

Practice Questions on Groups and Rings

1. If in a ring R every $x \in R$ satisfies $x^2 = x$, prove that R must be commutative.
2. Prove that subgroup of a cyclic group is itself a cyclic group.
3. How many generators does a cyclic group of size n have? (An element b is said to be a generator if $G = \{g, g^2, g^3, \dots, g^n\}$)
4. Prove or disprove the following:
 - Union of two subgroups of a group is always a subgroup.
 - Intersection of two subgroups of a group is always a subgroup.
5. For a subgroup H of G define the left coset aH of H in G as the set of all elements of the form ah , $h \in H$. Show that there is a one-to-one correspondence between the set of left cosets of H in G and the set of right cosets of H in G .
6. Prove that the units in a commutative ring with a unit element form an commutative group.