

```
a=randi([0,9],5,2)*randi([0,9],2,5)
```

```
a = 5x5
    0     5    47     5     5
    0     9    93     9     9
    0     0     0     0     0
    0     7    63     7     7
    0     1     8     1     1
```

```
[rr,ic]=rref(a)
```

```
rr = 5x5
    0     1     0     1     1
    0     0     1     0     0
    0     0     0     0     0
    0     0     0     0     0
    0     0     0     0     0
ic = 1x2
    2     3
```

```
l=length(ic);
r=rr(1:l,:)
```

```
r = 2x5
    0     1     0     1     1
    0     0     1     0     0
```

```
c=a(:,ic)
```

```
c = 5x2
    5    47
    9    93
    0     0
    7    63
    1     8
```

```
n=null(a)
```

```
n = 5x3
    1.0000    0.0020   -0.0000
   -0.0016    0.8165   -0.0000
    0.0000    0.0000   -0.0000
    0.0008   -0.4082   -0.7071
    0.0008   -0.4082    0.7071
```

```
s=size(n);
nullity=s(:,2)
```

```
nullity =
    3
```

```
a=randi([0,9],5,2)*randi([0,9],2,5)
```

```
a = 5x5
    48    56    44    14    82
    36    72    68    18    74
    30    20    10     5    45
    48    56    44    14    82
    39    56    48    14    71
```

```
[rr,ic]=rref(a)
```

```
rr = 5×5
    1.0000         0   -0.4444         0    1.2222
         0    1.0000    1.1667    0.2500    0.4167
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0

ic = 1×2
     1     2
```

```
l=length(ic);
r=rr(1:l,:)
```

```
r = 2×5
    1.0000         0   -0.4444         0    1.2222
         0    1.0000    1.1667    0.2500    0.4167
```

```
rsb1=r(1,:)'
rsb2=r(2,:)'
[rr,ic]=rref(a')
```

```
rr = 5×5
    1.0000         0    1.0000    1.0000    0.5500
         0    1.0000   -0.5000         0    0.3500
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0

ic = 1×2
     1     2
```

```
l=length(ic);
c=rr(1:l,:)
```

```
c = 2×5
    1.0000         0    1.0000    1.0000    0.5500
         0    1.0000   -0.5000         0    0.3500
```

```
csb1=c(1,:)'
```

```
csb1 = 5×1
    1.0000
         0
    1.0000
    1.0000
    0.5500
```

```
csb2=c(2,:)'
```

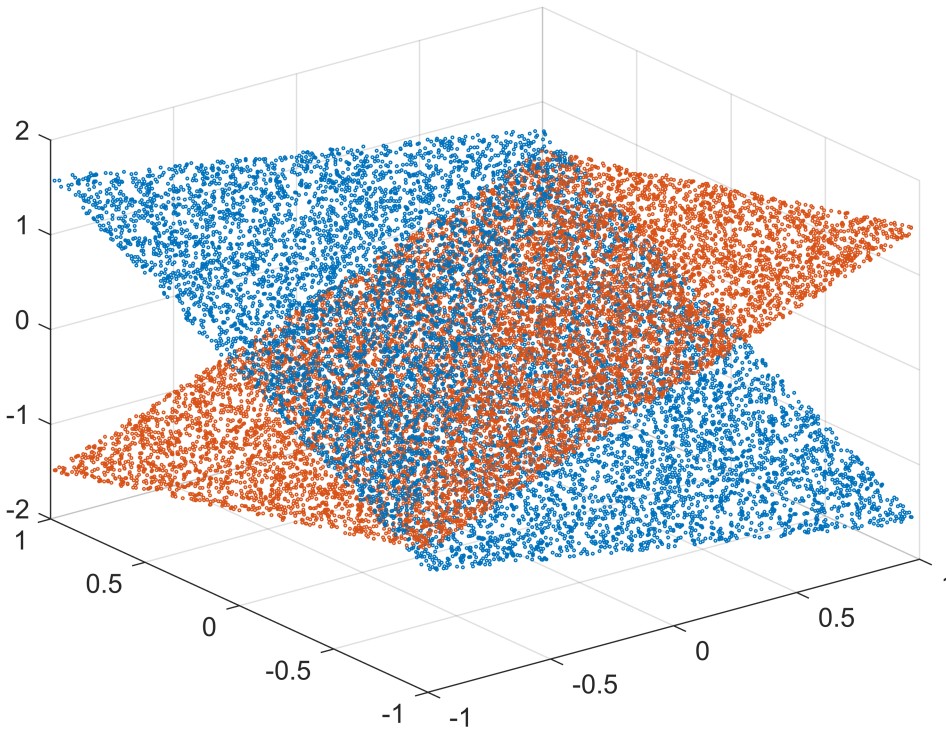
```
csb2 = 5×1
         0
    1.0000
   -0.5000
         0
    0.3500
```

```
rspts=[];
cspts=[];
for i=1:10000
    k1=-1+2*rand(1);
```

```

k2=-1+2*rand(1);
a1=-1+2*rand(1);
a2=-1+2*rand(1);
rspts=[rspts,k1*rsb1+k2*rsb2];
cspts=[cspts,a1*csb1+a2*csb2];
end
scatter3(rspts(1,:),rspts(2,:),rspts(3,:),1);
hold on
scatter3(cspts(1,:),cspts(2,:),cspts(3,:),1);
hold off

```



```

clc
a=[1,0,0;0,0,0;0,0,3]

```

```

a = 3x3
    1     0     0
    0     0     0
    0     0     3

```

```

[rr,ic]=rref(a)

```

```

rr = 3x3
    1     0     0
    0     0     1
    0     0     0
ic = 1x2
    1     3

```

```

l=length(ic);

```

```
r=rr(1:1,:)
```

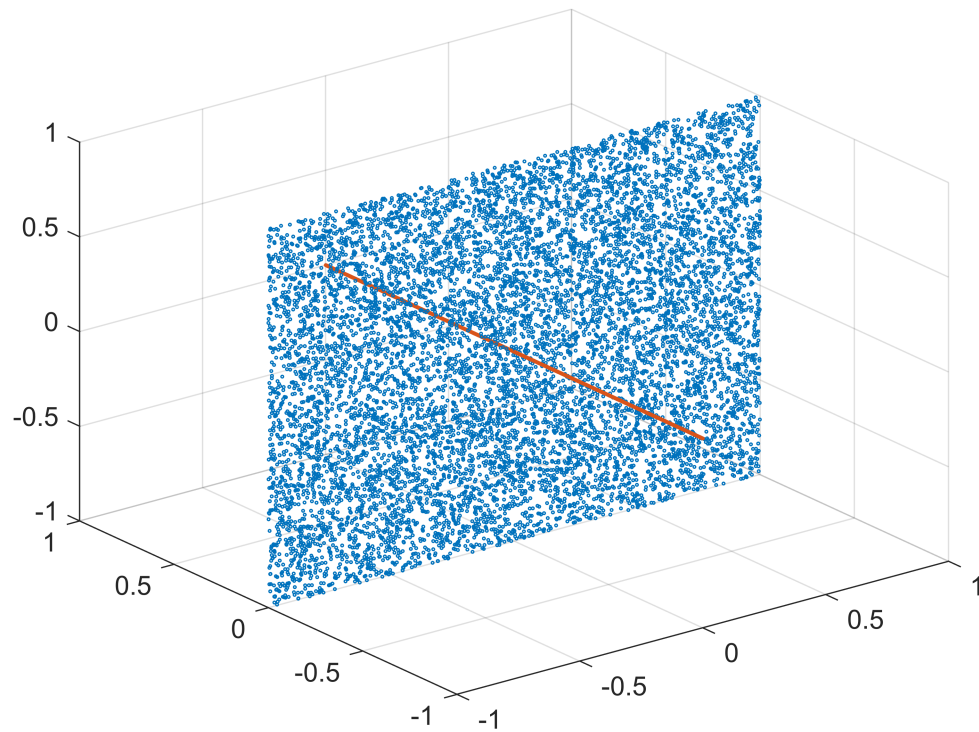
```
r = 2×3
    1    0    0
    0    0    1
```

```
rsb1=r(1,:);
rsb2=r(2,:);
n=null(a)
```

```
n = 3×1
    0
    1
    0
```

```
rspts=[];
nspts=[];

for i=1:10000
    k1=-1+2*rand(1);
    k2=-1+2*rand(1);
    a1=-1+2*rand(1);
    rspts=[rspts,k1*rsb1+k2*rsb2];
    nspts=[nspts,a1*n];
end
scatter3(rspts(1,:),rspts(2,:),rspts(3,:),1);
hold on
scatter3(nspts(1,:),nspts(2,:),nspts(3,:),1);
hold off
```



```
clc
a=[1 4;0 5];
b=[0 0;0 5];
c=[0 0;0 0];
an=null(a)
```

```
an =

    2×0 empty double matrix
```

```
b1=null(b)
```

```
b1 = 2×1
     1
     0
```

```
c1=null(c)
```

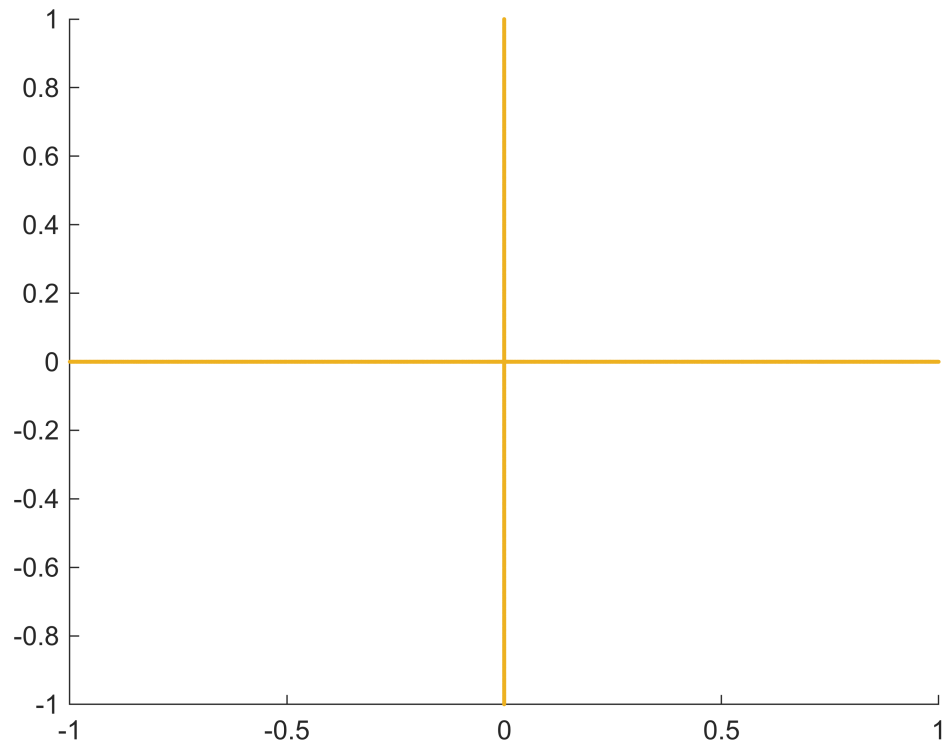
```
c1 = 2×2
     1     0
     0     1
```

```
apts=[];
bpts=[];
cpts=[];
for i=1:10000
    a1=-1+2*rand(1);
    a2=-1+2*rand(1);
```

```

a3=-1+2*rand(1);
apts=[apts,a1*an];
bpts=[bpts,a2*b1];
cpts=[cpts,a3*c1];
end
scatter(apts(1,:),apts(2,:),1);
hold on
scatter(bpts(1,:),bpts(2,:),1);
hold on
scatter(cpts(1,:),cpts(2,:),1);
hold off

```



```

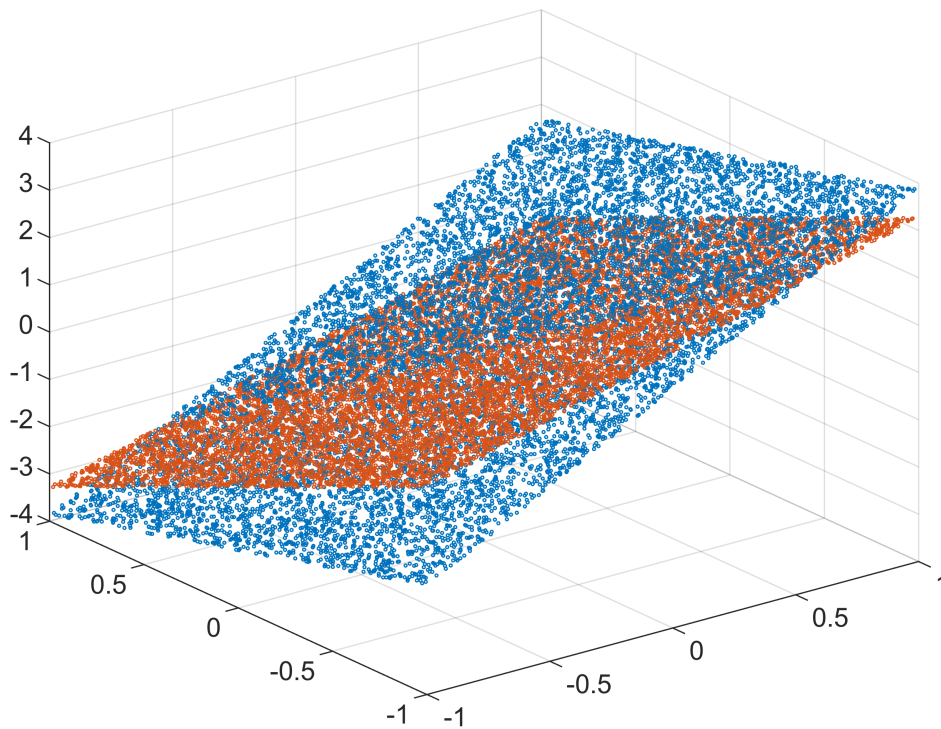
a=rand(3,2) * rand(2,3);
[rr,ic]=rref(a);
l=length(ic);
r=rr(1:l,:);
rsb1=r(1,:)';
rsb2=r(2,:)';
[rr,ic]=rref(a');
l=length(ic);
c=rr(1:l,:);
csb1=c(1,:)';
csb2=c(2,:)';
rspts=[];
cspts=[];
for i=1:10000

```

```

k1=-1+2*rand(1);
k2=-1+2*rand(1);
a1=-1+2*rand(1);
a2=-1+2*rand(1);
rspts=[rspts,k1*rsb1+k2*rsb2];
cspts=[cspts,a1*csb1+a2*csb2];
end
scatter3(rspts(1,:),rspts(2,:),rspts(3,:),1);
hold on
scatter3(cspts(1,:),cspts(2,:),cspts(3,:),1);
hold off

```



```

a=rand(3,1)*rand(1,3)
[rr,ic]=rref(a)
l=length(ic);
c=rr(1:l,:);
csb1=c(1,:)' ;
csb2=c(2,:)' ;
n=null(a)
cspts=[];
nspts=[];

for i=1:10000
    k1=-1+2*rand(1);
    k2=-1+2*rand(1);
    a1=-1+2*rand(1);

```

```

cspts=[cspts,k1*csb1+k2*csb2];
nspts=[nspts,a1*n];
end
scatter3(cspts(1,:),cspts(2,:),cspts(3,:),1);
hold on
scatter3(nspts(1,:),nspts(2,:),nspts(3,:),1);
hold off

```

```

M = randi([-3, 3], 3, 1) * randi([-3, 3], 1, 3)
[rr, ic] = rref(M');
l = length(ic);
C = RR(1:l, :)
ln=null(M')
csb1 = C(1, :)';
lnsb1=LN(:,1);
lnsb2=LN(:,2);
cspts = [];
lnspts=[];
for i = 1:10000
    k1 = -1 + 2 * rand(1);
    a2 = -1 + 2 * rand(1);
    a1=-1+2*rand(1);
    cspts = [cspts, k1 * csb1 ];
    lnspts=[lnspts,a1*lnsb1+a2*lnsb2];
end
scatter3(cspts(1, :), cspts(2, :), cspts(3, :), 1);
hold on
scatter3(lnspts(1, :), lnspts(2, :), lnspts(3, :), 1);
hold off

```

5a

```
A=[1 3 4 7;2 4 6 10;3 5 8 13;4 6 10 16]
```

```

A = 4×4
     1     3     4     7
     2     4     6    10
     3     5     8    13
     4     6    10    16

```

```
u=[-2;-3;1;1]
```

```

u = 4×1
    -2
    -3
     1
     1

```

```

if rank([A u])==rank(A)
disp('u is a column space of A')
elseif rank ([A; u'])==rank(A)
disp('u is a row space of A')

```



```
elseif A*u == 0
disp('u is in the null space of A')
elseif transpose(u)*A == 0
disp('u is in the left null space of A')
else
disp('u is not in any space of A')
end
```

u is in the null space of A

5b

```
A=[1 3 4 7;2 4 6 10;3 5 8 13;4 6 10 16]
```

```
A = 4×4
     1     3     4     7
     2     4     6    10
     3     5     8    13
     4     6    10    16
```

```
u=[5;8;11;14]
```

```
u = 4×1
     5
     8
    11
    14
```

```
if rank([A u])==rank(A)
disp('u is a column space of A')
elseif rank ([A; u'])==rank(A)
disp('u is a row space of A')
elseif A*u == 0
disp('u is in the null space of A')
elseif transpose(u)*A == 0
disp('u is in the left null space of A')
else
disp('u is not in any space of A')
end
```

u is a column space of A

5c

```
A=[1 3 4 7;2 4 6 10;3 5 8 13;4 6 10 16]
```

```
A = 4×4
     1     3     4     7
     2     4     6    10
     3     5     8    13
     4     6    10    16
```

```
u=[1;1;2;3]
```

```
u = 4×1
    1
    1
    2
    3
```

```
if rank([A u])==rank(A)
disp('u is a column space of A')
elseif rank ([A; u'])==rank(A)
disp('u is a row space of A')
elseif A*u == 0
disp('u is in the null space of A')
elseif transpose(u)*A == 0
disp('u is in the left null space of A')
else
disp('u is not in any space of A')
end
```

u is a row space of A

5d

```
A=[1 3 4 7;2 4 6 10;3 5 8 13;4 6 10 16]
```

```
A = 4×4
    1     3     4     7
    2     4     6    10
    3     5     8    13
    4     6    10    16
```

```
u=[-1;1;1;-1]
```

```
u = 4×1
   -1
    1
    1
   -1
```

```
if rank([A u])==rank(A)
disp('u is a column space of A')
elseif rank ([A; u'])==rank(A)
disp('u is a row space of A')
elseif A*u == 0
disp('u is in the null space of A')
elseif transpose(u)*A == 0
disp('u is in the left null space of A')
else
disp('u is not in any space of A')
end
```

u is in the left null space of A

6a

```
A=[1 -1 2 3;0 2 1 4;1 1 3 1;2 0 5 4]
```

```
A = 4×4
    1   -1    2    3
    0    2    1    4
    1    1    3    1
    2    0    5    4
```

```
u=[5;1;-2;0]
```

```
u = 4×1
    5
    1
   -2
    0
```

```
if rank([A u])==rank(A)
disp('u is a column space of A')
elseif rank ([A; u'])==rank(A)
disp('u is a row space of A')
elseif A*u == 0
disp('u is in the null space of A')
elseif transpose(u)*A == 0
disp('u is in the left null space of A')
else
disp('u is not in any space of A')
end
```

u is in the null space of A

6b

```
A=[1 -1 2 3;0 2 1 4;1 1 3 1;2 0 5 4]
```

```
A = 4×4
    1   -1    2    3
    0    2    1    4
    1    1    3    1
    2    0    5    4
```

```
u=[0;2;2;2]
```

```
u = 4×1
    0
    2
    2
    2
```

```
if rank([A u])==rank(A)
disp('u is a column space of A')
elseif rank ([A; u'])==rank(A)
disp('u is a row space of A')
elseif A*u == 0
disp('u is in the null space of A')
elseif transpose(u)*A == 0
disp('u is in the left null space of A')
```

```

else
disp('u is not in any space of A')
end

```

u is a column space of A

6c

```
A=[1 -1 2 3;0 2 1 4;1 1 3 1;2 0 5 4]
```

```

A = 4x4
     1    -1     2     3
     0     2     1     4
     1     1     3     1
     2     0     5     4

```

```
u=[-1;2;-1;1]
```

```

u = 4x1
    -1
     2
    -1
     1

```

```

if rank([A u])==rank(A)
disp('u is a column space of A')
elseif rank ([A; u'])==rank(A)
disp('u is a row space of A')
elseif A*u == 0
disp('u is in the null space of A')
elseif transpose(u)*A == 0
disp('u is in the left null space of A')
else
disp('u is not in any space of A')
end

```

u is not in any space of A

6d

```
A=[1 -1 2 3;0 2 1 4;1 1 3 1;2 0 5 4]
```

```

A = 4x4
     1    -1     2     3
     0     2     1     4
     1     1     3     1
     2     0     5     4

```

```
u=[3;-1;7;7]
```

```

u = 4x1
     3
    -1
     7
     7

```

```

if rank([A u])==rank(A)
disp('u is a column space of A')
elseif rank ([A; u'])==rank(A)
disp('u is a row space of A')
elseif A*u == 0
disp('u is in the null space of A')
elseif transpose(u)*A == 0
disp('u is in the left null space of A')
else
disp('u is not in any space of A')
end

```

u is a row space of A