providing mailbox. POP is used by client [illegible] is a simple HTTP [illegible].

4.40 (a)

In the [illegible] every node of [illegible] and updating distance vector. N will receive [illegible] to N1 and N3 and 0 [illegible] respectively.

4.41 (d)

[illegible] will [illegible] N3 to N1 [illegible]
So it will update [illegible] + 3 = 10.

4.42 (b)

Message → Application layer
Segment → Transport layer
Datagram → Network layer
Frame → Data link layer

4.43 (b)

UDP: Transport layer protocol which is unreliable [illegible]
TCP: Transport layer connection oriented protocol which is reliable but [illegible]
So [illegible] multiple [illegible] processing [illegible] suitable for [illegible]

[illegible]

4.34 (d):

[illegible] message must be network [illegible] be encrypted by a public key.

[illegible]

[illegible]

[illegible]

4.35 (c):

In the layered architecture sender and receiver [illegible] layer of the network layer.

[illegible] layer is [illegible] times [illegible] layer is [illegible] times.

4.36 (b):

S1 is TRUE: [illegible] protocol is high because of link state [illegible].

S2 is FALSE: [illegible] protocol avoids [illegible] routing loops.

S3 is TRUE: [illegible] convergence because of link state [illegible] network. This information is known to the source by the [illegible].

4.37 (a):

RIP-1 and RIP-2 are used to provide a [illegible] space.

4.38 (b):

[illegible]

[illegible]

4.39 (c):

Given [illegible] and [illegible] routers.

[illegible] introduce an error in [illegible] address of [illegible].

4.40 (a):

[illegible] connections between the same [illegible] and the [illegible] used with two separate TCP connections.

4.41 (c):

Every pair of users need a separate key. Hence [illegible] pairs are required for N people.

$$\text{[illegible]} = \frac{N(N-1)}{2} \text{ [illegible] required.}$$

4.42 (d):

In digital signature, [illegible] encrypted with the sender's private key [illegible] the receiver's public key [illegible] the [illegible] K_{pu} (sender).

4.43 (a):

Verification or authentication can be done by encrypting sender's private key and decrypting at receiver with [illegible] public key.

4.44 (c):

IDS: [illegible], TCP SYN, ICMP, UDP, TCP-UDP [illegible].

4.45 (c):

[illegible] mol/L = (0.1 mol/L) [illegible]

= [illegible] mol/L

= [illegible] mol/L

= [illegible] mol/L

= 1

4.46 (b):

[illegible] used for [illegible] but not for collision detection.

■■■■

Unit - XI

General Aptitude

UNIT XI General Aptitude

Syllabus :

Verbal Ability: English grammar, sentence completion, verbal analogy, word groups, critical reasoning and verbal deduction.

Numerical Ability: Numerical computation, numerical estimation, numerical reasoning and data interpretation.

Analysis of Previous GATE Papers

Exam Year	2010	2011	2012	2013	2014 Set 1	2014 Set 2	2014 Set 3	2015 Set 1	2015 Set 2	2015 Set 3	2016 Set 1	2016 Set 2
1 Mark Ques.	5	5	5	5	5	5	5	5	5	5	5	5
2 Mark Ques.	5	5	5	5	5	5	5	5	5	5	5	5
Total Marks	15	15	15	15	15	15	15	15	15	15	15	15

General Aptitude

1.1 Which of the following options is the closest in meaning to the word below:

Circumscribe

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2010: 1 Mark]

1.2 The question below consists of a pair of words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

Unemployed : Worker

(a) Driver : Road

(b) Teacher : School

(c) [illegible] : [illegible]

(d) Patient : Doctor

[2010: 1 Mark]

1.3 Choose the most appropriate word from the options given below to complete the following sentence.

If we manage to ..... our natural resources, we would leave a better planet for our children.

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2010: 1 Mark]

1.4 Choose the most appropriate word from the options given below to complete the following sentence.

His rather casual comments on politics ..... his lack of seriousness about the subject.

(a) [illegible] (b) belied

(c) [illegible] (d) [illegible]

[2010: 1 Mark]

1.5 Tagore scored in a cricket match 15 runs less than Gandhi and 10 runs more than Nehru. Gandhi and Nehru scored 10 runs each. The difference between the scores of Tagore and Nehru is

(a) 8 (b) 17

(c) 10 (d) 4

2010: 1 Mark]

1.6 Modern science has enabled man to travel from large cities to the moon in a matter of hours. Population explosion, coupled with the exploitation of natural resources, has led to the depletion of the earth's resources.

Which of the following statements best sums up the meaning of the above passage?

(a) Modern science has made life easier for man.

(b) Chemical sciences are needed to conserve resources.

(c) Use of science in everyday life would be uneconomic.

(d) Dangers in using science should be taken into account.

[2010: 2 Marks]

1.7 If 137 + 273 = 410 then which one of the following is true?

(a) 1.31 (b) 1.43

(c) 1.03 (d) 1.01

[2010: 2 Marks]

1.8 A machine produces 10 parts in 2 minutes. Each part has 20 machined surfaces. It takes 1 minute to mount a part on the machine. There are 5 machines, 6 operators, 3 helpers and 6 machined surfaces must be ready at the end of 1 hour. How many parts will be produced in 1 hour?

(a) 60 (b) 50

(c) 30 (d) 25

[2010: 2 Marks]

1.9 Given a set {2, 3, 7, 5, 9, [illegible]} how many distinct pairs of numbers can be added to it so that the median of the set remains 5?

(a) 0 (b) 1

(c) 2 (d) 3

[2010: 2 Marks]

1.10 Ram (R), Shyam (S), Jadu (J) and Madhu (M) are siblings. Ram is taller than Madhu. Jadu is the tallest. Madhu is taller than Shyam. Jadu is shorter than Ram. Shyam is the shortest among the four. Given the following facts, [illegible]

[illegible] Hatchback + Sedan + Station wagon + Sports car

3. The only difference between Sedan and Station wagon is that Sedan has a boot and Station wagon has a rear door.

4. The number of seats.

Which of the above are the correct differences?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2012 : 2 Marks]

1.11 Which of the following options is the closest in meaning to the word below:

Inexplicable

(a) Inexpressible (b) Inflexible

(c) Inestimable (d) Indelible

[2011 : 1 Mark]

1.12 If $\log(P) = \frac{1}{2}\log(Q) + \frac{1}{3}\log(R)$, then which of the following is TRUE?

(a) $P^2 = QR^3$ (b) $Q = P^2R$

(c) $P^3 = QR^2$ (d) $P = Q^2R$

[2011 : 1 Mark]

1.13 Choose the most appropriate word from the options given below to complete the following sentence.

Inconceivable Singapore for any vacation hotspot that is not within it.

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2011 : 1 Mark]

1.14 Choose the most appropriate word from the options given below to complete the following sentence.

If you are trying to make a strong impression on your audience, you cannot afford to be understated, certainly not

(a) [illegible] (b) [illegible]

(c) [illegible] (d) indifferent

[2011 : 1 Mark]

1.15 Choose the word from the options given below that is most nearly opposite in meaning to the given word:

Amalgamate

(a) [illegible] (b) split

(c) collapse (d) segregate

[2011 : 1 Mark]

1.16 Two schools of thought include a method how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.

Based on the above passage, which one of the following statements is most logically consistent?

(a) [illegible]

(b) what emotional [illegible] through the loss process

(c) what the leading [illegible]

(d) [illegible]

[2011 : 2 Marks]

1.17 P, Q, R and S are four types of dangerous microbes. [illegible] The area of each circle represents the diameter of the bacteria. [illegible] the dangerous microbes [illegible] the figure below.

A pharmaceutical company is developing the development of a vaccine against the most dangerous microbe. Which vaccine should be [illegible] to be most effective?

(a) P (b) Q

(c) R (d) S

[2011 : 2 Marks]

1.18 The variable cost (V) of manufacturing a product varies according to the equation $V = dq$, where q is the quantity produced. The fixed cost (F) of production of the same product is [illegible] according to the equation $F = (b/q)$. The [illegible] cost (V + F)?

(a) 5 (b) 4

(c) 7 (d) 6

[2011 : 2 Marks]

1.19 A company receives orders for some product for five consecutive days. Orders for the product are processed in batches of 5 units each. If an order is received, then it will be processed at the end of the day. The maximum number of orders that can be processed in a day is 5. If more than 5 orders are received in a day, then all the orders are delayed to be processed on the following day. What is the minimum number of orders that will be delayed, if the orders are received in the following order for each of the 5 days?

(a) 1 (b) 3
(c) 6 (d) 7

[2011 : 2 Marks]

1.20 A tank initially contains 10 litres of pure water. From this container 1 litre is taken out and replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much pure milk is now left in the container?

(a) 7.29 litres (b) 7.00 litres
(c) 7.1 litres (d) 8.29 litres

[2011 : 2 Marks]

1.21 The cost function for a product in a firm is given by $5q^2$, where q is the amount of the product. The firm can sell the product at a fixed price of ₹ 50 per unit. The number of units to be produced by the firm such that profit is maximized is

(a) 5 (b) 10
(c) 15 (d) 25

[2018 : 1 Mark]

1.22 Choose the most appropriate alternative from the options given below to complete the following sentence:

The given several _____ chemical-storage guidelines in its attempt to resolve the conflict.

(a) interpret (b) set aside
(c) mesmerize (d) disregard

[2018 : 1 Mark]

1.23 Which one of the following options is the closest in meaning to the word given below?

Mitigate

(a) Diminish (b) Devalue
(c) Endure (d) Worsen

[2018 : 1 Mark]

1.24 Choose the grammatically INCORRECT sentence:

(a) They saw the house and liked it immediately.
(b) Hardly had I reached the station when it started raining.
(c) This is the only one of the books I have read from the reading list.
(d) Each of the students was asked to do a reading of Fifty Lakh papers, but later selected from less than.
(e) This month's issue of the journal contains a short and a very long article.

[2018 : 1 Mark]

1.25 Choose the most appropriate alternative from the options given below to complete the following sentence.

Sherlock's dog is the one _____ was sure to the encyclopedia.

(a) that (b) which
(c) who (d) whom

[2012 : 1 Mark]

1.26 Wanted. Temporary. Data entry operator for the period of 6 months. Interviews on Saturday, personal interviews and short-listed candidates communicated by telephone. Interviews will be held at Kendriya Vidyalaya, Sector-4, Bokaro. Interviews will be available for 2 days. Evenings and weekday work. Transportation paid, expenses reimbursed.

What inference can be drawn from the above text for the job seeker?

(a) Contact the commander
(b) Be punctual
(c) Not disappointed by employer's poor attitude
(d) Not punctual when it comes to pay

[2012 : 2 Marks]

1.27 A personal computer acquires data from a remote monitoring field. The profile of the search follows the equation $x = 2t - 6t^2$, where x is the height of the search module. The maximum possible height of the search module is

(a) 3 units (b) 6 units
(c) 12 units (d) 24 units

[2012 : 2 Marks]

1.28 An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are

[illegible] is reliable. Of the [illegible] are reliable. They [illegible] check absences, which are found to be reliable in [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2018 : 2 Marks]

1.29 Which of the following statements is CORRECT?

P: Adding 7 to each entry in a list leaves the mean of the list unchanged.

Q: Adding 7 to each entry in a list leaves the standard deviation of the list unchanged.

R: Doubling each entry in a list doubles the mean of the list.

S: Doubling each entry in a list leaves the standard deviation of the list unchanged.

(a) P, Q (b) Q, R
(c) P, R (d) R, S

[2017 : 2 Marks]

1.30 Given that [illegible] is true, which of the following is true?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2012 : 2 Marks]

1.31 Which one of the following words is the closest in meaning to the word given below?

Hostile

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2018 : 1 Mark]

1.32 Complete the [illegible]

Universal [illegible] is to [illegible] as [illegible] is to

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2018 : 1 Mark]

1.33 The [illegible] will be the maximum [illegible] 44, 48, 40, [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2014 : 1 Mark]

1.34 [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2013 : 1 Mark]

1.35 Choose the grammatically INCORRECT sentence.

(a) [illegible]
(b) They [illegible] to Africa.
(c) [illegible]
(d) They might [illegible] to Australia.

[2014 : 1 Mark]

1.36 Find the value of the expression

$$\frac{1}{\sqrt{8+\sqrt{8+\cdots}}}$$

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2013 : 2 Marks]

1.37 [illegible] between [illegible] and [illegible] is selected randomly. What is the probability that the selected number is not divisible by [illegible]?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2013 : 2 Marks]

1.38 [illegible] spider [illegible] spider [illegible] spider [illegible] spider [illegible] failure. [illegible] spider [illegible] spider [illegible] spider [illegible]

Which one of the following statements is best supported by the above information?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2013 : 2 Marks]

1.39 A [illegible] of the journey is [illegible] [illegible] and [illegible] kinds. The average speed of the [illegible] during the [illegible] of the journey is

(c) 30 (d) 54
(e) 81 (f) 18

[2013 : 2 Marks]

1.40 The current population of a city is 1 million in the year 1990. If the population increases by 10% in the current year and 5% in the next year, the population in the current year is _____ and the population in the next year is _____

(a) 18 lakh (b) 16.5 lakh
(c) 11.55 (d) 10.125

[2013 : 2 Marks]

1.41 Which of the following options is closest in meaning to the phrase underlined in the sentence below?

It is the training to see the large image with minute visual observations.

(a) subtle (b) simple
(c) subtle (d) complex

[2013 (Set-1) : 1 Mark]

1.42 Choose the most appropriate word from the options given below to complete the following sentence.

It would be understated to say that scientific knowledge has grown exponentially. Such a phenomenon is the result of an inquiry _____.

(a) driven (b) motivated
(c) motivated (d) enhancing

[2013 (Set-1) : 1 Mark]

1.43 In a pre-industrial society, farmers formulated solid "theories" based on their observations. What did the anti-colonists by the subaltern?

(a) The war will be money
(b) Have the time [illegible]
(c) [illegible]
(d) [illegible]

[2013 (Set-1) : 1 Mark]

1.44 [illegible]

[2013 (Set-1) : 2 Marks]

1.45 The speed of a [illegible] and goal is [illegible] km/h. [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2013 (Set-1) : 1 Mark]

1.46 The [illegible] is a [illegible] km wide in the southern part of the Western Ghats in India. [illegible] The [illegible] in the [illegible] Tamil Nadu and [illegible] from [illegible] South West monsoon [illegible] higher [illegible]

What can be inferred from this passage?

(a) The [illegible] to high [illegible] and high temperature [illegible] Kodaikanal, Kerala
(b) The [illegible] Tamil Nadu and Kerala that are [illegible] the Palghat [illegible]
(c) The [illegible] the Palghat [illegible] significantly impacted [illegible] the eastern part of Tamil Nadu and Kerala
(d) Higher [illegible] temperature results in higher rainfall near the Palghat Gap area

[2013 (Set-1) : 2 Marks]

1.47 Consider a city [illegible] very [illegible] in the [illegible] the [illegible] of pollution [illegible] has depreciated [illegible] and [illegible] quality. [illegible] will be able to [illegible] the [illegible] early [illegible]

Consider the following assumptions [illegible] statement [illegible]

(a) [illegible] now available [illegible] during [illegible] illnesses
(b) [illegible] will [illegible]
(c) All [illegible] can be turned back [illegible] how they are expressed
(d) In the future, [illegible] will become [illegible] field for [illegible] psychiatric illnesses

[2013 (Set-1) : 2 Marks]

1.48 [illegible] of 10 [illegible] in a [illegible] of 4 [illegible] fare [illegible] Rs. 100 [illegible] will be charged Rs. _____

[2013 (Set-1) : 2 Marks]

1.49 In a survey of 300 students, it was found that they owned books in Hindi or English or both. The number of students who owned only Hindi books was twice the number who owned only English books. The number of students who owned both Hindi and English books was 40. The number of students who owned only Hindi books and the number of students who owned only English books are tabulated below. What percentage of students do not own any of these books?

Only Hindi	80	40
Only English	40	20
Both	40	40
Neither	20	20

[2014 (Set-1) : 2 Marks]

1.50 Water pipes are to be laid to connect a village with four other villages. A survey commissioned by a village has estimated the costs (in lakhs of rupees) of laying pipes between the villages as shown in the following table. Assume that the costs are proportional to the distance between the villages. How many pipes should be laid to connect all the villages with the least cost? _____

[2014 (Set 1) : 2 Marks]

1.51 Choose the most appropriate option from the given options to complete the following sentence.

(a) ... was a former British colony.
(b) Indian investment in Bangladesh positively influences economic growth.
(c) Trade deficit between two countries ...
(d) Income has helped other countries gain freedom.

[2014 (Set 1) : 1 Mark]

1.52 We ... were coming to see us this evening.

(a) good (b) this evening
(c) ... (d) had good

[2014 (Set 2) : 1 Mark]

1.53 Match the columns.

Column 1	Column 2
(P) ...	(1) micro-processor
(Q) diskette	(2) ...
(R) ...	(3) ...
(S) ...	(4) ...

Codes:

(a) P-3, Q-1, R-2, S-4
(b) P-2, Q-4, R-1, S-3
(c) P-1, Q-3, R-2, S-4
(d) P-3, Q-1, R-4, S-2

[2014 (Set 2) : 1 Mark]

1.54 What is the ... Wall implemented in China in 1982?

(a) ... (b) 140
(c) 160 (d) 170

[2014 (Set-2) : 1 Mark]

1.55 The value of $\sqrt{\ldots}$ is

(a) 0.484 (b) 0.8
(c) 0.09 (d) 4.44

[2014 (Set-2) : 1 Mark]

1.56 The territory of Poland was invaded by Germany and subjected to occupation before World War 2. Poland lost 20% of its population during the events of the war. The population was 35 million at the end of the war. The population was 18 million at the end of the war. Russian population in the Russian sector of Poland and the number of Poland in the south and west and Ukraine in the east perspective. Which of the statements below can be inferred from the passage?

(a) Kaliningrad was ceded to the Russian Federation as an enclave.
(b) Technique of ... Russian capital ... Russian ...
(c) Kaliningrad was annexed. Kaliningrad ... Russian ...
(d) Poland and Ukraine ... on the route from ... to the ... of Russia.

[2014 (Set-2) : 2 Marks]

1.57 The ... people diagnosed with dengue in ... in India in ... increased from ... in ... to ... in ... Municipal authorities have ... that ... the ... in the region.

Which one of the following statements, if true, would most ... the ...?

(a) Actual population ... affected population has returned from ... dengue is prevalent.
(b) More cases of dengue are now reported because of an increase in the ... efficiency.
(c) Many more cases of dengue are being diagnosed because ... and better ...
(d) The number of people with ... from ... dengue ...

[2014 (Set 2) : 3 Marks]

1.55 If x and y satisfy $|x - 2y - 3| = 11$, the possible value of $x - 2y$ is/are

(a) 8 & 2 (b) 14

(c) 4, 28 (d) [illegible]

[2014 (Set-2) : 2 Marks]

1.58 The ratio of male to female students in a college is 5 : 4. If 10% increase in the number of female students and 20% increase in the number of male students are expected, what is the percentage increase in the total number of students?

[2014 (Set-2) : 2 Marks]

1.59 [illegible] the minute hand of a clock moves at angle 360° in 60 min, the angle between the hour and minute hands of a clock at 6:45 is

(a) [illegible] (b) 6:27 a.m.

(c) [illegible] (d) 6:45 a.m.

[2014 (Set-3) : 2 Marks]

1.61 "While [illegible] 111 [illegible] and [illegible] 110 [illegible] 101 [illegible]."

The number by which the sentences 2001 appropriate is

(a) [illegible] (b) 1

(c) [illegible] (d) 16

[2015 (Set-1) : 1 Mark]

1.62 [illegible] how to arbitrate the [illegible] dome for [illegible]

(a) known, [illegible]

(b) [illegible]

(c) had [illegible] have

(d) should have known, would have

[2014 (Set-3) : 1 Mark]

1.65 Choose the word that is most nearly opposite in the meaning to 'obscure'.

(a) [illegible] (b) well [illegible]

(c) [illegible] (d) [illegible]

[2014 (Set-3) : 1 Mark]

1.66 Which number [illegible] in the series below?

[illegible] 10, 1, 26, 5, 50, 64

(a) 27 (b) 87

(c) 64 (d) 26

2014 (Set-4) : 1 Mark]

1.67 The table below has question wise data on the performance of students in an examination. In the examination, questions are classified. There is no negative or partial marking in the examination.

Q. No.	Marks	Correct Solutions	Attempted (Average)	Not Attempted
1	1	21	17	6
2	2	8	27	5
3	3	31	18	9

What is the average of the marks obtained by the class in the examination?

(a) 2.34 (b) [illegible]

(c) 4.03 (d) [illegible]

[2014 (Set-4) : 1 Mark]

1.68 A dance [illegible] participating in the programme and [illegible] from the [illegible] be assigned to [illegible] students and [illegible] the program [illegible]. The information you [illegible] is appropriate to choose

(a) [illegible] should come at 9.00 a.m. and [illegible] should come at 10.00 a.m.

(b) Participants [illegible] should come at 9.00 a.m. accompanied by a parent and other students and [illegible] should come by 10.00 a.m.

(c) [illegible] should come at 9.00 a.m. and they should accompany the parents. Participating students should come at 9.00 a.m.

(d) [illegible] should come before 8.00 a.m. Those who accompany them should come at 9.00 a.m. and others should come at 10.00 a.m.

[2014 (Set-4) : 2 Marks]

1.65 In the beginning of the [illegible] hypothesis was [illegible] a parallel growth [illegible] understanding of the [illegible] relationship. However, the [illegible] was [illegible] experimental [illegible] [illegible] system. Without [illegible] [illegible] [illegible] [illegible]

(i) [illegible] of the universe [illegible]

(ii) [illegible]

(iii) [illegible] important [illegible] for universe

(iv) Experimental evidence was important in developing this process [illegible]

(a) (i), (ii) and (iv) (b) (i) only

(c) (i), (iii) (d) (iv) only

[2014 (Set-II): 2 Marks]

1.66 The Gross Domestic Product (GDP) in Rajasthan grew at [illegible] during 2017-2018. An [illegible] component of the GDP is composed of US Dollar (USD) [illegible] based on the [illegible] exchange rate. During the year 2017-2018 the exchange rate for 1 USD [illegible] GDP in USD during the period 2013-2018

(a) increased by [illegible] than the rate of [illegible]

(b) decreased by [illegible] than the rate of [illegible]

[2014 (Set-II): 2 Marks]

1.67 There [illegible] in a village in [illegible] the following line graph [illegible] the number of people [illegible] 2018 is equal to [illegible]

(a) [illegible] (b) 2.1

(c) [illegible] (d) 2.5

[2014 (Set-II): 2 Marks]

1.68 Consider [illegible] $(x_1, x_2) = (x_1 + x_2)^2$, where (x_1, x_2) [illegible] X_1, X_2 [illegible]

(a) [illegible] (b) 17.7

(c) [illegible] (d) 5.32

[2014 (Set-II): 2 Marks]

1.71 Didn't you buy [illegible] shopping?

(a) any paper (b) [illegible] paper

(c) no paper (d) [illegible] paper

[2015 (Set-I): 1 Mark]

1.72 Which of the following options is the most [illegible] to the underlined [illegible]?

She enjoyed herself immensely at the party.

(a) She had a terrible time at the party

(b) She had a horrible time at the party

(c) She had a terrible time at the party

(d) She had a terrifying time at the party

[2015 (Set-I): 1 Mark]

1.73 Given $S_1 = \{1, 2, 3\}$ and $S_2 = \{1, 2, 3, 4, 5\}$, [illegible] relations from S_1 to S_2. Next to the cycle in line [illegible] the number of relations equals 1?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2015 (Set-I): 1 Mark]

1.74 Read the following statements and [illegible] more appropriate option [illegible] [illegible] is a common [illegible] How many steps are there in [illegible] the [illegible] from the [illegible] the building?

Statements:

1. [illegible] high.
2. [illegible] wide.

(a) Statement 1 alone is sufficient, but statement 2 alone is not sufficient

(b) Statement 2 alone is sufficient, but statement 1 alone is not sufficient

(c) Both statements together are sufficient, but neither statement alone is sufficient

(d) Statement 1 and 2 together are not sufficient

[2015 (Set-I): 1 Mark]

1.75 Which one of the following [illegible] is incorrect?

(a) [illegible] Swimmer

(b) [illegible] Tennis

(c) [illegible]

(d) [illegible] Kurukshetra

[2016 (Set-I): 1 Mark]

1.56 The number of students in a class who have answered correctly, wrongly, or not attempted each of the questions in an exam are listed in the table below. The marks for correct, wrong and not attempted are 5, -1, 0 respectively. There is no negative marking.

Question	Wrong	Correct	Attempted	Not attempted
1	4	21	17	3
2	3	7	27	4
3	1	17	29	4
4	2	22	10	3
5	3	37	12	1

What is the average score of the students in the class on this exam?

(a) 2.850 (b) 2.975

(c) 3.125 (d) 3.375

[2016 (Set-1) : 2 Marks]

1.57 Select the characteristic most appropriate to describe a good leadership.

The chief minister's focus is on his/her level-headed approach to problem solving.

(a) Ability of a person in a political position

(b) Ability to make a quick decision

(c) Ability to focus

(d) Ability to effectively manage

[2016 (Set-1) : 2 Marks]

1.58 The given statement is followed by some assumptions. Assuming the statement to be true, decide the assumption(s) to be taken.

Statement: There has been a significant drop in the water level in the lake supplying water to the city.

Assumptions:

1. The water supply authority should impose a partial cut in supply to ensure water availability.
2. The government should appeal to all the residents through mass media to use minimum water.
3. The government should cut the water supply to lower areas.

(a) Statement 1 and 2 follow

(b) Statement 1 and 3 follow

(c) Statement 2 and 3 follow

(d) All statements follow

[2015 (Set-1) : 2 Marks]

1.59 The pie chart below has the breakup of the number of students from different departments in an engineering college for the year 2012. The ratio of male to female students in each department is 5 : 1. There are 60 students in Electrical Engineering. What is the difference between the number of male students in the Civil Engineering and the female students in the Mechanical Engineering?

[2016 (Set-1) : 2 Marks]

1.60 The probabilities that a student passes in Mathematics, Physics and Chemistry are p, q and r respectively. It is known that the probability that the student fails in Mathematics is the same as the probability of passing in all three and the probability of passing in exactly two is the same as the probability of passing in Physics. The following relations are known to hold between p, q and r.

1. $p = qr$
2. $p + q + r = 1$
3. $pqr = 0.24$

(a) Only relation 1 is true

(b) Only relation 2 is true

(c) Relations 1 and 2 are true

(d) Relations 1 and 3 are true

[2015 (Set-1) : 2 Marks]

1.61 We _____ our friends' birthday and we _____ how to make it up to him.

(a) forgot ... don't know

(b) forgot ... don't know

(c) forgot ... don't know

(d) forgot completely ... don't know

[2016 (Set-3) : 1 Mark]

1.62 Choose the statement where underlined word is used correctly.

(a) The indicator is at half a percentage point

(b) I will be my experience in engineering relatively

(c) All personnel are based on the division

(d) Being religious is a personal aspect

[2016 (Set-3) : 1 Mark]

1.83 Consider a function $x(t) = 1 - |t|$, $-1 \le t \le 1$. The value of t at which the function attains maximum and the maximum value of the function are

(a) 0, 1 (b) 1, 0

(c) 0, 1 (d) 1, 2

[2016 (Set-2) : 1 Mark]

1.84 A scientific term that means the science of clothing and how it is related to a person's lifestyle is

(a) home (b) fashion

(c) wear (d) apparel

[2016 (Set-2) : 1 Mark]

1.85 Based on the given statements, select the most appropriate option to solve the given question. What will be the total weight of gold in the alloy?

Statements:

1. Gold and silver in the alloy are in the ratio 5 : 3.
2. The total weight of these pieces is 120 kg more than the total weight of two pieces.

(a) Statement 1 alone is sufficient

(b) Statement 2 alone is sufficient

(c) Either 1 or 2 alone is sufficient

(d) Both statements 1 and 2 together are not sufficient

[2016 (Set-2) : 1 Mark]

1.86 If the letters of the word P, Q, R, S, T, U is an arithmetic sequence, which of the following are true in the same sequence?

1. P, R, S, U
2. P, Q, R, S, T, U
3. T, U, P, Q

(a) 1 only (b) 1 and 2

(c) 2 and 3 (d) 1 and 3

[2016 (Set-2) : 2 Marks]

1.87 Man in a boat is at a point marked at M. X. Distance P. M is 4 km. P is south of O at a distance of 8 km. N is east of O by 3 km. What is the distance between M and P in km?

(a) 6.54 (b) 8.74

(c) 2.46 (d) 5.66

[2016 (Set-2) : 2 Marks]

1.88 In a triangle PQR, PS is the angle bisector of $\angle QPR$ and $\angle QPS = \angle RPS$. What is the length of PS?

(a) $\frac{pq-r^2}{pr}$ (b) $\frac{pr}{(p+q)}$

(c) $\sqrt{pq(p+q+r)}$ (d) $\frac{(p+q)^2}{qr}$

[2016 (Set-2) : 2 Marks]

1.89 The $\oplus$ and $\otimes$ are defined as follows:

$p \oplus q = \max(p, q, r)$

$p \otimes q = \min(p, q, r)$

Input $x = \min(\max(p, q, r), \max(p, q, r))$, $y = \max(\min(p, q, r), \min(p, q, r))$

Also, $z = \min(\max(p, q, r), \max(p, q, r)) \oplus \max(\min(p, q, r), \min(p, q, r))$

Also the same operation is applied on the variable z and the variable z is the input of the form (p, q).

What is the value of $f(x, y, z)(0, 1, 2)$, $(1, 2, 0)$, $(2, 0, 1)$?

[2016 (Set-2) : 2 Marks]

1.90 Out of the following four sentences, select the most suitable sentence with respect to grammar and usage:

(a) Since a general lacked essential information, it was difficult to know

(b) The report was useless to them because there were no useful information in it

(c) Since the report did not contain any useful information, it was not worth reading

(d) Since the report lacked essential information, it would not had been useful to them

[2016 (Set-2) : 2 Marks]

1.91 If ROAD is written as UKDG, then BRAIN should be written as:

(a) YEIQ (b) YEDQ

(c) YFJP (d) [illegible]

[2016 (Set-2) : 1 Mark]

1.98 The Tamil version of _____ John Abraham's [illegible] _____ [illegible] with [illegible] last week. But Film's distributors _____ [illegible] the [illegible] release in Tamil Nadu [illegible] this Friday.

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[2016 (Set-B) : 1 Mark]

1.99 [illegible] syllabus and study [illegible] has become [illegible] Indian students [illegible] to the sequencing of [illegible].

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 (Set-B) : 1 Mark]

1.100 Select the pair that [illegible] is expressed in the pair

Chicken : Pedigree Dog

(a) Adidas : Orthopaedic
(b) [illegible] : Chemotherapy
(c) Walker : Nephrologist
(d) [illegible] : [illegible]

[2016 (Set-B) : 1 Mark]

1.101 [illegible] Find the value of [illegible].

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 (Set-B) : 1 Mark]

1.102 Alexander [illegible] India [illegible] Persia. Which one of the statements below is logically valid and can be inferred from the above sentence?

(a) Alexander would not have [illegible] India if India had been conquered by Persia.
(b) Alexander was ready to [illegible] India, and wanted to march to India.
(c) Alexander was completely [illegible] Persia and could [illegible] towards India.
(d) Since Alexander's kingdom was divided after the conquest of Persia, he was forced to move further.

[2016 (Set-B) : 2 Marks]

1.103 The [illegible] P, Q, R, S. [illegible] of [illegible], Power, Defense, Telecom, and Finance. Q does not [illegible] P [illegible] Q [illegible] Which is the right distribution of portfolios?

(a) P: Home, Q: Power, R: Defense, S: Telecom, T: Finance
(b) P: Home, Q: Power, R: Defense, S: Finance, T: Telecom
(c) P: Home, Q: Power, R: Defense, S: Telecom, T: Finance
(d) Q: Power, R: Defense, S: Finance, T: Telecom, T: Finance

[2016 (Set-B) : 2 Marks]

1.104 Choose the most appropriate equation for the [illegible] shown in the [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 (Set-B) : 2 Marks]

1.105 [illegible] of possessing all the essential skills required to be a [illegible] of the [illegible] order, [illegible] is [illegible] to be [illegible] lack of [illegible] [illegible]

[illegible] of the following [illegible] [illegible] Which [illegible] [illegible] the highest [illegible] [illegible]

Which of the statements, below, [illegible] logically valid [illegible] information from the above passage?

(a) [illegible] successful [illegible] the highest level
(b) [illegible] improvement [illegible] because [illegible]

(iii) The failure to make more of his wealth shows [illegible]

(iv) Improving his [illegible] skills will give him success

(a) (iii) and (iv) (b) (ii) and (iii)

(c) (i), (ii) and (iii) (d) (iv) only

[2015 (Set-4) : 2 Marks]

1.100 The [illegible] imports and exports of a country [illegible] 2000 to 2007 are given in the following bar chart. In which year [illegible] combined percentage increase in imports and exports was the highest?

[2015 (Set-2) : 2 Marks]

1.101 Out of the following four sentences, select the most suitable sentence with respect to common usage.

(a) I will be at your office until five minutes [illegible]

(b) I will be waiting for you in the office [illegible]

(c) I will be waiting for you in the office [illegible]

(d) I will be waiting for you in the office [illegible]

[2016 : 1 Mark, Set-1]

1.102 A crowd thing of something will never [illegible]

(a) precipitous (b) precious

(c) precedent (d) prevalent

[2016 : 1 Mark, Set-1]

1.103 Archimedes said, "Give me a lever long enough and a fulcrum on which to place it, and I will move the world." This sentence alone is an example of a _____ statement.

(a) [illegible] (b) [illegible]

(c) [illegible] (d) figurative

[2016 : 1 Mark, Set-4]

1.104 "Everything [illegible] machine." [illegible] which of the following could [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[2016 : 1 Mark, Set-4]

1.105 A [illegible] is [illegible] of [illegible] is removed from [illegible] of the other. The resulting [illegible] is [illegible] the body [illegible] is [illegible] of the same-sized [illegible]

(a) 68 (b) [illegible]

(c) 72 (d) 76

[2016 : 1 Mark, Set-1]

1.106 A [illegible] company [illegible] Electronics, Electronics [illegible] and [illegible] Electronics [illegible] Rs. 45 [illegible] Rs. 60, Rs. 50 and Rs. 70 and [illegible] Rs. 270 [illegible]. The [illegible] shows the [illegible] in each quarter of a year.

Quarter / Product	Electronics	Speech	Both	Neither
Q1	[illegible]	2000	1700	[illegible]
Q2	[illegible]	[illegible]	[illegible]	[illegible]
Q3	[illegible]	[illegible]	1000	1074
Q4	[illegible]	[illegible]	1600	[illegible]

Which product [illegible] the maximum [illegible] in that year?

(a) Electronics (b) Speech [illegible]

(c) Speech (d) [illegible]

[2016 : 2 Marks, Set-1]

1.107 In a currency notes show the denomination [illegible] of [illegible] in India [illegible] the country's [illegible]. Which of the following can be [illegible] from the above [illegible]?

(a) [illegible] is the country of [illegible] language

(b) Hindi is the [illegible] indicator of [illegible] diversity.

(c) [illegible] currency notes [illegible] the Indian [illegible]

(d) The [illegible] indicates [illegible] of [illegible] diversity.

[2016 : 2 Marks, Set-4]

1.108 Consider the following statements relating to the levels of polynomial time problems P, NP, NP-C, NP-H. [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 : 2 Marks, Set-II]

1.109 If $f(x) = [illegible]$, which of the following is true?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 : 2 Marks, Set-II]

1.110 In a process, the number of [illegible] is initially with an increasing load. At a load of 30 units, it takes [illegible] for a turn. When the load is reduced, it takes [illegible] to turn. When the load for a [illegible] will happen [illegible] 70 units is _____

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 : 2 Marks, Set-I]

1.111 The name [illegible] is now Microsoft. Gates worked as _____.

(a) the security guard at a university
(b) [illegible] at a university
(c) [illegible] guard at a university
(d) [illegible] at a university

[2016 : 1 Mark, Set-II]

1.112 Nobody's [illegible] how the [illegible] team is [illegible] the difficult [illegible] friendly [illegible]. Choose the option which is closest in meaning to the underlined phrase in the above sentence.

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 : 1 Mark, Set-II]

1.113 Find the odd [illegible] from the following group of words.

[illegible], decide, [illegible], [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 : 1 Mark, Set-II]

1.114 Pick the odd one from the following options.

(a) CONDEF (b) JDLACL
(c) [illegible] (d) CINTMQ

[2016 : 1 Mark, Set-II]

1.115 In a quadratic function, the [illegible] of the roots is [illegible]. Find the value of

$$\frac{\alpha^2 - \beta^2}{\alpha^2 - \beta^2}$$

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 : 1 Mark, Set-II]

1.116 Sandra, 150 faculty members in an institute 55 are [illegible] Facebook and 85 are [illegible] through WhatsApp. 30 faculty members [illegible] Facebook or WhatsApp accounts. The number of faculty members is [illegible] Facebook accounts is _____.

(a) [illegible] (b) 15
(c) [illegible] (d) [illegible]

[2016 : 2 Marks, Set-II]

1.117 Computers [illegible] computers in [illegible] ubiquitous. There is [illegible] is an unintended consequence of [illegible]. With the advent of more computing [illegible] are good or more important [illegible]

Which of the statements below are logically valid and can be inferred from the above [illegible]?

(i) [illegible] believe that computers [illegible]
(ii) [illegible] computers and the [illegible]

(a) (i) only (b) (ii) only
(c) [illegible] (i) and (ii) (d) neither (i) nor (ii)

[2016 : 2 Marks, Set-II]

1.118 [illegible] Choose the [illegible] sentence.

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[2016 : 1 Mark, Set-II]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
(e) [illegible]

[2016 : 2 Marks, Set-II]

1.119 In a [illegible] rectangle is [illegible] rectangle can be observed in [illegible]?

(a) 21 (b) 27
(c) 30 (d) 36

[2016 : 2 Marks, Set-II]

1.120

Choose the correct expression for $f(x)$ [illegible] the graph.

(a) $f(x) = 1 - |x|$
(b) $f(x) = 1 + |x - 1|$
(c) $f(x) = 2 - |x - 1|$
(d) $f(x) = 2 + |x - 1|$

[2016 : 2 Marks, Set-II]

Answers General Aptitude

1.1	(b)	1.2	(a)	1.3	(d)	1.4	(c)	1.5	(d)	1.6	(d)	1.7	(c)	1.8	(d)	1.9	(b)
1.10	(b)	1.11	(a)	1.12	(b)	1.13	(c)	1.14	(b)	1.15	(d)	1.16	(c)	1.17	(d)	1.18	(a)
1.19	(c)	1.20	(d)	1.21	(a)	1.22	(b)	1.23	(c)	1.24	(b)	1.25	(a)	1.26	(d)	1.27	(b)
1.28	(b)	1.29	(a)	1.30	(a)	1.31	(b)	1.32	(a)	1.33	(c)	1.34	(a)	1.35	(c)	1.36	(b)
1.37	(a)	1.38	(d)	1.39	(c)	1.40	(b)	1.41	(b)	1.42	(c)	1.43	(a)	1.44	(d)	1.45	(c)
1.47	(b)	1.48	(a)	1.49	(b)	1.50	(a)	1.51	(b)	1.52	(c)	1.53	(b)	1.54	(c)	1.55	(d)
1.56	(a)	1.57	(d)	1.58	(c)	1.59	(c)	1.60	(c)	1.61	(a)	1.62	(a)	1.63	(d)	1.64	(d)
1.65	(c)	1.66	(c)	1.67	(a)	1.68	(a)	1.69	(a)	1.70	(a)	1.71	(c)	1.72	(c)	1.73	(c)
1.74	(a)	1.75	(d)	1.76	(a)	1.77	(a)	1.78	(c)	1.79	(d)	1.80	(c)	1.81	(b)	1.82	(a)
1.83	(b)	1.84	(a)	1.85	(c)	1.86	(b)	1.87	(b)	1.88	(c)	1.89	(d)	1.90	(b)	1.91	(b)
1.92	(b)	1.93	(d)	1.94	(a)	1.95	(a)	1.96	(c)	1.97	(b)	1.98	(b)	1.99	(b)	1.100	(d)
1.101	(d)	1.102	(a)	1.103	(a)	1.104	(c)	1.105	(d)	1.106	(b)	1.107	(d)	1.108	(d)	1.109	(b)
1.110	(b)	1.111	(b)	1.112	(a)	1.113	(c)	1.114	(c)	1.115	(d)	1.116	(a)	1.117	(d)	1.118	(d)
1.119	(c)	1.120	(c)														

Explanations : General Aptitude

1.1 (b)

(a) Circumstance — Deviating from a straight course or path
(b) Cycle — Bicycle
(c) Indecent — Not according to what is fit or proper
(d) Circle — Triangle
(e) Circumference — Diameter

1.2 (d)

Unemployed — Worker — Hence — Employer
(a) Farmer — Land — Follow — Farmer
(b) Teacher — Student — Defeat — Teacher
(c) Wife — Husband — Sister — Brother
(d) Professor — Hence — Remove — Professor

1.3 (d)

(a) Diplomatic — Diplomatic — Appropriate
(b) Inactive — Active — Inactive — Not appropriate
(c) Conservative — Conservative — Appropriate
(d) Conservative — Conservative — Appropriate

1.4 (d)

(a) Mistake — Hide — Under a false pretence — Mistake
(b) Defined — Defined — Appropriate
(c) Revered — Revered — Appropriate
(d) Supposed — Supposed — Supposed — Appropriate

1.5 (d)

Using Venn diagram formula

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 15 + 17 - 6 = 26$

Hence, people who play cricket or hockey = 26

Alternative Method

Using the Venn diagram given below:

1.6 (d)

(a) Modern warfare has increased the scale of destruction. There is no direct consequence of warfare on civilian population.
(b) Chemical agents can be used in modern warfare. Damage done and the whole chemical system are useful as well as not useful.
(c) Conflicts are sometimes more destructive than wars. Conflicts are not necessarily more destructive than wars.
(d) Even a military establishment is not immune to change. Change is inevitable. Change is not necessarily detrimental to military people should that change be needed. Military establishment is necessarily subservient to warfare.

1.7 (b)

$[illegible] + [illegible] = [illegible]$

Therefore, total [illegible]

$[illegible] = [illegible]$

Alternative Method

[illegible] is determining the mean [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$\begin{array}{r} (781)_9 \\ +(212)_9 \\ \hline (1025)_9 \end{array}$$

Hence, the correct problem was based on nonsystematic base which was a nine-digit number on 9 base.

3.8 (a)

Per day work done by 6 skilled workers $= \frac{1}{20}$

$\Rightarrow$ Per day work done by one skilled worker $=$

$$\frac{1}{6 \times 20} = \frac{1}{120}$$

Similarly, per day work done by one semi-skilled worker

$$= \frac{1}{8 \times 30} = \frac{1}{240}$$

And per day work done by one unskilled worker

$$= \frac{1}{30}$$

$\Rightarrow$ Per day work done by two semi-skilled workers

$$= \frac{1}{120} + \frac{1}{240} = \frac{1}{80}$$

Thus, one day work done by skilled, 6 semi-skilled and 5 unskilled workers

$$= \frac{5}{120} + \frac{6}{240} + \frac{5}{30} = \frac{10 + 18 + 40}{240}$$

$$= \frac{68}{240} = \frac{17}{60}$$

Therefore, work completed in 3 days.

Alternative Method:

Let, no. of skilled, semi-skilled and unskilled workers be x, y, z respectively.

$20x = 30y = 40z = 240$ Total work done

$6x = 30y = 240$

$x = 8 \quad y = 8$

$\Rightarrow \quad z = \frac{240}{40} = 6$

Given that 5 skilled, 8 semi-skilled and 6 unskilled workers are working. Let they finish the work in t days.

$(5x + 8y + 6z)t = 240$

$\Rightarrow$ Total work done

$$\left[5 \times 4 + 8 \times 8 + 6 \times \frac{40}{3}\right]t = 240$$

$$\frac{520}{3}t = 240$$

$$t = 1.7 \text{ days}$$

3.9 (b)

We have to make 4 digit numbers, so the number should be even with 2 or 4 as the last digit possible.

Case (i): Thousands digit is 1

Now, only one digit is left, so leaving 1, 2, 3, 4, 5, 6, 7, 8, 9

(a) Using 1, 2, 3

$\Rightarrow$ 1123, 1213, 1312, 1132

$\left[\binom{3}{1} = 3 \text{ numbers are possible}\right]$

(b) Using 1, 2, 4 $\Rightarrow$ 1124, 1214, 1412, 1142

$\left[\binom{3}{1} = 3 \text{ numbers are possible}\right]$

(c) Using 1, 2, 6 $\Rightarrow$ 1126, 1216, 1612, 1162

$\left[\binom{3}{1} = 3 \text{ numbers are possible}\right]$

(d) Using 1, 3, 4 $\Rightarrow$ 1324, 1234, 1432, 1342

$\left[\binom{3}{1} = 3 \text{ numbers are possible}\right]$

(e) Using 1, 3, 6 $\Rightarrow$ 1336, 1316, 1613, 1363

$\left[\binom{3}{1} = 3 \text{ numbers are possible}\right]$

(f) Using 1, 4, 6 $\Rightarrow$ 1464, 1446, 1644, 1646

$\left[\binom{3}{1} = 3 \text{ numbers are possible}\right]$

(g) Using 1, 5, 6 $\Rightarrow$ 1556, 1565, 1655, 1656

$\left[\binom{3}{1} = 3 \text{ numbers are possible}\right]$

Therefore, total number of 4 digit numbers

$= 3 + 3 + 3 + 3 + 3 + 3 + 3 = 21$

Case (ii): [illegible] digit numbers. Now digits [illegible] digit numbers = any of 1, 2, 3, 4, 5, 6, 7, 8, 9

(v) Using 1, 2, 3 — 123, 132, 213, 231, 312, 321

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(vi) Using 1, 2, 4 — 124, 142, 214, 241, 412, 421

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(vii) Using 1, 2, 5 — 125, 152, 215, 251, 512, 521

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(viii) Using 1, 2, 6 — 126, 162, 216, 261, 612, 621

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(ix) Using 1, 2, 7 — 127, 172, 217, 271, 712, 721

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(x) Using 1, 2, 8 — 128, 182, 218, 281, 812, 821

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xi) Using 1, 2, 9 — 129, 192, 219, 291, 912, 921

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xii) Using 1, 3, 4 — 134, 143, 314, 341, 413, 431

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xiii) Using 1, 3, 5 — 135, 153, 315, 351, 513, 531

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xiv) Using 1, 3, 6 — 136, 163, 316, 361, 613, 631

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xv) Using 1, 3, 7 — 137, 173, 317, 371, 713, 731

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xvi) Using 1, 3, 8 — 138, 183, 318, 381, 813, 831

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xvii) Using 1, 3, 9 — 139, 193, 319, 391, 913, 931

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xviii) Using 1, 4, 5 — 145, 154, 415, 451, 514, 541

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xix) Using 1, 4, 6 — 146, 164, 416, 461, 614, 641

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xx) Using 1, 4, 7 — 147, 174, 417, 471, 714, 741

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxi) Using 1, 4, 8 — 148, 184, 418, 481, 814, 841

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxii) Using 1, 4, 9 — 149, 194, 419, 491, 914, 941

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxiii) Using 1, 5, 6 — 156, 165, 516, 561, 615, 651

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxiv) Using 1, 5, 7 — 157, 175, 517, 571, 715, 751

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxv) Using 1, 5, 8 — 158, 185, 518, 581, 815, 851

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxvi) Using 1, 5, 9 — 159, 195, 519, 591, 915, 951

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxvii) Using 1, 6, 7 — 167, 176, 617, 671, 716, 761

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxviii) Using 1, 6, 8 — 168, 186, 618, 681, 816, 861

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxix) Using 1, 6, 9 — 169, 196, 619, 691, 916, 961

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxx) Using 1, 7, 8 — 178, 187, 617, 671, 716, 761

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxxi) Using 1, 7, 9 — 179, 197, 719, 791, 917, 971

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

(xxxii) Using 1, 8, 9 — 189, 198, 819, 891, 918, 981

$$\left[\frac{3!}{(3-3)!} = 6 \text{ numbers are possible}\right]$$

Total 3 digit numbers in case (ii)

= 6 + 6 + 6 + 6 + 6 + 6

= 36 + 36 + 36 = 108

Thus total 4 digit numbers in case (i), (ii) and (iii)

= 23 + 108 = 131

Alternative Method / Short-cut method

As this number is greater than 4000, so [illegible]

Let's consider the 4 digit numbers

Case (i): When thousand's place is 5

[illegible]

[illegible] there are no restrictions on hundred's, ten's and unit's. These such [illegible] can be selected with repetition and [illegible]

So 5 × 9 × 9 = 405 numbers are there. Out of which 4000, [illegible] are invalid as 5 can be used twice. [illegible]

[illegible] = 9 × 9 = 81

Case (ii): When thousand's place is 6

[illegible]

Without restriction, number of 4 digit numbers [illegible] 27 ways.

Out of these 27 numbers, 4 [illegible] have [illegible] only

So, valid numbers are 27 − 1 = 26.

Total numbers from Case (i) & Case (ii) 25 + 26 = 51.

1.11 (a)

Suppose : Height of A : H, Color of eyes : C, Salary : S, Intelligence : I

- H = C + I = d
- Using Statement (i) both C and S are [illegible] S = 1, so it will be [illegible] for sure
- There are no [illegible] C = 1 for 8 possible
- (b) IIIC: not possible as there are [illegible] C and I [illegible] possible [illegible]
- (c) SCIII: [illegible] H = 1 and C = H + S = 1 possible. Because H = 1, C = H + 2 = 1 + 2 = 3, I = C − 2 = 3 − 2 = 1
- (d) HSCI: [illegible] C = 8 and S = 11 [illegible] H = 8 + 3 = 11 which is opposite of statement (i) which is not possible.
- (e) HSFC: [illegible] adding both we get [illegible] = 11 − 6 = 5 [illegible] statement (i) [illegible] possible

1.12 (b)

$$\log_P Q = \frac{(\log_R Q)(\log_P R)}{\log_R P}$$

$$Q = P^x$$

$$R = P^y$$

$$Q = R^z$$

$$\Rightarrow \quad P^x = R^z = P^{yz} = (P^y)^z = (P^z)^y = P^{zy}$$

1.17 (a)

As we know that the slope of the tangent to the curve represents the marginal cost. At the same time it will be inversely proportional to the marginal revenue. It will achieve the minimum acceptable level of frequency

$\propto \frac{P\uparrow}{P\downarrow}$ where, P_1 and P_2 are the prices

growth and revenue are defined by equation.

So $D_1 = \frac{100}{q}$ ...(i)

Where K is a constant of proportionality.

Suppose demand curve shifts to the right

by $D_2 = \frac{0.5 \times 100}{2.0}$.

Similar calculation for D_3, D_4, D_5 can be done

In case of (iii) Total revenue D_3 is maximum so

we must choose the price corresponding to this.

1.18 (b)

$TC = \text{Variable cost} + \text{Fixed cost}$

$$TC = 4q + \frac{100}{q}$$

As we have to minimize total cost

Using options

(a) $q = 6$, $TC = 4 \times 6 + \frac{100}{6} = 30$

(b) $q = 5$, $TC = 4 \times 4 + \frac{100}{4} = 41$

(c) $q = 7$, $TC = 4 \times 7 + \frac{100}{7} = \text{Least}$

(d) $q = 8$, $TC = 4 \times 8 + \frac{100}{8} = 40.66$

Hence, TC is minimum at $q = 7$ (c) ans.

Alternate Solution

$$TC = 4q + \frac{100}{q}$$

For minimum $\frac{d}{dq}(TC) = \frac{100}{q^2}\Big|_{q=7} = 0$

$\Rightarrow q^2 = 25$ or $q = 5$

$$\frac{d^2}{dq^2}(TC) = -\frac{200}{q^3}\Big|_{q=7} < 0$$

Hence TC (total cost) is minimum at $q = 7$ (c) ans.

1.19 (b)

Let us write the balancing equations of the

[illegible] reaction each [illegible]. So as per

conditions we can write

$4x + y = 20$ ...(i)

$10x + 2y = 30$ ...(ii)

Solving (i) and (ii),

$$x = \frac{5}{2} \text{ and } y = \frac{10}{2}$$

Since we need to find out number of moles so that corresponding order will be [illegible] [illegible] stoichiometry,

$C_x + y = 10 \times 6$

So we need to find C_x where C_x is the number of moles of oxygen

$$n = \frac{\text{Sol. Vol}}{\text{Molar Vol}} = \frac{5 \times 6}{2} = \frac{30}{2} = 15$$

Hence, 15 moles of O_2 are required. As [illegible] of [illegible] is natural [illegible] [illegible] moles will be [illegible].

1.20 (a)

Short cut Method

Every time if we replace 1 litre of mixture with 1 litre of water the number of [illegible] decreases by 10%.

So, final quantity of spirit after 3 such operations is

$= 10 \times 0.9 \times 0.9 \times 0.9 = 7.29$ litre

Alternate Solution

$$\frac{\text{Quantity of spirit left after 3 operations}}{\text{Total quantity of mixture}}$$

$$= \frac{10 - 1 - \frac{1}{10} - \frac{1}{10} - \frac{1}{10}}{10} = \frac{10 - 3 - \frac{3}{10}}{10}$$

[illegible] quantity of pure spirit in 10 L is quantity taken out and replaced every time. Hence, quantity of spirit left after 3 [illegible] [illegible]

[illegible]

1.21 (a)

Total Cost $C = 5q^2$

Total Sales Revenue

$S = 70q$

Profit $(\pi) = S - C = 70q - 5q^2$

As the profit function is a concave

$P = 70q - 5q^2$

Differentiation

(a) $q = 0$

$P = 70 \times 0 - 5 \times 0^2 = 0$

(b) $q = 10$

$P = 70 \times 10 - 5 \times 10^2 = 200$

(c) $q = 20$

$P = 70 \times 20 - 5 \times 20^2 = 200$

Hence the maximum profit will happen at $q = 10$

Alternative solution:

$$P = 70q - 5q^2$$

For maximum $\frac{dP}{dq} = 0$

$\Rightarrow 70 - 10q = 0$

$q = 10$

$\left.\frac{d^2P}{dq^2}\right|_{q=10} = -10$ which is negative

Hence maximum profit happens at

$q = 10$

1.22 (b)

Setbacks

Despite several setbacks the mission successful in its attempt to reach the moon.

[illegible]

Hence the word setbacks indicates that despite many problems the mission was successful

1.23 (a)

Diminish

Mitigate means to make something less severe or painful. Diminish means to reduce. Reduce means to make it smaller. Hence the given word

Thus the answer is diminish or reduce which are similar with the given word

1.24 (b)

[illegible]

The correct statements should be: The country's expenditure in [illegible] Bangladesh. [illegible] should be compared with Bangladesh's expenditure.

1.25 (a)

Sustainable development is the [illegible] development

[illegible]

1.26 (c)

Not enough information

Choice (a) is incorrect because it is one word and that of form [illegible] with the given argument

Choice (b) is wrong [illegible] profile [illegible] machine designed to make [illegible] effective [illegible] make the model [illegible] which are [illegible] of the government. Hence [illegible]

1.27 (b)

$$y = 2x - 0.1x^2$$

For y to be maximum $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 2 - 0.2x = 0$$

$\Rightarrow x = 10$

$$\left.\frac{d^2y}{dx^2}\right|_{x=10} = -0.2 < 0 \quad \left.\frac{d^2y}{dx^2}\right|_{x=0} > 0$$

[illegible]

$y = 2 \times 10 - 0.1 \times 10^2 = 10$

Maximum height = 10 meters

1.28 (b)

Total 100 black shirts are supplied

[illegible] = 60 (60% of 100)

[illegible] = 40 (40% of 100)

[illegible]

[illegible]

So the required probability [illegible]

Required probability [illegible]

$$= \frac{20.5}{[illegible] + 20.0} = 0.[illegible]$$

1.89 (c)

Adding constant to every observation by [illegible] increases the arithmetic mean by [illegible] and standard deviation [illegible] by constant multiplied to the mean [illegible] same constant. Hence the standard deviation will not show similarity.

Alternative Solution:

$$[illegible]$$

$$[illegible]$$

$$\text{and} \quad \frac{[illegible]}{6} = [illegible]$$

Hence [illegible]

1.90 (a)

ATCGPSCP is a prime term. The first letter on each term A, C, T, S are respectively 1, 3, 20, 19 values of alphabet Letters.

A = [illegible] letter of alphabetical sequence

C = [illegible] letter of alphabetical sequence

T = [illegible] letter of alphabetical sequence

[illegible]

[illegible] letter of alphabetical sequence. So next term should begin with G.

Also in alphabetical order, [illegible] letters in between them respectively. Hence next term should be GV as there should be a gap of [illegible] letters between G and V.

Hence answer is GV.

1.91 (b)

Lowest

[illegible]

1.92 (b)

Specificity

[illegible]

Differences [illegible] to specifying [illegible] [illegible]

1.93 (c)

[illegible]

Alternate:

Sum of first N [illegible] of Numbers

[illegible]

1.94 (a)

Would fly

[illegible]

1.95 (c)

She is an Engineer

[illegible] but the second [illegible] of the word. [illegible] with L which is a vowel but it does not [illegible]

So [illegible] correct sentence should be, 'She is a Ranger'.

1.96 (c)

The expression can be written as

$$\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \cdots \frac{1}{\sqrt{9}+\sqrt{80}}$$

Rationalisation [illegible] multiplying [illegible] with [illegible]

$$\frac{1}{\sqrt{2}+\sqrt{1}} \times \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \cdots$$

[illegible]

We have to be received in

[illegible]

[illegible]

Now [illegible] will be left with

[illegible]

1.37 [illegible]

[illegible] numbers between 1 and 100 [illegible] 10 [illegible]

[illegible]

[illegible] equal probability that the number [illegible] divisible by

[illegible]

1.38 [illegible]

No [illegible] justifies giving my hope.

Option [illegible] [illegible] concerning [illegible] relationship with the given [illegible].

Option [illegible] [illegible] [illegible] remember, [illegible] [illegible] [illegible] Europe with [illegible].

1.39 [illegible]

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{D}{\frac{D}{80} + \frac{D}{30} + \frac{D}{40}} = 24 \text{ km/h}$$

1.40 [illegible]

[illegible]

[illegible]

[illegible]

1.41 [illegible]

[illegible]

[illegible]

[illegible]

1.42 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

1.43 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[

Now following venn diagram can be drawn

So % of people not owing a scooter

$$\frac{[\text{illegible}]}{800} \times 100 = \frac{44}{800} \times 100 = 15.5$$

1.30 Sol:

Because satellite [illegible] will be [illegible].

1.31 (b):

There are 15 multiples of [illegible] from 2 to 198 typically [illegible]

Average of these [illegible] will be

$$= \frac{(2 + 20 + 30 + \cdots + 198)}{15}$$

$$= \frac{(2 + 3 + \cdots + 15)}{15}$$

$$= \frac{(15 \times 20)}{15 \times 2} = 10$$

1.32 (d):

$$\text{Time} = \sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \cdots}}}$$

$$\text{Let } y = \sqrt{2 \times \sqrt{2 \times \cdots}}$$

Squaring $y^2 = 2x$

$y^2 = 2x$

$\Rightarrow 2x = y^2$

$x = 1$

Option (d) is correct.

1.33 (d):

$$x^2 - 2x - 3 = 1$$

$$\Rightarrow x^2 - 2x - 3 = 7$$

$$\Rightarrow x^2 - 2x - 3 - 8 = 0$$

$$\Rightarrow x^2 - 4x + 2x - 4 = 0$$

$$\Rightarrow x = 2, -4$$

(i) Put $x = 1$ in $x^2 + x^3 - x$

$$\Rightarrow 1 + 1 + 1 - 1 = 2$$

(ii) Put $x = -2$ in $x^2 + x^3 - x$

$$\Rightarrow 4 - 8 + 2 = -2$$

Possible values are $-2, 2$

1.34 Sol:

Given ratio [illegible] = 2.5 in year 2008.

Number of students in 2008 = [illegible]

$$\text{Number} = \frac{M}{2.5} \text{ in } 2008 = 2.5$$

Now [illegible] in 2009

Number of [illegible] in 2009 = 2 × 200 = 400

$$\frac{M}{2.5} \text{ value in } 2009 \text{ is } 4$$

$$\frac{M}{400} = 4 \Rightarrow M = 1600$$

So number of [illegible] in 2009 is 1600

% increase in number of [illegible] in 2008 to 2009

$$\% \text{ increase} = \frac{1600 - 200}{200} \times 100 = 400\%$$

1.35 (a):

Angle made and corresponding time can be driven by

$$\theta = \frac{H}{5} \left| \frac{M}{11} \right| \times \frac{12}{11}$$

Between final Time [illegible] 30

H = 6 and M = 60

$$\theta = \frac{60}{5} \times \frac{5}{11} - 30 \times \frac{12}{11} = \frac{5}{11}$$

So at 6:21 $\frac{5}{11}$ angle between minutes and hour hand will be 60°.

In given question angle close to closer to 60° measurement gives is 5:00

1.36 (d):

Given series: 2, 5, 10, 17, 26, 37, 50, 65

$1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1, 5^2 + 1, 6^2 + 1, 7^2 + 1, 8^2 + 1$

$\Rightarrow 2, 5, 10, 17, 26, 37, 50, 65$

So (d) is correct option

1.37 (c):

Subject	Marks	Average Marks	Standard Deviation	Max Marks	Min Marks
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$$\text{Average} = \frac{\text{Total number of students}}{\text{Total number of lectures}}$$

$$= \frac{[illegible]}{44} = [illegible] = 9.76$$

1.68 (b):

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$\text{So, new GDP in 2012} = [illegible] \times \frac{[illegible]}{[illegible]} = [illegible]$$

[illegible]

[illegible]

1.69 (a):

Turnout male in Tamil Nadu 2011 is given.

[illegible]

[illegible]

Male turnout in 2012 [illegible]

$$\frac{M}{F} = [illegible]$$

$$= \frac{M}{F} = [illegible] \Rightarrow M = [illegible]$$

[illegible]

[illegible]

$$= \frac{[illegible]}{[illegible]} = [illegible]$$

1.70 (a):

$$CTAP = (V_1 - V_2)I_c$$

$$= [illegible] - [illegible]$$

$$= [illegible] - [illegible] = [illegible]$$

1.71 (a):

This is you [illegible] paper which you were dropping!

1.72 (a):

The employed herself is [illegible]

[illegible]

1.73 (b):

Total number of ways of selecting two numbers

$$= {}^{10}C_1 \times {}^{2}C_1 = 4 \times 5 = 20$$

Favourable outcomes are (2, 1), (2, 3), (4, 1), (4, 3)

$\therefore$ Probability $= \frac{4}{20} = 0.20$

1.74 (a):

Statement 1:

$$\text{Number of steps} = \frac{[illegible]}{[illegible]} = 22 \text{ steps}$$

$\therefore$ Statement 1 is alone sufficient.

Statement 2:

Only with two steps given which are [illegible]

$\therefore$ Option (a) is correct.

1.75 (a):

The [illegible] combination is

[illegible] = 8 [illegible]

1.76 (b):

Total [illegible] student = 44

Marks obtained by [illegible] 1 question correctly $= 2 \times 32 = 32$

From 2nd [illegible] answered correctly

$\Rightarrow 3 \times 8 = 16$

From 3rd question [illegible] correctly

$= 11 \times 1 = 11$

From 4th question [illegible] correctly

$= [illegible] \times [illegible] = 18$

From 5th question [illegible] correctly

$= 0 \times 21 = 150$

Total $= 230$

$\therefore$ Average marks $= \frac{230}{44} = 5.23$

1.77 (b):

The [illegible] are taught rather than by [illegible]

[illegible]

[illegible] as 'tools to Right'.

1.78 (a):

Only statement 1 and 3 follows.

1.79 (Bonus)

Let total number of students in Theory be T_t

Also, students in Theory = 60% of T_t = 0.60 × T_t = 0.60 × T_t = 0.60 × T_t

[illegible] Electrical students [illegible] 20% of [illegible]

Total = [illegible] × Total = [illegible]

Number of students in Civil

= [illegible] × 40% = [illegible]

Number of students in Mechanical

= [illegible] × 40% = [illegible]

Difference between the number of Mechanical students and Electrical students

= [illegible] − [illegible] = [illegible]

1.80 (Bonus)

[illegible]

[illegible] = [illegible] = [illegible] = 50%

[illegible] + [illegible] = [illegible] = 180%

∴ Statement 3 is correct.

[illegible] = 35

A ∩ Maths ∩ Physics ∩ Chemistry

= [illegible]

= [illegible]

= [illegible]

∴ Statement 1 is correct.

1.81 (c)

We completely forgot [illegible] and we just don't know how to make it [illegible]

1.82 (c)

All persons are brown the day [illegible]

1.83 (c)

[illegible]

[illegible]

[illegible]

Substituting in the given function we get

[illegible]

[illegible]

Which is same as

[illegible]

[illegible]

The graphical [illegible] is shown below

From the graph we can easily see that the function attains the maximum at x = 0 and the maximum value of the function is 1.

1.84 (Bonus)

[illegible]

1.85 (a)

Statement [illegible] Total [illegible] weight of 1 [illegible] = 3 kg ⇒ Weight of 1 [illegible] = [illegible] kg

∴ Statement 1 is also [illegible]

Weight of 10 [illegible] = [illegible] = 30 kg

⇒ Weight of [illegible] = 60 kg

⇒ Weight of [illegible] = 20 kg

∴ Statement 2 is also sufficient.

1.86 (b)

Adding by [illegible], subtracting by [illegible], and multiplying by [illegible] constant to the Arithmetic sequence does not change the property of arithmetic progression.

1.87 (b)

$\frac{1}{x}$ = [illegible]; $\frac{1}{y}$ = [illegible]

$x = \frac{1}{[illegible]}$ $y = \frac{1}{[illegible]}$

[illegible]

1.83 [illegible]

Area of $\Delta PQS = \frac{1}{2} \times 5 \times 5 \times \sin 60°$

Area of $\Delta PRS = \frac{1}{2} \times 5 \times 5 \times \sin 60°$

Area of $\Delta PQR = \frac{3}{2} \times 5 \times 5 \times \frac{\sqrt{3}}{2}$ (i)

Also $\Delta PQR = \frac{1}{2} \times 10 \times 5 \times \sin 120°$ (ii)

Equating (i) and (ii) $\Delta PQR = \frac{25\sqrt{3}}{4}$

1.84 [illegible]

$\min\{2, 6, 7, 8\} = \log_{[illegible]} [illegible]$

$(x \times y) > (y \times z)$

Therefore $\min\{2, 6, 7, 8\}$ [illegible]

$\frac{7 \times 5}{2 \times 5} = 1$

Now $f_2(1, 4, 3, 8) = f(1, 4, 6, 8) \times f(1, 4, 6, 8)$

$= 3 \times 1 = 3$

1.85 [illegible]

Since the [illegible] $P_{[illegible]} =$

1.86 [illegible]

$K \quad O \quad S \quad E$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$L \quad M \quad T \quad G$

Replace [illegible] alphabet [illegible]

Therefore [illegible]

1.87 [illegible]

[illegible] Board [illegible] found no takers among the [illegible] for a release [illegible] on this Friday.

1.88 [illegible]

[illegible] students [illegible] they [illegible] requirements of [illegible].

1.89 [illegible]

[illegible]

1.90 [illegible]

[illegible]

$f(x) = [illegible] + 2x - 9 = 29$

$f(3) = [illegible] + 6 - 9 = 30$

$\Rightarrow [illegible] = 20$

$\Rightarrow [illegible] = 2$

$\Rightarrow [illegible] = 30$

$f(x) = [illegible] = [illegible] = (30) - 2 \times 3 = 54 - 6$

1.91 [illegible]

[illegible]

The [illegible] statement is true [illegible]

[illegible] Over the [illegible] Parvin, [illegible] move further.

1.92 [illegible]

P: House, S: Power, T: Defence, Q: Economy, R: [illegible]

1.93 [illegible]

From the given [illegible] $x = [illegible]$, $y = -1$

Put [illegible] each of the four options [illegible].

Option (a) $2 = 1 \quad 1 = 2$ (False)

Option (b) $2 = [illegible] = 2$ (True)

Option (c) $2 = 1 + [illegible] = 0$ (False)

Option (d) $2 = [illegible] = 0$ (False)

[illegible]

1.100 (b)

The following statements are logically valid and can be inferred from the given passage.

(i) [illegible] improve his performance in order to succeed in [illegible].

(iii) He failed to make more of his potential [illegible].

1.101 (a)

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Thus, the conditional statement is true in 2000.

1.102 (b)

Original number of [illegible] = 8 (49 – 35)

Two [illegible] are removed. Since [illegible] are removed which will [illegible] original [illegible] will remain unchanged. [illegible]

1.103 (b)

Elegance:

(27%) [illegible]

Excellence:

[illegible]

Smooth:

[illegible]

Soft:

[illegible]

Which is [illegible].

1.104 (d)

[illegible] conditions [illegible] following inference can be drawn.

[illegible] Q

Q always beats R

[illegible] R [illegible]

Thus, attention [illegible] performing between performance is

[illegible]

[illegible] condition [illegible] beats [illegible]

[illegible] times. [illegible]

[illegible] performing [illegible]

[illegible] performance [illegible] performance [illegible]

[illegible]

1.105 (b)

$f(x) = ax^2 + bx + c$

Use option (a). If $f(x) = 0$ will be true then on putting $x = 1$ [illegible] $b = 0$ [illegible].

$f(1) = 2(1)^2 - 3 = 0 - 5 = -3 \neq 0$

1.106 (d)

Statements [illegible] A, B, C only (C) doesn't follow this pattern.

Hence statement not [illegible].

1.107 (d)

$$\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2-\beta^2} = \frac{\alpha^2-\beta^2}{\alpha^2-\beta^2} = 2\alpha^2$$

Given $\alpha \times \beta = 4$

Hence $2\alpha^2 = 8$

1.108 (a)

Reference venn diagram can be drawn as

[illegible]

So only [illegible].

1.109 (c)

[illegible]

[illegible]

As whenever any two horizontal and any two vertical [illegible] will [illegible].

■■■■

Conventional Questions & Solutions

■ **Contents**

Sl.	Topic	Page No.
1.	Engineering Mathematics	56
2.	Theory of Computation	600
3.	Digital Logic	608
4.	Computer Organization & Architecture	611
5.	Programming & Data Structures	615
6.	Algorithms	623
7.	Compiler Design	639
8.	Operating System	650

Part I Engineering Mathematics

1. Mathematical Logic

Q.1 [illegible] to show that the following argument is incorrect:

[illegible]

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

[illegible]

[1992 : 5 Marks]

Solution:

Given that

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(iv) [illegible]

From (i) [illegible]

[illegible] (v)

From (v), by universal specification

[illegible] ...(vi)

From (iii) and (iv) [illegible] disjunctive syllogism

[illegible] (vii)

From (vi) and (vii) [illegible] by modus ponens

[illegible] ...(viii)

From (ii) and (viii) [illegible] disjunctive syllogism.

[illegible] (ix)

However, (vi) and (ix) contradict each other

[illegible]

Q.2 Show that proposition C is a logical consequence of the formula

[illegible]

[illegible]

[1993 : 5 Marks]

Solution:

| [illegible] | | |

Solution:

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

2. Graph Theory

Q.5 How many minimum spanning trees does the following graph have? Draw them. (Weights are assigned to the edges.)

[1995 : 5 Marks]

Solution:

Using Kruskal's algorithm

Accepted edges: [illegible]

Now [illegible] edges with unit weight [illegible]

Consider the two different minimum spanning trees.

(i) [illegible]

(ii) [illegible]

Q.6 [illegible]

(i) Which of the following sets are independent?

(ii) What is the chromatic number of this graph?

(iii) Does this graph have a Hamiltonian cycle? [illegible]

[1998 : 5 Marks]

Solution:

(i) [illegible]

Also [illegible]

(ii) [illegible]

Also [illegible]

(b) [illegible] is a simple graph. The given graph contains [illegible] distinct vertices. So, [illegible] minimum cardinality of minimal vertex cover in given graph should be [illegible] than

Because only the vertices $\{B, D, E\}$, has simple [illegible] minimal vertex cover.

Therefore, the cardinality of minimal vertex cover = [illegible].

(c) [illegible] minimum cardinality is, [illegible] degree of [illegible] vertex is k.

Try Handshaking lemma:

Sum of all degrees of vertices

$= 2 \times$ [illegible]

$= 2k$ [illegible]

$\therefore$ G has minimum [illegible] edges.

3. Set Theory and Algebra

Q.7 (a) Let G be a group of order 6. Show that G has a subgroup of order 3. Justify your answer.

(b) Consider the set of integers {1, 2, 3, 4, 5, 6, 7, 8, 9} together with the two binary operations LCM (lowest common multiple) and GCD (greatest common divisor).

Which of the following algebraic structures does this set form?

(i) group (ii) ring (iii) field (iv) cyclic group

Justify your answer.

[1995 : 6 Marks]

Solution:

Let G be a group of order 6. Assume G has a subgroup of order 3. Let H be a proper subgroup of G. Then for [illegible] $\in G$, [illegible] $\in H$. Then [illegible] $\in H$. Since H is a proper subgroup, [illegible] $\in G$, which has contradiction. Therefore, H contains only 3 elements in G.

So it cannot be an element of order 6. Let [illegible] be an element of G.

So [illegible] $\Rightarrow$ [illegible]

$\Rightarrow$ [illegible]

$\Rightarrow$ [illegible]

Therefore, there exists an element [illegible] such that [illegible]

(b) Let The given set of integers is a finite set. Let [illegible] be the set of integers. Let [illegible] be the set of integers. Let [illegible] be the set of integers.

Q.8 Let S be the set of all integers and let [illegible] be a multiple of n. Show that S is an equivalence relation and find its equivalence classes.

[1998 : 6 Marks]

Solution:

Given for [illegible], $a \equiv b \pmod{n}$ if a is a multiple of n.

$a \equiv b \pmod{n}$ $\Rightarrow$ $a - b = nk$

where n is a fixed and k is an integer.

(i) Relation R is reflexive

$(a, a) = (a - a) = 0 \in S$

So, R is reflexive relation. Reflexive.

(ii) Relation R is Symmetric

If $a \equiv b \pmod{n}$ [illegible]

Then $a - b = nk$ [illegible] $b - a = -nk$ is also solved

Therefore, relation is symmetric.

(iii) Transitive

If $a \equiv b \pmod{n}$ and

$a - b = nk_1$ where k_1 and k_2 are integers.

$b - c = nk_2$

Then adding relations we get

$a - c = n(k_1 + k_2)$

$a - c = nk$ where $k = k_1 + k_2$

Therefore, if $a \equiv b$ and $b \equiv c$ then $a \equiv c$.

Since relation R is reflexive, symmetric and transitive, it is equivalence relation. This relation R is also known as "congruent modulo n".

The equivalence classes for n = 3 are

$[0]_3 = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$

$[1]_3 = \{\ldots, -5, -2, 1, 4, 7, \ldots\}$

$[2]_3 = \{\ldots, -4, -1, 2, 5, 8, 11, \ldots\}$

$[3]_3 = \{\ldots, -3, 0, 3, 6, 9, 12, \ldots\}$

$[4]_3 = \{\ldots, -2, 1, 4, 7, 10, 13, \ldots\}$

Q.9 Let G be a group, p be a prime, and p^n be the order of G.

(a) $g^{p^n} = e$ for all $g \in G$. Show that G is a [illegible]

(b) $g^{p^n} = e$ for all $g \in G$. Show that G is a [illegible]

(c) $g^{p^n} = e$ for all $g \in G$. Show that G is a [illegible]

[1998 : 6 Marks]

Solution:

Given, $p^2 = p = e$

$(pq)^2 = e$ (equipotent)

Thus, it is abelian and satisfies associative operation.

(a) $p^{-1}q^{-1}q^{-1}p$

$= (p^{-1}q^{-1})(q^{-1}p)$ (associative)

$= (q^{-1}p^{-1})(q^{-1}p)$

$= q^{-1}q^{-1}$

$= e$ (Hence)

Q.10 Let G_1 and G_2 be subgroups of a group G.

(a) Show that $G_1 \cap G_2$ is a subgroup of G.

(b) Let $G_1 \cup G_2$ be a subgroup of G.

[1995 : 6 Marks]

Solution:

(a) Consider two elements

$x \in (G_1 \cap G_2)$ and

$y \in (G_1 \cap G_2)$

$\Rightarrow x \in G_1, x \in G_2$ and $y \in G_1$, $y \in G_2$

Since G_1 is a subgroup of G

$x, y \in G_1$

$\Rightarrow xy^{-1} \in G_1$

Since G_2 is a subgroup of G and

$x \in G_1, y \in G_2$

$\Rightarrow xy^{-1} \in G_2$

So $xy^{-1} \in (G_1 \cap G_2)$

This proves that $(G_1 \cap G_2)$ is a subgroup of G.

(b) $(G_1 \cup G_2)$ is a subgroup of G. Then G_1 is contained in G_2 or G_2 is contained in G_1.

Therefore $(G_1 \cup G_2)$ is equal to one of the subgroups G_1 or G_2.

Q.11 Let $S = \{0, 1, 2, 3, 4, 5, 6\}$ and S denotes multiplication modulo 7. Find $x \in S$ such that $x^2 = 4$.

(a) Prove that $(S, \cdot)$ is a cyclic group.

(b) Write all distinct groups of order 6 up to isomorphism.

[2000 : 4 Marks]

Solution:

(a) A cyclic group is a group that is generated by a single element. Thus, a cyclic group is a group that can be generated by a single element. Therefore, $(S, \cdot)$ is a cyclic group.

(b) $\{1, 2, 3, 4, 5, 6\}$ is a cyclic group and also a cyclic subset of S.

Q.12 A multiplicative commutative group of order 6 is given. The elements of the group are the cube roots of unity.

(a) Write down the table of the group.

(b) Show that the group is cyclic.

(c) How many multiplicative inverses are there in the group?

[2000 : 6 Marks]

Solution:

(a) There are exactly four planes in the field in the intersection using the distinct points. Let the four points be A, B, C, D. Let the four planes be P_1, P_2, P_3, P_4. Let the four points be A, B, C, D. Let the four planes be P_1, P_2, P_3, P_4. Let the four points be A, B, C, D. Let the four planes be P_1, P_2, P_3, P_4. Let the four points be A, B, C, D. Let the four planes be P_1, P_2, P_3, P_4.

Case 1: Exactly one element is in the intersection of each. There are $\binom{4}{1}$ ways.

Case 2: Exactly two elements in the intersection of each. There are $\binom{4}{2}$ ways.

Case 3: Exactly three elements in the intersection of each. There are $\binom{4}{3}$ ways.

Case 4: Exactly four elements in the intersection of each. There are $\binom{4}{4}$ ways.

Total ways for the intersection

$= \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}$

$= 4 + 6 + 4 + 1 = 15$ ways

Case 5: Exactly two elements in the intersection of each. There are $\binom{4}{2}$ ways. Fill up the intersection of each pair of distinct elements in the intersection of each pair of distinct elements in the intersection of each pair of distinct elements in

Q.13 Let A be a set of n elements. Let N_1 be the number of binary relations on A and let N_2 be the number of functions from A to A.

(a) Give a simple expression for N_1 in terms of n.

(b) Give a simple expression for N_2 in terms of n.

(c) Which is larger, for all possible n, N_1 or N_2?

[2008 : 5 Marks]

Solution:

(a) Number of binary relations on set A with n elements is $N_1 = 2^{n^2}$

(b) Number of functions from set A with n elements is $N_2 = n^n$.

(c) Since every function is a relation, however, every relation need not be a function, the number of relations is larger than number of functions. Thus

$N_1 > N_2$ for all possible n.

Q.14 (a) $S = \{x \mid x \text{ is a positive integer and } x \le 10\}$ is a binary relation on set $A = \{1, 2, 3\}$. Find the minimum number of ordered pairs to be added to make it an equivalence relation.

Give the modified S.

(b) Let $S = \{a, b\}$ and let $P(S)$ be the power set of S. Consider the binary relation $\subseteq$ (subset) on $P(S)$. Draw the Hasse diagram corresponding to the poset $(P(S), \subseteq)$.

[2008 : 5 Marks]

Solution:

(a) Relation $S = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$

Since $(1, 1), (2, 2), (3, 3)$ are present in S, the relation S is reflexive.

Relation S is already symmetric, transitive. However it is not reflexive. To make relation S an equivalence

$\Rightarrow S = \{(x, y) \mid x \le 3\}$

$\Rightarrow S = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

So the modified S.

$S = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Since S is reflexive, symmetric and transitive, the relation S is an equivalence relation.

(b) $S = \{a, b\}$

The power set of S, $P(S)$ is given as

$P(S) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

The Hasse diagram corresponding to the poset $(P(S), \subseteq)$ is given as follows

Hasse diagram for $(P(S), \subseteq)$

4. Calculus

Q.15 (a) Find the points of local maxima and minima if any of the following function defined for $-\infty < x < \infty$.

$x^3 - 6x^2 + 9x + 15$

(b) Evaluate $\int_{-\pi}^{\pi} x \cos x \, dx$

[1999 : 5 Marks]

Solution:

(a) Given $f(x) = x^3 - 6x^2 + 9x + 15$

For local maxima and minima, we put $f'(x) = 0$

$f'(x) = 0$

$f'(x) = 3x^2 - 12x + 9 = 0$

$x = 1, 3$

$f''(x) = 6x - 12$

$f''(1) = -6 < 0$

$f(x)$ is a point of local maximum

$f''(3) = 6 > 0$

$f(x)$ is a point of local minimum

So, $x = 1$ is a point of local maximum

$x = 3$ is a point of local minimum

(b) $\int_{-\pi}^{\pi} x \cos x \, dx$

$f(x) = x \cos x$

$f(-x) = (-x)\cos(-x)$

$= -x \cos x$

$= -f(x)$

Since $f(-x) = -f(x)$, then $f(x)$ is odd function.

$\int_{-a}^{a} f(x) = 0$ if $f(x)$ is odd function

So, $\int_{-\pi}^{\pi} x \cos x \, dx = 0$

[illegible] odd function.

Unit 2 Theory of Computation

1. Finite Automata : RL, RE and RG

Q.1 Give a minimal DFA that gives an output 1 each time the input sequence ends with a multiple of 3 1's in the input sequence is a multiple of 3.

[Year : 6 Marks]

Solution:

The minimal DFA has seven states.

Transition diagram is [illegible]

Q.2 Given the regular expression (0+1)*(0+11) denote the second proposition and substrings of the strings [illegible]

[1997 : 6 Marks]

Solution:

The proper non null substrings of the string 0110 are [illegible]

So the regular expression of proper non null substrings of L is denoting 0110 is

$R_1 = 0 + 1 + 01 + 11 + 10 + 011 + 110 + 0110$

Q.3 Is the class of regular sets closed under infinite union? Justify.

[1996 : 6 Marks]

Solution:

No, the class of regular sets is not closed under infinite union.

If regular sets are closed under infinite union, $L = \{a^i b^i \mid i \geq 0\}$ must be regular because $a^i b^i$ is regular. But it is not regular as follows

$L_0 = \phi$

$L_1 = ab$

$L_2 = aabb$

$L_3 = aaabbb$

$\vdots$

$L = \bigcup_{i=0}^{\infty} a^i b^i$

This is a context free language.

Thus

$L = \{a^i b^i \mid i \geq 0\}$

is not regular language. It is context free language.

Q.4 Given that language L_1 is regular and language $L_2 \subseteq L_1$ is a proper subset. Is the language L_2 always regular?

[1994 : 6 Marks]

Solution:

No, it need not be always regular. Given

$L_1 = \{ab\}^*$ which is regular language

$L_2 = \{a^n b^n \mid n \geq 0\}$ which is not a regular language. (Subset)

$L_1 \cap L_2 = \{ab\}$ which is regular language

So L_2 need not be a regular always.

2. Push Down Automata: CFL & DCFL

Q.5 State whether the following statements are true or false.

1) A regular language is generated by a context free grammar.

2) [illegible] is a regular language.

3) A minimal DFA that is equivalent to an NDFA with n states has exactly 2ⁿ states.

4) The intersection of two CFL's is a CFL.

5) A language accepted by a Turing machine is a recursively enumerable language. (Turing Machine Model)

6) The problem arises when a Turing machine never stops on input x is undecidable.

[Year : 6 Marks]

Solution:

1) True, since a regular language is generated by a regular grammar and context free grammar is a superset of regular grammar.

2) False, since a subset of regular set is also regular but not all.

3) False, a minimal DFA that is equivalent to an NDFA with n states has at most 2ⁿ states, not always.

(c) [illegible]

(d) True. All recursive [illegible] can be accepted by Turing machine M and every w which is not in A, M will never halt and [illegible] rejected by the machine M.

(e) True. Intersection of Turing machine and M is recognizable.

3. Turing Machine: RS & RES

4.8 Which of the following statement is/are true?

(a) Union of two recursive languages is recursive.

(b) The language $\{0^n \mid n \text{ is prime}\}$ is not regular.

(c) Regular languages are closed under infinite union.

[2002 : 5 Marks]

Solution:

(a) Union of recursive languages are closed under union and therefore recursive languages is recursive.

(b) The language $L = \{0^n \mid n \text{ is prime}\}$ is not regular.

(c) Suppose we have an infinite set of regular languages $L_1, L_2, L_3, \ldots$ which is infinite union of following regular languages should be regular.

$L = \{a^i b^i \mid i = 1, 2, 3, \ldots\}$

However L is CFL.

■■■

Unit 3 Digital Logic

1. Logic Functions & Minimization

Q.1 Find the minimum product of sums of the following expression

$f = ABC + ABC$

[1990 : 5 Marks]

Solution:

Given SOP expression is

$f = ABC + \overline{A}\overline{B}\overline{C}$

The required minimal POS form is

$f = (B + C)(A + C)(A + B)$

Q.2 Show how the Boolean function can be implemented using a multiplexer.

$f = AB + BC + CA$

[1990 : 5 Marks]

Solution:

$f = B + BC + CA$

Using 3-D method

Implement $f = f(A, B, C)$ using 4 × 1 MUX

2. Combinational Circuits

Q.3 A logic network has two data inputs A and B, and two control inputs C_1 and C_0. Implement the functions given below as indicated in the following table.

C_1	C_0	F
0	0	$A + B$
0	1	$A + B$
1	0	$A \oplus B$
	1	AB

Implement the circuit using one 4 × 1 Multiplexer, one 2-input Exclusive-OR gate, one 2-input AND gate, one 2-input OR gate and one Inverter.

[1990 : 5 Marks]

Solution:

Q.4 Find the minimum sum of products form of the logic function.

$f(A, B, C, D) = \Sigma m(0, 2, 8, 10, 13) + \Sigma d(4, 1, 12, 14)$

Where m and d denote the minterms and don't care terms respectively.

[1991 : 5 Marks]

Solution:

$f(A, B, C, D) = \Sigma m(0, 2, 8, 10, 13) + \Sigma d(4, 1, 12, 14)$

The minimum POS form of the logic function is given as $f(A, B, C, D) = (D + A)(C + A)$.

Q.5 Express the function $f(x, y, z) = xz' + yz'$ with only a complement operation and one or more AND/OR operations. Draw the logic circuit implementing the expression, using a single NOT gate and one or more AND/OR gates.

[2002 : 6 Marks]

Solution:

To use only single NOT gate expression with only one complementation and one or more AND/OR operation

$$f(x, y, z) = xz' + yz'$$

Additional terms are

$$f(x, y, z) = xz' + yz' + xz'$$
$$= yz' + xz' + z'$$
$$= z(y' + x) + z(y' + x)$$

Logic circuit implementing the expression obtained is given as below

3. Sequential Circuits

Q.6 Consider the synchronous sequential circuit in figure

(a) Draw state diagram which is implemented by the circuit. Use the following notation: the state corresponding to the values of flip flops as given below.

Q1	Q2	Q0	State
0	0	0	S_0
0	0	1	S_1
	1		

(b) Given that for initial state of the circuit is S_0, identify the state sequence which can not reach the

[1996 : 5 Marks]

Solution:

$D_2(Q_2) = Q_2'$	$D_1(Q_1) = Q_2$	$D_0(Q_0) = Q_1$	Q_2	Q_1	Q_0	State
0	0	0	0	0	0	(S_0)
0	0	1	0	0	1	(S_1)
1	0	0	1	0	0	(S_4)
0	1	0	0	1	0	(S_2)
1	0	1	1	0	1	(S_5)
1	1	0	1	1	0	(S_6)
1	1	1	1	1	1	(S_7)

(c) The state diagram which is implemented by the circuit is as follows

(d) If the initial state of the counter is S_0 then the state which is not reachable is S_3

$S_0 \to S_4 \to S_5 \to S_2 \to S_1 \to S_6 \to S_7 \to S_0$

Q.7 For the synchronous counter shown in figure write down the table for Q_2, Q_1 and Q_0 taking initial state as $Q_2 = Q_1 = Q_0 = 0$ and determine the counting sequence and also the modulo of the counter.

What is the modulus of the counter with initial state $Q_2 Q_1 Q_0 = 000$

(a) 3 (d) 7

(b) 5 (d) 6

[1990 : 5 Marks]

Solution: (c)

[illegible]								
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

There are 7 valid count states in 3-bit ring counter.

Therefore, the modulus of the counter is 7.

Q.8 Analyse the circuit in figure and construct the truth table.

a	b	Q_n
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[1991 : 6 Marks]

Solution:

The output of the circuit is given as

$Q_n = aQ_{n-1} + \bar{a}\bar{b}Q_{n-1}$

$Q_n = Q_{n-1}(a + \bar{b}) + \bar{a}\bar{b}$

a	b	Q_n
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

■■■■

Unit 4 Computer Organization & Architecture

1. Cache and Main Memory

Q.1 State the Booth's algorithm for multiplication of two numbers. Draw a flowchart for the implementation of the Booth's algorithm for determining the product of two 8 bit signed numbers.

[1999: 5 Marks]

Solution:

The Booth's algorithm generates the 2's complement of the operands automatically. Booth's algorithm is a sequential multiplication algorithm.

The algorithm has two distinctive features:

(i) It can handle both positive and negative multiplicands.

(ii) Second, it achieves some speed improvement when adjacent bits are identical since it skips over blocks of zeros.

Booth's algorithm for multiplication of two 8-bit signed numbers.

Q.2 A computer system has a three level memory hierarchy with access time and hit ratios as shown below:

Size	Hit ratio
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Size	Hit ratio
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

Size [illegible]

[illegible]

(i) What should be the minimum sizes of level 1 and 2 memories to achieve an average access time of [illegible] ns?

(ii) What is the average access time achieved using the chosen sizes of level 1 and level 2 memories?

[1999: 5 Marks]

Solution:

Taking level 1 memory cache size 8 MB

Level 2 memory size 4 MB and

Level 3 memory size 200 MB

Average access time,

$T_{avg} = 1.80(ns) = 0.2 \times 0.98 \times 200 + 0.2 \times 0.98 \times (8000 + 50 + 200)$ ns

$= 110.2 + 1.96 \times 200$ ns

So taking level 1 memory size 18 MB

Level 2 memory size 2 MB and

Level 3 memory size 200 MB

$T_{avg} = 0.80(ns) = 0.1 \times 0.89 \times 200 + 0.1 \times 0.08 \times (8000 + 50 + 200)$ ns

$= 80 + 0.1 \times 0.08 \times 8250$ ns

(i) The minimum sizes of level 1 and level 2 memories are 16 MB, 2 MB respectively.

(ii) The average access time achieved is 80.7 ns.

Q.3 A CPU has 32-bit memory address and a 256 KB cache memory. The cache is organized as a 4-way set associative cache with block size of 16 words.

(a) What is the number of sets in the cache?

(b) What is the size (in bits) of the tag field per cache block?

(c) What is the ratio of the size of tag memory to the cache memory?

(d) How many address bits are required to find the byte within a cache block?

(e) What is the total amount of memory (in bits) required for the tag bits?

[2000 : 5 Marks]

Solution:

Main memory address size = 32-bit

Cache size = 256 KB

Block size = 16 bits

(a) $\text{# of sets} = \frac{\text{# of blocks in cache}}{\text{# of blocks in a set}}$

$$= \frac{256\ \text{KB} \div (16 \times 4)}{4} = 4096$$

(b) [illegible]

Size of the tag field per cache block is 16 bits.

(c) [illegible] = 1/16 or 6.25%

(d) Address bits required for word offset = 4 bits

(e) # of blocks in cache = [illegible]

$$= \frac{256\ \text{KB}}{4 \times 16 \times 4} \times 16 \times 4 = 256\ \text{KB} \times 4 = 64\ \text{KB}$$

2. Instructions : Pipelining and Addressing Modes

Q.4 An instruction pipeline consists of 4 stages – Fetch (F), Decode (D), Execute (E), Write (W). Write (W) the fetch unit is a cache memory, execution unit takes one stage for the ALU instructions and two stages for the load instructions.

Number of clock cycles needed to execute the 5 instructions is

Instruction	F	D	E	W
1	1	[illegible]	[illegible]	1
2	2	2	2	1
3	3	1	2	2
4	4	3	2	
5	[illegible]	2	1	2

Find the number of clock cycles needed to execute the 5 instructions.

[1999 : 6 Marks]

Solution:

The number of clock cycles needed to perform the 5 instructions are 11 cycles.

Q.5 An instruction pipeline has five stages where each stage takes 2 nanoseconds and all instructions use all five stages. Branch instructions are not overlapped, i.e., the instruction after the branch is not fetched till the branch instruction is completed. Under ideal conditions,

(a) Calculate the average instruction time assuming that 20% of all instructions executed are branch instructions.

(b) For branch instructions, conditional branch instructions are executed. When 80% of all branch instructions are conditional branch instructions and 20% of the conditional branch instructions are taken, the average instruction execution time.

[2000 : 6 Marks]

Solution:

[illegible]

(b) [illegible]

[illegible]

3. Secondary Memory & DMA

Q.6 Hard disk is connected to a 32 MHz processor through a DMA controller. Assume that it initially takes 10 cycles of the DMA controller to set up a block transfer in the processor, and assume that the handling of the interrupt at DMA completion requires 5 processor cycles for the interrupt. The hard disk has a transfer rate of 2000 KB/sec and sector size of 1000 bytes. What fraction of the processor time is consumed by the disk, if the disk is actively transferring data at the full rate?

(1996; 5 Marks)

Solution:

Clock time of processor

$$T = \frac{1}{32 \times 10^6} = 20 \text{ ns}$$

DMA transfer time

DMA + CPU = Total time

Overhead time

$$= (10 \times 20 + 5 \times 20) \text{ ns} = 30 \mu\text{sec}$$

Transfer time of hard disk

$$= \frac{1000 \text{ B}}{2000 \text{ KB/s}} = 2 \text{ ms}$$

Fraction time of hard disk

$$= \frac{30 \mu\text{sec}}{2 \text{ ms}} = 1.5\%$$

Q.7 Consider the following program segment in the assembly language of a certain hypothetical processor. The processor has two general purpose registers R1 and R2. The mnemonics of the instructions are self-explanatory, with DST as the destination.

X: CMP R1, R2 ; Compare R1 and R2, set flags accordingly in status register

JGE Y ; Jump if zero or greater to Y

MOV R2, R1 ; Copy contents of R1 into R2

SHR R1 ; Shift right R1 by 1 bit

SHR R2 ; Shift right R2 by 1 bit

CMP R2, R1 ; Compare R2 and R1 and set flags in status register

JGE Y ; Jump if zero or greater to Y

INC R2 ; Increment R2 by 1

Z: SHLR1 ; Shift right R1 by 1 bit

JMP X ; Jump to label X

?

(a) Let the initial values of R1 and R2 be 10 and 5 respectively. What are the final values of R1 and R2 when control reaches Z?

(b) In general, if R1 and R2 initially contain the values m and n respectively, what are the final values of R1 and R2 when control reaches Z?

(1999; 5 Marks)

Solution:

(a) The instructions execute

$R_1 = 10$, $R_2 = 5$ and $R_2 = 2$

(b) $R_1 = m$, $R_2 = n$ and $R_2 = \log_2 n$

Q.8 Consider the following 8086 program segment, where registers P and Q contain the 16-bit offsets.

S1:	MVI	A, 06H
	MVI	B, 06H
	SUB	C
	ADD	D
	DAA	

[illegible]

(b) Find the [illegible] when [illegible] accesses S2.

(c) Find the values required for a Belady [illegible] machine S1.

(d) What [illegible] is the value of D and E as a function of B and C when [illegible] machine S1.

[2009 : 6 Marks]

Solution:

(a) The [illegible] performs [illegible] operations per [illegible] C + b

(i) [illegible] C = 25

(ii) B = 88 and C = 8

(b) (i) [illegible]

(ii) [illegible]

Q.9 Consider a disk with 20 surfaces, 800 tracks/surface, 50 sectors/track, 512 bytes/sector, and rotation speed 3000 rpm. The average seek time is 30 ms. The average seek time between the disk tracks is 1 ms. [illegible] The disk controller [illegible] the data. The [illegible] transfer time [illegible] the disk controller [illegible] transfer the data to memory. The disk controller [illegible] processing.

Suppose a random file block [illegible] seek time + [illegible] rotational delay, [illegible] data block [illegible] 1 microseconds, the DMA initialization and termination overheads [illegible] the [illegible] DMA controller [illegible]

(a) What is the [illegible]?

(b) What is the data transfer rate?

(c) What is the percentage of time the CPU [illegible] for this disk [illegible] for [illegible]?

(d) What is the maximum percentage of time the CPU is held up for this disk [illegible] using DMA transfer?

[2001 : 5 Marks]

Solution:

Total surface = 20 surface

# of tracks per surface = 800 tracks/surface

# of sectors per track = 50 sectors/track

# of bytes per sector = 512 Bytes/sector

Rotational speed = 3000 rpm

(a) Total capacity of the disk

$$= \#\text{ of surface} \times 800 \frac{\text{tracks}}{\text{surface}} \times 50 \frac{\text{sectors}}{\text{track}} \times 512 \frac{\text{Bytes}}{\text{sectors}}$$

$$= 512.0\ \text{MB}$$

(b) Data transfer rate

Data size = 512 kb

$$\text{Time} = \frac{512}{3000 \times 10^{-3}}$$

$$= 170.7\ \text{kbps}$$

Therefore data transfer rate = 170.7 kbps

(c) % CPU time $= \frac{0.1}{0.5} \times 100 = 20\%$

(d) Max % time CPU is held up for disk access $= 5\%$

■■■■

Part 3 Programming & Data Structures

1. Programming Language Concepts

Q.1 Consider the following pseudo-code for illustrating use of type concept:

```
procedure P(x, y);
    x := 2;
    y := x + 1;
end P;
begin
    x := 1;
    y := 5;
    z := 100;
    P(x, x*y, z);
    Write(x, y, z);
end
```

Determine the output, if the parameters are passed as the procedure P by

(a) value, (b) reference and (c) name.

[1991 : 6 Marks]

Solution:

(a) Passed by value: 1, 10

(b) Passed by reference: 2, 2

(c) Passed by name: 2, 5

Q.2 Consider the following pseudo code. Indicate the output if dynamic scope rules are used.

```
var n, a: integer;
Procedure P;
        a := 2; b := 10
end P;
procedure Q;
var n, a: integer;
P;
a := 10;
begin
n := 1; a := 2;
Q;
Write(n, a, b);
end.
```

[1991 : 6 Marks]

Solution:

(i) n, 10, 10 (ii) 1, 2

Q.3 State whether the following statements are True or False with reasons for your answer:

(a) Compiler is a machine dependent program.

(b) A two pass assembler takes machine opcode tables for first pass of assembly.

[1994 : 5 Marks]

Solution:

(a) False

Compiler is a machine independent program which converts instructions written in high level language into machine program. Component such as computer instructions, variables, infinite loop, while statement, etc. allow multiple paths for execution using conditional programming execution control conditions.

(b) False

In first pass assembler generates the storage for tokens.

Q.4 State whether the following statements are True or False with reasons for your answer:

(a) A macro instruction cannot always be used to replace a subroutine in an assembler language program.

(b) A symbolic debugger helps in assembly language program if a statement in a high level language program is executed an address mapping is required to provide the source code to the user.

[1994 : 5 Marks]

Solution:

(a) True

(b) True

Q.5 Consider the program below

```
Program main
    var i: integer;
    procedure P(var
    begin { procedure P }
    procedure Q(var
    var i: integer;
    begin { procedure Q }
    end { procedure Q }
    end { procedure P }
    end.
```

What is printed by the following program?

(i) [illegible]

(ii) [illegible]

[illegible]

[1991 : 5 Marks]

Solution:

[illegible]

[illegible]

In solution [illegible] all [illegible] are passed [illegible] [illegible] [illegible] [illegible] [illegible]

Q.6 Consider the following recursive function [illegible]

```
function Thf(n: integer): integer;
begin
  [illegible]
  [illegible]
  [illegible]
end
```

The above function [illegible] with a stack of [illegible]. Assume that only [illegible] and parameters are [illegible] [illegible] and that an integer [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[1994 : 6 Marks]

Solution:

[illegible]

[illegible]

[illegible]

Q.7 Consider the following Pascal function where A [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

```
function GETA(A, B: [illegible]);
begin
    if P = 0 then
        GETA := [illegible]
    else if A[illegible] then
        GETA := [illegible]
    else
        GETA := GETA(A, [illegible], [illegible]);
end
```

[1994 : 6 Marks]

Solution:

[illegible]

Value of GETA(5, 2) = 2

Q.8 What will be the output of the following program assuming that parameters are passed [illegible]

```
procedure [illegible];
    [illegible]
    [illegible]
    [illegible]
    [illegible]
    [illegible]
    [illegible]
end;
```

[1998 : 5 Marks]

Solution:

(i) [illegible]

(ii) [illegible] = 18

[illegible]

(iii) [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Q.9 Consider the following [illegible] [illegible]

```
program [illegible];
    var [illegible],
    procedure Q(var [illegible]);
    begin
        [illegible]
        [illegible]
    end;
procedure P(var [illegible]);
        [illegible];
    begin
        [illegible],
        Q(x);
        [illegible]
    end;
    begin
    x:=5;
    P(y);
    Q(x);
    [illegible]
end.
```

What is the output of the program when

(a) The parameter passing mechanism is call-by-value and the scope rules are static scoping?

(b) The parameter passing mechanism is call-by-reference and the scope rules are static scoping?

[2000 : 5 Marks]

Solution:

(a) [illegible] (b) [illegible]

Q.10 Consider the following C program.

```
void abc(int a*)
    {int [illegible];
    [illegible];
    abc(&[illegible]);
    [illegible]
    }
main()
    {
    [illegible];
    }
```

(a) What will be the output of the program?

(b) If abc() is called with a character string as the argument (not mentioning the [illegible] is expected), how many characters will be printed by abc()?

[2001 : 5 Marks]

Solution:

main()

{

[illegible]

{

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

printf("[illegible]", [illegible])

Now again prog

```
| abc_28
| [illegible]
```

Dump: [illegible] line was [illegible] prog.obj(50).

[illegible]

[illegible]

(b) [illegible]

2. Arrays

Q.11 For an array of positive integers, segment finds the largest segment sum in the array. Assume the array contains integer values. Fill in the blanks to complete the program and [illegible] [illegible] and [illegible] in your answer sheet. Assume that [illegible] is variable to store the largest value and [illegible] is a variable to store the array.

```
begin
    [illegible]
    while [illegible]
        begin
        [illegible]
        [illegible]
        end begin
```

[illegible]

[illegible]

[illegible]

(b) [illegible]

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

4. Linked List

Q.16 [illegible] linked list [illegible] stores d1, d2, ..., dn are stored in those nodes. Let F be a given integer [illegible] 1 ≤ i ≤ n, an [illegible] is stored. A new node is inserted in the [illegible] address Y is to be inserted. Give an algorithm to insert a node [illegible] the list to obtain a list having stored d1, d2, ..., di, d, di+1, ..., dn in the [illegible] using [illegible]

[1995 : 5 Marks]

Solution:

[illegible]

Q.18 Let [illegible] given as shown in the figure [illegible] single [illegible] list.

What do the following assignment statements [illegible]?

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(iv) [illegible]

[1996 : 5 Marks]

Solution:

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(iv) [illegible]

[illegible] cell [illegible] deleted [illegible] from the [illegible] free [illegible]

5. Trees

Q.17 Consider the binary tree in Figure.

(a) What [illegible] is [illegible]?

(b) Give the [illegible] in the following tree [illegible] key is stored in the order in which it is processed.

[1991 : 5 Marks]

Solution:

(a) It is [illegible].

(b) The node has 2 children. [illegible] we replace it by either [illegible] in the [illegible].

Q.18 [illegible] induction [illegible] any binary tree in which [illegible] degree 2 [illegible] the number of leaves in the tree is one more than the number of [illegible] nodes.

[1993 : 6 Marks]

Solution:

Step-I: [illegible]

Here, [illegible]

[illegible]

$\therefore$ [illegible]

Step-II: [illegible] having 2 [illegible]

$(2 - 1) = 1$ [illegible] internal node [illegible] only one [illegible]

iii. [illegible] with 8 nodes. Level 2 [illegible]

For an internal node [illegible] by mathematical induction. By [illegible] having 2 descendants [illegible] leaf nodes.

Q.19 [illegible] internal node [illegible] three children. Use induction [illegible] with n internal nodes [illegible].

[1994 : 6 Marks]

Solution:

Step-I: [illegible] with one internal node:

Step-II: [illegible] with 2 internal nodes:

We [illegible]

Step-III: Therefore, if we have [illegible] internal nodes and l_{n-1} [illegible]

$$l_n = l_{n-1} + 2 \text{ [illegible]}$$

So

$$l_n = l_{n-1} + 2$$

$$l_n = l_{n-2} + 2 + 2$$

$$l_n = l_{n-3} + 2 + 2 + 2$$

$$l_n = l_{n-\text{[illegible]}} + \underbrace{2 + 2 + \ldots + 2}_{\text{[illegible] times}}$$

$$l_n = l_1 + 2n$$

When $n = 1$:

$$l_n = l_1 + 2n$$

$$l_n = l_1 + 2n$$

$$l_n = 3 + 2n \quad \text{[illegible]} \quad l_1 = 3$$

Q.20 [illegible] with 9 nodes [illegible] postorder [illegible] A, B, C [illegible] these binary trees.

[1994 : 5 Marks]

Solution:

[illegible] binary trees with 9 nodes:

Q.21 A tree balanced, [illegible] which for every node the difference between the number of nodes in the left and right subtrees is at most 1. [illegible] the height of the tree. The height of a binary tree is the maximum distance of a leaf node from the root.

(a) Prove by using induction on h, that a tree balanced binary tree of height h contains at least 2^h nodes.

(b) In a tree balanced binary tree of height h, h = 1, how many nodes are present in the tree? Give a formula. (Write only the answer without any explanation.)

[1987 : 5 Marks]

Solution:

(a) Base case: h = 0

→ total no. of nodes = $2^0 = 1$

Induction: Assume for a tree of height h = k

minimum no. of nodes = 2^k

[illegible]

We must show that it holds for h = k + 1.

h = k + 1 → total no. of nodes = $2 \cdot 2^k = 2^{k+1}$

Hence it is proved.

(b) $2^{h+1} - 1$

Q.22 Draw the binary tree with nodes labelled a, b, c, d, e, f and g for which the inorder and postorder traversals result in the following sequences.

Inorder : a f b c d e g

Postorder : a f b g e d c

[1995 : 5 Marks]

Solution:

Q.23 (a) Define the height of a binary tree or subtree and also define a height balanced (AVL) tree.

(b) Find the balance factor of each of the tree given in figure and state whether it is height-balanced.

(c) Insert the same sequence given above in AVL and show the new balance factors that would result if the tree is not balanced. Finally, write down the required rebalancing of the tree and show the new tree with the balance factor of each node.

[1992 : 5 Marks]

Solution:

(a) The height of a tree or a subtree is defined as the length of the longest path from the root to a leaf.

A tree is said to be height balanced if all the nodes are having a balance factor (–1, 0 or 1).

The balance factor is the height of the left subtree minus height of the right subtree.

(b)

(c) Insert 18.

Q.24 (a) In binary tree, a full node is defined as a node with 2 children. Show that the number of full nodes plus one is equal to the number of leaf nodes.

(b) Draw the min-heap after inserting the following elements in order in an initially empty min-heap: 7, 6, 5, 4, 3, 2, 1. Show the result after the deletion of the root of this heap.

[1999 : 5 Marks]

Solution:

(a) Let n_0 = # of leaf nodes, n_2 = # of full nodes

$n_0 = n_2 + 1$ or $n_0 - n_2 = 1$ [Proved]

Assume that a tree

$\Rightarrow$ # leaf nodes = # full nodes

A binary tree = 1.

In this level assume there are n nodes.

Let m of these n nodes are leaf nodes.

Then (n − m) nodes are full nodes.

If we delete one full node from the tree, then the tree becomes non-full.

If we delete a node from (n − m) full nodes, then one child then no effect on no. of leaves.

Binary tree with height h = 2:

$\Rightarrow$ # leaves = # full nodes + 1

(b) Min heap tree

Now delete the root node

6. Hashing

Q.25 [illegible] hash table with [illegible] buckets, using external chaining. The hash function is such that the probability that a key value is hashed to a particular bucket is $\frac{1}{n}$. The hash table is initially empty and K distinct values are inserted in the table.

(i) What is the probability that exactly K buckets remain empty after the K insertions?

(ii) What is the probability that exactly K buckets are used in any of the K insertions?

(iii) What is the probability that the length of the longest chain is K after K insertions?

[1999 : 6 Marks]

Solution:

(i) Probability that a key goes to a bucket $= \frac{1}{n}$

Probability that a key does not go to a bucket $= 1 - \frac{1}{n} = \frac{n-1}{n}$

Probability that a bucket remains empty

$= \frac{n-1}{n} \cdot \frac{n-1}{n} \cdot \frac{n-1}{n} \cdots$ K times

$= \left(\frac{n-1}{n}\right)^K$

$= \left(1 - \frac{1}{n}\right)^K$

(ii) Probability that exactly one key goes to one of K buckets is

$n(n-1)(n-2)\cdots(n-K+1)$

(iii) Probability that the K keys all hash to the K buckets is

$\frac{n(n-1)(n-2)\cdots(n-K+1)}{n^K}$

Unit 6 Algorithms

1. Algorithm Analysis and Asymptotic Notations

Q.1 Consider the following pseudo C code fragment that removes duplicates from an ordered linked list.

```
struct node{
    int val;
    struct node *next;
};

typedef struct node *Node;
Node *remove_duplicates(Node *head){
    Node *p, *q, *r;
    if (head == NULL)
    return head;
    p = head;
    while (p->next != NULL) {
    q = p->next;
    if (p->val == q->val) {
    p->next = q->next;
    free(q);
    }
    else {
    p = p->next;
    }
    }
    return head;
}
```

Assume the list contains n elements in the following questions.

(a) How many times is the comparison in statement S_1 made?

(b) What is the minimum and maximum number of times statement marked S_2 gets executed?

(c) What is the condition of the value of the integer pointed to by p when the function terminates?

[1997: 5 Marks]

Solution:

(a) Can't

S_1 statement every time check the value of all nodes of linked list, so it executes O(n) times.

(b) O(max), O(n)

(c) The integer value pointed by p is the different value present in linked list.

Q.2 Consider the following algorithm. Assume that the time complexity of Rank(i, j) and Select(i) is O(1) units of time respectively. Derive the time complexity of the algorithm in O notation.

```
algorithm Mystery(n)
begin
    if n = 1 then call A
    else begin
    while n > 1;
    call Rank(n, 1);
    call Select(n)
    end
end.
```

[1999: 5 Marks]

Solution:

Can't

2. Divide and Conquer

Q.3 An integer array of size 10 has the values as given below.

25 [illegible] 30 8 61 26 48 29

This is the array in sorted ascending order.

(a) [illegible] input to the binary search algorithm to find the value 26 in the array. Show the subarrays examined at each step. The algorithm terminates when the value is found.

(ii) [illegible] sorting [illegible] Trace the execution [illegible] Also [illegible] time complexity.

[1999 : 6 Marks]

Solution:

(i) [illegible]

Step 1: [illegible]

Step 2: [illegible]

Step 3: [illegible]

Step 4: [illegible]

Note: Circled [illegible] which is one of the [illegible] in the previous step.

(ii) [illegible]

Q.4 Find a solution for the following recurrence relation.

$T(n) = [illegible] + T([illegible])$

$T(1) = 1$

[1989 : 8 Marks]

Solution:

By using substitution method

$W = [illegible]$

Q.5 Give an optimal algorithm in pseudo code for sorting a sequence of numbers of which exactly k distinct numbers occur more than [illegible] times. Also find the time complexity of your algorithm.

[1997 : 12 Marks]

Solution:

Use Merge sort or Heap sort. It will have $O(n \log n)$ complexity.

3. Greedy Method

Q.6 A language uses an alphabet of the letters a, b, c, d, e, f. The relative frequency of occurrence of the alphabet is given in the table below.

Letter	Relative frequency of occ
a	0.29
b	0.15
c	0.17
d	0.06
e	0.40
f	0.11

Design a prefix binary code for the language which minimizes the average length of the encoded words of the language.

[1996 : 5 Marks]

Solution:

a: 0.29 - 10

b: 0.15 - 001

c: 0.17 - 110

d: 0.06 - 111

e: 0.40 - 01

f: 0.11 - 000

a - 10	0.29 × 2 = 0.58
b - 1010	0.15 × 4 = 0.60
c - 110	0.17 × 3 = 0.51
d - 1011	0.06 × 4 = 0.24
e - 0	0.40 × 1 = 0.40
f - 100	0.11 × 3 = 0.33
	2.66

No. of bits $= \frac{2.66}{1} = 2.66$ bits/seq

Q.7 An independent set is a set of vertices in a graph such that no two of them are adjacent. A greedy algorithm for a greedy approximation to find a maximum independent set in a tree is as follows.

W = Set of all vertices in the tree, I = ∅

While W ≠ ∅

{

Select a vertex v from W and add it

to I

Remove v and its neighbours from W

}

V = V − {v}
if u is [illegible]
[illegible]
end.
output(T);

(ii) Complete the algorithm by specifying the property of vertex u in each case.

(iii) What is the time complexity of the algorithm.

[1994 : 5 Marks]

Solution:

(i) Complete algorithm is as:
V = set of all vertices in the graph
[illegible] = false;
While V ≠ ∅ do
begin
Select a vertex u in V such that
V = V − {u}
u is a member of V and not reached in {T}
Reached[u] = true;
end
output(T);

(ii) Time complexity:
O(|E|) if vertices present in T
O(|V|) if edges present in T

Q.8 Find minimum cost spanning tree for the following graph. Show the steps. Weights are assigned to the edges.

[1995 : 5 Marks]

Solution:

Three spanning trees are possible

Q.9 A simple undirected weighted graph G is given on the vertices {1, 2, ..., n} for any fixed n. Draw the conclusions assuming each edge

(a) the weight of the edge (1, 2) is |1 − 2|

(b) the weight of the edge (1, 2) is 1/2

[1998 : 6 Marks]

Solution:

(a) Weight of the edge is [illegible]

MST is

Weight of MST [illegible] = [illegible]

(b) Weight of the edge is [illegible]

MST is

Weight of MST is [illegible] = [illegible] = 132.

Q.10 The following is the directed, weighted graph. Show how will the algorithm [illegible]

We are interested in the shortest path from A to [illegible].

(a) Output the sequence of vertices inside S when the Dijkstra's algorithm [illegible] source vertex when the algorithm is started at node A.

(b) Write down the sequence of vertices in the shortest path from A to B.

(c) What is the cost of the shortest path from A to B?

[2008 : 6 Marks]

Solution:

(a) Apply Dijkstra's [illegible] algorithm

A → B → C → E

(b) Sequence of vertices between A to E

A → D → C → F

(c) Cost = 31

Q.11 Consider a graph whose vertices are points in the plane with integer coordinates (x, y) such that $1 \le x \le n$ and $1 \le y \le n$, where n is a fixed integer. Two vertices (x_1, y_1) and (x_2, y_2) are adjacent if $|x_1 - x_2| + |y_1 - y_2| = 1$. The weight of an edge $((x_1, y_1), (x_2, y_2))$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

(a) What is the weight of a minimum weight spanning tree in this graph? Write only the numerical value, rounded up to the nearest integer.

(b) What is the weight of a maximum weight spanning tree in this graph? Write only the numerical value, rounded up to the nearest integer.

[1997 : 6 Marks]

Solution:

For $n = 5$ the graph can be sketched as

(a) Minimum weight will be [illegible] and n is edges in minimum spanning tree is $n - 1$. Therefore, the weight of minimum spanning tree is $(n - 1)$.

(b) If $n = 2$, the weight of maximum weight spanning tree is $\sqrt{2} + 2$.

If $n = 3$, the weight is $2\sqrt{2} + 2$.

If $n = 4$, the weight is $3\sqrt{2} + 2$.

So the weight of maximum weight spanning tree is $(n - 1)\sqrt{2} + 2$.

Q.12 Consider a weighted undirected graph with vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and edge set $E = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_1, v_5), (v_2, v_3), (v_2, v_4), (v_3, v_4), (v_3, v_5), (v_4, v_5), (v_4, v_6), (v_5, v_6)\}$. The edge weights are given by the matrix below, where the $(i, j)^{th}$ entry in the matrix represents the weight of the edge (v_i, v_j) if it exists.

$$\begin{bmatrix} 0 & 2 & 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(a) [illegible] the weight of a minimum spanning tree of this graph?

(b) How many different minimum spanning trees does this graph have?

(c) [illegible] possible minimum spanning trees of this graph?

(d) [illegible] the maximum [illegible] weight of a minimum spanning tree [illegible] possible [illegible] spanning trees of this graph?

Solution:

First draw the given graph according to question.

(a) Minimum spanning tree

(b) Number of minimum spanning trees = 2 [illegible] edges [illegible] connected to [illegible] MST for the above graph.

(c) Yes

(d) Yes

[2001 : 5 Marks]

4. Dynamic Programming

Q.13 Tilak is the [illegible] following sequence of [illegible] algorithm to compute all pairs shortest paths [illegible] coordinates [illegible] matrix $A[i, j]$ equals 1 if there is an edge from i to j, and 0 otherwise. The [illegible] filling [illegible] the [illegible] of the algorithm is correct.

INITIALIZATION: For i = 1 ... n

For j = 1 ... n

if A[i, j] = 0 then P[i, j] = ... else

P[i, j] = ...

ALGORITHM: For i = 1 ..... n

For j = 1 ... n

For k = 1 ... n

P[..... ...] = min [..... ...]

}

}

(a) Copy the incomplete lines of the algorithm in the initialization step and fill in the blanks.

(b) Copy the complete line corresponding to the core in the algorithm and fill in the blanks.

(c) Fill in the blanks: The time complexity of the Algorithm is $O($_____$)$.

[2002 : 5 Marks]

Solution:

(a) Initialization:

for i = 1 to n

{

for j = 1 to n

if A[i, j] = 1 then P[i, j] = ...

else P[i, j] = ...

}

(b) Algorithm

for k = 1 to n

{

for i = 1 to n

{

for j = 1 to n

{

P[i, j] = min(P[i, k] + P[k, j], P[i, j]);

}

}

}

(c) The complexity time of the algorithm is $O(n^3)$

5. Misc Topics

Q.14 Consider the function Fibonacci(n) which takes an integer n as input and returns the Fibonacci number as defined below:

Fibonacci(n);

begin

if n = 0 then F ← 0

else if n = 1 then F ← 1

else

begin

x ← 0

for i = 1 to Fibonacci(n-2) do

begin

temp = x

x = x + Fibonacci(n-1)

Fibonacci(n-1) = temp

end

F = x + F1

end

F1 = F

end

(a) is a positive integer greater than zero.

(b) The space requirement of the function Fibonacci(n) is

(c) Assume that multiplication of two integers takes $O(n^2)$ time. Assume that addition of two integers takes $O(n)$ time. Then the time complexity of Fibonacci(n).

[1998 : 5 Marks]

Solution:

(a) The n > 0 or n = 1

(b) $O(n)$

(c) $O(n^3)$

Q.15 Consider the following pseudocode for a procedure bubblesort(n);

procedure bubblesort(n);

var i, j: integer; temp: real;

begin

for i = 1 to n-1 do

if A[i] > A[i+1] then

begin

temp = A[i];

A[i] = A[i+1];

A[i+1] = temp;

end

bubblesort(n-1);

end

Let a_n be the number of times the statement "temp = A[i]" is executed by bubblesort(n). Write a recurrence relation for a_n by defining a_n in terms of a_{n-1}. Solve for a_n.

[1998 : 5 Marks]

Solution:

Recurrence relation:

$$a_n = \begin{cases} \text{return } A[1] & \text{if } n = 1 \\ \text{swap } A[i] \text{ and } A[i+1] & \text{if } A[i] > A[i+1] \end{cases}$$

Q.16 Assume A contains integers in nondecreasing order. A[1] ≤ A[2] ≤ .... ≤ A[n]. Describe using Pascal like pseudocode, an $O(n)$ time algorithm to find i, j such that A[i] + A[j] = a given integer M, if such i, j exist.

[1994 : 6 Marks]

Solution:

Take 3 variable : A, First, B, Last

```
if (A[First] == B[Last], return (A, B)
else
if (A[First] > A[Last])
    B = B - 1;
else
    A = A + 1;
O(n)
```

Q.17 A row of n non-negative integers A[1 ... n] is said to be sorted if $A[i] \le A[i+1]$ for $1 \le i \le n-1$. Given a sorted array A[1 ... n] and a number x, we want to find the smallest index i such that $A[i] \ge x$. Complete the following code using binary search.

```
procedure binsearch (A, x, n)
    var i, j, k: integer
    begin
    i := 1; j := n;
    while i < j do
    begin
    k := ⌊(i + j)/2⌋;
    if A[k] < x then i := k + 1
    else j := k;
    end;
    if A[i] = x then writeln("x is present at", i)
    else writeln("x is not present");
end binsearch;
```

[1996 : 6 Marks]

Solution:

(a) i = 1, j = n

(b) Line 8: while (i < j) do

Line 7: A[k] = A[k] + 1

j = j - 1;

Line 5: A[i] = x;

Q.18 An array A contains n+1 positive integers, i.e. the elements A[1], A[2], ..., A[n+1] are positive integers. The following program fragment prints the length of the shortest contiguous subsequence of A[1], A[2], ..., A[n] such that the sum of the elements is greater than x. x is a given positive number. It assumes n+1 is the total sequence length. Complete the program by filling the blanks. Your code should be in the simplest possible expression. Write only the number and the contents of the blanks.

```
1.  begin
2.  i := 1; j := 1;
3.  sum := A[1];
4.  min := n+1; while sum <= x do
5.  while sum <= x do
6.  begin
7.  sum := sum + A[j];
8.  j := j + 1;
9.  end;
10. if j - i < min then min := j - i;
11. sum := ____
12. i := i + 1;
13. end;
14. writeln(min);
15. end.
```

[1997 : 5 Marks]

Solution:

line 7: sum = sum - A[i]

line 6: if (sum > x)

line 11: sum = sum - A[i]

Unit 7 Compiler Design

1. Parsing Techinques

Q.1 Consider the following grammar:

S → [illegible]

S → S(S)S | ε

(a) Construct the canonical collection of LR(0) items for the grammar and draw the goto graph.

(b) Indicate the shift, reduce and reduce reduce conflicts if any in the canonical collection of the LR(0) parser.

[1992, 5 Marks]

Solution:

(a) Step-1: The augmented grammar is

S' → S

S → S(S)S | ε

Step-2: [illegible]

Step-3: Using I_0 constructing goto graph

(b) Shift reduce conflict state is [illegible] I_0 and I_1 and I_2 but there is no reduce reduce conflict.

Unit 3 Operating System

1. Process Management-II

Q.1 Consider the following concurrent [illegible] and [illegible] [illegible] [illegible] and semaphores

[illegible]

```
mutex, [illegible], [illegible] : semaphore
[illegible] = 0
[illegible] = 0
mutex.value = 1          procedure writer;
procedure reader;        begin
  begin                  [illegible]
  [illegible]              begin
    [illegible]              [illegible]
    [illegible]              [illegible] := [illegible] + 1;
    [illegible]              [illegible]
    [illegible]              [illegible]
    [illegible]              [illegible]
```

[illegible] process immediately. There is no preemption of resources. A resource is released back to the system only when the process explicitly releases it.

(i) Can the system of four processes get into deadlock? If so, give an example of a sequence of requests for resources and releasing resources of these processes which leads to a deadlock.

(ii) Will the processes always get into deadlock? If not, give a sequence of resource requests which leads to a complete run of the processes.

(iii) Will the strategy work if it is used to prevent deadlock in a system in which processes are allowed to use shared resources of preemptive or non-preemptive nature? Justify your answer.

[1999 : 5 Marks]

Solution:

(i) Yes, there is a possibility of deadlock in above process.

Concurrent	P	Q	R	S
	req. A	req. C		
	req. B	req. A		
	release A		req. B	
	release B	release C		req. A
		release A	req. C	
			release A	req. C
			release C	
				release A
				release C

(ii) No, not always, given a total sequence as a.

P	Q	R	S
req. A	req. C		
req. B	req. A		
release A		req. B	
release B	release C		req. A
	release A	req. C	
		release A	req. C
		release C	
			release A
			release C

(iii) The process access release of the resource before request by the resource manager.

Q.8 Consider the following solution to the mutual exclusion problem in a situation with three processes P_0, P_1 and P_2.

```
P:
repeat
  flag[j]=true
  while flag[j] do flag[j]=false
    if flag[j] then
      begin
      flag[j]=false
      while turn ≠ j do skip;
      flag[j]=true
      end
    k:=flag[k] div
      begin
      flag[j]=false;
      while turn = j do skip;
      flag[j]=true
      end
  end
  critical section
    turn = j; flag[j]=false
    flag[j]=false
  remainder section
end
```

(i) Does this solution ensure mutual exclusion in the critical section? Briefly explain.

(ii) Is there a situation in which a waiting process never enters the critical section? If so, explain and suggest modifications to the algorithm to solve this problem.

[1991 : 5 Marks]

Solution:

Yes, mutual exclusion is satisfied, because entrance into CS is dependent on the value of 'turn' and 'flag' of the process which is currently in the critical section.

Q.9 Write a [illegible] program using parbegin/parend and semaphores to represent the precedence constraints of the statements S_1 to S_6 as shown in figure below.

[1993 : 5 Marks]

Solution:

S_1
cobegin
S_2
parbegin
S_4
S_5
parend
S_3
begin
S_6
end
parend
S_7
cobegin
S_8
S_9
parend

Q.6 (a) Draw a precedence graph for the statements using parallel notation. The statements are numbered from S_1 to S_9.

S_1 read a
S_2 b = 5
S_3 if (b > 0) goto
S_4 c = a + b
S_5 d = a - b
S_6 e = c * d
S_7 print "Write a"

(b) Can the graph be converted to a cobegin-parbegin-parend construct using parbegin parend statements only?

[1991 : 6 Marks]

Solution:

Q.8 Consider the following program segment for concurrent processes using semaphore operations P and V for synchronization. Draw the precedence graph for the statements S1 to S9.

var
a, b, c, d, e, f, g, h, i, j, k : semaphore;
begin
cobegin
begin S1; V(a); V(b); end
begin P(a); S2; V(c); V(d); end;
begin P(b); S3; V(e); end;
begin P(c); S4; V(f); end;
begin P(d); P(e); S5; V(g); end;
begin P(f); S6; V(h); V(i); end;
begin P(g); S7; V(j); end;
begin P(h); P(j); S8; V(k); end;
begin P(i); P(k); S9; end;
coend
end.

[1998 : 5 Marks]

Solution:

Q.7 Write a concurrent program using cobegin-coend to represent the precedence graph shown below.

[1998 : 5 Marks]

Solution:

S
parbegin
S_2
begin
S_3
end

[illegible]

Q.8 (a) A certain processor supports a test and set instruction which is used as follows.

TSL R1, mem; R1 = mem; mem = 1;

The instruction atomically copies the memory location into register R1 and sets the memory location to 1. The instruction is used to implement mutual exclusion as follows.

[illegible]

(b) Consider the following solution to the producer-consumer problem using a buffer of size 1. Assume that the initial value of count is 0. Assume that the [illegible] and increments to count are atomic operations.

Producer
Begin
Produce an item;
if count = 1 then sleep;
place item in buf;
Count = 1;
Wake up (Consumer);
End Producer

Consumer
Begin
Sleep;
If count = 0 then sleep;
Remove item from buffer;
Count = 0;
Wake up (producer);
Consume an item;
End Consumer

Show that in this solution it is possible for both the producer and the consumer to sleep at the same time.

[1998 : 3 Marks]

Solution:

(a) Busy waiting

1. TSL R1, MUTEX
2. cmp R1, #0
3. JNZ [illegible]
4. CS
5. Store 0[illegible]

(b) At the 3rd line in producer code the producer gets preempted. Consumer code is running and when it gets exited [illegible] = 0 and the producer process has [illegible] = 1. Now the consumer [illegible] = 1, so it sleeps. Producer [illegible] and [illegible] sleep.

Q.9 Two concurrent processes P1 and P2 want to use two resources R1 and R2 in a mutually exclusive manner. Initially, R1 and R2 are free. The programs executed by the two processes are given below.

P1 program loop L1
S1: While (R1 is busy) do nop;
S2: Set R1 = busy;
S3: While (R2 is busy) do nop;
S4: Set R2 = busy;
S5: Use R1 and R2;
S6: Set R1 = free;
S7: Set R2 = free;

P2 program loop L2
Q1: While (R2 is busy) do nop;
Q2: Set R2 = busy;
Q3: While (R1 is busy) do nop;
Q4: Set R1 = busy;
Q5: Use R1 and R2;
Q6: Set R2 = free;
Q7: Set R1 = free;

(a) Is mutual exclusion guaranteed for R1 and R2? If not, show a possible interleaving of the statements of P1 and P2 such that mutual exclusion is violated for R1 and R2 or R1 or R2 but the same [illegible].

(b) Can a deadlock occur in the above program? If yes, show a possible interleaving of the statements of P1 and P2 [illegible] deadlock.

(c) Replacing the statements Q1 and Q3 with the statements Q2 and Q4, is mutual exclusion guaranteed now? Can deadlock occur?

[2001 : 4 Marks]

Solution:

(a) Mutual exclusion guaranteed.

(b) Deadlock occurs when first 2 lines of both P_1 and P_2 are executed then after system continues in progress.

(c) No, now mutual exclusion is also not guaranteed. Deadlock continues at this situation.

Q.10 (a) Fill in the boxes below to get a solution for the reader-writer problem, using a single binary semaphore mutex (initialized to 1) and busy waiting. Write the numbers (1-8) and their contents in your answer book.

```
int R = 0, W = 0;
Reader() {
L1: wait(mutex);
    while (W == 0) {
    R = R + 1;
        [      ]_____(1)
    }
    else {
        [      ]_____(2);
    goto L1;
    }
    /* Critical Section */
    wait(mutex);
    R = R - 1;
    signal(mutex);
    }
    Writer() {
L2: wait(mutex);
    If (_____) {_____(3);
        signal(mutex);
        goto L2;
    }
    W = 1
    signal(mutex);
    /* Critical Section */
    wait(mutex);
    W = 0;
    signal(mutex);
```

(b) Give the novel solution and state reason of writer's?

[2002 : 5 Marks]

Solution:

(a) 1. wait(mutex)
2. signal(mutex)
3. w = 1 or R = 1

(b) No, starvation is not taken care for writers because whenever reader value becomes 0 it is possible w = 1 immediately.

Q.11 Consider the following precedence graph (fig.) of processes where a node represents a process and a directed edge from node P_i to node P_j implies that P_i must complete before P_j commences. Implement the DAG-graph using FORK and JOIN constructs. The actual computation done by a process may be indicated by a comment.

[2002 : 6 Marks]

Solution:

```
Int count = 0, b = 2
Lock class variable count and two processes
Join class variable variables which decrements the
associated variable and terminates the process if
new value is not zero.
S = P1
Fork L1
Fork L2
Fork L3
Fork L4
S1
Join P2, P3
Execute P4
S2
Join P4, P5
goto L5
L1 : S3
Next.
```

Q.12 The functionality of an [illegible] TEST-AND-SET assembly language instruction is given by the following C function

```
int TEST-AND-SET (int *x)
{
    int y;
    A1: y = *x;
    A2: *x = 1;
    A3: return y;
}
```

(i) Complete the following C functions for implementing mutual exclusion using busy waiting based on the above TEST-AND-SET instruction.

```
int mutex = 0;
void enter_cs()
{
while (.................................) ;
.
void leave_cs()
{
    ..............  ..............  ,
```

(ii) Is the above solution to the critical section problem deadlock free and convenient? Why?

(iii) For the above solution show by an example that mutual exclusion is not preserved if TEST-AND-SET instruction is not atomic.

[2003 : 6 Marks]

Solution:

```
void enter_cs()
{
while(TEST-AND-SET(mutex));
}
void leave_cs()
{
    mutex = 0;
}
```

(i) Solution is deadlock free but not convenient.

(ii) If process gets preempted after A1, then mutual exclusion is not ensured. The solution guarantees to lock variable, which in turn gives rise to mutual exclusion.

2. Deadlock

Q.13 Consider the resource allocation graph shown in the figure.

(a) Find out the system is in a deadlock state.

(b) Otherwise, find a safe sequence.

[1998 : 6 Marks]

Solution:

(a) No deadlock, every process sequence satisfy.

(b) P_2 P_0 P_1 P_3

Q.14 A computer system uses the Banker's Algorithm to deal with deadlock. Its current state is shown in the tables below, where P0, P1, P2 are processes, and R0, R1, R2 are resource types.

	Maximum Need				Currently Allocated		
	R0	R1	R2		R0	R1	R2
P0	7	5	3	P0	0	1	0
P1	3	2	2	P1	2	0	0
P2	9	0	2	P2	3	0	2

Available		
R0	R1	R2
3	3	2

(a) Show that the system can be in a safe state.

(b) When will system do so if a request by process P0 for one unit of resource type R0?

[1998 : 5 Marks]

Solution:

(a) 1. Find first remaining need = maximum need − current allocation

2. Then try to satisfy the remaining need with help of available value. If a process, it it is satisfied then safe state otherwise not safe sequence = P_1 P_0 P_2

P_1 P_0 P_2

(b) When P_0 asks for one unit of resource R_0 then first check available first R_0, if it is ok then give that to P_0

3. Memory Management and Virtual Memory

Q.15 Let [illegible] page reference and the working [illegible] respectively. The initial working set at time [illegible] page [illegible] was referenced at time [illegible] and was referenced at time [illegible] determine the total number of page faults and the average number of page frames used by using the working set strategy.

[1995 : 5 Marks]

Solution:

Next [illegible]

So no change

Now [illegible] change

Now no change

Total page faults = 4

Avg. no. of page frames are used :

Q.16 The following [illegible] in the given sequence were generated by a program:

[illegible]

This program [illegible] page size [illegible] page. Indicate the page numbers [illegible] for the following page replacement algorithms.

(a) LRU (b) FIFO (c) OPT

Assume that the main memory is empty initially.

[1995 : 5 Marks]

Solution:

(a) LRU: Initially [illegible] page faults.

[illegible] page faults

(b) FIFO

[illegible] page faults

Q.17 A [illegible] has [illegible] of main memory. The [illegible] and finish in the following sequence.

Job 1 requiring 60k memory

Job 2 requiring 500k memory

Job 3 requiring 400k memory

Job 1 finishes

Job 4 requiring 100k memory

Job 5 requiring 300k memory

Job 6 requiring 600k memory

(i) Draw the memory allocation using Best Fit and First fit algorithms

(ii) Which algorithm performs better for the [illegible]?

[1996 : 6 Marks]

Solution:

(i) Using Best Fit

Now [illegible] finished

[illegible]

Using Direct Pic

Delete Location.

All is totally gone memory

(b) Worst fit algorithm is best for the sequence.

Q.15 A demand paged virtual memory system has 16 bit virtual address, page size of 256 bytes, and has [illegible] of main memory. LRU page replacement is implemented using [illegible] whose current contents (in decimal) is

[illegible]

LRU page

Use such hexadecimal addresses in the following sequence given below

0087F, 0100, 00FF, 0180

Then state,

(i) The new status of the first [illegible] page table, if any, and

(ii) [illegible]

[1996 : 5 Marks]

Solution:

Main memory = [illegible]

Page size = [illegible]

$$\text{No. of frames} = \frac{2^{11}}{2^{8}} = 2^{3} = 8$$

Assume page number in PMT

Currently page no. 1, 3 & 6 are shown on the first row. Let number be numbered as [illegible]

(i) If (convert the page no. into decimal) = [illegible]

Page [illegible] is the line.

Replace the page no. [illegible]

So,

(ii) The new status of the table

[illegible]

iii. No. of page faults = [illegible]

iii. No. of page replacements = 0

Q.16 In a computer system which uses pure demand paging, the following page [illegible] were noted:

Page table access time in ns	[illegible]
Disk access time in ms	[illegible]
Page fault rate	[illegible]

What will be the EAT in nanoseconds?

[1998 : 5 Marks]

Solution:

Now EAT. Have to work all 24 hours completely which takes 10 units of time.

After this 10% page faults and 90% [illegible] completion time unit, then it has 20% page faults [illegible]

Total time = 25 units.

Q.17 A uniprocessor system has the segmented page memory structure for virtual memory. The memory is byte addressable. Both virtual and physical address space consists of 32 bytes each. The virtual address space is divided into [illegible] equal size segments. The Memory Management Unit (MMU) has a segment table, each entry of which contains the physical address of the page table for the segment. Page table entries are stored in the main memory and occupies 4 byte page table entries.

(i) What is the minimum page size in bytes that is required for a segment to require no more than one page table page? Assume that the page size can only be a power of 2?

(ii) Now suppose that the page size is 512 bytes. To speed up address translation, the processor TLB will be organized as a single set of [illegible] entries. Assuming the recently referenced set of pages, [illegible] will use the direct mapping

[illegible] with each cache entry?

(iv) Assume that each [illegible] entry [illegible] for page protection [illegible]. How many bits are available in page table entry for storing the paging information for the page? Assume that the page size [illegible] bytes.

[1999 : 6 Marks]

Solution:

(ii) Minimum page size = 28 bytes

Let page size = 2^n bytes

Page table size = $2^{24} \times 2$ bytes

$= 2^{24+1}$ bytes

Page size $\geq \frac{2^{24+1}}{2^{24}} = 2$

If page size = 2 bytes then page size

$= 2^1 = 28$ bytes

(iv) [illegible]

Page table entry size

$= 1000 - 2^{24} = 7$ bits

So [illegible]

Therefore available for storage of other information [illegible] paging information [illegible].

4. File System and Device Management

Q.21 A certain moving arm disk storage has the following specifications:

Number of bytes per track = 20 K

Rotation time = 16.67 ms

Disk speed = 3600 rpm

Average seek time = 30 ms

Return the average latency, the disk storage capacity and the data transfer rate.

[1990 : 6 Marks]

Solution:

Disk storage capacity = Track storage capacity × No. of tracks

$= 20 \times 4004 \times 200$ bytes

Data transfer rate

$\frac{20 \text{ K}}{16.67 \text{ ms}} = 60$ [illegible]

$t = \frac{60}{3600} = \frac{1}{60}$ [illegible]

Average latency $= \frac{1}{2n} = 8.33$ [illegible]

[illegible] = 20000 B

1 sec ⟶ 60 × 1000.00 B

= 20 × 1000.00 bytes/sec

Q.22 Assuming [illegible] disk [illegible] the cylinders are by 1, 36, 16, 34, 9, 12, 10, 36, 8, 30 and 50 find the sequence [illegible] using

(i) shortest seek time first (SSTF)

(ii) [illegible] scheduling policy

[1999 : 6 Marks]

Solution:

(i) SSTF

Sequence: 30, 40, 36, 50, 34, 36, 38, 20, 18, 8, 9, 1

(ii) [illegible] scheduling. Arm moves towards [illegible]

Sequence: 30, 34, 36, 38, 36, 40, 40, 20, 18, 8, 9

Q.28 [illegible]

[illegible] the following parameters:

[illegible]

[illegible] = 12 cm

[illegible] = 4 cm

Inter track space: 0.1 mm

[illegible] = 20

[1995 : 5 Marks]

Solution:

[illegible]

$$= \text{[illegible]} = \frac{4\text{cm}}{0.1\text{mm}}$$

$$= 400$$

[illegible]

$$= 400 \times 20 \times 4096 \text{ bytes}$$

Disk capacity $= 0 \times 400 \times 20 \times 4096$ bytes

$= 2^{27}$ MB

Q.34 [illegible] 100 [illegible] a sequence of track 55. Before queue of requests kept in FIFO order is 100, 50, [illegible], 55, 85.

Which of [illegible] algorithm [illegible] 1995

[illegible] and SSTF [illegible] total head movement for each of the algorithms.

[1995 : 5 Marks]

Solution:

FIFO:

Total number of head movement = 4

SSTF:

Total number of head movement = 2

Q.35 A [illegible] system [illegible] implemented on a disk with disk block size of 2 Kbyte. The disk [illegible]

Disk block 0	[illegible]
Disk block 1	[illegible]
Disk block 2	Data block 1:
Disk block 3	Data block [illegible]

(a) [illegible] maximum possible number of files

(b) [illegible] maximum possible file size in blocks?

[2002 : 5 Marks]

Solution:

Disk block size = 2 Kb

(a) Maximum possible number of files

$$= \frac{\text{Disk block size}}{\text{File size}} = \frac{2^{11}}{2^{2}} = 2^{9} = 512$$

(b) Maximum possible file size

$= \text{Disk block size} = 2^{2}$