Social Network Analysis – Network Dynamics

Cascade

Network Effects

- Application – Viral Marketing
- Information Cascade
- **Cascading Behavior in Networks**
- Epidemics

Views of SNA

- An individual's choices depend on what other people do
  - Example: Information cascades, Rich-get-richer etc.
- When we perform analysis of how innovations are adopted by a population the underlying social network can be considered at two conceptually very different levels of resolution:
  - We view the network as a relatively amorphous population of individuals, and look at effects in aggregate
  - We move closer to the fine structure of the network as a graph, and look at how individuals are influenced by their particular network neighbors.

Diffusion of Innovation

- Studies have shown that innovation can be adopted in two ways:
  - As people observed the decisions of their network neighbors, it provided indirect information that led them to try the innovation as well.
  - Decisions about adoption were driven primarily by direct-benefit effects rather than informational ones.
- An innovation can fail to spread through a population, even when it is has significant *relative advantage* compared to existing practices [2]
  - The success of an innovation also depends on its *complexity* for people to understand and implement
  - Its *observability*, so that people can become aware that others are using it
  - Its *trialability*, so that people can mitigate its risks by adopting it gradually
  - Its overall *compatibility* with the social system that it is entering
- *The* principle of homophily can sometimes act as a barrier to diffusion
  - Since people tend to interact with others who are like themselves, while new innovations tend to arrive from "outside" the system, it can be difficult for these innovations to make their way into a tightly-knit social community

Modeling Diffusion through a Network

- Network models based on direct-benefit effects involve the following underlying consideration:
  - You have certain social network
  - The benefits to you of adopting a new behavior increase as more and more of these neighbors adopt it
  - Simple self-interest will dictate that you should adopt the new behavior once a sufficient proportion of your neighbors have done so

A Networked Coordination Game (2 players)

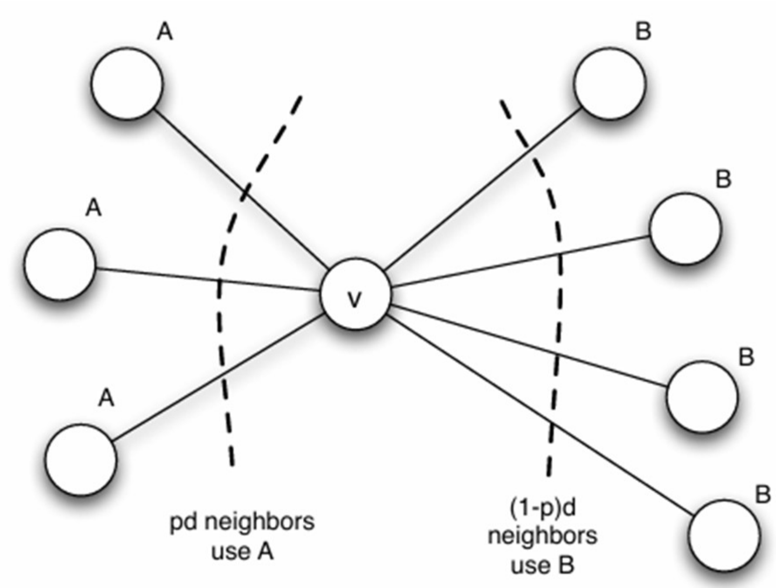
- Each node v and w has a choice between two possible behaviors, labeled A and B.
- If they are linked by an edge, then there is an incentive for them to have their behaviors match.
- We represent this using a game in which v and w are the players and A and B are the possible strategies.
- The payoffs are defined as follows:
  - If v and w both adopt behavior A, they each get a payoff of $a > 0$
  - If they both adopt B, they each get a payoff of $b > 0$
  - If they adopt opposite behaviors, they each get a payoff of 0

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{w} \\ & & A & B \\ \cline{3-4} \multirow{2}{*}{v} & A & a, a & 0, 0 \\ \cline{3-4} & B & 0, 0 & b, b \\ \cline{3-4} \end{array}$$

A Networked Coordination Game

- Node v is playing a copy of this game with each of its neighbors, and its payoff is the sum of its payoffs in the games played on each edge.
- If p fraction of v's neighbors have behavior A, and $(1 - p)$ fraction have behavior B
- if v has d neighbors, then pd adopt A and $(1 - p)d$ adopt B,
- So if v chooses A, it gets a payoff of pda, and if it chooses B, it gets a payoff of $(1 - p)db$.
- Thus, A is the better choice if $pda \geq (1 - p)db$, or $p \geq b/(a + b)$
- *From now on $q=b/(a+b)$*

Cascading Behavior

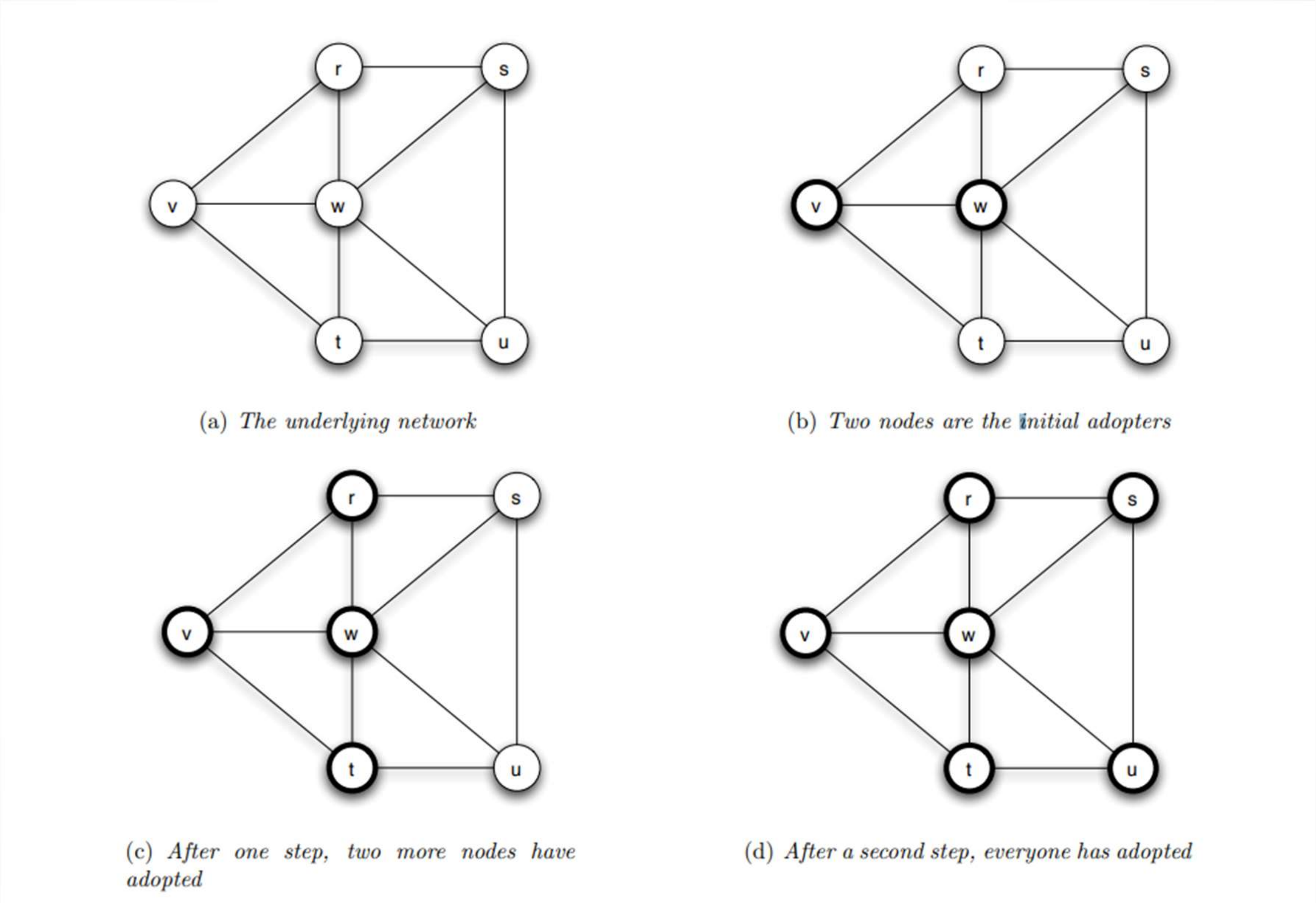


v must choose between behavior A and behavior B, based on what its neighbors are doing

Cascading Behavior

- Consider a set of initial adopters who start with a new behavior A, while every other node starts with behavior B.
- Nodes then repeatedly evaluate the decision to switch from B to A using a threshold of q.
- If the resulting cascade of adoptions of A eventually causes every node to switch from B to A, then we say that the set of initial adopters causes a **complete cascade** at threshold q

Cascading Behavior



(a) *The underlying network*

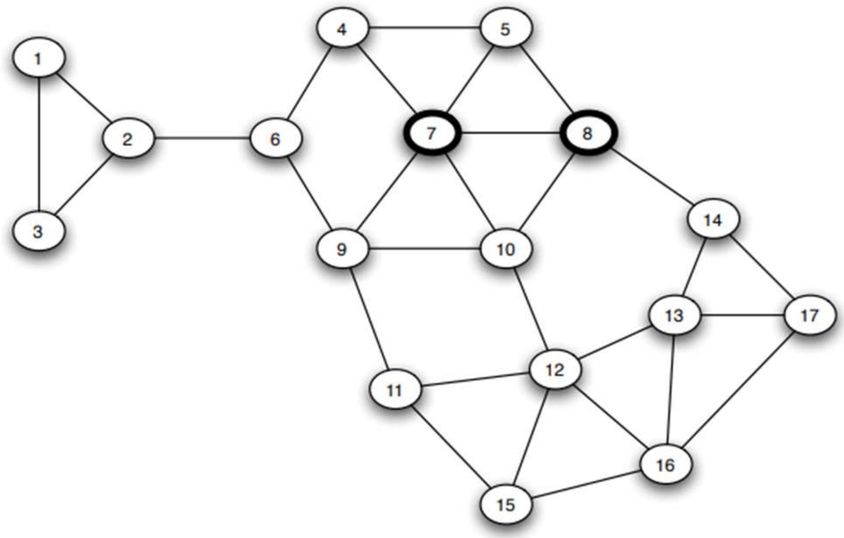
(b) *Two nodes are the initial adopters*

(c) *After one step, two more nodes have adopted*

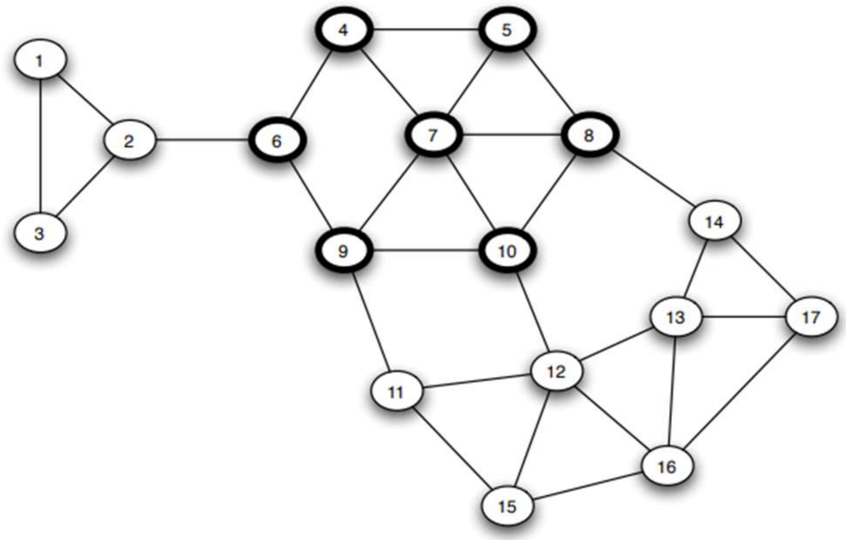
(d) *After a second step, everyone has adopted*

Starting with *v* and *w* as the initial adopters, and payoffs *a* = 3 and *b* = 2, the new behavior *A* spreads to all nodes in two steps.

Cascading Behavior - Incomplete



(a) *Two nodes are the initial adopters*



(b) *The process ends after three steps*

Restarting Incomplete Cascades

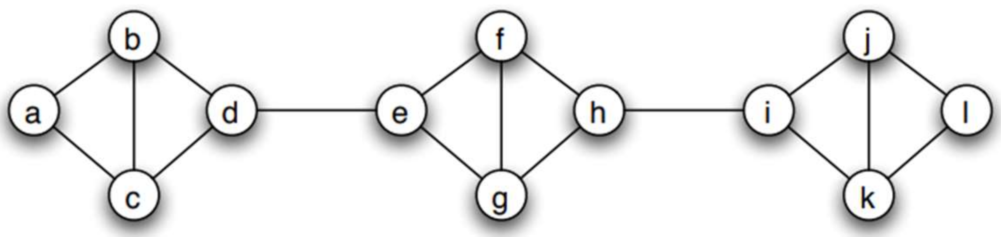
- Tightly-knit communities in the network can work to hinder the spread of an innovation.
- Homophily can often serve as a barrier to diffusion.
- However if the payoff increase the situation will change.
  - For example if we change the payoff of A from 3 to 4.
- Viral Markrting: The marketer can target particular nodes
  - For example, nodes 12 or 13 to again start the cascade effect.
  - No effect if node 11 or 14 are chosen

Differences between Population-level cascading and Network-level cascading

- In a population-level model, when everyone is evaluating their adoption decisions based on the fraction of the entire population that is using a particular technology, it can be very hard for a new technology to get started, even when it is an improvement on the status quo.
- In a network, however, where you only care about what your immediate neighbors are doing, it's possible for a small set of initial adopters to essentially start a long fuse running that eventually spreads the innovation globally.
- This idea that a new idea is initially propagated at a local level along social network links is something one sees in many settings where an innovation gains eventual widespread acceptance

Clusters

- A cluster of density p is a set of nodes such that each node in the set has at least a p fraction of its network neighbors in the set.



4 node clusters of density 2/3

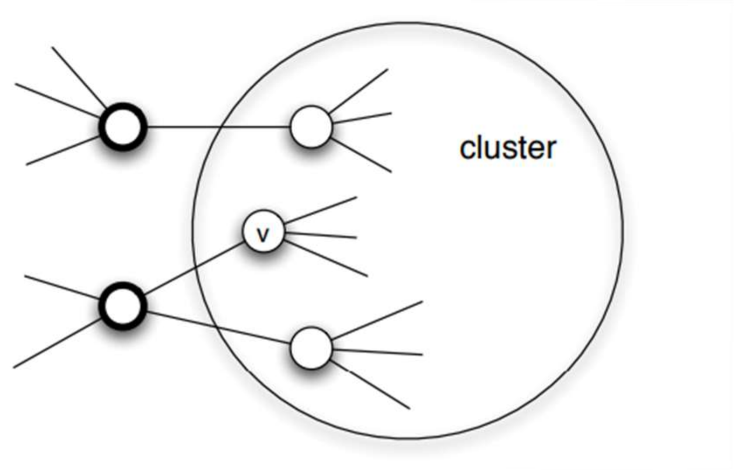
Clusters & Cascade

- Consider a set of initial adopters of behavior A, with a threshold of q for nodes in the remaining network to adopt behavior A.
  - If the remaining network contains a cluster of density greater than $1 - q$, then the set of initial adopters will not cause a complete cascade.
  - Moreover, whenever a set of initial adopters does not cause a complete cascade with threshold q, the remaining network must contain a cluster of density greater than $1 - q$.

- Thus clusters truly are natural obstacles to cascades [3]:
  - Clusters block the spread of cascades
  - Whenever a cascade comes to a stop, there's a cluster that can be used to explain why.

Clusters are obstacles of Cascade (1/2)

- The spread of a new behavior, when nodes have threshold q, stops when it reaches a cluster of density greater than (1 − q)
- Proof:
  - Assume the opposite — that some node inside the cluster does eventually adopt A
  - Consider the earliest time step t at which some node inside the cluster does so.
  - Let v be the name of a node in the cluster that adopts A at time t.

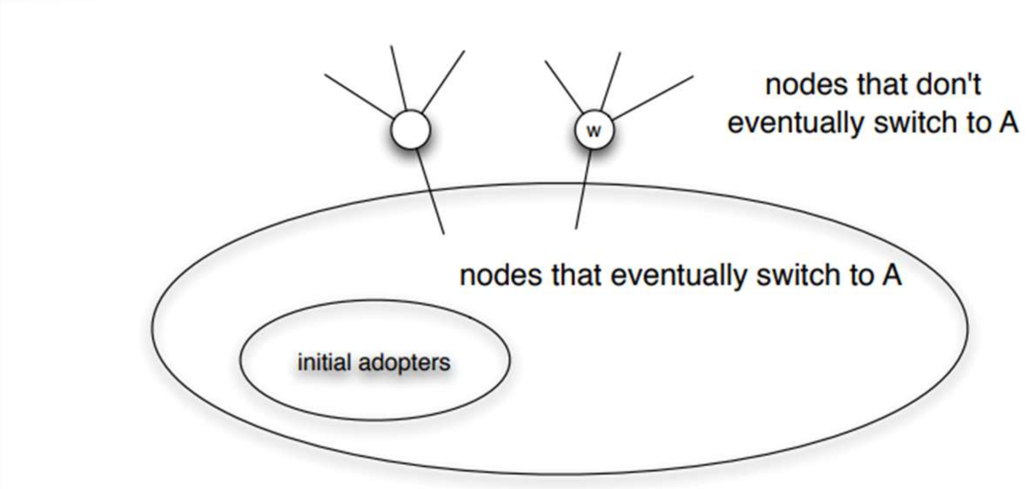


Clusters are obstacles of Cascade (2/2)

- At the time that v adopted A, its decision was based on the set of nodes who had adopted A by the end of the previous time step.
- Since no node in the cluster adopted before v did, the only neighbors of v that were using A at the time it decided to switch were *outside* the cluster.
- But since the cluster has density greater than $1-q$, more than a $1-q$ fraction of v's neighbors are inside the cluster, and hence less than a q fraction of v's neighbors are outside the cluster.
- Since these are the only neighbors who could have been using A, and since the threshold rule requires at least a q fraction of neighbors using v, this is a contradiction.
- Hence our original assumption, that some node in the cluster adopted A at some point in time, must be false.

Clusters are the only obstacles to Cascades (1/2)

- If the spread of A stops before filling out the whole network, the set of nodes that remain with B form a cluster of density greater than 1 – q.

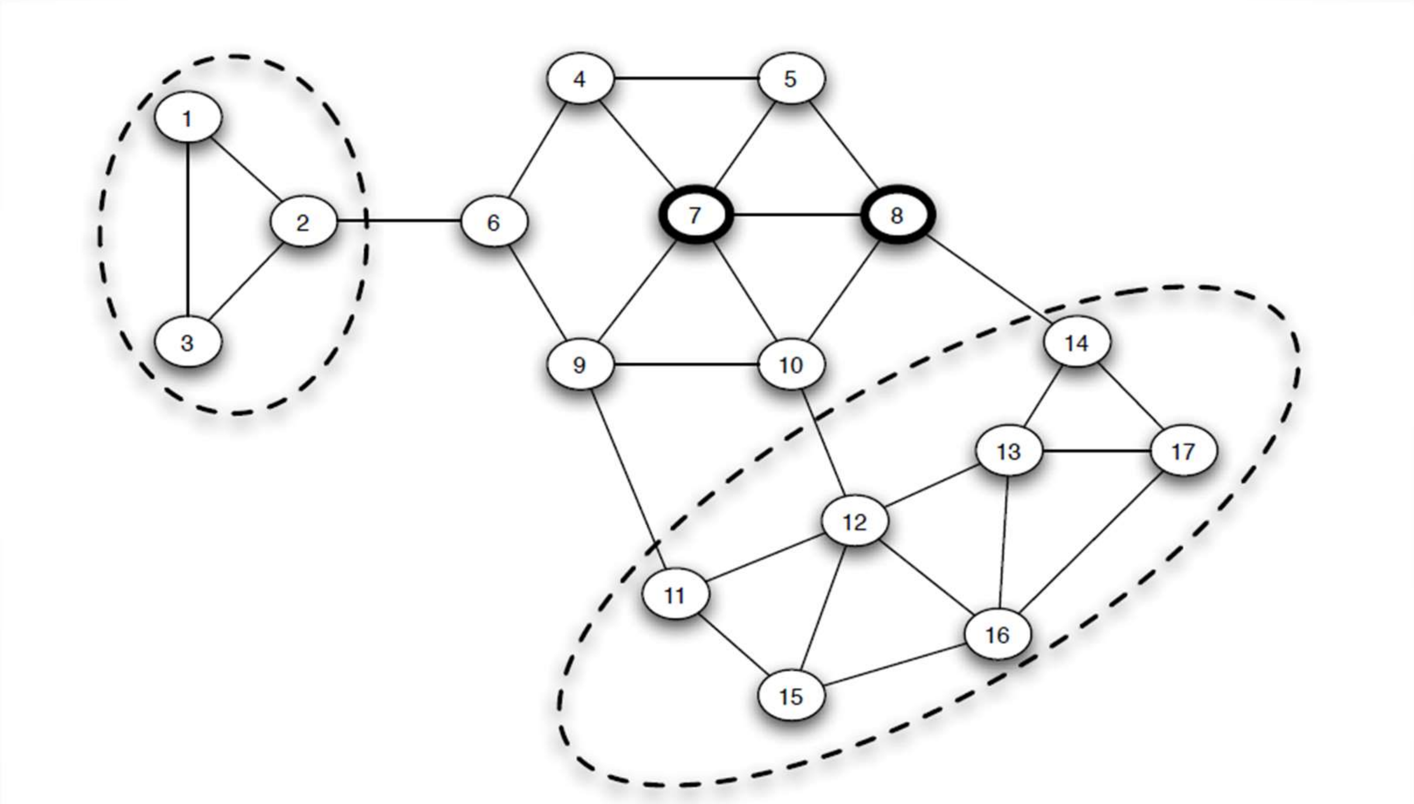


- Proof:
  - We show that whenever a set of initial adopters fails to cause a complete cascade with threshold q, there is a cluster in the remaining network of density greater than (1-q).
  - Consider running the process by which A spreads, starting from the initial adopters, until it stops.
  - It stops because there are still nodes using B, but none of the nodes in this set want to switch

Clusters are the only obstacles to Cascades (2/2)

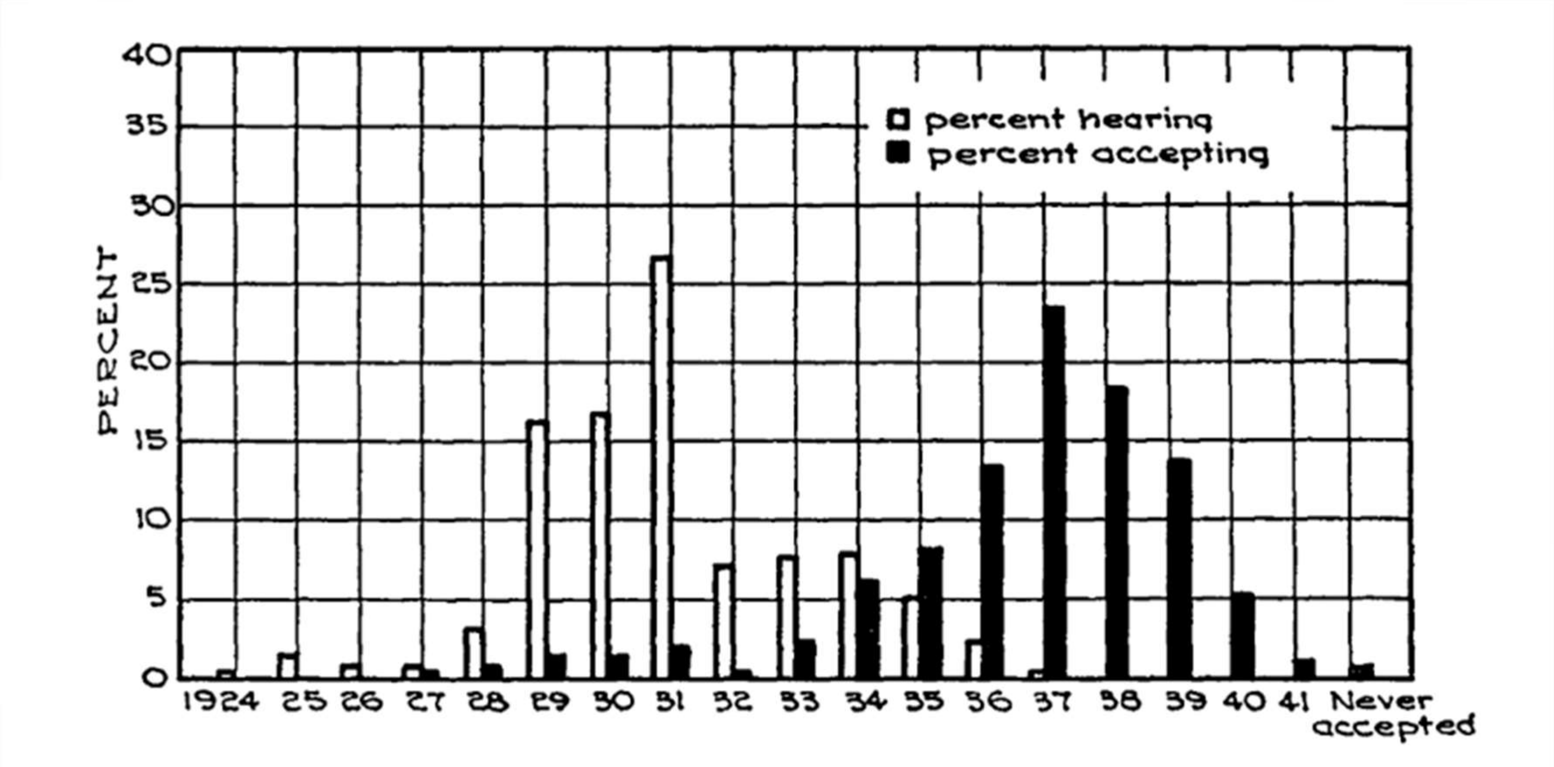
- Proof:
  - Let S denote the set of nodes using B at the end of the process.
  - To see why S is a cluster of density greater than $1 - q$, consider any node w in this set S.
  - Since w doesn't want to switch to A, it must be that the fraction of its neighbors using A is less than q — and hence that the fraction of its neighbors using B is greater than $1 - q$.
  - But the only nodes using B in the whole network belong to the set S, so the fraction of w's neighbors belonging to S is greater than $1-q$.
  - Since this holds for all nodes in S, it follows that S is a cluster of density greater than $1 - q$.

Cluster - Example



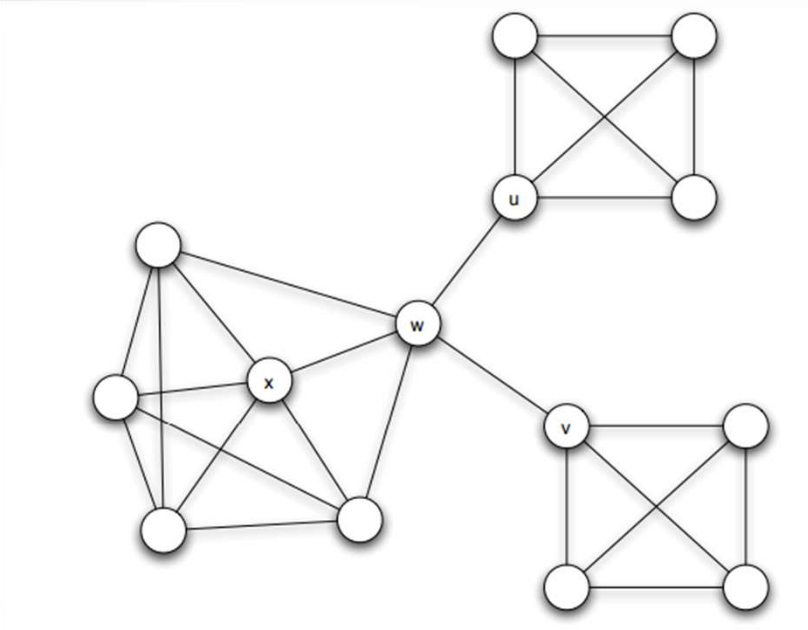
Clusters of density greater than 3/5 block the spread of *A* at threshold 2/5.

Difference between learning about and adopting innovation



The years of first awareness for hybrid seed corn significantly precedes the wave of adoptions [4]

Weak ties form local bridges in a social network



The u-w and v-w edges are more likely to act as conduits for information than for high-threshold innovations

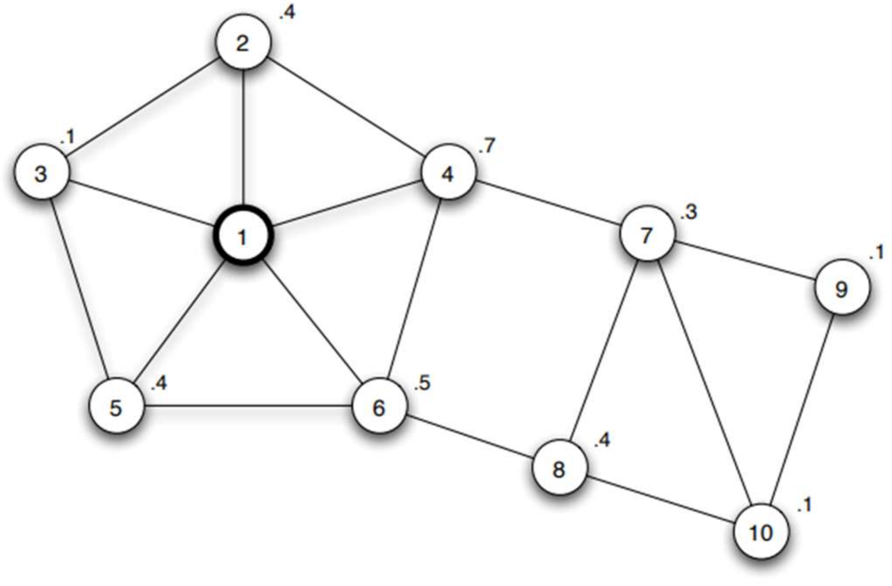
- Bridges in a social network: are powerful ways to convey awareness of new things
- However they are weak at transmitting behaviors that are in some way risky or costly to adopt — behaviors where you need to see a higher threshold of neighbors doing it before you do it as well.

Heterogenous Thresholds

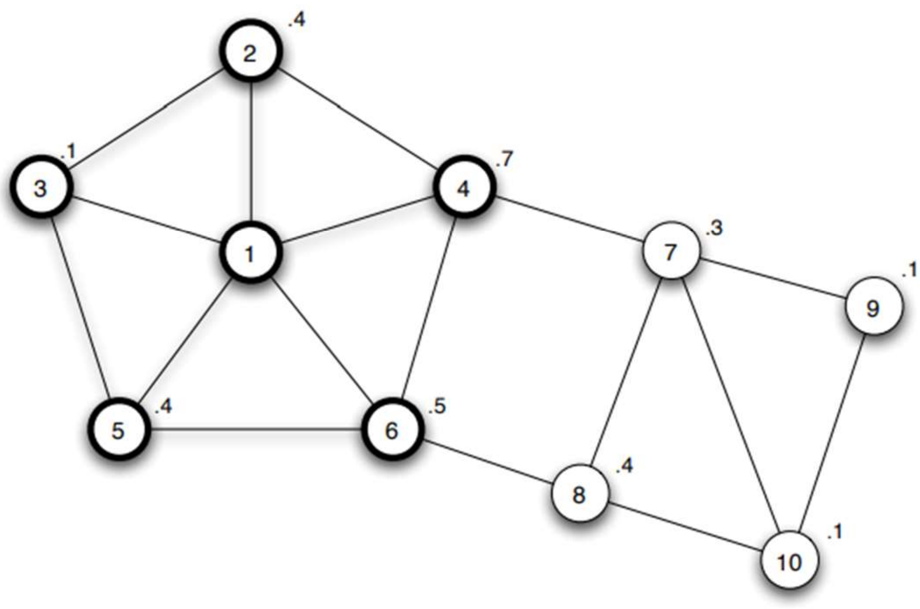
- Suppose that each person in the social network values behaviors A and B differently.
- Thus, for each node v, we define a payoff a_v — labeled so that it is specific to v — that it receives when it coordinates with someone on behavior A,
- We define a payoff b_v that it receives when it coordinates with someone on behavior B.

		w: A	w: B
v	A	a_v, a_w	$0, 0$
	B	$0, 0$	b_v, b_w

Heterogenous Thresholds



(a) *One node is the initial adopter*



(b) *The process ends after four steps*

$$p \geq \frac{b_v}{a_v + b_v}.$$

Heterogenous Thresholds Observations

- For understanding the spread of behaviors in social networks, we need to take into account not just the power of influential nodes, but also the extent to which these influential nodes have access to easily *influenceable* people
  - Example: Despite node 1's "central" position, it would not have succeeded in converting anyone at all to *A* were it not for the extremely low threshold on node 3

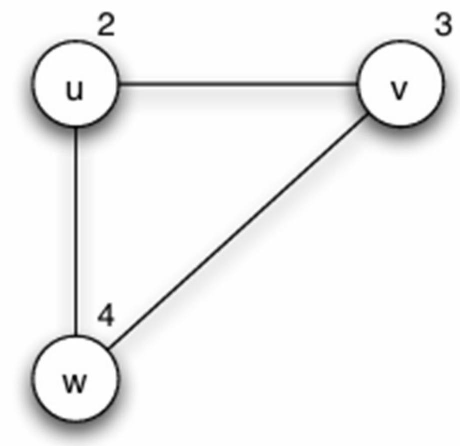
- Given a set of node thresholds, *blocking cluster* in the network is a set of nodes for which each node *v* has more than a $1-q_v$ fraction of its friends also in the set.
  - A set of initial adopters will cause a complete cascade — with a given set of node thresholds — if and only if the remaining network does not contain a blocking cluster.

Knowledge & Thresholds

- Integrating network effects at both the population level and the local network level.
- Situations where coordination across a large segment of the population is important
- The underlying social network is serving to transmit information about people's willingness to participate
- **Collective Action problem**: An activity produces benefits only if enough people participate.
- **Pluralistic Ignorance**: People have wildly erroneous estimates about the prevalence of certain opinions in the population at large [5]

Example

Each node in the network has a threshold for participation, but only knows the threshold of itself and its neighbors.

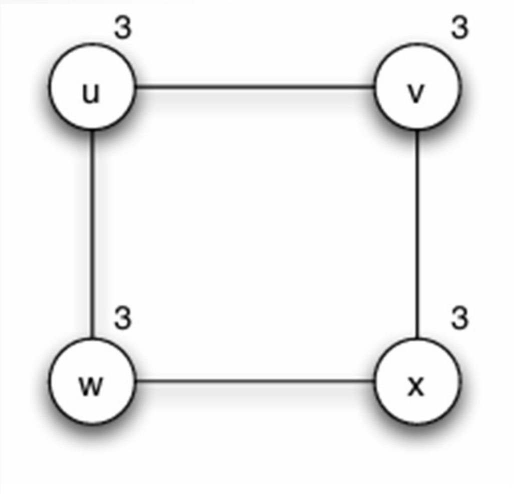


(a) *An uprising will not occur*

- Node w would only join the protest if at least four people do; since there are only three people in total, this means he will never join.
- Node v knows that w's threshold is four, so v knows that w won't participate. Since v requires three people in order to be willing to join, v won't participate either.
- Finally, u only requires two people in order to participate, but she knows the thresholds of both other nodes, and hence can determine that neither will participate. So she doesn't either.

Example

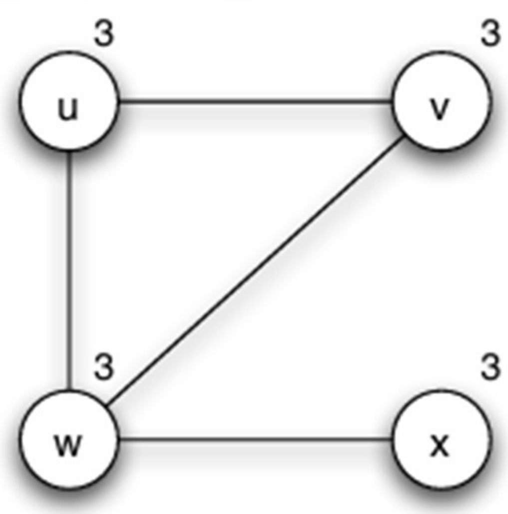
Sometimes nodes must reason about what other nodes know in order to reason about what they will do.



(b) *An uprising will not occur*

- u knows that v and w each have a threshold of three, and so each of u, v, and w would feel safe taking part in a protest that contained all three of them.
- But she also knows that v and w don't know each other's thresholds, and so they can't engage in the same reasoning that she can.
- Since u doesn't know x's threshold, there's the possibility that it's something very high, like 5. In this case, node v, seeing neighbors with thresholds of 3 and 5, would not join the protest. Neither would w. So in this case, if u joined the protest, she'd be the only one — a disaster for her. Hence, u can't take this chance, and so she doesn't join the protest.

Example



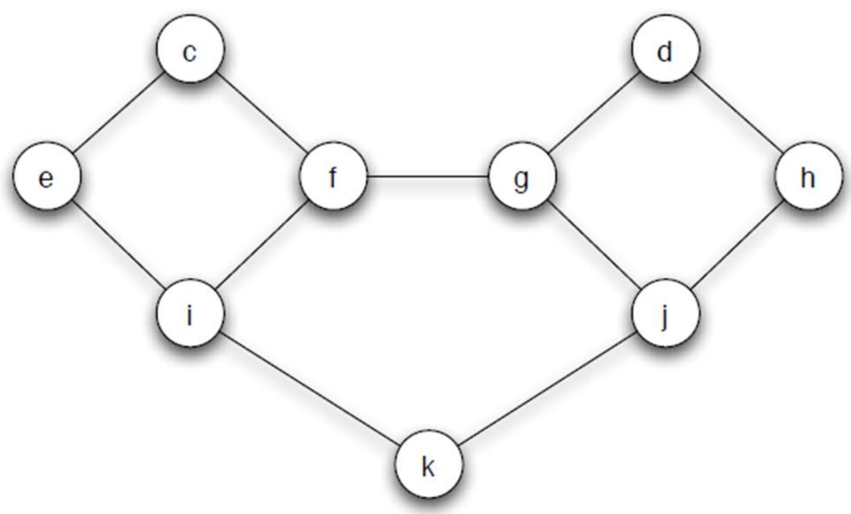
(c) *An uprising can occur*

- Now, each of u, v, and w not only knows the fact that there are three nodes with thresholds of 3
- This fact is common knowledge: among the set of nodes consisting of u, v, and w, each node knows this fact, each node knows that each node knows it, each node knows that each node knows that each node knows it, and so on indefinitely.

The differences between the examples in (b) and (c) are subtle, and come down to the different networks' consequences for the knowledge that nodes have about what others know.

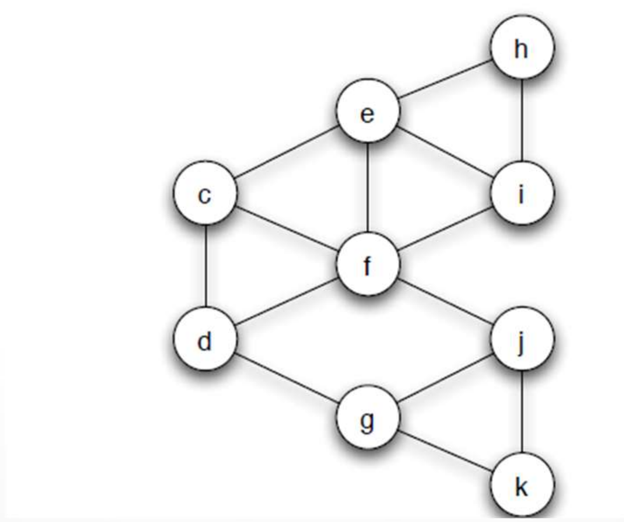
Exercises

1. Consider the network depicted below; suppose that each node starts with the behavior *B*, and each node has a threshold of $q = ½$ for switching to behavior *A*.
   a. Now, let *e* and *f* form a two-node set *S* of initial adopters of behavior *A*. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to *A*?
   b. Find a cluster of density greater than $1 - q = 1/2$ in the part of the graph outside *S* that blocks behavior *A* from spreading to all nodes, starting from *S*, at threshold *q*.



Exercises

2. Suppose we have the social network depicted below; suppose that each node starts with the behavior *B*, and each node has a threshold of $q = 2/5$ for switching to behavior *A*.
   a. Now, let *c* and *d* form a two-node set *S* of initial adopters of behavior *A*. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to *A*?
   b. Find a cluster of density greater than $1 - q = 3/5$ in the part of the graph outside *S* that blocks behavior *A* from spreading to all nodes, starting from *S*, at threshold *q*.
   c. Suppose you were allowed to add a single edge to the given network, connecting one of nodes *c* or *d* to any one node that it is not currently connected to. Could you do this in such a way that now behavior *A*, starting from *S* and spreading with a threshold of 2/5 would reach all nodes?



References

1. Lisa R. Anderson and Charles A. Holt. Classroom games: Information cascades. Journal of Economic Perspectives, 10(4):187–193, Fall 1996
2. Everett Rogers. Diffusion of Innovations. Free Press, fourth edition, 1995.
3. Stephen Morris. Contagion. Review of Economic Studies, 67:57–78, 2000.
4. Bryce Ryan and Neal C. Gross. The diffusion of hybrid seed corn in two Iowa communities. Rural Sociology, 8:15–24, 1943
5. Hubert J. O'Gorman. The discovery of pluralistic ignorance: An ironic lesson. Journal of the History of the Behavioral Sciences, 22:333–347, 1986

Readings

- David Easley and Jon Kleinberg. Networks, Crowds, and Markets: Reasoning About a Highly Connected World

  https://www.cs.cornell.edu/home/kleinber/networks-book/

  - Chapter 16
  - Chapter 19.1-19.6

- Social Network Analysis Tanmoy Chakraborty. Chapter 7