23MAT124
ENGINEERING MATHEMATICS-1

Elementary row transformations and Rank of Matrices

Dr. R. Naveen Kumar R

Asst. Professor,

Department of Mathematics

Amrita School of Engineering, Bengaluru

Elementary Row Operations of Matrices

- Elementary Transformation

1. Interchange two equations.
2. Multiply both sides of an equation by a nonzero constant.
3. Add a multiple of one equation to another equation.

- Elementary Row Operation

1. Interchange two rows of a matrix.
2. Multiply the elements of a row by a nonzero constant.
3. Add a multiple of the elements of one row to the corresponding elements of another row.

Row Echelon Form

Matrix is said to be in row echelon form, if the following conditions hold:

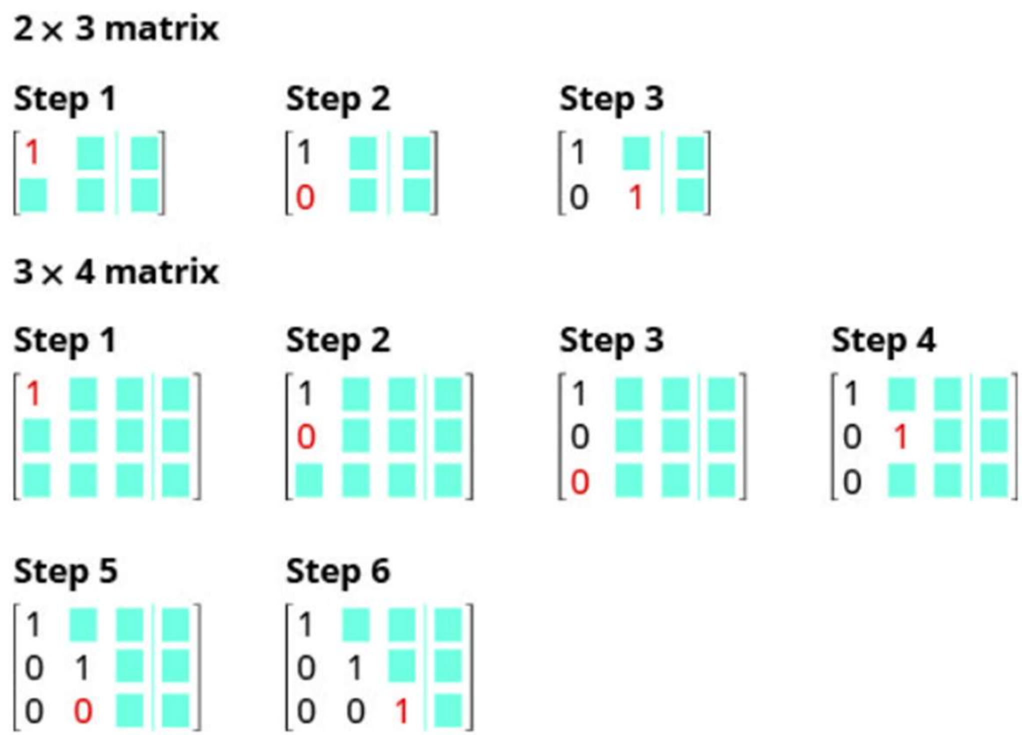
- The first non-zero element in each row, called the leading coefficient, is 1.
- Each leading coefficient is in a column to the right of the previous row leading coefficient.
- Rows with all zeros are below rows with at least one non-zero element.

Reduced Row Echelon Form

A matrix is in reduced row echelon form if its entries satisfy the following conditions.

- The first nonzero entry in each row is a 1 (called a leading 1).
- Each leading 1 comes in a column to the right of the leading 1s in rows above it.
- All rows of all 0s come at the bottom of the matrix.
- If a column contains a leading 1, then all other entries in that column are 0.

Visual representation to show the order for getting the 1's and 0's in the proper position for row-echelon form



Let's try...

Which of the following matrices are in row reduced echelon form?

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

f) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

g) $\begin{bmatrix} 0 & 1 & 2 & 3 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

h) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rank of a matrix

- Rank of a matrix is defined as the number of non-zero rows in a echelon form of a matrix.
- Denoted by $\rho(A)$, where A is any matrix.
- Rank of the matrix cannot exceed the total number of rows in a matrix.

Finding rank using Echelon form

The minor method becomes very tedious if the order of the matrix is very large. So in this case, we convert the matrix into Echelon Form.

- Convert the given matrix into its Echelon Form.
- The number of non-zero rows obtained in the Echelon form of the matrix is the rank of the matrix.

NOTATIONS FOR FUTURE REFERENCE

Symbol	Meaning
$R_i \leftrightarrow R_j$	Interchange rows i and j
cR_i	Multiply the ith row by the nonzero constant c
$cR_i + R_j$	Multiply the ith row by c and add to the jth row

Properties of Rank of Matrix

- Rank of a matrix is equal to the order of the matrix if it is a non-singular matrix.
- Rank of a matrix is equal to the number of non-zero rows if it is in Echelon Form.
- Rank of matrix is equal to the order of identity matrix in it if it is in normal form.
- Rank of matrix < Order of matrix if it is singular matrix.
- Rank of matrix < minimum {m, n} if it is a rectangular matrix of order m x n.
- Rank of identity matrix is equal to the order of the identity matrix.
- Rank of a zero matrix or a null matrix is zero.

$$1.\ A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

Sol:

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 4 & 0 & 2 & 1 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$= \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & -2 & -4 & -7 \\ 0 & 2 & 1 & 3 \\ 0 & 2 & -2 & 0 \end{bmatrix} \quad R_2 = R_2 - 2R_1 \quad , \quad R_3 = R_3 - R_1 \ , R_4 = R_4 - R_1$$

$$= \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & -2 & -4 & -7 \\ 0 & 0 & -3 & -4 \\ 0 & 0 & -6 & -7 \end{bmatrix} \quad R_3 = R_3 + R_2 \qquad ,R_4 = R_4 + R_2$$

$$= \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & -2 & -4 & -7 \\ 0 & 0 & -3 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_4 = R_4 - 2R_3$$

$$= \begin{bmatrix} 1 & 1/2 & 3/2 & 2 \\ 0 & 1 & 2 & 7/2 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_1 = R_1/2, \quad R_2 = -R_2/2, \quad R_3 = -R_3/3$$

Since all the four rows are non-zero in row echelon form ‘A’ $\therefore$ $\rho(A)=4$

2. Find ‘K’ so that the rank of matrix $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ 3 & 5 & K \end{bmatrix}$ is ‘2’.

Sol:

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ 3 & 5 & K \end{bmatrix} = 0$$

2(4k-10) – (k - 6) – (5 – 12) =0

7k – 7=0

K=1

3. Determine the value of ‘k’ such that the rank of matrix ‘A’ is 3

Sol:

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ k-2 & 0 & 4 & 2 \\ 0 & 0 & k+9 & 3 \end{bmatrix} \quad R_2 = R_2 - 2R_1 \quad , \quad R_3 = R_3 - 2R_1 \ , R_4 = R_4 - 9R_1$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ k-2 & 0 & 0 & -2 \\ 0 & 0 & k+6 & 0 \end{bmatrix} \quad R_3 = R_3 - 4R_2 \;, R_4 = R_4 - 3R_2$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & k+6 & -2 \\ k-2 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \leftrightarrow R_4$$

If k=2 , ρ(A)=3

If k=-6, number of non-zero rows is 3,∴ ρ(A)=3

Exercise

Find rank of matrices

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 6 & 7 \\ 1 & 5 & 0 & 10 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & -1 & 0 & 5 \\ 0 & 3 & 1 & 4 \end{bmatrix}$


Thank
you