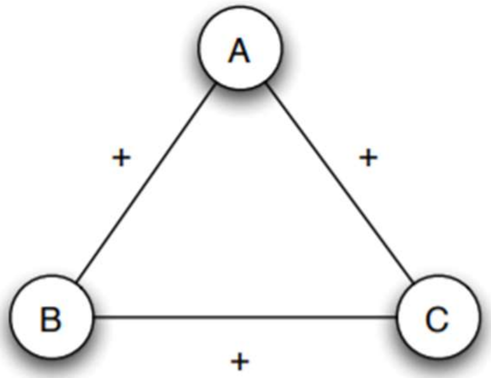
Social Network Analysis

Positive & Negative Relationships

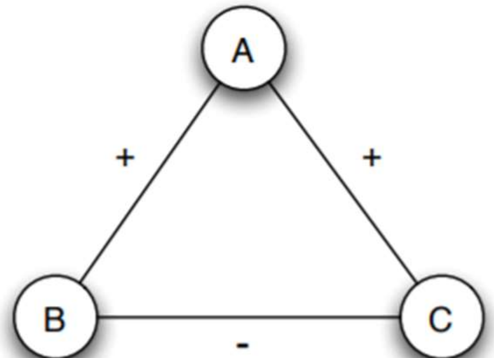
Positive & Negative Relationships

- Required in many scenarios
  - Friends/Enemies
  - International Relations
  - Opinion

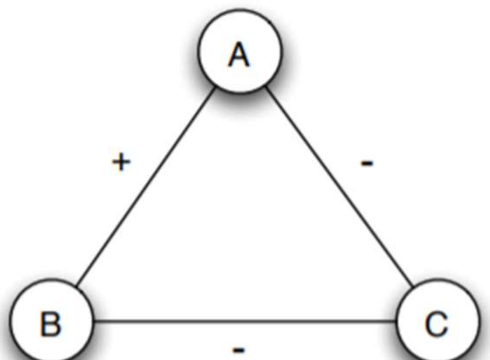
3 Nodes – 4 Scenarios



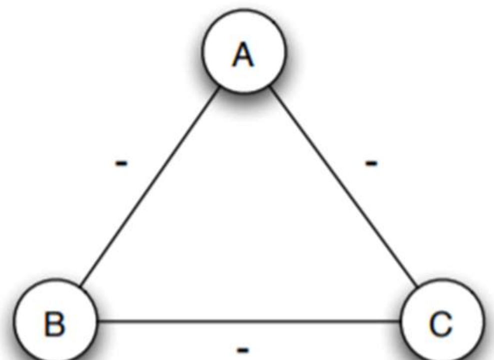
(a) *A, B, and C are mutual friends: balanced.*



(b) *A is friends with B and C, but they don't get along with each other: not balanced.*



(c) *A and B are friends with C as a mutual enemy: balanced.*



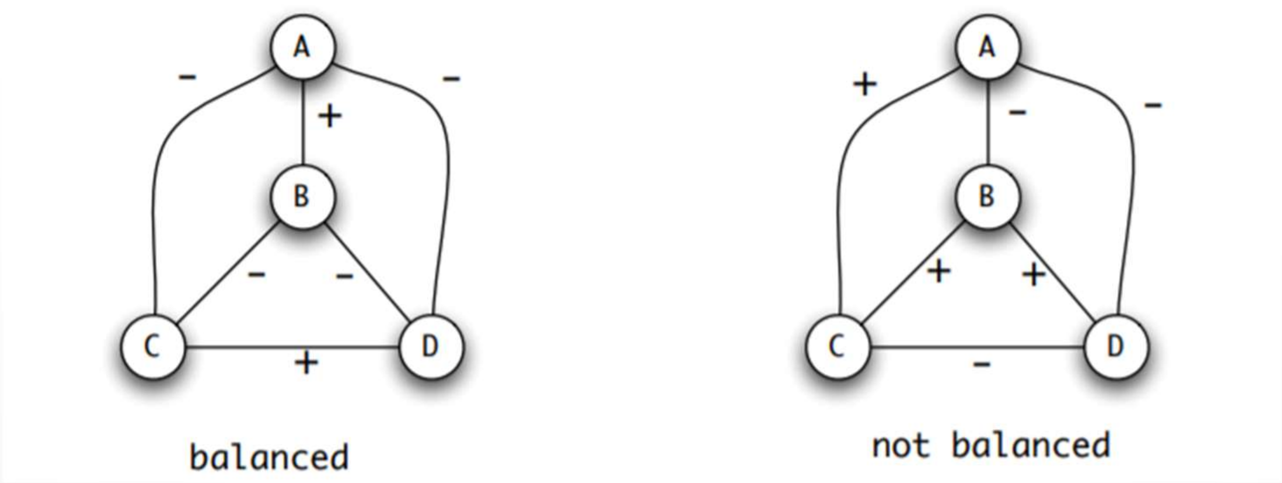
(d) *A, B, and C are mutual enemies: not balanced.*

3 Nodes – 4 Scenarios

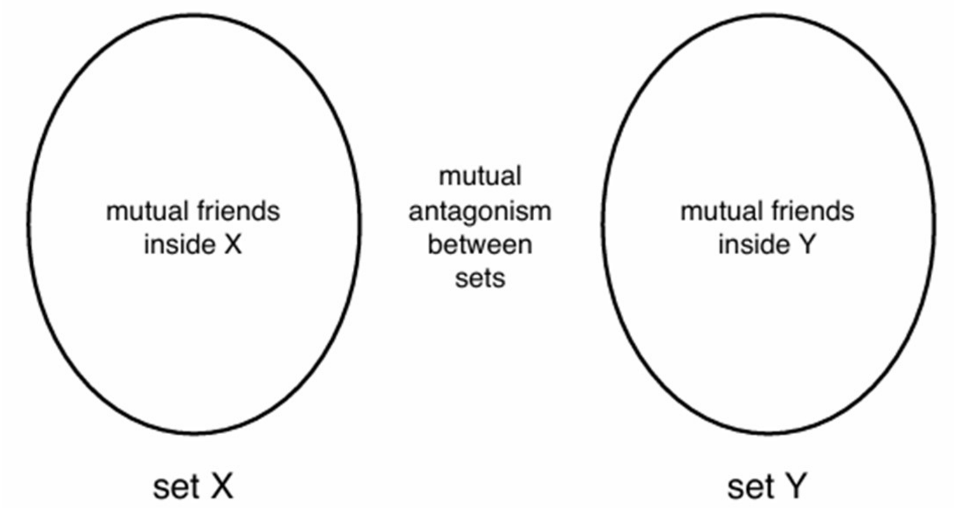
1. Given a set of people A, B, and C, having three pluses among them is a very natural situation: it corresponds to three people who are mutual friends.
2. Having a single plus and two minuses in the relations among the there people is also very natural: it means that two of the three are friends, and they have a mutual enemy in the third.
3. A triangle with two pluses and one minus corresponds to a person A who is friends with each of B and C, but B and C don't get along with each other.
   - In this type of situation, there would be implicit forces pushing A to try to get B and C to become friends (thus turning the B-C edge label to +);
   - or else for A to side with one of B or C against the other (turning one of the edge labels out of A to a -).
4. Similarly, there are sources of instability in a configuration where each of A, B, and C are mutual enemies
   - In this case, there would be forces motivating two of the three people to "team up" against the third (turning one of the three edge labels to a +).

Structural Balance Property

- A labeled complete graph is balanced if every one of its triangles is balanced — that is, if it obeys the Structural Balance Property
  - For every set of three nodes, if we consider the three edges connecting them, either all three of these edges are labeled +, or else exactly one of them is labeled +.



Structural Balance – Complete Graph



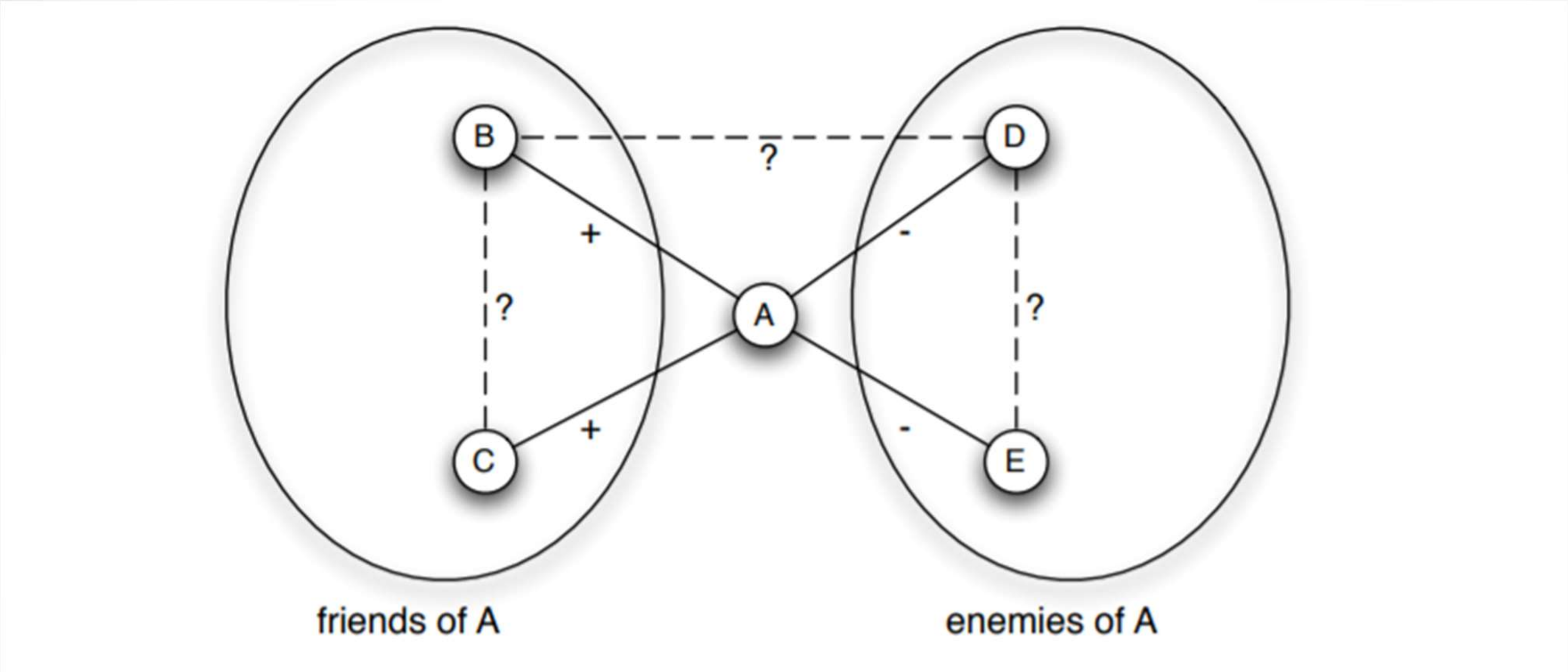
If a complete graph can be divided into two sets of mutual friends, with complete mutual antagonism between the two sets, then it is balanced. Furthermore, this is the only way for a complete graph to be balanced.

Balance Theorem

- If a labeled complete graph is balanced, then:
  - either all pairs of nodes are friends, or
  - else the nodes can be divided into two groups, X and Y , such that every pair of nodes in X like each other, every pair of nodes in Y like each other, and everyone in X is the enemy of everyone in Y

- Another example of:
  - Local effects — phenomena involving only a few nodes at a time — can have global consequences that are observable at the level of the network as a whole.
  - A recurring issue in the analysis of networked systems

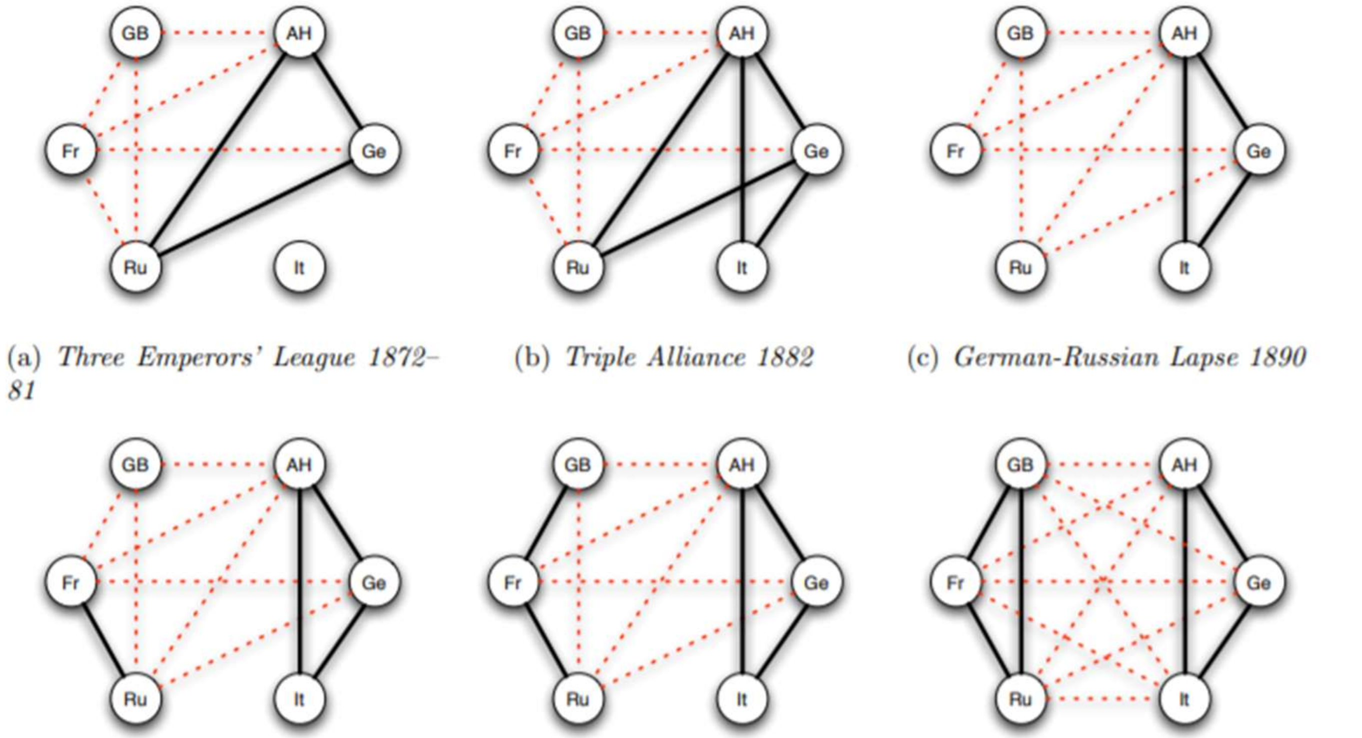
Proof



- To Prove

i. Every two nodes in X are friends
ii. Every two nodes in Y are friends
iii. Every node in X is an enemy of every node in Y

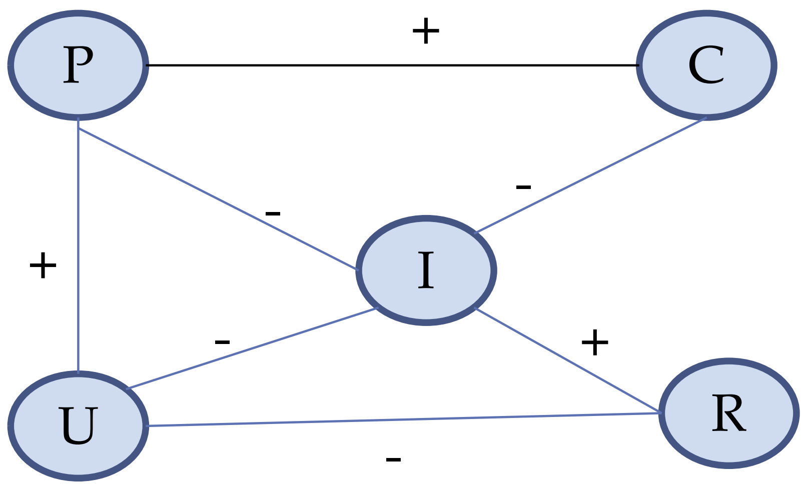
Application of Structural Balance – World War I



(a) *Three Emperors' League 1872–81*
(b) *Triple Alliance 1882*
(c) *German-Russian Lapse 1890*
(d) *French-Russian Alliance 1891–94*
(e) *Entente Cordiale 1904*
(f) *British Russian Alliance 1907*

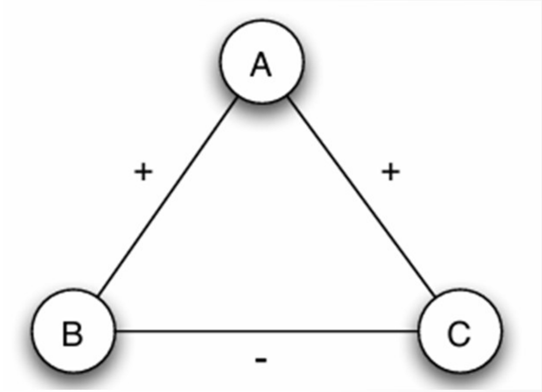
The evolution of alliances in Europe, 1872-1907 leading to World War I: Positive and Negative relations between Great Britain, France, Russia, Italy, Germany, and Austria-Hungary formed to ensure balance

Application of Structural Balance – Formation of Bangladesh

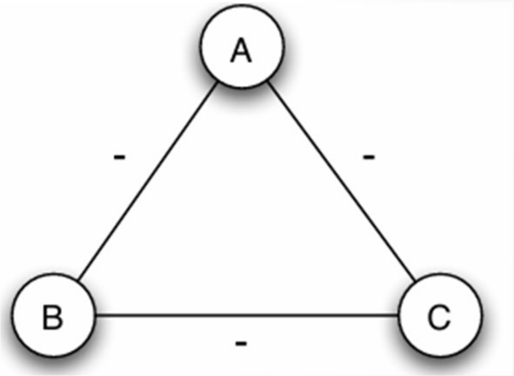


Weakly Balanced Property

- we may see friends of friends trying to reconcile their differences (resolving the lack of balance) - Unbalanced



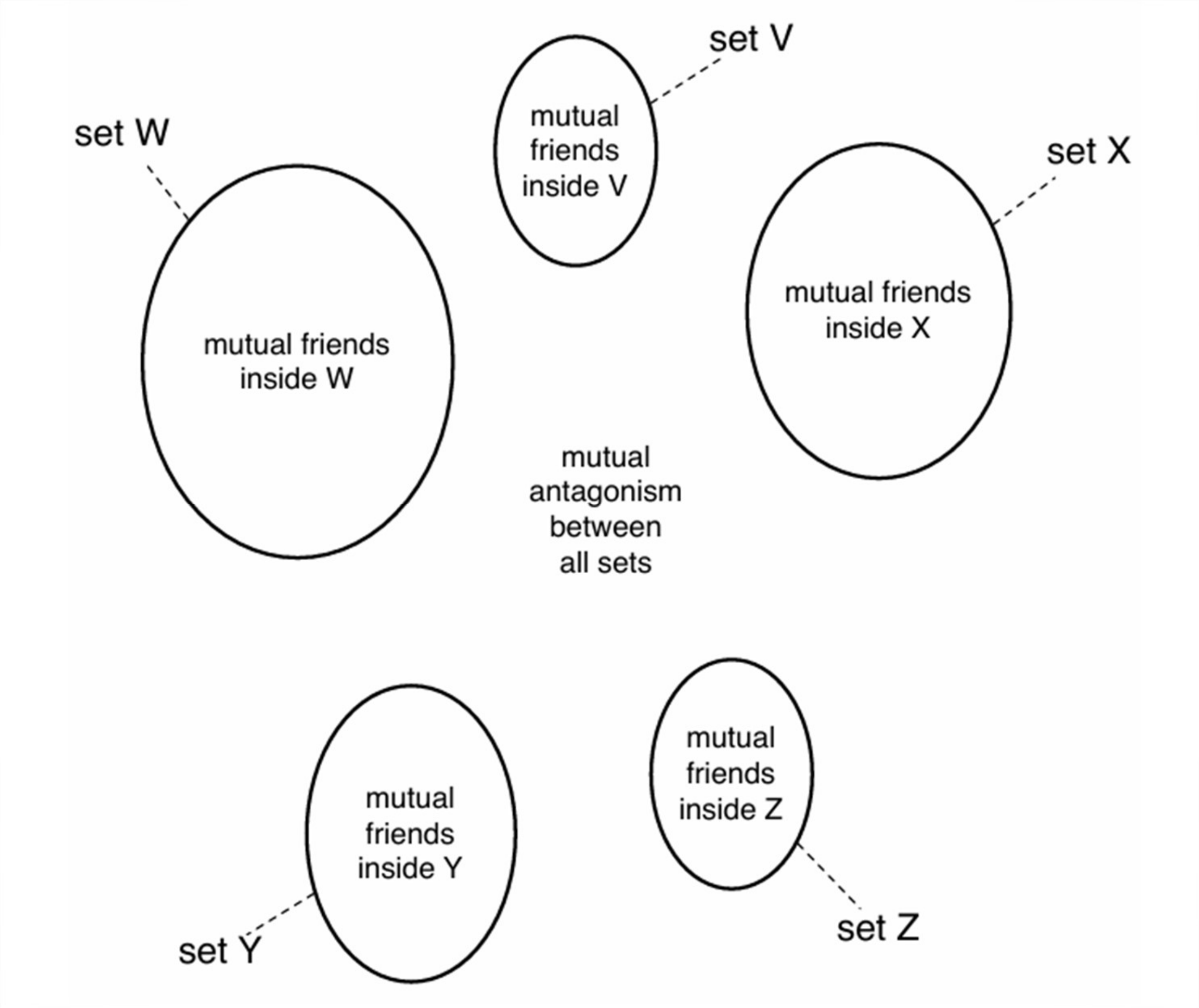
- There could be less of a force leading any two of three mutual enemies – Weakly balanced



Weakly Balanced Network

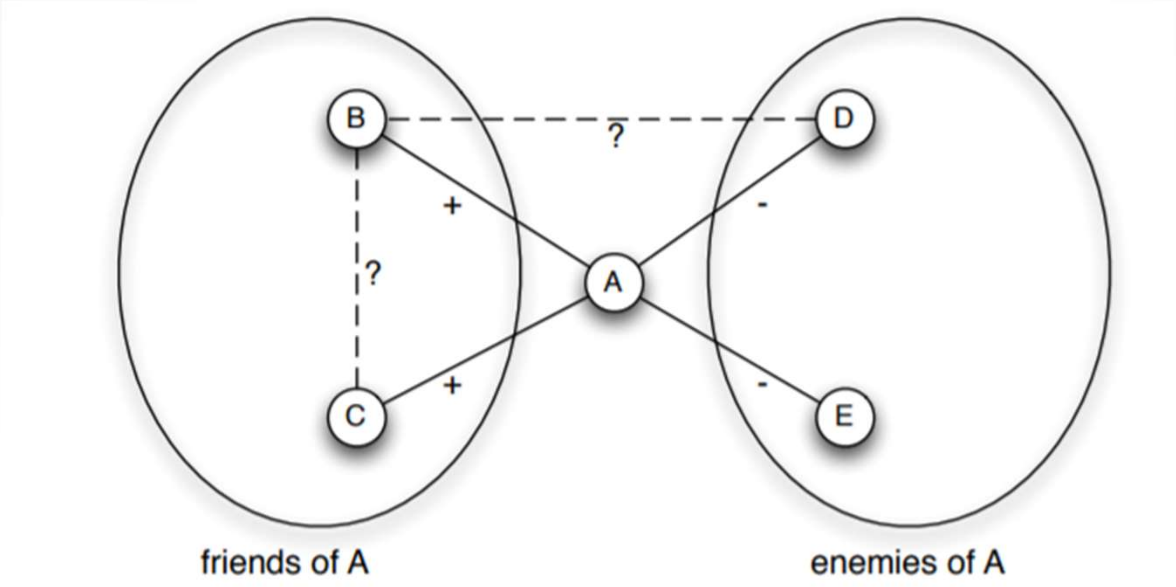
- **Weak Structural Balance Property**: There is no set of three nodes such that the edges among them consist of exactly two positive edges and one negative edge
- **Characterization of Weakly Balanced Networks:** If a labeled complete graph is weakly balanced, then its nodes can be divided into groups in such a way that:
  - Every two nodes belonging to the same group are friend, and
  - Every two nodes belonging to different groups are enemies

Weakly Balanced Complete Graph



A complete graph is weakly balanced precisely when it can be divided into multiple sets of mutual friends, with complete mutual antagonism between each pair of sets

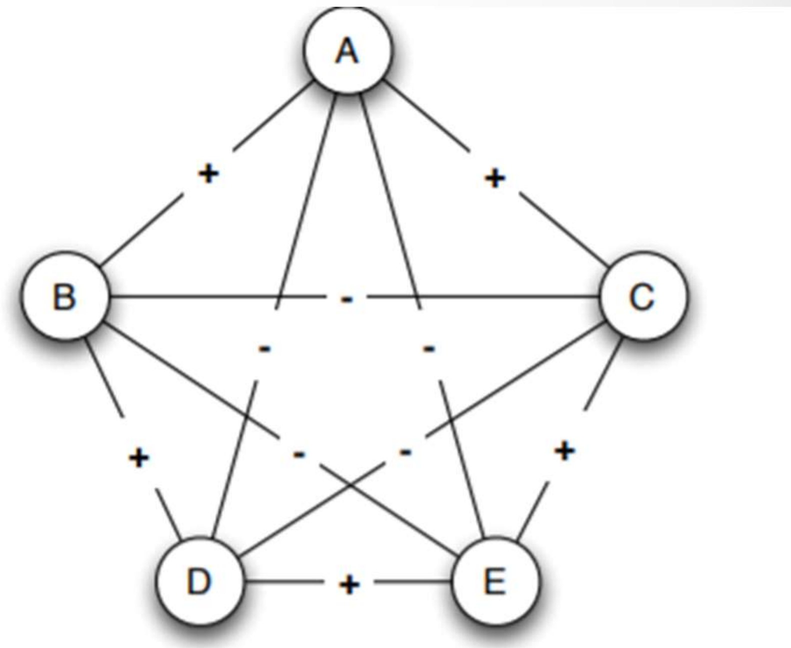
Proof



i. All of A’s friends are friends with each other. (This way, we have indeed produced a group of mutual friends).
ii. A and all his friends are enemies with everyone else in the graph. (This way, the people in this group will be enemies with everyone in other groups, however we divide up the rest of the graph).

Exercises

- Each edge in this network participates in three triangles: one formed by each of the additional nodes who is not already an endpoint of the edge. (For example, the A-B edge participates in a triangle on A, B, and C, a triangle on A, B, and D, and a triangle on A, B, and E. We can list triangles for the other edges in a similar way.) For each edge, how many of the triangles it participates in are balanced, and how many are unbalanced.



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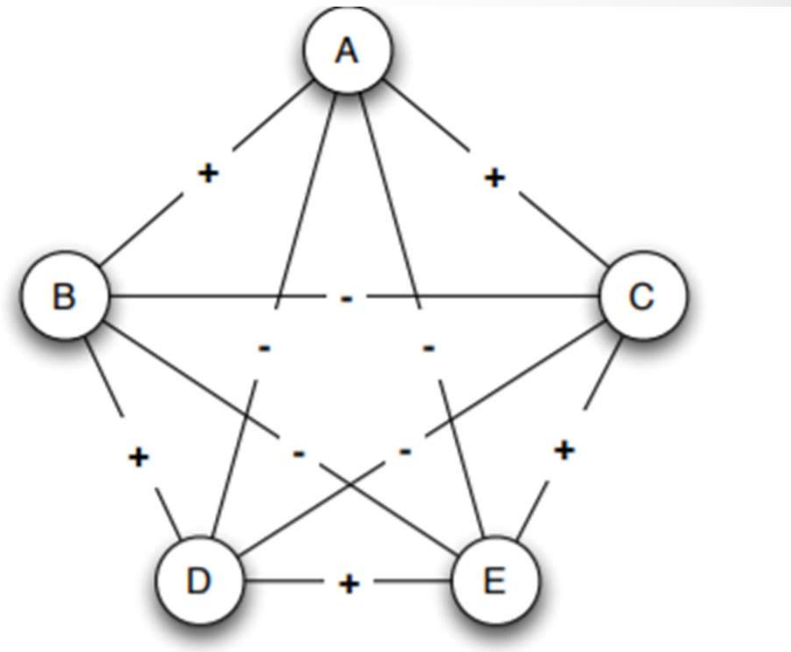

ABC: N ABD: N ABE: Y
ACD: Y ACE: N ADE: Y
BCD: N BCE: Y BDE: Y
CDE: N

References

1. https://www.cs.cornell.edu/home/kleinber/networks-book/
   - **Chapter 5.1-5.4**