Practice Questions - Groups

1. A group can be constructed by using the rotations and reflections of a pentagon into itself.
   - How many elements are in the group?
   - Is it a commutative group?
   - Construct the group.
2. Show that only two groups exist with four elements. Construct them and show that they are commutative.
3. Show that if every element of the group $\mathbf{G}$ is its own inverse, then $\mathbf{G}$ is commutative
4. If the cardinality (size) of the group $\mathbf{G}$ is an even number, prove it has an element $a \neq e$ satisfying $a * a = e$, where $*$ is the operator and e is the identity element of $\mathbf{G}$.
5. If $\mathbf{G}$ is a group such that $(a * b) * (a * b) = (a * a) * (b * b)$ for all $a, b \in \mathbf{G}$, show that $\mathbf{G}$ must be commutative.