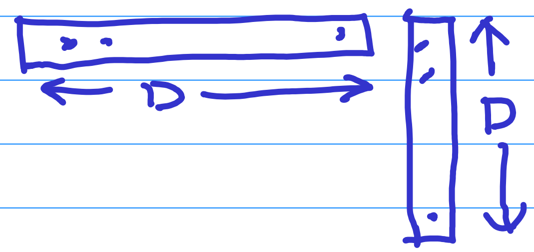
Squared Mahalanobis distance

$$d_M^2 \triangleq (\underline{x}_1 - \underline{x}_2)^T \Sigma^{-1} (\underline{x}_1 - \underline{x}_2)$$

$$d_E^2 \triangleq (\underline{x}_1 - \underline{x}_2)^T (\underline{x}_1 - \underline{x}_2)$$

↳ Euclidean

$$(x_{1_1} - x_{2_1})^2 + \cdots (x_{1_j} - x_{2_j})^2 + \cdots (x_{1_D} - x_{2_D})^2$$



Metric property
1. $d(\underline{a}, \underline{b}) \geq 0$
2. $d(\underline{a}, \underline{b}) = d(\underline{b}, \underline{a})$
3. Triangle inequality.

Example

$$\begin{bmatrix} \text{temp} \\ \text{pressure} \\ \vdots \end{bmatrix} \begin{matrix} \rightarrow 100^\circ C \\ \rightarrow 0{\cdot}01 \text{ mbars.} \\ \end{matrix}$$

Euclidean distance is swamped by a term whose variation is large.

$$X = \frac{x - \mu}{\sigma}, \text{ for each feature/dimension}$$

$$\begin{bmatrix} \vdots \\ x \\ \vdots \end{bmatrix}$$

Euclidean distance cannot account for 2 or more features/dimensions being correlated.

$$\left[(x_{1_1}-x_{2_1}) \cdots (x_{1_j}-x_{2_j}) \cdots (x_{1_D}-x_{2_D})\right] \Sigma^{-1} \begin{bmatrix} x_{1_1}-x_{2_1} \\ \vdots \end{bmatrix}$$

off-diagonal covariance terms

$$\begin{bmatrix} \frac{1}{\sigma_1^2} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \frac{1}{\sigma_2^2} & & \cdot & \cdot \\ \cdot & \cdot & & \ddots & \\ \cdot & \cdot & \cdot & & \frac{1}{\sigma_D^2} \end{bmatrix}_{D\times D}$$

Mahalanobis distance $\rightarrow$ Distance between distributions

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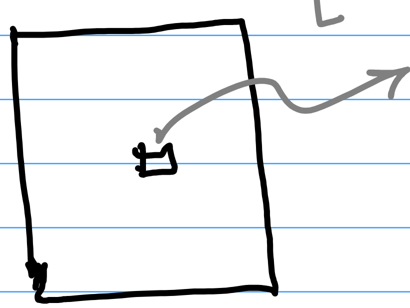
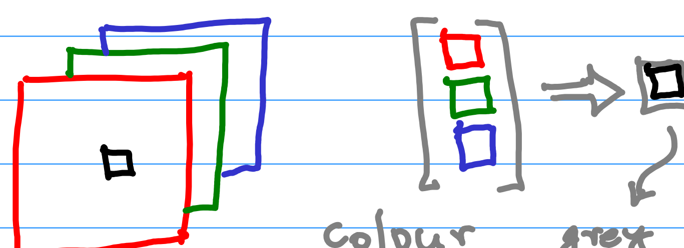
[row, column] 512×512 image



(r,c)

each pixel : 1 byte

Greyscale $\sim\frac{1}{4}$ MB



colour at this pixel (r,c)

grey scale

suppose that we work with greyscale images (scalars)

Model $x = \sum_{j=1}^{K} \pi_j \mathcal{N}(x \mid \mu, \sigma^2)$

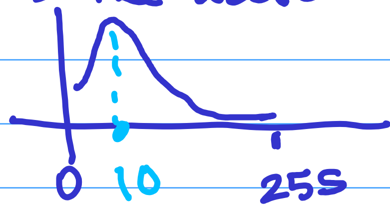
colour images $\underline{x}_{3\times1}$, $\underline{\mu}_j$ 3×1, Σ_j 3×3

Example: movie camera staring at the sea.

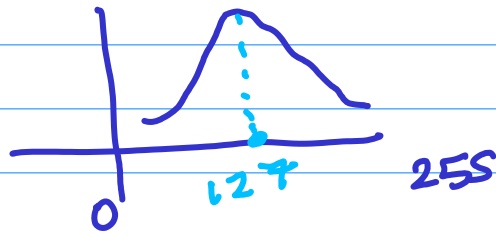
Consider the (r, c) pixel.

Realistic: (1) it sees the water (dark)

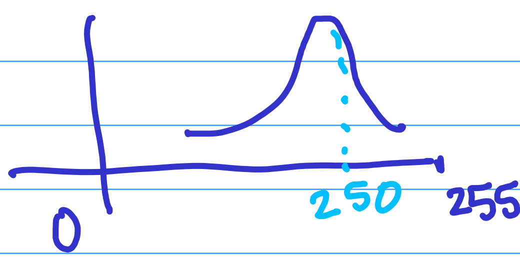
low grey value, close to 0



(2) a wave breaks, foam (higher grey value)



(3) Sunlight could be reflected off the water (very bright)



Mixture of $K = 3$ gaussians.